Future of Particle Physics

D$^{\mathrm{r}}$ Steve Muanza: CPPM Marseille, CNRS-IN2P3 & AMU

Physics of the two infinities:
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Plan

Rationale

- In order to grasp the future of anything, one has to understand where it comes from and how it currently stands
- Section I adresses the past and the present of particle physics, i.e. the Standard Model
- Section II presents theoretical ideas on how to transcend the Standard Model and to answer some of its open questions
- Section III presents instrumental facilities aimed at testing ideas of Section II

Foundations of the Standard Model

- The Standard Model (SM) of Particle Physics is a Quantum Field Theory (QFT)
- It complies simultaneously to:
  - Special Relativity (SR) and Quantum Mechanics (QM)
  - see lecture by Y. Coadou → Link
- QFT is defined by its:
  - Particle content
  - Continuous symmetries:
    - External symmetries:
      - Lorentz group: rotations and boosts in space-time (t, x_i)
      - Poincaré group: Lorentz group + translations in space-time
    - Internal quantum symmetries:
      - Gauge symmetries: origin of fundamental interactions
      - Flavour symmetries ($SU(2)_{Flav}$ & $SU(3)_{Flav}$ symmetries of hadrons,...)
  - Discrete symmetries:
    - C: charge conjugation
      $$Q \leftrightarrow -Q$$
    - P: space reversal
      $$x_i \leftrightarrow -x_i$$
    - T: time reversal
      $$t \leftrightarrow -t$$

Particle Content

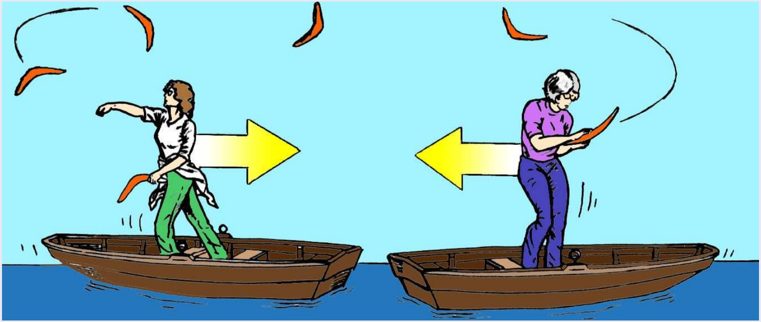
- Elementary bricks of matters are fermions ($S = 1/2$)

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2			Quarks spin =1/2		
Flavor	Mass GeV/c^2	Electric charge	Flavor	Approx. Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0	u up	0.002	2/3
e electron	0.000511	−1	d down	0.005	−1/3
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0	c charm	1.3	2/3
μ muon	0.106	−1	s strange	0.1	−1/3
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0	t top	173	2/3
τ tau	1.777	−1	b bottom	4.2	−1/3

Fundamental Interactions: Illustrations

- Interactions proceed through the exchange of bosons ($S = 1$) serving as mediators



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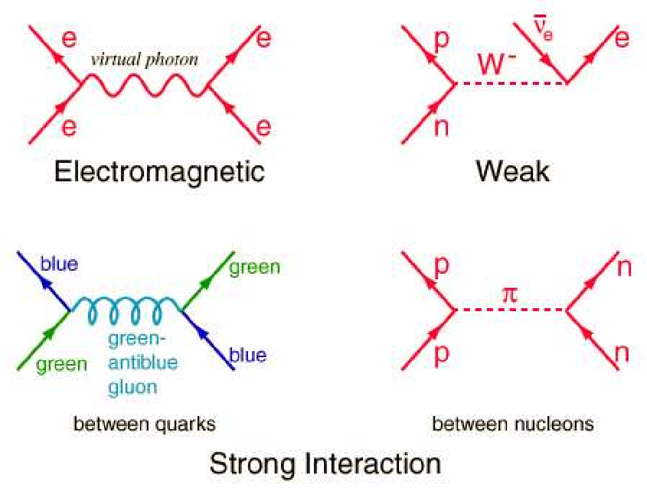

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Basis of QFT in a Nutshell

- Elementary particles are quanta of fields
- Lagrangian: real scalar function encoding the dynamics of a physical system

Classical Case

- Lagrangian Mechanics is a variational version of Newtonian Mechanics
- Lagrangian: $L = T(\dot{q}_i^2) - V(q)$
- Action is a functional of the system path defined as $S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i)dt$
- The equation of motion are obtained by minimizing the action: $\delta S = 0$ (leaving start and end points fixed)
- From there, the Euler-Lagrange equations follow: $\frac{\delta L}{\delta q_i} - \frac{d}{dt}\frac{\delta L}{\delta \dot{q}_i^2} = 0$

QFT Case

- Correspondence principle: $p_i \to \frac{\partial}{\partial q_i}$ and $E \to \frac{\partial}{\partial t} \Longrightarrow \partial_\mu = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x_i}\right)$; $\hbar = c = 1$
- Lagrangian and its density : $L(q_i, \dot{q}_i) \to L(\phi(x_\mu), \partial_\mu \phi(x_\mu)) = \int \mathcal{L} d^3x$
- Action: $S = \int Ldt = \int \mathcal{L} d^4x$
- Euler-Lagrange equations: $\frac{\delta \mathcal{L}}{\delta \phi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} = 0$
- Symmetry: $\mathcal{L}$ unchanged under transfos of a symmetry group: $\mathcal{L} \to \mathcal{L}' = \mathcal{L} \leftrightarrow \delta S = 0$

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Fundamental Interactions: properties (1/2)

- Strong Interaction
  - Applies between quarks and gluons (attraction)
  - Responsible for matter cohesion at nuclear and nucleon scales
  - QFT: Quantum Chromo Dynamics (QCD)
  - Symmetry group: $SU(3)_C$
    - 8 generators: the gluons g (S=1) which couple to color charge
    - Non-abelian group: gluons have self-interactions (they bare both color and anti-color charges)
  - Range:
    - In principle: $\frac{1}{m_g} \to \infty$
    - Confinement: $\frac{1}{m_\pi} \approx 1\ fm = 10^{-15} m$
  - Relative intensity: 1 (reference)
  - Typical time scale: $10^{-23}\ s$

- Weak Interaction
  - Applies between quarks/leptons and W/Z bosons (attraction or repulsion)
  - Responsible for β-decays, for some hadron decays, for nucleosynthesis in stars
  - QFT: Electroweak Theory
  - Symmetry group: $SU(2)_L$
    - 3 generators: $W^{\pm}$, Z^0 (S=1) which couples to weak isospin
    - Non-abelian group: $W^{\pm}$, Z^0 have self-interactions
    - Z boson couples differently to L-handed and R-handed fermions (parity violation)
    - W boson does not couple to R-handed fermions at all
  - Range: $\frac{1}{m_Z} \approx 10^{-3} fm = 10^{-18} m$
  - Relative intensity: 10^{-14}
  - Typical time scale: $\geq 10^{-12}\ s$

Fundamental Interactions: properties (2/2)

- EM Interaction
  - Governs all: electric, magnetic and optical phenomena
  - Classically described by the Maxwell's equations
  - EM Interactions attractive or repulsive
  - Responsible for matter cohesion at atomic and molecular scales
  - QFT: Quantum Electro Dynamics (QED)
  - Symmetry group: $U(1)_{EM}$
    - Single generator: the photon γ (S=1) which couples to eletric charge
    - Abelian group: photon has no self-interaction (it is neutral)
  - Range: $\frac{1}{m_\gamma} \to \infty$
  - Relative intensity: 10^{-3}
  - Typical time scale: $10^{-16} - 10^{-20}$ s

- Gravitation
  - Attraction of any type of matter-energy
  - Responsible for matter cohesion at large scales (planets, solar system, galaxies, galaxies clusters,...)
  - QFT: No satistfactory such theory yet!
  - Symmetry group: ???
    - Hypothetical generator: the graviton $G_{\mu\nu}$ (S=2) which couples to $(E, \vec{p})$ tensor
  - Range: $\frac{1}{m_{G_{\mu\nu}}} \to \infty$
  - Relative intensity: 10^{-39}

Two Master Equations

Klein-Gordon Equation

- Applies to (real scalar) bosons (S=0)
- Lagrangian (density) for a free boson ϕ writes: $\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right)$
- Derived from Euler-Lagrance equations by applying the correspondence principle to: $E^2 = p^2 + m^2$
- $(\partial_\mu\partial^\mu + m^2)\phi = 0$

Dirac Equation

- Applies to fermions (S=1/2)
- Lagrangian (density) for a free fermion ψ writes: $\mathcal{L} = \bar{\psi}\gamma_\mu\partial^\mu\psi$
- Derived from $E^2 = p^2 + m^2$ too, but trying to linearize this equation:
  - Start from: $H\psi = i\frac{\partial\psi}{\partial t}$, rewrite H as $H = -i\alpha_i\frac{\partial}{\partial x^i} + \beta m$
  - This yields: $(\alpha \cdot \mathbf{p} + \beta \cdot \mathbf{m})\psi = \mathbf{i}\frac{\partial}{\partial \mathbf{t}}\psi$
  - The square of this equation is calculated and developed, leading to
- Dirac equation: $(i\gamma_\mu\partial^\mu - m)\psi = 0$
- provided that: $\{\alpha_i, \alpha_j\} = 2\delta_{ij}$, $\{\alpha_i, \beta\} = 0$, and $\beta^2 = \hat{1}$
- Gamma matrices: $\gamma^\mu \equiv (\gamma^0, \gamma^i)$ where $\gamma^0 = \beta$, $\gamma^i = \beta\alpha_i$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

Covariant ElectroMagnetism

- EM four-potential: $A_\mu(x) = (V(x), \vec{A}(x))$
- External EM current: $j_\mu(x) = (\rho(x), \vec{j}(x))$
- EM tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
  - Mixes components of $\vec{E}$ and $\vec{B}$:

$$F_{\mu\nu}F^{\mu\nu} = 2(\vec{B}^2 - \vec{E}^2)$$

- Maxwell's equations: applying Euler-Lagrange to $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_\mu j^\mu$

$$\partial_\mu F^{\mu\nu} = j^\nu \leftrightarrow \begin{cases} \vec{\nabla}\vec{E} = \rho \\ \vec{\nabla} \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t} \end{cases}$$

$$\partial_\lambda F^{\mu\nu} + \partial_\mu F^{\nu\lambda} + \partial_\nu F^{\lambda\mu} = 0 \leftrightarrow \begin{cases} \vec{\nabla}\vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \end{cases}$$

Quantum Electro-Dynamics Lagrangian

Free Charged Particle (1/2)

- Model: free charged particle
- Represented by: a spinor ψ of S=1/2, mass m and charge e

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$$

- A unitary transfo of ψ writes: $\psi'(x) = e^{iq\theta}\psi(x)$
  - Global transfo: θ constant wrt space-time coordinates x
  - Local transfo: $\theta(x)$ depends on x (aka gauge transfo)
- Corresponding symmetry group: $U(1)$
- Gauge transformations:
  - Spinor particle:

$$\psi \longrightarrow \psi'(x) = e^{i\cdot e\cdot\theta(x)}\psi(x)$$

  - Spinor anti-particle:

$$\bar{\psi} \longrightarrow \bar{\psi}'(x) = e^{-i\cdot e\cdot\theta(x)}\bar{\psi}(x)$$

Quantum Electro-Dynamics Lagrangian

Interlude: Noether's Theorem & Global Invariance

- Let's consider a free particle with: $\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi$
- Let's consider an infinitesimal gauge transformation: $\psi \longrightarrow \psi' = (1 + i \cdot \alpha)\psi$
- $\psi' = (1 + i \cdot \alpha)\psi \Rightarrow \Delta\psi = \psi' - \psi = i \cdot \alpha\psi \Rightarrow \delta\psi = \psi' - \psi = i \cdot \alpha\psi$
- $\bar{\psi}' = (1 - i \cdot \alpha)\bar{\psi} \Rightarrow \ldots \Rightarrow \delta\bar{\psi} = -i \cdot \alpha\bar{\psi}$
- $\mathcal{L} = \mathcal{L}(\psi, \bar{\psi}, \partial_\mu\psi, \partial_\mu\bar{\psi})$ is invariant under this global transfo, if:
- $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta(\partial_\mu\psi) + \delta\bar{\psi}\frac{\partial\mathcal{L}}{\partial\bar{\psi}} + \delta(\partial_\mu\bar{\psi})\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}$
- $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\psi}(i\alpha\psi) + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}(i\alpha\partial_\mu\psi) + h.c.$
- $\delta\mathcal{L} = 0 = i\alpha\left[\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right)\right]\psi + i\alpha\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right) + h.c.$
- ... (calculations ongoing, try it for yourself)
- $\delta\mathcal{L} = 0 = \partial_\mu j^\mu$, with $j^\mu = \frac{i \cdot e}{2}\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi - \bar{\psi}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\right]$
- Hence $j^\mu = -e\bar{\psi}\gamma^\mu\psi$, according to the free lagrangian above
- $\partial_\mu j^\mu = 0 \Rightarrow Q = \int j^0 d^3x$
- Each global invariance implies a conserved current, hence a conserved charge

Quantum Electro-Dynamics Lagrangian

Free Charged Particle (2/2)

- Is $\mathcal{L}$ invariant under such a local symmetry?
- $\mathcal{L}' = \bar{\psi}'(i\gamma_\mu\partial^\mu - m)\psi' = i\bar{\psi}'\gamma_\mu\partial^\mu\psi' - m\bar{\psi}'\psi'$
- ... (calculations ongoing, try it for yourself)
- $\mathcal{L}' = \mathcal{L} + e\bar{\psi}\gamma^\mu A_\mu\psi$, not invariant...

Charged Particle in EM Field

- To preserve $\mathcal{L}$ invariance, add field $A_\mu(x)$ to the model, corresponding to the EM 4-potential (covariant)
- Replace usual derivative by covariant derivative D_μ, which transforms like ψ:

$$D_\mu = \partial_\mu - i \cdot e \cdot A_\mu(x) \text{ and } \mathrm{A}_\mu(\mathrm{x}) \longrightarrow \mathrm{A}'_\mu(\mathrm{x}) = \mathrm{A}_\mu(\mathrm{x}) + \frac{1}{\mathrm{e}}\partial_\mu\theta(\mathrm{x})$$

- $\mathcal{L}' = \bar{\psi}'(i\gamma_\mu D^\mu - m)\psi' = i\bar{\psi}'\gamma_\mu D^\mu\psi' - m\bar{\psi}'\psi'$
- $\mathcal{L}' = \mathcal{L}$, is invariant...
- Interpretation:
  - A_μ is a S=1 field, corresponding to the photon γ
  - Local invariance of $\mathcal{L}$ under symmetry operators provides the gauge interaction!
  - $\mathcal{L}_{QED} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi + e\bar{\psi}\gamma_\mu A^\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

Quantum Electro-Dynamics Lagrangian

Mass term for the photon

- The interaction term between the photon and the charged fermion, owed to local gauge invariance
- We had to complete the $\mathcal{L}_{QED}$ by adding a propagation term for the photon
- The next question is: could we also add a mass term for the photon, such as the covariant

$$\frac{1}{2} m_A A_\mu A^\mu ?$$

- How this additional term transforms?

$$A^\mu A_\mu \rightarrow \left(A^\mu + \frac{1}{e}\partial^\mu \theta(x)\right)\left(A_\mu + \frac{1}{e}\partial_\mu \theta(x)\right) \neq A^\mu A_\mu, \text{ not invariant}$$

- Preserving gauge invariance forbids to add a mass term for the gauge boson!

Mass of fermions

Chiral fermions

- Weak interactions couple differently to left-handed and to right-handed fermions:

$$\psi_L = (1+\gamma^5)/2 \text{ and } \psi_{\rm L} = (1-\gamma^5)/2$$

- This accounts for the violation of parity in weak interaction
  - Predicted by C.N. Yang and T.D. Lee in 1956 (Nobel Prize in 1957)
  - Discovered by C.S. Wu in 1957
- Adding a mass term for the fermions in the QED (or other non chiral theory) would not cause problems
- However in the SM, weak interactions are based on $SU(2)_L$ where this mass term writes:

$$m\bar{\psi}\psi = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

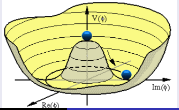
- But in $SU(2)_L$:
  - ψ_L is a weak isospin doublet: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$
  - ψ_R is a weak isospin singlet: $(e^+)_R$
  - therefore, it's impossible to form the above scalar: $m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$
- Chirality of weak interactions forbids to include a mass term for charged fermions in the Standard Model!

Spontaneous Electroweak Symmetry Breaking

Higgs Mechanism

- Gauge invariance $\Rightarrow$ no mass terms for the gauge bosons
  - Fine for the gluons g and the photon γ
  - Problem for $W^{\pm}$ and Z^0: weak interactions are short-ranged $\leftrightarrow$ heavy mediators
- Fermions chirality $\Rightarrow$ no mass terms for the charged fermions
  - Problem: we know it cannot be this way
- Hint: supposed level of symmetry is too high
- Solution: find a mechanism which provides mass to
  - provides mass to $W^{\pm}$ and Z^0
  - leaves g and γ massless
- Higgs mechanism: spontaneous breaking of $SU(2)_L \otimes U(1)_Y \to U(1)_{EM}$
  - Higgs field: introduce an $SU(2)_L$ doublet of complex fields $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_L$ with
    $\phi^+ = \frac{\phi^1 + i\phi^2}{\sqrt{2}}$ and $\phi^0 = \frac{\phi^3 + i\phi^4}{\sqrt{2}}$
    each component accounts for (2s+1)=1 degree of freedom, 4 in total
  - Higgs potential:

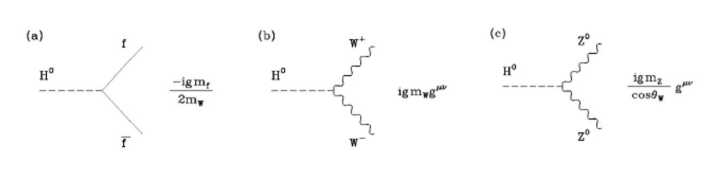
$$V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$$



Spontaneous Electroweak Symmetry Breaking

Consequences of Higgs Mechanism

- Higgs field does not have a vanishing vacuum expectation value:
  $< 0|\Phi|0 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ with $v = 243$ GeV
- Higgs potential has an infinity of equiprobable minima
- Actual choice of a specific minimum breaks the EWK symmetry
- Interactions of all SM particle with the Higgs field provide their mass
  - Gauge EWK interactions: $m_W = g_2 \frac{v}{2}, \quad m_Z = \frac{g_2}{cos\theta_W} \frac{v}{2}$
  - Yukawa interactions: $m_f = y_f \frac{v}{2}$
  - Gauge EWK self-interactions: $m_H = \sqrt{2\lambda}v = \sqrt{2}\mu$
  - Photons do not couple to Φ: $m_\gamma = 0$
  - Gluons do not couple to Φ: $m_g = 0$
- The particles coupling to the Higgs field are proportional to their mass



Lagrangian of the Standard Model

- I presented some basic features of QFT based upon $U(1)$ symmetry group
- SM Symmetry Group: $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_C \otimes U(1)_{EM}$

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Dirac} + \mathcal{L}_{\rm mass} + \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm gauge/\psi} \, . \tag{1}$$

Here,

$$\mathcal{L}_{\rm Dirac} = i\bar{e}^i_{\rm L}\partial\!\!\!/ e^i_{\rm L} + i\bar{\nu}^i_{\rm L}\partial\!\!\!/ \nu^i_{\rm L} + i\bar{e}^i_{\rm R}\partial\!\!\!/ e^i_{\rm R} + i\bar{u}^i_{\rm L}\partial\!\!\!/ u^i_{\rm L} + i\bar{d}^i_{\rm L}\partial\!\!\!/ d^i_{\rm L} + i\bar{u}^i_{\rm R}\partial\!\!\!/ u^i_{\rm R} + i\bar{d}^i_{\rm R}\partial\!\!\!/ d^i_{\rm R} \, ; \tag{2}$$

$$\mathcal{L}_{\rm mass} = -v\left(\lambda^i_e \bar{e}^i_{\rm L} e^i_{\rm R} + \lambda^i_u \bar{u}^i_{\rm L} u^i_{\rm R} + \lambda^i_d \bar{d}^i_{\rm L} d^i_{\rm R} + \text{h.c.}\right) - M^2_{\rm W} W^+_\mu W^{-\mu} - \frac{M^2_{\rm W}}{2\cos^2\theta_W} Z_\mu Z^\mu \, ; \tag{3}$$

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}(G^a_{\mu\nu})^2 - \frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{WZA} \, , \tag{4}$$

where

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_3 f^{abc} A^b_\mu A^c_\nu \\ W^\pm_{\mu\nu} &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \, , \end{aligned} \tag{5}$$

and

$$\begin{aligned} \mathcal{L}_{WZA} = \; & i g_2 \cos\theta_{\rm W}\left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu\right)\partial^\mu Z^\nu + W^+_{\mu\nu}W^{-\mu}Z^\nu - W^-_{\mu\nu}W^{+\mu}Z^\nu\right] \\ & + ie\left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu\right)\partial^\mu A^\nu + W^+_{\mu\nu}W^{-\mu}A^\nu - W^-_{\mu\nu}W^{+\mu}A^\nu\right] \\ & + g_2^2 \cos^2\theta_{\rm W}\left(W^+_\mu W^-_\nu Z^\mu Z^\nu - W^+_\mu W^{-\mu} Z_\nu Z^\nu\right) \\ & + g_2^2\left(W^+_\mu W^-_\nu A^\mu A^\nu - W^+_\mu W^{-\mu} A_\nu A^\nu\right) \\ & + g_2 e \cos\theta_{\rm W}\left[W^+_\mu W^-_\nu\left(Z^\mu A^\nu + Z^\nu A^\mu\right) - 2W^+_\mu W^{-\mu} Z_\nu A^\nu\right] \\ & + \frac{1}{2}g_2^2\left(W^+_\mu W^-_\nu\right)\left(W^{+\mu}W^{-\nu} - W^{+\nu}W^{-\mu}\right) \, ; \end{aligned} \tag{6}$$

and

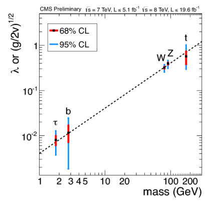
$$\mathcal{L}_{\rm gauge/\psi} = -g_3 A^a_\mu J^{\mu a}_{(3)} - g_2\left(W^+_\mu J^\mu_{W^+} + W^-_\mu J^\mu_{W^-} + Z_\mu J^\mu_Z\right) - eA_\mu J^\mu_A \, , \tag{7}$$

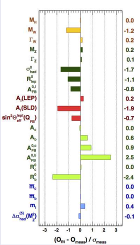
where

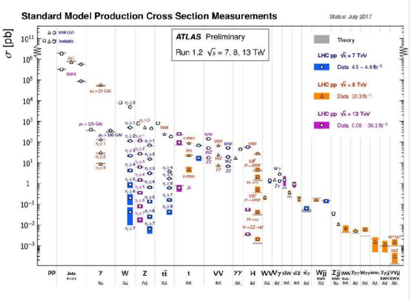
$$\begin{aligned} J^{\mu a}_{(3)} &= \bar{u}^i\gamma^\mu T^a_{(3)} u^i + \bar{d}^i\gamma^\mu T^a_{(3)} d^i \\ J^\mu_{W^+} &= \frac{1}{\sqrt{2}}\left(\bar{\nu}^i_{\rm L}\gamma^\mu e^i_{\rm L} + V^{ij}\bar{u}^i_{\rm L}\gamma^\mu d^j_{\rm L}\right) \\ J^\mu_{W^-} &= (J^\mu_{W^+})^* \\ J^\mu_Z &= \frac{1}{\cos\theta_{\rm W}}\left[\frac{1}{2}\bar{\nu}^i_{\rm L}\gamma^\mu\nu^i_{\rm L} + \left(-\frac{1}{2} + \sin^2\theta_{\rm W}\right)\bar{e}^i_{\rm L}\gamma^\mu e^i_{\rm L} + (\sin^2\theta_{\rm W})\bar{e}^i_{\rm R}\gamma^\mu e^i_{\rm R}\right. \\ & \quad + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_{\rm W}\right)\bar{u}^i_{\rm L}\gamma^\mu u^i_{\rm L} + \left(-\frac{2}{3}\sin^2\theta_{\rm W}\right)\bar{u}^i_{\rm R}\gamma^\mu u^i_{\rm R} \\ & \quad \left. + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_{\rm W}\right)\bar{d}^i_{\rm L}\gamma^\mu d^i_{\rm L} + \left(\frac{1}{3}\sin^2\theta_{\rm W}\right)\bar{d}^i_{\rm R}\gamma^\mu d^i_{\rm R}\right] \\ J^\mu_A &= (-1)\bar{e}^i\gamma^\mu e^i + \left(\frac{2}{3}\right)\bar{u}^i\gamma^\mu u^i + \left(-\frac{1}{3}\right)\bar{d}^i\gamma^\mu d^i \, . \end{aligned} \tag{8}$$

Successes of the Standard Model

- Discoveries: top quark (1995), tau neutrino (2000), Higgs boson (2012)
- Precise measurements:







Limitations of the Standard Model

- No quantum theory of gravity
- No candidate for Cold Dark Matter, no explanation for Dark Energy
- Non-natural Higgs sector:

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 - 4M_t^2\right]$$

- No explanation for neutrinos mass
- No unification of fundamental interactions
- 19 (26) free parameters

BACK-UP

Lie Group Theory: basic concepts

Lie Group Theory: Representations

- $U(N)$: $N \times N$ unitary ($U^\dagger = U^{-1}$) matrices
- $SU(N)$: subgroup of $U(N)$ for which $det(U(N)) = 1$ (special unitary)
- $O(N)$: $N \times N$ orthogonal ($O^t = O^{-1}$) matrices
- $SO(N)$: subgroup of $O(N)$ for which $det(O(N)) = 1$ (special orthogonal)