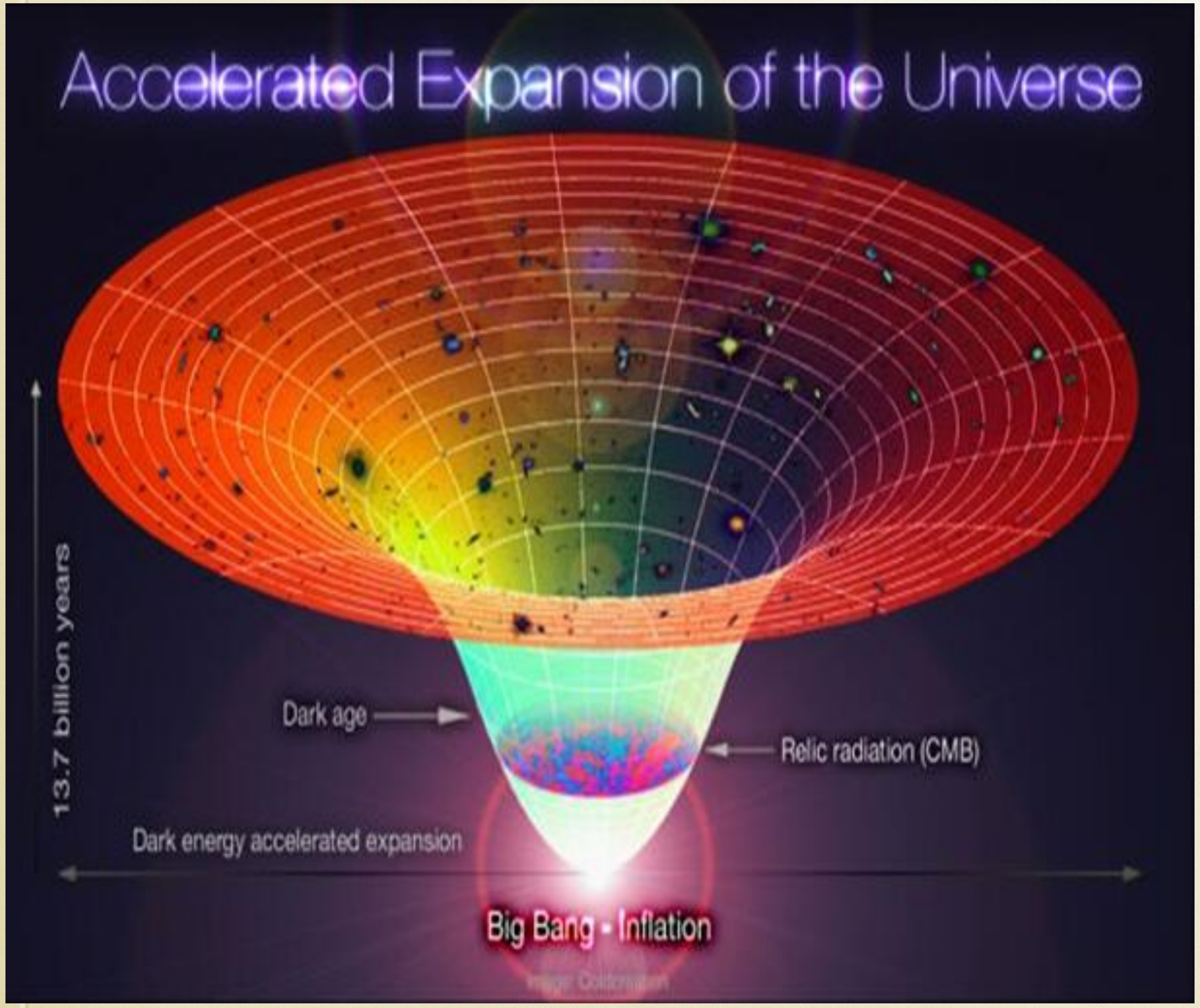
Baryon Acoustic Oscillations (BAO)

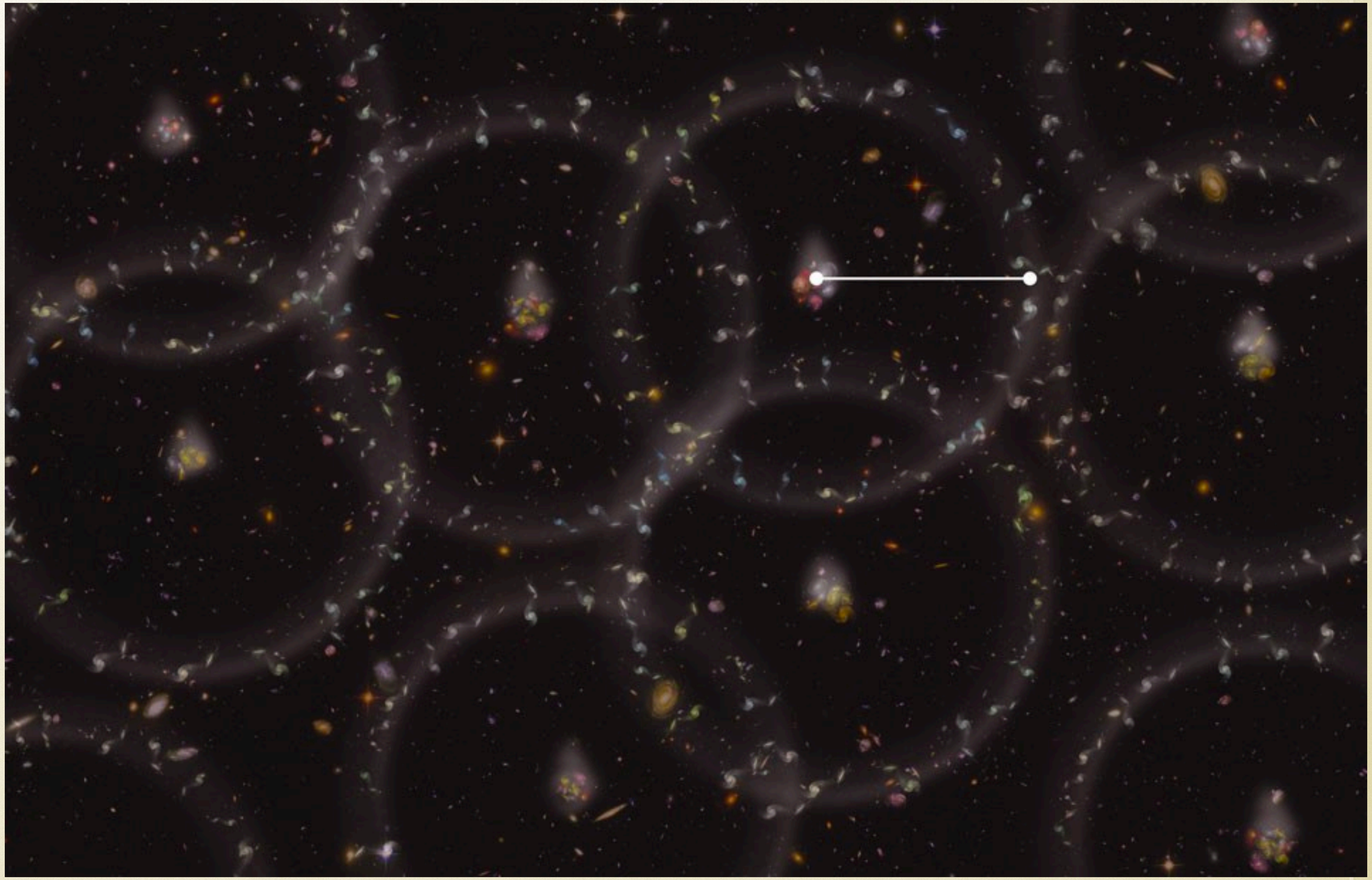
XinMeng Ye
YuTong Yang
Supervisor : Pierros Ntelis

Outline

- Introduction of BAO
- A feature to observe BAO——Two Point Correlation Function
    - Theory
    - Observation
- Detection of BAO Peak Position
- Estimation of Cosmological Parameters with BAO

Introduction of BAO





Two Point Correlation Function

- The number overdensity field or density contrast

$$\delta(t,x) = \frac{n(t,x) - \bar{n}(t)}{\bar{n}(t)}$$

- The two point correlation function

$$\xi(t, r_1, r_2) = \langle \delta(t, r_1)\delta(t, r_2) \rangle = \xi(t, r_1 - r_2) = \xi(t, |r_1 - r_2|)$$

- A simpler defination: the excess probability of finding a galaxy at a finite volume $d^3 r_2$ with radius $r = r_1 - r_2$ from a observed one in $d^3 r_1$.

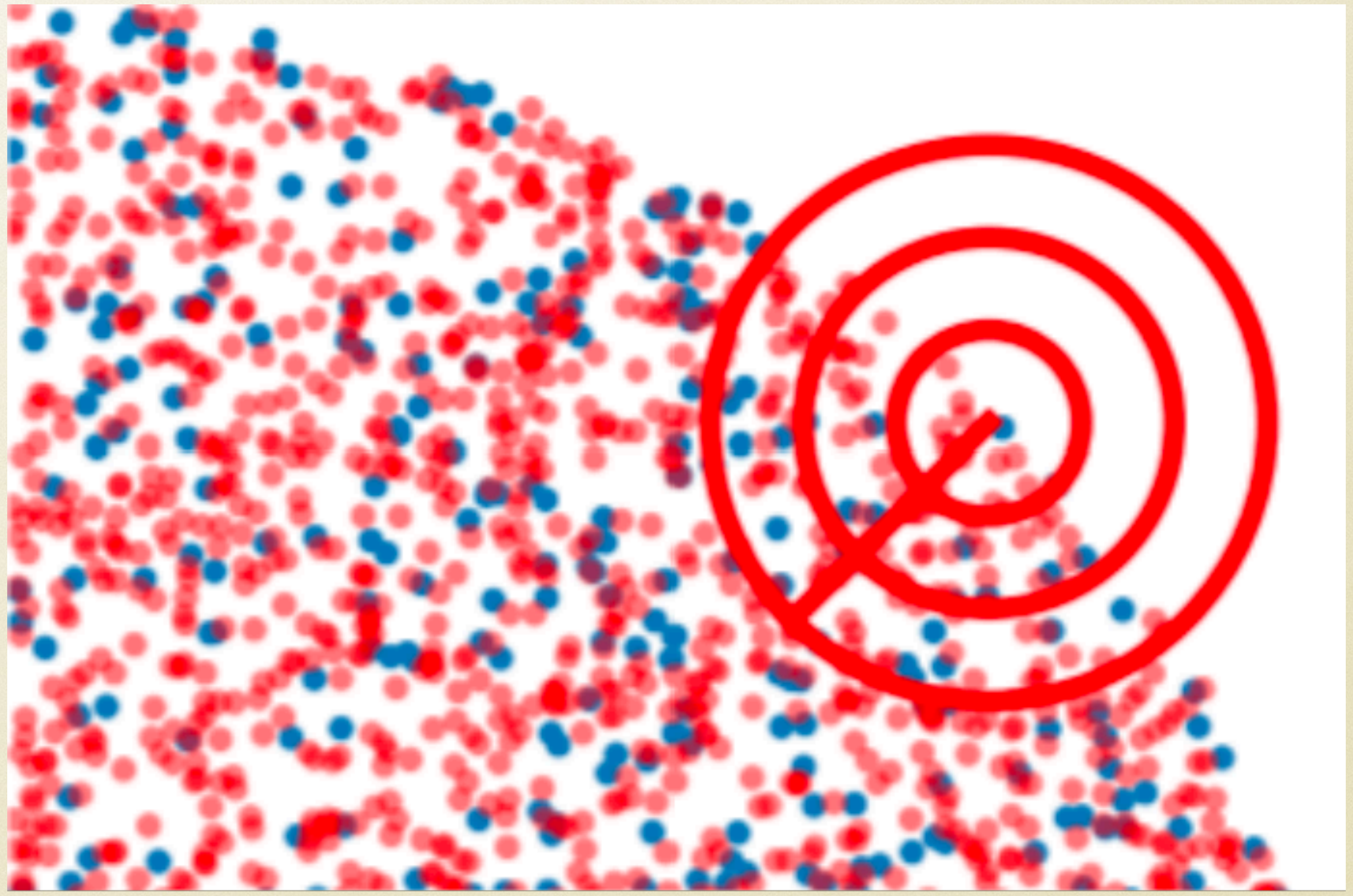
$$dP_{pair} = \bar{n}^2 \, [1 + \xi(r)] \, d^3 r$$

Estimators from Observation

- The estimators of the correlation function from actual galaxy data
- What we can observe is the counts-in-cells
- Landy and Szalay estimator:

$$\hat{\xi}_{ls}(r) = \frac{dd(r) - 2dr(r) + rr(r)}{rr(r)}$$

where $dd(r), dr(r), rr(r)$ are the normalized number of galaxy, galaxy random-point, random-point pairs with distance r.



Prediction from theory

- Theory Prediction of number density field
  - Perturbed Einstein Boltzmann Equation

$$D_t[f_X(\overrightarrow{x}, \overrightarrow{p}, t)] = C[f_X(\overrightarrow{x}, \overrightarrow{p}, t)]$$

  - Pertubed FLRW metric

$$ds^2 = -[1 + 2\Psi(\overrightarrow{x})]dt^2 + a^2(t)[1 + 2\Phi(t)]d\overrightarrow{x}^2$$

- Power Spectrum:

$$P(k_1, k_2) = \frac{1}{(2\pi)^3}\langle \delta(k_1)\delta(k_2)\rangle$$

- The correlation function are the Fourier transformation of the Power Spectrum

$$\xi(k) = \int \frac{d^3 r}{(2\pi)^3} P(k) e^{-ikr}$$

$$\dot{\delta} + ikv = -3\dot{\Phi}$$

$$\dot{v} + \frac{\dot{a}}{a} v = -ik\mu\Psi$$

$$\dot{\delta}_b + ikv_b = -ik\mu\Psi$$

$$\dot{v}_b + \frac{\dot{a}}{a} v_b = -ik\mu\Psi + \frac{\dot{\tau}}{R}[v_b + 3i\Theta_1]$$

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu v_b - \frac{1}{2}L_2(\mu)\Pi\right]$$

$$\Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$$

$$\dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}\left[-\Theta_P + \frac{1}{2}(1 - L_2(\mu))\Pi\right]$$

$$\dot{\Theta}_v + ik\mu\Theta_v = -\dot{\Phi} - ik\mu\Psi$$

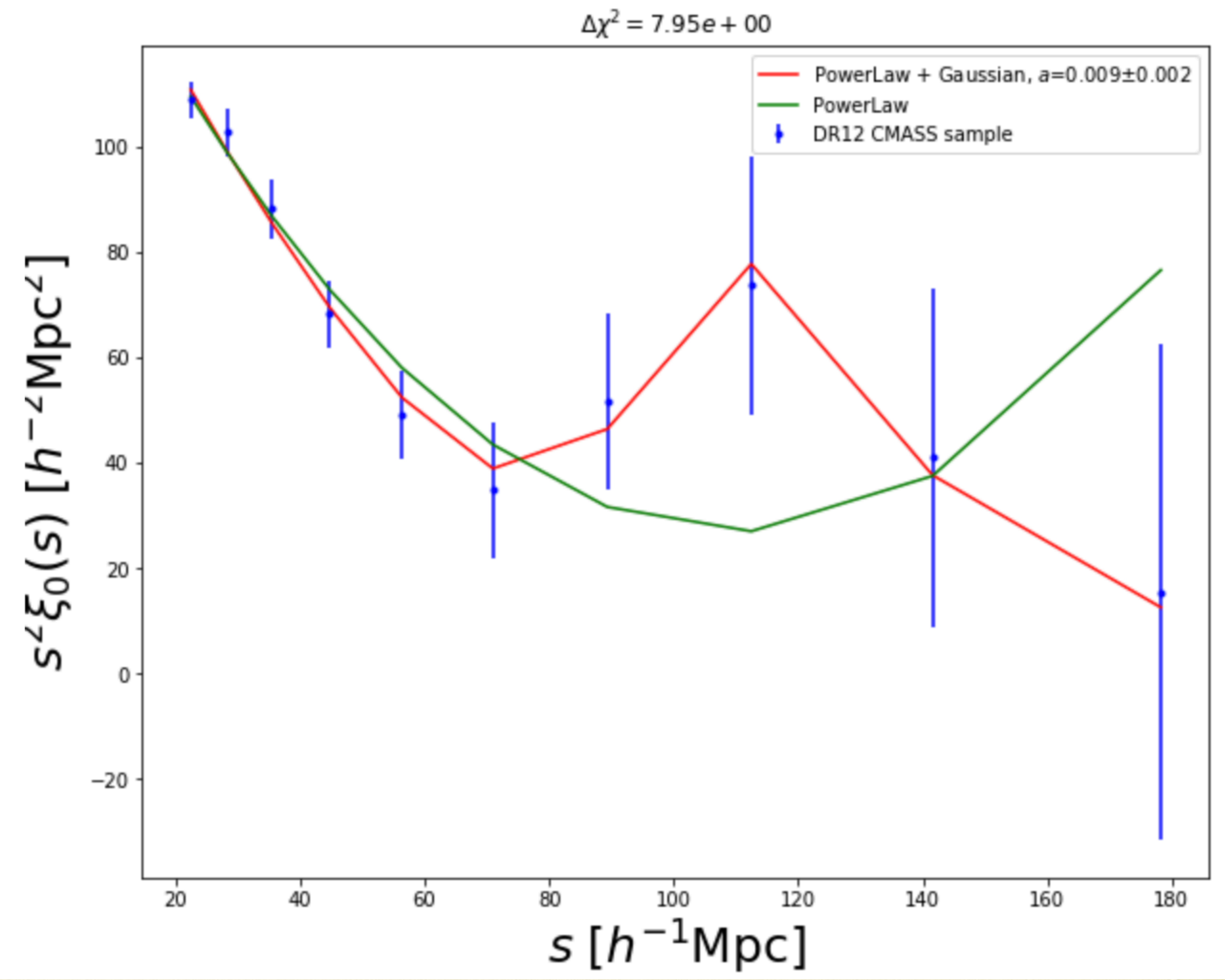
Extraction of Cosmology from BAO Peak Position

- Detect the BAO Peak Position
  - Data: current & more efficient telescope
  - covariance matrix & the diagonal of the covariance matrix
  - Significance method 1 & 2 : $\Delta\chi^2$ & χ^2
- Estimate cosmological parameters using BAO

Detect the BAO Peak Position

- $GaussianLaw(r) = A_{BAO} * exp(-0.5 * (\frac{r - \alpha_{iso} * r_d^{th}}{\sigma_{BAO}})^2)$
  - r_d^{th} - the theoretical value of the BAO peak
- $PowerLaw(r) = A_0 + A_1/r + A_2/r^2$
- Significance of the detection of the peak position:

$$\Delta\chi^2 = \chi^2_{Gaussian+PowerLaw} - \chi^2_{PowerLaw}$$



Method 1

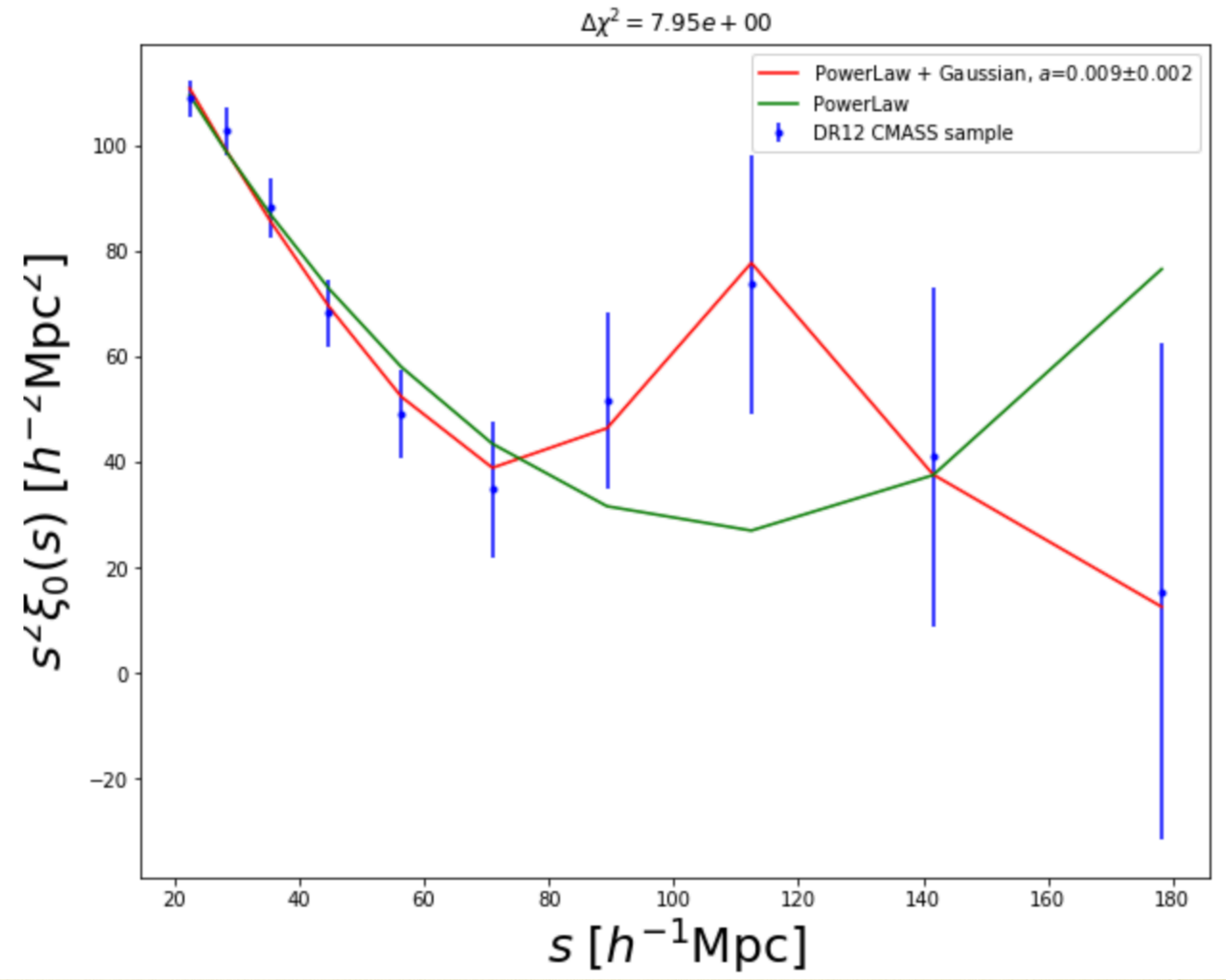
1. Fix the Gaussian on the Gaussion+Powerlaw model ➡️ Fit the Rest Parameters

- $GaussianLaw(r) = A_{BAO} * exp(-0.5 * (\frac{r - \alpha_{iso} * r_d^{th}}{\sigma_{BAO}})^2)$
- $PowerLaw(r) = A_0 + A_1/r + A_2/r^2$
- Result: α_{iso} = 1.114e+00, A_{BAO} = 6.269e-03, σ_{BAO} = 2.189e+01
  A_0 = 9.126e-03, A_1 = -2.471e+00, A_2 = 1.616e+02

⬇️

$\chi^2_{Gaussian+PowerLaw} = 1.667$ $\chi^2_{PowerLaw} = 9.619$

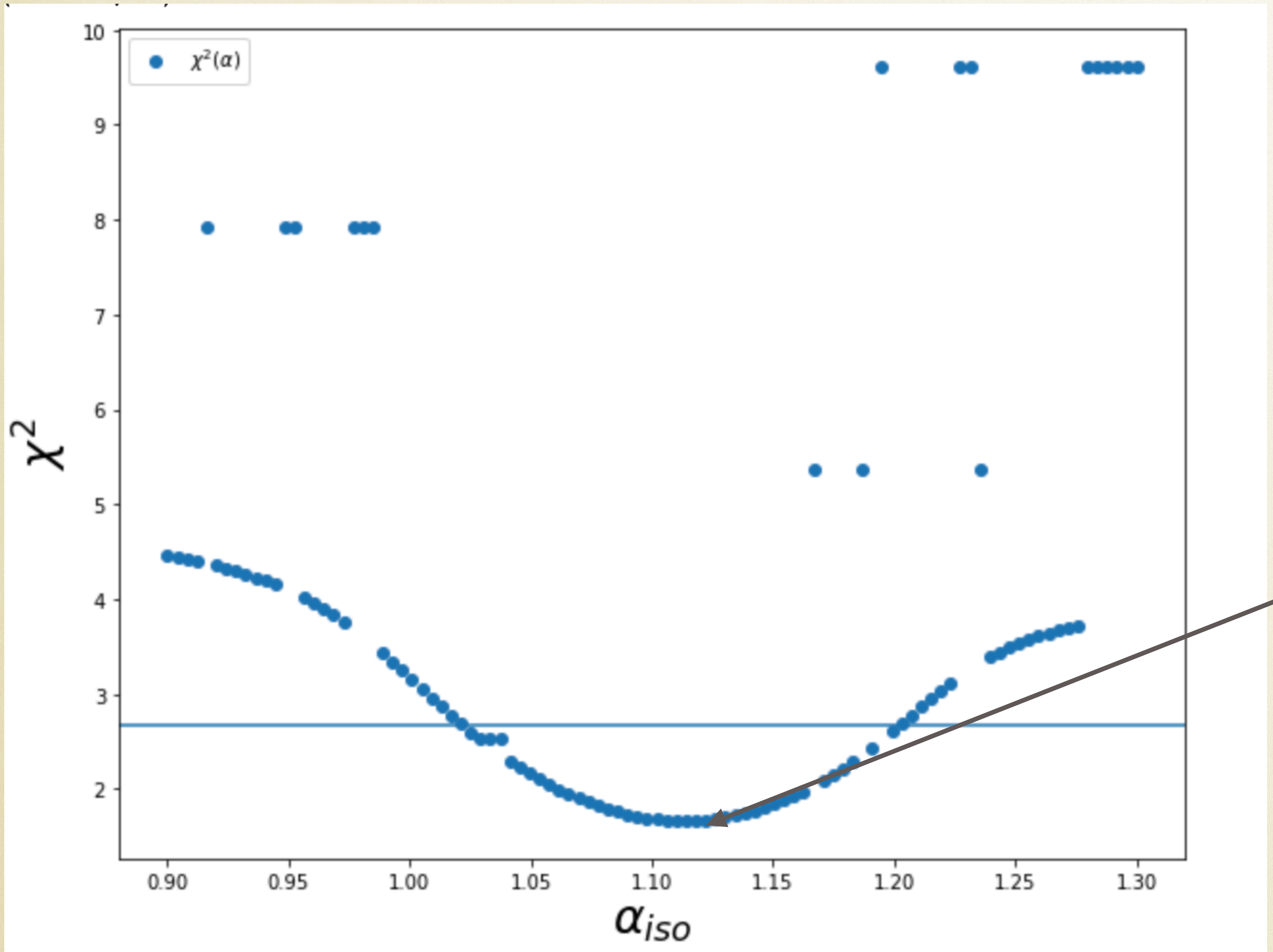
$\Delta\chi^2 = 7.952$ 👉 2 σ for the detection



the diagonal of the covariance matrix
the current telescope data

Method 1

2. Scan α_{iso} to Get $\chi^2(\alpha_{iso})$



Range of α_{iso} : from 0.9 to 1.3

$$\chi^2(\alpha_{iso}) = \chi^2_{min}(\alpha_{iso}) + 1 = 2.667$$

⬇️

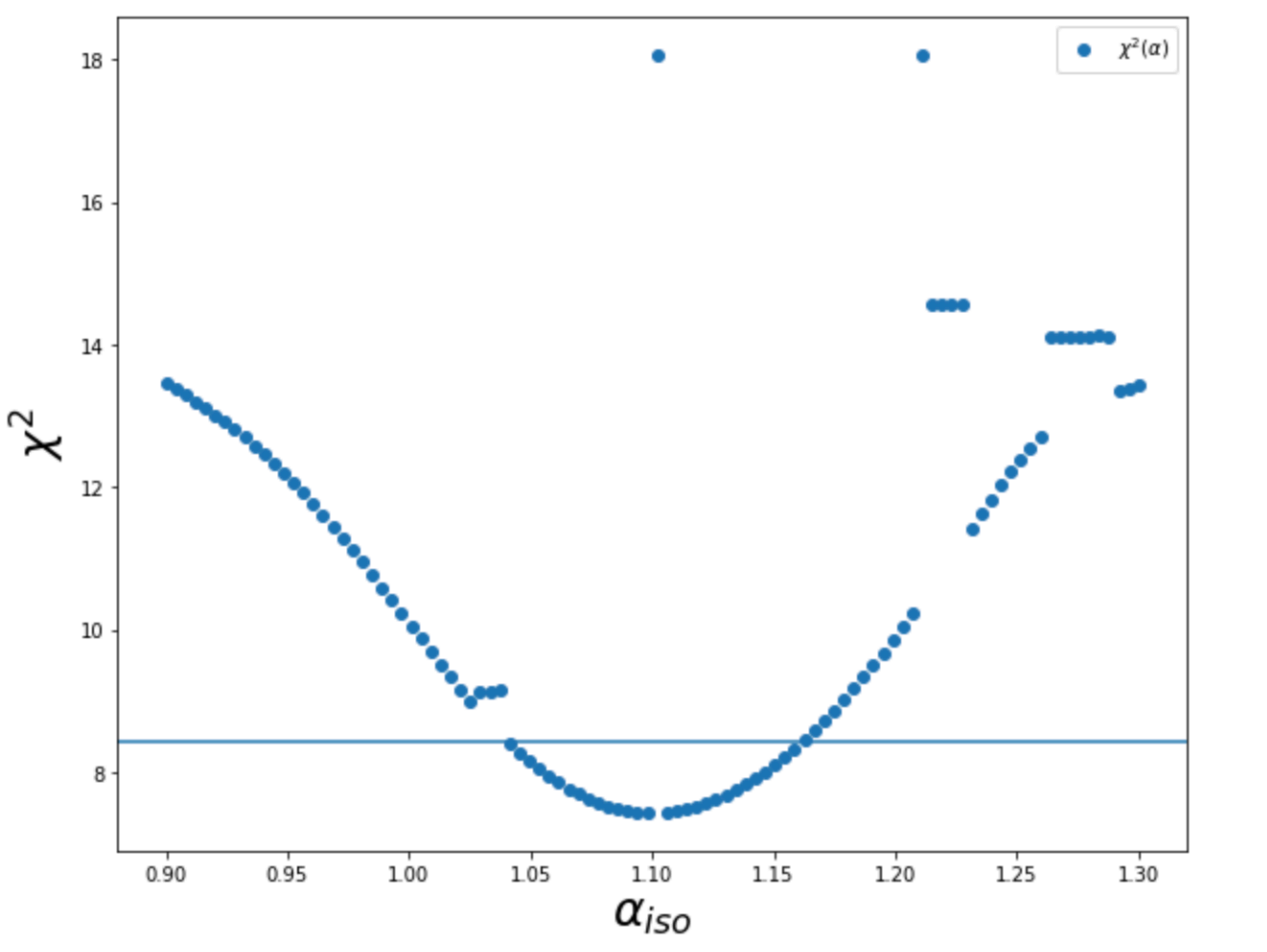
Method 2

1σ deviation of α_{iso} : $1.114^{+0.089}_{-0.093}$

2σ deviation of α_{iso} : $1.114^{+0.178}_{-0.186}$

the diagonal of the covariance matrix
the current telescope data

3. Using the Covariance Matrix of the Data



Range of α_{iso} : from 0.9 to 1.3

$$\chi^2(\alpha_{iso}) = 8.432$$

⬇️

Method 2

1σ deviation of α_{iso} : $1.098^{+0.065}_{-0.057}$

2σ deviation of α_{iso} : $1.098^{+0.129}_{-0.113}$

the covariance matrix
the current telescope data

👉 Covariance gives better estimates
👉 Significance: 0.5 σ

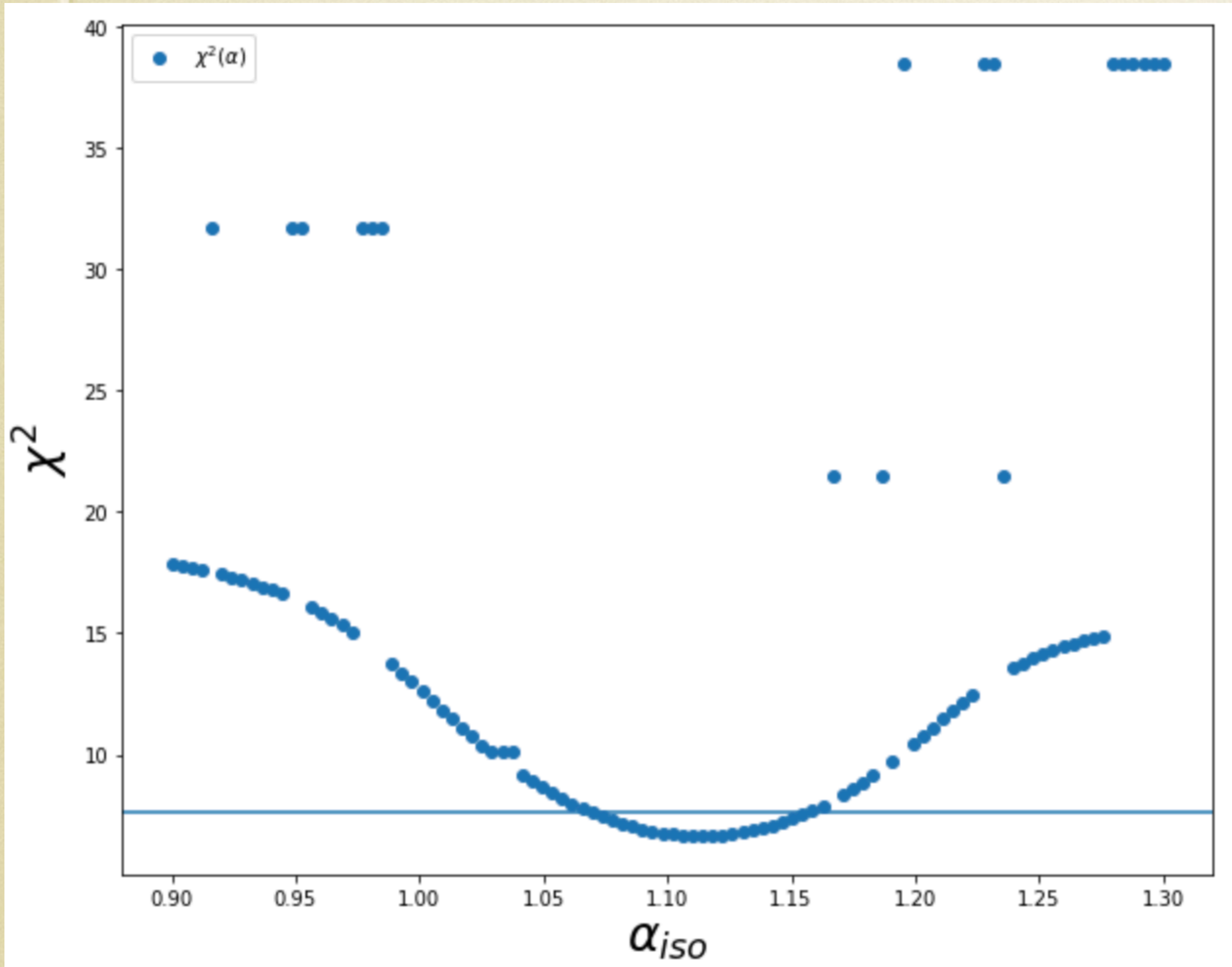
4. Detect BAO with Data from a more Efficient Telescope

🔭 NEW telescope 👉 extract the information of Trillions of Galaxies

👉 measure two point correlation function more efficiently

Method 2

the diagonal of the covariance matrix
the future telescope data



1σ deviation of α_{iso} :

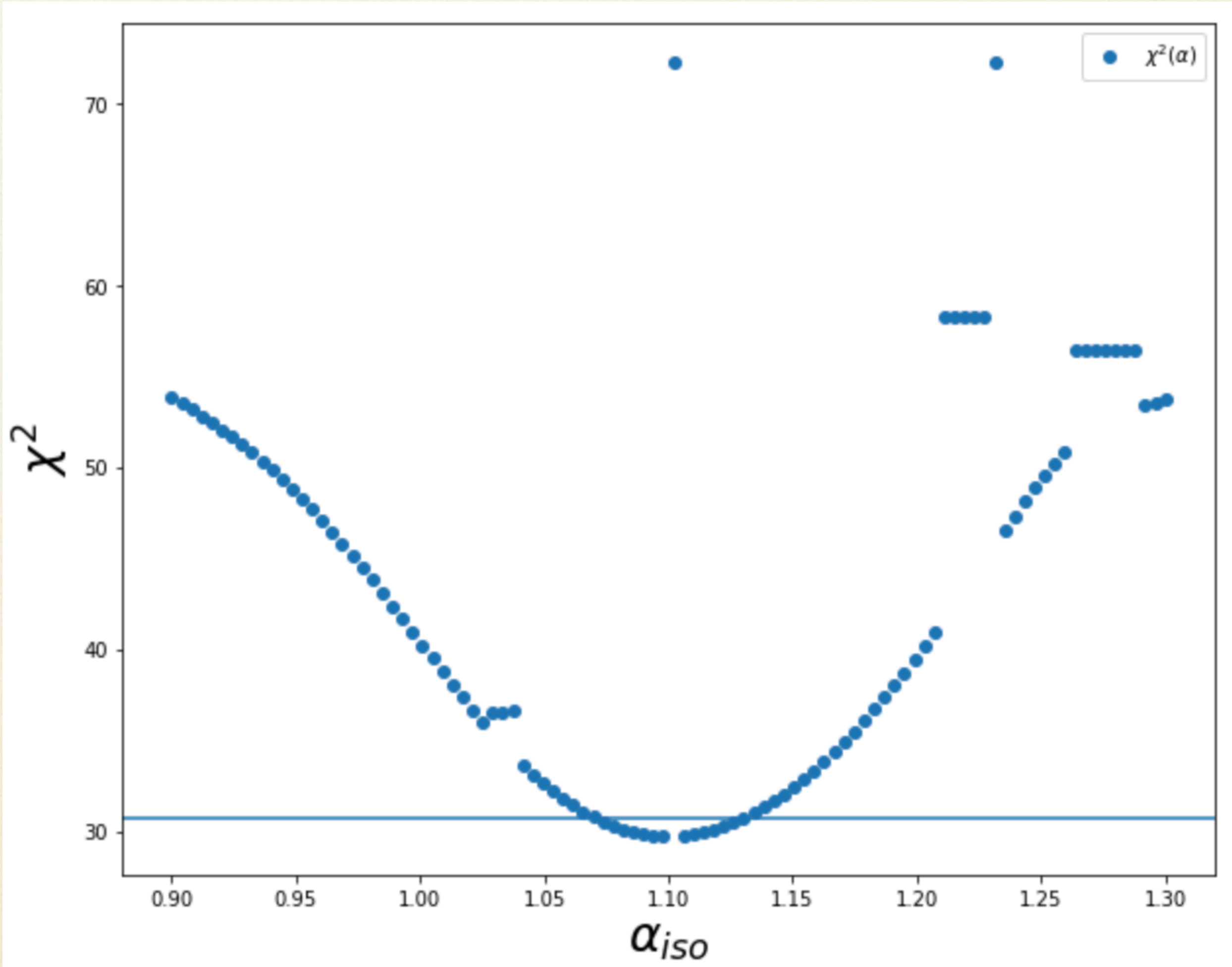
$1.114^{+0.044}_{-0.044}$

2σ deviation of α_{iso} :

$1.114^{+0.089}_{-0.089}$

$\chi^2(\alpha_{iso}) = \chi^2_{min}(\alpha_{iso}) + 1 = 7.669$

the covariance matrix
the future telescope data



1σ deviation of α_{iso} :

$1.098^{+0.032}_{-0.028}$

2σ deviation of α_{iso} :

$1.098^{+0.065}_{-0.057}$

$\chi^2(\alpha_{iso}) = \chi^2_{min}(\alpha_{iso}) + 1 = 30.729$

👉 Significance: 3-4σ

Estimate Cosmology Parameters

Add cosmological parameters contribution to α_{iso}

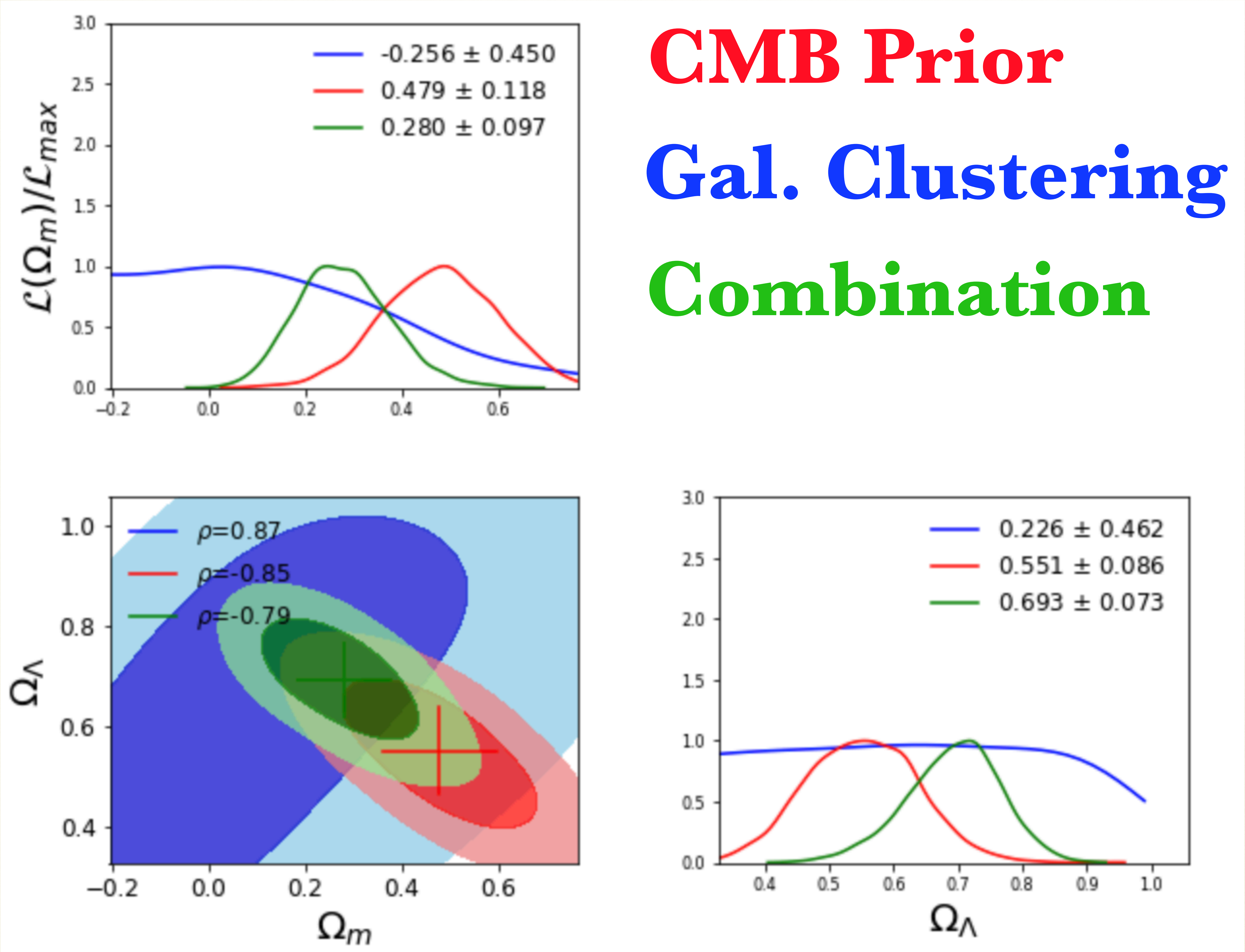
$$\xi^{Model}(r,\vec{\Omega}_F) = Gaus\left(r;\alpha_{iso}, A_{BAO}, \sigma_{BAO}, r_d^{th}(\vec{\Omega}_F)\right) + PowerLaw(r; A_0, A_1, A_2)$$

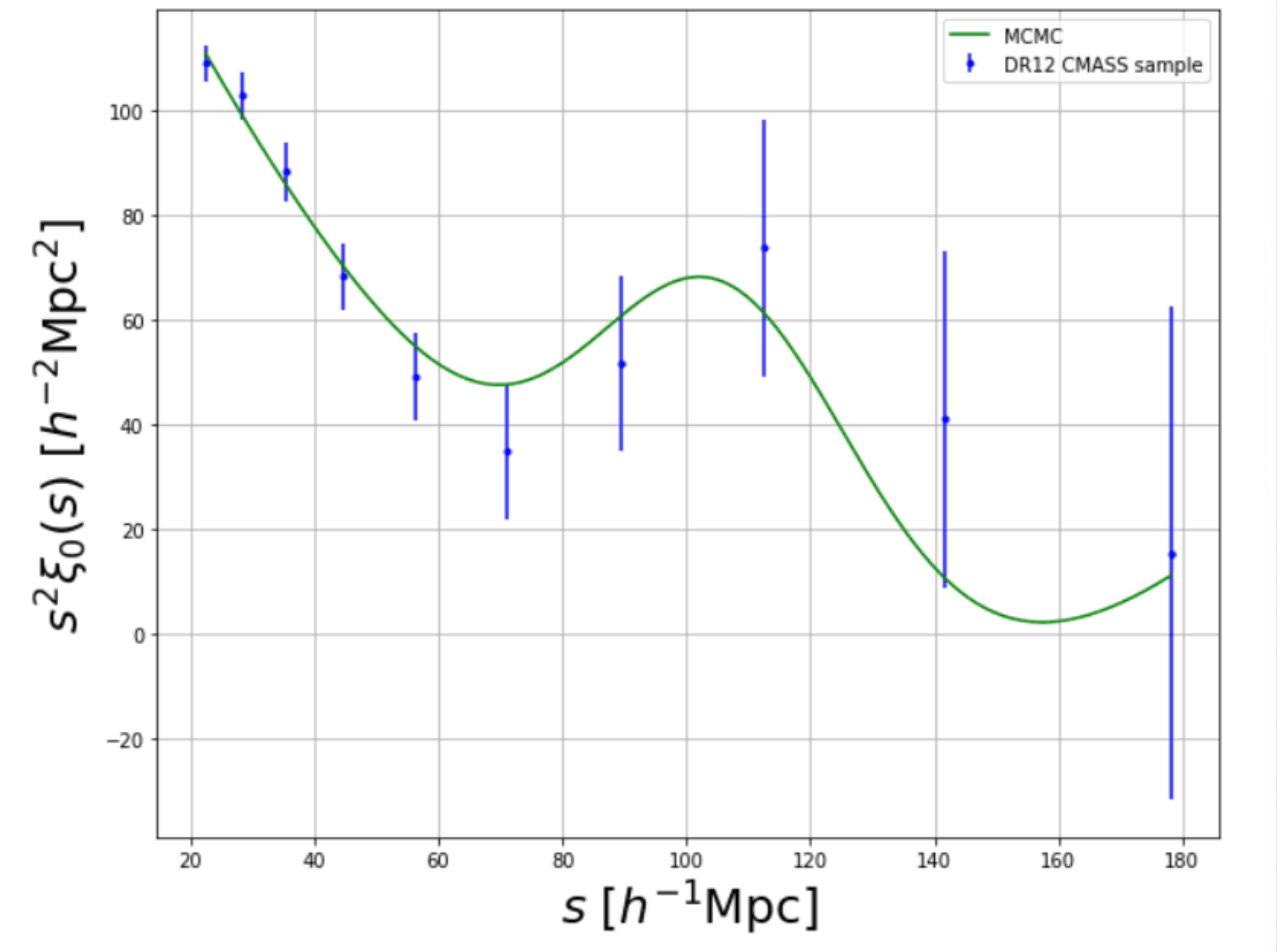
$$\alpha_{iso} = D_V(z_{eff};\Omega_m,\Omega_\Lambda,w_0) / D_V(z_{eff};\Omega_m = \Omega_m^F,\Omega_\Lambda = \Omega_\Lambda^F, w_0 = w_0^F)$$

Cosmological Parameters :

Ω_m : Matter density ratio
Ω_Λ : Dark energy density ratio
w_0 : Dark energy equation of state

Estimate Cosmology Parameters





Combination increases precision!

Conclusion

- Detect BAO peak position in the two point correlation function with current telescope :
    - Method 1 with 2σ significance
    - Method 2 with 0.5σ significance
- Compare the result of diagonal matrix and covariance matrix.
- Future telescope will detect the BAO peak with higher significance:
    - Method 2 with 3 to 4 σ
- Measure the cosmological parameters with BAO using both Galaxy Clustering and CMB data.

Thank You

Back up

Significance method 1

Using the $\Delta\chi^2$ method
Need a null model(green) and
a physical model (red)

Table 39.2: Values of $\Delta\chi^2$ or $2\Delta \ln L$ corresponding to a coverage probability $1-\alpha$ in the large data sample limit, for joint estimation of m parameters.

$(1-\alpha)$ (%)	$m=1$	$m=2$	$m=3$
68.27	1.00	2.30	3.53
90.	2.71	4.61	6.25
95.	3.84	5.99	7.82
95.45	4.00	6.18	8.03
99.	6.63	9.21	11.34
99.73	9.00	11.83	14.16

Significance method 2

Using the χ^2 method
Need a physical model

Calculate the p-value

=> p-value = $\alpha/2$
=> α= 2 p-value

Back up

Significance method 2

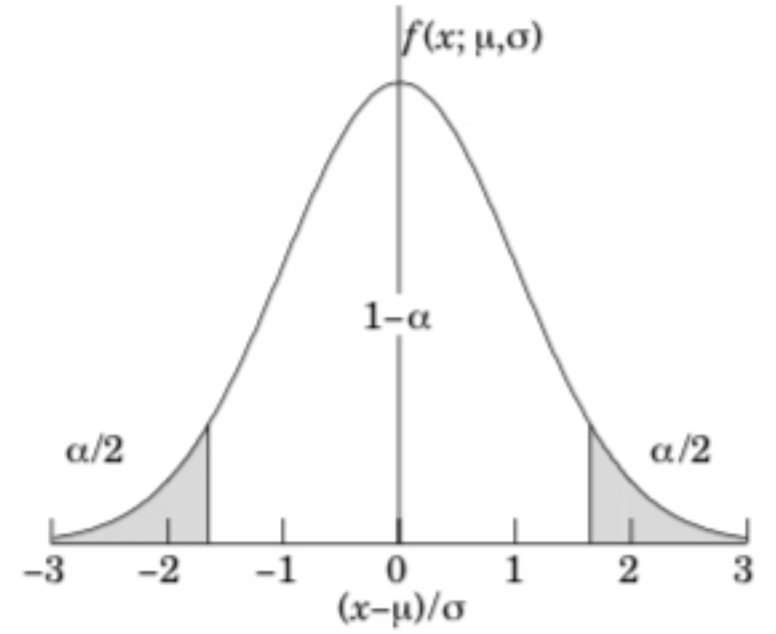


Figure 39.4: Illustration of a symmetric 90% confidence interval (unshaded) for a Gaussian-distributed measurement of a single quantity. Integrated probabilities, defined by $\alpha = 0.1$, are as shown.

=> p-value = α/2

=> α= 2 p-value

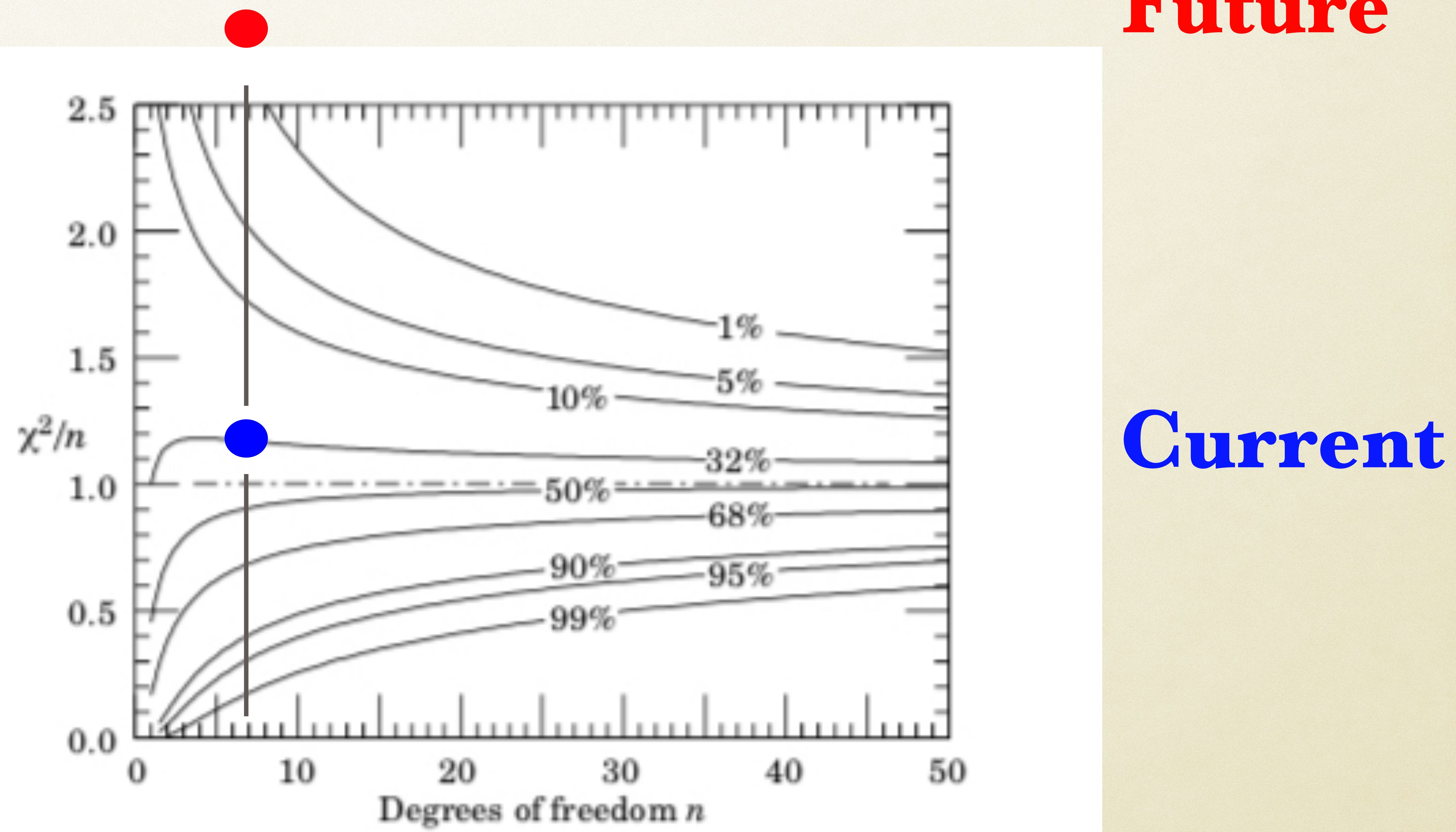


Figure 39.2: The 'reduced' χ^2, equal to χ^2/n, for n degrees of freedom. The curves show as a function of n the χ^2/n that corresponds to a given p-value.

Back up: Significance

=> Number of parameters to the fitter 5

Current telescope

=> $\chi^2 =$ 8 for degrees of freedom n=12-5 =7

=> $\chi^2 =$ 8/7 = 1.2

=> p-value = 0.30

=> Significance 1-α = 1-2p-value = 1- 2.*0.32 = 0.36

=>0.5σ detection

Future telescope

=> $\chi^2 =$ 30 for degrees of freedom n=12-5 =7

=> $\chi^2 =$ 30/7 = 4.2

=> p-value =< 0.01

=> Significance 1-α = 1-2p-value = 1- 2.*0.01 = 0.98

=> 3-4σ detection

Conclusion

- Detect BAO peak position in the two point correlation function with 2σ significance.
- Compare the result of diagonal matrix and covariance matrix.
- Measure the cosmology parameters with BAO using both galaxy clustering and CMB data.
- The current model is a bit too simple. It cannot give a good significance in more complicated measurement.