Gravitational Waves: where do they come from and what do they tell us

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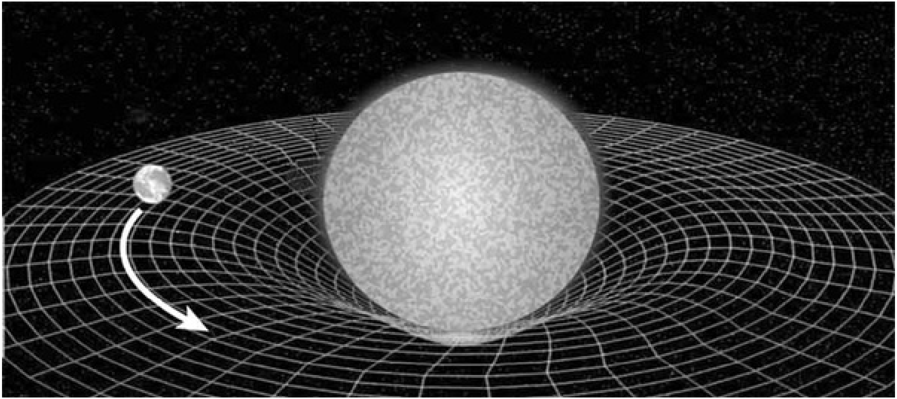
- What are gravitational waves?
- Gravitational wave sources
- Gravitational wave data analysis
- Observed gravitational waves signals

Part 1: What are gravitational waves?

What are gravitational waves?



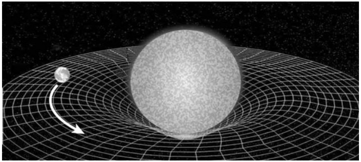
Einstein, Theory of General Relativity 1915

gravity $\neq$ force

$\Rightarrow$ deformation of space-time

- Masses bend space-time
- Objects follow curved space-time

What are gravitational waves?



gravitational waves: gravity is not instantaneous propagating perturbation

General relativity → Gravitational Waves

- General Relativity: space-time is a Riemann space

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu,$$

with the metric created by the matter/energy (Einstein's equation)

$$\underbrace{G_{\mu\nu}}_{\text{non-linear second order derivative of } g_{\mu\nu}} = \frac{8\pi G}{c^4} \underbrace{T_{\mu\nu}}_{\text{energy-momentum tensor}}$$

- Gravitational Waves (GW) → usually seen as linear limit of General Relativity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \ll 1 \quad \eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ – flat metric

$$\Rightarrow \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) h_{\mu\nu} = 0$$

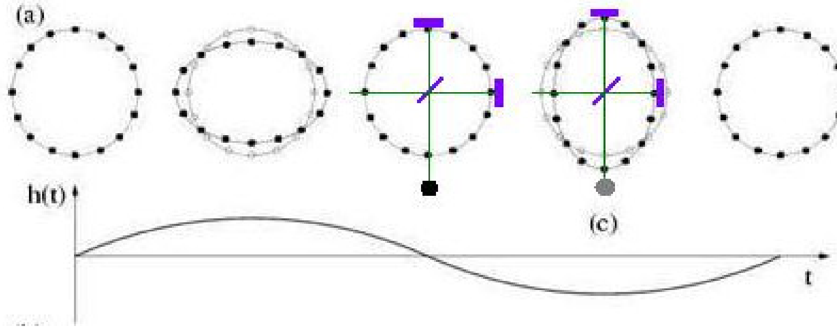
Waves propagating at speed of light

- Tensorial waves → 10 degrees of freedom (symmetric tensor)
gauge freedom → 2 polarizations

$$h_{\mu\nu} = h_+ A_{\mu\nu} + h_\times B_{\mu\nu}$$

What is the effect of a gravitational wave?

What is the effect of a gravitational wave?



Principle:

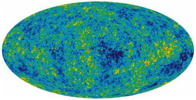
Compare the distance in two perpendicular directions

Difficulty: relative deformation is 10^{-22}

Change in Earth-Sun distance by one atomic diameter

Part 2: gravitational wave sources

4 families of potential GW signal morphologies

	precisely modeled	uncertain form
permanent	Deformed rotating neutron stars 	Incoherent sum of unresolved sources Primordial GW background 
transient	Cosmic strings cusps, kinks Coalescence of neutron stars or black holes 	Star quakes, Non spherically symmetric stellar collapse, ... 

4 families of potential GW signal morphologies

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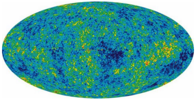
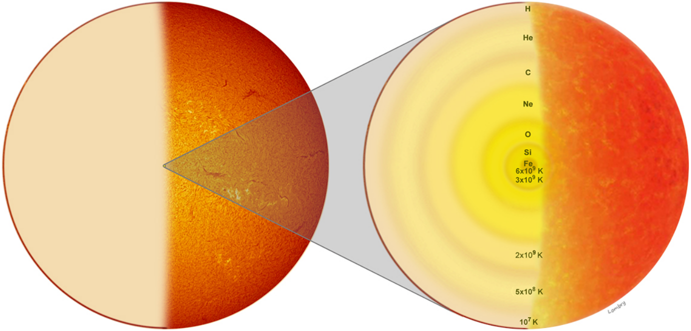
permanent	Deformed rotating neutron stars 	Incoherent sum of unresolved sources Primordial GW background 
transient	Cosmic strings cusps, kinks Coalescence of neutron stars or black holes 	Star quakes, Non spherically symmetric stellar collapse, ... 

Compact binary coalescence

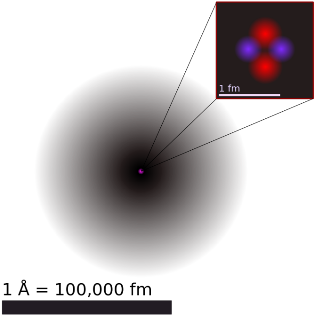
- Only source detected so far
- Black hole - black hole (BH-BH) binary
- BH - Neutron star (NS)
- NS - NS
- duration $\sim$ min, frequency $\sim$ 10 Hz - 1 kHz, amplitude $h \sim 10^{-23}$ at 10 Mpc

Compact stars



- Origin of compact objects:
  - Pressure from nuclear reaction preventing gravitational collapse
  - For stellar masses greater than $10M_\odot$ no reaction in iron core
  - ⇒ core supported by electron degeneracy
  - ∼1000 km iron core collapses ⇒ supernova
  - Depending on amount of matter falling back on collapsed core
    - Neutron star, 1-3 $M_\odot$
    - Black hole, 5-50 $M_\odot$
  - Neutron star, black hole size ~ 10 km in radius ⇒ compact
  - Stellar graveyard

Neutron star

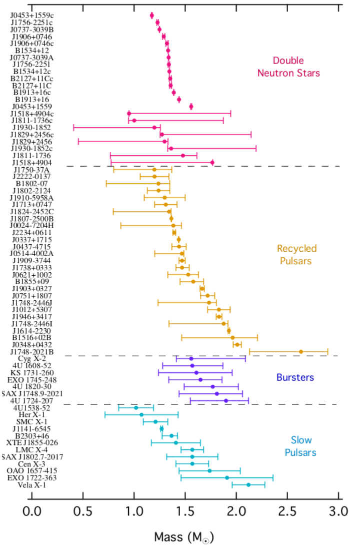


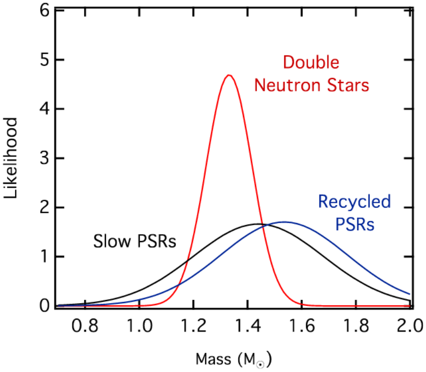
- Atoms composed of three ingredients
  - neutron
  - proton
  - electrons
- Electron capture during core collapse

$$p + e^- \rightarrow n + \nu_e$$

- neutron star $\sim$ 10 km ball with nuclear density
- pulsar: special case of neutron stars with large magnetic fields and radio emission pulsing with rotation

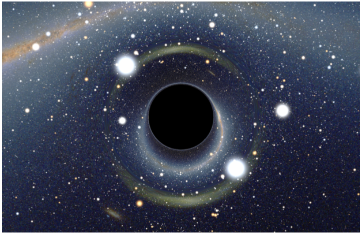
Neutron star

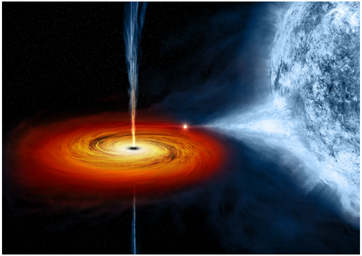




- Neutron stars (pulsars) in binary system
- Masses measured from orbital parameters
- For double neutron stars mass distribution much more narrow
  $\Rightarrow$ Particular evolution conditions needed to form double neutron stars?

Black holes

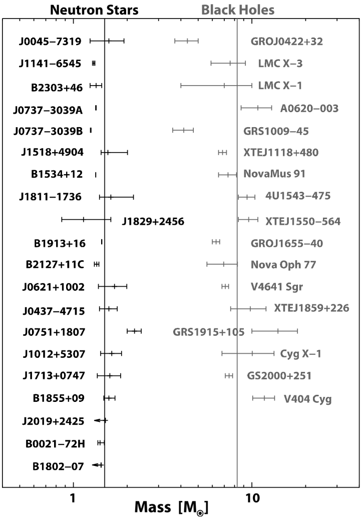


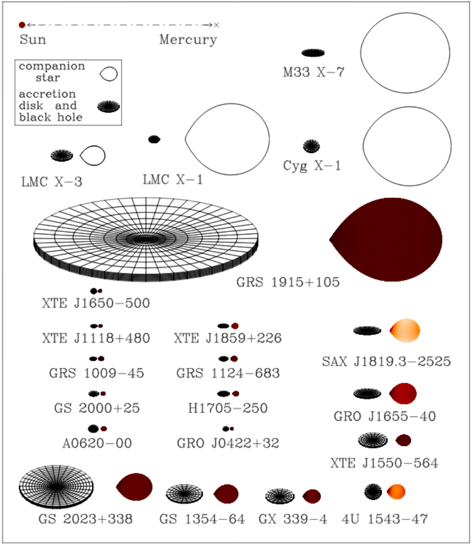


- Gravitation strong enough that photons can't escape

  $\Rightarrow$ Black hole horizon
- What happens inside the black hole horizon is not known but no influence on outside universe
- Geometrical objects defined by two quantities
  - Mass (scalar)
  - Spin (vector)
- Black holes were observed from X-ray emission of gas falling into the black hole

Black holes





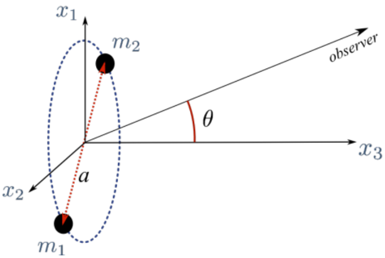
- mass gap?
- BH mass measured from orbital parameters

Binary system: GW generation

- Binary system of two compact objects
  - Masses m_1 and m_2
  - Distance between objects a
  - Total mass $M = m_1 + m_2$
  - Reduced mass $\mu = \frac{m_1 m_2}{M}$
- Newtonian approximation
  - 3rd Kepler's law $\omega = \sqrt{\frac{GM}{a^3}}$
- Point object coordinates

$$x_1(t) = \frac{a}{2}\cos\omega t, \quad x_2(t) = \frac{a}{2}\sin\omega t, \quad x$$

- Assume circular orbit and observer at large distance $R \gg a$



Gravitational source quadrupolar approximation

Approximation: far field + slow moving source

- Mass distribution quadrupolar moment

$$I_{ij} = \int (x_i x_j - \tfrac{1}{3}\delta_{ij}\delta_{km}x^k x^m)\rho(x)\mathrm{d}^3x$$
$$= \frac{\mu a^2}{2}\begin{pmatrix} \left(\frac{1}{3}+\cos(2\omega t)\right) & \sin(2\omega t) & 0 \\ \sin(2\omega t) & \left(\frac{1}{3}-\cos(2\omega t)\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

- Source of gravitational waves

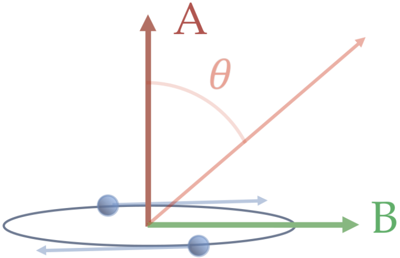
$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad \longrightarrow \quad h_{jk}^{TT} = \frac{2G}{Rc^4}\underbrace{P_{jkmn}}_{\text{projection}}\ddot{I}^{mn}(t-\tfrac{R}{c}),$$

- Resulting waveform

$$h_+ = -\frac{4G}{Rc^4}\mu a^2\omega^2\frac{1+\cos^2\theta}{2}\cos 2\omega t$$
$$h_\times = -\frac{4G}{Rc^4}\mu a^2\omega^2\cos\theta\sin 2\omega t,$$

Binary coalescence: GW generation geometry

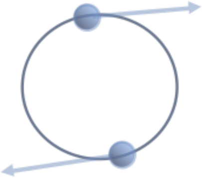
$$h_{jk}^{TT} = \frac{2G}{Rc^4} \underbrace{P_{jkmn}}_{\text{projection}} \ddot{I}^{mn}(t - \tfrac{R}{c}),$$



$$h_+ = -\frac{4G}{Rc^4}\mu a^2 \omega^2 \frac{1+\cos^2\theta}{2}\cos 2\omega t$$

$$h_\times = -\frac{4G}{Rc^4}\mu a^2 \omega^2 \cos\theta \sin 2\omega t,$$

- Observer A sees two polarizations, $\cos\theta = 1$



- Observer B sees one polarizations, $\cos\theta = 0$

Binary coalescence: GW power

$$h_+ = -\frac{4G}{Rc^4}\mu a^2\omega^2 \frac{1+\cos^2\theta}{2}\cos 2\omega t$$
$$h_\times = -\frac{4G}{Rc^4}\mu a^2\omega^2 \cos\theta \sin 2\omega t,$$

- Radiated power per unit solid angle

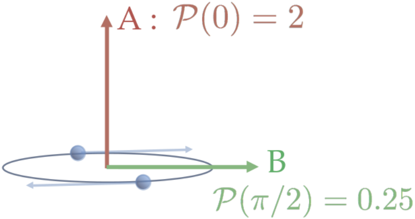
$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{c^3}{16\pi G}\left\langle \left(\dot{h}_+\right)^2 + \left(\dot{h}_\times\right)^2 \right\rangle = \frac{2G\mu^2 a^4\omega^6}{\pi c^5}\mathcal{P}(\Omega)$$

$$\mathcal{P}(\Omega) = \frac{1}{4}\left(1 + 6\cos^2\theta + \cos^4\theta\right)$$

- Radiated power non-zero in all directions

- Total radiated power

$$P_{\mathrm{GW}} = \frac{32G\mu^2 a^4\omega^6}{5c^5}$$



GW power: some examples

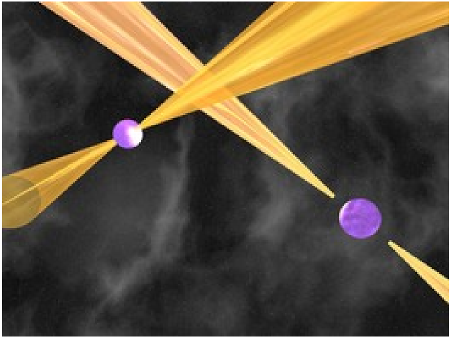
- Sun-Jupiter system

$$m_J = 1.9 \times 10^{27}, \quad a = 7.8 \times 10^{11} \quad \omega = 1.68 \times 10^{-7} s^{-1}$$

$$\Rightarrow P_{\mathrm{GW}} = 5 \times 10^3 J/s$$

  - Negligible compared to the sun $L_\odot \simeq 3.8 \times 10^{26} J/s$
- Binary pulsar PSR 1913+16 (Hulse and Taylor)

$$P_{\mathrm{GW}} = 7.35 \times 10^{24} J/s$$



Radiated power: orbit shrinks, emission frequency increases

- Potential energy and Kepler's law

$$E = -G\frac{m_1 m_2}{2a}, \quad \omega^2 = \frac{GM}{a^3}$$

$$\Rightarrow \dot{E} = -G^{2/3}\frac{m_1 m_2}{2M^{1/3}}\frac{2}{3}\dot{\omega}\omega^{-1/3}$$

- Match orbital energy loss to radiated GW energy

$$\dot{E} = -P_{\mathrm{GW}} \quad \Rightarrow \quad G^{2/3}\frac{m_1 m_2}{2M^{1/3}}\frac{2}{3}\dot{\omega}\omega^{-1/3} = \frac{32G\mu^2 a^4\omega^6}{5c^5}$$

- Use Kepler's law to substitute a by ω

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\frac{G^{5/3}}{c^5}\frac{\mu}{M}(M\omega)^{5/3}$$

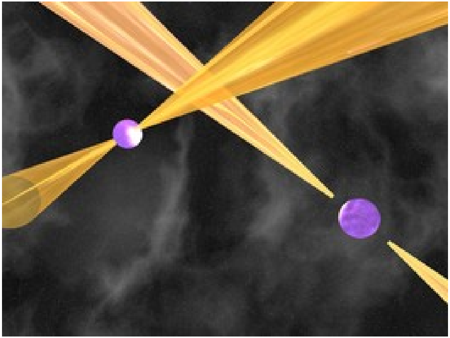
- GW frequency is $2\pi f_{\mathrm{GW}} = 2\omega$

$$\dot{f}_{\mathrm{GW}} = \frac{96}{5}\frac{G^{5/3}}{c^5}\pi^{8/3}\mathcal{M}^{5/3}f_{\mathrm{GW}}^{11/3}$$

- Where we define the chirp mass that drives the frequency evolution

$$\mathcal{M} = \mu^{3/5}M^{2/5}$$

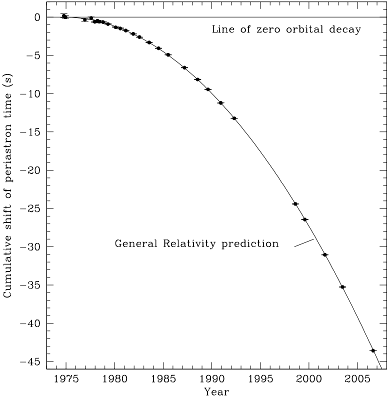
Indirect observation of GWs



Indirect observation of gravitational radiation

- $\phi(t) = \int^t \omega(t)\mathrm{d}t$
- Orbital period measured through Doppler effects on radio pulses
- Follows GR with $\sim 10^{-3}$ precision

"double" pulsar PSR1913+16



Hulse-Taylor Nobel Prize 1993

Post-newtonian (PN) corrections needed

- Development of GR around the newtonian limit $\epsilon = \left(\frac{v}{c}\right)^2$
  - v – speed of the two stars, $v = (GM\omega)^{1/3}$
- For example orbital phase development

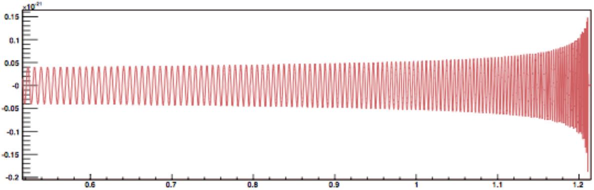
$$\phi(t) = \phi_N \times \sum_k \phi_{\frac{k}{2}PN} v^k$$

- High order correction become rapidly complex

k	N	2	3	4	5
$\mathcal{F}_k$	$\frac{32\eta^2 v^{10}}{5}$	$-\frac{1247}{336} - \frac{35\eta}{12}$	4π	$-\frac{44711}{9072} + \frac{9271\eta}{504} + \frac{65\eta^2}{18}$	$-\left(\frac{8191}{672} + \frac{535\eta}{24}\right)\pi$
t_k^v	$-\frac{5m}{256\eta v^8}$	$\frac{743}{252} + \frac{11\eta}{3}$	$-\frac{32\pi}{5}$	$\frac{3058673}{508032} + \frac{5429\eta}{504} + \frac{617\eta^2}{72}$	$-\left(\frac{7729}{252} + \eta\right)\pi$
ϕ_k^v	$-\frac{1}{16\eta v^5}$	$\frac{3715}{1008} + \frac{55\eta}{12}$	-10π	$\frac{15293365}{1016064} + \frac{27145\eta}{1008} + \frac{3085\eta^2}{144}$	$\left(\frac{38645}{672} + \frac{15\eta}{8}\right)\pi \ln\left(\frac{v}{v_{\rm lso}}\right)$
ϕ_k^t	$-\frac{2}{\eta\theta^5}$	$\frac{3715}{8064} + \frac{55\eta}{96}$	$-\frac{3\pi}{4}$	$\frac{9275495}{14450688} + \frac{284875\eta}{258048} + \frac{1855\eta^2}{2048}$	$\left(\frac{38645}{21504} + \frac{15\eta}{256}\right)\pi \ln\left(\frac{\theta}{\theta_{\rm lso}}\right)$
F_k^t	$\frac{\theta^3}{8\pi m}$	$\frac{743}{2688} + \frac{11\eta}{32}$	$-\frac{3\pi}{10}$	$\frac{1855099}{14450688} + \frac{56975\eta}{258048} + \frac{371\eta^2}{2048}$	$-\left(\frac{7729}{21504} + \frac{3}{256}\eta\right)\pi$
τ_k	$\frac{3}{128\eta}$	$\frac{5}{9}\left(\frac{743}{84} + 11\eta\right)$	-16π	$2\phi_4^v$	$\frac{1}{3}\left(8\phi_5^v - 5t_5^v\right)$

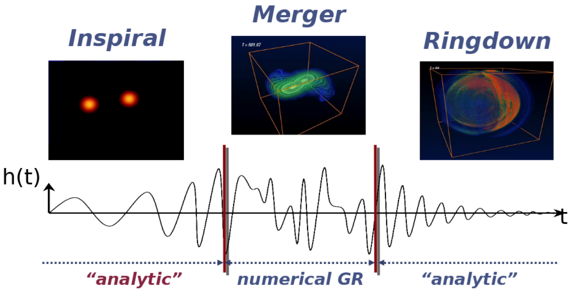
Example waveform

- Analytical part we looked at



$$P_{\mathrm{GW}} = \frac{32 G \mu^2 a^4 \omega^6}{5 c^5}$$

- Full picture



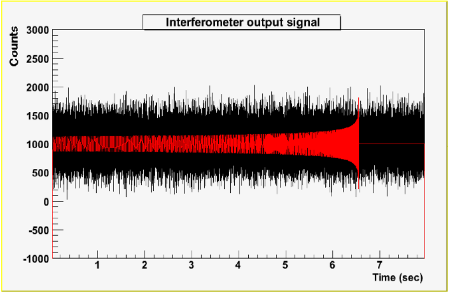
Part 3: gravitational wave data analysis

The problem

- Signal buried in noise

$$d(t) = n(t) + s(t)$$

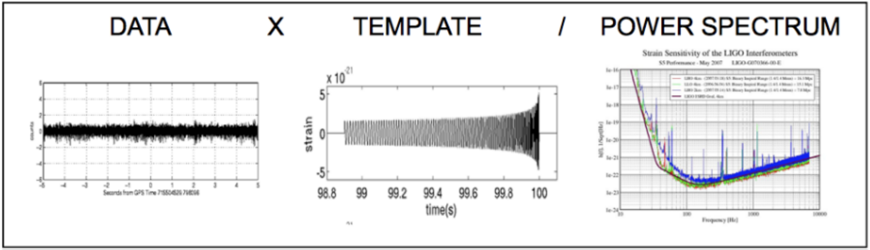
$$\text{Data} = \text{Noise} + \text{Signal}$$

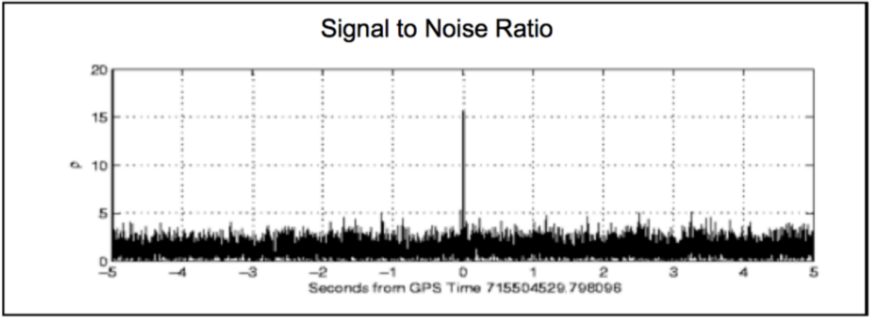


- Noise is stochastic (random variable)
- Signal time evolution is known (post-newtonian expansion)

$\Rightarrow$ Use signal shape knowledge (templates)

Solution: matched filtering





Known signal in independent Gaussian noise

$$d(t) = n(t) + s(t)$$

- Independent Gaussian noise

$$P(n(t_0)) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n(t_0)^2}{2\sigma^2}}$$

- H_0 hypothesis – there is only noise

$$P(d(t)|H_0) \propto e^{-\frac{d_1^2}{2\sigma^2}} \times e^{-\frac{d_2^2}{2\sigma^2}} \times ... = \exp\left(-\sum_{i=1}^{N} \frac{d_i^2}{2\sigma^2}\right)$$

- H_1 hypothesis – there is a signal s in the noise

$$P(d(t)|H_1) \propto e^{-\frac{((d_1-s_1)^2}{2\sigma^2}} \times e^{-\frac{(d_2-s_2)^2}{2\sigma^2}} \times ... = \exp\left(-\sum_{i=1}^{N} \frac{(d_i - s_i)^2}{2\sigma^2}\right)$$

- Likelihood ratio of each hypothesis

$$\frac{P(d(t)|H_1)}{P(d(t)|H_0)} = \exp\left(-\sum_{i=1}^{N} \frac{(d_i - s_i)^2 - d_i^2}{2\sigma^2}\right) = \exp\left(\sum_{i=1}^{N} \frac{2d_i s_i - s_i^2}{2\sigma^2}\right)$$

Known signal in independent Gaussian noise

- Likelihood ratio of each hypothesis

$$L = \log \frac{P(d(t)|H_1)}{P(d(t)|H_0)} = 2 \sum_{i=1}^{N} \frac{d_i s_i}{2\sigma^2} - \sum_{i=1}^{N} \frac{s_i^2}{2\sigma^2}$$

⇒ Correlation between data and expected signal tells which hypothesis is more likely

- Unknown parameters – signal amplitude (source distance)

$$s_i \rightarrow A s_i$$

$$L(A) = 2A \sum_{i=1}^{N} \frac{d_i s_i}{2\sigma^2} - A^2 \sum_{i=1}^{N} \frac{s_i^2}{2\sigma^2}$$

- Find analytically the maximum of L, most likely signal amplitude A

$$\frac{\partial L}{\partial A} = 2 \sum_{i=1}^{N} \frac{d_i s_i}{2\sigma^2} - 2A \sum_{i=1}^{N} \frac{s_i^2}{2\sigma^2} = 0 \quad \Rightarrow \quad A = \frac{\sum_{i=1}^{N} \frac{d_i s_i}{2\sigma^2}}{\sum_{i=1}^{N} \frac{s_i^2}{2\sigma^2}}$$

$$\max_A L(A) = 2 \frac{\left(\sum_{i=1}^{N} \frac{d_i s_i}{2\sigma^2}\right)^2}{\sum_{i=1}^{N} \frac{s_i^2}{2\sigma^2}} - \frac{\left(\sum_{i=1}^{N} \frac{d_i s_i}{2\sigma^2}\right)^2}{\sum_{i=1}^{N} \frac{s_i^2}{2\sigma^2}} = \frac{\left(\sum_{i=1}^{N} \frac{d_i s_i}{2\sigma^2}\right)^2}{\sum_{i=1}^{N} \frac{s_i^2}{2\sigma^2}}$$

Signal templates

$$\max_A L(A) = \frac{\left(\sum_{i=1}^{N} \frac{d_i s_i}{2\sigma^2}\right)^2}{\sum_{i=1}^{N} \frac{s_i^2}{2\sigma^2}}$$

- Normalized signal template

$$u_i = \frac{s_i}{\sqrt{\sum_{i=1}^{N} \frac{s_i^2}{2\sigma^2}}}$$

- Standard form of detection statistic

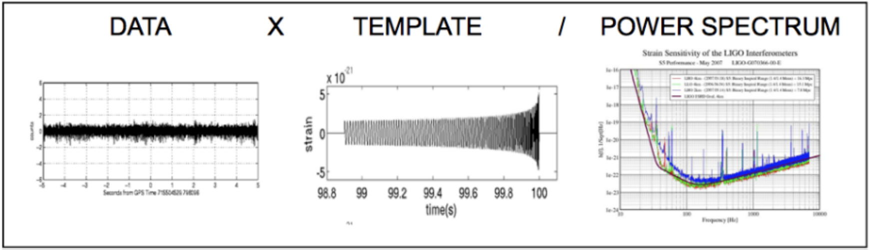
$$\max_A L(A) = \left(\sum_{i=1}^{N} \frac{d_i u_i}{2\sigma^2}\right)^2 = \text{SNR}^2$$

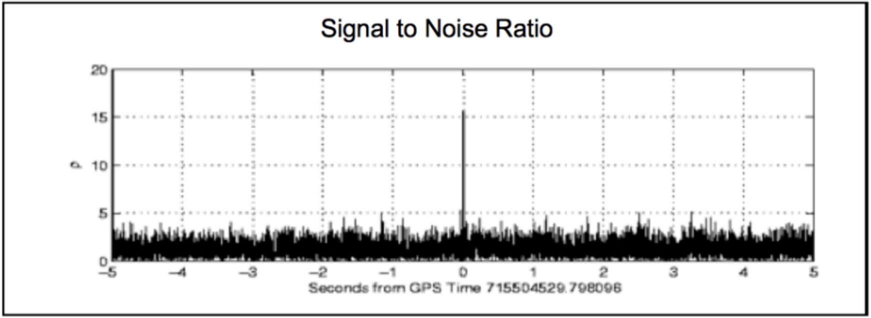
- In practice noise is correlated in time but independent in frequency domain

$$\max_A L(A) = \left(\sum_{i=1}^{N} \frac{\tilde{d}_k \tilde{u}_k}{2\sigma_k^2}\right)^2$$

- Maximization (masses of objects, ...) on other parameters is done numerically

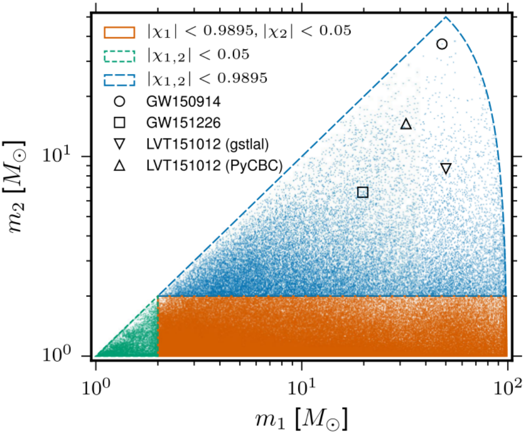
Solution: matched filtering





$$\max_A L(A) = \left(\sum_{i=1}^{N} \frac{\tilde{d}_k \tilde{u}_k}{2\sigma_k^2} \right)^2$$

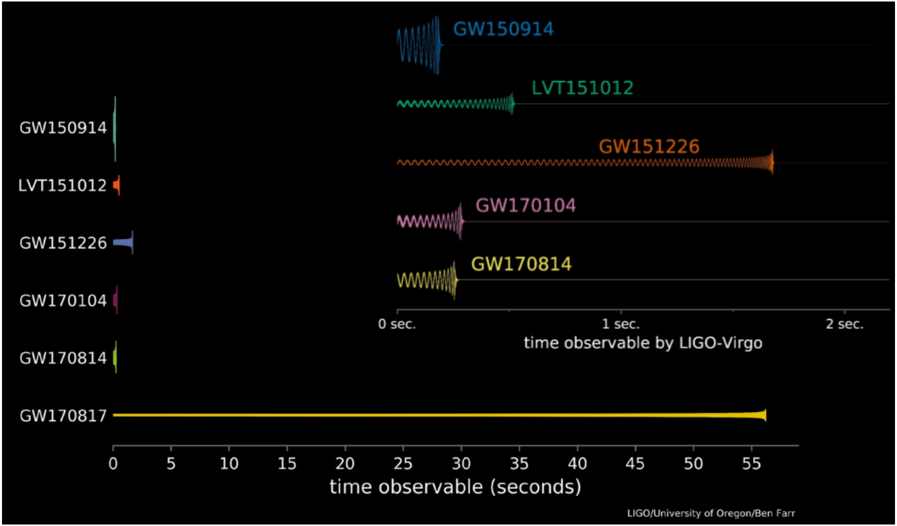
Search over large parameter space



$$\max_{m_1, m_2} L(m_1, m_2) = \left(\sum_{i=1}^{N} \frac{\tilde{d}_k \tilde{u}_k(m_1, m_2)}{2\sigma_k^2} \right)^2$$

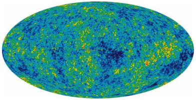
- $\sim 250 \times 10^3$ templates
- Include NS-NS, NS-BH, BH-BH

Signal diversity



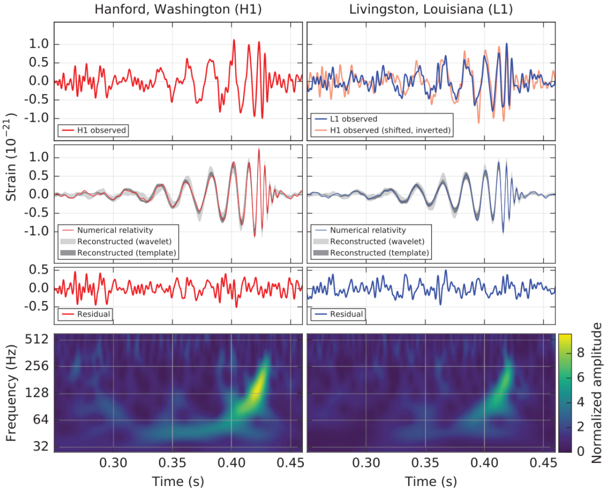
- Waveforms of signals detected so far

4 families of potential GW signal morphologies

	precisely modeled	uncertain form
permanent	Deformed rotating neutron stars 	Incoherent sum of unresolved sources Primordial GW background 
transient	Cosmic strings cusps, kinks Coalescence of neutron stars or black holes 	Star quakes, Non spherically symmetric stellar collapse, ... 

Part 4: gravitational wave results

GW150914: First direct detection of GWs

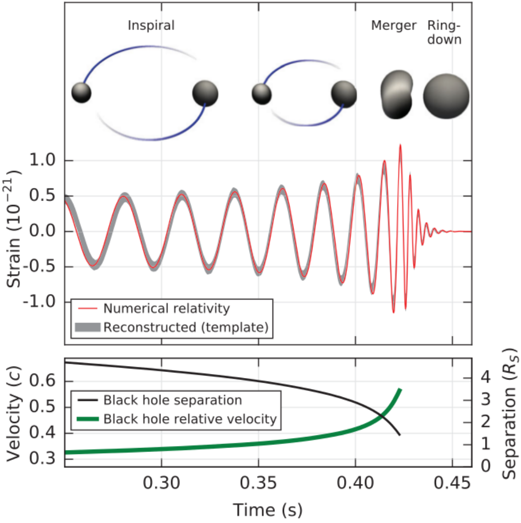


- GW150914 – 2015 September 14
- short signal: 0.1 second
- highest frequency is ~ 250 Hz

GW150914: First direct detection of GWs

Waveforms without noise

Waveform shape matches general relativity prediction



- GW frequency $\Rightarrow$ twice the orbital frequency $\Rightarrow$ $\sim$ orbit from Kepler's law
- Relativistic collision $v \sim 0.5c$, fastest double neutron star known $v/c \simeq 2 \times 10^{-3}$

Gravitational wave carry away energy

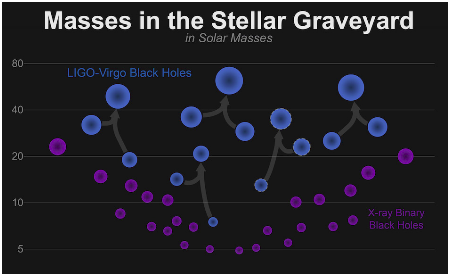
Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

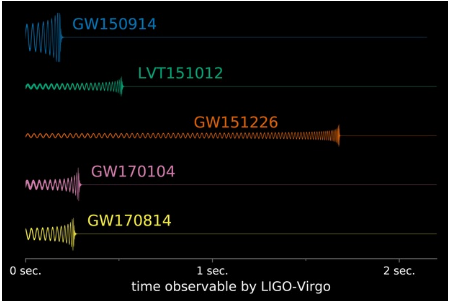
- GW amplitude correspond to 3 solar masses emitted

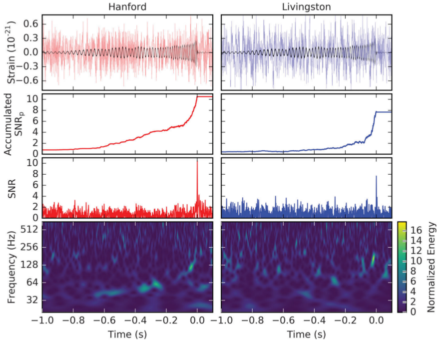
$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{c^3}{16\pi G} \left\langle \left(\dot{h}_+\right)^2 + \left(\dot{h}_\times\right)^2 \right\rangle$$

- Nuclear reaction 0.1-0.3% mass conversion, here it is $\sim 4\%$
- Not a suprise, known for 40 years

Several other binary BH detected so far





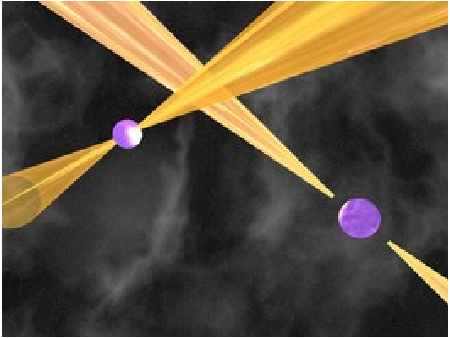


- Heavier than most black hole observed through X-rays
- Templates necessary to detect weaker signals

Testing general relativity (GR)

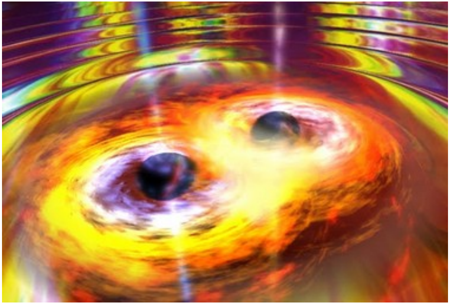
PSR J0737-3039

- Most relativistic binary pulsar known
- orbital velocity $\frac{v}{c} \sim 2 \times 10^{-3}$



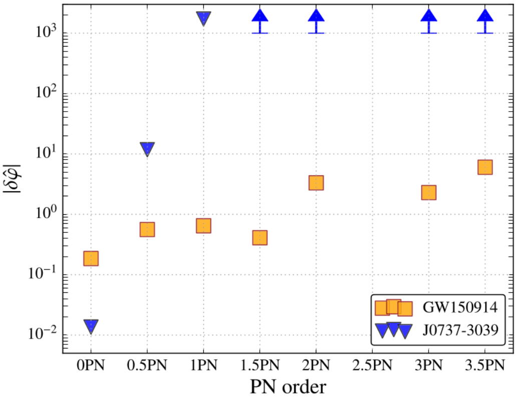
Binary black hole GW

- large velocity, strong gravitational field
- orbital velocity $\frac{v}{c} \sim 0.5$



Deviation from general relativity

k	N	2	3	4	5
$\mathcal{F}_k$	$\frac{32\eta^2 v^{10}}{5}$	$-\frac{1247}{336} - \frac{35\eta}{12}$	4π	$-\frac{44711}{9072} + \frac{9271\eta}{504} + \frac{65\eta^2}{18}$	$-\left(\frac{8191}{672} + \frac{535\eta}{24}\right)\pi$
t_k^v	$-\frac{5m}{256\eta v^8}$	$\frac{743}{252} + \frac{11\eta}{3}$	$-\frac{32\pi}{5}$	$\frac{3058673}{508032} + \frac{5429\eta}{504} + \frac{617\eta^2}{72}$	$-\left(\frac{7729}{252} + \eta\right)\pi$
ϕ_k^v	$-\frac{1}{16\eta v^5}$	$\frac{3715}{1008} + \frac{55\eta}{12}$	-10π	$\frac{15293365}{1016064} + \frac{27145\eta}{1008} + \frac{3085\eta^2}{144}$	$\left(\frac{38645}{672} + \frac{15\eta}{8}\right)\pi \ln\left(\frac{v}{v_{\rm lso}}\right)$
ϕ_k^t	$-\frac{2}{\eta\theta^5}$	$\frac{3715}{8064} + \frac{55\eta}{96}$	$-\frac{3\pi}{4}$	$\frac{9275495}{14450688} + \frac{284875\eta}{258048} + \frac{1855\eta^2}{2048}$	$\left(\frac{38645}{21504} + \frac{15\eta}{256}\right)\pi \ln\left(\frac{\theta}{\theta_{\rm lso}}\right)$
F_k^t	$\frac{\theta^3}{8\pi m}$	$\frac{743}{2688} + \frac{11\eta}{32}$	$-\frac{3\pi}{10}$	$\frac{1855099}{14450688} + \frac{56975\eta}{258048} + \frac{371\eta^2}{2048}$	$-\left(\frac{7729}{21504} + \frac{3}{256}\eta\right)\pi$
τ_k	$\frac{3}{128\eta}$	$\frac{5}{9}\left(\frac{743}{84} + 11\eta\right)$	-16π	$2\phi_4^v$	$\frac{1}{3}\left(8\phi_5^v - 5t_5^v\right)$



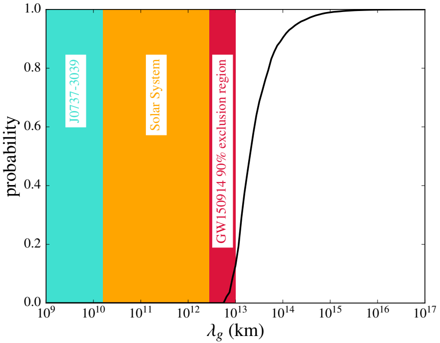
- Double neutron star has small $(v/c) \simeq 2 \times 10^{-3}$
- $\delta\varphi$ in units of GR prediction

Dispersion relation and graviton mass

$$E^2 = p^2c^2 + m_g^2c^4$$

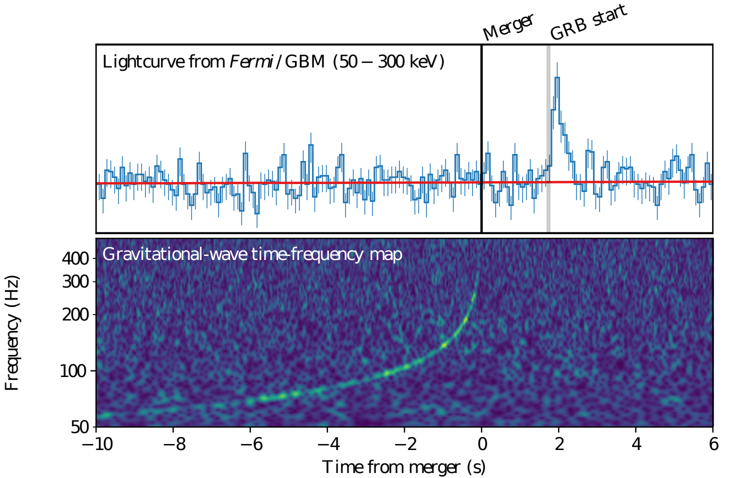
scale at which G/r^2 fails: $\lambda_g = \dfrac{h}{m_g c}$

$$\frac{v_g^2}{c^2} \equiv \frac{c^2p^2}{E^2} \simeq 1 - \frac{h^2c^2}{\lambda_g^2 E^2}$$



- $\Rightarrow$ Low energy (frequency) GW propagate slower (slowed down by mass)
- $\Rightarrow$ Low frequency GW would arrive after instead of before high frequency GWs!
- $\Rightarrow$ $\lambda_g > 1 \times 10^{13}$ km ~ 0.5 parsec $\Leftrightarrow m_g < 1.2 \times 10^{-22}$eV/c^2

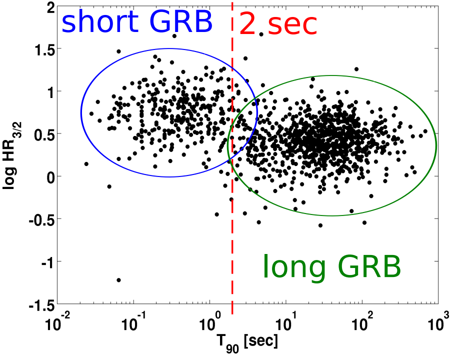
GW170817 and GRB 170817A



- Gamma-ray bursts starts 1.74 s after the merger

Gamma-ray bursts

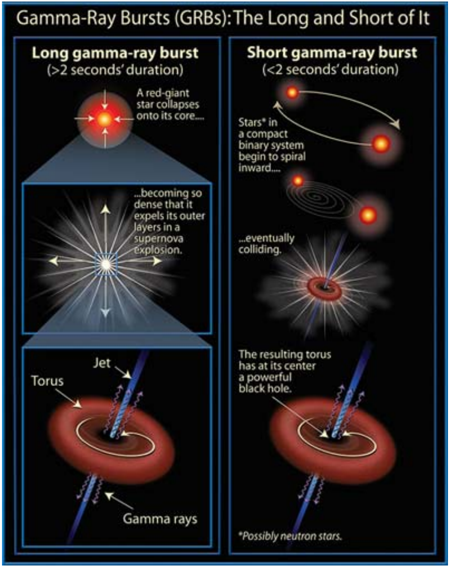
- Observational definition $\longrightarrow$ a burst of γ-rays $(10\,\text{keV} - 1\,\text{MeV})$
- Discovered in the 70's by nuclear bomb test surveillance satellites



BATSE 4B catalog

- T_{90} - duration of 90% of photon counts ($\sim 15 - 300\,\text{keV}$)
- Two observational populations:
  - short-hard GRBs $T_{90} \lesssim 2\,\text{s}$ spectrum peaks at higher energy
  - long-soft GRBs $T_{90} \gtrsim 2\,\text{s}$ spectrum peaks at lower energy

Gamma-ray burst models



credit: Ute Kraus

- Long GRBs

⇒ Massive rapidly spinning star collapse and explosion

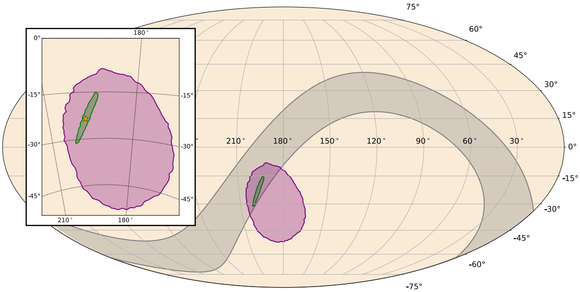
⇒ Confirmed by several association to Supernovae

- Short GRBs

⇒ Coalescence of a neutron star and a compact object

⇒ Common in the outskirts of old galaxies

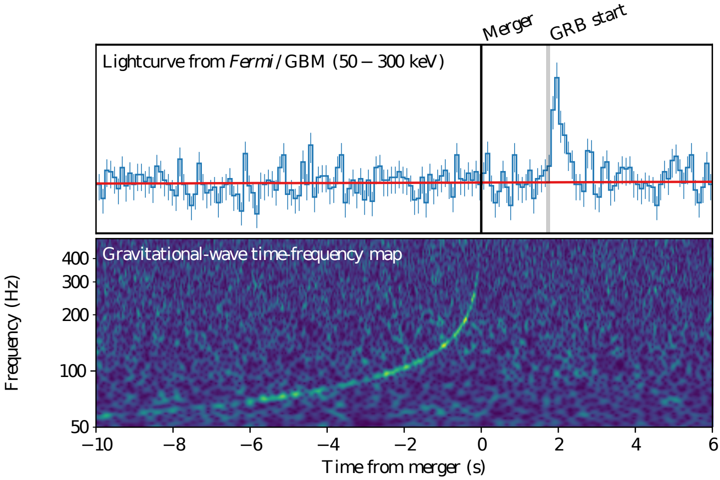
GW170817 / GRB 170817A have a common origin



- 1.74 s time delay vs 0.12 short GRB per day ⇒ p-value 5×10^{-6}
- sky location overlap ⇒ p-value 0.01
- p-value 5×10^{-8} or 5.3σ

⇒ (Some) short gamma-ray bursts are indeed due to binary neutron star mergers

GW170817 and GRB 170817A

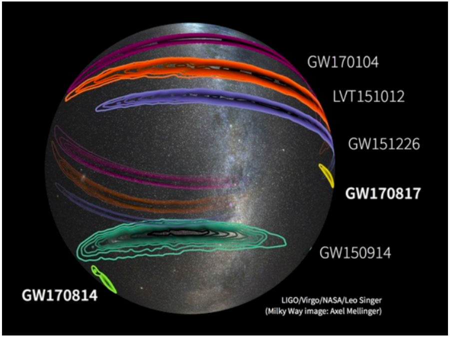


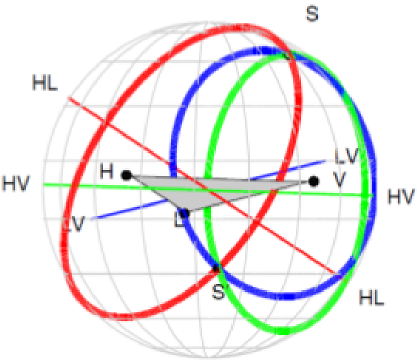
- Gamma-ray bursts starts 1.74 s after the merger

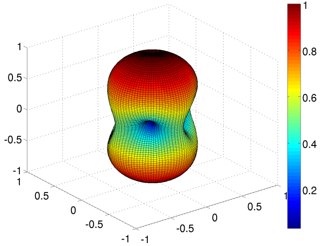
⇒ (Some) short gamma-ray bursts are indeed due to binary neutron star mergers

Gravitational wave sky localization

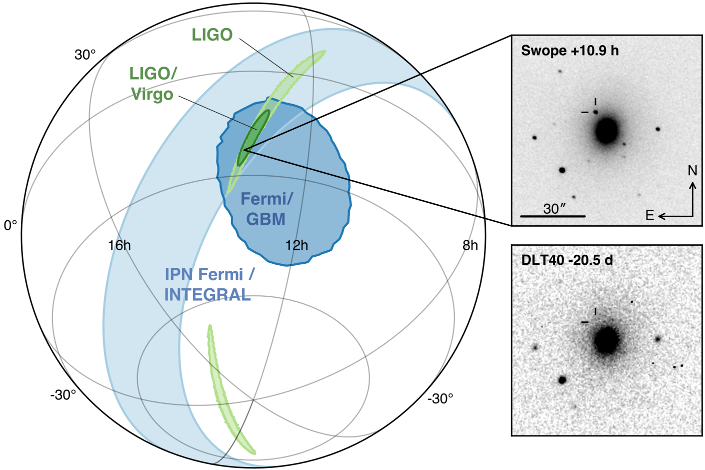
- Primarily time delay
  - 2 detectors: ring on the sky
  - 3 detectors: intersection of 2 rings
- Amplitude information helps







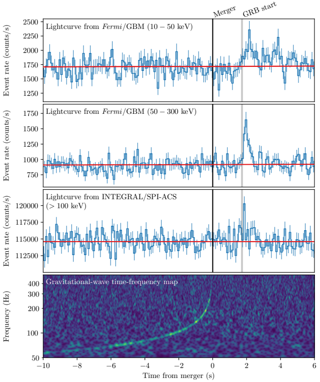
An optical counterpart



- Localized in the sky by 3 GW detectors
- Observed near a galaxy (NGC 4993) 130 million light years away (40 Mpc)

⇒ A kilonova

GW170817 / GRB 170817A - fundamental physics test



- 1.74 s delay over 130 million years of propagation
- Assuming gamma emission delayed by [0,10] s

$$-3 \times 10^{-15} \leq \frac{v_{\mathrm{GW}} - v_{\mathrm{EM}}}{v_{\mathrm{EM}}} \leq 7 \times 10^{-16}$$

- Shapiro effect: gravitational potential slows clocks down

$\Rightarrow$ Equivalence principle test, GW and EM clocks are affected the same, $\gamma_{\mathrm{GW}} = \gamma_{\mathrm{EM}} = 1$

- Only using Milky Way potential at large distances (100 kpc)

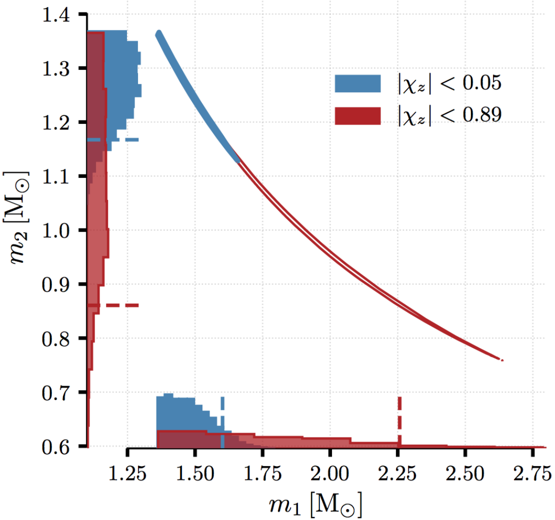
$$-2.6 \times 10^{-7} \leq \gamma_{\mathrm{GW}} - \gamma_{\mathrm{EM}} \leq 1.2 \times 10^{-6}$$

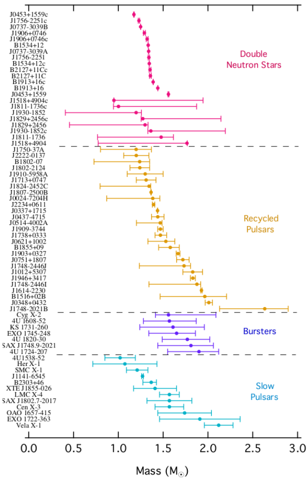
GW170817 / GRB 170817A - fundamental physics test

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [46] Brans-Dicke/$f(R)$ [47], [48] Kinetic Gravity Braiding [50]	quartic/quintic Galileons [13], [14] Fab Four [15] de Sitter Horndeski [49] $G_{\mu\nu}\phi^\mu\phi^\nu$ [51], $f(\phi)$·Gauss-Bonnet [52]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

- Prediction of $\frac{v_{\mathrm{GW}} - v_{\mathrm{EM}}}{v_{\mathrm{EM}}} \simeq 10^{-4}$ ruled out by 10 orders of magnitude
- Many GR modification to explain dark matter or dark energy are excluded

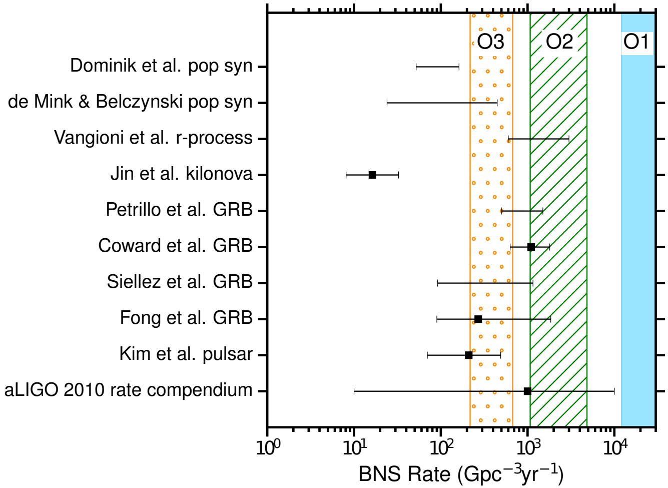
A collision of two neutron stars



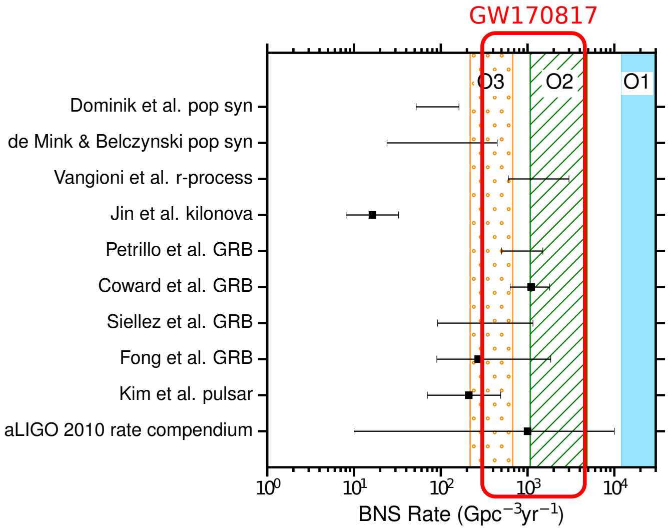


- Very small possibility that heavier object is a rapidly spinning light black hole

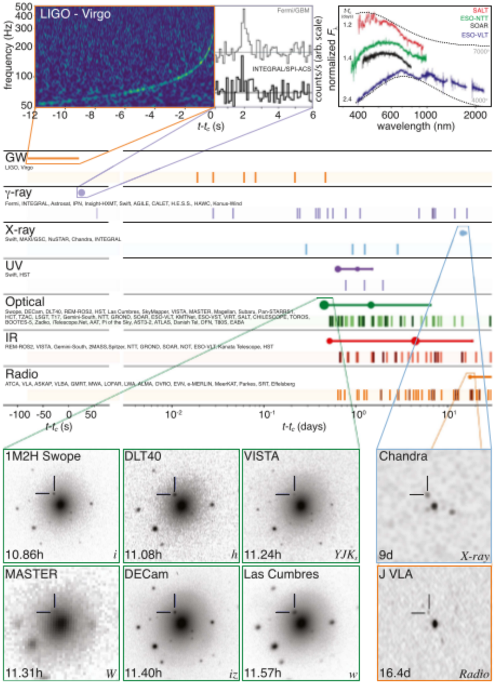
Single event ⇒ a measure of the BNS merger rate



Single event ⇒ a measure of the BNS merger rate

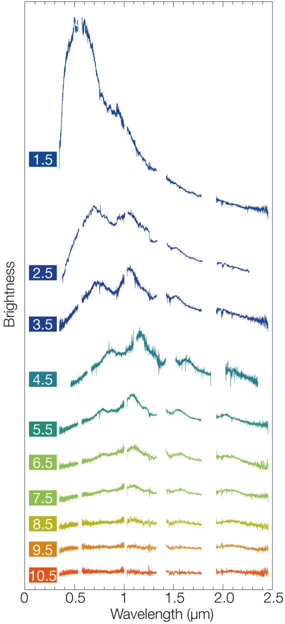


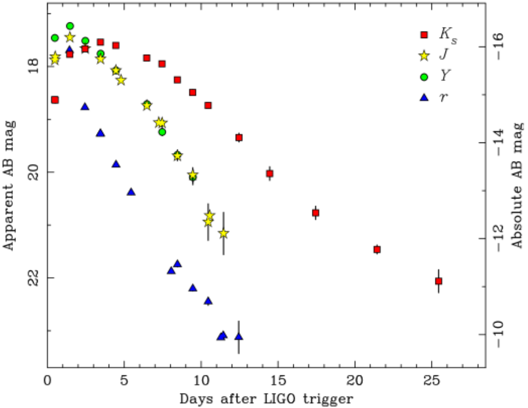
- Measured merger rate: $300 - 5000\,\text{Gpc}^{-3}\text{yr}^{-1}$
- compatible with O1 upper limit, r-process nucleosynthesis and GRB rate


LIGO - Virgo
frequency (Hz)
t-t_c (s)
Fermi/GBM
INTEGRAL/SPI-ACS
counts/s (arb. scale)
normalized F_λ
SALT
ESO-NTT
SOAR
ESO-VLT
wavelength (nm)
GW
LIGO, Virgo
γ-ray
Fermi, INTEGRAL, Astrosat, IPN, Insight-HXMT, Swift, AGILE, CALET, H.E.S.S., HAWC, Konus-Wind
X-ray
Swift, MAXI/GSC, NuSTAR, Chandra, INTEGRAL
UV
Swift, HST
Optical
IR
Radio
t-t_c (s)
t-t_c (days)
1M2H Swope
10.86h
DLT40
11.08h
VISTA
11.24h
Chandra
9d
X-ray
MASTER
11.31h
DECam
11.40h
Las Cumbres
11.57h
J VLA
16.4d
Radio

A complicated astrophysical event

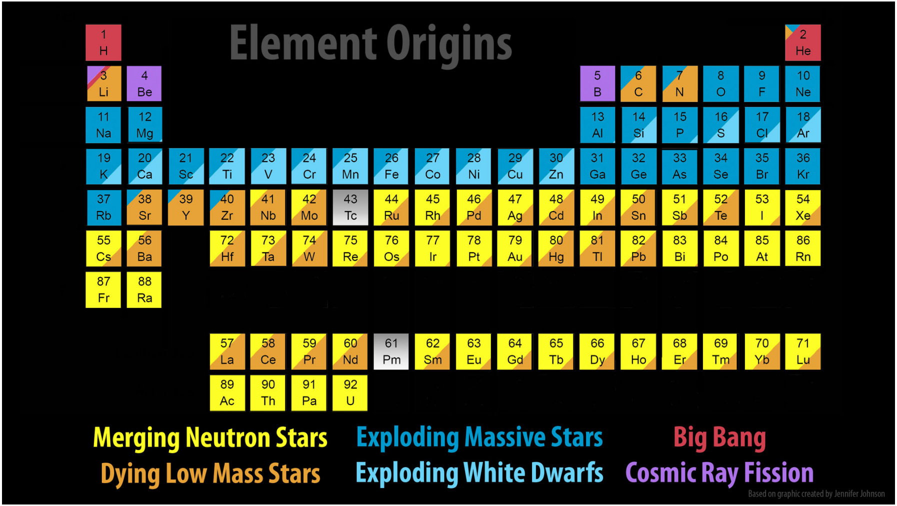
A very well studies optical transient





- "kilonova" – 1% of supernova
- much faster evolution, days instead of weaks
- spectral lines broadening measures eject speed $\sim 0.1c$
- previously 2 tentative observations in 15 years

Where do expensive metals come from



- r-process (rapid neutron capture) power the optical transient
- Good explanation of origin of heavy elements in the universe

- What are gravitational waves?
- Gravitational wave sources
- Gravitational wave data analysis
- Observed gravitational waves signals

⇒ Tuesday, how gravitational wave detectors are build

r-Process simulation

EM measurement of NS mass-radius

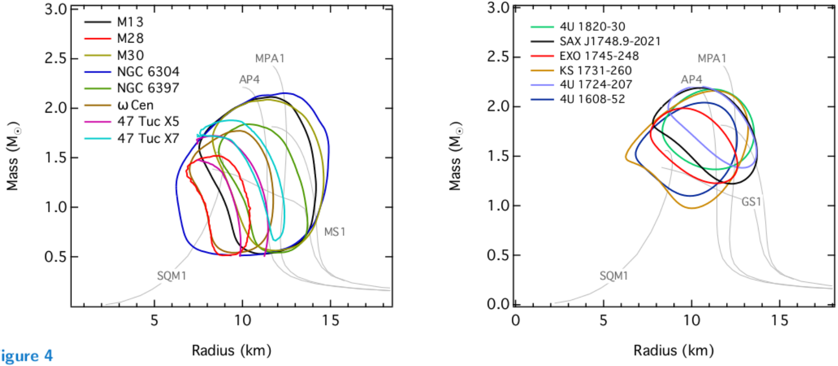
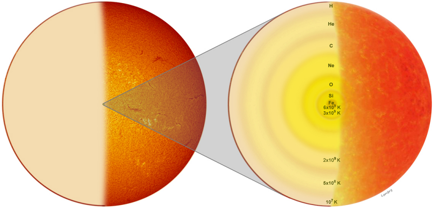
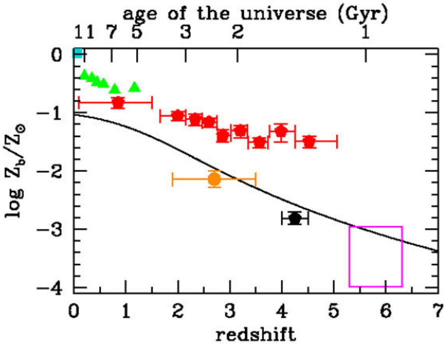


Figure 4

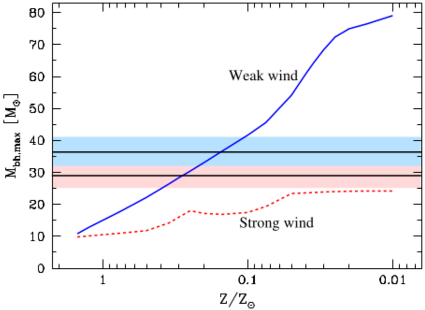
Black hole merger masses are large

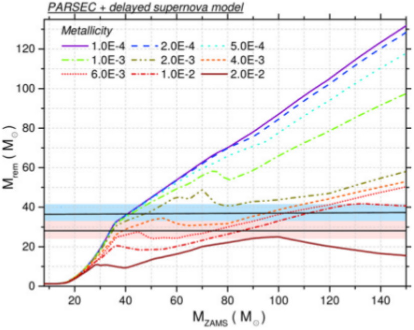




- Photons from nuclear reaction push stellar envelope outwards
- Cross section is higher if envelope contains “metals” (not hydrogen or helium)
- Supernova produce and disseminate metals
  $\longrightarrow$ metallicity of stars increases with universe age

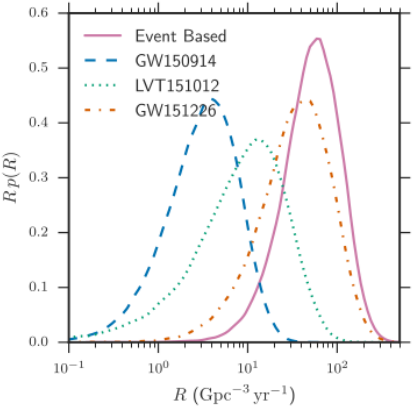
Black hole merger masses are large

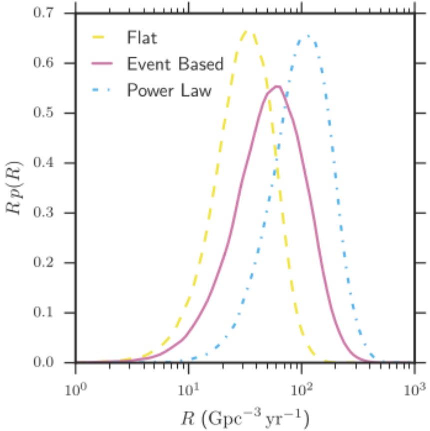




- Models of stellar winds did not allow BH masses larger than $25 \mathrm{M}_\odot$
- Confirms recent models of stellar wind
- The binary BH system formed in an environment with $Z < Z_\odot/2$

Black hole merger rate

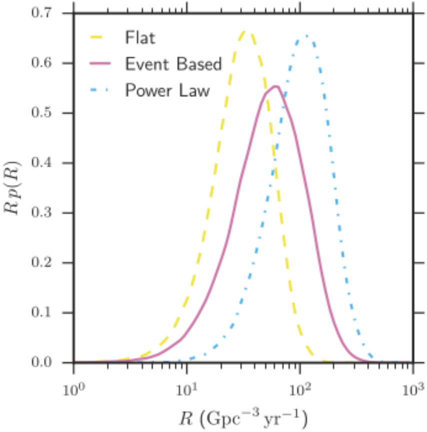


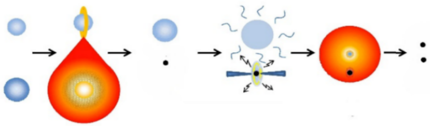


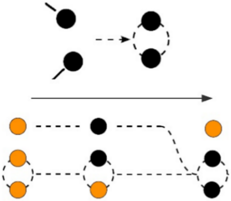
$$R_{\text{eventbased}} \simeq \frac{1}{T_{\text{obs}} V_{\text{obs,GW150914}}} + \frac{1}{T_{\text{obs}} V_{\text{obs,LVT151012}}} + \frac{1}{T_{\text{obs}} V_{\text{obs,GW151226}}}$$

- Detectors horizon is smaller for lighter BH binaries $\Rightarrow$ smaller volume
- flat in logarithm $p(m_1, m_2) \propto m_1^{-1} m_2^{-2}$
- powerlaw $p(m_1) \propto m_1^{-2.35}$, $p(m_2) \propto \theta(m_1 - m_2)$

Black hole merger rate

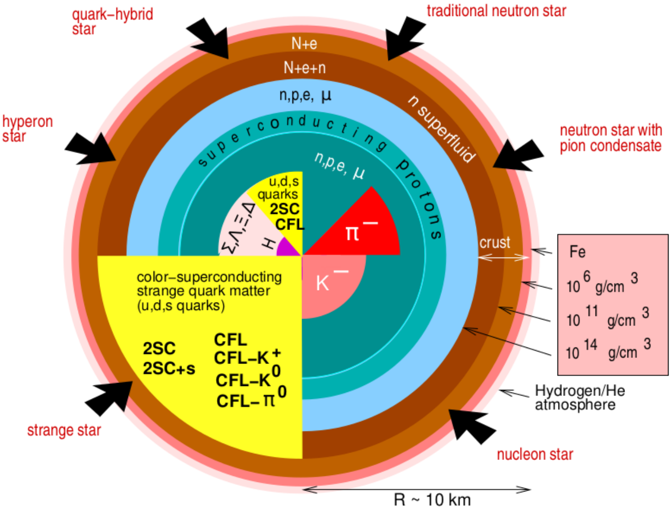






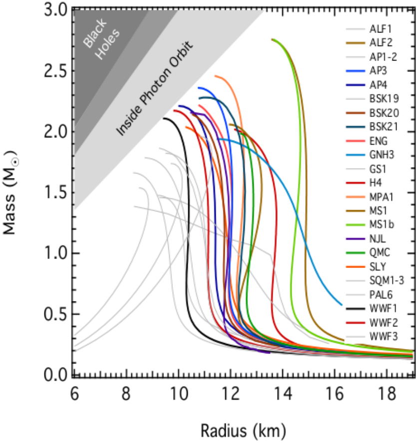
- Measured rate $R = 9 - 240\,\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$
- Models were predicting $R = 0.1 - 300\,\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$
- Exclude a few models and parameter space that were predicting $R \lesssim 1\,\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$

Neutron star structure and equation of state



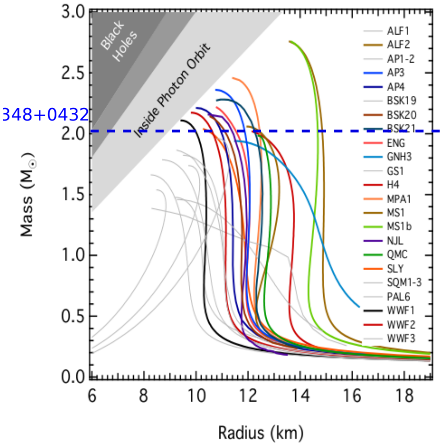
- Strange state of matter may exist in neutron star cores

Neutron star equation of state



- EOS: Pressure = f(density)
- Governs relation between neutron star mass and radius
- Heavier neutron stars are smaller!
- Stiff equation of state (rapid pressure increase) $\rightarrow$ large neutron star
- Soft equation of state (slow pressure increase) $\rightarrow$ small neutron star

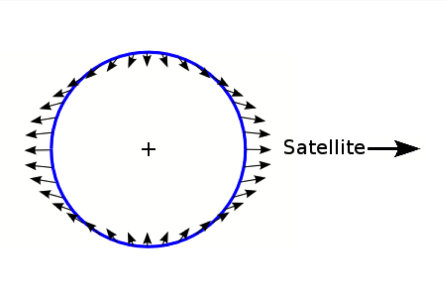
Neutron star equation of state



- J0348+0432 (MSP-WD) mass $2.01 \pm 0.04 M_\odot$
- Soft equation of state has small maximum NS mass

Tidal deformability

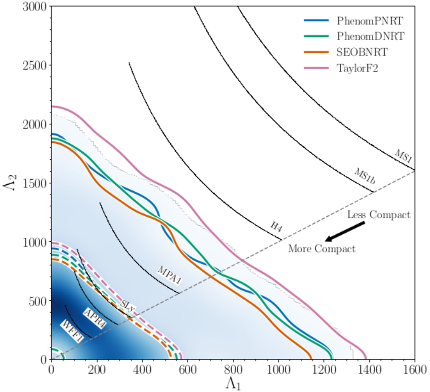
$$\Lambda = \frac{2}{3} k_2 \left(\frac{c^2 R}{Gm} \right)^5$$



- Λ parameter changing gravitational wave phase
- k_2 dimensionless quantity (Love number) characterizing tidal deformability
- R neutron star radius
- m neutron star mass
- Large neutron star (stiff EOS) have higher tidal effect
- Tidal deformation causes neutron stars to merger faster (additional energy loss)
- GW encodes a combination of both stars deformabilities

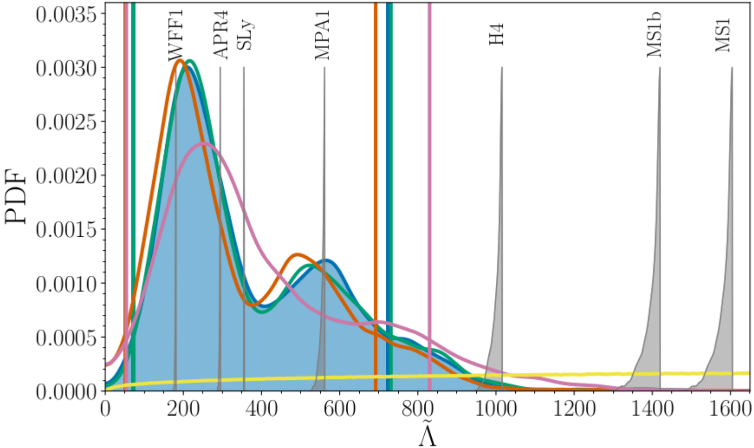
$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12 m_2) m_1^4 \Lambda_1 + (m_2 + 12 m_1) m_2^4 \Lambda_2}{(m_1 + m_2)^5}$$

Tidal deformability



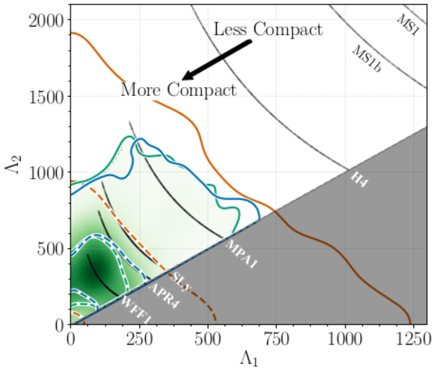
- NS tidal deformation speeds up binary coalescence
- Disfavors stiff equations of states that result in large neutron stars

Tidal deformability



- $50 < \tilde{\Lambda} < 830$

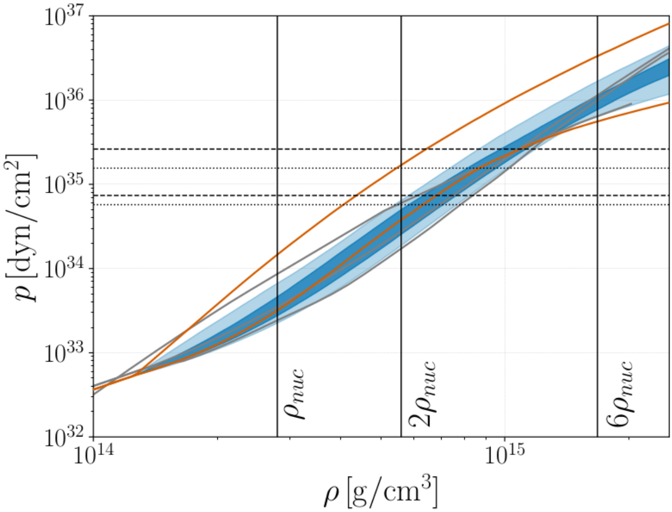
Assuming equal EOS for both neutron stars



- Λ_1, Λ_2 can still be different because of unequal mass
- $70 < \tilde{\Lambda} < 580$
- Green: same EOS, max mass > 1.97 $M_\odot$
- Blue: same EOS
- Red: independent EOS

⇒ Not a proof that objects are not BHs, boson stars, ...

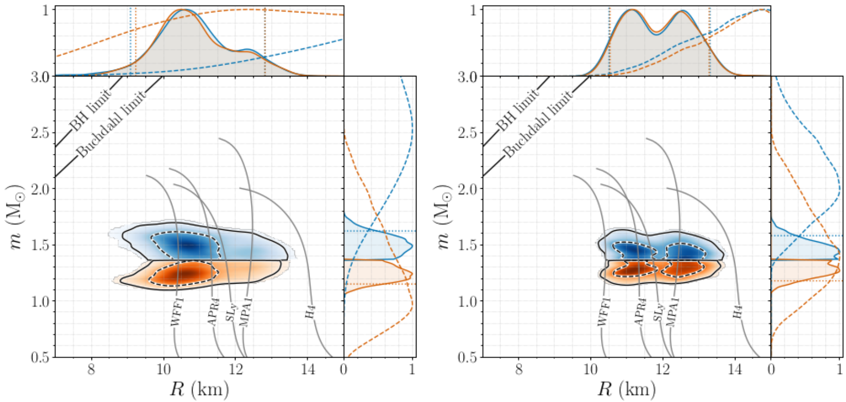
A measurement of the equation of state



- Soft equation of state are favored

A measurement of the equation of state

- Tidal deformability only
- Parametrized EOS & EOS allow $M_{NS} > 1.97M_{\odot}$

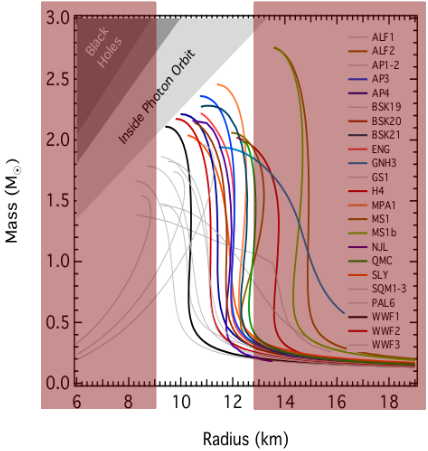


$9.1\,\text{km} < R_1 < 12.8\,\text{km}$ $\quad$ $10.5\,\text{km} < R_1 < 13.3\,\text{km}$

$9.2\,\text{km} < R_2 < 12.8\,\text{km}$ $\quad$ $10.5\,\text{km} < R_2 < 13.3\,\text{km}$

A measurement of the equation of state

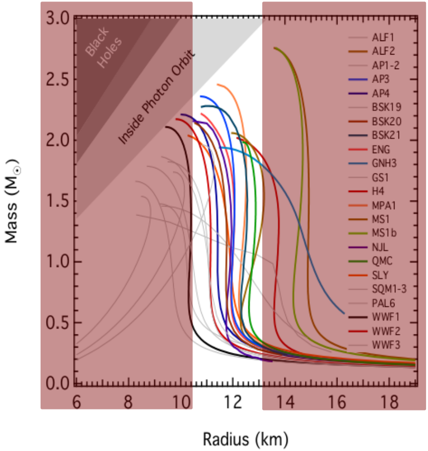
- Tidal deformability only



$9.1\,\text{km} < R_1 < 12.8\,\text{km}$

$9.2\,\text{km} < R_2 < 12.8\,\text{km}$

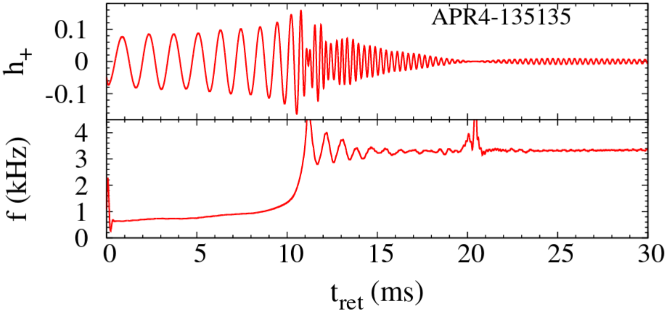
- Parametrized EOS
  & EOS allow $M_{\text{NS}} > 1.97\text{M}_\odot$



$10.5\,\text{km} < R_1 < 13.3\,\text{km}$

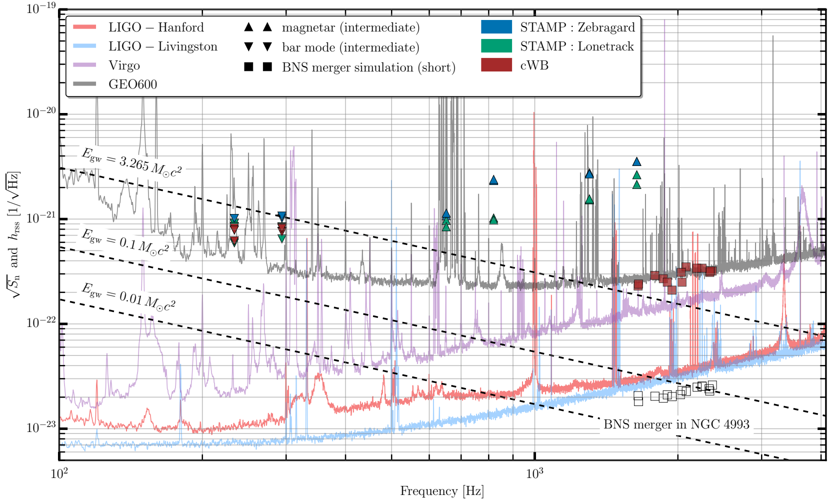
$10.5\,\text{km} < R_2 < 13.3\,\text{km}$

Post-merger scenarios



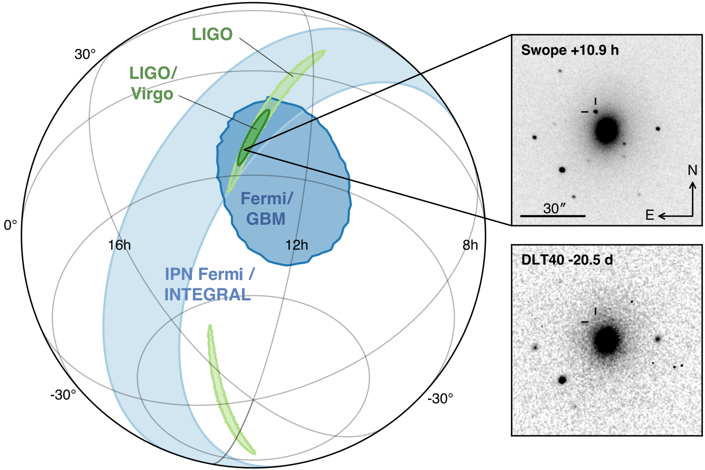
- prompt collapse to a BH
  - Small amplitude and high frequency signal $\rightarrow$ not detectable
- hypermassive NS collapsing to a BH $\lesssim 1\,s$
  - Numerical relativity simulations, short signal
- supramassive or stable NS with $\gtrsim 10$ s lifetime
  - Semi-analytical computation of unstable modes

No direct information on post-merger signal



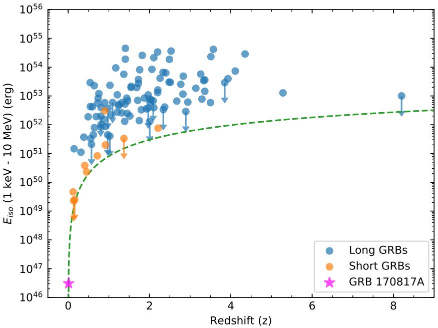
- A detectable signal $\sim$ most of remnant evaporating in gravitational waves

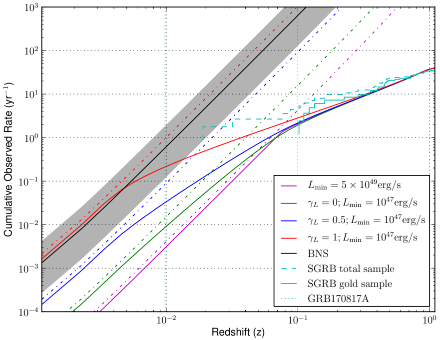
An optical counterpart



- Localized in the sky by 3 GW detectors
- Observed near a galaxy (NGC 4993) $\Rightarrow$ known redshift

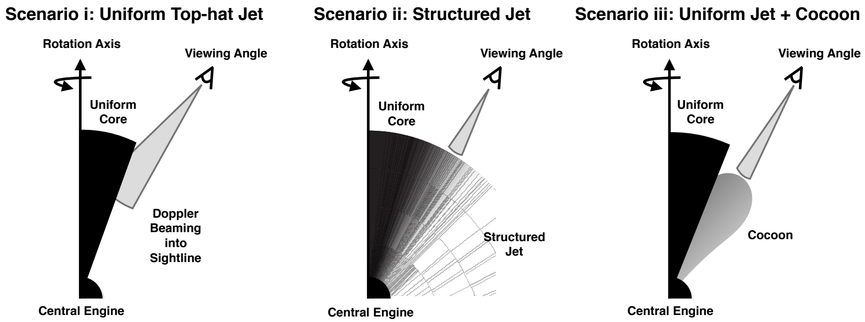
A very faint gamma-ray burst



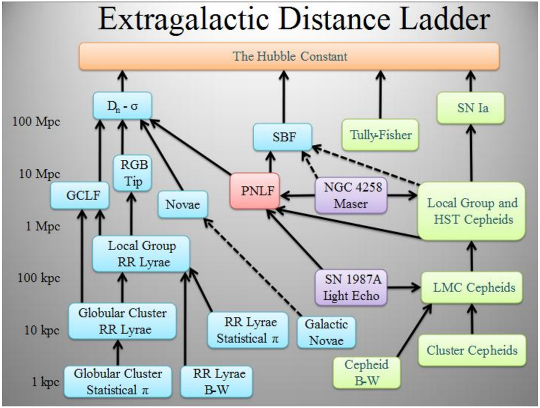


- More nearby faint GRBs than previously thought
- Gamma-ray detectors (satellites) miss most of them

A very faint gamma-ray burst – seen off axis?



Hubble constant rely on a long chain of measurements



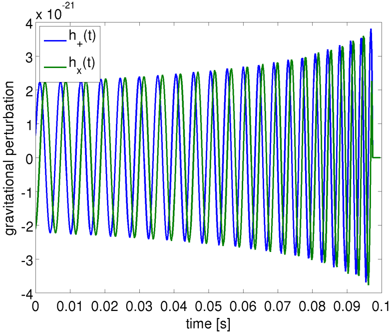
- Potential for systematic error at each step
- Gravitational wave measure distance directly

Measuring Hubble's constant with GWs

All potential GWs sources $z \lesssim 0.1$: $H_0 = c\frac{z}{D_L}$

$$\begin{bmatrix} h_+(t) \\ h_\times(t) \end{bmatrix} = \underbrace{\frac{A(t;\,(1+z)\mathcal{M})}{D_L}}_{\text{enveloppe}} \underbrace{\begin{bmatrix} (1+\cos^2\iota)\cos(\Psi(t)) \\ 2\cos\iota\sin(\Psi(t)) \end{bmatrix}}_{\text{polarized oscillations}}$$

- $A(t;\,(1+z)\mathcal{M})$ - GW shape sets absolute amplitude of the waveform
- D_L - luminosity distance
- ι - binary inclination angle - degenerate with luminosity distance (polarization is hard to measure)
- z - redshift - degenerate with the mass of the binary



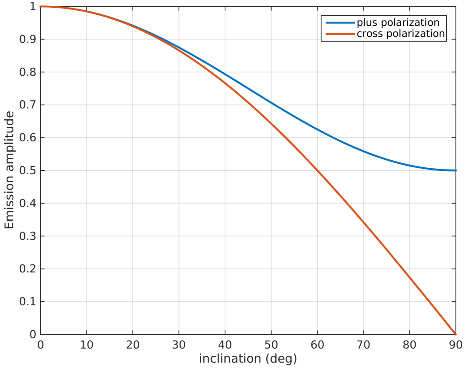
Measuring Hubble's constant with GWs

$$\begin{bmatrix} h_+(t) \\ h_\times(t) \end{bmatrix} = \frac{A(t; (1+z)\mathcal{M})}{D_L} \begin{bmatrix} (1+\cos^2\iota) cos(\Psi(t)) \\ 2\cos\iota\sin(\Psi(t)) \end{bmatrix}$$

Several approaches

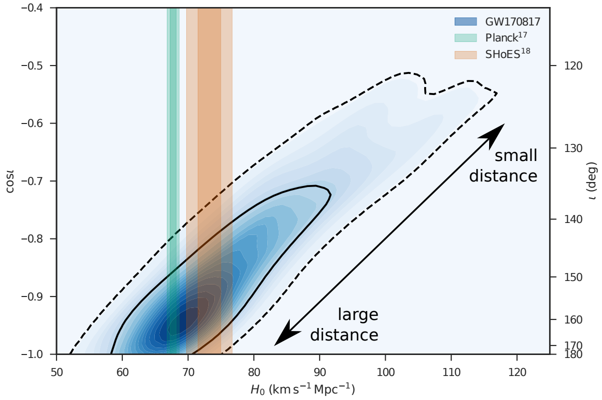
- Combine GW and GRB observation
  - redshift given by EM observations
  - GW shape yields absolute amplitude
    $\rightarrow$ Measure D_L from GW amplitude
  - D_L vs inclination degeneracy
- Use GW information alone
  - Assume $\mathcal{M}$ known - binary neutron star system
    $\rightarrow$ Measure redshift from GW shape
  - GW shape yields absolute amplitude
    $\rightarrow$ Measure D_L from GW amplitude
  - Dozens of events per year
    $\rightarrow$ helps breaking the D_L vs inclination degeneracy

Distance vs inclination degeneracy – direct measurement



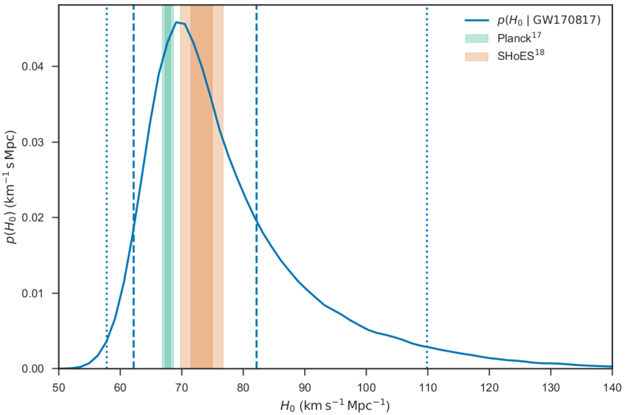
- LIGO Hanford and Livingston aligned
  - $\Rightarrow$ sensitive to only one polarization
- A strong signal in 3 detectors
  - $\Rightarrow$ measure polarization: circular vs linear
  - $\Rightarrow$ direct measurement of system inclination only for inclination > 50 deg

Distance vs inclination degeneracy



- Clear degeneracy $\Rightarrow \cos\iota \propto 1/D$

Hubble constant measurement



- Inclination degeneracy is limiting but more events will statistically reduce it