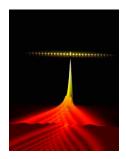
Relaxion dark matter & the precision front

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Weizmann Inst.



New Paths towards New Physics, 2019 CNRS/LAPTh-Annecy mini-workshop

Outline

Intro: brief motivation for light physics & dark matter (DM).

Relaxion & coherent DM, \w dynamical misalignment.

Prelim: probing heavyish-light-scalar/relaxion DM \w clocks.

Very prelim: probing scalar-stars \w clocks (earth & space).

Conclusions.

Introduction

New particles/forces must exists as Standard Model can't account for baryon asym., dark matter (DM), Higgs-hierarchy, etc.

Conventional particle-TeV-physics wisdom is challenged by the null results of the LHC & DM experiments.

New paradigms recently proposed suggest alternative solutions.

"Cosmic attractors", "dynamical relaxation", "N-naturalness", "relating the weak-scale to the CC" & "inflating the Weak scale".

Presence of light scalar/s is common to most.

Benchmarking-relaxion

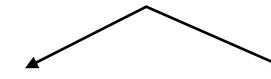
Relaxion-models => interesting & concrete: solves the hier' strong CP problems in a simple & computable way, \w definite Lagrangian.

Graham, Kaplan & Rajendran (15); Hook, Marques-Tavares; Gupta, Komargodski, GP & Ubaldi (16); Davidi, Gupta, GP, Redigolo & Shalit; Gupta; Nelson & Prescod-Weinstein (17)

The relaxion is light because it is axion-like particle but due to
 CP violation it mixes with the Higgs => has scalar interactions.

it interacts \w atoms

Flacke, Frugiuele, Fuchs, Gupta & GP; Choi & Im (16)

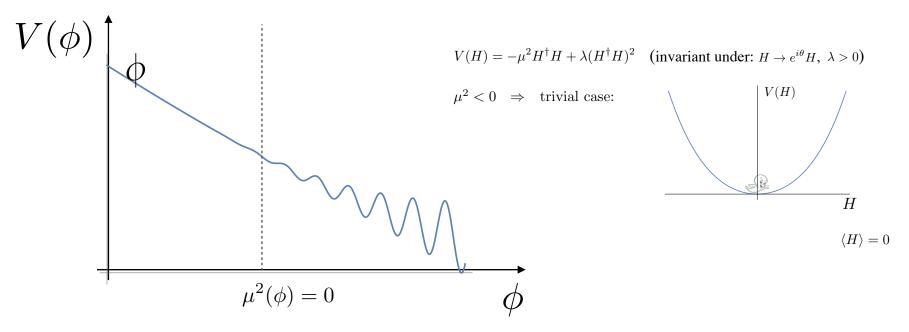


its variation <=> variation of all constants

Use as benchmark to compute sensitivity of variety of exp' & compare them to scalar-new-physics.

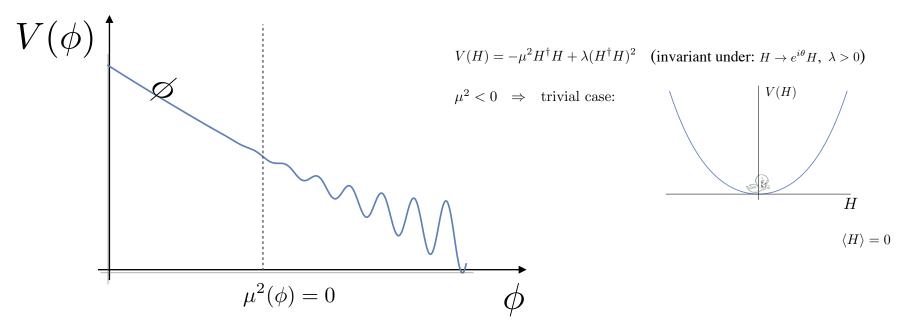
Graham, Kaplan & Rajendran (15)

- A dynamical solution/amelioration of the Higgs fine-tuning problem: $\mu^2(\phi)$
 - (*i*) Add a scalar (relaxion) Higgs dependent mass:
 - (*ii*) ϕ roles till μ^2 changes sign $\Rightarrow \langle H \rangle \neq 0 \Rightarrow$ stops rolling.



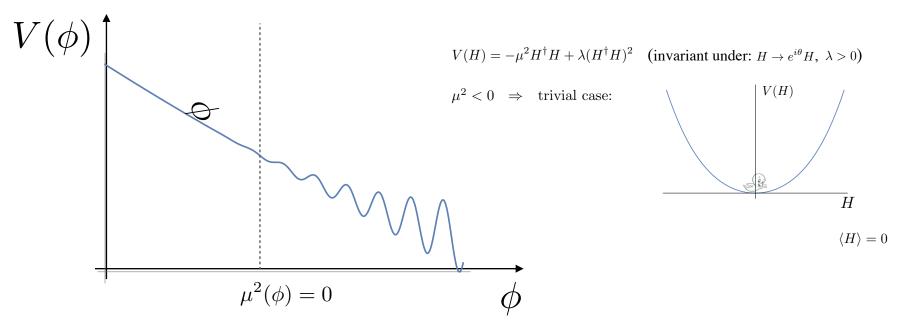
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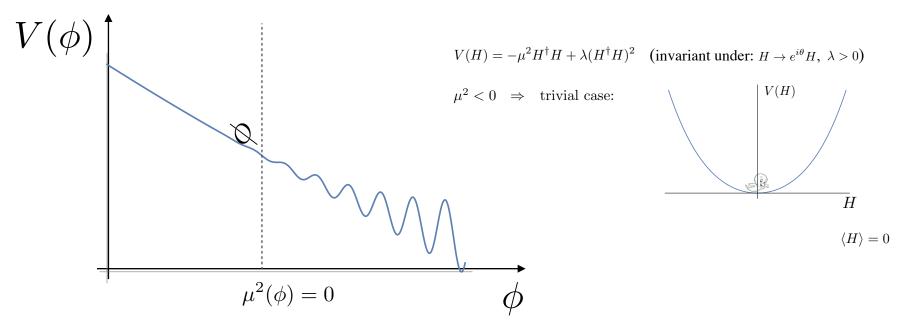
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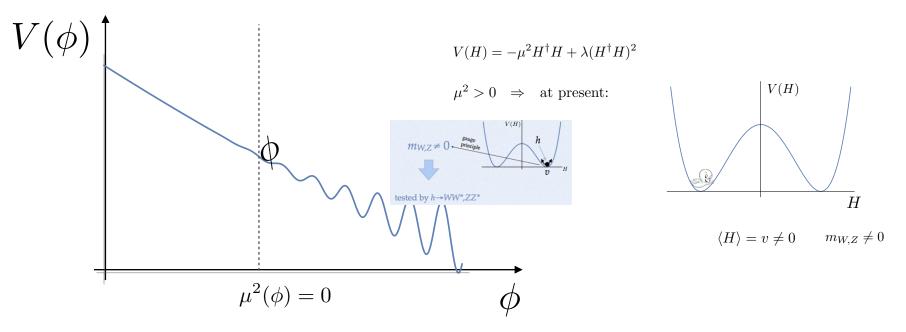
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A dynamical solution/amelioration of the Higgs fine-tuning problem: $\mu^2(\phi)$ (*i*) Add a scalar (relaxion) Higgs dependent mass: $(\Lambda^2 - g^2 \phi^2) H^{\dagger} H$ (*ii*) ϕ roles till μ^2 changes sign $\Rightarrow \langle H \rangle \neq 0 \Rightarrow$ stops rolling. $V(\phi)$ $V(H) = -\mu^2 H^{\dagger} H + \lambda (H^{\dagger} H)^2$ $\mu^2 > 0 \Rightarrow \text{at present:}$ V(H)evolution ends H $\langle H \rangle = \underbrace{v}_{m_{wz} \neq 0} \neq \underbrace{0}_{m_{wz} \neq 0}$ $\mu^{2}(\phi) = 0$

Graham, Kaplan & Rajendran (15)

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 $\underbrace{\mu^2(\phi)}_{\left(\Lambda^2 - g^2\phi^2\right)} H^{\dagger}H$

(*ii*) ϕ roles till μ^2 changes sign $\Rightarrow \langle H \rangle \neq 0 \Rightarrow$ stops rolling.

Focus shifts from TeV Higgs dynamics to relaxion, which is light & weakly coupled ...

Motivation to hunt & compare sensitivity to broad class of scalar-new-physics in:

If it is heavy, mass >> eV

(i) can be copiously produced & detected at colliders;

(ii) can affect cosmological history + astrophysics dynamics.

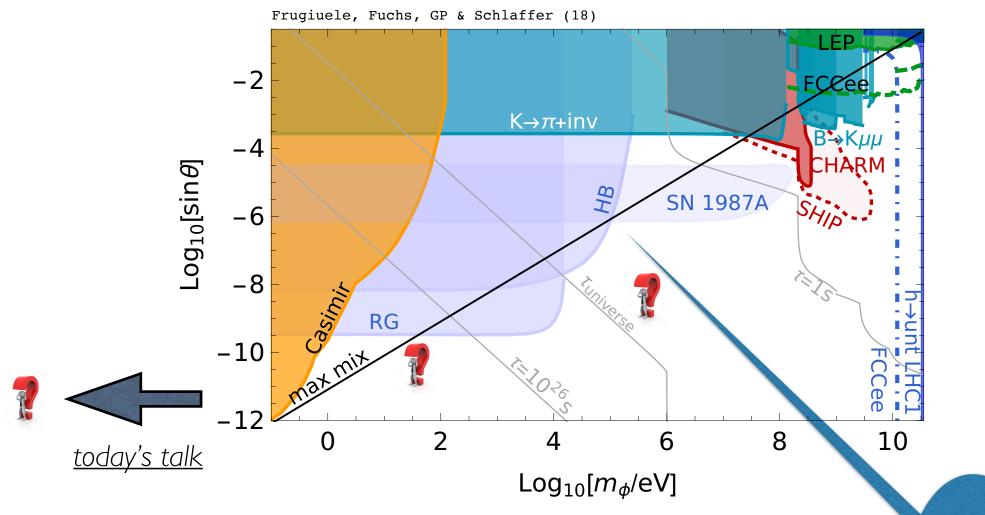
Flacke, Frugiuele, Fuchs, Gupta & GP; Choi & Im (16); Frugiuele, Fuchs, GP & Schlaffer; Fonseca, Morgante & Servant; Fonseca & Morgante(18)

<u>If it is ultra-light, mass < eV (today's talk)</u>

(i) virtual processes searching for long-range "Yukawa" force;

(ii) time-depend. background if relaxion/scalar = dark matter (DM):
 (a) for average DM density; (b) for relaxion stars.

Hunting a "heavy" relaxion/scalar-portal



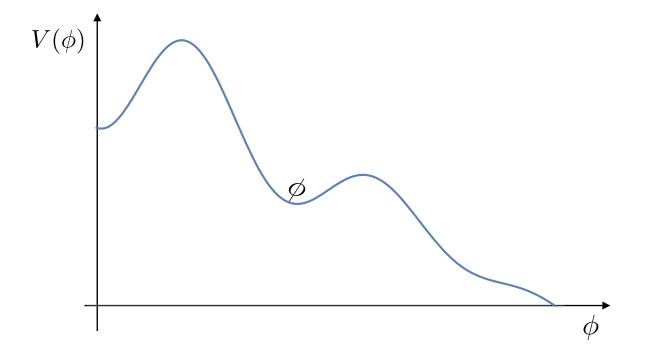
"Physical" region: below diagonal

Relaxion/scalar light dark matter

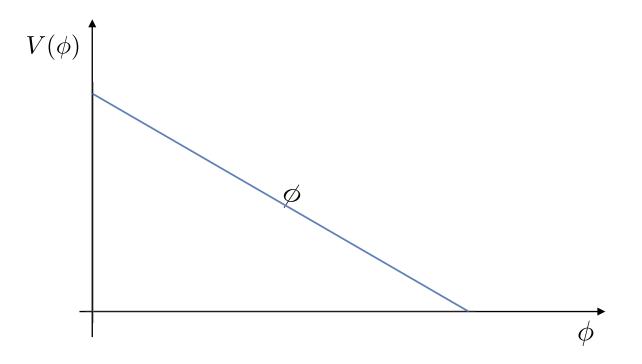
Banerjee, Kim & GP (18)

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Basic idea is similar to axion DM (but avoiding missalignment problem):

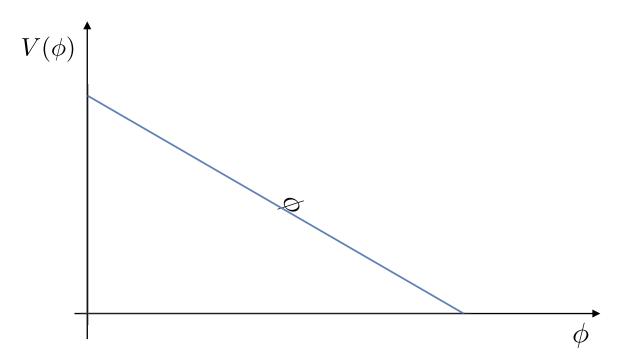


Basic idea is similar to axion DM (but avoiding missalignment problem): After reheating the wiggles disappear (sym' restoration):



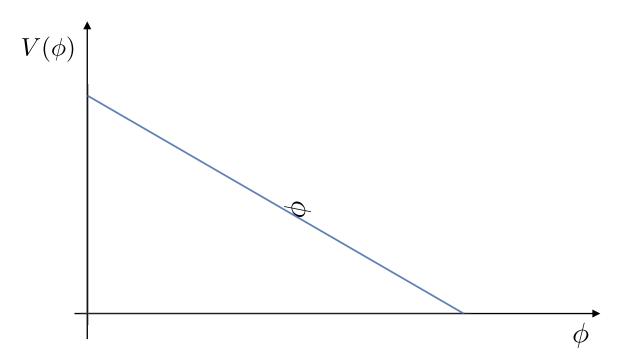
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After reheating the wiggles disappear: and the relaxion roles a bit.

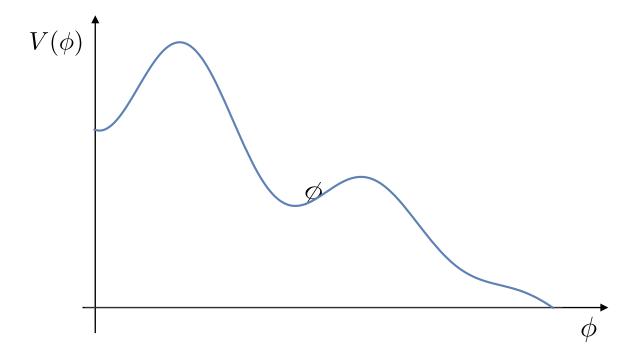


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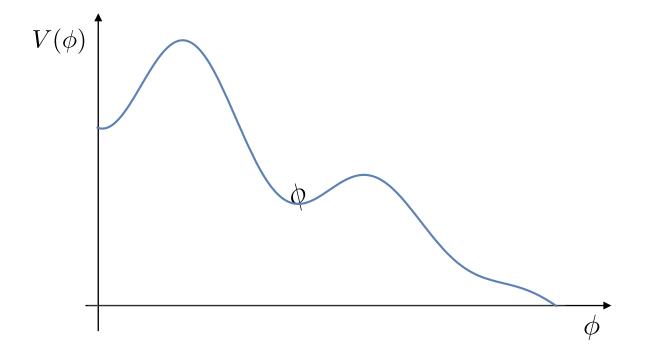
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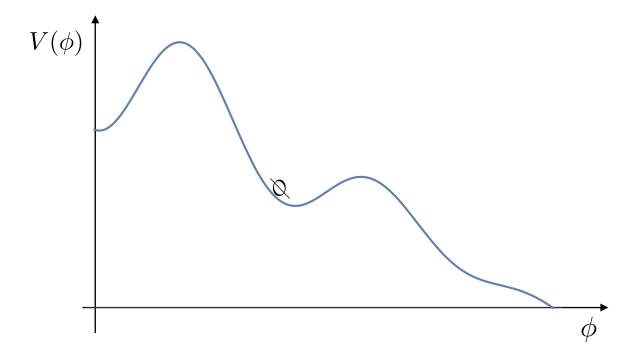
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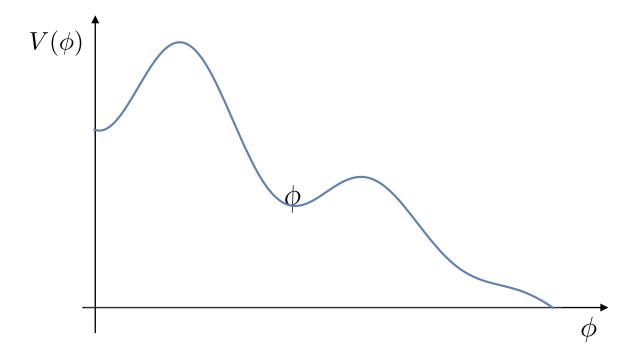
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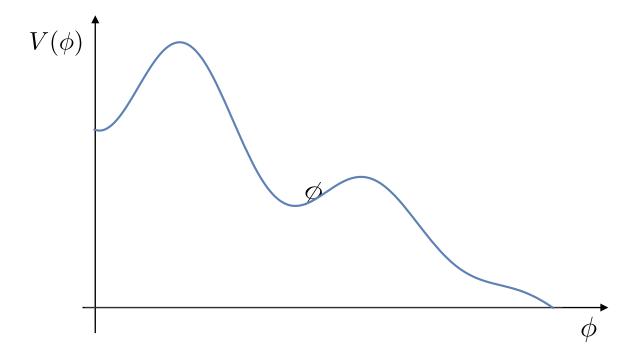
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Coherent relaxion DM relic density

Series And Antication And Antication Basic idea is similar to axion DM (but avoiding missalignment problem):

Now the relaxion not at the min' and start to oscillates = DM.

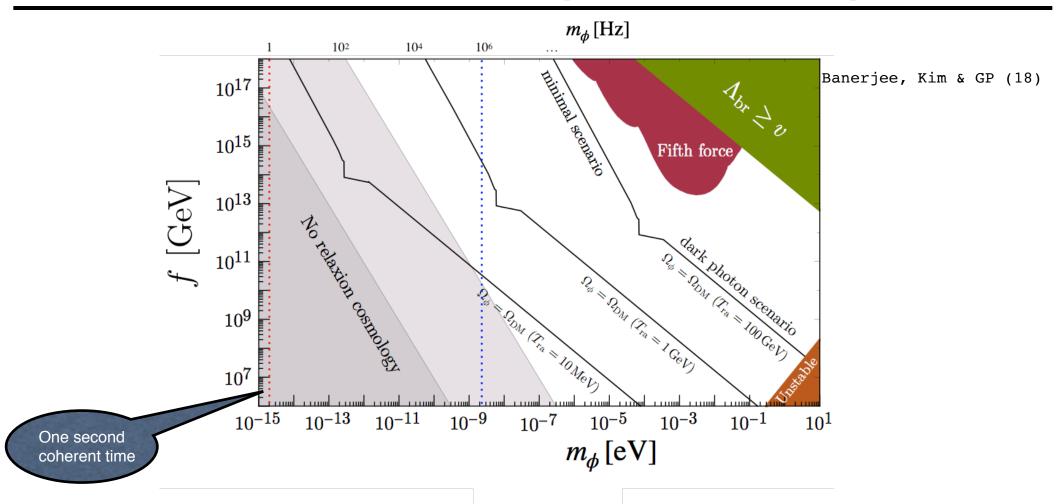
Light-coherent DM abundance: $\rho_{\rm DM}^{\rm cos} \sim m^2 \Delta \phi^2$

For
$$m_{\phi} \gtrsim H(T_{\rm ra})$$
: $\rho_{\rm DM}^{\rm cos} \sim \Omega_{\phi} h^2 \approx 3 \times (\Delta \theta)_{T=T_{\rm os}}^2 \left(\frac{\Lambda_{\rm br}}{1 \,{\rm GeV}}\right)^4 \left(\frac{100 \,{\rm GeV}}{T_{\rm os}}\right)^3$

where the observed DM abundance is $\Omega_{\rm DM} h^2 \simeq 0.12$

For $m_{\phi} < H(T_{ra})$: extra suppression is obtained as oscilation starts when $H(T_{osc}) \sim m_{\phi}$.

Relaxion dark matter, parameter space



If the relaxion oscillates due to its mixing with the Higgs all Banerjee, Kim & GP (18)
Constants of nature + masses now oscillates.

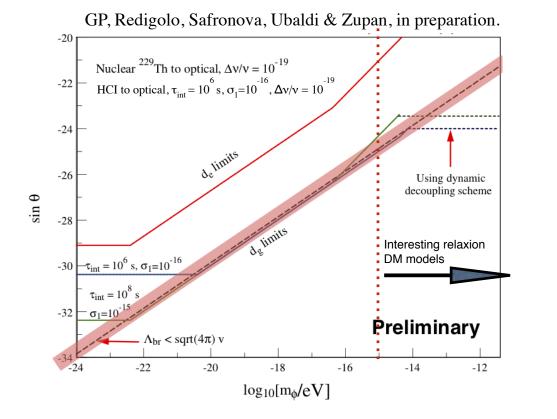
$$\frac{\delta m_e}{m_e} \lesssim y_e \sin_{\phi h} \frac{\sqrt{\rho_{\rm DM}}}{m_e m_{\phi}} \sin(m_{\phi} t) \text{ Arvanitaki, Huang & Van Tilburg (15)}$$

Two relevant questions

- (i) Notice that relevant models have osc. freq. I 10¹⁴ Hz. Can we probe these?
- (ii) Is the amplitude large enough to probe meaningful models?

Constraining sub-Hz relaxion DM

Graham, Kaplan, Mardon, Rajendran & Terrano; Arvanitaki, Dimopoulos & Van Tilburg; Van Tilburg, Leefer, Bougas & Budker (15)



 $(d_{e,\alpha} \sim \delta m_e, \delta \alpha / m_e, \alpha)$

 d_e stands for the time dependent component of the fine coupling constant, the bound on d_g (the coefficient of the time dependent component of α_s , the strong coupling) assumes a working ²²⁹Th nuclear clock with a 1 : 10¹⁹ precision, τ_{int} stands for the total assumed integration time and σ_1 stands for the corresponding stability. The dashed-red line on the diagonal corresponds to the maximal mixing allowed in this scenario, Λ_{br} corresponds to a coupling in the relaxion model.

Back to our 2 questions

- (i) Notice that relevant models have osc. freq. I 10¹⁴ Hz.
 Can we probe these?
- (ii) Is the amplitude large enough to probe meaningful models?

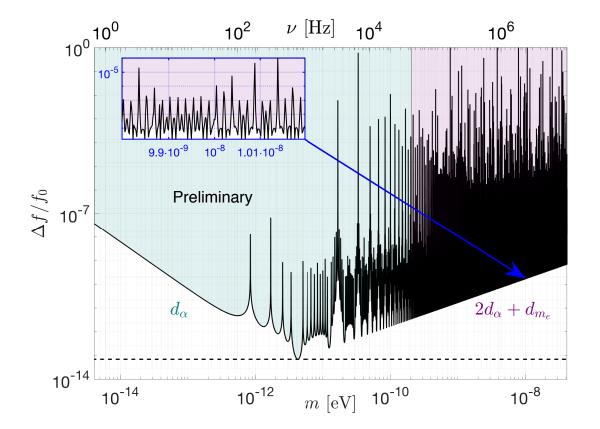
However, gravity can help: dark matter might form "relaxionplanets" that might be trapped around earth-gravitational field.

Banerjee, Budker, Eby, Kim, GP, in Prep.

(similar to axion-stars requiring stability and assuming capturing & coherence)

Kimball, et al. (17)

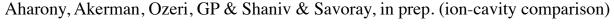
Beyond IHz DM mass \w dynamical decoupling (DD)

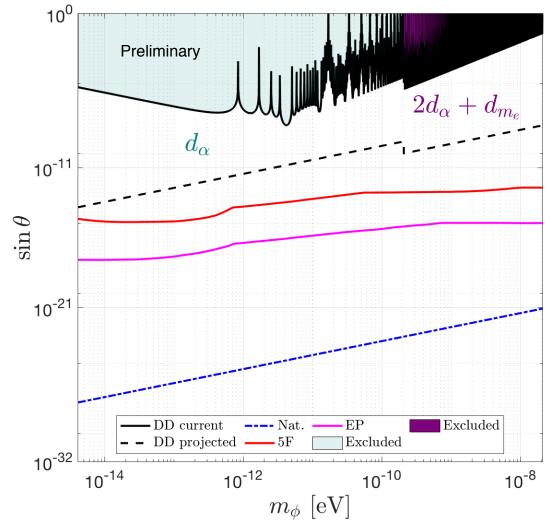


Aharony, Akerman, Ozeri, GP & Shaniv & Savoray, in prep. (ion-cavity comparison)

Current bound on the relative modulation of the transition frequency from a DD experiment, placed at 95% CL. The dashed line marks the current sensitivity reach, corresponding to scanning over ν_m . The inset is a magnified view of $m \sim 10^{-8} \text{eV}$.

Beyond IHz DM mass \w dynamical decoupling





The bounds on the mixing angle of a relaxion DM: Black – current and projected bounds from DD experiments at 95% CL. Red – Bounds from fifth force experiments. Magenta – EP-tests bounds. Dash-dotted – Bounds from Naturalness.

Searching for a relaxion DM planet around us

Assume small DM density & large radius => mass-radii relation:

$$R_{\rm star} \approx \frac{M_{\rm Pl}^2}{m_{\phi}^2} \frac{1}{M_{\rm Earth}} \qquad (M_* \ll M_{\rm Earth}) \,.$$

Eby, Leembruggen, Street, Suranyi & Wijewardhana (18);

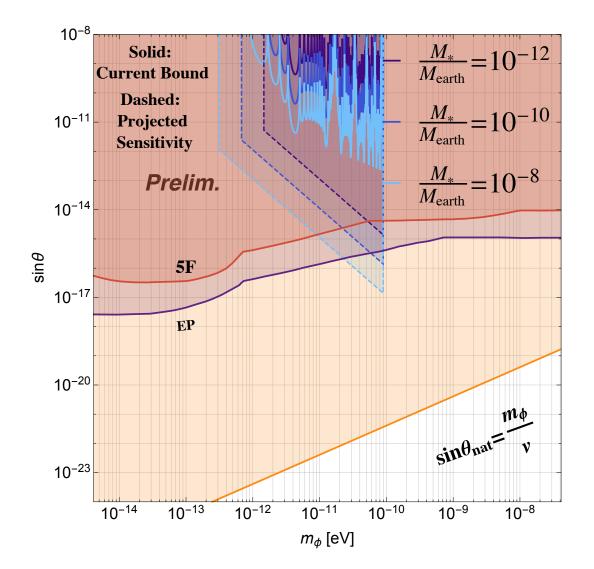
Banerjee, Budker, Eby, Kim, GP, in Prep.

Can obtain large density enhancement:

$$r \equiv \frac{\rho_{\text{star}}}{\rho_{\text{loc}-\text{DM}}} \sim \xi \frac{M_{\text{Earth}}^4 m_{\phi}^6}{M_{\text{Pl}}^6 \rho_{\text{loc}-\text{DM}}} \sim \xi \times 10^{28} \times \left(\frac{m_{\phi}}{10^{-10}}\right)^6 \qquad \xi \equiv M_{\text{star}}/M_{\text{Earth}}$$

Large star DM density => visible effect

Aharony, Akerman, Ozeri, GP & Shaniv & Savoray, in prep. (ion-cavity comparison); Banerjee, Budker, Eby, Kim, GP, in Prep.



Bounds for a relaxion star centered around earth.

Conclusions

Null-LHC + new paradigms + incredible sensitivity => new era!

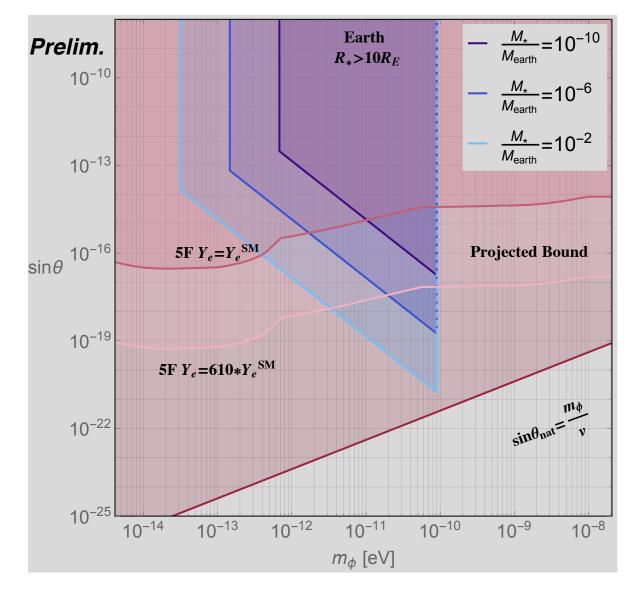
Relaxion-benchmarking allows to compare sensitivities.

Relaxion-DM: dynamic decoupling -> strong bounds but cannot
 compete \w 5th force & can't probe physical region.

Relaxion-DM-stars: table-top probe physical region, stronger than 5th force & can/should compare \w space.

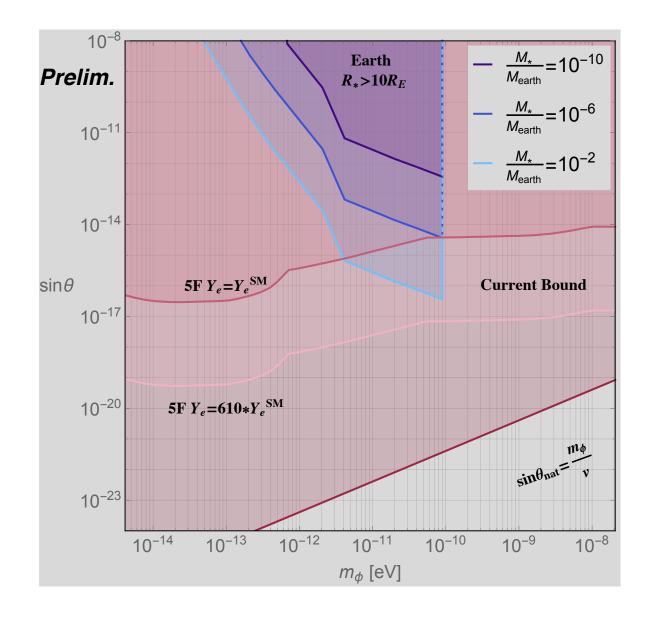
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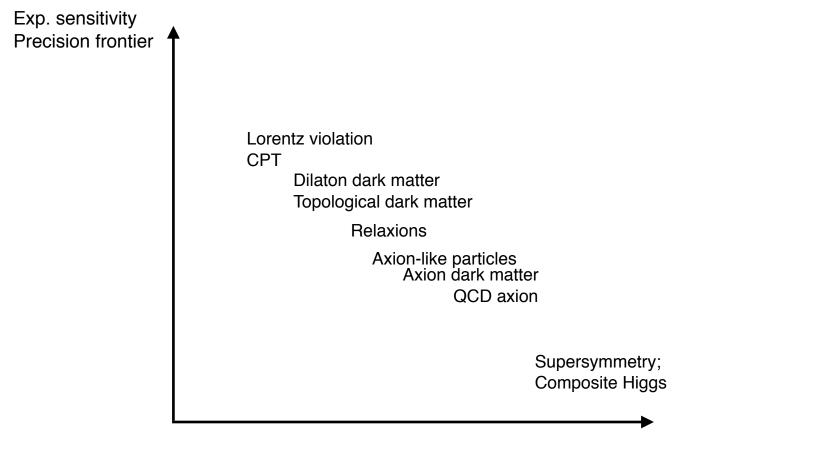


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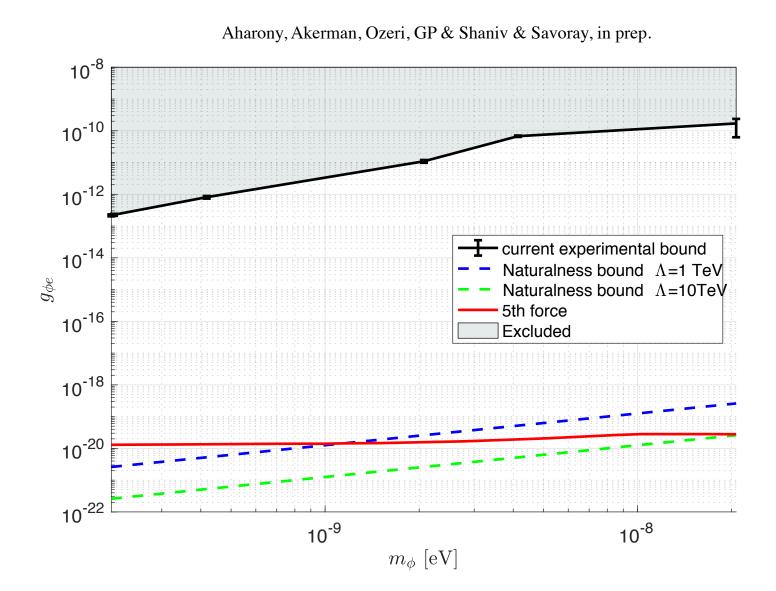


Backups



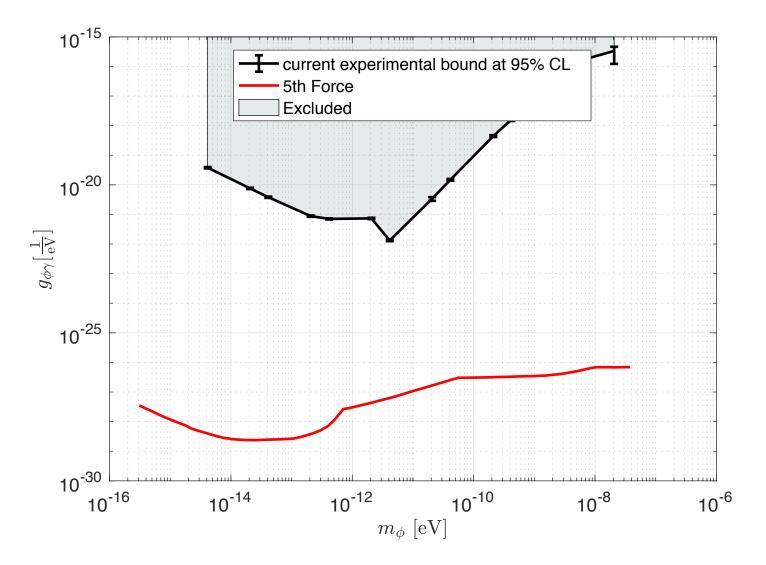
Theoretical motivation

Beyond IHz DM mass \w dynamical decoupling



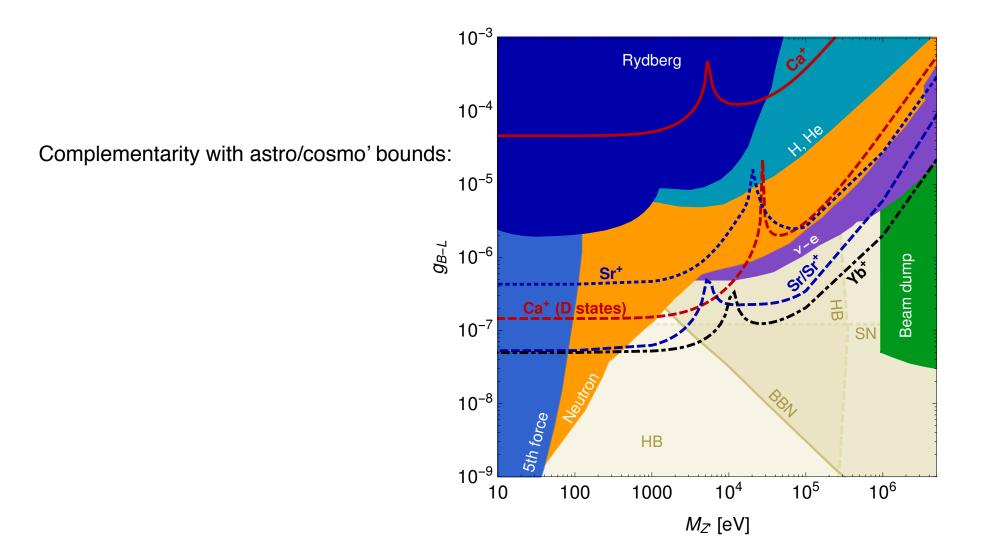
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U(I)_{B-L}

Frugiuele, Fuchs, GP & Schlaffer (16)



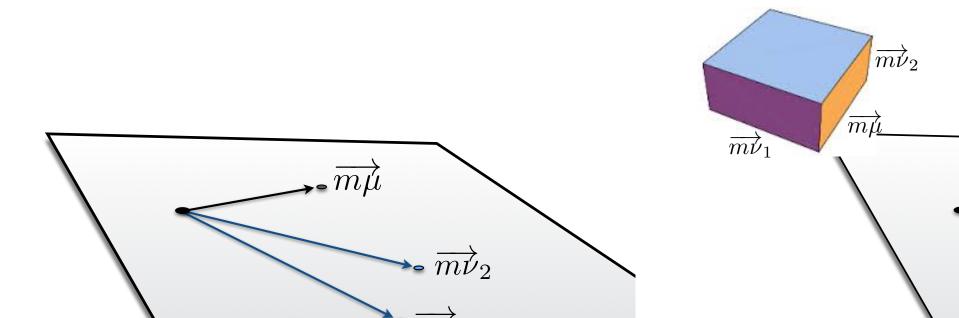
King comparison

• Level of linearity can be quantified by comparing area of triangle to that of a cube: $NL/|\vec{m\nu}_2||\vec{m\nu}_1| \ll 1$.

$$\rightarrow$$
 _ (1 1 1)

$$\mathrm{NL} = \frac{1}{2} \left| (\overrightarrow{m\nu}_1 \times \overrightarrow{m\nu}_2) \cdot \overrightarrow{m\mu} \right| \,.$$

Or volume of prallelepiped:



King linearity implications

• Linearity implies that $\overrightarrow{m\nu}_2 \& \overrightarrow{m\nu}_1$ must be linearly dependent:

$$\overrightarrow{m\nu}_{2} = K_{2} \, \overrightarrow{m\mu} + F_{2} \, \vec{v} + \mathcal{O} \left(10^{-4} \right)$$
$$\overrightarrow{m\nu}_{1} = K_{1} \, \overrightarrow{m\mu} + F_{1} \, \vec{v} + \mathcal{O} \left(10^{-4} \right)$$
$$\overrightarrow{m\nu}_{2} \cong K_{21} \, \overrightarrow{m\mu} + F_{21} \, \overrightarrow{m\nu}_{1} ,$$
with $F_{21} \equiv F_{2}/F_{1}$ and $K_{21} \equiv K_{2} - F_{21}K_{1}.$

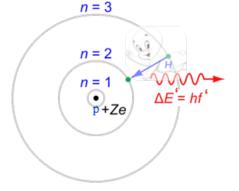
 $F_i \& \vec{v}$ are unknown but $F_{21} \& K_{21}$ can be measured precisely.

Adding light new physics (NP)

New forces acts on electron & quarks leads to change of energy levels.

$$\begin{array}{c} \overset{n=2}{\underset{p+Ze}{}} & \downarrow \\ & \downarrow$$

n = 3



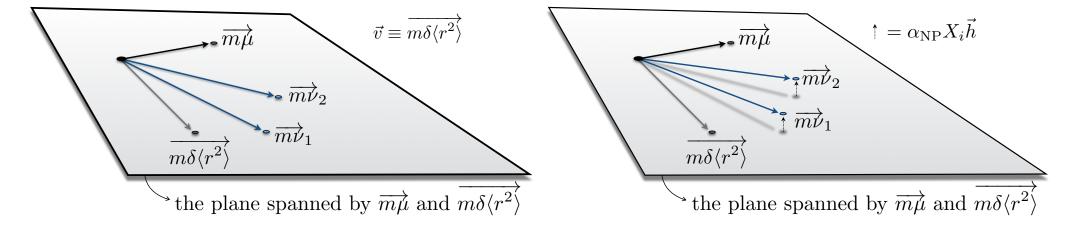
New physics part known, precisely calculated:

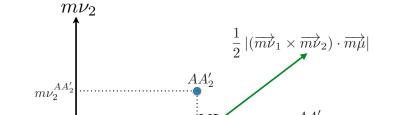
CI+MBPT: Dzuba, Flambaum & Kozlov (96) Berengut, Flambaum & Kozlov (06); GRASP2K: Jonsson, Gaigalas, Biero, Fischer & Grant (2013) (Combination of the many-body perturbation theory with the configuration-interaction method)

$$\overrightarrow{m\nu}_{i} = K_{i} \overrightarrow{m\mu} + F_{i} \overrightarrow{v} + y_{e} y_{n} X_{i} \overrightarrow{h},$$

$$\bigvee^{\text{Delaunay, Ozeri, GP & Soreq (16)}} \overrightarrow{m\nu}_{2} = K_{21} \overrightarrow{m\mu} + F_{21} \overrightarrow{m\nu}_{1} + \alpha_{\text{NP}} \overrightarrow{h} X_{1} (X_{21} - F_{21}),$$
and $X_{21} \equiv X_{2}/X_{1}.$

Illustration: adding light new physics (NP)





Light mediators

If mediator's mass, m_X , is smaller than inverse of outer electrons than the potential is Coulombic.

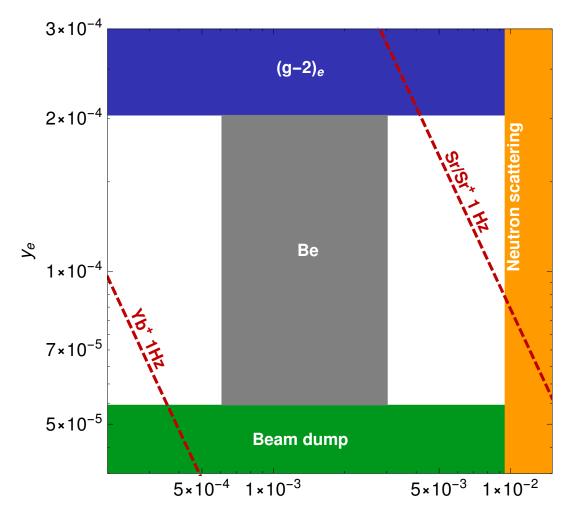
If mediator's mass is smaller than inverse distance of most inner electron from the nucleus then the full Yukawa potential is required.

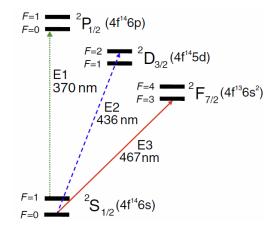
Otherwise the potential is described via a delta function.

$$V(r) = \begin{cases} \frac{1}{r} \text{ for } m_X \lesssim \alpha m_e ,\\ \frac{e^{-rm_X}}{r} \text{ for } \alpha m_e \lesssim m_X \lesssim \alpha m_e Z ,\\ \frac{1}{m_X^2} \delta^3(r) \text{ otherwise }. \end{cases}$$

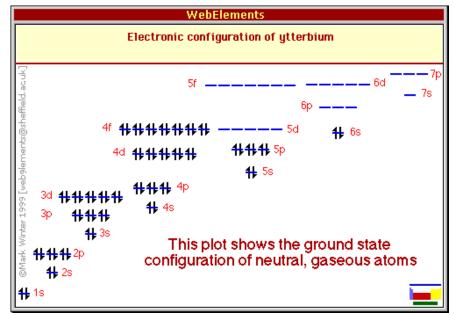
Be 17 MeV anomaly

Frugiuele, Fuchs, GP & Schlaffer v2 (16)









The electronic configuration of ytterbium.

nuclide symbol	Z(p)	N(n)	isotopic mass (u)	half-life	decay mode(s) ^{[2][n 1]}	daughter isotope(s) ^[n 2]	nuclear spin	representative isotopic composition	range of natura variation
	excitation energy						-	(mole fraction)	(mole fraction)
⁴⁸ Yb	70	78	147.96742(64)#	250# ms	β*	¹⁴⁸ Tm	0+		
¹⁴⁹ Yb	70	79	148.96404(54)#	0.7(2) s	β*	¹⁴⁹ Tm	(1/2+,3/2+)		
¹⁵⁰ Yb	70	80	149.95842(43)#	700# ms [>200 ns]	β*	¹⁵⁰ Tm	0+		
151					β*	¹⁵¹ Tm	(1/2+)		
¹⁵¹ Yb	70 81	81	150.95540(32)	1.6(5) s	β ⁺ , p (rare)	¹⁵⁰ Er			
^{151m1} Yb					β*	¹⁵¹ Tm			
Yb	7	50(100	D)# keV	1.6(5) s	β ⁺ , p (rare)	¹⁵⁰ Er	(11/2-)		
151m2Yb	1	790(50	00)# keV	2.6(7) µs			19/2-#		
^{151m3} Yb	2	450(50	00)# keV	20(1) µs			27/2-#		
¹⁵² Yb					β*	¹⁵² Tm			
- YD	70	82	151.95029(22)	3.04(6) s	β ⁺ , p (rare)	¹⁵¹ Er	0+		
¹⁵³ Yb	70		83 152.94948(21)#	4.2(2) s	a (50%)	¹⁴⁹ Er	7/2-#		
		83			β* (50%)	¹⁵³ Tm			
					β ⁺ , p (.008%)	152Er			
153mYb	2	700(10	00) keV	15(1) µs	P . P ((27/2-)		
					a (92.8%)	¹⁵⁰ Er			
¹⁵⁴ Yb	70	84	153.946394(19)	0.409(2) s	β ⁺ (7.119%)	154Tm	0+		
					a (89%)	¹⁵¹ Er			
¹⁵⁵ Yb	70	85	154.945782(18)	1.793(19) s	β ⁺ (11%)	155Tm	(7/2-)		
					β [*] (90%)	¹⁵⁶ Tm			
¹⁵⁶ Yb	70	86	155.942818(12)	26.1(7) s 38.6(10) s	α (10%)	¹⁵² Er	0+		
						¹⁵⁷ Tm			
¹⁵⁷ Yb	70	87	156.942628(11)		β* (99.5%)	¹⁵³ Er			
					a (.5%)	¹⁵⁸ Tm			
¹⁵⁸ Yb	70	88	157.939866(9)	1.49(13) min	β* (99.99%)	154 _{Er}	0+		
159					α (.0021%)				
159Yb	70	89	158.94005(2)	1.67(9) min	β*	¹⁵⁹ Tm	5/2(-)		
¹⁶⁰ Yb	70	90	159.937552(18)	4.8(2) min	β*	¹⁶⁰ Tm	0+		
¹⁶¹ Yb	70	91	160.937902(17)	4.2(2) min	β*	¹⁶¹ Tm	3/2-		
¹⁶² Yb	70	92	161.935768(17)	18.87(19) min	β*	¹⁶² Tm	0+		
¹⁶³ Yb	70	93	162.936334(17)	11.05(25) min	β*	¹⁶³ Tm	3/2-		
¹⁶⁴ Yb	70	94	163.934489(17)	75.8(17) min	EC	¹⁶⁴ Tm	0+		
¹⁶⁵ Yb	70	95	164.93528(3)	9.9(3) min	β*	¹⁶⁵ Tm	5/2-		
¹⁶⁶ Yb	70	96	165.933882(9)	56.7(1) h	EC	¹⁶⁶ Tm	0+		
¹⁶⁷ Yb	70	97	166.934950(5)	17.5(2) min	β*	¹⁶⁷ Tm	5/2-		
¹⁶⁸ Yb	70	98	167.933897(5)	Observ	ationally Stable		0+	0.0013(1)	
¹⁶⁹ Yb	70	99	168.935190(5)	32.026(5) d	EC	¹⁶⁹ Tm	7/2+		
^{169m} Yb	2	4.199(3) keV	46(2) s	IT	¹⁶⁹ Yb	1/2-		
¹⁷⁰ Yb	70	100	169.9347618(26)	Observ	ationally Stable	[n 4]	0+	0.0304(15)	
170mYb	1	258.46	6(14) keV	370(15) ns			4-		
¹⁷¹ Yb	70	101	170.9363258(26)	Observ	ationally Stable	[n 5]	1/2-	0.1428(57)	
^{171m1} Yb	9	5.282(2) keV	5.25(24) ms	IT	¹⁷¹ Yb	7/2+		
	1	22.416	5(2) keV	265(20) ns			5/2-		
171m2Yb		102	171.9363815(26)	Observ	ationally Stable	[n 6]	0+	0.2183(67)	
	70				vationally Stable		5/2-	0.1613(27)	
^{171m2} Yb	70 70	103	172.9382108(26)	Observ					
^{171m2} Yb ¹⁷² Yb	70						1/2-		
171m2Yb 172Yb 173Yb 173mYb 173mYb	70	98.9(5		2.9(1) µs		[n 8]	1/2-	0.3183(92)	
171m2Yb 172Yb 173Yb 173mYb 173mYb	70 3 70	98.9(5 104) keV 173.9388621(26)	2.9(1) µs Observ	ationally Stable	(n 8) 175 _{Lu}	0+	0.3183(92)	
171m2Yb 172Yb 173Yb 173mYb 173mYb 174Yb 175Yb	70 3 70 70	98.9(5 104 105) keV 173.9388621(26) 174.9412765(26)	2.9(1) µs Observ 4.185(1) d			0+ 7/2-	0.3183(92)	
171m2үb 172үb 173үb 173mүb 173mүb 175үb 175үb	70 3 70 70 5	98.9(5 104 105 14.865) keV 173.9388621(26) 174.9412765(26) 5(4) keV	2.9(1) µs Observ 4.185(1) d 68.2(3) ms	ationally Stable β ⁼	¹⁷⁵ Lu	0+ 7/2- 1/2-		
171m2Yb 172Yb 173Yb 173mYb 173mYb 174Yb 175Yb 175mYb 176yb	70 3 70 70 5 70	98.9(5 104 105 14.865 106) keV 173.9388621(26) 174.9412765(26) 5(4) keV 175.9425717(28)	2.9(1) µs Observ 4.185(1) d 68.2(3) ms Observ	ationally Stable	¹⁷⁵ Lu	0+ 7/2- 1/2- 0+	0.3183(92)	
171m2yb 172yb 173yb 173myb 174yb 175yb 175yb 175myb 176yb 176yb	70 3 70 70 5 70 1	98.9(5 104 105 14.865 106 050.0() keV 173.9388621(26) 174.9412765(26) 5(4) keV 175.9425717(28) 3) keV	2.9(1) µs Observ 4.185(1) d 68.2(3) ms Observ 11.4(3) s	p ⁻ β ⁻ vationally Stable	175 _{Lu} (n 9)	0+ 7/2- 1/2- 0+ (8)-		
171m2үb 172үb 173үb 173mγb 174үb 174үb 175үb 175mγb	70 3 70 70 5 70 1 70	98.9(5 104 105 14.865 106 050.0() keV 173.9388621(26) 174.9412765(26) 5(4) keV 175.9425717(28) 3) keV 176.9452608(28)	2.9(1) µs Observ 4.185(1) d 68.2(3) ms Observ	ationally Stable β ⁼	¹⁷⁵ Lu	0+ 7/2- 1/2- 0+		

Precision mass measurements: 10⁻¹⁰



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The most precise atomic mass measurements in Penning traps

6.1

Table 10

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Edmund G. Myers*

Florida State University, Department of Physics, Tallahassee, FL 32306-4350, USA

Atom	FSU mass (u)	σ_m/m (ppt)
⁸⁶ Sr	85.909 260 730 9(91)	105
⁸⁷ Sr	86.908 877 497 0(91)	105
⁸⁸ Sr	87.905 612 257 1(97)	110
¹⁷⁰ Yb	169.934 767 241(18)	105
¹⁷¹ Yb	170.936 331 514(19)	110
¹⁷² Yb	171.936 386 655(18)	105
¹⁷³ Yb	172.938 216 213(18)	105
¹⁷⁴ Yb	173.938 867 539(18)	105
¹⁷⁶ Yb	175.942 574 702(22)	125

and the state of the

Partial solution, comparing different isotope shift, searching of nonlinearity in "King plot"

King's factorisation formula (King, 1963):

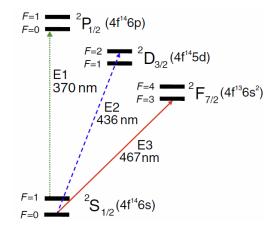
$$\delta \nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'} = \overline{K_i \, \mu_{AA'} + F_i \delta \langle r^2 \rangle_{AA'}},$$

$$(\mu_{AA'} \equiv 1/m_A - 1/m_{A'} = (A' - A)/(AA') \, \text{amu}^{-1}, \text{ where amu} \approx 0.931 \, \text{GeV})$$
only depend on nucleus

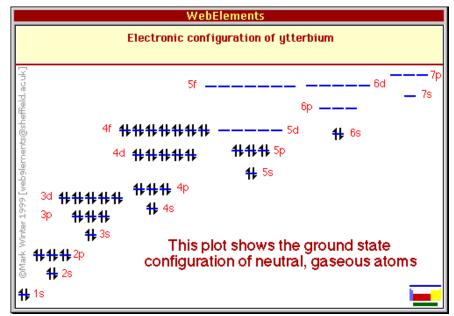
We can solve for $\delta \langle r^2 \rangle_{AA'}$ to get a linear relation:

$$m\delta\nu_{AA'}^2 = F_{21}m\delta\nu_{AA'}^1 + K_{21} \,,$$

(with $K_{21} \equiv (K_2 - F_{21}K_1)$ and $F_{21} \equiv F_2/F_1$ and $m\delta\nu_{AA'}^i \equiv \delta\nu_{AA'}^i/\mu_{AA'}$.)







The electronic configuration of ytterbium.

nuclide symbol	Z(p)	N(n)	isotopic mass (u)	half-life	decay mode(s) ^{[2][n 1]}	daughter isotope(s) ^[n 2]	nuclear spin	representative isotopic composition	range of natura variation
	excitation energy			1				(mole fraction)	(mole fraction)
¹⁴⁸ Yb	70	78	147.96742(64)#	250# ms	β*	¹⁴⁸ Tm	0+		
¹⁴⁹ Yb	70	79	148.96404(54)#	0.7(2) s	β*	¹⁴⁹ Tm	(1/2+,3/2+)		
¹⁵⁰ Yb	70	80	149.95842(43)#	700# ms [>200 ns]	β*	¹⁵⁰ Tm	0+		
¹⁵¹ Yb					β*	¹⁵¹ Tm			
Yb	70 81	81	150.95540(32)	1.6(5) s	β ⁺ , p (rare)	¹⁵⁰ Er	(1/2+)		
(fint)					β*	¹⁵¹ Tm			
^{151m1} Yb	7	50(100	D)# keV	1.6(5) s	β ⁺ , p (rare)	¹⁵⁰ Er	(11/2-)		
151m2Yb	1	790(50	00)# keV	2.6(7) µs	P 1 P 0		19/2-#		
^{151m3} Yb			00)# keV	20(1) µs			27/2-#		
	2400(000)# No F			20(1) 40	B*	¹⁵² Tm			
¹⁵² Yb	70	70 82	151.95029(22)	3.04(6) s	β ⁺ , p (rare)	¹⁵¹ Er	0+		
¹⁵³ Yb	70		83 152.94948(21)#	4.2(2) s	a (50%)	149Er			
					β ⁺ (50%)	¹⁵³ Tm	7/2-#		
		63				152 _{Er}			
153myb		700/4/		45(4)	β [*] , p (.008%)	Er			
	2	700(10	00) keV	15(1) µs		¹⁵⁰ Er	(27/2-)		
¹⁵⁴ Yb	70	84	4 153.946394(19)	0.409(2) s	α (92.8%)		0+		
					β* (7.119%)	¹⁵⁴ Tm			
155 _{Yb}	70	85	5 154.945782(18)	1.793(19) s	a (89%)	¹⁵¹ Er	(7/2-)		
					β* (11%)	¹⁵⁵ Tm			
¹⁵⁶ Yb	70	86	86 155.942818(12) 87 156.942628(11)	26.1(7) s 38.6(10) s	β* (90%)	¹⁵⁶ Tm	0+		
	10	00			α (10%)	¹⁵² Er			
¹⁵⁷ Yb	70	87			β* (99.5%)	¹⁵⁷ Tm	7/2-		
10	70 8	01			α (.5%)	¹⁵³ Er	1)2-		
158 _{Yb}			157.939866(9)	1.49(13) min	β* (99.99%)	¹⁵⁸ Tm	0+		
	70	70 88			a (.0021%)	¹⁵⁴ Er			
¹⁵⁹ Yb	70	89	158.94005(2)	1.67(9) min	β*	¹⁵⁹ Tm	5/2(-)		
¹⁶⁰ Yb	70	90	159.937552(18)	4.8(2) min	β*	¹⁶⁰ Tm	0+		
¹⁶¹ Yb	70	91	160.937902(17)	4.2(2) min	β*	¹⁶¹ Tm	3/2-		
¹⁶² Yb	70	92	161.935768(17)	18.87(19) min	β*	¹⁶² Tm	0+		
¹⁶³ Yb	70	93	162.936334(17)	11.05(25) min	β*	¹⁶³ Tm	3/2-		
¹⁶⁴ Yb	70		163.934489(17)	75.8(17) min	EC	¹⁶⁴ Tm	0+		
¹⁶⁵ Yb	70		164.93528(3)	9.9(3) min	β*	165 _{Tm}	5/2-		
100Vh						¹⁶⁶ Tm	5/2-		
166 _{Vb}	70	96		56 7(1) h	EC.		0+		
¹⁶⁶ Yb	70		165.933882(9)	56.7(1) h	EC R ⁺		0+		
¹⁶⁶ Yb ¹⁶⁷ Yb	70	97	165.933882(9) 166.934950(5)	17.5(2) min	β*	¹⁶⁷ Tm	5/2-	0.0013(1)	
¹⁶⁶ Yb ¹⁶⁷ Yb ¹⁶⁸ Yb	70 70	97 98	165.933882(9) 166.934950(5) 167.933897(5)	17.5(2) min Observ	β ⁺ vationally Stable	¹⁶⁷ Tm [n 3]	5/2- 0+	0.0013(1)	
166Yb 167Yb 168Yb 169Yb	70 70 70	97 98 99	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5)	17.5(2) min Observ 32.026(5) d	β ⁺ ationally Stable EC	¹⁶⁷ Tm [n 3] ¹⁶⁹ Tm	5/2- 0+ 7/2+	0.0013(1)	
166Yb 167Yb 168Yb 169Yb 169mYb	70 70 70 2	97 98 99 24.199(165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV	17.5(2) min Observ 32.026(5) d 46(2) s	β [*] vationally Stable EC IT	¹⁶⁷ Tm ^(n 3) ¹⁶⁹ Tm ¹⁶⁹ Yb	5/2- 0+ 7/2+ 1/2-		
165Yb 167Yb 168Yb 169Yb 169mYb 169mYb	70 70 70 2 70	97 98 99 24.199(100	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26)	17.5(2) min Observ 32.026(5) d 46(2) s Observ	β ⁺ ationally Stable EC	¹⁶⁷ Tm ^(n 3) ¹⁶⁹ Tm ¹⁶⁹ Yb	5/2- 0+ 7/2+ 1/2- 0+	0.0013(1)	
166 _{Yb} 167 _{Yb} 168 _{Yb} 169 _{Yb} 169m _{Yb} 169m _{Yb} 170 _{Yb}	70 70 70 2 70 1	97 98 99 24.199(100 258.46	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26) \$(14) keV	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns	β ⁺ ationally Stable EC IT vationally Stable	167 _{Tm} [n 3] 169 Ym [n 4]	5/2- 0+ 7/2+ 1/2- 0+ 4-	0.0304(15)	
165 Yb 167 Yb 168 Yb 169 Yb 169 WYb 170 Yb 170 Wb 171 Yb	70 70 70 2 70 1 70	97 98 99 24.199(100 258.46 101	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26) \$(14) keV 170.9363258(26)	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ	β ⁺ EC IT vationally Stable vationally Stable	167 _{Tm} [n 3] 169 Ym 169Yb [n 4] [n 5]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2-		
166 yb 167 yb 168 yb 169 yb 169 myb 169 myb 170 yb 170 myb 171 yb 171 miyb	70 70 70 2 70 1 70 9	97 98 99 24.199(100 258.46 101 35.282(165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26) 8(14) keV 170.9363258(26) 2) keV	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms	β ⁺ ationally Stable EC IT vationally Stable	167 _{Tm} [n 3] 169 Ym [n 4]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+	0.0304(15)	
166 Yb 167 Yb 168 Yb 169 Yb 169 mYb 169 mYb 169 mYb 170 Yb 171 mYb 171 mYb 171 mYb 171 mYyb	70 70 70 2 70 1 70 9 9	97 98 99 24.199(100 258.46 101 35.282(22.416	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26) 8(14) keV 170.9363258(26) 2) keV 3(2) keV	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns	β ⁺ rationally Stable EC IT rationally Stable rationally Stable	167 Tm (n 3) 169 Yb (n 4) (n 5) 171 Yb	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2-	0.0304(15)	
166γb 167γb 168γb 169γb 169ηγb 170γb 170γb 171γb 171m1γb 171m2γb	70 70 70 2 70 1 70 9 1 70	97 98 99 24.199(100 258.46 101 35.282() 22.416 102	165 933682(9) 166 934950(5) 167 933897(5) 168 935190(5) 3) keV 169 9347618(26) 9(14) keV 170 9363258(26) 2) keV 3(2) keV 171 9363815(26)	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ	β* ationally Stable EC IT ationally Stable IT rationally Stable	167 Tm [n 3] 169 YD [n 4] [n 6] 171 YD [n 6]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+	0.0304(15) 0.1428(57) 0.2183(67)	
166γb 167γb 168γb 168γb 169mγb 170mγb 171m2γb 171m2γb 172γb 172γb	70 70 2 70 1 70 9 1 70 70 70	97 98 99 24.199(100 258.46 101 5.282() 22.416 102 103	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 168.935190(5) 3) keV 169.9347618(26) 5(14) keV 170.9363258(26) 2) keV 171.9363815(26) 172.9382108(26)	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ	β ⁺ rationally Stable EC IT rationally Stable rationally Stable	167 Tm [n 3] 169 YD [n 4] [n 6] 171 YD [n 6]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2-	0.0304(15)	
166yb 167yb 168yb 168yyb 168myb 170yb 171yb 171m2yb 172yb 172yb 173yb 173myb	70 70 2 70 1 70 9 1 70 70 70 3	97 98 99 4.199(100 258.46 101 5.282(22.416 102 103 98.9(5	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 169.9347618(26) 3) keV 170.9363258(26) 3(14) keV 170.9363258(26) 2) keV 171.9363815(26) 171.9363815(26) 171.2382108(26) 2) keV	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ Observ 2.9(1) µs	β* rationally Stable EC IT rationally Stable IT rationally Stable rationally Stable	167 m [n 3] 169 Yb [n 4] [n 6] [n 6] [n 6] [n 7]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2-	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27)	
166үр 167үр 168үр 168үр 169үр 169үр 169үр 169үр 170үр 170үр 171үр 171м2үр 172үр 172үр 173мүр	70 70 2 70 1 70 9 1 70 70 70	97 98 99 4.199(100 258.46 101 5.282(22.416 102 103 98.9(5	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 168.935190(5) 3) keV 169.9347618(26) 5(14) keV 170.9363258(26) 2) keV 171.9363815(26) 172.9382108(26)	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ Observ 2.9(1) µs	β* ationally Stable EC IT ationally Stable IT rationally Stable	167 m [n 3] 169 YD [n 4] [n 6] [n 6] [n 6] [n 6] [n 8]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2-	0.0304(15) 0.1428(57) 0.2183(67)	
166 yb 167 yb 167 yb 168 yb 168 yb 169 yb 169 yb 170 yb 170 yb 171 yb 171 yb 171 m2 yb 173 yb 173 yb 173 yb 173 yb 173 yb	70 70 2 70 1 70 9 1 70 70 70 3	97 98 99 24.199(100 258.46 101 35.282() 22.416 102 103 398.9(5) 104	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 169.9347618(26) 3) keV 170.9363258(26) 3(14) keV 170.9363258(26) 2) keV 171.9363815(26) 171.9363815(26) 171.2382108(26) 2) keV	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ Observ 2.9(1) µs	β* rationally Stable EC IT rationally Stable IT rationally Stable rationally Stable	167 m [n 3] 169 Yb [n 4] [n 6] [n 6] [n 6] [n 7]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2-	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27)	
166 yb 167 yb 167 yb 168 yb 168 yb 169 yb 169 yb 170 yb 170 yb 171 yb 171 m2 yb 173 yb 173 yb 173 yb 175 yb 175 yb 175 yb 175 myb	70 70 70 2 70 1 1 70 9 1 1 70 70 70 3 70 70	97 98 99 24.199(100 258.46 101 55.282(22.416 102 103 398.9(5 104 105	to6 933862(9) to6 933862(9) to6 933450(5) to7 933897(5) to7 933897(5) to8 935190(5) 3) keV to9 9347618(26) 8(4) keV to9 9347618(26) 2) keV to9 934258(26) to171 9363815(26) to172 9362108(26) to8V to73 938862(126) to8V to73 938862(126)	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns 0525(24) ms 265(20) ns Observ Observ 2.9(1) µs	β* rationally Stable EC IT rationally Stable rationally Stable IT rationally Stable rationally Stable	167 m [n 3] 169 YD [n 4] [n 6] [n 6] [n 6] [n 6] [n 8]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2- 0+	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27)	
166γb 167γb 167γb 168γb 168wyb 168wγb 170γb 171γb 171w1γb 171m1γb 171m2γb 173wb 173wyb 174γb 175γb 173wyb 175γb	70 70 70 2 70 1 1 70 9 1 1 70 70 70 3 70 70	97 98 99 24.199(100 1258.46 101 105 5.282(102 103 103 198.9(5 104 105 514.865	165 933882(9) 166 934950(5) 167 933897(5) 168 935190(5) 3) keV 169 9347618(26) 8(14) keV 170 9363258(26) 170 9363258(26) 172 9363815(26) 172 9382108(26)) keV 173 9388621(26) 173 9388621(26)	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ Observ 4.185(1) d 68.2(3) ms	β* rationally Stable EC IT rationally Stable rationally Stable IT rationally Stable rationally Stable	1467 Trm [n 3] 109 Trm 109 YD [n 4] [n 6] [n 6] [n 7] [n 8] 175 Lu	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2- 0+ 7/2-	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27)	
166 yb 167 yb 167 yb 168 yb 168 yb 169 yb 169 yb 170 yb 170 yb 171 yb 171 m2 yb 173 yb 173 yb 173 yb 175 yb 175 yb 175 yb 175 myb	70 70 2 70 1 70 9 9 1 1 70 70 70 3 70 70 70 5 70	97 98 99 24.199(100 1258.46 101 105.282(102 103 998.9(5 104 105 104 105 104 105 106	165 933882(9) 166 934950(5) 167 933897(5) 168 935190(5) 3) keV 169 9347618(26) 5(14) keV 170 9363258(26) 2) keV 171 9363815(26) 172 9382108(26)) keV 173 9388621(26) 174 9412765(26) 5(4) keV	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ Observ 4.185(1) d 68.2(3) ms	β ⁺ rationally Stable EC IT rationally Stable rationally Stable IT rationally Stable rationally Stable β ⁻ β ⁻	1467 Trm [n 3] 109 Trm 109 YD [n 4] [n 6] [n 6] [n 7] [n 8] 175 Lu	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2- 0+ 7/2- 1/2-	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27) 0.3183(82)	
166 yb 167 yb 167 yb 168 yb 168 yb 168 yb 168 yb 168 yb 170 yb 170 yb 171 yb 171 yb 171 yb 172 yb 173 yb 173 yb 173 yb 175 yb 177 yb 175 yb	70 70 2 70 1 70 9 9 1 1 70 70 70 3 70 70 70 5 70	97 98 99 24.199(100 1258.46 101 125.282(102 103 398.9(5 104 105 514.865 514.865 514.865 0050.0(165 933882(9) 166 934950(5) 167 933897(5) 168 935190(5) 3169 9347618(28) (14) keV 169 9347618(28) (14) keV 170 9363258(26) (2) keV 171 9363815(26) 172 9382108(26)) keV 173 938621(26) 174 9412765(26) (4) keV 175 9425717(28)	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns S 25(24) ms 265(20) ns Observ Observ 2.9(1) µs Observ 4.185(1) d 68.2(3) ms	β ⁺ rationally Stable EC IT rationally Stable rationally Stable IT rationally Stable rationally Stable β ⁻ β ⁻	147 Tm (n 3) 149 Tm (n 4) 149 Tm (n 4) 171 Yb (n 4) 171 Yb 175 Lu 177 Lu 177 Lu	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 0+ 7/2- 1/2- 0+ 0+	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27) 0.3183(82)	
166γD 167γD 167γD 168γD 169γD 170γD 170γD 171γD 171m1γD 171m2γD 171m2γD 171m2γD 171m2γD 175mγD 175γD 175γD 176γD 176mγD	70 70 2 70 1 70 9 9 1 70 70 70 70 70 70 1 70	97 98 99 24.199(100 1258.46 101 125.282(102 103 398.9(5 104 105 514.865 514.865 514.865 0050.0(165 933882(9) 166 933897(5) 167 933897(5) 167 933897(5) 168 935190(5) 3) keV 169 9347618(26) 3) keV 170 9363258(26) 170 9363258(26) 172 936315(26) 172 936315(26) 173 9388621(26) 174 9412765(26) 4) keV 175 9425717(28) 3) keV 176 9452608(28)	17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns 5.25(24) ms 265(20) ns Observ Observ 2.9(1) µs Observ 4.185(1) d 68.2(3) ms Observ 11.4(3) s	β* ationally Stable EC IT ationally Stable IT ationally Stable ationally Stable ationally Stable p* ationally Stable p* ationally Stable ationally Stable	147 Tm 179 Tm 149 Tm 149 Yb 169 Yb 161 171 Yb 171 Yb 175 Lu 175 Lu 175 Lu	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 0+ 5/2- 0+ 7/2- 1/2- 0+ (6)-	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27) 0.3183(82)	

Ex.: Sr⁽⁺⁾ with Z=38, n=5 and A=84-88 (90).

- Electron Configuration: 1s² 2s²p⁶ 3s²p⁶d¹⁰ 4s²p⁶ 5s²⁽¹⁾
- Electrons per Energy Level: 2,8,18,8,2(1)



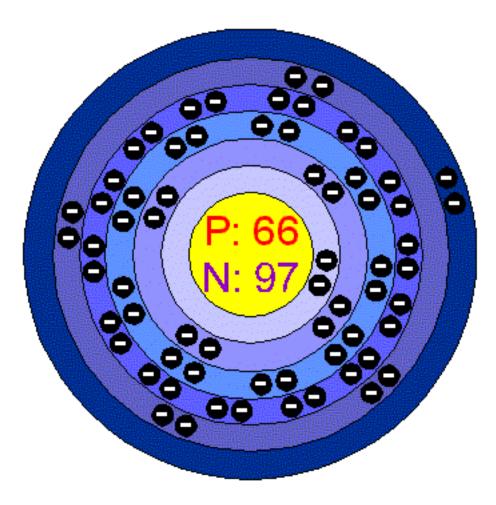
Ex.: $Ca^{(+)}$ with Z=20, n=4 and A=40-48.

- Electron Configuration:1s² 2s²p⁶ 3s²p⁶ 4s¹
- Electrons per Energy Level: 2,8,8,2(1)



•

Ex.: Dy with *Z*=66, n=6 and *A*=158-164.



Number of Energy Levels: 6 First Energy Level: 2 Second Energy Level: 8 Third Energy Level: 18 Fourth Energy Level: 28 Fifth Energy Level: 8 Sixth Energy Level: 2

The observables

• We have 3 isotope shifts $(AA'_{1,2,3})$ for 2 transitions (*i*=1,2):

$$\overrightarrow{m\nu}_{i} \equiv \left(m\nu_{i}^{AA_{1}^{\prime}}, m\nu_{i}^{AA_{2}^{\prime}}, m\nu_{i}^{AA_{3}^{\prime}}\right)$$

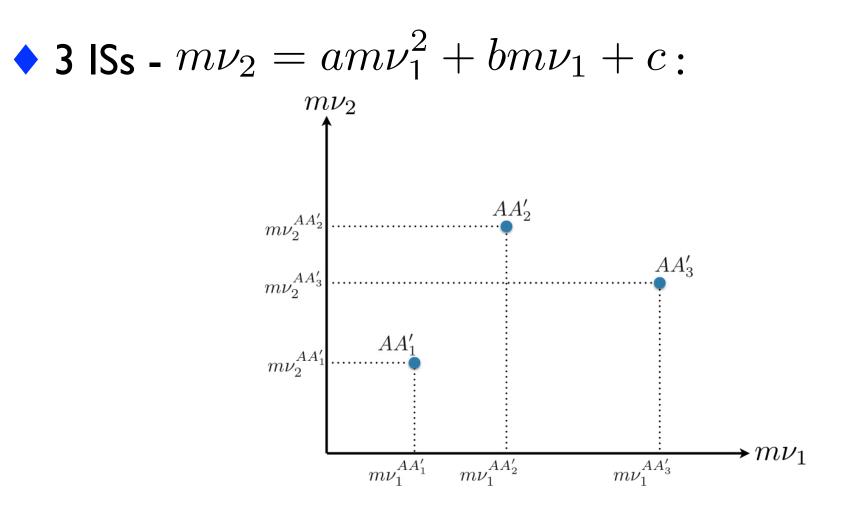
$$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'} \,. \qquad m\nu_i^{AA'} \equiv \nu_i^{AA'} / \mu_{AA'}$$

$$\mu_{AA'} \equiv m_A^{-1} - m_{A'}^{-1}$$

Target accuracy: $\Delta m \nu_i^{AA'} / m \nu_i^{AA'} \lesssim 10^{-6}$. (currently: 10^{-4} , projected $< 10^{-9}$)

The observable: King comparison (1964)

• What would be the generic form of $\overrightarrow{m\nu}_2$ vs. $\overrightarrow{m\nu}_1$?



What about existing data ?

Limitation of method

$$\alpha_{\rm NP} = \frac{(\overrightarrow{m\nu}_1 \times \overrightarrow{m\nu}_2) \cdot \overrightarrow{m\mu}}{(\overrightarrow{m\mu} \times \vec{h}) \cdot (X_1 \, \overrightarrow{m\nu}_2 - X_2 \, \overrightarrow{m\nu}_1)}$$

Berengut, Budker, Delaunay, Flambaum, Frugiuele, Fuchs, Grojean, Harnik, Ozeri, GP & Soreq (17)

- Only useful to bound new physics (barring cancellation).
- Short range NP: $X_i \propto F_i \Rightarrow \vec{v}$ is redefined to absorb NP; requires extra carefulness when approaching this limit.

As long as linearity holds bounds are limited by exp' accuracy:

$$\alpha_{\rm NP} \lesssim \sigma_{\alpha_{\rm NP}} = \sqrt{\sum_k (\partial \alpha_{\rm NP} / \partial O_k)^2 \sigma_k^2},$$

(O_K various exp' observables.)

Once non-linearity observed bound will be set by observation.