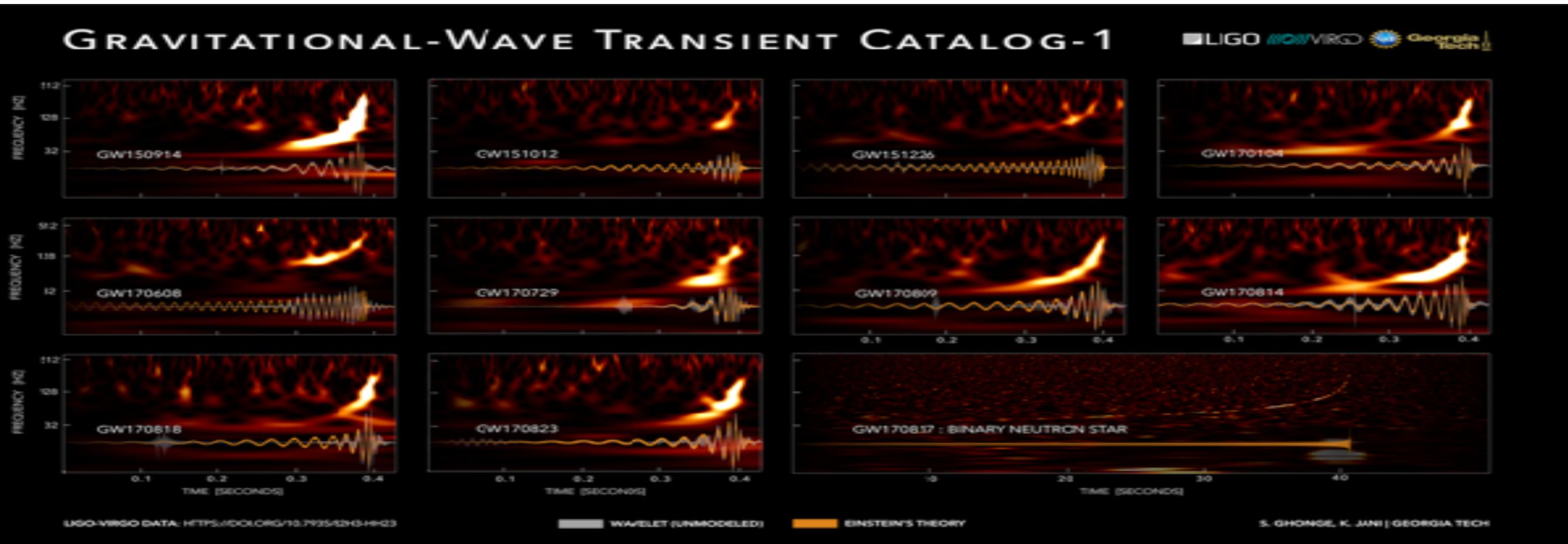


Detecting Gravitational Waves



*E. K. Porter
APC/CNRS*

Overview



- Detection
 - Detection / Selection criteria
 - Time-Frequency analysis
 - Matched Filtering
 - Low-latency announcement
- Parameter Estimation
 - Bayesian Inference
- Conclusion



What do we measure with GWs?



- Redshift masses - need an EM counterpart or assume a cosmology to get detector frame masses, i.e. $m_i^{det} = (1+z)m_i$
- Direct measure of luminosity distance, D_L
- Spins
- All sky detector - sky position and orientation via triangulation / time delays
- inclination - highly correlated with luminosity distance
- tidal deformation (NSs)



Two Methods For Detection of GWs



- Un-modelled - Time-Frequency
 - cWB
 - searches for CBC sources and supernovae

- Modelled - Matched Filtering
 - GstLAL, MBTA (LAPP,Urbino), PyCBC, spiir
 - searches for CBC sources



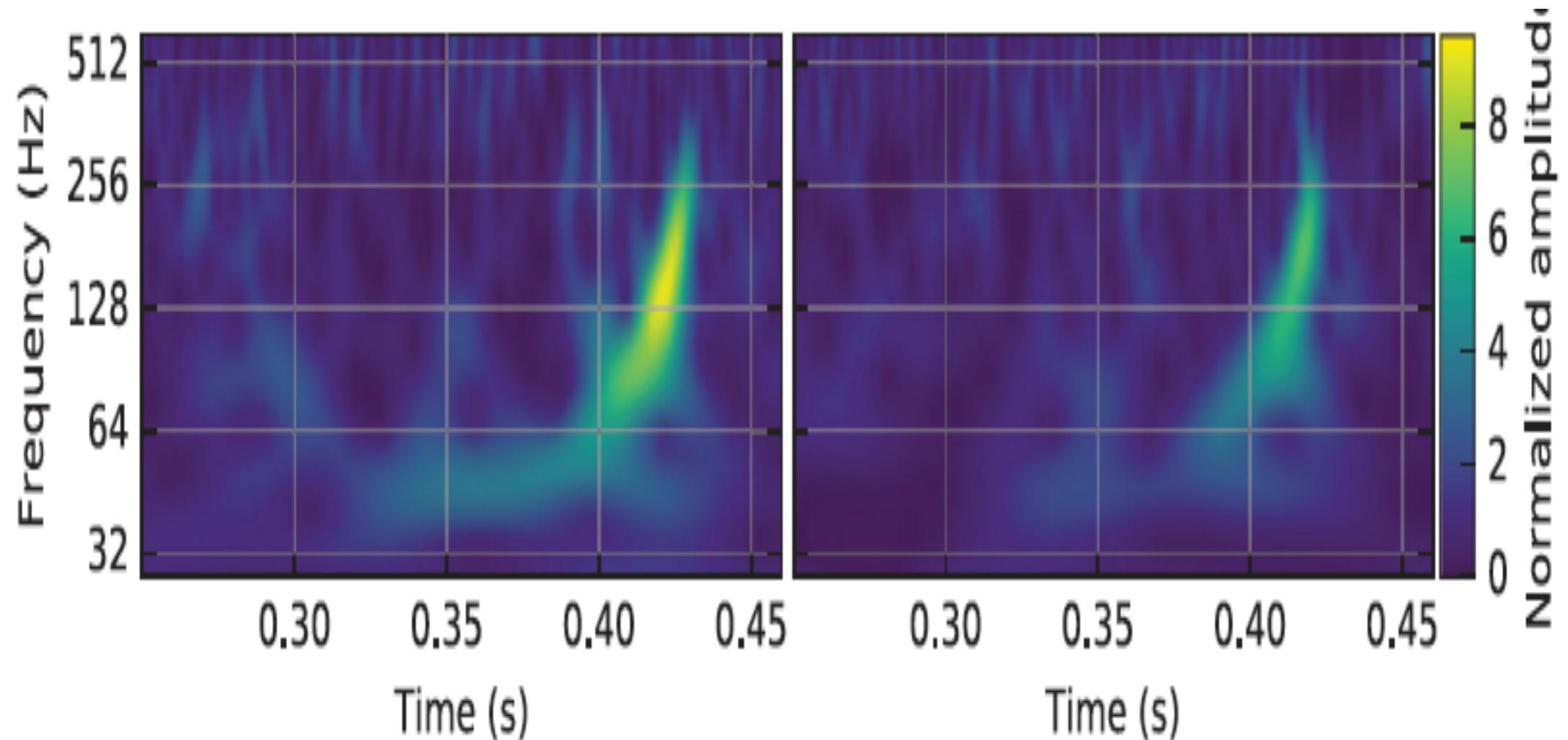
- “Burst” search
- Coherent search over multiple detectors
- Search for $m < 100 M_{\odot}$
- Identifies coincident excess power in a multi-resolution TF plan
- Pros
 - Un-modelled search
 - Can be used for multiple source-types
- Cons
 - Has difficulty with long duration signals (BNS)
 - Difficult to extract astrophysical parameters

Time-Frequency Analysis



GW150914

Abbott et al, PRL 116, 061102 (2016)



First GW detection was made by the cWB pipeline

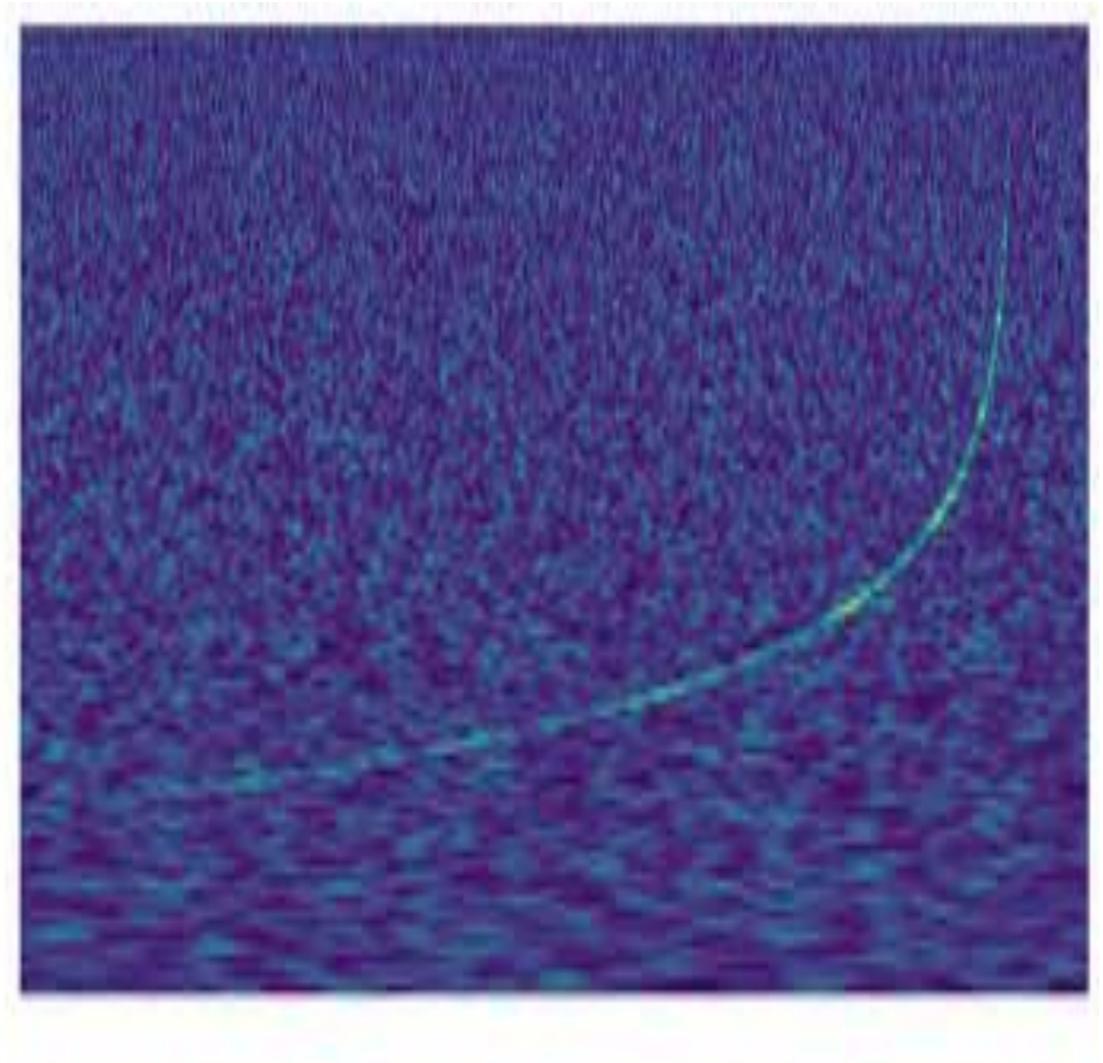
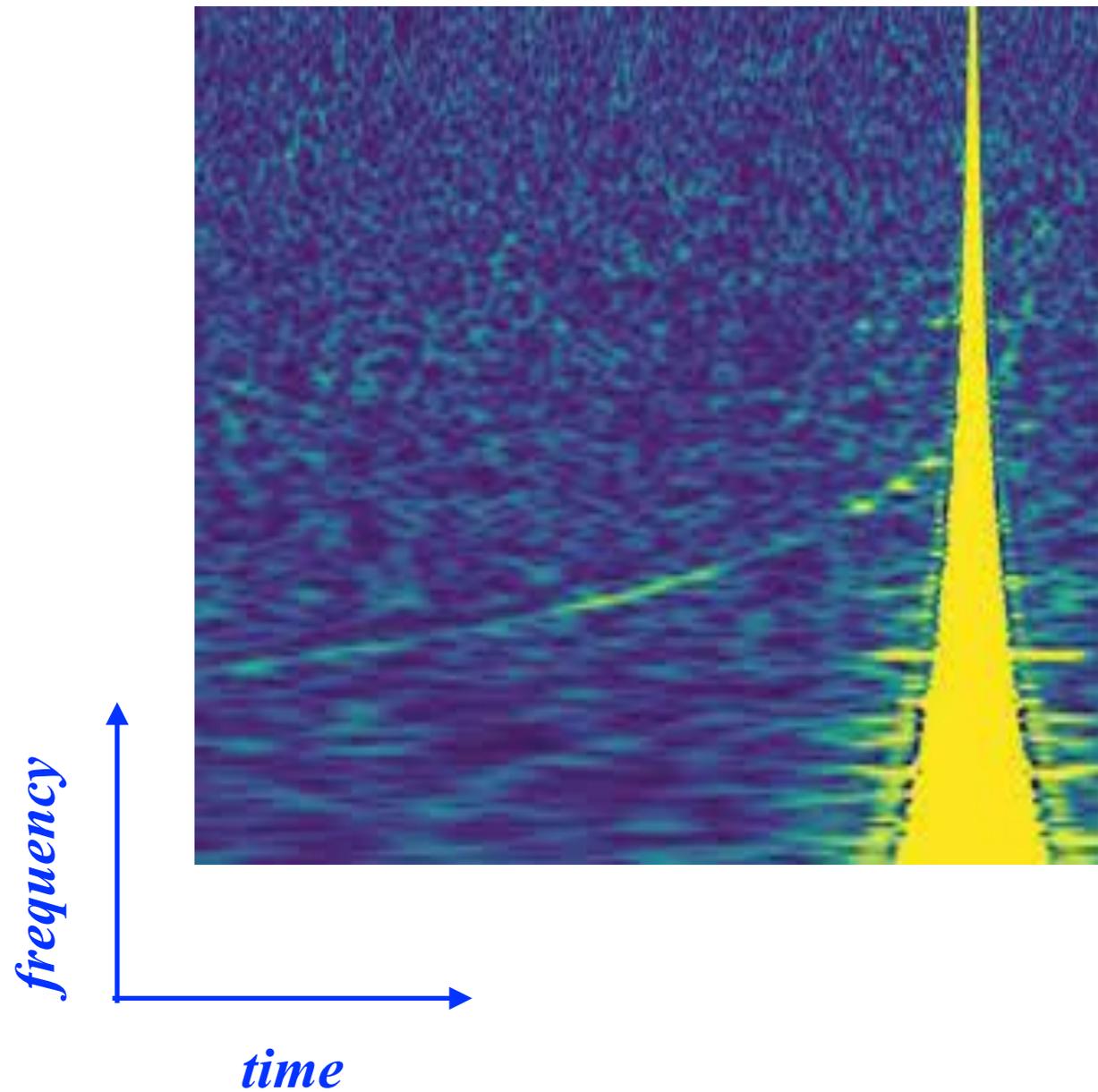


Time-Frequency Analysis



GW170817+glitch

GW170817+glitch removed



Abbott et al, PRL 116, 061102 (2016)



Time-Frequency Analysis



Event	m_1/M_\odot	m_2/M_\odot	M/M_\odot	χ_{eff}	M_f/M_\odot	a_f	$E_{\text{rad}}/(M_\odot c^2)$	$\ell_{\text{peak}}/(\text{erg s}^{-1})$	d_L/Mpc	z	$\Delta\Omega/\text{deg}^2$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4} \times 10^{56}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	180
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$	$0.04^{+0.28}_{-0.19}$	$35.7^{+9.9}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.5^{+0.5}_{-0.5}$	$3.2^{+0.8}_{-1.7} \times 10^{56}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$	1555
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7} \times 10^{56}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$	$-0.04^{+0.17}_{-0.20}$	$49.1^{+5.2}_{-3.9}$	$0.66^{+0.08}_{-0.10}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-0.9} \times 10^{56}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$	924
GW170608	$10.0^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.0^{+0.2}_{-0.2}$	$0.02^{+0.19}_{-0.07}$	$17.8^{+3.2}_{-0.7}$	$0.60^{+0.04}_{-0.04}$	$0.0^{+0.05}_{-0.1}$	$2.5^{+0.4}_{-1.3} \times 10^{56}$	220^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	206
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$	$0.36^{+0.21}_{-0.25}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5} \times 10^{56}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$	1033
GW170809	$33.2^{+8.7}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$23.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$	$30.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.3^{+0.6}_{-0.9} \times 10^{56}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$	340
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.11}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5} \times 10^{56}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1 \times 10^{56}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7} \times 10^{56}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$3.3^{+0.9}_{-0.8}$	$3.6^{+0.6}_{-0.9} \times 10^{56}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$	1651

✳ GW170729: most massive source yet detected

✳ higher significance in cWB than in any of the matched filter searches

Abbott et al, arXiv:1811.12907 (2018)



Matched Filtering



- GWs are analogous to 1D sound waves
- A matched filter is an optimal linear filter for signals buried in noise
- Very sensitive to phase evolution
- Need very accurate models of the waveform
- No solution to the 2-body problem in GR
- Require waveforms from analytical/numerical relativity



GstLAL, MBTA, PyCBC



- Matched filtering pipelines
- Search between 2 and 400-500 M_{\odot}
- Also search over spins < 0.999
- Pros
 - Very accurate if GR is correct
 - Good initial estimation of astrophysical parameters
- Cons
 - Assumes GR is correct
 - Requires accurate waveform models
 - Cross-correlating templates with the data is expensive



Morphology of a CBC signal



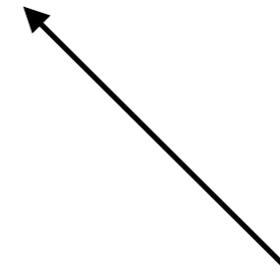
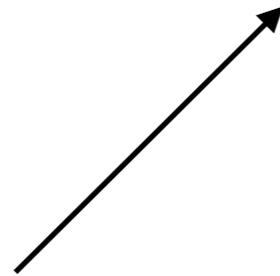
$$h_i(t) = h_+(t + \tau_i)F_i^+ + h_\times(t + \tau_i)F_i^\times$$



Morphology of a CBC signal



$$h_i(t) = h_+(t + \tau_i)F_i^+ + h_\times(t + \tau_i)F_i^\times$$



$$A_+ \cos \Phi(t)$$

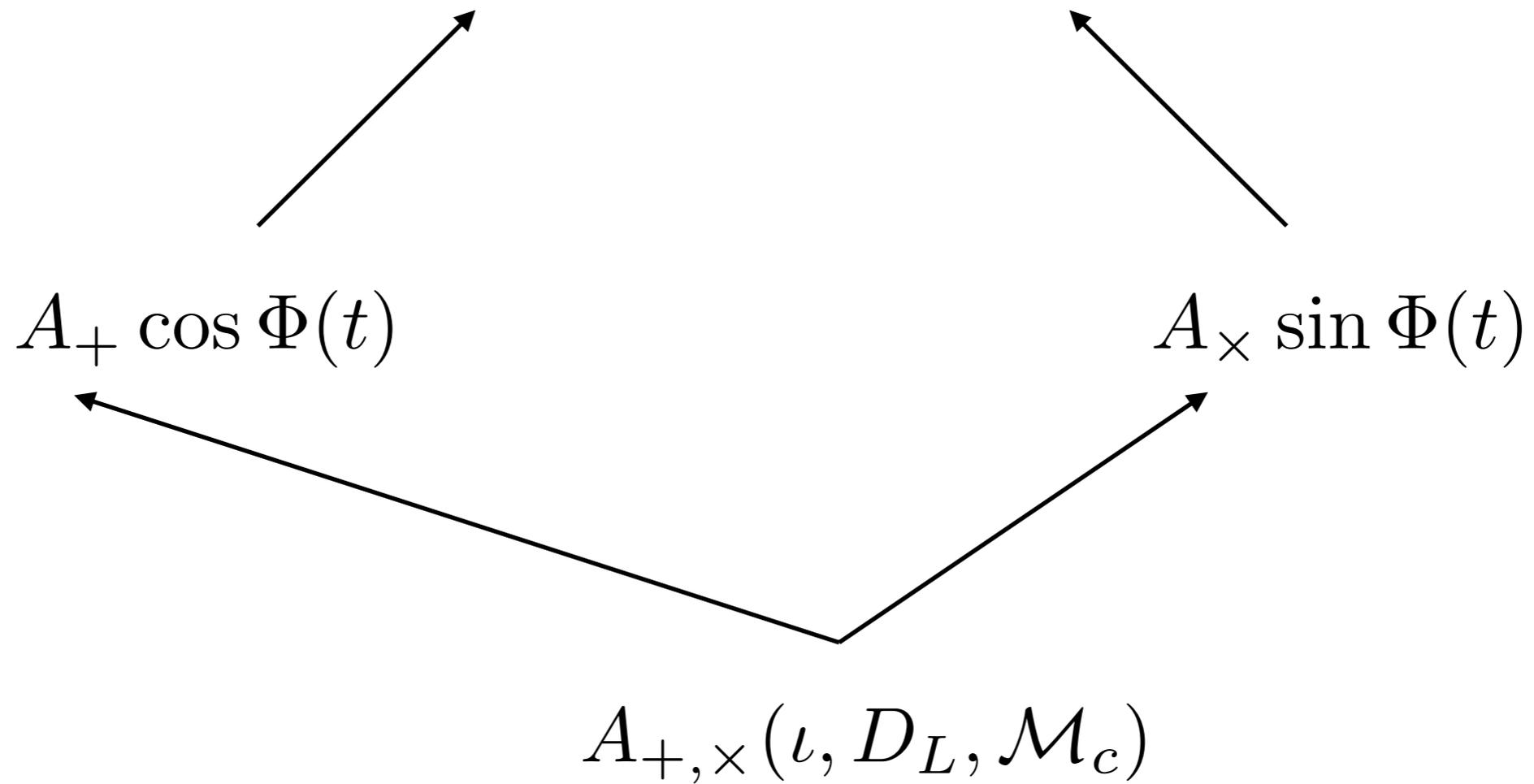
$$A_\times \sin \Phi(t)$$



Morphology of a CBC signal



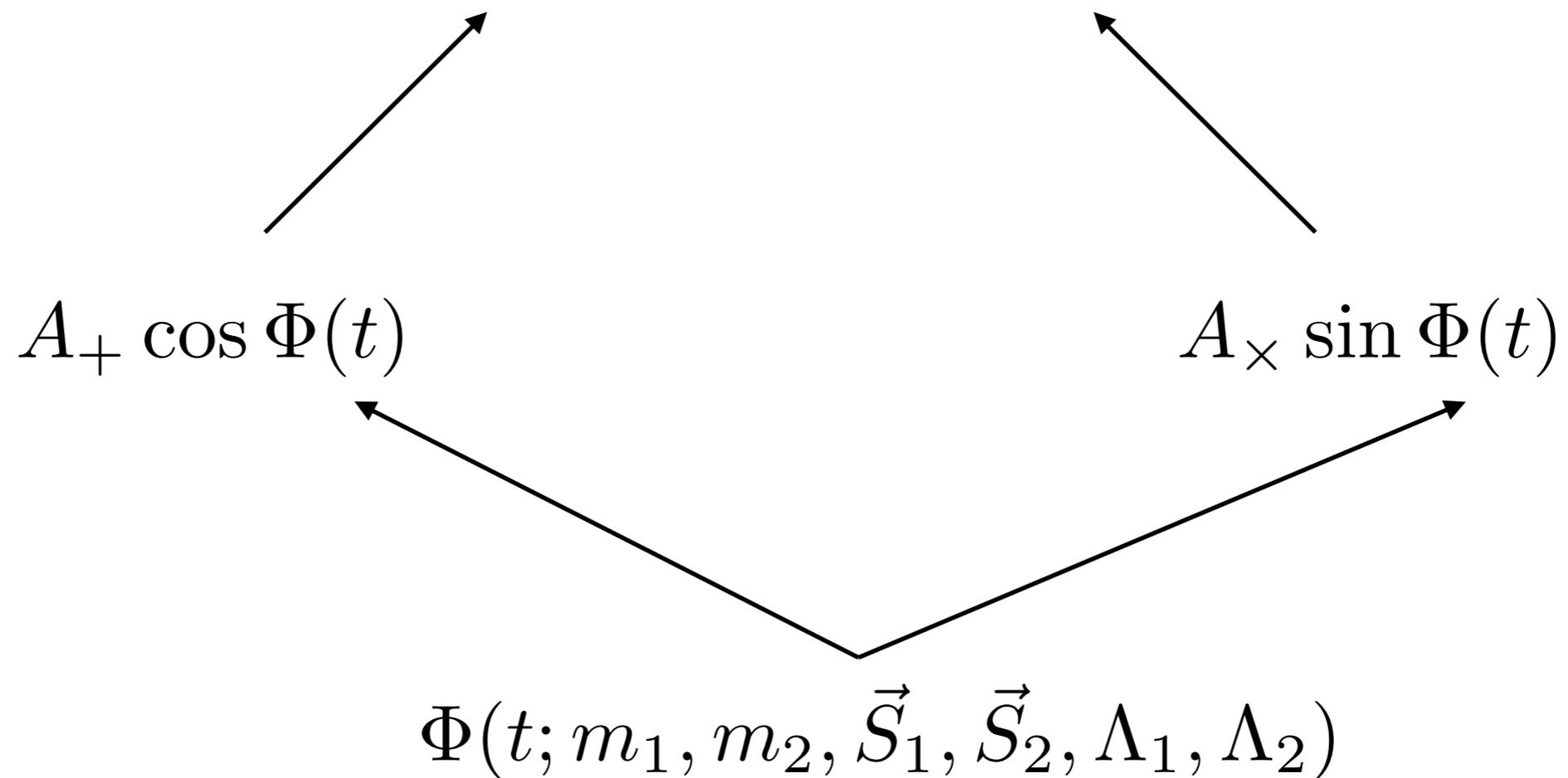
$$h_i(t) = h_+(t + \tau_i)F_i^+ + h_\times(t + \tau_i)F_i^\times$$



Morphology of a CBC signal



$$h_i(t) = h_+(t + \tau_i)F_i^+ + h_\times(t + \tau_i)F_i^\times$$



Morphology of a CBC signal

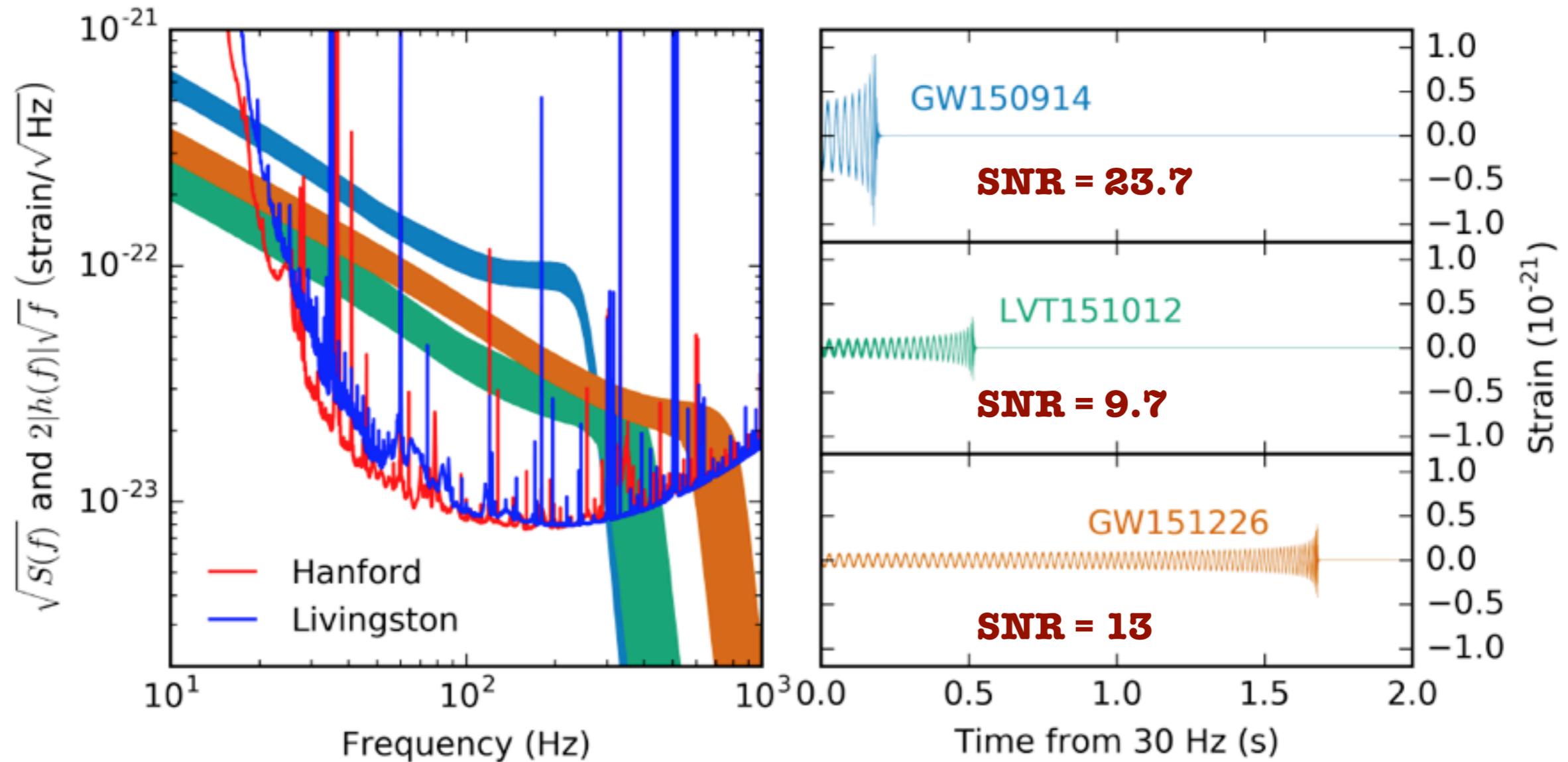


$$h_i(t) = h_+(t + \tau_i)F_i^+ + h_\times(t + \tau_i)F_i^\times$$

$$F^{+, \times}(\alpha, \delta, \psi)$$



ADOR - 1



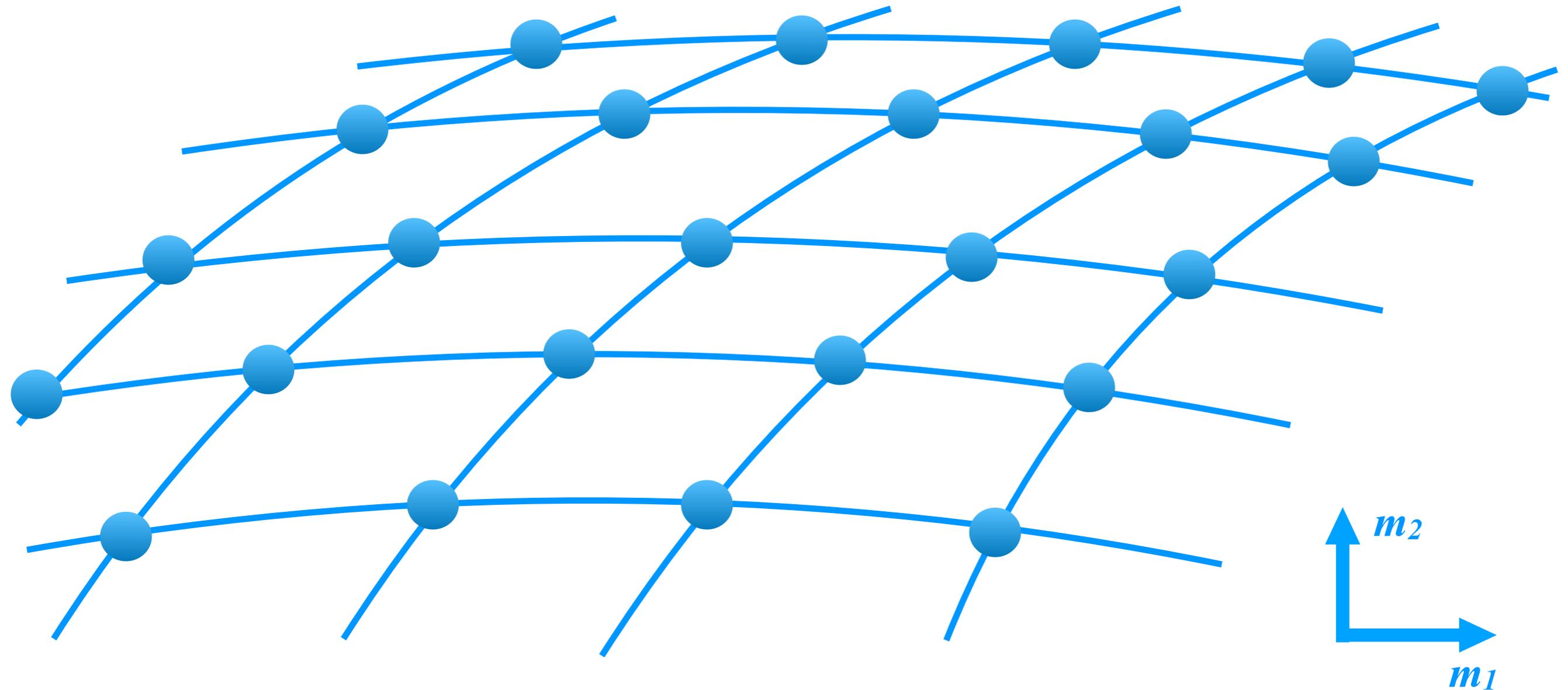
Abbott et al, PRX 6, 041015 (2016)



Matched filter search for GWs

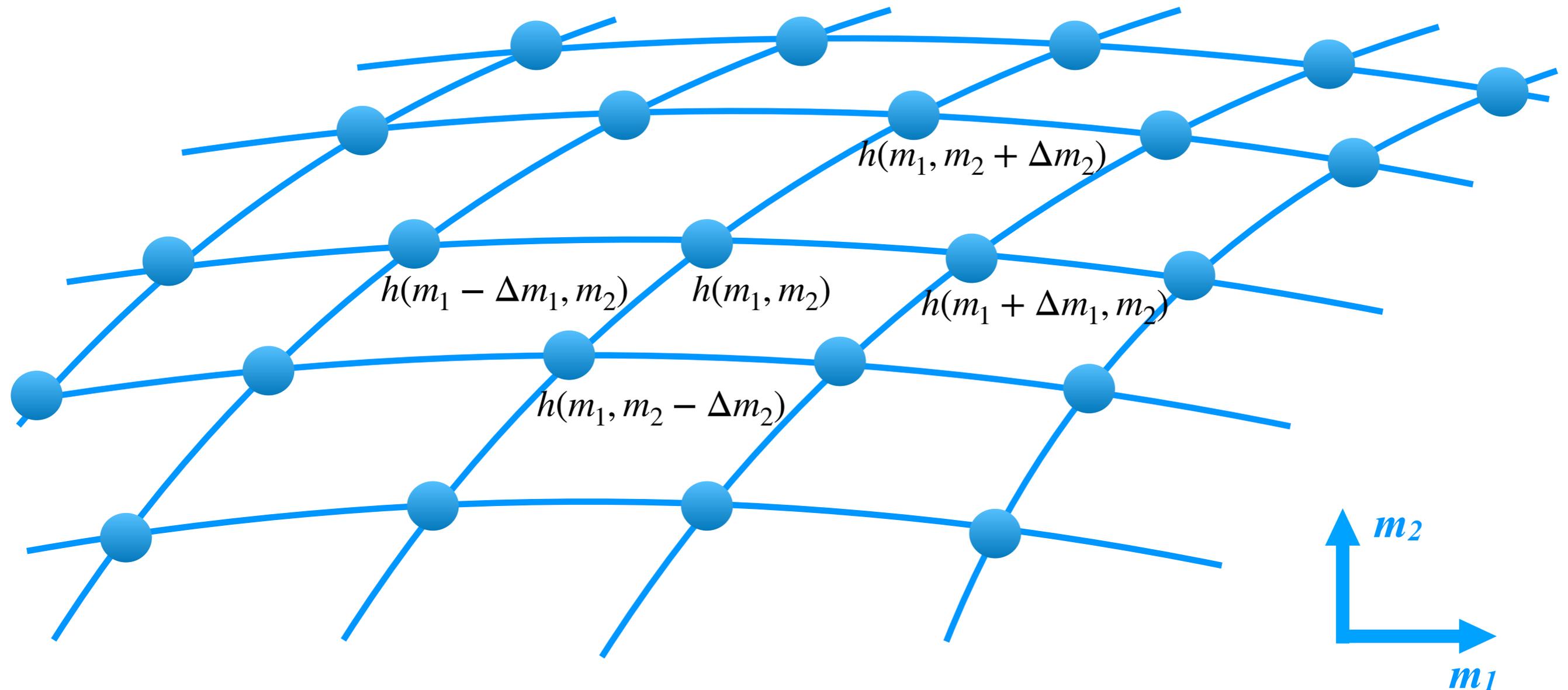


- Template Grid : assume 2D



Matched filter search for GWs

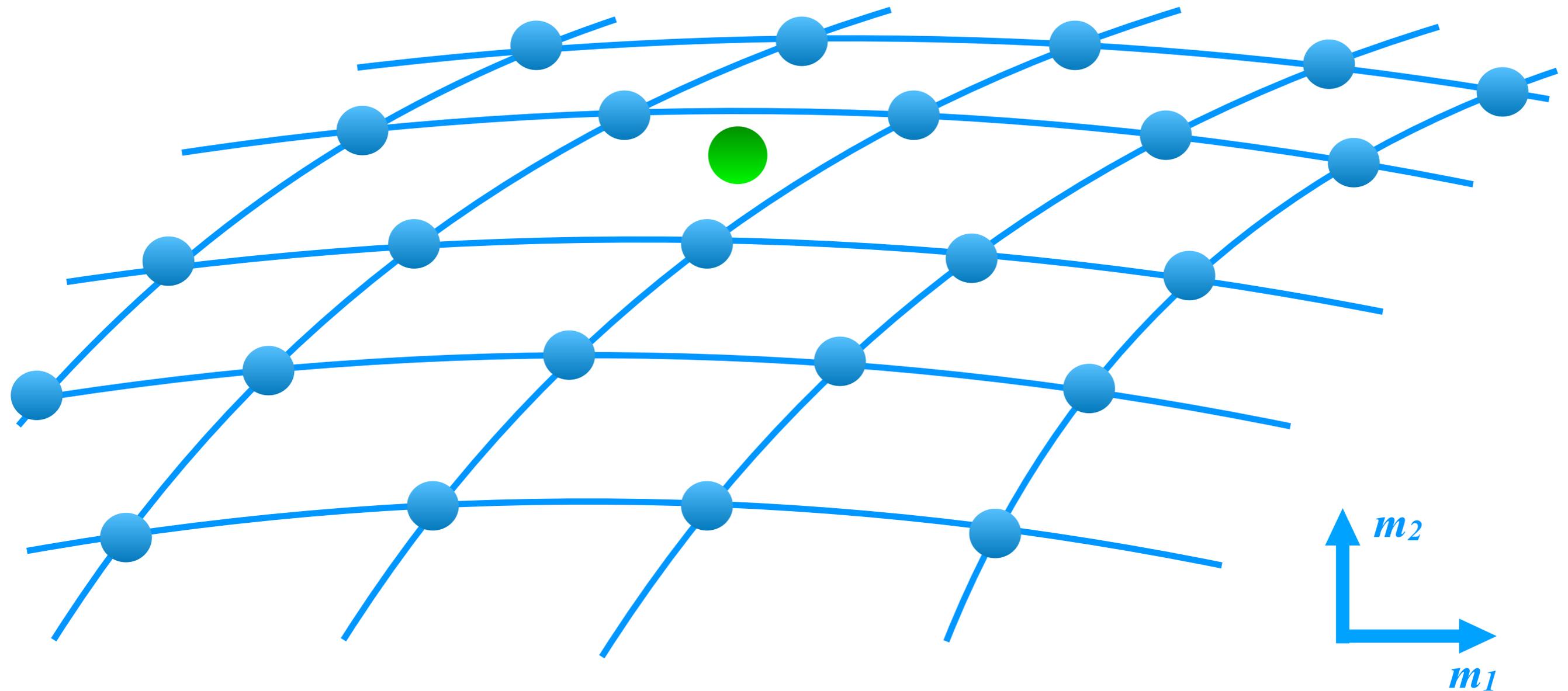
- Template Grid



Matched filter search for GWs



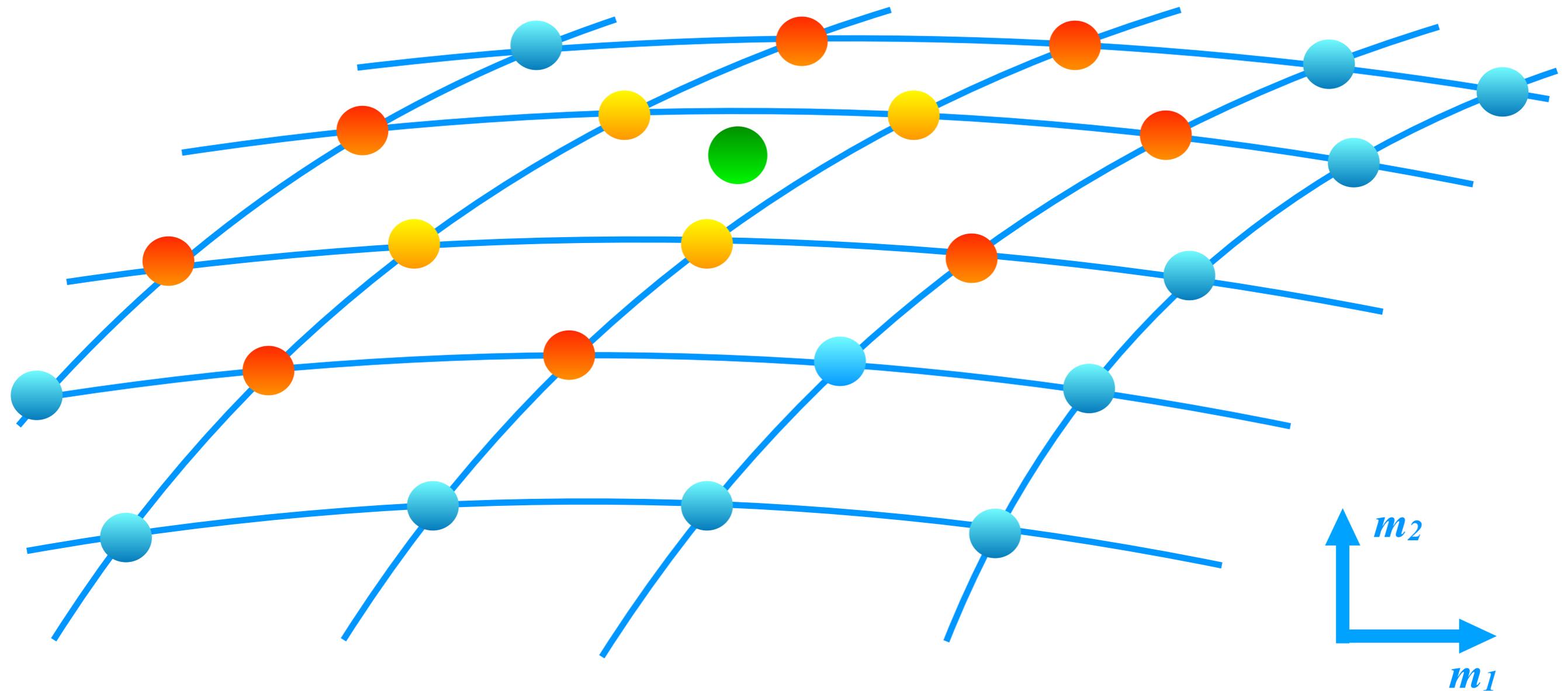
- Template Grid



Matched filter search for GWs



- Template Grid



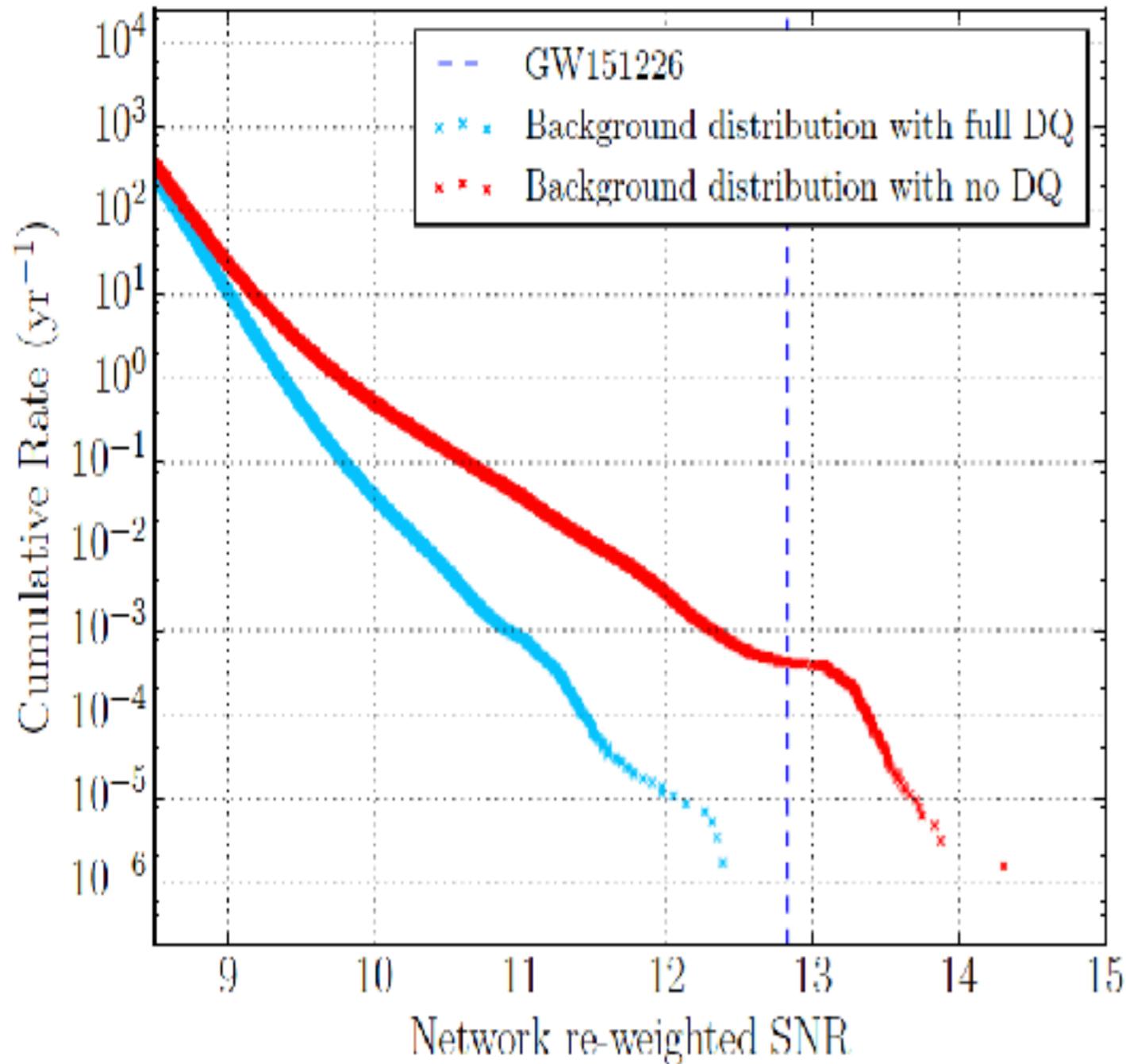
Detection Criteria



- cWB : $\text{FAR} < 1/1000$ years
- MF : $\text{SNR} > \text{threshold}$
 - $\text{FAR} < 1/100$ years (O1)
 - For O2, a trigger with a $\text{FAR} > 1/30$ days = 50% noise
 - $\text{FAR} < 1 / 30$ days and $p_{\text{astro}} > 50\%$ (O2)



Data Quality & Detector Characterisation



Detector Characterisation improved the FAR of GW151226 by a factor of 500: from 1/320 years to 1/183000 years

Also need to calibrate the time series: reduce the error in amplitude and phase measurement

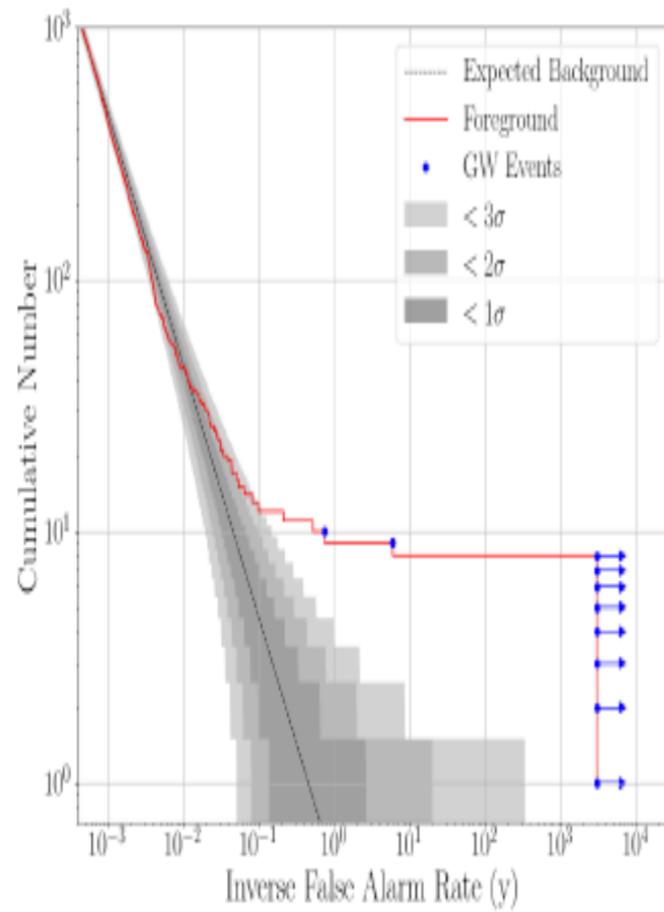
B. Abbott, CQG 2018



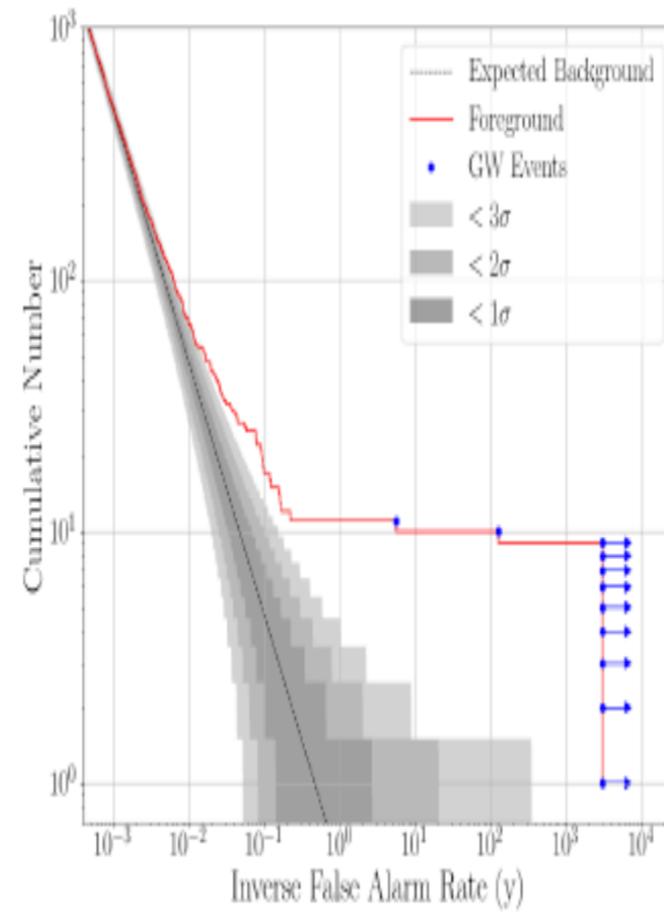
False Alarm Rates



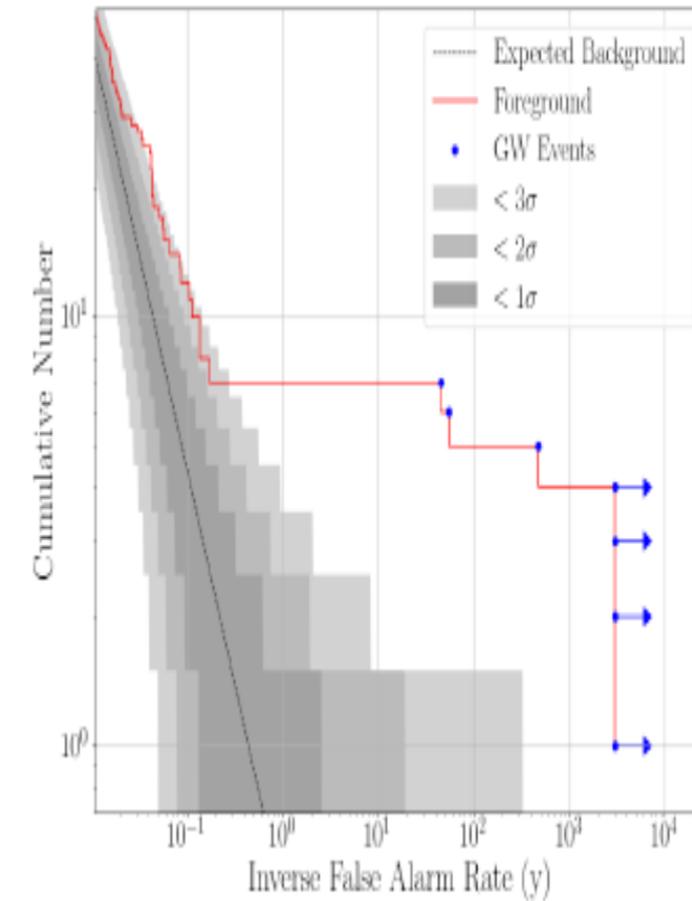
PyCBC



GstLAL



cWB



Abbott et al, arXiv:1811.12907 (2018)



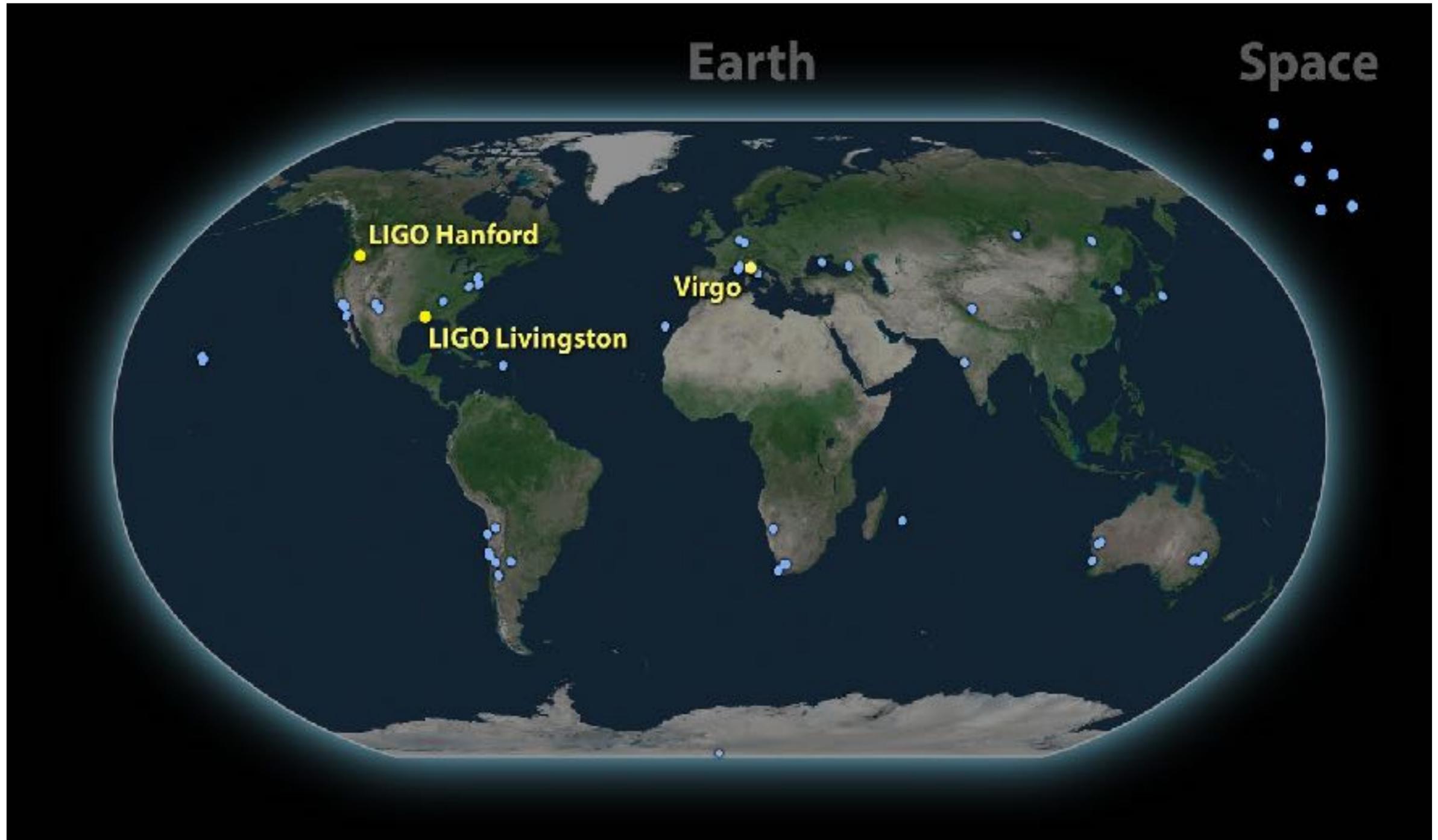
Low-latency Analysis



- First alert goes out to the astronomical community within minutes
- Alert contains a 3D sky-map, an “EM-bright” flag, and for O3 a data quality statement
- Further circulars are sent as data comes in
- Final inference sky-maps and parameters not sent to external community



EM FOLLOW-UP



www.ligo.caltech.edu/images



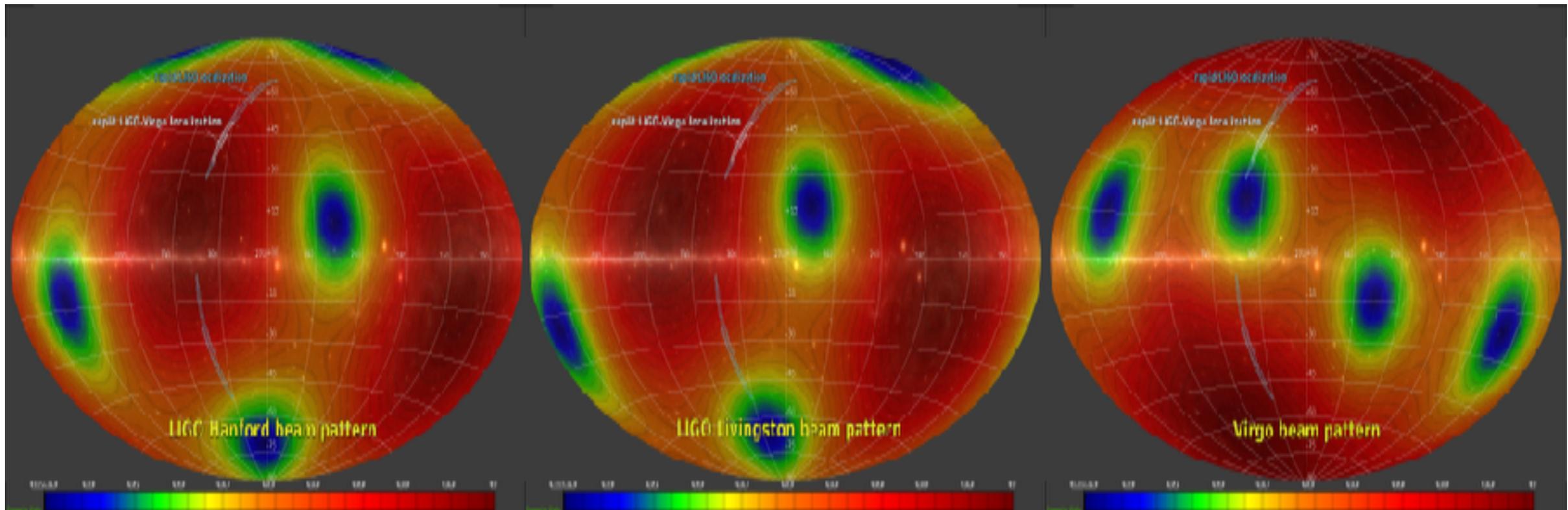
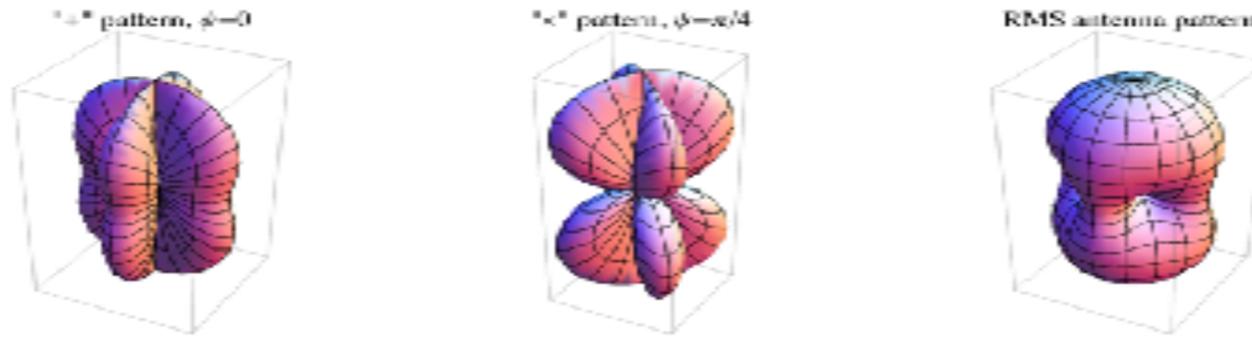
Journées Thématiques Ondes Gravitationnelles à Lyon, 12/02/2019

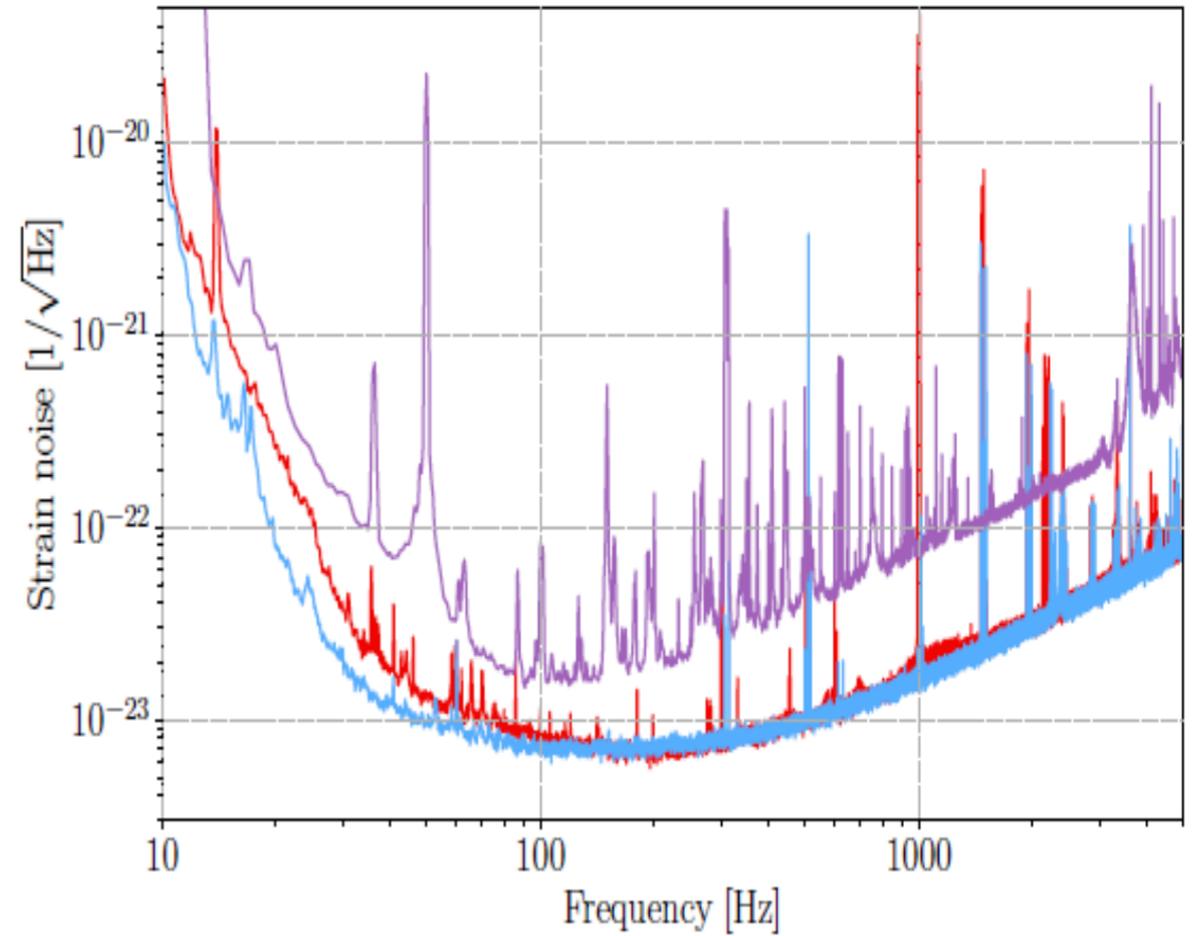
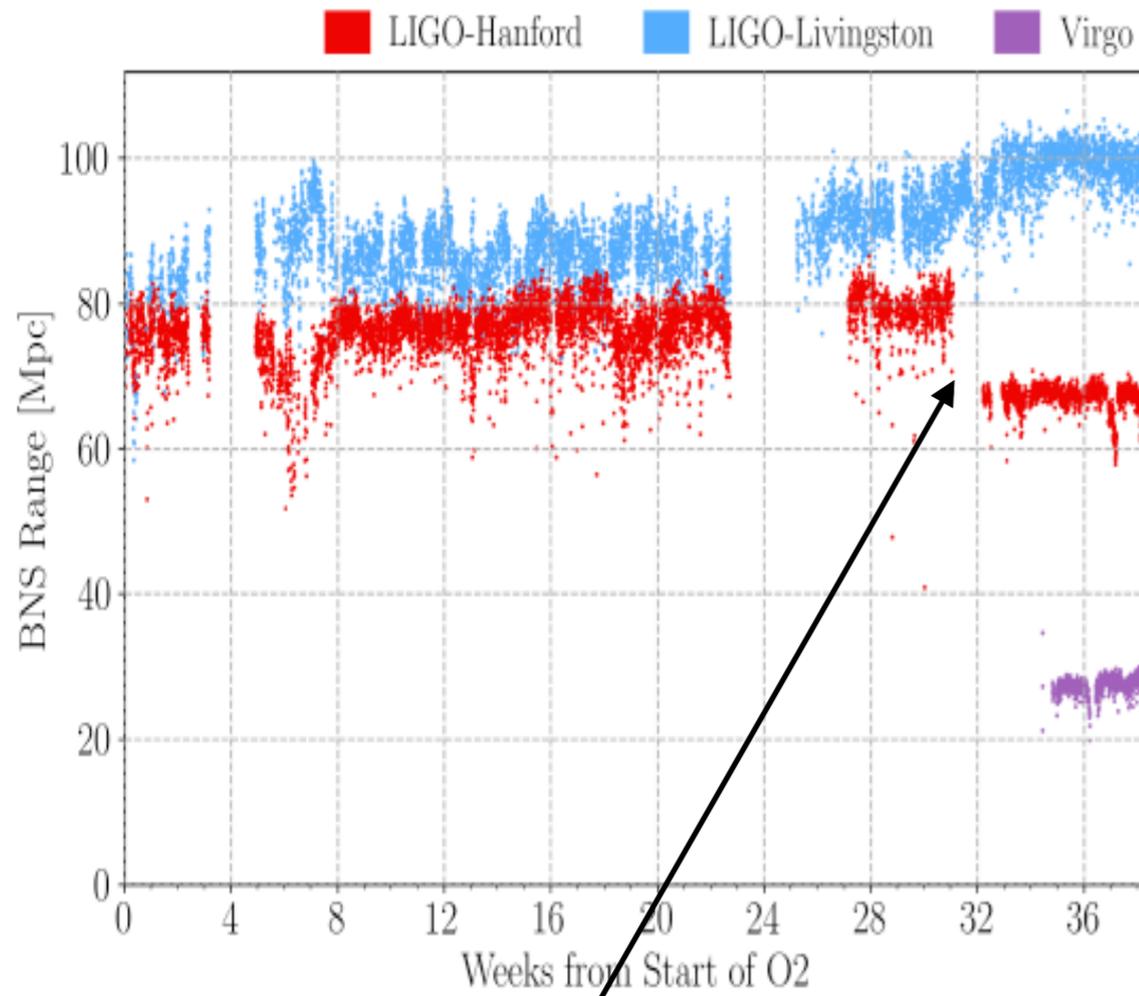


Sky Localisation

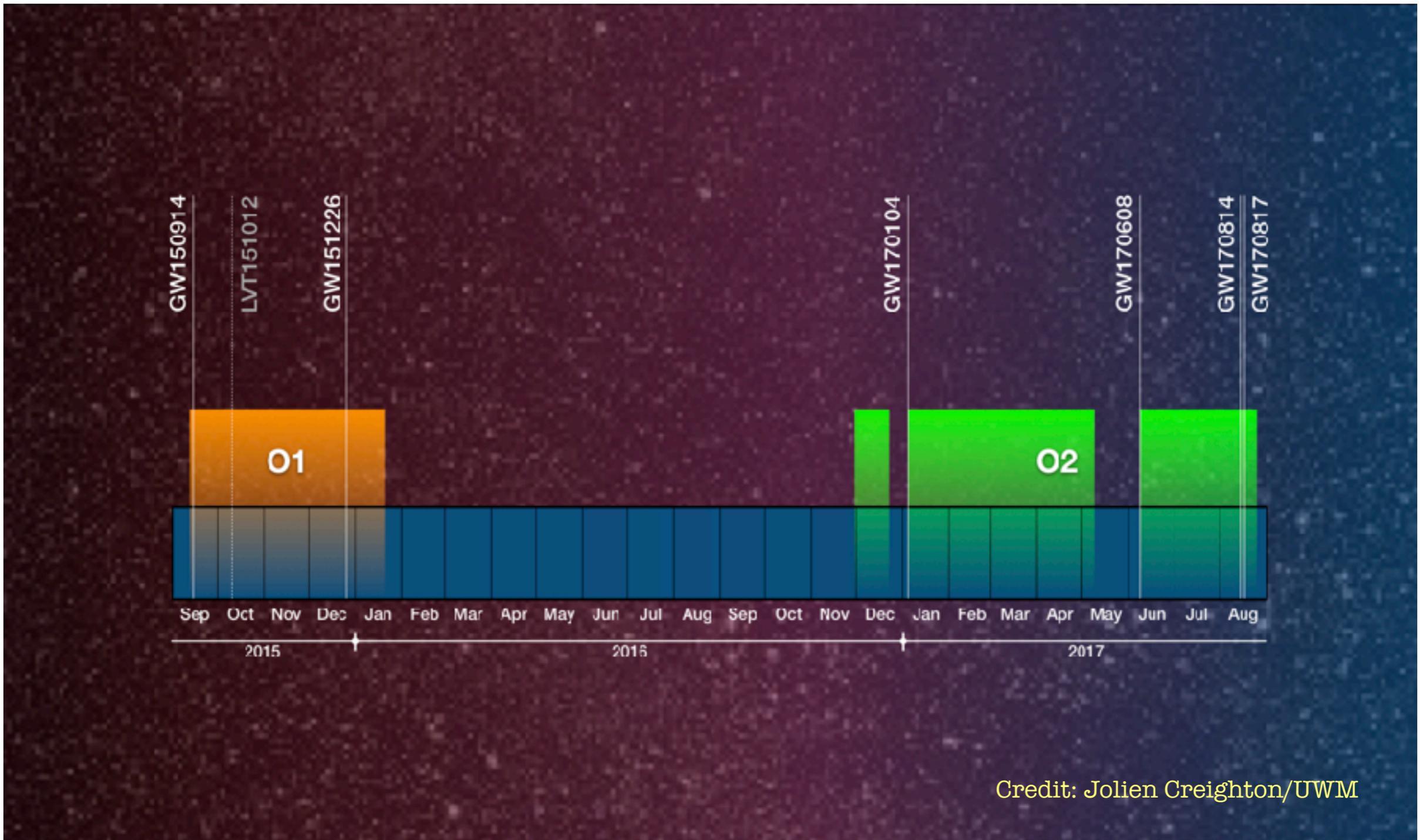


$$h_i(t) = h_+(t + \tau_i)F_i^+ + h_\times(t + \tau_i)F_i^\times$$



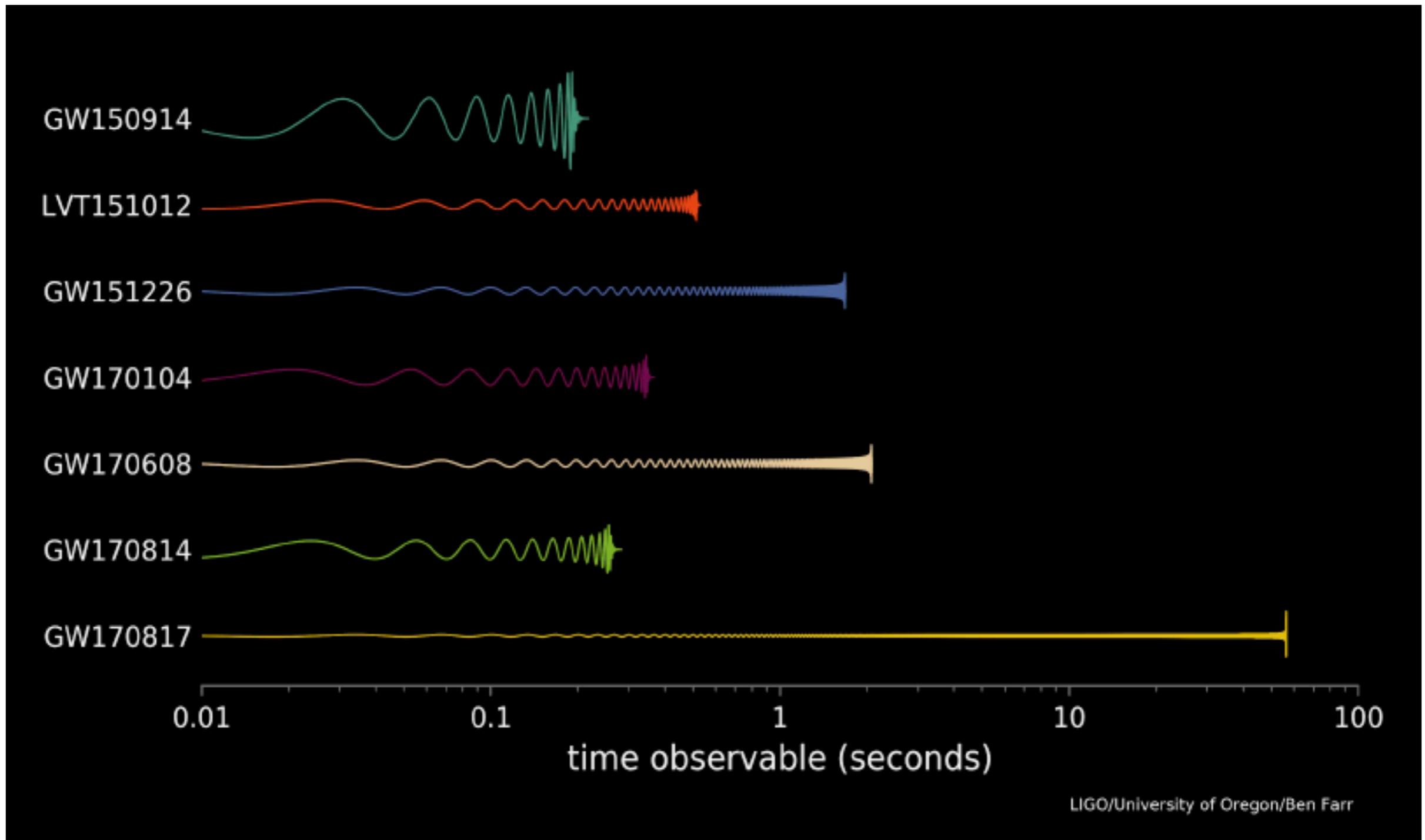


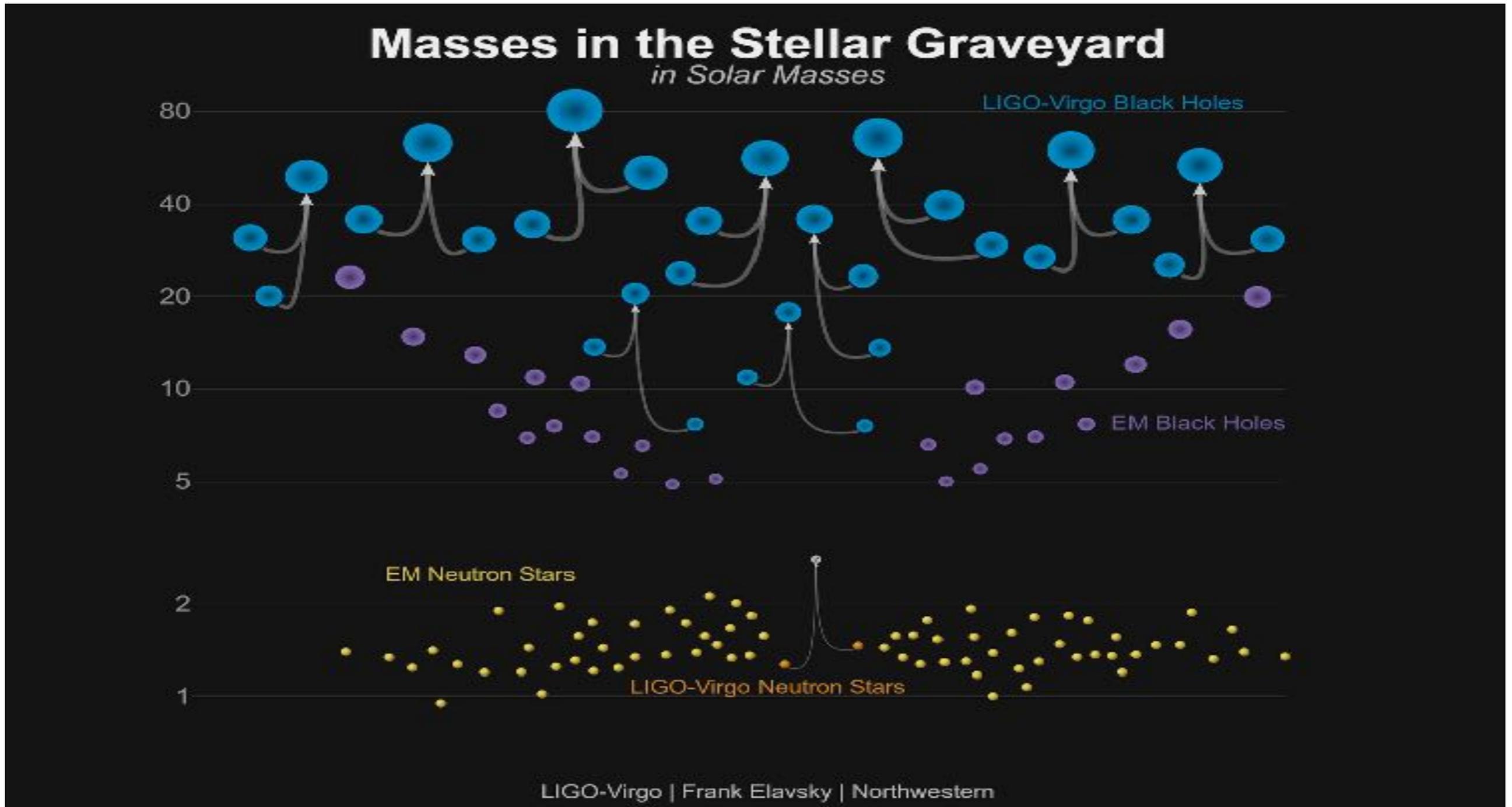
Earthquake in Montana (~700kms)



Credit: Jolien Creighton/UWM

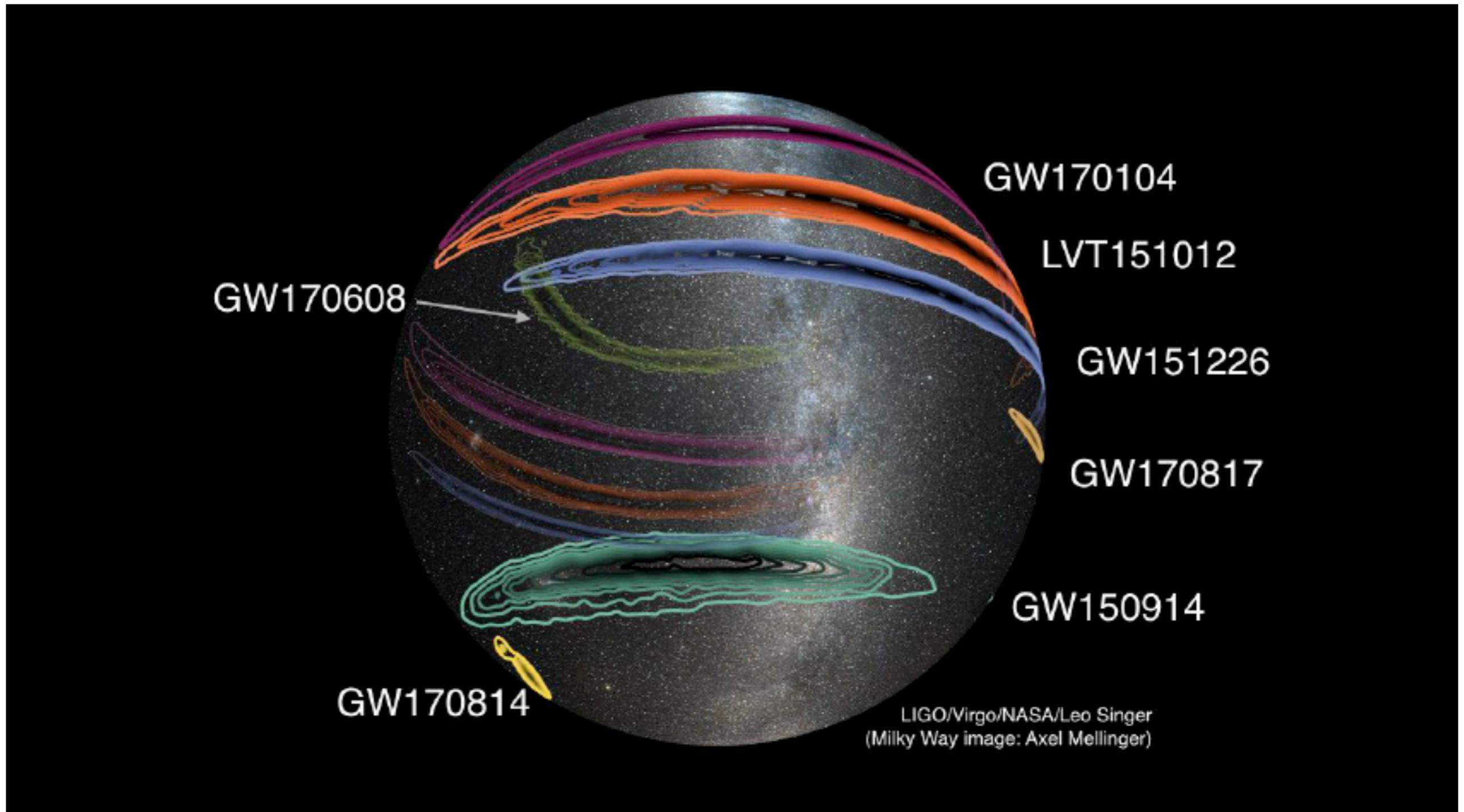
WAS GW170817 A BNS?





www.ligo.caltech.edu/images





Why Bayesian Inference?



- 2 options
 - Frequentist
 - Bayesian
- GW mergers are one-time events
- Perfectly suited for Bayesian analysis



What is Bayesian Inference?



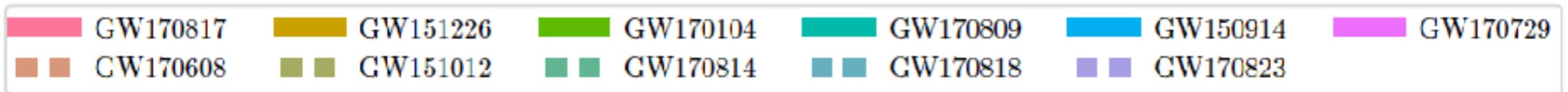
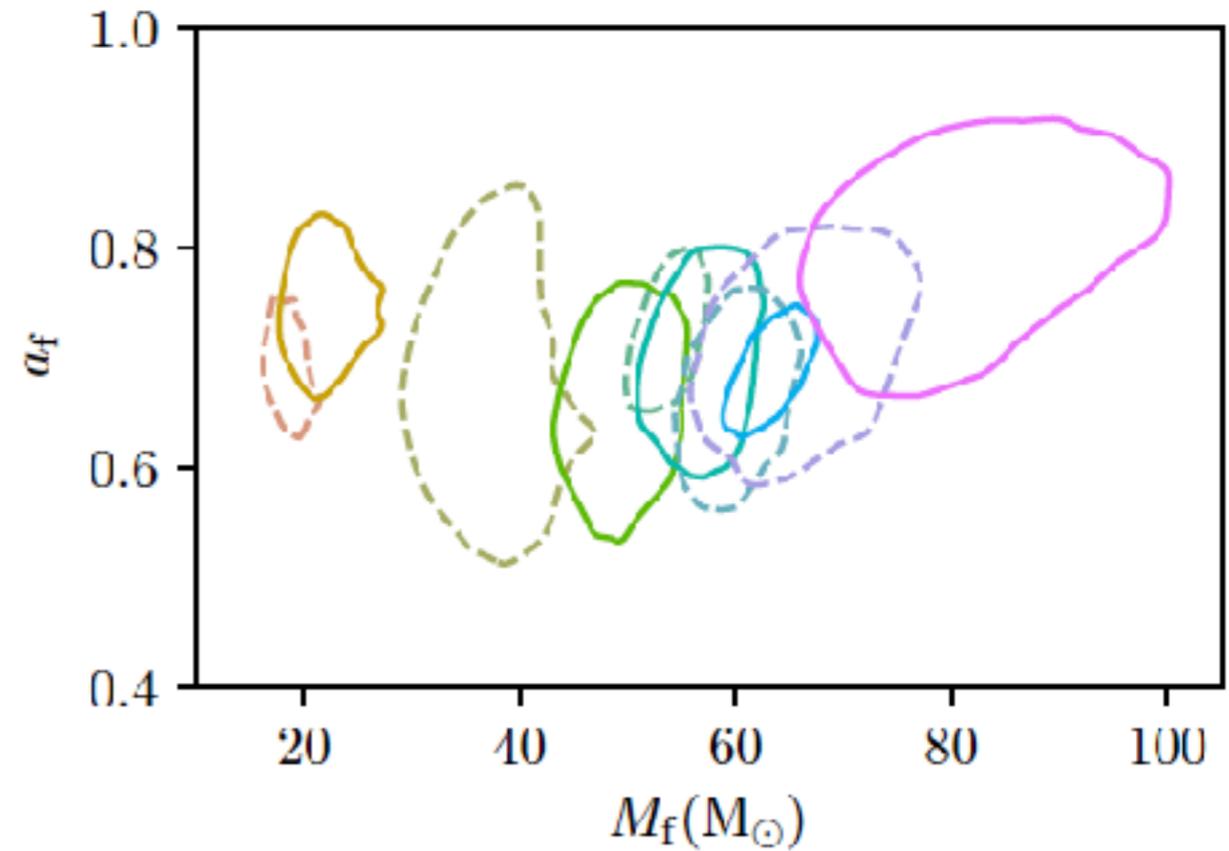
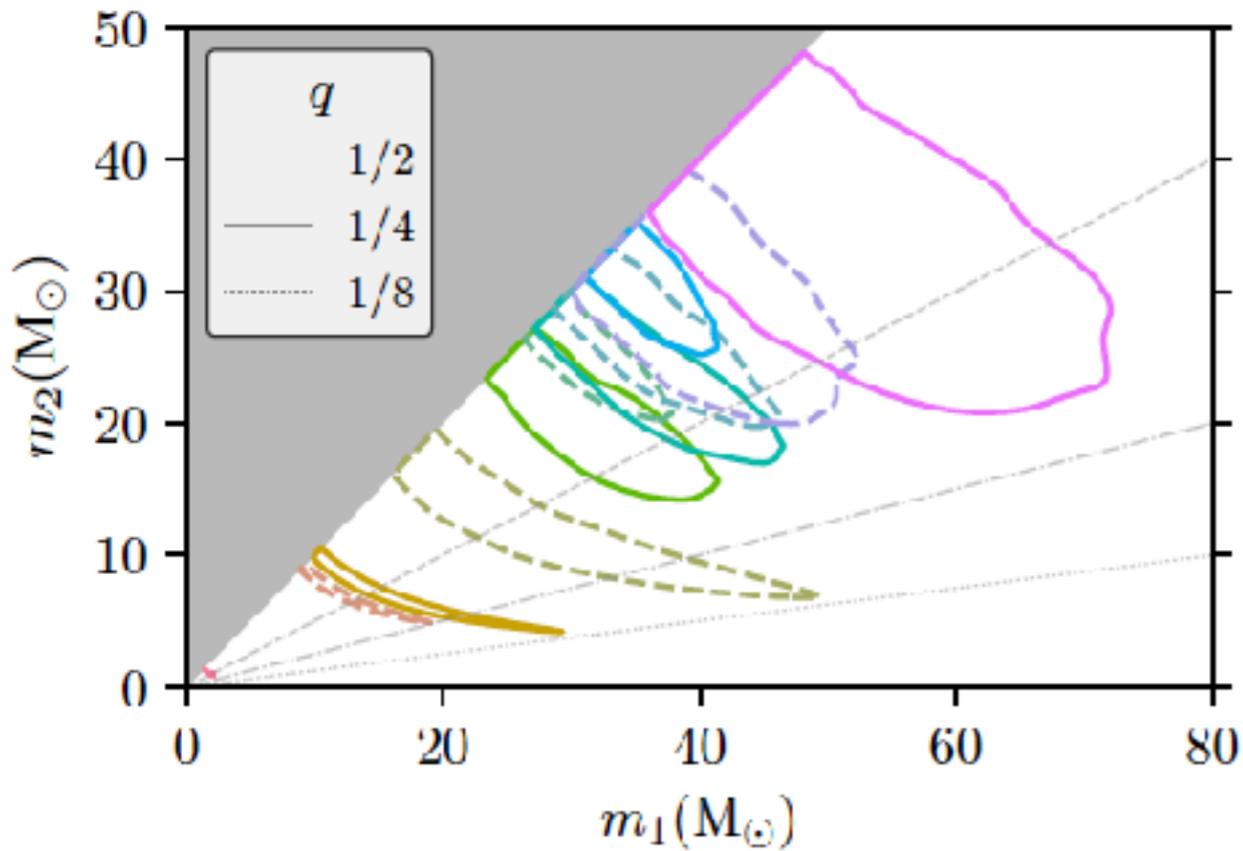
- Bayes' theorem $p(\lambda^\mu | s) = \frac{\pi(\lambda^\mu)p(s|\lambda^\mu)}{p(s)}$
- No single point estimate
- Estimate the posterior distribution
- Define the likelihood $\mathcal{L}(\lambda^\mu) = \exp \left[-\frac{1}{2} \langle s - h(\lambda^\mu) | s - h(\lambda^\mu) \rangle \right]$



How do we do Bayesian Inference?

- Stochastic samplers
 - Markov chain Monte Carlo
 - sequential sampler
 - suffers from high autocorrelations
 - guaranteed to converge to target density, but unclear how long it will take
 - can get stuck on local minima
 - Nested sampling
 - population based sampler
 - wider exploration
 - can get stuck on local minima
 - requires a large number of sample solutions

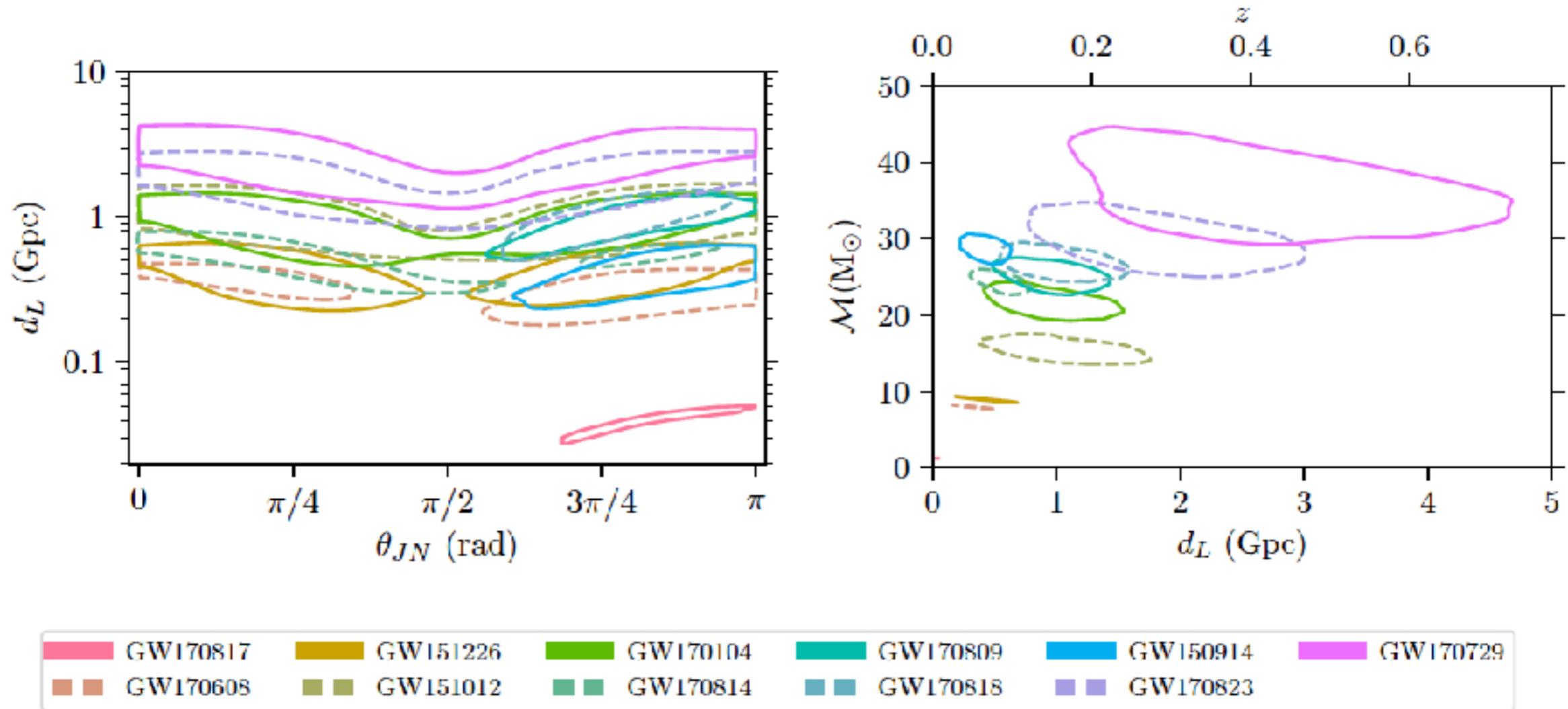
Exploring the Posterior



Abbott et al, arXiv:1811.12907 (2018)



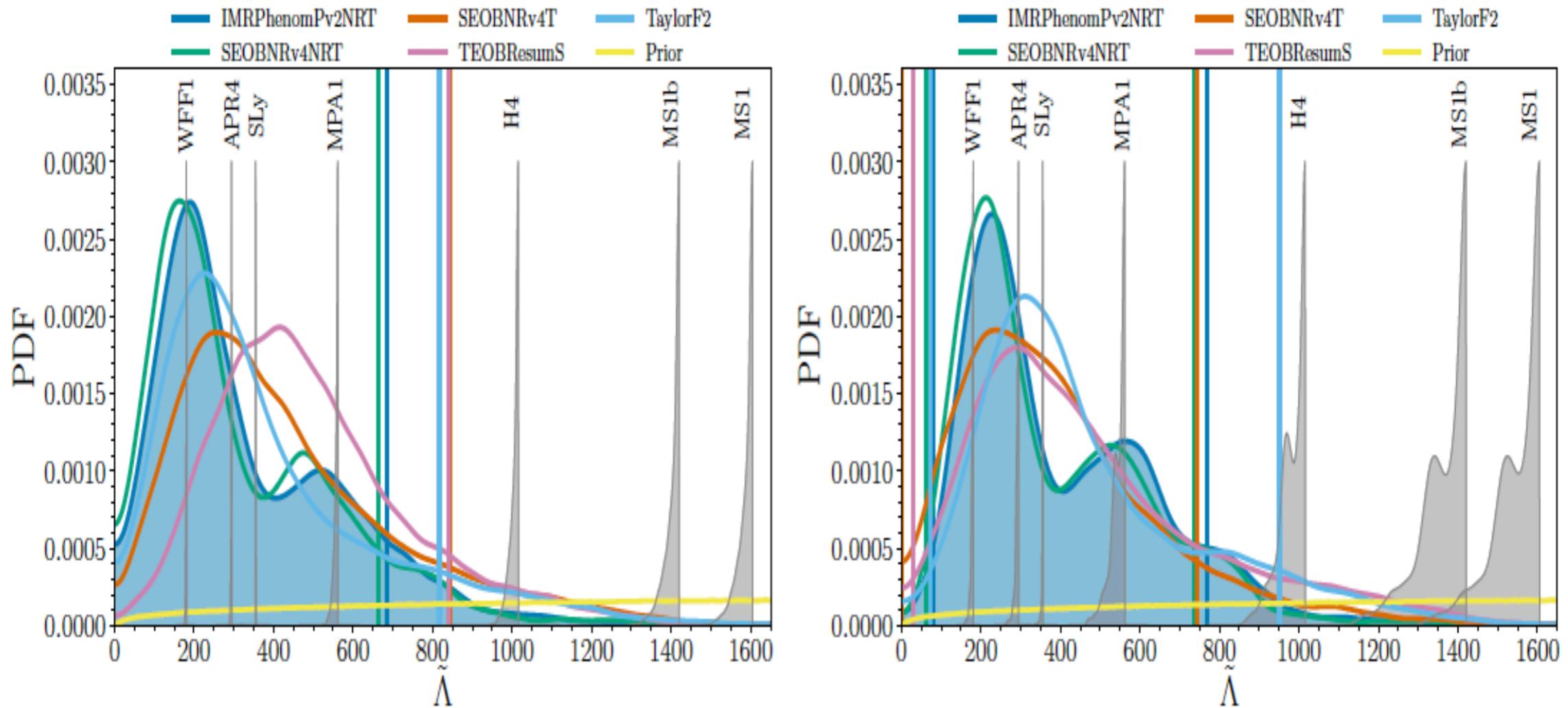
Exploring the Posterior



Abbott et al, arXiv:1811.12907 (2018)



Exploring the Posterior



Abbott et al, arXiv:1811.12907 (2018)



Bayesian Inference



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GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$	$0.36^{+0.21}_{-0.25}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5} \times 10^{56}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$	1033
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9} \times 10^{56}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$	340
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GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7} \times 10^{56}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$3.3^{+0.9}_{-0.8}$	$3.6^{+0.6}_{-0.9} \times 10^{56}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$	1651

Abbott et al, arXiv:1811.12907 (2018)



Cost of Parameter Estimation



- MCMC
 - requires around 10^7 iterations, i.e. 10^7 waveform generations and likelihood calculations
 - takes days (BBH) to weeks (BNS)
 - relatively small number of statistically independent samples
- Nested sampling
 - requires evolving 1024 (2048) solutions for N generations, i.e. $N \times 1024(2048)$ waveform generations and likelihood calculations
 - again takes days (BBH) to weeks (BNS)
 - larger number of statistically independent samples
 - Evidence



Conclusion



- GW astronomy is now open
- However, it is difficult to do
- Need further development in detector characterisation, waveform modelling, search algorithms, inference samplers etc.
- Need to be more efficient
- Computational cost will explode as event rates increase

