The measurement problem in quantum theory and possible implications for the expectations of primordial tensor modes from inflation.

Daniel Sudarsky, ICN-UNAM, México.
Collaborations with: A. Perez, H. Sahlmman, P. Perale, P. Cañate, G. León García, E. Okon, and A. Majhi
Marseille, France

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PLAN OF THIS TALK:

1) The problem with QM and tinkering with it. We consider applying to the whole universe because we think it applies to all physics.

2) Observations about issues in Quantum Theory, in particular as they concern gravitational contexts.

3) Exploring the Gravity/ Quantum Interface.

4) The Primordial Fluctuations in Inflationary Cosmology. Different treatment, similar predictions but in a more clear conceptual framework.

5) The Tensor Modes (Polarization B Modes). The same treatment as before but now the predictions are different.

6) Other applications of the line of research and some final comments.

The measurement problem in Quantum Theory:
2 rules determining the change in the quantum state: U and R.
No satisfactory rule specifying which one applies. (i.e. what exactly constitutes a measurement?)

The Mini-Mott *gedanken-experiment* : Consider a 2 level detector $|-\rangle$ (ground) & $|+\rangle$ (excited), and take two of them located at $x = x_0$ & $x = -x_0$. They are both initially in the ground state. Take a free particle with initial wave function $\psi(x, 0)$ given by a simple gaussian centered at $x = 0$ (so the whole set up is symmetric w.r.t $x \to -x$).

The particles's Hamiltonian: $\hat{H}_P = \hat{p}^2/2M$ while that of each detector is

$$\hat{H}_i = \epsilon \hat{I}_p \otimes \{|+\rangle^{(i)}\langle+|^{(i)} - |-\rangle^{(i)}\langle-|^{(i)}\}. \qquad (1)$$

where $i = 1, 2$. The interaction of particle and detector 1 is

$$\hat{H}_{P1} = \frac{g}{\sqrt{2}}\delta(\hat{x} - x_0\hat{I}_p) \otimes (|+\rangle^{(1)}\langle-|^{(1)} + |-\rangle^{(1)}\langle+|^{(1)}) \otimes \hat{I}_2 \qquad (2)$$

and similar expression for the particle's interaction with detector 2.

Schrödinger's equation can be solved for the initial condition

$$\Psi(0) = \sum_x \psi(x,0)|x\rangle \otimes |-\rangle^{(1)} \otimes |-\rangle^{(2)}$$

and it is clear that after some time t we have

$$\Psi(t) = \sum_x \psi_1(x,t)|x\rangle \otimes |+\rangle^{(1)} \otimes |-\rangle^{(2)} + \sum_x \psi_2(x,t)|x\rangle \otimes |-\rangle^{(1)} \otimes |+\rangle^{(2)}$$

$$+ \sum_x \psi_0(x,t)|x\rangle \otimes |-\rangle^{(1)} \otimes |-\rangle^{(2)} + \sum_x \psi_D(x,t)|x\rangle \otimes |+\rangle^{(1)} \otimes |+\rangle^{(2)}$$

One can interpret the last two terms easily: no detection and double detection (involving bounce) which is small $O(g^2)$.
One might think the first two terms indicate the initial symmetry was broken with high probability: Either detector 1 was excited or detector 2 was.
We just use some kind of " natural interpretation" and everything is fine, ...really?

The problem can be seen by considering instead the:
alternative state basis for the detectors (or “context")

$$|U\rangle \equiv |+\rangle^{(1)} \otimes |+\rangle^{(2)} \tag{3}$$

$$|D\rangle \equiv |-\rangle^{(1)} \otimes |-\rangle^{(2)} \tag{4}$$

$$|S\rangle \equiv \frac{1}{\sqrt{2}}[|+\rangle^{(1)} \otimes |-\rangle^{(2)} + |-\rangle^{(1)} \otimes |+\rangle^{(2)}] \tag{5}$$

$$|A\rangle \equiv |\frac{1}{\sqrt{2}}[|+\rangle^{(1)} \otimes |-\rangle^{(2)} - |-\rangle^{(1)} \otimes |+\rangle^{(2)}] \tag{6}$$

In fact these are more convenient for describing issues related to symmetries of the problem.

It is then easy to see that the $x \to -x$ and $1 \to 2$ symmetry of the initial setting and of the dynamics prevents the excitation an asymmetric term.
Often **decoherence** is presented as **a solution** but it is not! We can talk extensively about this at the end .

The issue is thus: can we or can we not describe things in this basis? And, if not, why not?

An experimental physicist in the Lab has no problem. He/She knows by practice (FAPP) to use the other basis (he/she knows that the detectors are always either exited or un-exited.. never perceives them in superposition). The measurement problem can be seen as : Exactly how does our theory account for that *experience* of our experimental colleague? Often we just do not care.

However, if we contend with situation where there is no experimentalist.... and nothing else in the universe (except perhaps for, say, a Maxwell field which is also in its vacuum state), we simply do not know what to do.

In that situation, why would we believe the conclusions drawn in the first context but not those of the second?. i.e. How do we account for the breakdown of the symmetry?

Ideas about addressing the MP can be ordered using the following result Tim Maudlin (*Topoi* **14**, 1995).

The following 3 premises can not be held simultaneously in a self consistent manner:

i) **The characterization of a system by its wave function is complete.** Its negation leads, for instance to hidden variable theories.

ii)**The evolution of the wave function is always according to Schrödinger's equation.** Its negation leads, for instance to spontaneous collapse theories.

iii) **The results of experiments lead to definite results.** Its negation leads for instance to Many World/ Minds Interpretations, Consistent histories approach, etc.

We will focus on ii). Specifically on spontaneous collapse theories.

Collapse Theories: Large amount of work: GRW, Pearle, Diosi, Penrose, Bassi (recent advances to make it compatible with relativity Tumulka, Bedningham, Pearle).

The basic idea is to unify U and R. The changes are small when a few DOF are involved and become large when something like 10^{23} are entangled (and delocalized).

These address the problem successfully and are empirically viable.

As with any reasonable proposal capable of dealing with (experimentally confirmed) violations of Bell's inequalities, they involve a degree of non-locality, occurring in the "probability laws".

Continuous Spontaneous Localization (CSL) P. Pearle .

Continuous version of GRW . *The theory is defined by :*

i) A modified Schrödinger equation, whose solution is:

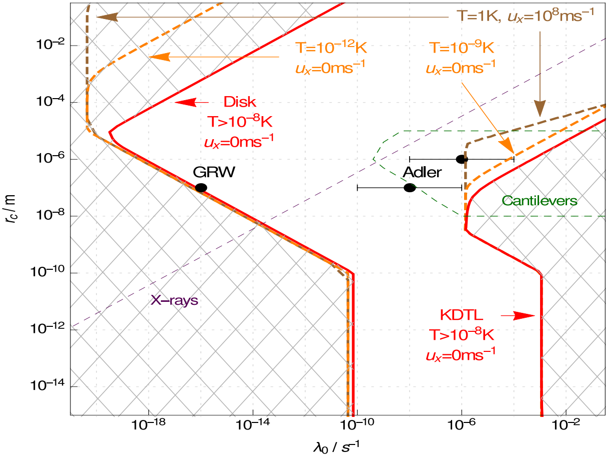
$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\gamma}[w(t') - 2\gamma\hat{A}]^2 \right]} |\psi, 0\rangle. \qquad (7)$$

($\hat{\mathcal{T}}$ is the time-ordering operator). $w(t)$ is a random classical function of time, of white noise type, whose probability is given by the second equation, ii) the Probability Rule:

$$PDw(t) \equiv {}_w\langle\psi, t|\psi, t\rangle_w \prod_{t_i=0}^{t} \frac{dw(t_i)}{\sqrt{2\pi\gamma/dt}}. \qquad (8)$$

The processes U and R (corresponding to the observable $\hat{A}$) are unified. For non-relativistc QM the proposal assumes : $\hat{A} = \hat{\vec{X}}_{r_c}$ (smeared with scale $r_c \sim 10^{-5} cm$). The combination of γ and r_c leads to an effective collapse rate for individual particle's wavefunctions λ_0 which must be small enough (not to conflict with tests of QM) and big enough to result in rapid localization of "macroscopic objects". The suggested range: $\lambda_0 \sim 10^{-17} sec^{-1}$.

The theory is being experimentally tested.



We need to adapt this to the gravitational and field theory contexts.

The exploration of the GR/ QT regime is done here in a bottom -up approach.

Usual top-down approach: (String Theory , LQG, Causal sets, dynamical triangulations, etc.) and attempt to connect to regimes of interest of the "world out there" : Cosmology, Black Holes, etc.

The bottom- up approach, push existing, well tested and developed theories, to address open issues that seem to lie beyond their domain. Possible modifications can serve as clues about the nature of the more fundamental theory .

The idea is to push GR together with QFT (i.e. semi-classical gravity) into realms/questions usually not explored.

Let us note that: The interface between QT and Gravitation need not involve the Planck regime: (space-time associated with a macroscopic body in quantum superposition of being in two locations).

Page and Gleiker (PRL 1981) consider an experiment and claim it to show semi-classical GR is not viable.

They argue that:

1) If there are no Quantum Collapses, then semi-classical GR conflicts with their experiment.

2) If there are Quantum Collapses, then semi-classical GR equations are inconsistent.

HOW DO MAKE SENSE OF OUR APPROACH THEN ?

Regard semi-classical GR as an approximated description with limited domain of applicability and to push that domain beyond what is usual : incorporate quantum collapses. It is clear that during the collapse the equations can not be valid. The proposal is to adopt an hydro-dynamical analogy:

Navier-Stokes equations for a fluid can not hold in some situations but they can be taken to hold before and after . Take Semi-classical GR equations to hold before and after a collapse but not during the collapse.

The approach will require providing a recipe to join the descriptions just before and just after the collapse.

Incorporate collapse to GR. At the formal level we rely on the notion of *Semi-classical Self-consistent Configuration* (SSC).

DEFINITION: The set $g_{\mu\nu}(x), \hat{\varphi}(x), \hat{\pi}(x), \mathcal{H}, |\xi\rangle \mathrm{in} \mathcal{H}$ represents a SSC iff $\hat{\varphi}(x)$, $\hat{\pi}(x)$ y $\mathcal{H}$ corresponds to QFT in CS over the space-time with metric $g_{\mu\nu}(x)$, and MOREOVER the state $|\xi\rangle$ in $\mathcal{H}$ is such that:

$$G_{\mu\nu}[g(x)] = 8\pi G \langle \xi | \hat{T}_{\mu\nu}[g(x), \hat{\varphi}(x), \hat{\pi}(x)] | \xi \rangle.$$

Note that this is a kind of GR version of the Schödinger -Newton system (and, as non-linear !).

Collapse: a transition for one complete SSC to another one. That is, we do not have simple jumps in states but jumps of the form $....SSC1.... \to ....SSC2....$

Matching conditions: for space-time and states in the Hilbert space. Involves delicate issues. Will become highly nontrivial when using a theory like CSL.

Could this fit with our current views regarding quantum gravity? Outstanding issues and conceptual difficulties:

I) The Problem of Time. Canonical Q.G. leads to timeless theory.

II) More generally how do we recover space-time from canonical approaches to QG ? (i.e. LQG).

Solutions to I) use a dynamical variable as a physical clock and consider relative probabilities (and wave functions). Following that line might lead to approx. Schrödinger eq. with corrections that violate unitarity.

Regarding II) there are many suggestions indicating space-time might be an emergent phenomena... T. Jacobson, R. Sorkin, N. Seiberg and many others.... It is not clear that, g_{ab}, as such, should be "quantized ".

Talk about space-time concepts implies a classical description. Some traces of QG regime might remain relevant, and "look like collapses"?

COSMIC INFLATION

Contemporary cosmology includes inflation as one of its most attractive components.
Its biggest success: the account for emergence of the seeds of cosmic structure and a correct estimate of the corresponding spectrum.
The standard account at the theoretical/conceptual level is not truly satisfactory.

1) At the mathematical level it relies on perturbation theory and can not be framed as an approximation to a well defined, and, in principle exact treatment, to which the perturbative methods just provide a suitable approximation. Sometimes one can do no better than that. However whenever possible, the alternative is clearly desirable!

2) Recall that the staring point of the analysis is a RW space-time background

$$dS^2 = a(\eta)^2\{-d\eta^2 + d\vec{x}^2\}$$

inflating under the influence of an inflaton background field $\phi = \phi_0(\eta)$ (taken as $\langle\hat{\phi}_0(\eta)\rangle$ in a sharply peaked state for the zero mode). The scale factor then behaves approximately as $a(\eta) = \frac{-1}{\eta H_I}$.

On top of this, one considers "space dependent perturbations": $\delta\phi, \delta\psi, ..., \delta h_{ij}$, treated quantum mechanically & assumed to be characterized by the " vacuum state" (essentially the BD vacuum) $|0\rangle$.

Inflation dilutes all preexisting features and drives all space dependent fields towards their vacuum states.

Thus, Quantum-Mechanically the zero mode of the field is taken to be in highly excited (and sharply peaked) state, while the space dependent modes are in the vacuum state.

The state of the quantum field is *"also characterized"* by the so called "quantum fluctuations" or "uncertainties".
Unfortunate use of a single word to refer to various things: *i)Statistical variations in an otherwise symmetric ensemble, ii) Spatial variations on a single extended object which is homogeneous at large scales, and iii) quantum indeterminancies.*
In our case, these are uncertainties or indeterminancies **of** the quantum state, **for** the field and conjugate momentum operators.

These are then **unjustifiably** (contentious view)identified as the primordial inhomogeneities which eventually evolved into all the structure in our Universe: galaxies, stars planets, etc...

In fact, note that, according to this picture: The Universe was H&I, (both in the part that could be described at the “classical level", and the quantum level) as a result inflation.

[A displacement of the state by $\vec{D}$ is $e^{i\hat{\vec{P}}.\vec{D}}|0\rangle = |0\rangle$ so it is completely homogeneous.]

However the end situation is not.

How does this happen if the dynamics of the closed system does not break those symmetries.?

OUR APPROACH:

Required: a physical process occurring in time, explaining the emergence of the seeds of structure. Emergence means : Something that was not there at a time, is there at a latter time. We need to explain the breakdown of the symmetry of the initial state: Collapse can do this.

Thus we Add to the standard inflationary paradigm, a quantum collapse of the wave function as a self induced processes.

The collapse is described by a modification of the dynamics of fields in space-time, so **we should rely on a classical description of the space-time geometry.**

Adapt to the GR context through the SSC formalism.

Space-time is treated classically (using a specific gauge):

$ds^2 = a^2(\eta)\{-(1+2\Psi(x,\eta))d\eta^2 + [(1-2\Psi(x,\eta))\delta_{ij} + h_{ij}(x,\eta)]dx^i dx^j\}$

Set $a = 1$ at the “present cosmological time", assume that inflationary regime ends at a value of $\eta = \eta_0$.

$$a(\eta) = \frac{-1}{\eta H_I} \text{ with } \eta \text{ in } (-\mathcal{T}, \eta_0), \eta_0 < 0.$$

The scalar field must be treated using QFT in curved space-time. The quantum state of the scalar field and the space-time metric satisfy Einstein's semi-classical eq.

$$G_{\mu\nu} = 8\pi G \langle\xi|\hat{T}_{\mu\nu}|\xi\rangle.$$

We have shown how to deal with it in a single discrete collapse using SSC in *JCAP*. **045**, 1207, (2012); 1108.4928 [gr-qc]) .

Treatment extended to second order (*JCAP*,**1808** 43 (2018) arXiv:1802.02238 [gr-qc]).

We will concentrate next on the $\vec{k} \neq 0$ modes .

Early stages of inflation $\eta = -\mathcal{T}$, the state of the $\hat{\delta}\phi$ is the Bunch-Davies vacuum, and the space-time is 100 % homogeneous and isotropic.

In the vacuum state , the operators $\hat{\delta}\phi_k$ $\hat{\pi}_k$ are characterized by gaussian wave functions centered on 0 with uncertainties $\Delta\delta\phi_k$ and $\Delta\pi_k$.

The collapse modifies the quantum state, and the expectation values of $\hat{\delta\phi_k}(\eta)$ and $\hat{\pi}_k(\eta)$.

Assume the collapse occurs mode by mode and described by an adapted version of collapse theories.

Our universe would correspond to one specific realization of the stochastic functions (one for each $\vec{k}$).

Let us study first the scalar metric perturbations $\Psi(\eta, x)$.

The Fourier decomposition of the semi classical Einstein's Equations give:

$$-k^2\Psi(\eta,\vec{k}) = \frac{4\pi G\phi_0'(\eta)}{a}\langle\hat{\pi}(\vec{k},\eta)\rangle \quad (9)$$

At ($\eta = -\mathcal{T}$) state is the vacuum, so $\langle\hat{\pi}(\vec{k},\eta)\rangle = 0$, and THUS the space-time is 100% homogeneous and isotropic. Now, the quantity of main observational interest is:

$$\frac{\Delta T(\theta,\varphi)}{\bar{T}} = \Psi(\eta_D, R_D, \theta, \varphi) = c\int d^3k e^{i\vec{k}\cdot\vec{x}}\frac{1}{k^2}\langle\hat{\pi}(\vec{k},\eta_D)\rangle, \quad (10)$$

corresponding to the point on the intersection of our past light cone with the last scattering surface ($\eta = \eta_D$) in the direction specified by θ, φ.Thus:

$$\alpha_{lm} = c\int d^2\Omega Y_{lm}^*(\theta,\varphi)\int d^3k e^{i\vec{k}\cdot\vec{x}}\frac{1}{k^2}\langle\hat{\pi}(\vec{k},\eta)\rangle. \quad (11)$$

No analogous to this expression in the standard approaches!

Again,

$$\alpha_{lm} = c \int d^2\Omega Y^*_{lm}(\theta, \varphi) \int d^3 k e^{i\vec{k}\cdot\vec{x}} \frac{1}{k^2} \langle \hat{\pi}(\vec{k}, \eta) \rangle. \qquad (12)$$

The eq. above shows that the quantity of interest can be thought of as a result of a “random walk" on the complex plane.

One can’t predict the end point of such “walk", but can focus on the magnitude of the total displacement and estimate its ML value by an ensemble average. Compute the ensemble average at “late times"

$$\overline{(\langle \hat{\pi}(\mathbf{k}, \eta) \rangle \langle \hat{\pi}(\mathbf{k}', \eta) \rangle^*)} = f(k)\delta(\mathbf{k} - \mathbf{k}').$$

Then,

$$\overline{|\alpha_{lm}|^2} = (4\pi c)^2 \int_0^\infty dk j_l(kR_D)^2 \frac{1}{k^2} f(k). \qquad (13)$$

Agreement with observations requires $f(k) \sim k$. The oscillations are generated by late time physics on top of the primordial flat spectrum.

With reasonable choices in the details of the collapse theory this can be achieved:

In CSL version: Collapse in the field operator or the momentum conjugate operators with $\lambda = \tilde{\lambda} k^{\pm 1}$ fixed by dimensional considerations (or collapse in the operators $(-\nabla^2)^{-1/4}\hat{\pi}(\vec{x})$ or $(-\nabla^2)^{1/4}\hat{\phi}(\vec{x})$) . Why is this the right thing?

The resulting prediction for the power spectrum is:

$$P_S(k) \sim (1/k^3)(1/\epsilon)(V/M_{Pl}^4)\tilde{\lambda}\mathcal{T} \tag{14}$$

Taking GUT scale for the inflation potential, and standard values for the slow-roll, leads to agreement with observation for: $\tilde{\lambda} \sim 10^{-5} MpC^{-1} \approx 10^{-19} sec^{-1}$.

Not very different from GRW suggestion ! .

[*PRD*, **87**, 104024 (2013)] .Other treatments with similar spirit by J. Martin, V. Vennin & P. Peter, [*PRD*, **86** , 103524 (2012)], and S. Das, K. Lochan, S. Sahu & T. P. Singh [*PRD*, **88**, 085020 (2013)]

TENSOR MODES

Similarly, the equation of motion for the tensor perturbations is:

$$(\partial_0^2 - \nabla^2)h_{ij} + 2(\dot{a}/a)\dot{h}_{ij} = 16\pi G \langle (\partial_i \delta\phi)(\partial_j \delta\phi) \rangle_{Ren}^{tr-tr} \qquad (15)$$

$tr - tr$ stands for the transverse trace-less part of the expression (retaining only dominant terms).

Note that it is quadratic in the collapsing quantities !!

Passing to a Fourier decomposition, we solve the eq.

$$\ddot{\tilde{h}}_{ij}(\vec{k},\eta) + 2(\dot{a}/a)\dot{\tilde{h}}_{ij}(\vec{k},\eta) + k^2 \tilde{h}_{ij}(\vec{k},\eta) = S_{ij}(\vec{k},\eta), \qquad (16)$$

with zero initial data, and source term:

$$S_{ij}(\vec{k},\eta) = 16\pi G \int \frac{d^3x}{\sqrt{(2\pi)^3}} e^{i\vec{k}\vec{x}} \langle (\partial_i \delta\phi)(\partial_j \delta\phi) \rangle_{Ren}^{tr-tr}(\eta,\vec{x}). \qquad (17)$$

The result is formally divergent, however we must introduce a cut-off *(the last scale exiting the horizon during inflation: $a_{end-inf} p_{UV}/2\pi = H_I$, or more realistically the scale of diffusion (Silk) dumping with $p_{UV} \approx 0.078 MpC^{-1}$).*

After a long calculation the prediction for the power spectrum of tensor perturbations is:

$$P_h(k) \sim (1/k^3)(V/M_{Pl}^4)^2(\tilde{\lambda}^2 \mathcal{T}^4 p_{UV}^5/k^3) \tag{18}$$

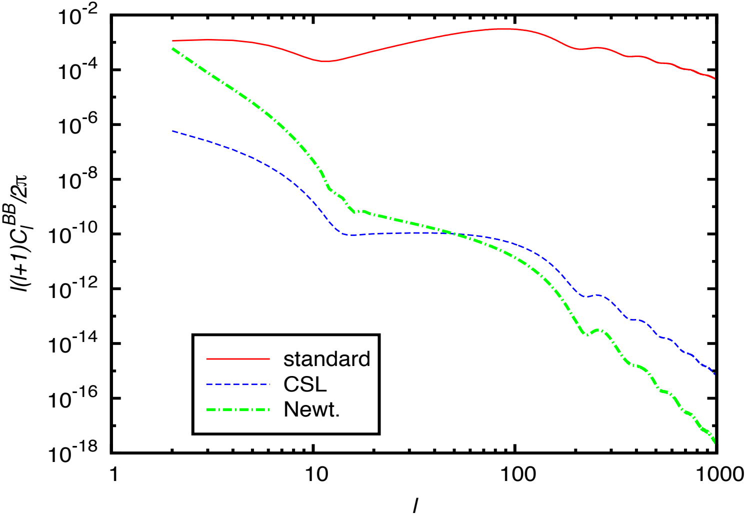
($\mathcal{T}$ the conformal time at the start of inflation taken for standard inflationary parameters as 10^4 MpC) while the power spectrum for the scalar perturbations is (as we saw):

$$P_S(k) \sim (1/k^3)(1/\epsilon)(V/M_{Pl}^4)\tilde{\lambda}\mathcal{T} \tag{19}$$

That is very different relation between them than usual. Thus, expected not to see tensor modes at the level they are being looked for!!

PRD **96**, 101301(R) (2017); *PRD* **98** 023512 (2018) .

We also considered a simpler collapse model, and again obtained **reduced tensor mode amplitude** but with slightly different shape.



OTHER APPLICATIONS OF THIS APPROACH

1) Proposal for resolution of the Black Hole Information Puzzle [with E. Okon, S. K. Modak, I. Pena, & D. Bedinghm].

2) Possibility of accounting for the anomalous low power in CMB spectrum at large angles [with G. L. García].

3) Promising in dealing with the Problem of Time in Quantum Gravity [with E. Okon].

4) Possible explanation for Penrose's Weyl Curvature hypothesis, for the initial state of the Universe [with E. Okon] .

5) Possible explanation for the value of Λ (in the context of unimodular gravity) [works with T. Josset A. Perez & J Bjorken] .

6) Making Higgs Inflation less problematic [with S. Rodriguez].

Ignoring "the measurement problem" in problems at the Gravity/Quantum interface can be a serious source of confusion, while incorporation of proposals to address it might lead to resolution of seemingly unconnected problems.

THANKS

SOME DETAILS:

We work with a rescaled field $y(\eta, \vec{x}) \equiv a\delta\phi(\eta, \vec{x})$ and its momentum conjugate $\pi_y(\eta, \vec{x}) = a\delta\phi'(\eta, \vec{x})$.

For the evolution of the state of the quantum filed we use a CSL type dynamics with parameter λ_χ and collapse operators based on such objects.

$$|\psi, t\rangle = \mathcal{T} e^{-i\int_{-\mathcal{T}}^{\eta} d\eta' [\hat{H} - \frac{1}{4\lambda_\chi} \int d^3x [w(\vec{x}, \eta') - 2\lambda_\chi \chi(\vec{x})]^2} |\psi, -\mathcal{T}\rangle. \quad (20)$$

Clearly the dimension of $2\lambda_\chi \chi^2$ must be L^{-1} so $\chi = y$ $([y] = L^{-1})$, we have $[\lambda_\chi] = L^{-2}$, while for $\chi = \pi$,$([y] = L^{-2})$, we have $[\lambda_\chi] = 1$.

The GRW λ_{GRW} has dimensions of L^{-1}.

Collapse on Field Operators

We would like to understand how the collapse looks when described in terms of the space-time field operators. In one case we can start by defining

$$\tilde{y}(\vec{x}) \equiv \frac{1}{(2\pi)^{3/2}} \int d^3k e^{i\vec{k}\cdot\vec{x}} k^{1/2} y(\vec{k}) = (-\nabla^2)^{1/4} \hat{y}(\vec{x}), \qquad (21)$$

The state vector evolution given by

$$|\psi, t\rangle = \mathcal{T} e^{-i\int_{-\mathcal{T}}^{\eta} d\eta' \hat{H} - \frac{1}{4\tilde{\lambda}} \int_{-\mathcal{T}}^{\eta} d\eta' \int d^3x [w(\vec{x},\eta') - 2\tilde{\lambda}\tilde{y}(\vec{x})]^2} |\psi, -\mathcal{T}\rangle. \qquad (22)$$

This is just the standard CSL state-vector evolution, where the collapse-generating operators (toward whose joint eigenstates collapse tends) are $\tilde{y}(\vec{x})$ for all $\vec{x}$.

Similarly, in the case where we take $\hat{\Pi}$ as Generator of Collapse we have.

$$|\psi,\eta\rangle = \mathcal{T}e^{-i\int_{-\mathcal{T}}^{\eta} d\eta'\hat{H} - \frac{1}{4\tilde{\lambda}}\int_{-\mathcal{T}}^{\eta} d\eta' \int d^3x[w(\vec{x},\eta') - 2\tilde{\lambda}\tilde{\pi}(\vec{x})]^2}|\psi, -\mathcal{T}\rangle. \tag{23}$$

where $\tilde{\pi}(\vec{x}) \equiv (-\nabla^2)^{-1/4}\hat{\pi}(\vec{x})$.

This is just the standard CSL state-vector evolution, where the collapse-generating operators (toward whose joint eigenstates collapse drives all states) are $\tilde{\pi}(\vec{x})$ for all $\vec{x}$.

What are the fundamental reasons determining the appearance of the operators $(-\nabla^2)^{-1/4}\hat{\pi}(\vec{x})$ (or $(-\nabla^2)^{1/4}\hat{y}(\vec{x})$)?

A satisfactory answer will have to wait for a general theory expressing, in all situations, from particle physics, to cosmology, the exact form of the CSL-type of modification to the evolution of quantum states. Such generic theory would likely involve gravitation playing a fundamental role. The research must continue.