A primer on LIGO and Virgo gravitational wave detectors

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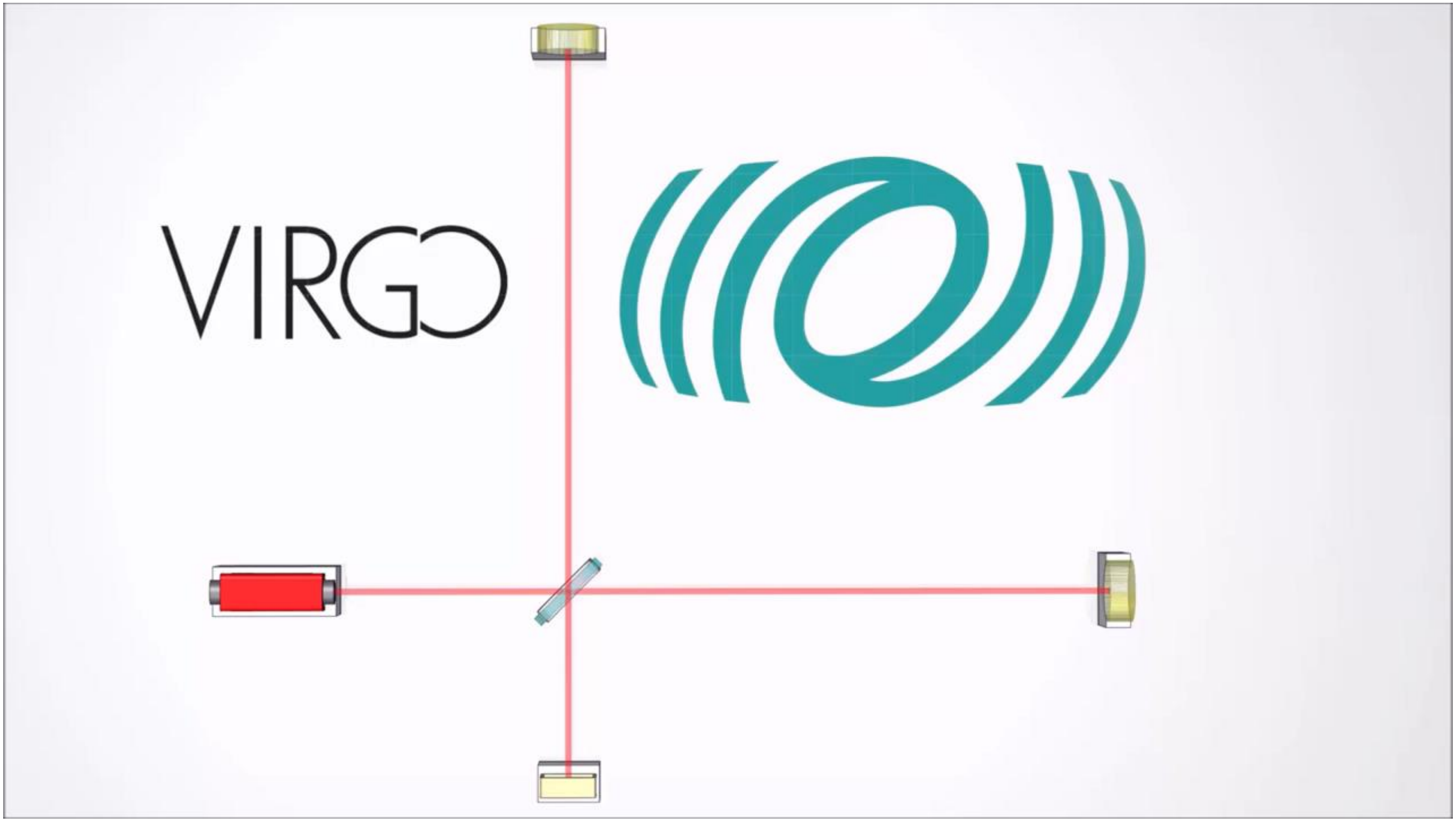
LIGO Scientific Collaboration

Advanced Virgo

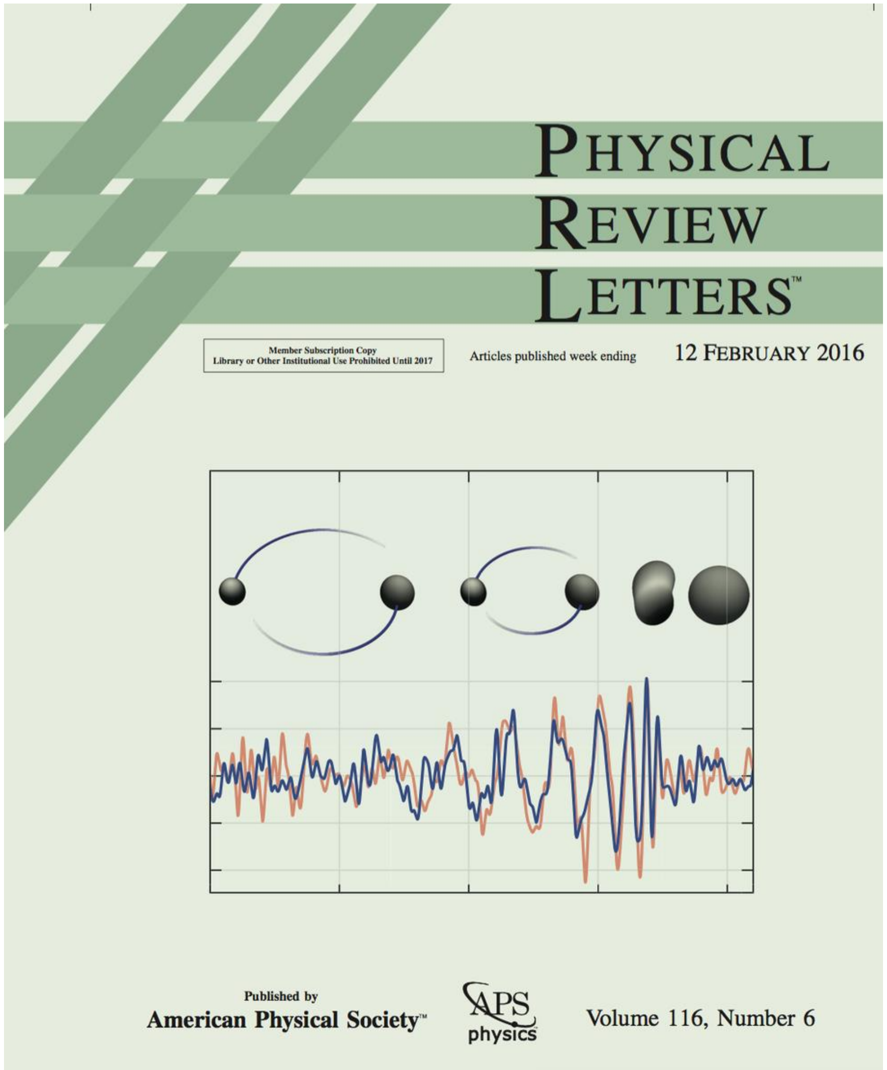
Interferometric gravitational wave detectors

Tiny vibrations in space can now be observed by using the kilometer-scale laser-based interferometers of LIGO and Virgo. This enables an entire new field in science



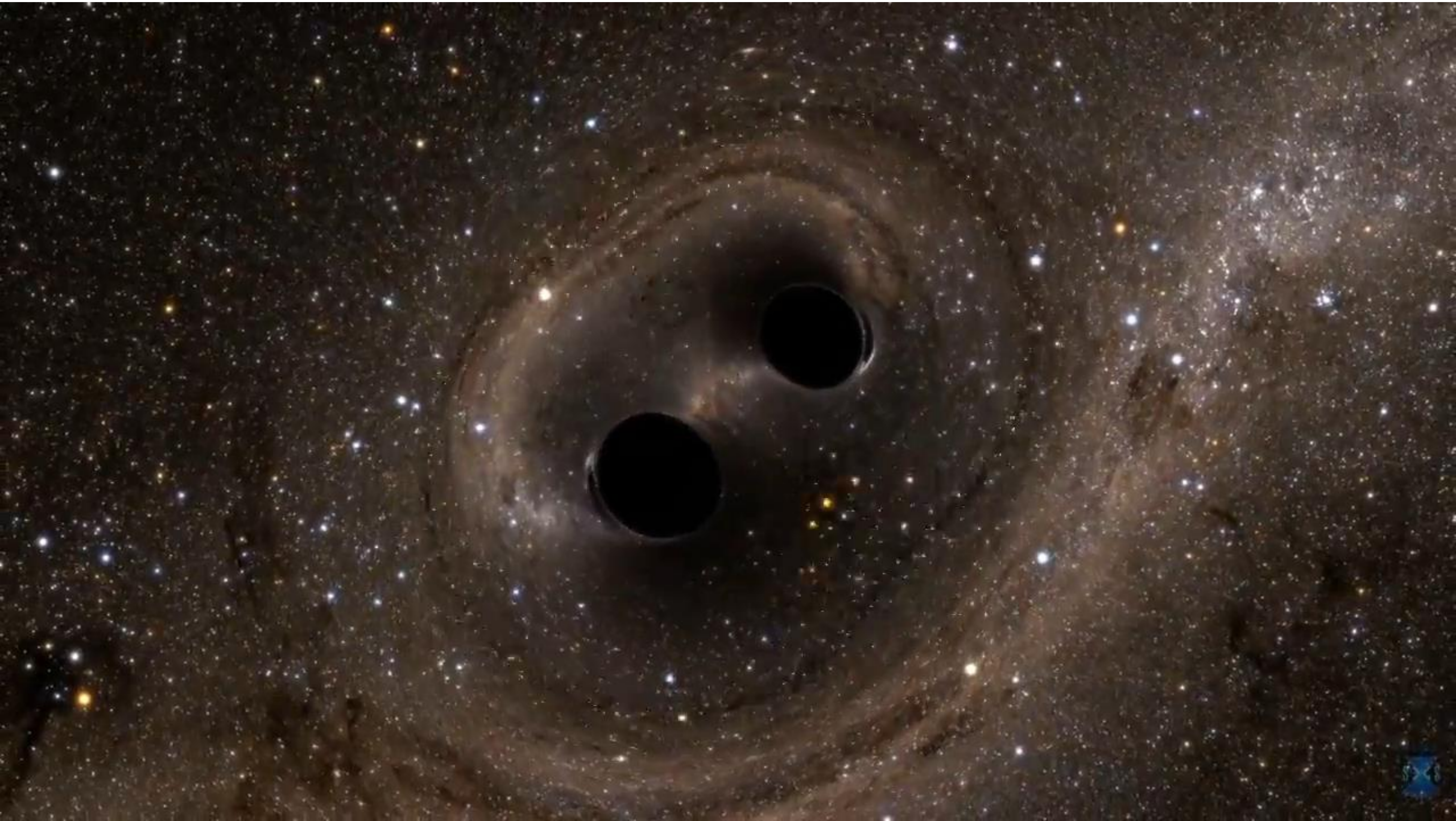
Eleven detections…so far

First gravitational wave detection with GW150914 and first binary neutron star GW170817



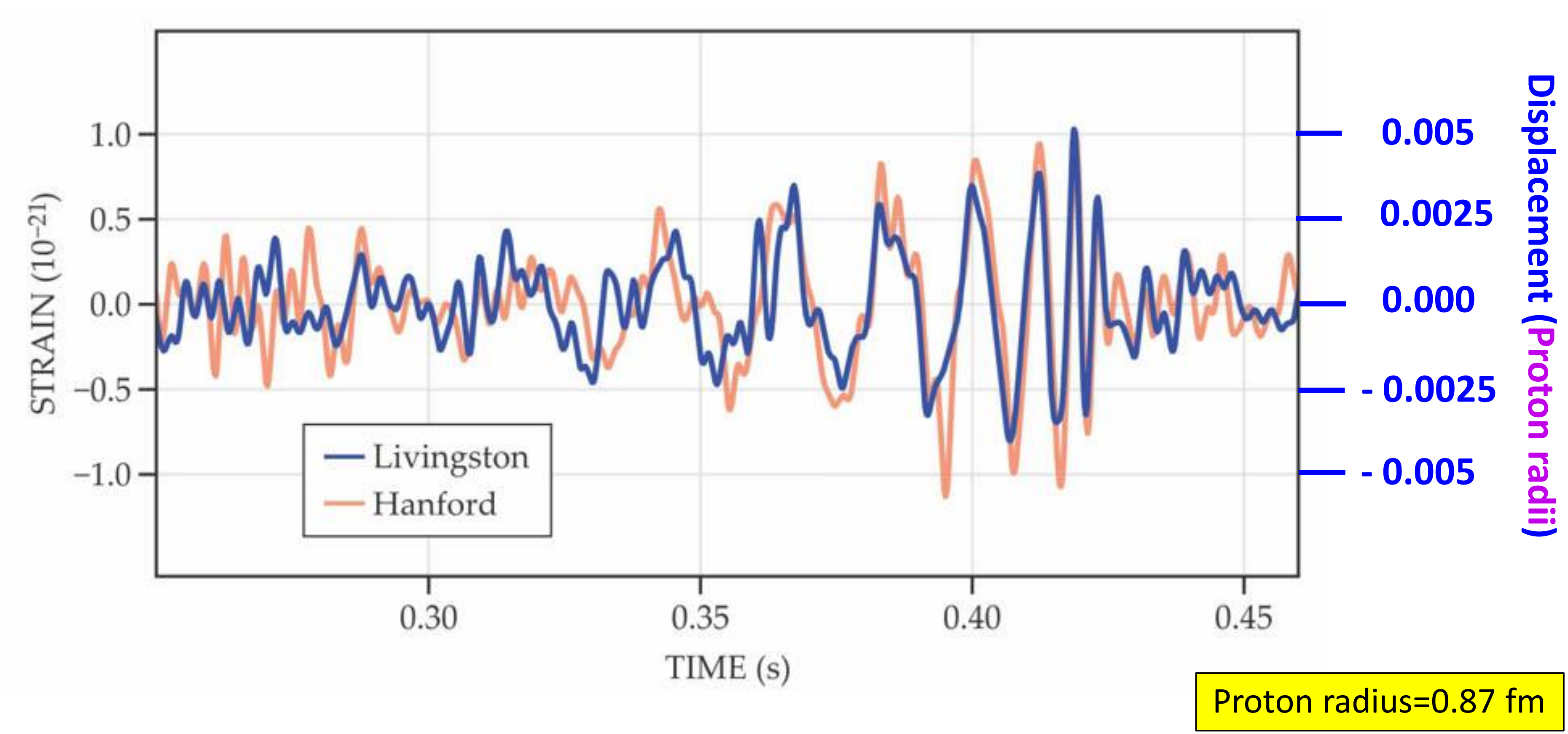
Event GW150914

Chirp-signal from gravitational waves from two coalescing black holes were observed with the LIGO detectors by the LIGO-Virgo Consortium on September 14, 2015



Event GW150914

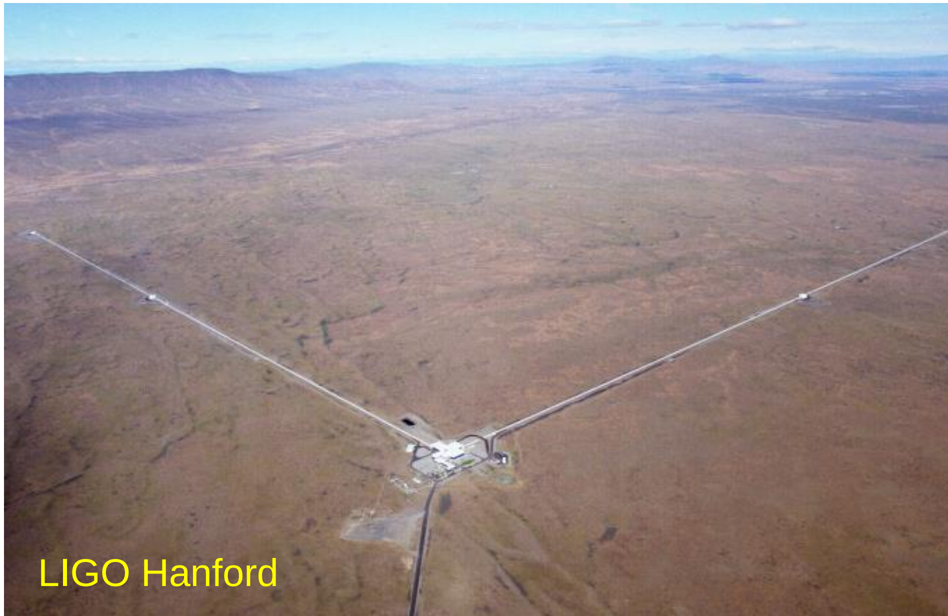
On September 14th 2015 the gravitational waves generated by a binary black hole merger, located about 1.4 Gly from Earth, crossed the two LIGO detectors displacing their test masses by a small fraction of the radius of a proton



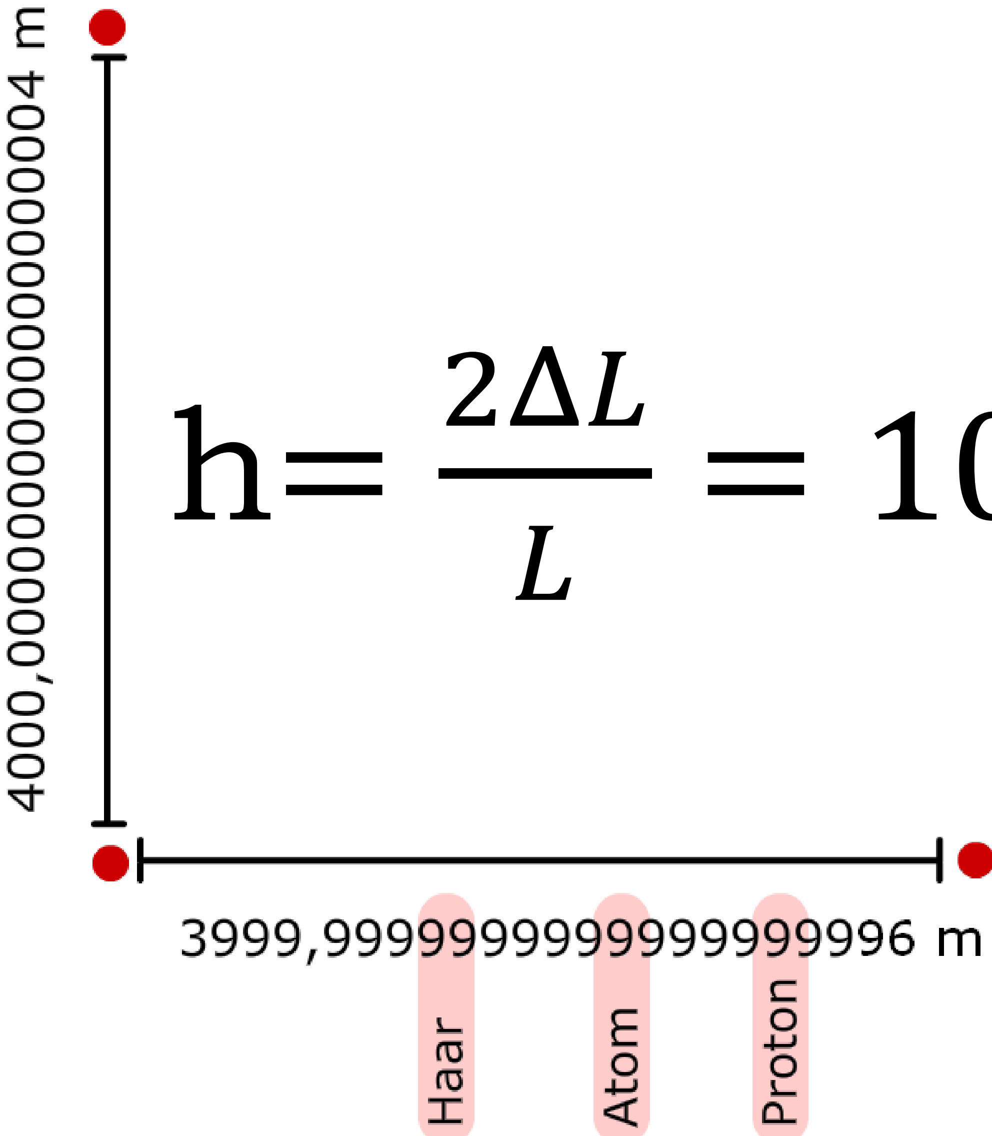
Measuring intervals must be smaller than 0.01 seconds

Laser interferometer detectors

Laser interferometer detectors



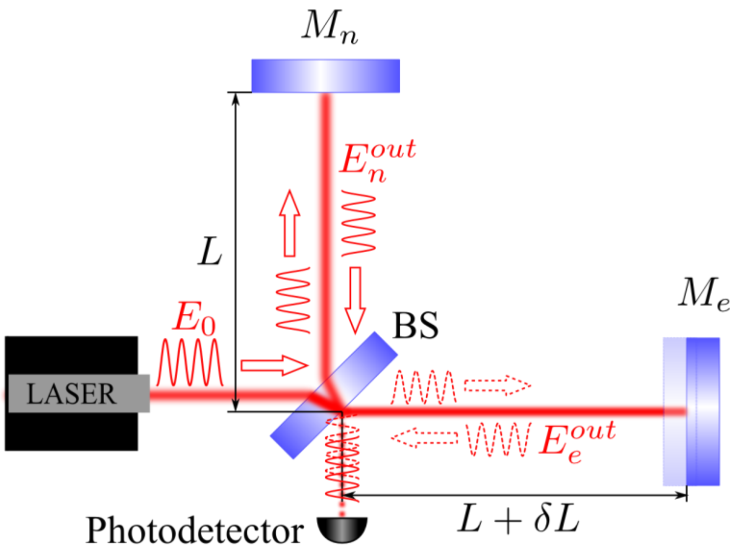
Effect of a strong gravitational wave on ITF arm

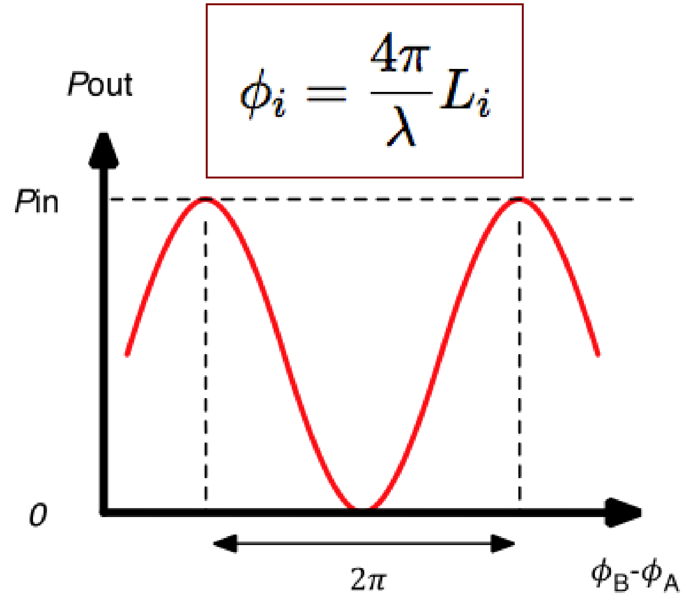


$$h = \frac{2\Delta L}{L} = 10^{-22}$$

Laser interferometer detectors

Michelson interferometer





$$E_{out} = \frac{1}{2}\left(e^{-i\phi_A} - e^{-i\phi_B}\right) E_{in}$$

$$E_{out} = \left[ie^{-i(\phi_A+\phi_B)/2} \sin\left(\frac{\phi_A - \phi_B}{2}\right)\right] E_{in}$$

$$P_{out} = E_{out} E_{out}{}^{*} = P_{in} \sin^2\left(\frac{\phi_A - \phi_B}{2}\right) = \left[1 - \cos\left(\phi_A - \phi_B\right)\right] \frac{P_{in}}{2}$$

If we would use a single photon and only distinguish between bright and dark fringe, then such an ITF would not be sensitive:

$$h_{\text{crude}} \approx \frac{\lambda/2}{L_{\text{optical}}} = \frac{0.5 \times 10^{-6}\ \text{m}}{4000\ \text{m}} \approx 10^{-10}$$

Need to do 10^{12} times better. This would required 10^{24} photons in each 0.01 s interval

Power required $P_{in} = \frac{2\pi\hbar c}{\lambda} \overline{N} \approx 20$ MW

Virgo uses 18 W of input power

Michelson (Nobel Prize in 1907) could read a fringe to $\lambda/20$, yielding h_{rms} of a few times 10^{-9}

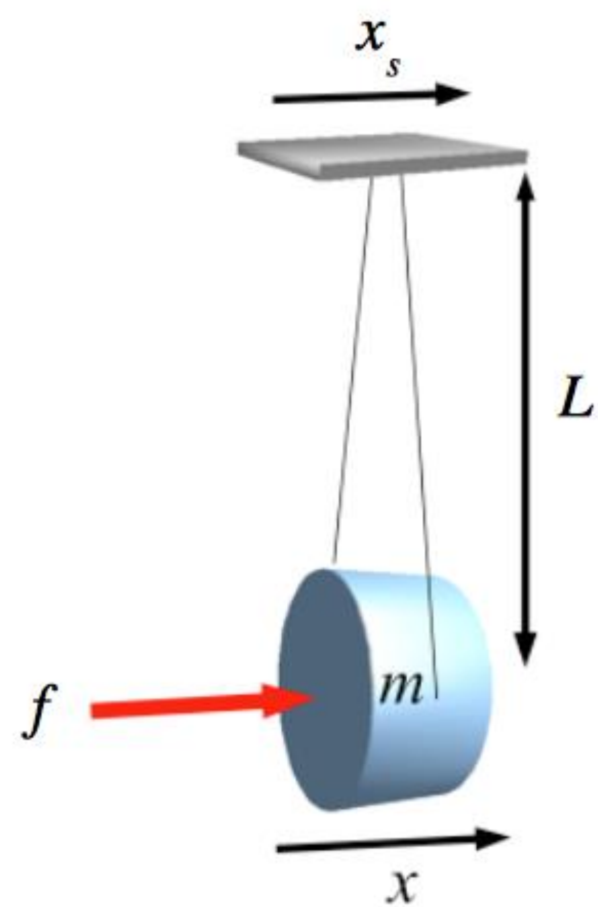
Laser interferometer detectors

Free-falling interferometers on Earth

Earth is not an inertial reference frame

nevertheless

It's always possible to make a test mass 'free falling' in a certain frequency range by 'suspending' it, i.e. by connecting it to the Earth as a pendulum



Laser interferometer detectors

Simple pendulum transfer function

The equation of motion is:

$$m\ddot{x} + b\dot{x} + \frac{mg}{L}(x - x_s) = f$$

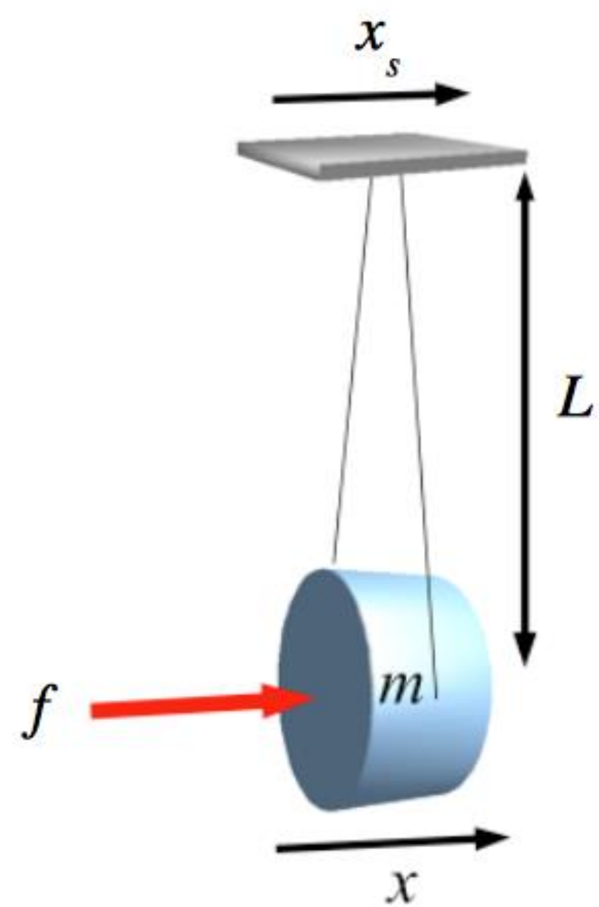
which is in the frequency domain is

$$-m\omega^2 X + i\omega X + \frac{mg}{L}(X - X_s) = F$$

by solving it we find

$$X = \frac{{\omega_0}^2}{\omega_0^2 - \omega^2 + i\frac{\omega\omega_0}{Q}} \cdot X_s + \frac{F/m}{{\omega_0}^2 - \omega^2 + i\frac{\omega\omega_0}{Q}}$$

where $\omega_0 = \sqrt{\frac{g}{L}}$ and $Q = \frac{m\omega_0}{b}$



Laser interferometer detectors

Simple pendulum transfer function

at low frequencies ($\omega \ll \omega_0$)

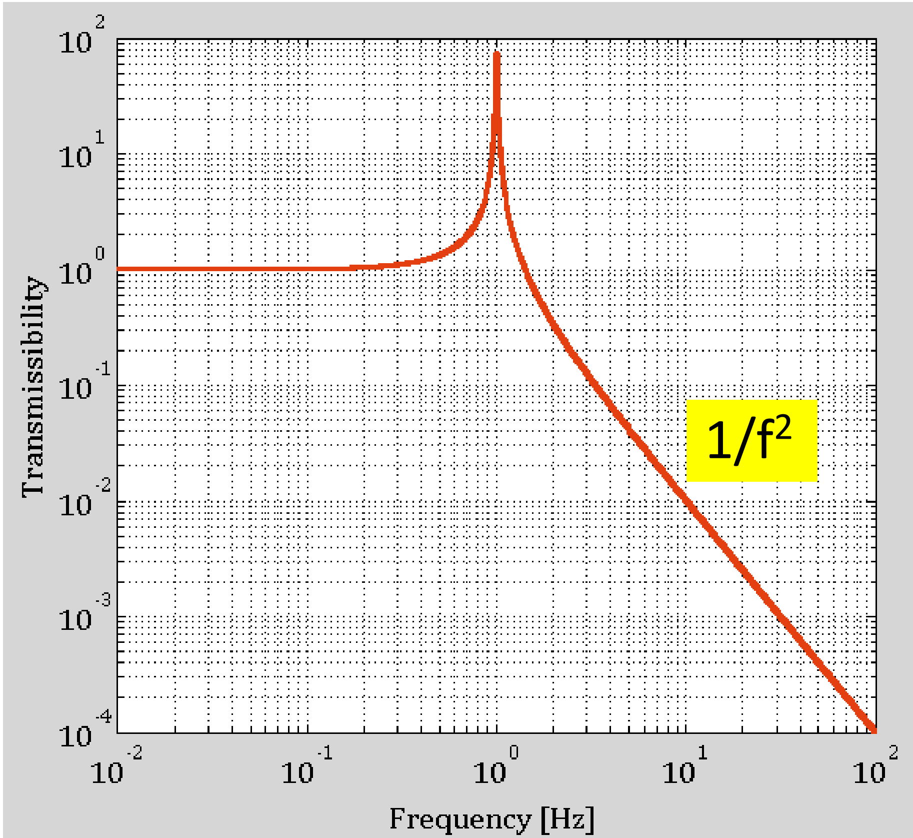
$$X \simeq X_s + F/m{\omega_0}^2$$

at the natural frequency ($\omega = \omega_0$)

$$X \simeq Q(X_s + F/m{\omega_0}^2)$$

while at high frequency ($\omega \gg \omega_0$)

$$X \simeq \frac{{\omega_0}^2}{\omega^2} X_s + F/m\omega^2$$



At high frequency the mirrors respond to external forces only with their inertia. They act as if the suspension is not there! In that sense they are ‘’freely falling’’

At low frequency we use feedback control to mimic rigidly mounting of our mirrors

Laser interferometer detectors

Michelson interferometer: response to a gravitational wave

Gravitational waves propagating through flat space are described by $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

A gravitational wave propagating in the z-direction can be described by $h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Two free parameters implies two polarizations

For light moving along the x axis, we are interested in the interval between points with non-zero *dx* and *dt*, but with *dy* = *dz* = *0*: $ds^2 = 0 = -c^2 dt^2 + (1 + h_{11})dx^2$

Strain *h(t)* can have any time dependence, but for now **assume that *h(t)* is constant** during light's travel through ITF. Rearrange, take square root, and replace square root with 1st two terms of binomial expansion.

We find $\int dt = \frac{1}{c}\int_{x=0}^{x=L}\left(1 + \frac{1}{2}h_{11}\right)dx = \frac{h_{11}L}{2c}$

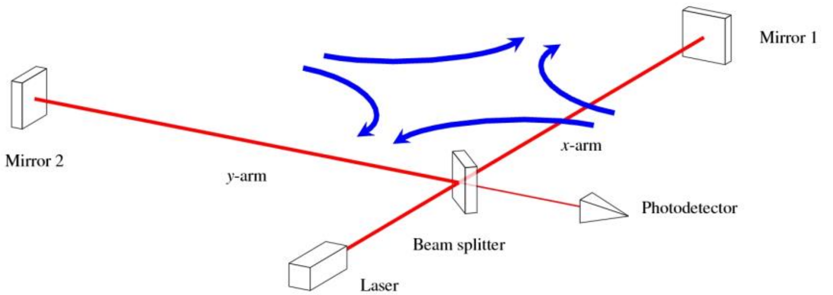
We choose coordinates that are marked by free masses: "Transverse-traceless (TT) gauge"
Beamsplitter at $x = 0$ and end mirror is always at $x = L$

Round trip along x-arm: $\Delta t = h_{11}L/c$ and for the y-arm (with $h_{22} = -h_{11} = -h$): $\Delta t_y = -hL/c$

Difference between x and y round-trip times: $\Delta\tau = 2hL/c$ and $\phi_x - \phi_y = \frac{4\pi L}{\lambda}h$

Laser interferometer detectors

Michelson interferometer: response to a gravitational wave



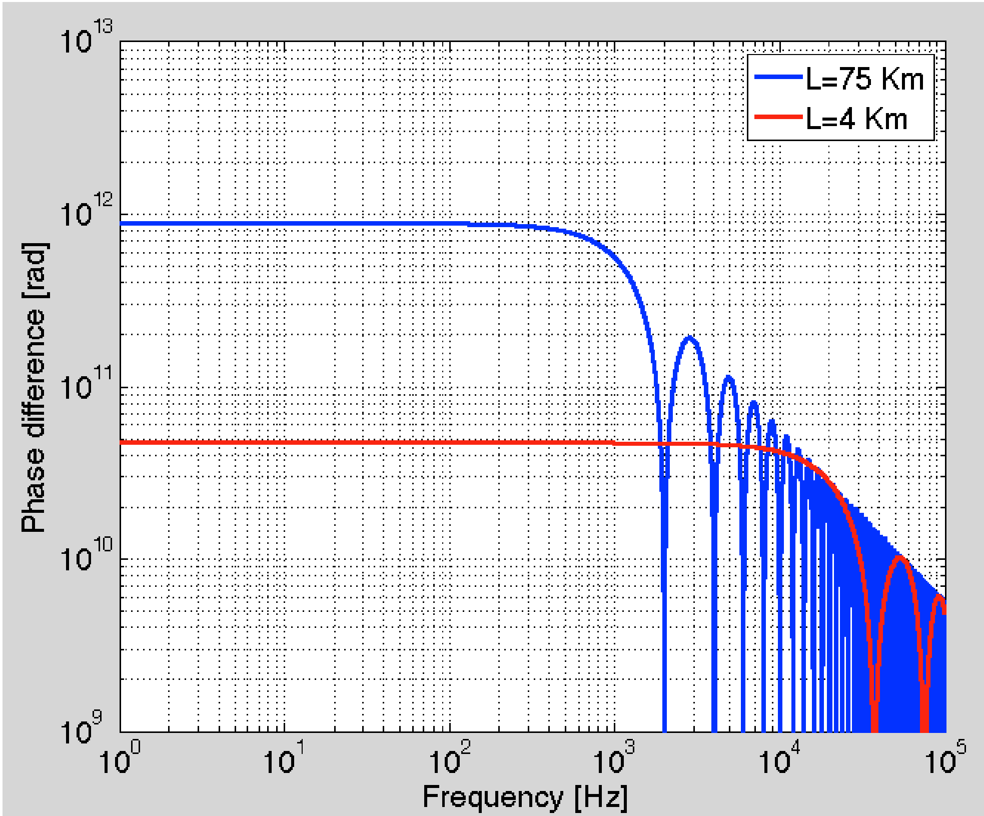
$$\phi_x - \phi_y = \Omega(\tau_x - \tau_y) = \Omega \int_0^{2L/c} h(\tau) d\tau$$

which for $h(t) = h_0 e^{i\omega t}$ is

$$\phi_x - \phi_y = \frac{4\pi L}{\lambda} \left[\frac{\sin \omega L/c}{\omega L/c} e^{-i\omega L/c} \right] h_0 e^{i\omega t}$$

for $f_{gw} \ll c/2L$ the response is flat with

$$\phi_x - \phi_y = \frac{4\pi L}{\lambda} h$$



One would love to have *L = 75* km but ....

Laser interferometer detectors

Fabry-Perot arm cavities *(idea from Ron Drever)*

$$E_{in}(t,x) = E_{in}^0 e^{i(\Omega t - \frac{2\pi}{\lambda}x)}$$

for the **circulating** field we find

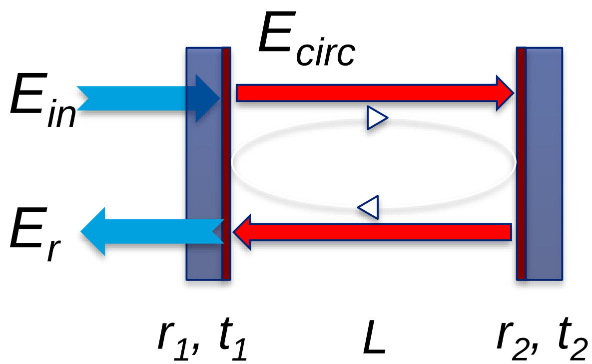
$$E_{circ} = t_1 E_{in} + r_1 r_2 E_{circ} e^{-i4\pi L/\lambda}$$

$$\frac{E_{circ}}{E_{in}} = \frac{t_1}{1 - r_1 r_2 e^{-i4\pi L/\lambda}}$$

while for the **reflected** field we find

$$E_r = -t_1 E_{in} + r_2 t_1 E_{circ} e^{-i4\pi L/\lambda}$$

$$\frac{E_r}{E_{in}} = -r_1 + \frac{t_1^2 r_2 e^{-i4\pi L/\lambda}}{1 - r_1 r_2 e^{-i4\pi L/\lambda}}$$



Circulating power builds up for $L=n\lambda/2$

Laser interferometer detectors

Fabry-Perot arm cavities

The ''sharpness'' of the peaks is defined by the finesse

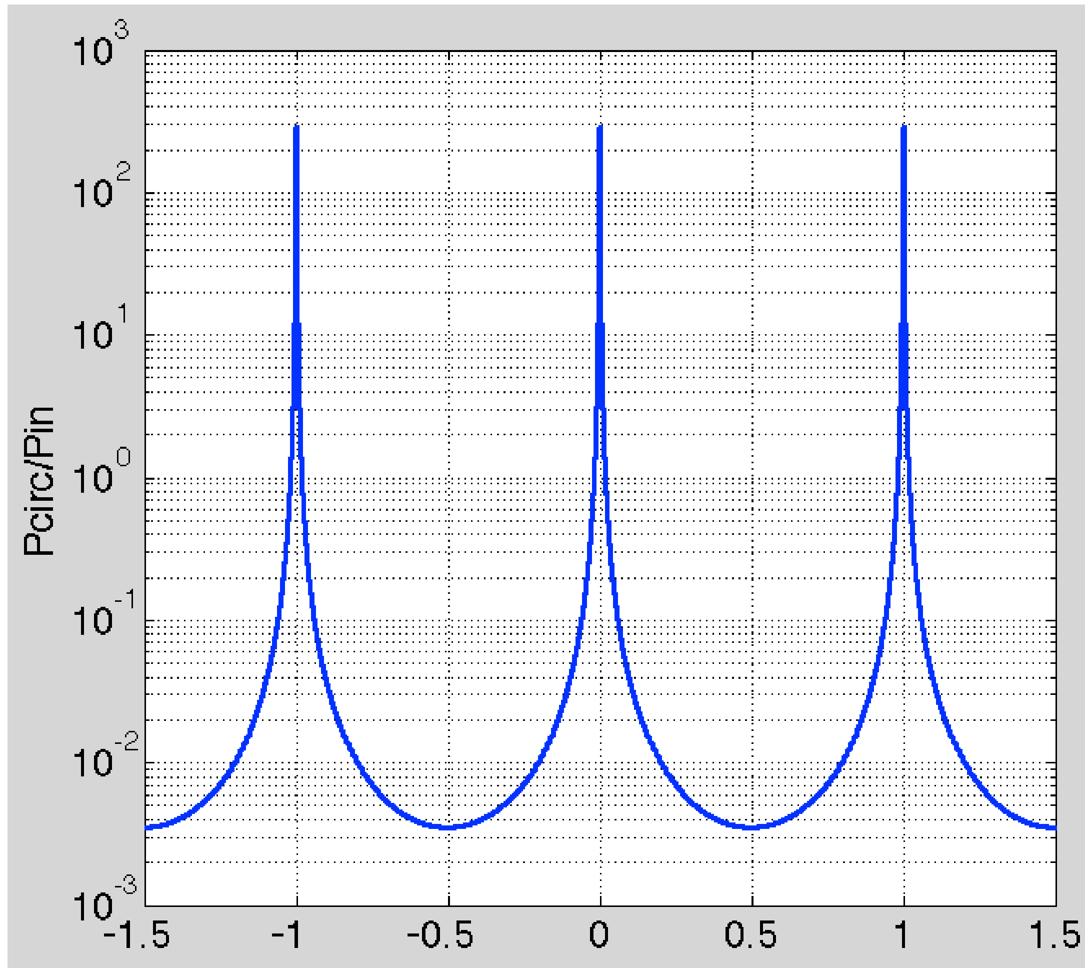
$$F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

The finesse is related to the number of round trips the light makes inside the cavity

$$N = \frac{2F}{\pi}$$

For Advanced Virgo

$$T_1 = t_1^2 = 0.014$$
$$T_2 = t_2^2 = 10 \text{ ppm}$$
$$L = 3 \text{ km}$$
$$F = 440$$
$$P_{\text{circ}} = 650 \text{ kW}$$
$$N = 280$$



Laser interferometer detectors

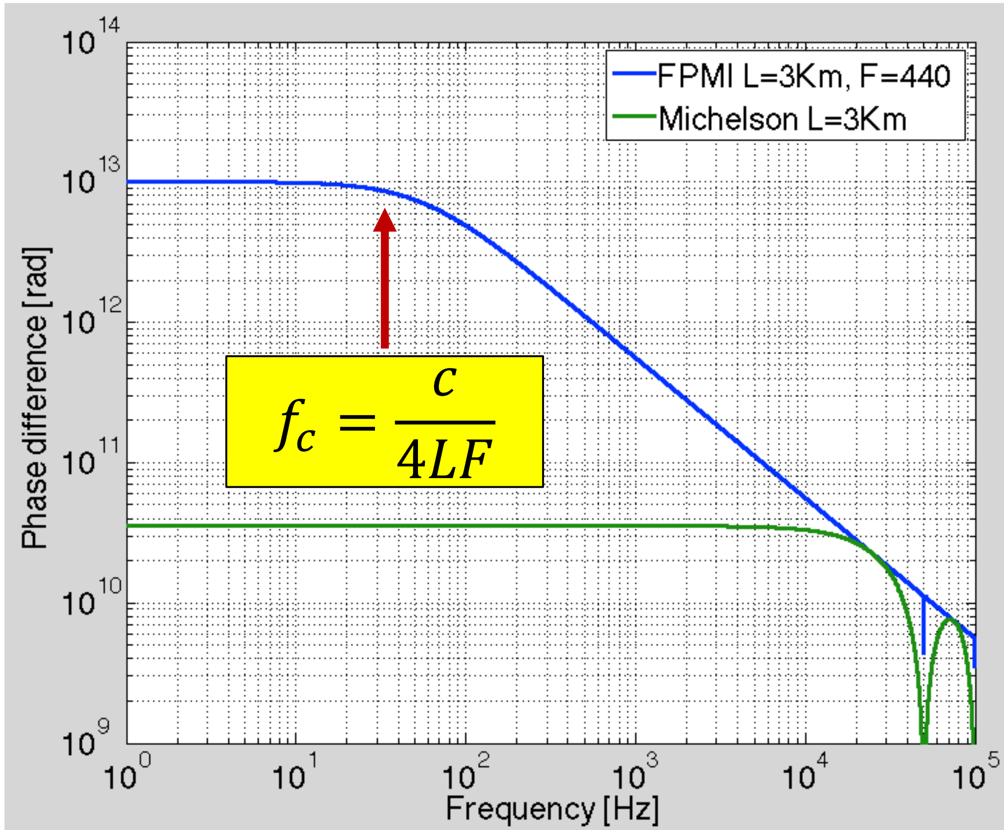
Fabry-Perot arm cavities

$$\phi_x - \phi_y = \frac{4\pi L}{\lambda}\left[\frac{\sin \omega L/c}{\omega L/c} e^{-i\omega L/c}\right]\left[\frac{t_1^2 r_2/(1-r_1 r_2)}{1-r_1 r_2 e^{-i2L\omega/c}}\right] h_0 e^{i\omega t}$$

which, for $f_{gw} \ll c/2L$, is well approximated by

$$\phi_x - \phi_y = \frac{8LF}{\lambda} \cdot \frac{h_0 e^{i\omega t}}{1 + i\omega \frac{2LF}{\pi c}}$$

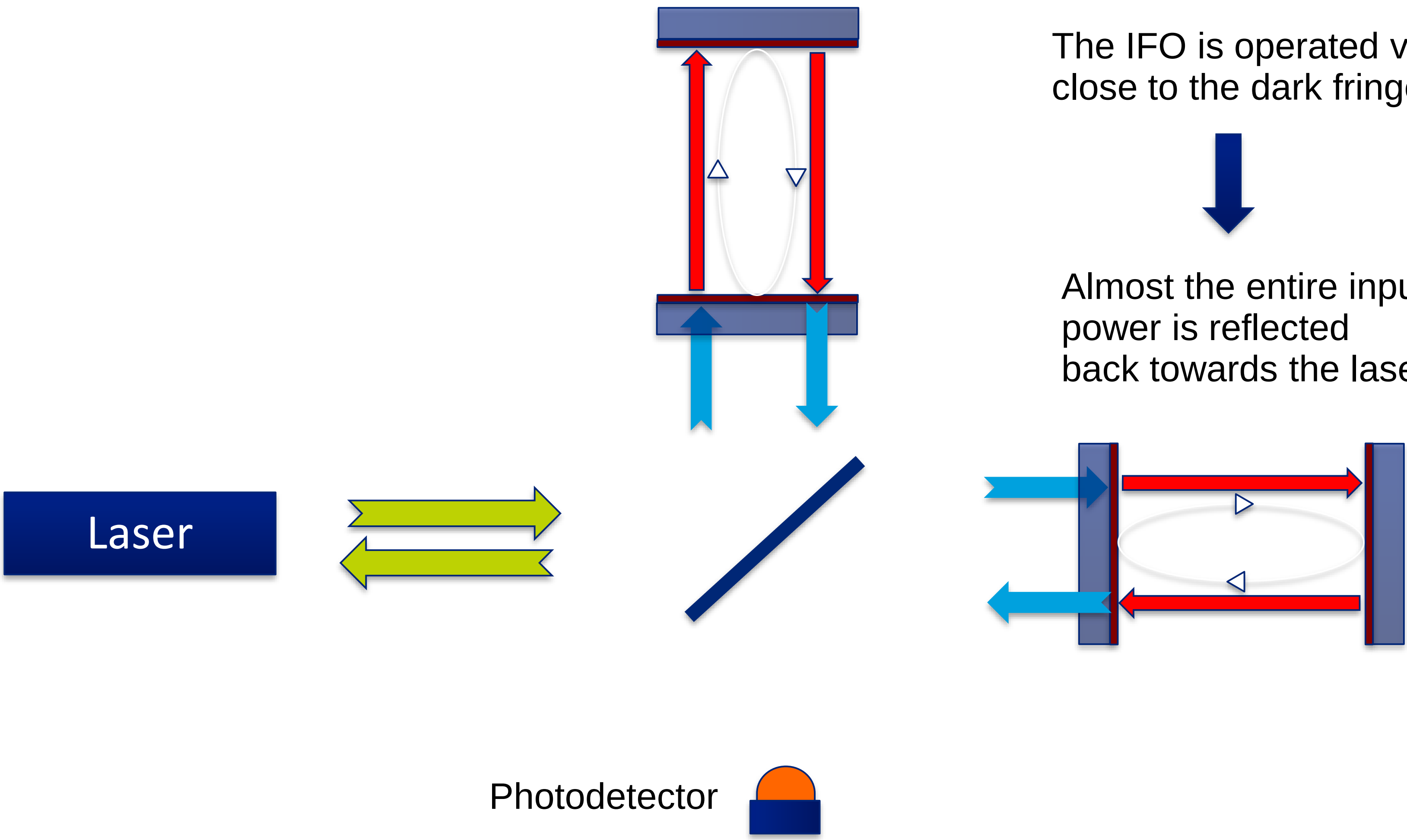
The "effective" arm length increases by a factor *N*



What else can we do to enhance the signal?

Laser interferometer detectors

Power recycling *(idea from Ron Drever)*



The IFO is operated very close to the dark fringe

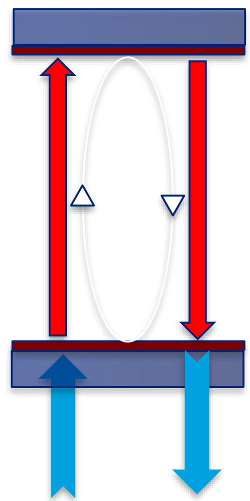
Almost the entire input power is reflected back towards the laser

Laser interferometer detectors

Power recycling *(idea from Ron Drever)*

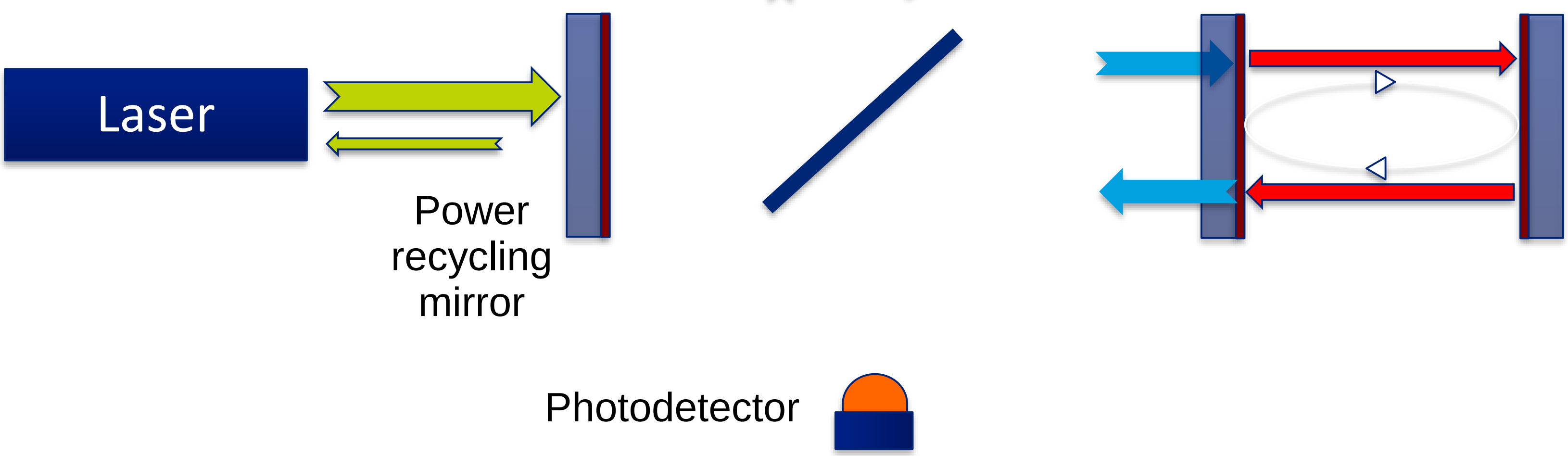
A partially transmitting mirror (PRM) is placed between Laser and beam splitter forming a three-mirror cavity with ITMs

Light reflected back from the IFO is summed coherently with ‘fresh’ photons from the laser



The IFO is operated very close to the dark fringe

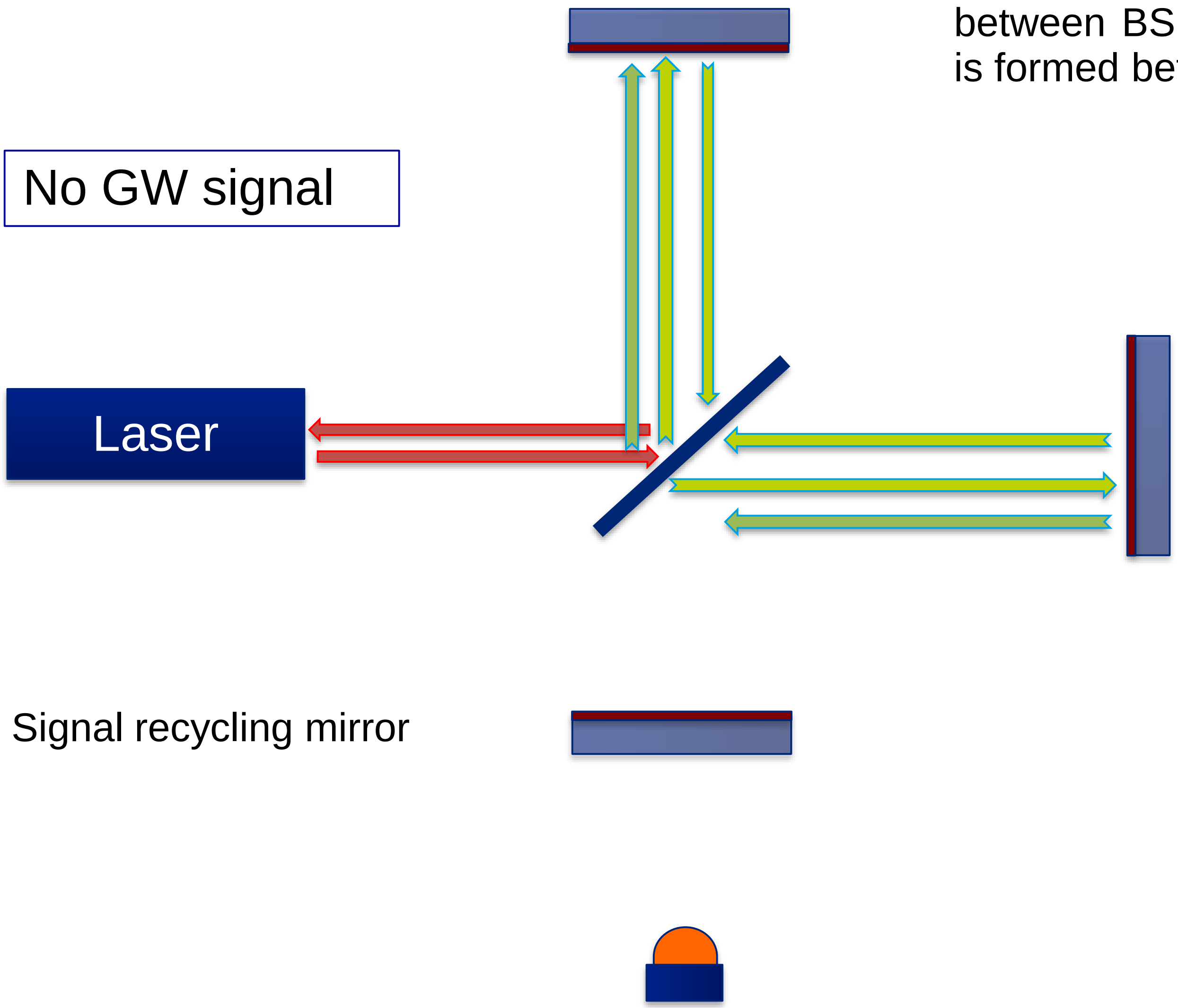
Almost the entire input power is reflected back towards the laser



Laser interferometer detectors

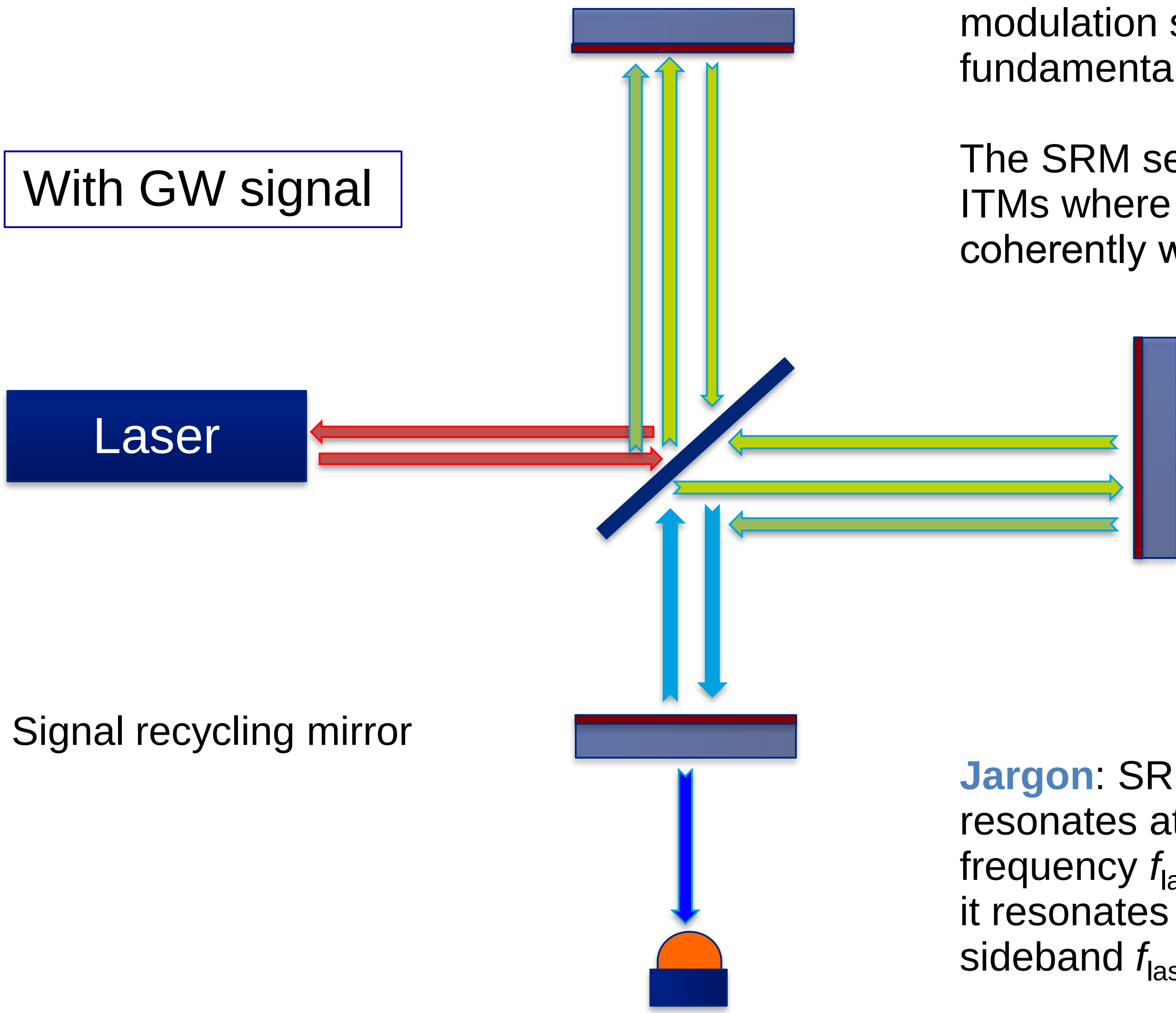
Signal recycling *(idea from Brian Meers)*

By introducing a partially reflective mirror between BS and PD a three-mirror resonator is formed between the end mirrors and SRM



Laser interferometer detectors

Signal recycling *(idea from Brian Meers)*



Idea: the GW signal appears as an amplitude modulation sidebands around the laser fundamental frequency

The SRM sends the signal back towards the ITMs where it gets reflected and summed coherently with 'fresh' signal from the arms

Signal builds up !!

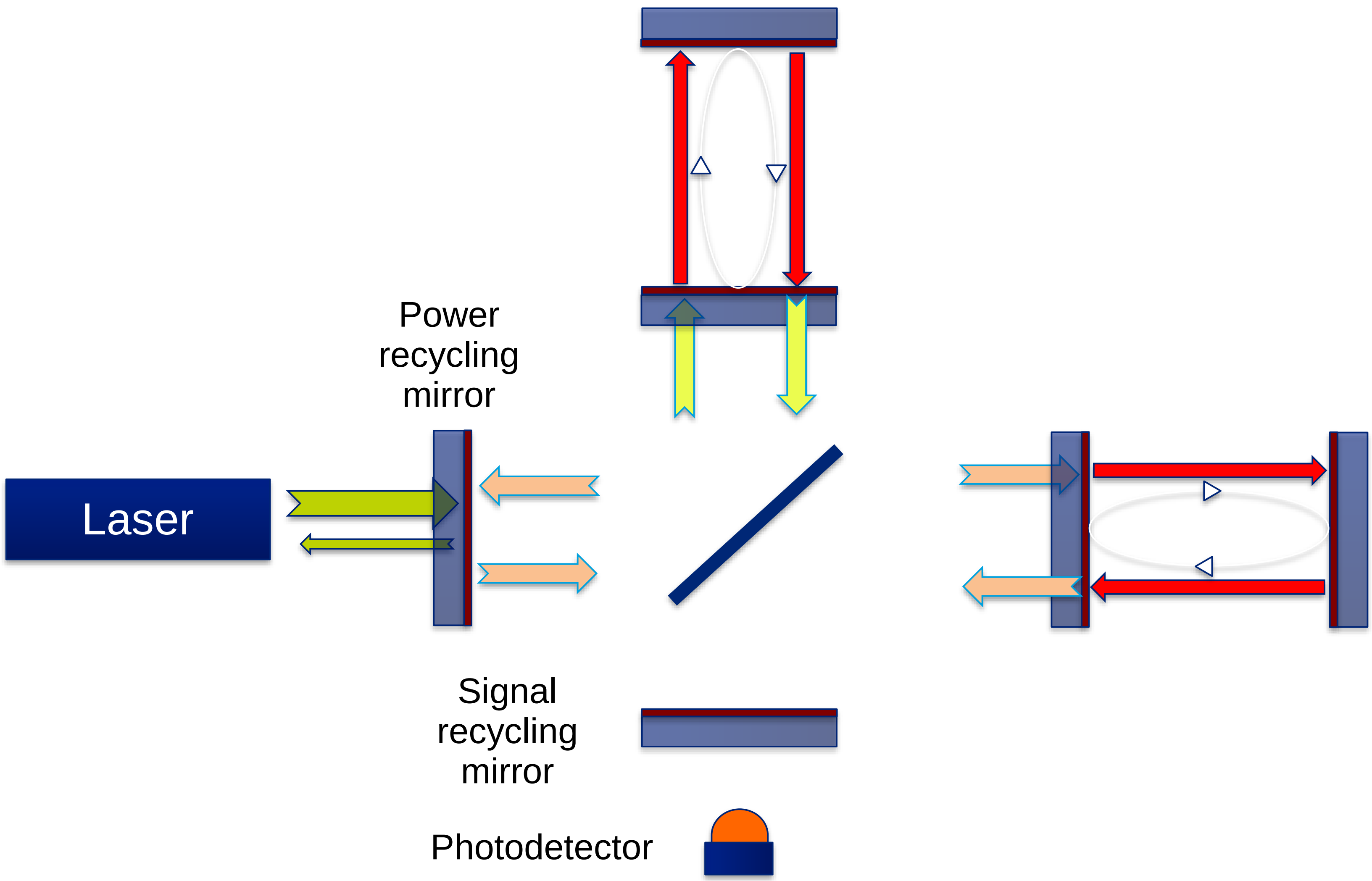
Advanced Virgo plans

$$T_{SRM} = 0.20$$
$$F = 26$$

Jargon: SR cavity is called tuned when it resonates at the laser fundamental frequency f_{laser}; it is called detuned when it resonates at a specific GW induced sideband $f_{laser}+f_{GW}$

Laser interferometer detectors

Dual recycled Fabry-Perot Michelson interferometer

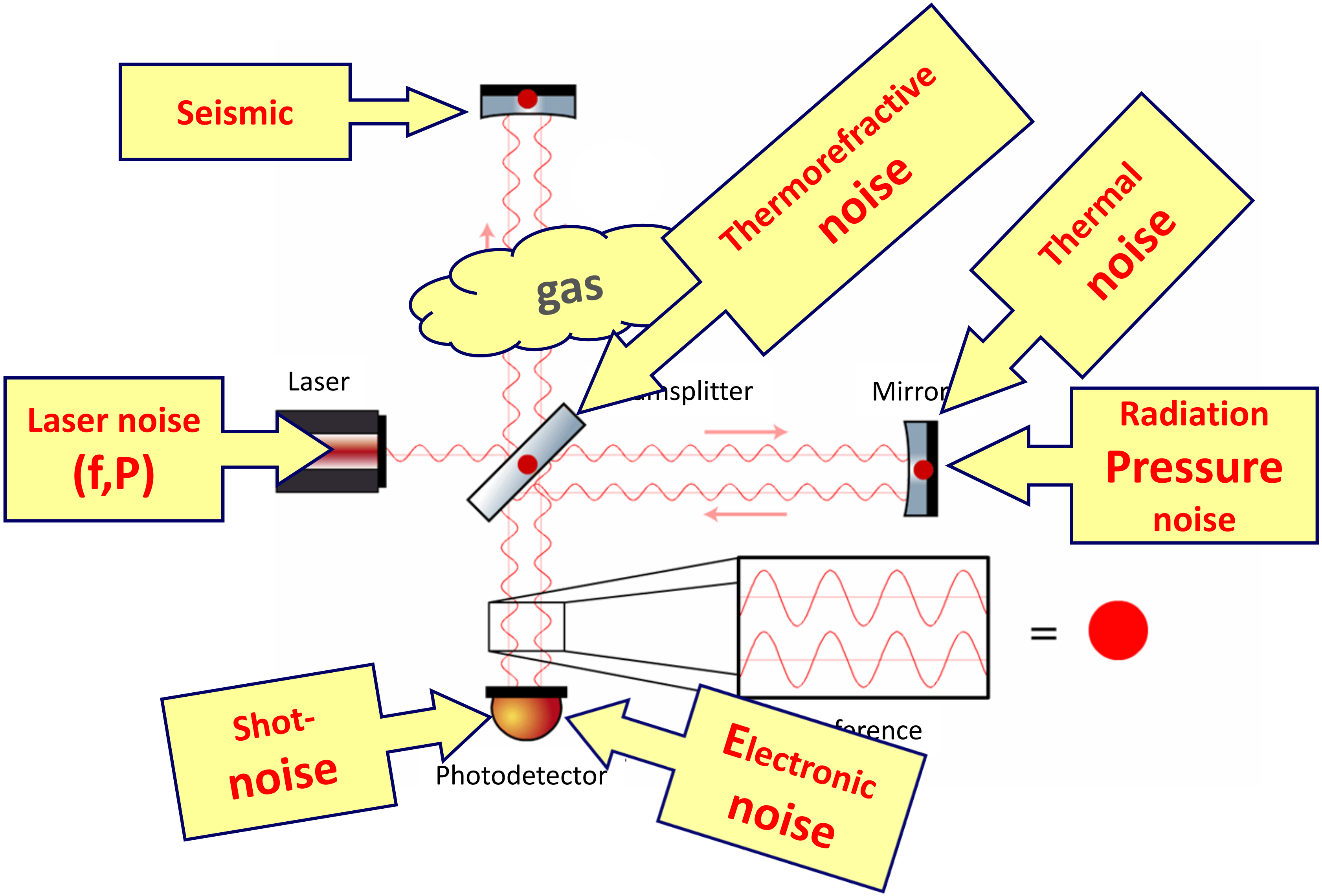


Laser interferometer detectors

Lock acquisition in Advanced Virgo (*signal recycling will be implemented after run O3*)

Interferometer: noise sources

Fundamental and technical noise sources limit the sensitivity of our instruments



Laser interferometer detectors

Fundamental limits: shot noise

A light beam consists of a stream of photons: a beam with power *P* has a photon flux (photons/sec)

$$\bar{N} = \frac{P}{h\nu}$$

We know that

- nothing guarantees that *N* photons will arrive every second; some seconds there will arrive more, and in other seconds fewer photons will arrive at the photodiode;
- experiments show that the behavior is regulated by a Poisson statistics;
- then, if we expect *N* independent events on average, the standard deviation is $\sigma = \sqrt{N}$
- then, the higher the power, the lower the relative fluctuation

In frequency domain the photon counting error appears as white noise with rms value

$$\Delta P_{shot} = \sqrt{2h\nu P \Delta f}$$

The corresponding minimum GW signal observable over 1 Hz bandwidth is (close to the dark fringe)

$$h_{min}^{GW} = \frac{\lambda}{4\pi L_e}\sqrt{\frac{2h\nu}{P_{in}}} = 1 \cdot 10^{-24}$$

ν = *300* THz
P_{in} = *3.9* kW
L_e = 840 Km

Laser interferometer detectors

Fundamental limits: radiation pressure

Photons carry momentum and exert a mechanical pressure on the mirrors (static and dynamic)

$$F_{static} = 2\bar{N}\frac{h\nu}{c}$$

$$F_{noise} = 2\frac{h\nu}{c}\sqrt{\bar{N}} = 2\frac{h\nu}{c}\sqrt{\frac{P}{h\nu}} = 2\sqrt{\frac{Ph}{\lambda c}}$$

$$x_{noise} = \frac{2}{m\omega^2}\sqrt{\frac{Ph}{\lambda c}}$$

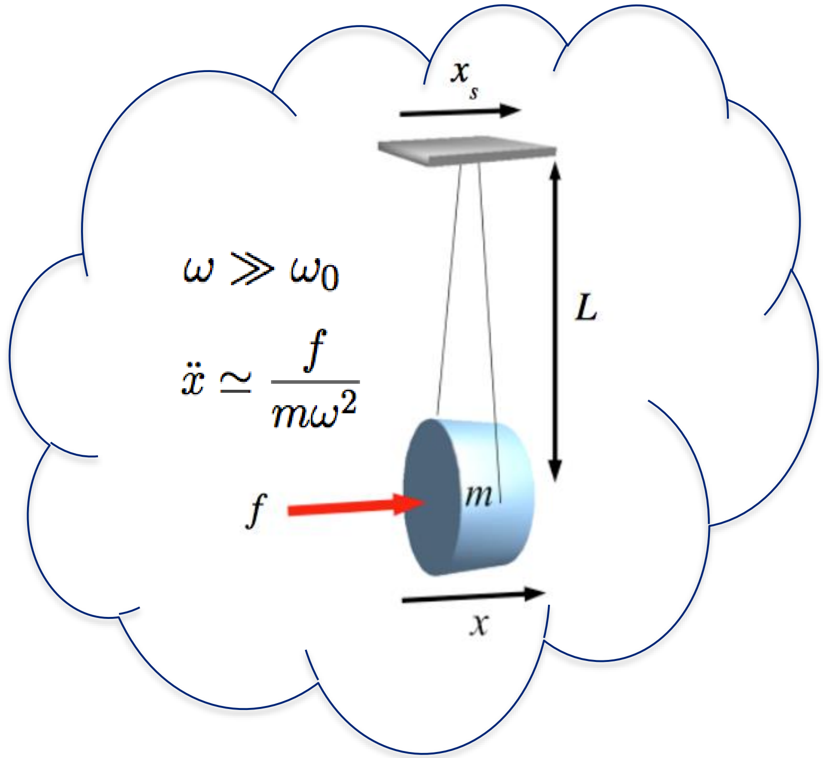
for a simple **Michelson** interferometer

$$h_{GW}^{rp} = \frac{2x_n}{L}$$

for a **FP Michelson** interferometer

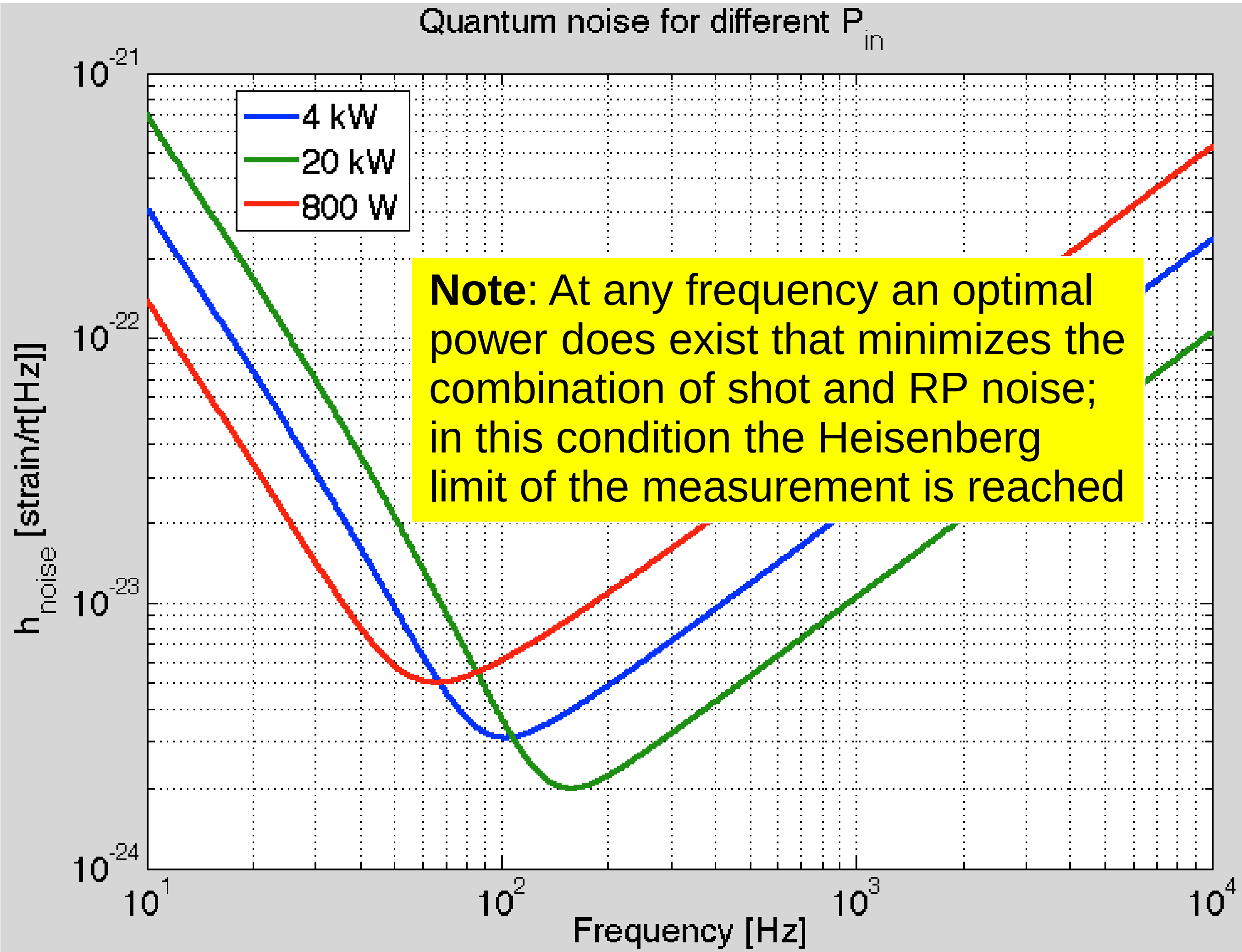
$$h_{GW}^{rp} = 2 \cdot \sqrt{2} \cdot \frac{2F}{\pi}\frac{x_n}{L} = \frac{8F}{\pi L m\omega^2}\sqrt{\frac{2P_{in}h}{\lambda c}}$$

Reminder



Laser interferometer detectors

Fundamental limits: radiation pressure



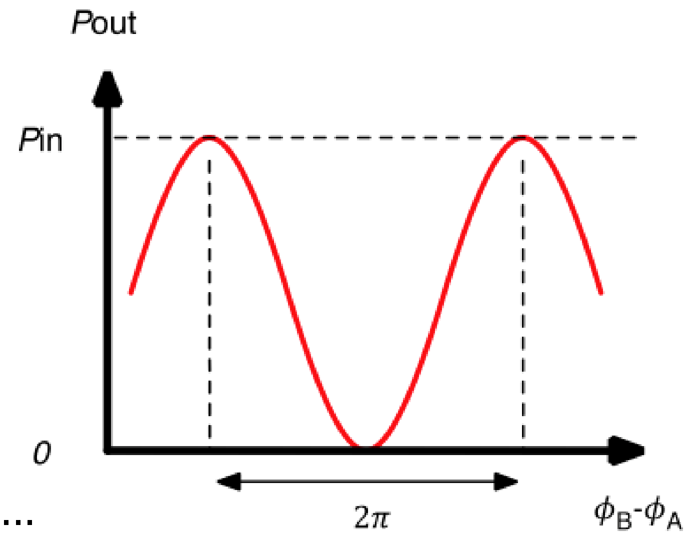
Note: At any frequency an optimal power does exist that minimizes the combination of shot and RP noise; in this condition the Heisenberg limit of the measurement is reached

Laser interferometer detectors

Working point (why the dark fringe?)

$$P_{out} = \frac{P_{in}}{2}\left[1 - \cos\left(\phi_0 + \frac{4\pi L_e}{\lambda}h\right)\right]$$

$$\Delta P_{out} = \frac{2\pi L_e}{\lambda} P_{in} \sin\phi_0 h$$



The max response to *h* is at π/2 but laser power fluctuations...
Even if the lasers used in GW detectors are the best ever made (dP/P < 10^{-8} at f > 10 Hz)

$$\Delta P_{out} = \frac{\Delta P_{in}}{2}(1 - \cos\phi_0) \Longrightarrow h_{min} = 10^{-8}\frac{\lambda}{4\pi L_e} = 8 \cdot 10^{-22}$$

not good enough !!

… better with a little offset from the dark fringe

$$\Delta P_{out} \simeq \frac{2\pi L_e}{\lambda} P_{in}\phi_0 h$$

$$h_{min} = 10^{-8}\phi_0\frac{\lambda}{4\pi L_e} = 8 \cdot 10^{-26}$$

Laser interferometer detectors

Seismic noise

Earth crust moves relentlessly in a wide frequency range from nHz to hundreds of Hz:

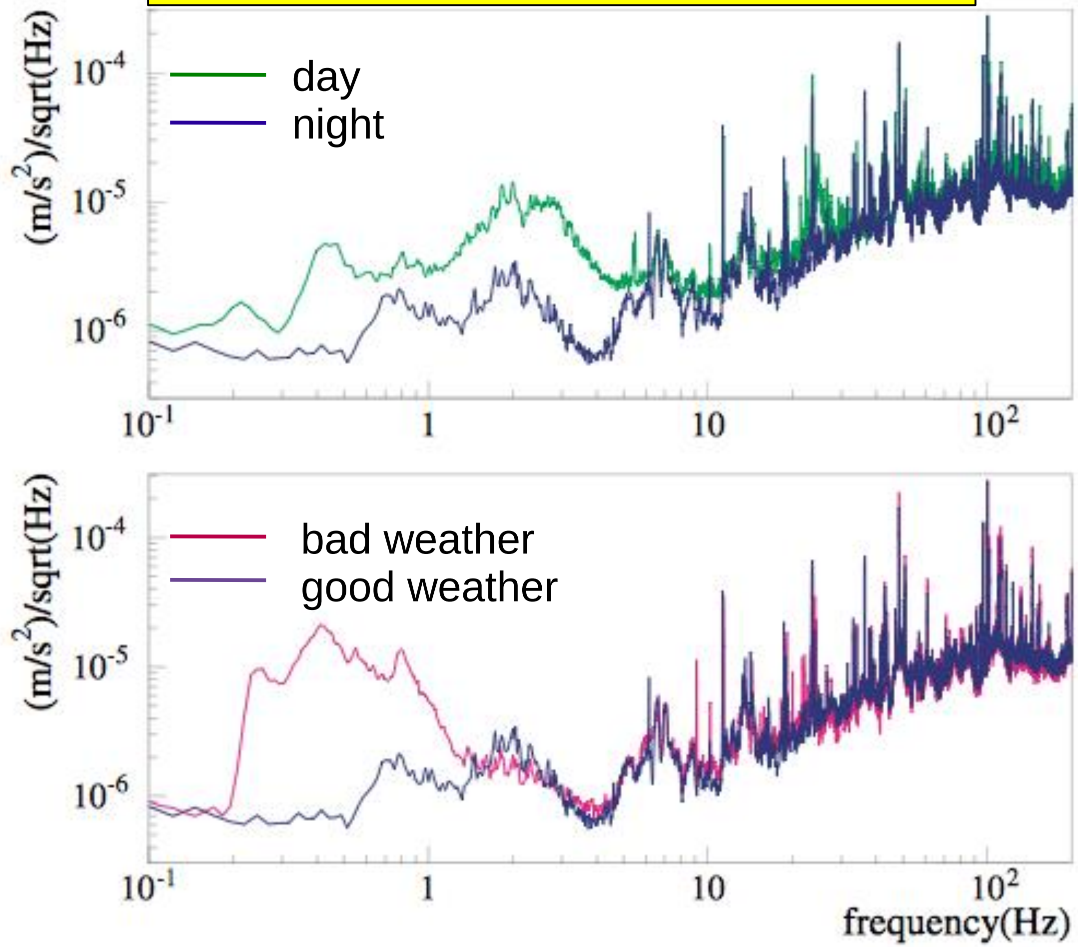
- Tectonic movements
- Lunar tides (few μHz)
- Microseismic peak from ocean waves (0.1-0.3 Hz)
- Anthropogenic and wind induced noise (f>1 Hz)

Amplitude exceeds by several orders of magnitude the test mass background motion aimed for GW detection ($<10^{-18}$m)

At the Virgo site:

at f > 10 Hz $$x_s \simeq \frac{10^{-7}}{f^2}\left[\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}\right]$$

Typical ground motion spectra at the Virgo site



Laser interferometer detectors

Simple pendulum transfer function

Reminder

at low frequencies ($\omega \ll \omega_0$)

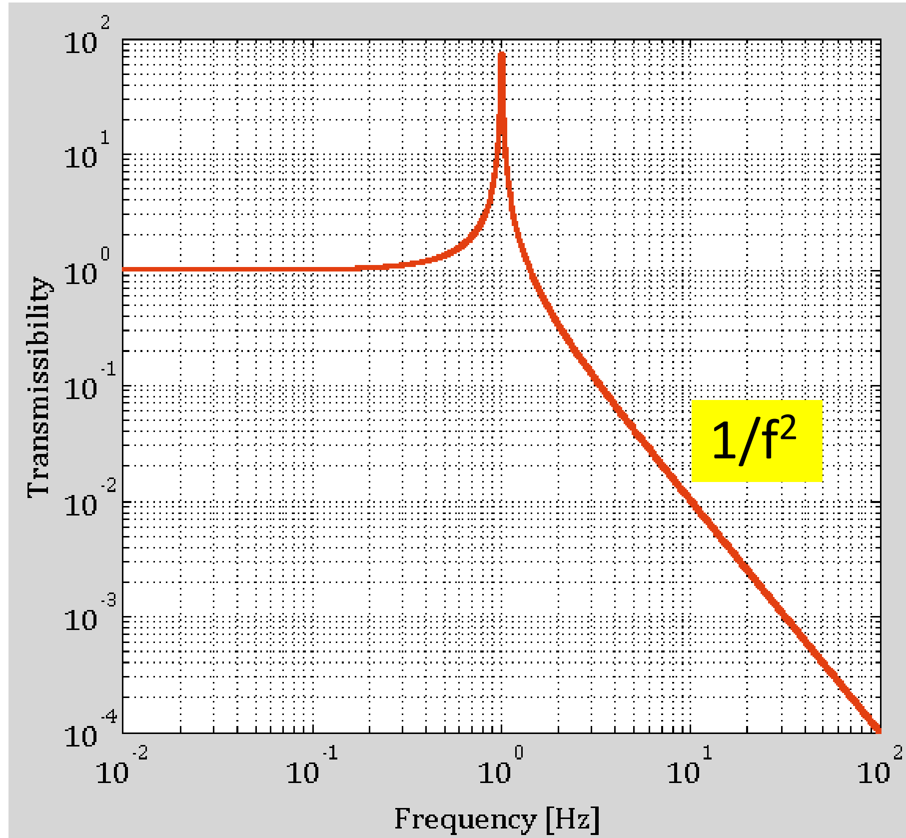
$$X \simeq X_s + F/m{\omega_0}^2$$

at the natural frequency ($\omega = \omega_0$)

$$X \simeq Q(X_s + F/m{\omega_0}^2)$$

while at high frequency ($\omega \gg \omega_0$)

$$X \simeq \frac{{\omega_0}^2}{\omega^2} X_s + F/m\omega^2$$



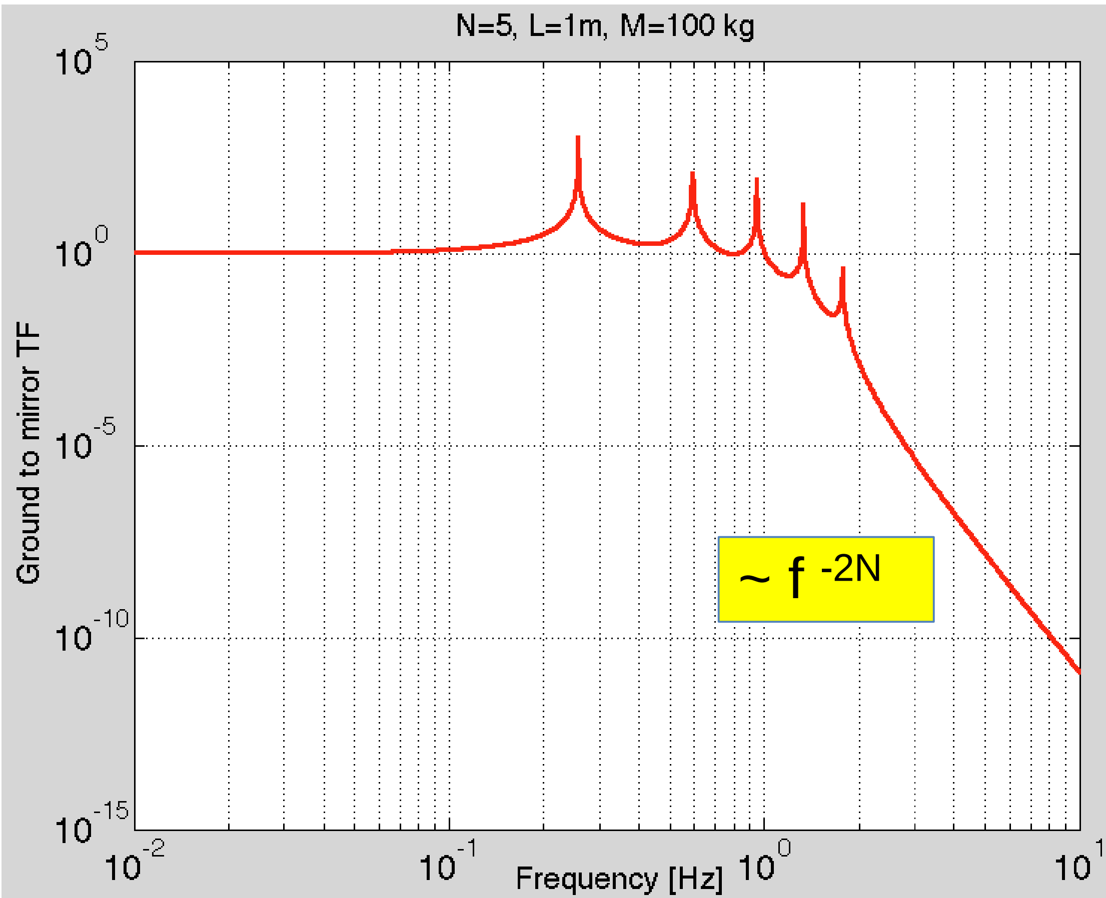
The suspension also provides attenuation of ground vibrations, ...but far from the 10^8-10^{10} seismic attenuation required in the GW detection band (10 Hz - 3 kHz)

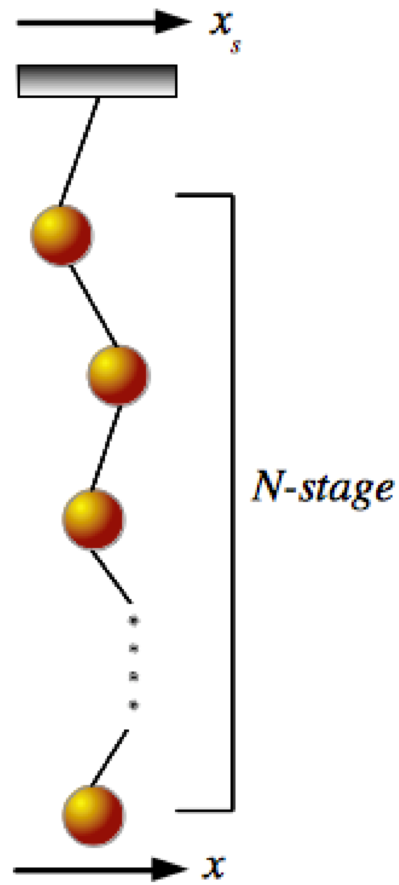
Laser interferometer detectors

Solution: cascading mechanical filters (*seismic filters*) with uncoupled natural frequencies sufficiently lower than 10 Hz

The Lagrangian of a chain of simple pendulums is:

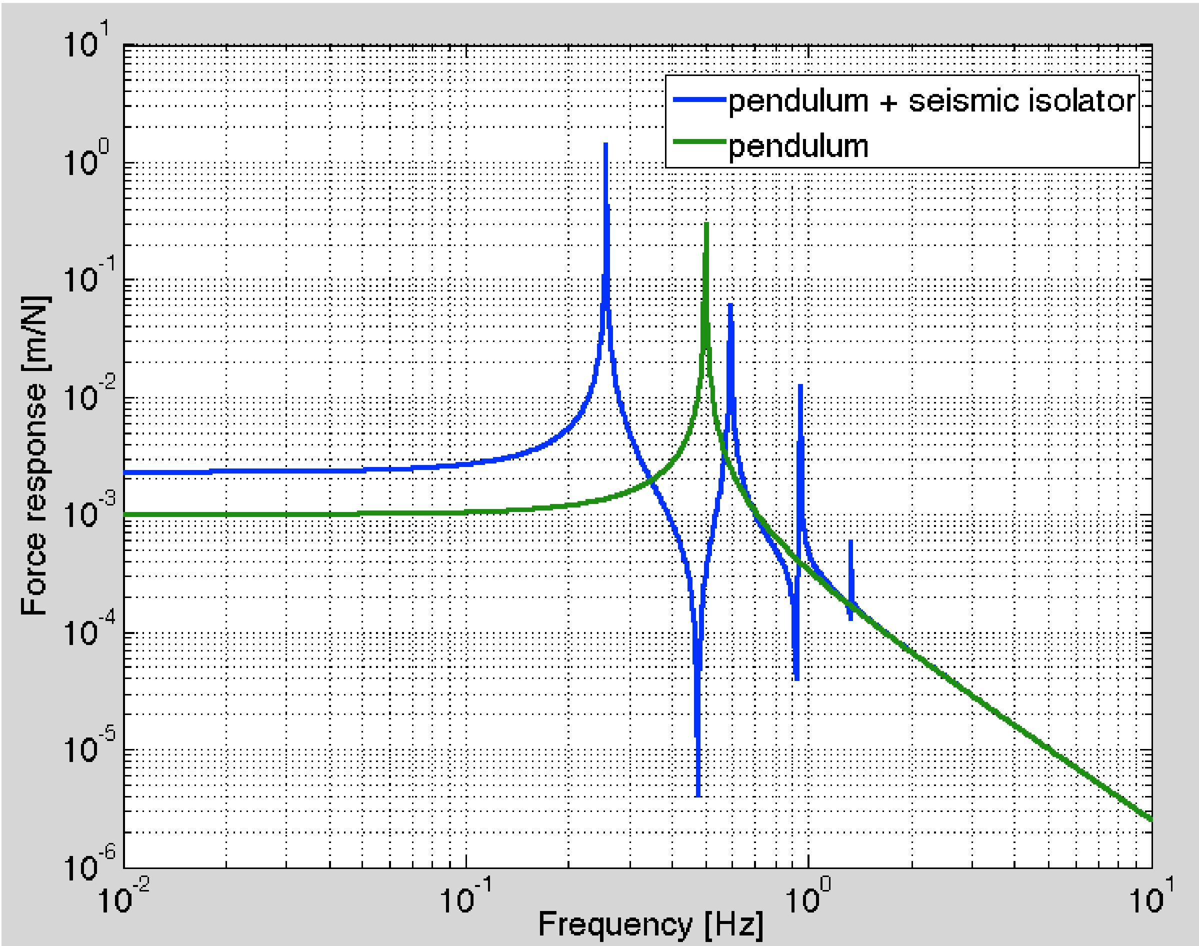
$$\mathcal{L} = T - V = \frac{1}{2}\sum_{i=1}^{N} m_i \dot{x}_i^2 - \frac{1}{2} g \sum_{i=1}^{N} \frac{\sum_{j=i}^{N} m_j}{L_i} (x_i - x_{i-1})^2$$

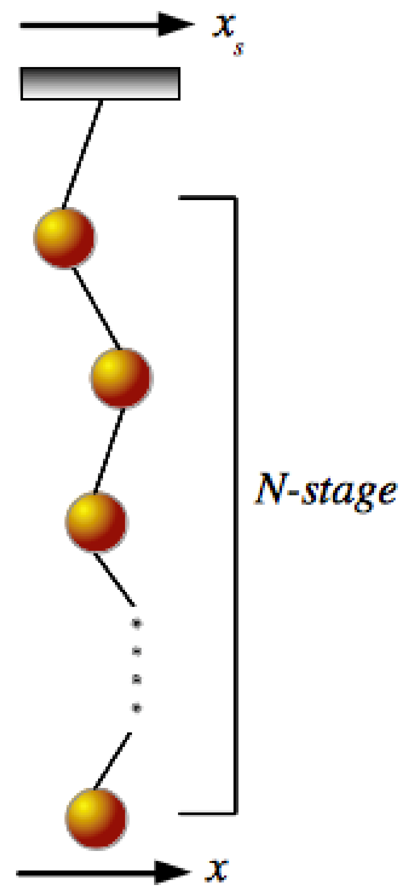




Laser interferometer detectors

Applying a force to the test-mass





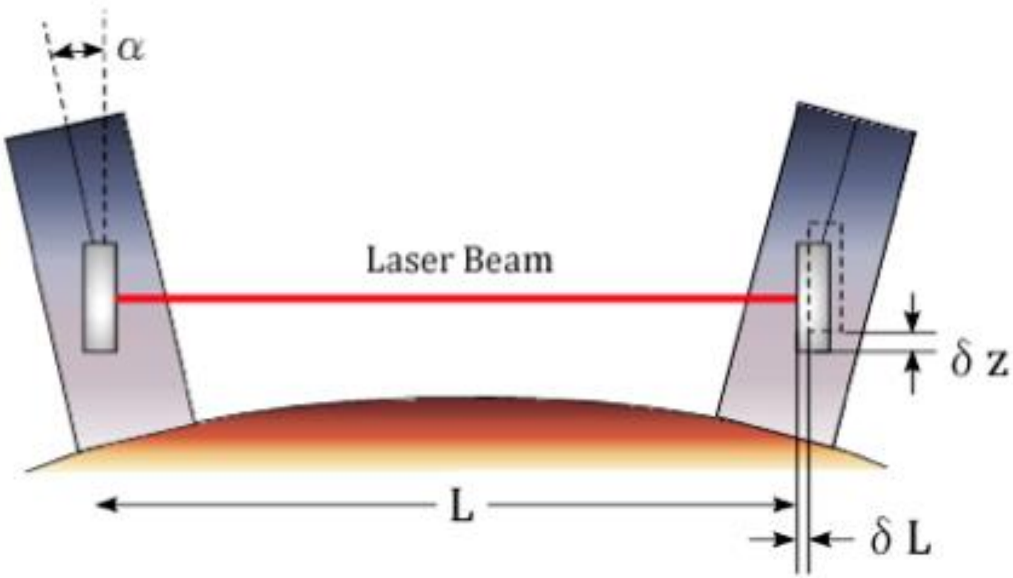
Above the seismic isolator cut-off the mirror responds as a single simple pendulum

Laser interferometer detectors

… but life is hard …

Horizontal seismic filtering is not sufficient because:

1. Non-parallelism of verticality between objects a few km apart channels vertical seismic noise along the GW sensing axis ($2*10^{-4}$ coupling over 3 km)

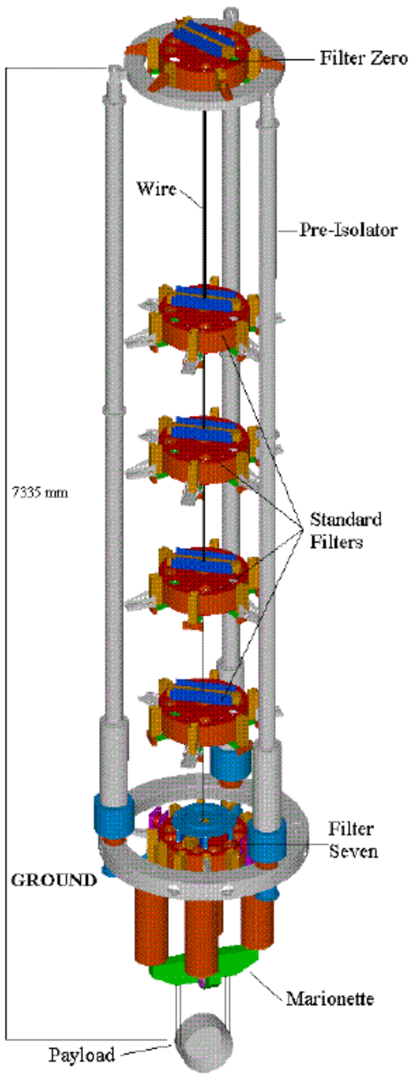


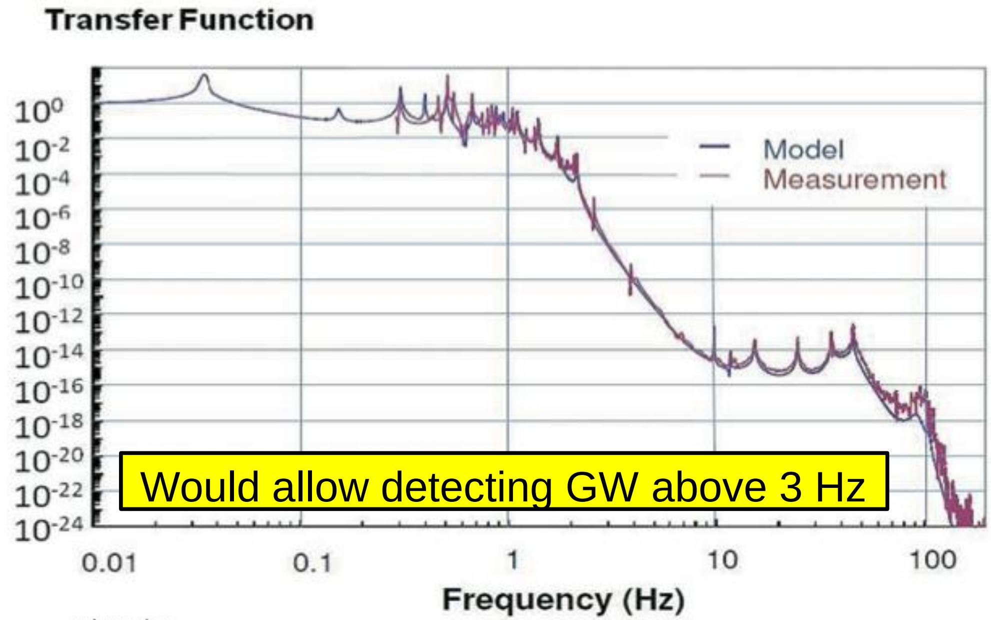
2. Imperfections in the mechanical assembly may cause even larger couplings (up to 1%)

Vertical seismic isolation is necessary !!

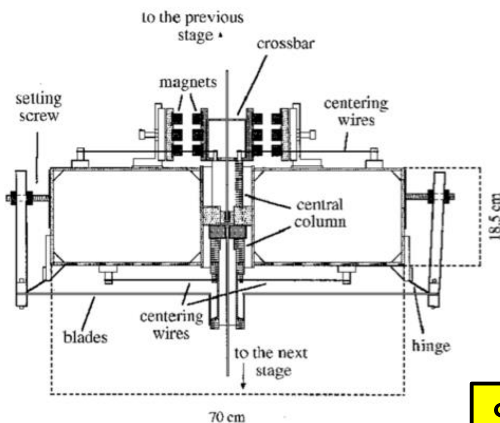
Laser interferometer detectors

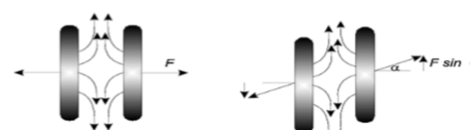
The Virgo superattenuator





Would allow detecting GW above 3 Hz





working principle of the magnetic anti-spring system. When the magnets are displaced in the vertical direction (right side) a vertical component of the repulsive force appears.

SA Magnetic anti-spring vertical filters

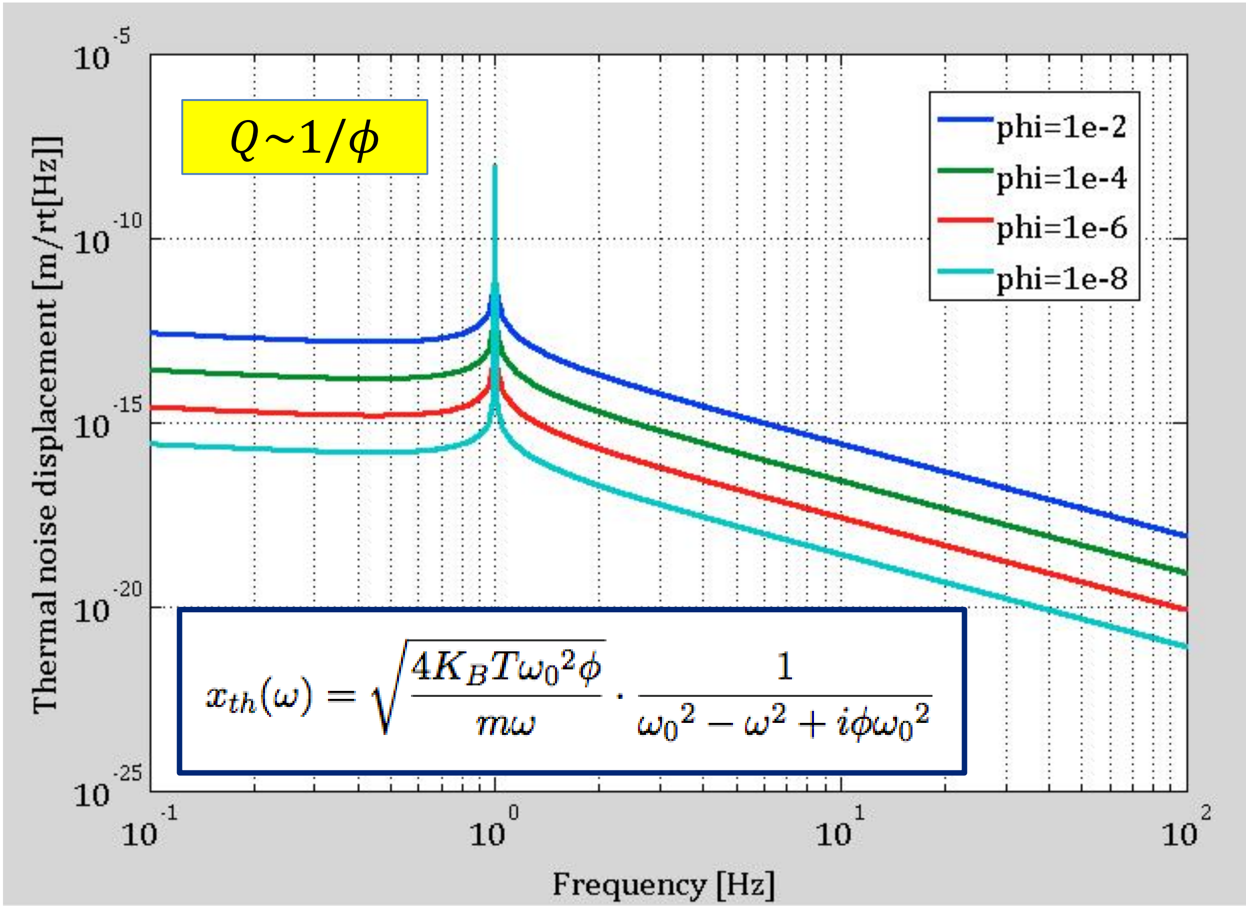
Laser interferometer detectors

Brownian motion

- Internal friction in the material of the suspension wires causes the mirrors to move
- The effect dominates over filtered seismic at f > 3 Hz

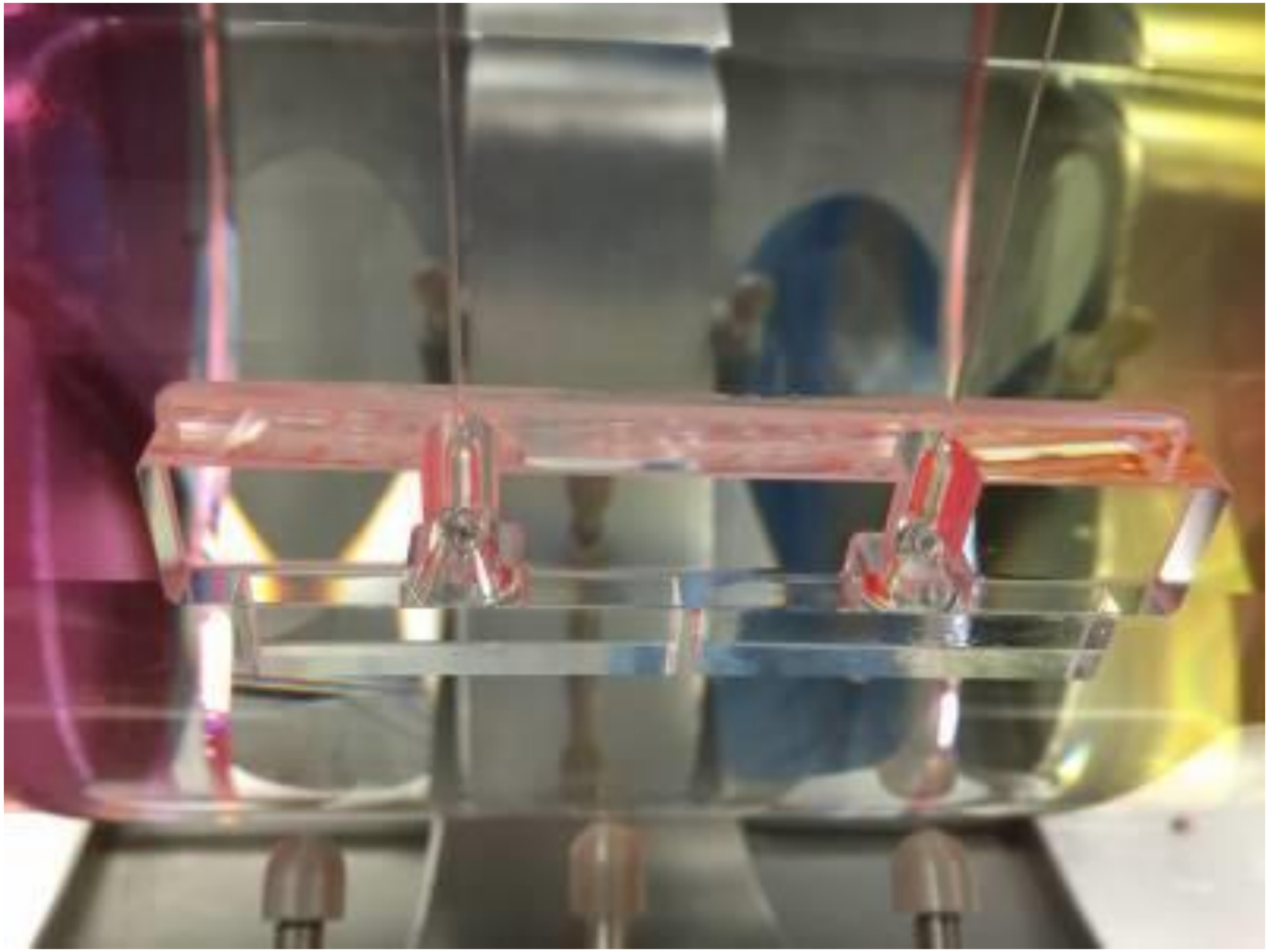
Two possible choices for the wire material:

- ❑ a 'perfect' crystal
- ❑ a 'perfect' glass



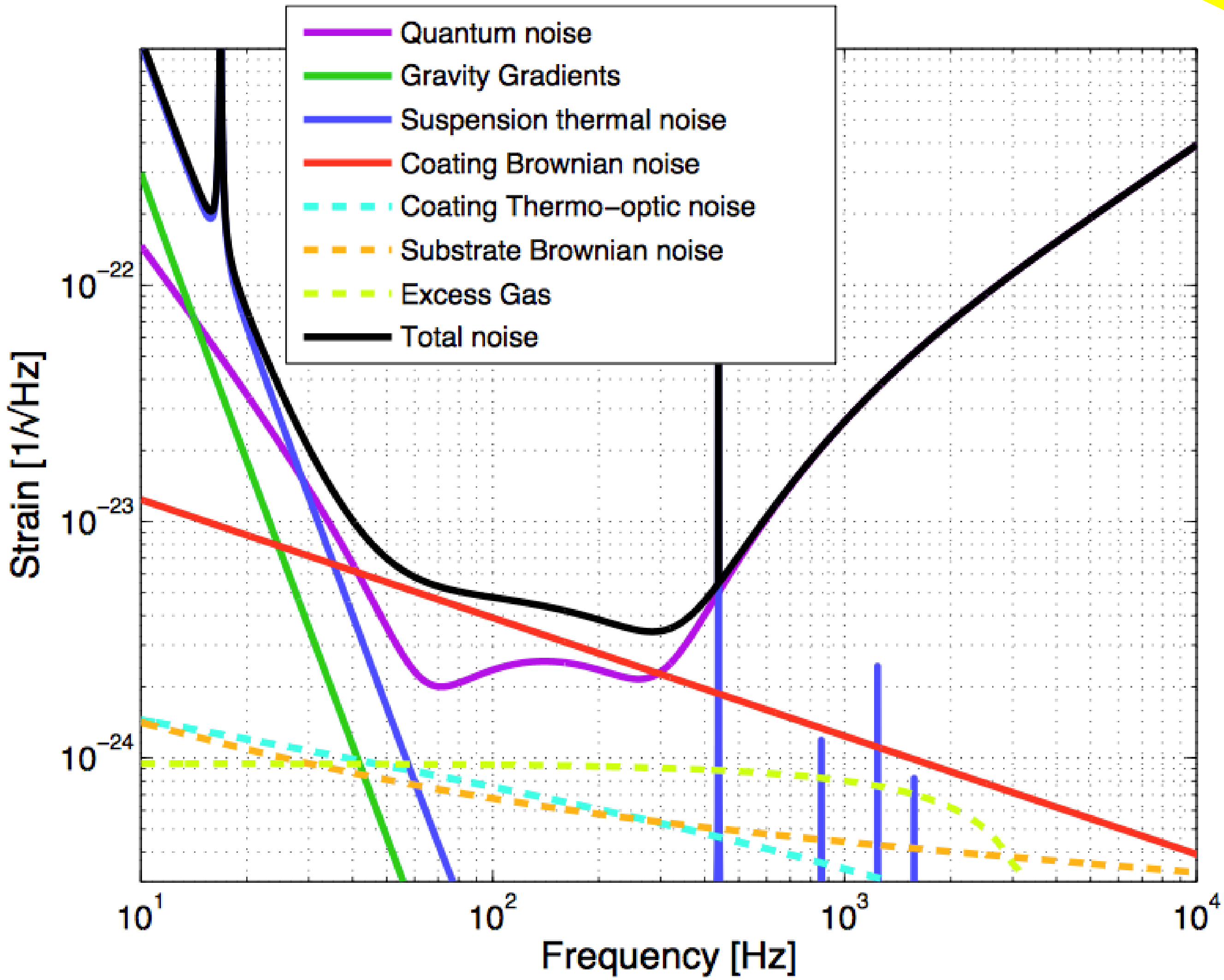
Thermal noise

Monolithic suspensions. High Q-values, but now sensitive to parametric instabilities



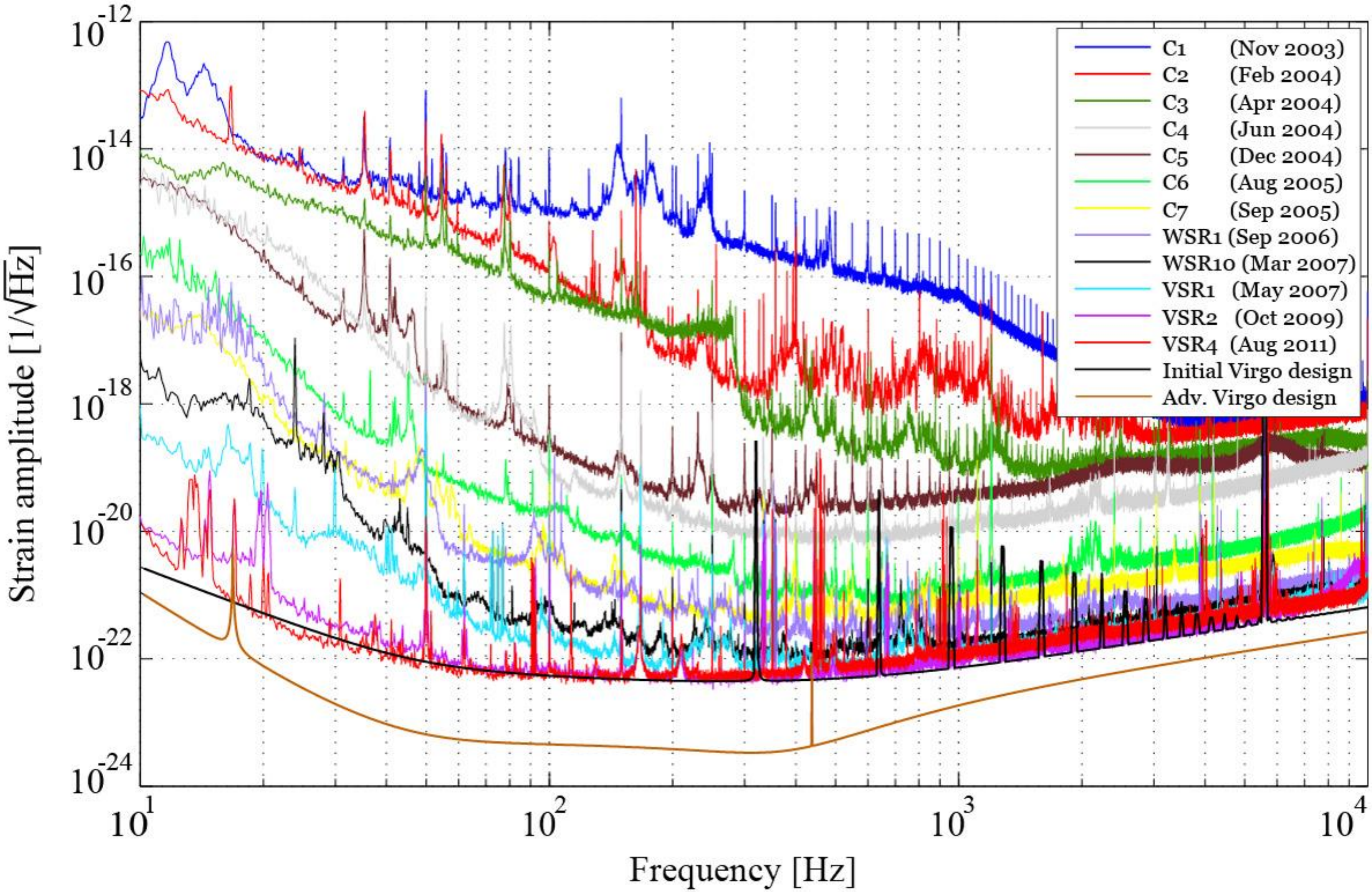
Laser interferometer detectors

Advanced Virgo sensitivity curve



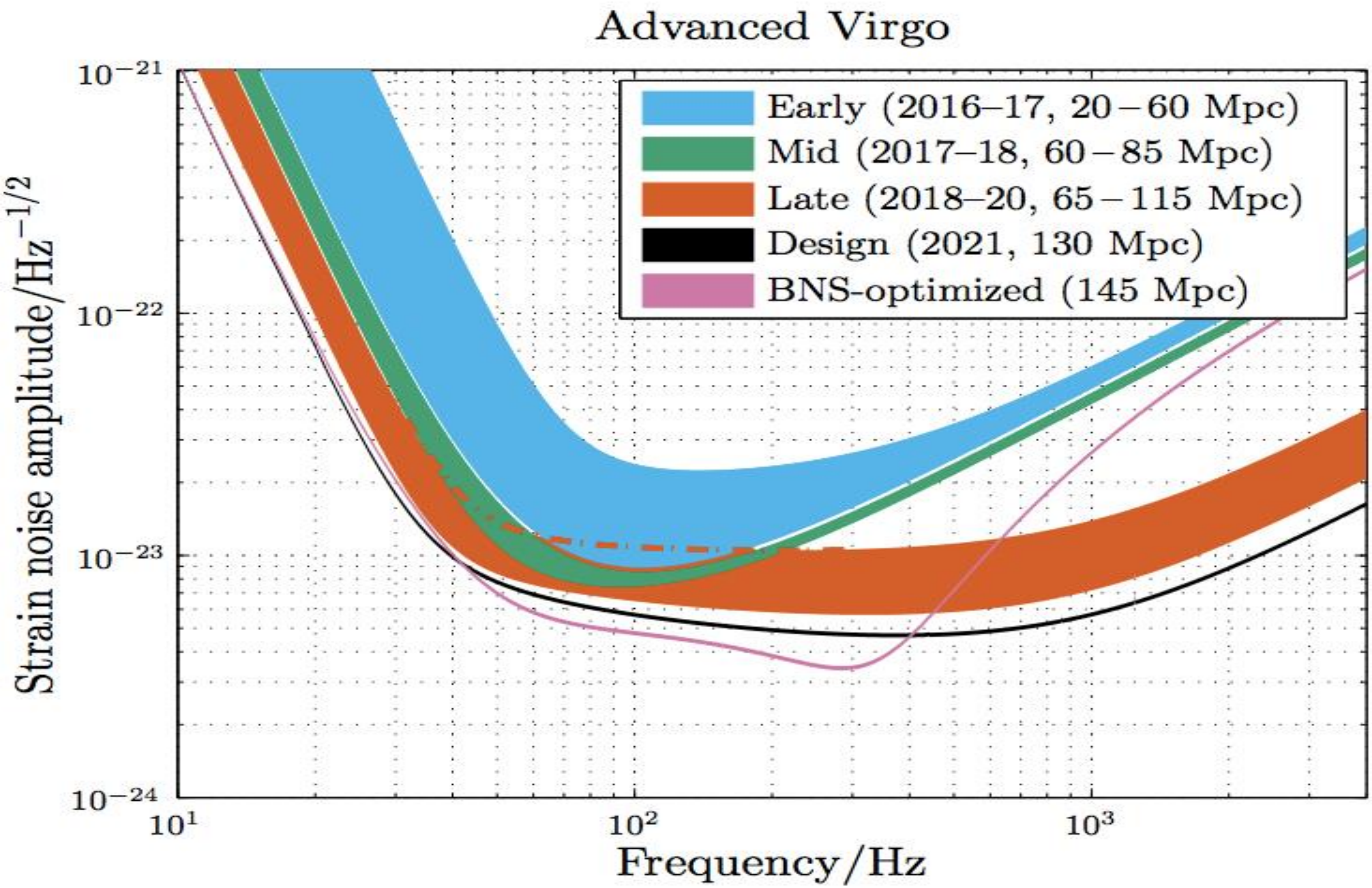
… now working hard to reach it ….

Virgo: sensitivity evolution



Projected sensitivity evolution for Virgo

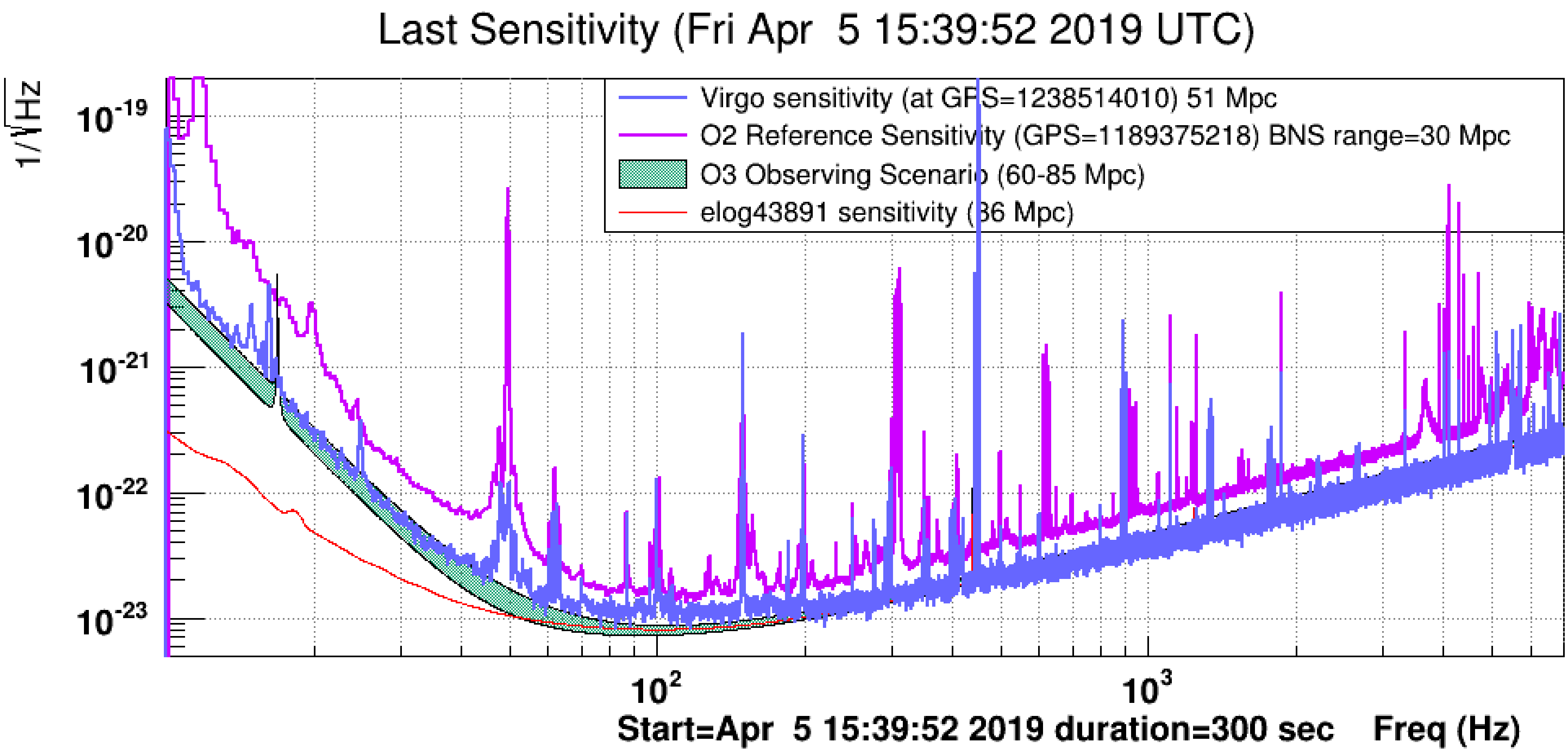
From the 2013 ‘‘Observing Scenario’’, arXiv:1304.0670. We projected at least 60 Mpc for 2018
The average sensitivity during O2 was about 26 Mpc



April 1, 2019: LIGO and Virgo started Observation run O3

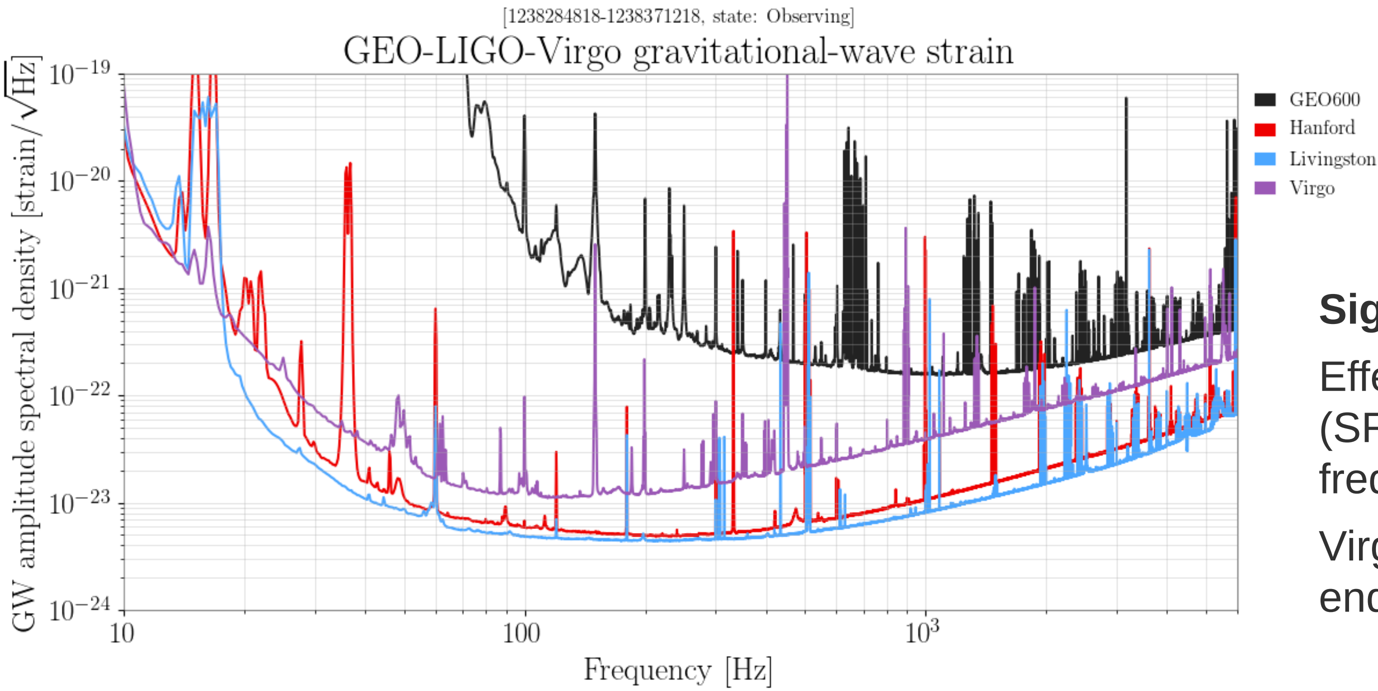
Virgo sensitivity: typically around 50 Mpc

Significant improvement (> 90%) with respect to the average sensitivity (26 Mpc) obtained in O2. We see a flat noise contribution at mid-frequencies, and significant 50 Hz noise. Power amounts to 18 W



Gravitational-Wave Observatory Status

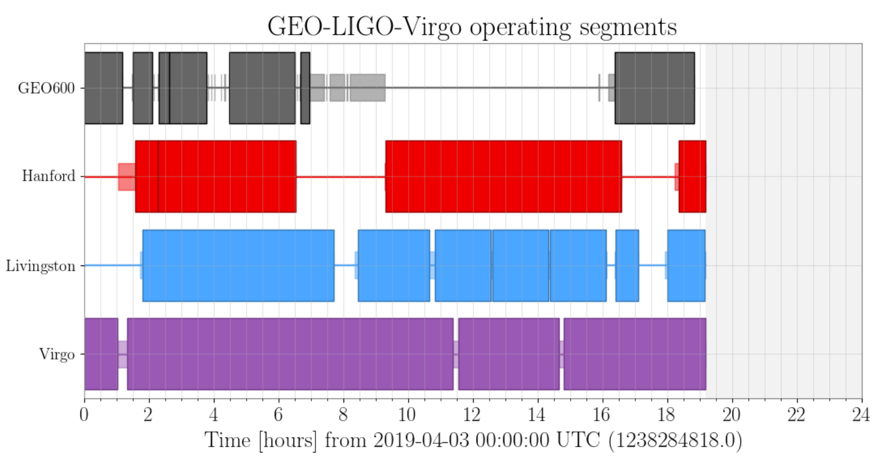
https://www.gw-openscience.org/summary_pages/detector_status/

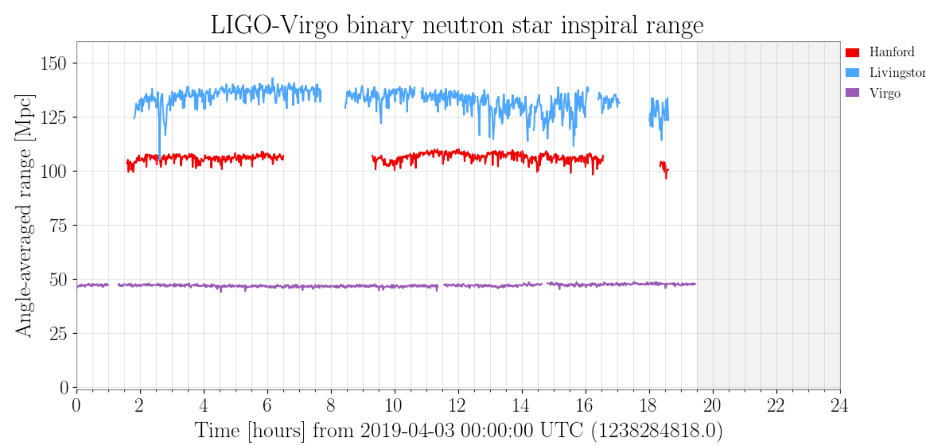


Signal recycling

Effect of missing signal recycling (SR) in Virgo is visible at high frequency

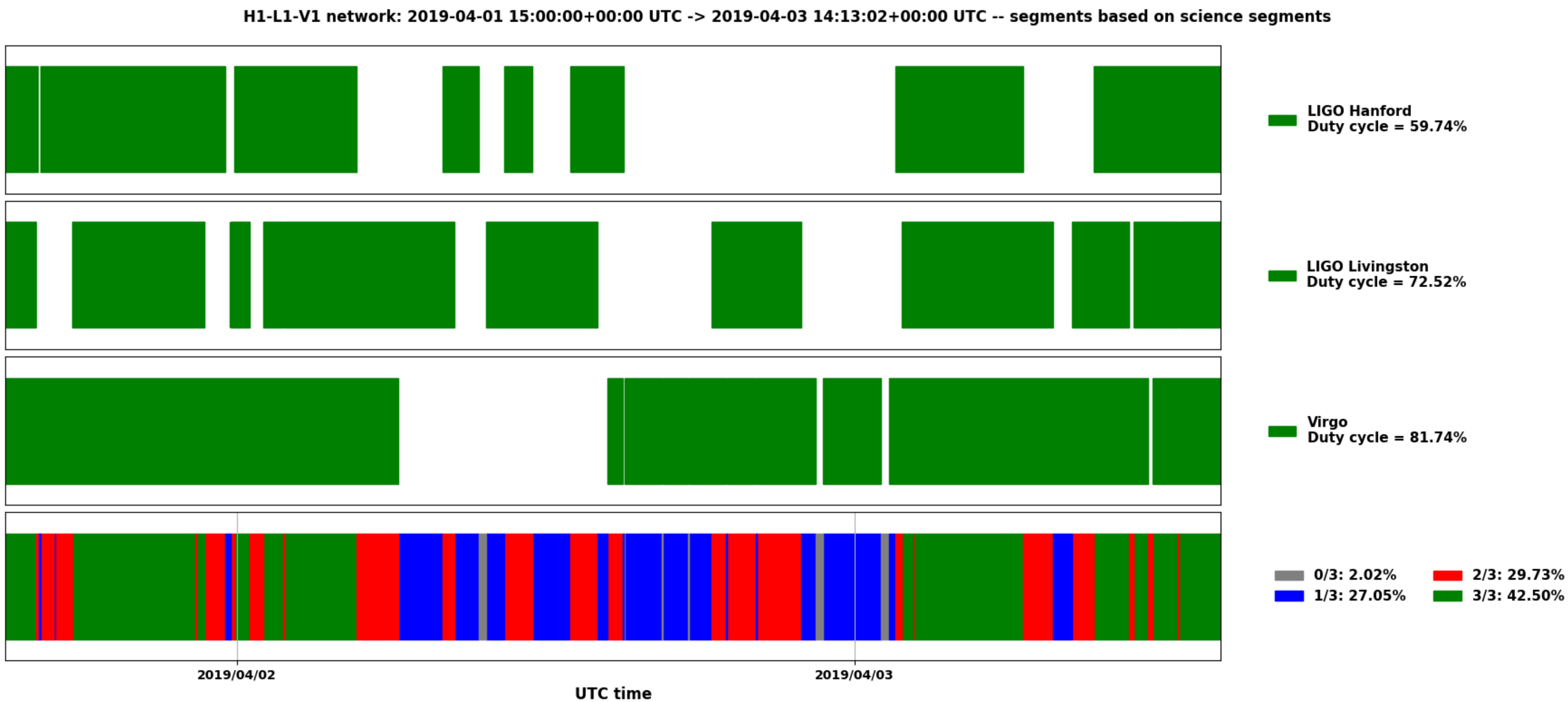
Virgo will implement SR at the end of O3





O3: Network performance

Three-fold coincidences represent a significant fraction of the data. Two- and three-detector events represent about 70% of the data



Squeezing

Virgo has a collaboration with AEI on squeezing

Squeezing results

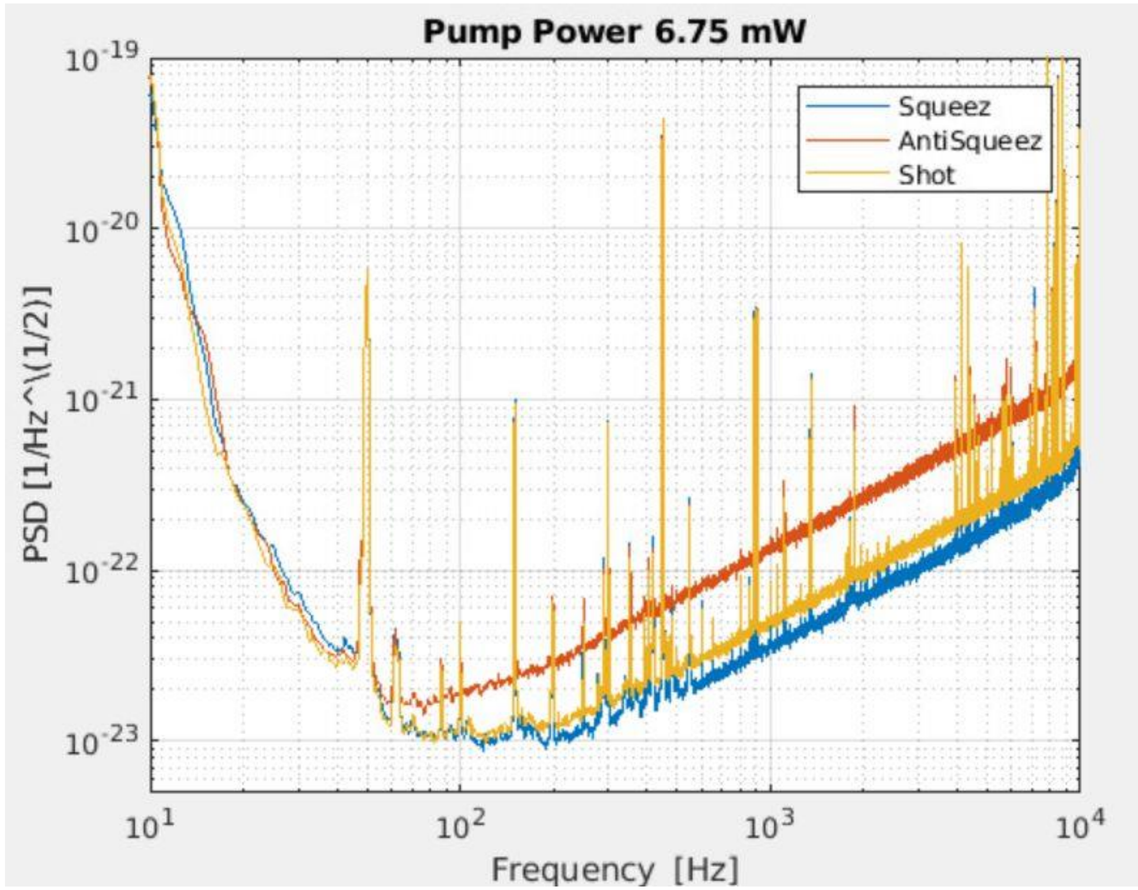
Target of the squeezing project has been reached: Virgo is ready to take advantage of the injection of squeezed light in AdV during O3

Squeezing results
Best present value of the high frequency sensitivity gain is about 3 dB

Maximum increase of the BNS range is achieved when the HF gain is kept to about 2.5-2.7 dB (injecting less squeezing)

Limits
Currently optical losses about 43%

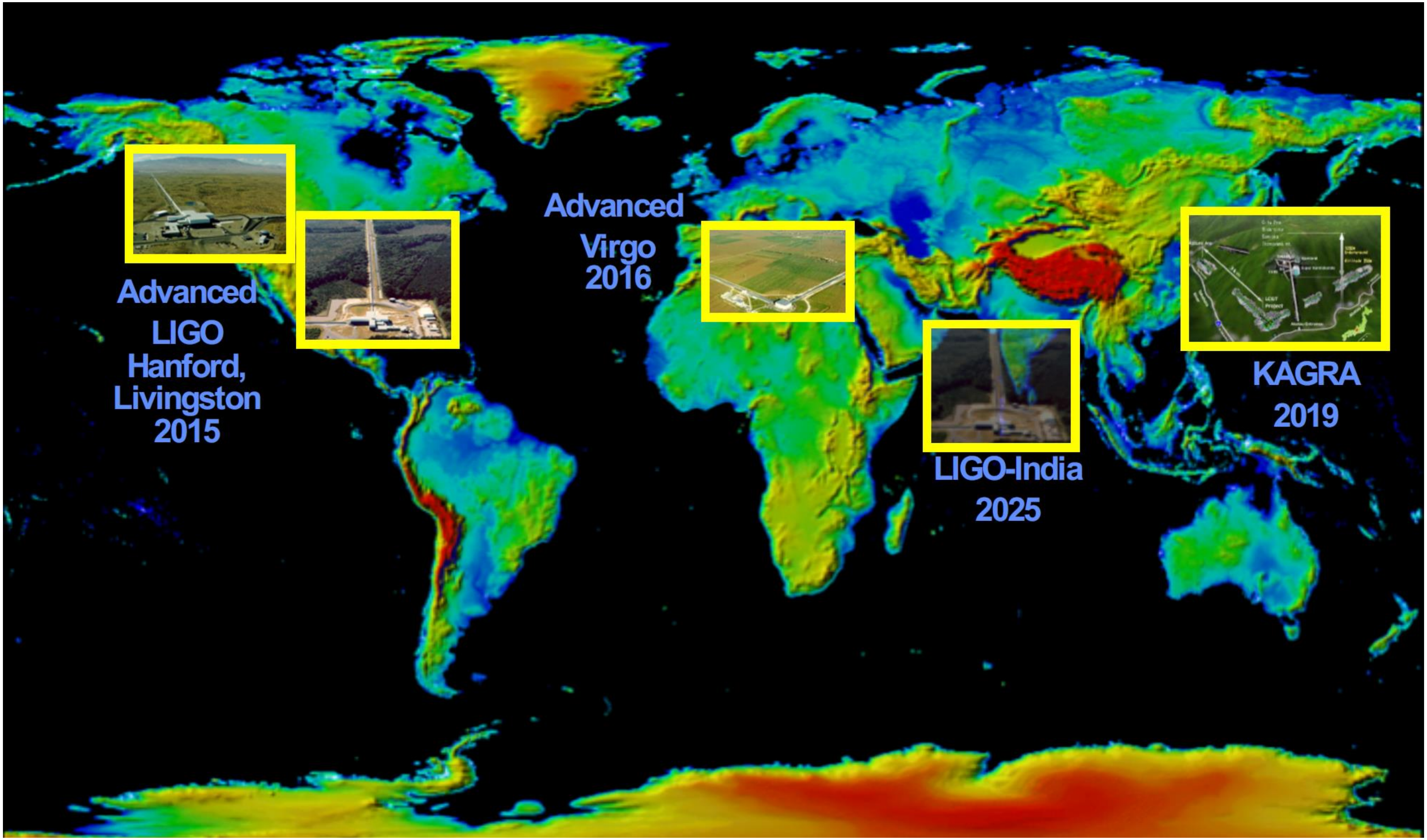
Losses will decrease by about 10% due to newly installed high-QE PDs



Next steps: upgrade project AdV+

Towards a global network

Expected to join LIGO and Virgo in Observation run 3

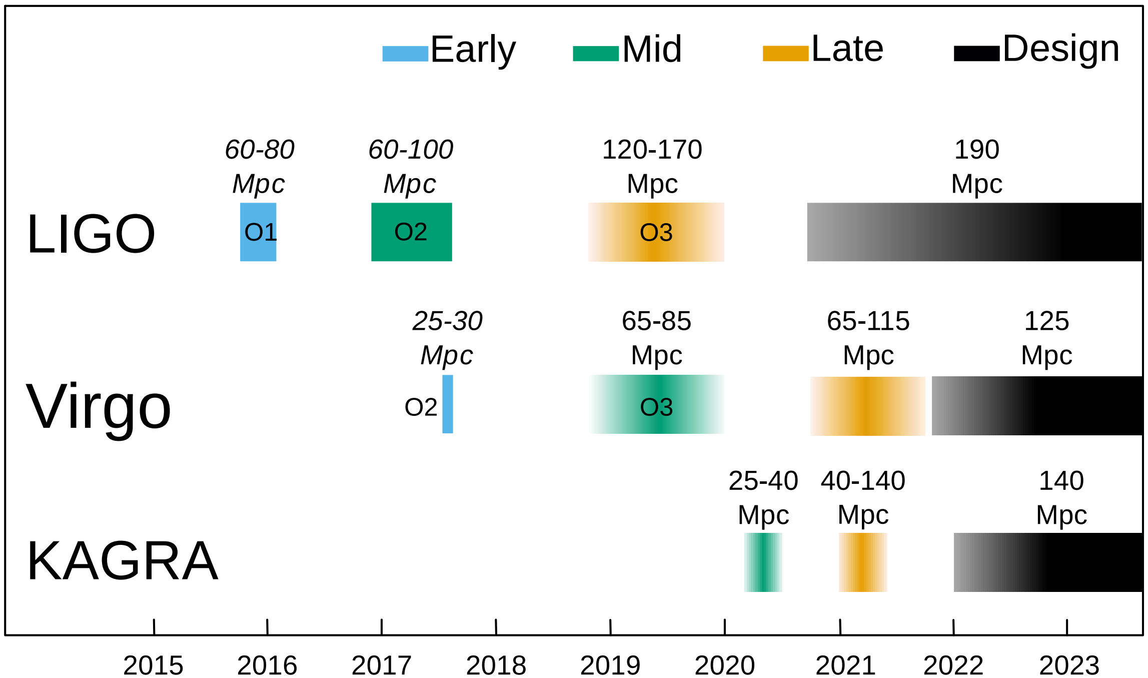


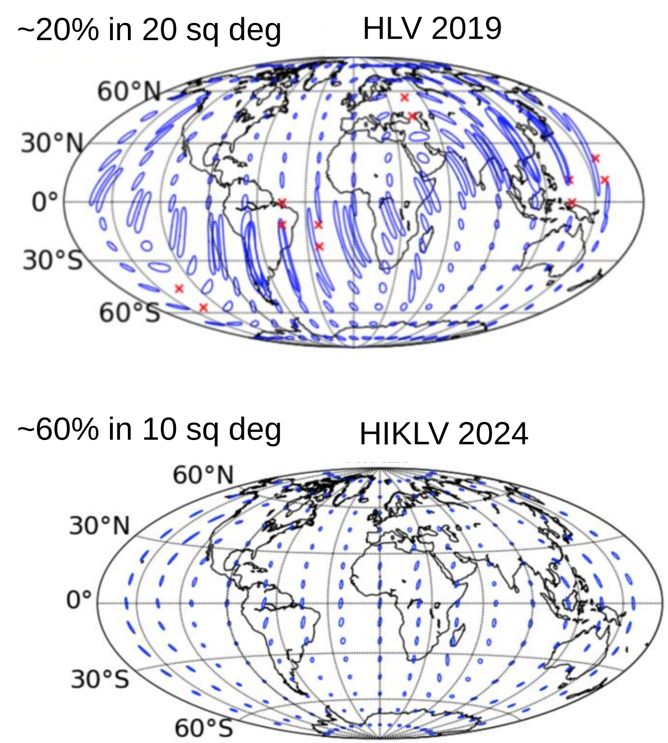
Planned observing timeline

One-year O3 planned to start in April 2019 with about twice the sensitivity in O3 (thus about 2^3 in rate).
In O3 LIGO and Virgo will release Open Public Alerts

Observation run O3

Three detectors and perhaps 1 event per week
KAGRA expected to join at the end of O3
Contribute to sky localization and PE





B. P. Abbott et al., *Prospects for Observing and Localizing Gravitational-Wave Transients with Advanced LIGO, Advanced Virgo and KAGRA*, 2016, Living Rev. Relativity 19

AdV+ as the next incremental step forward in sensitivity

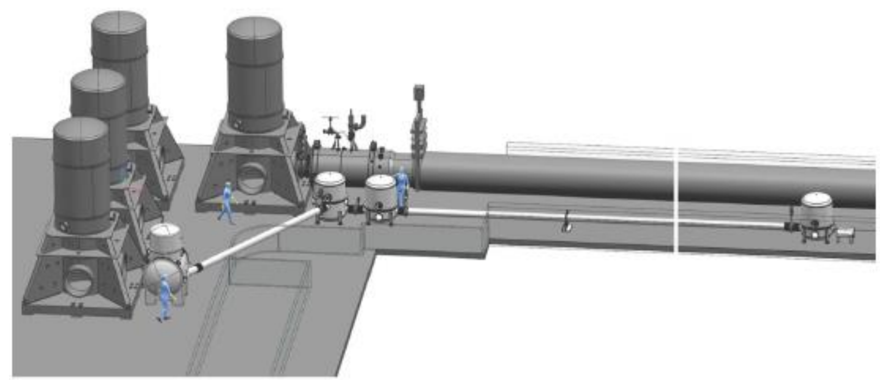
AdV+ is the plan to maximize Virgo's sensitivity within the constrains of the EGO site. It has the potential to increase Virgo's detection rate by up to an order of magnitude

AdV+ features

- Maximize science
- Secure Virgo's scientific relevance
- Safeguard investments by scientists and funding agencies
- Implement new innovative technologies
- De-risk technologies needed for third generation observatories
- Attractive for groups wanting to enter the field

Upgrade activities

- Tuned signal recycling and HPL: 120 Mpc
- Frequency dependent squeezing: 150 Mpc
- Newtonian noise cancellation: 160 Mpc
- Larger mirrors (105 kg): 200-230 Mpc
- Improved coatings: 260-300 Mpc

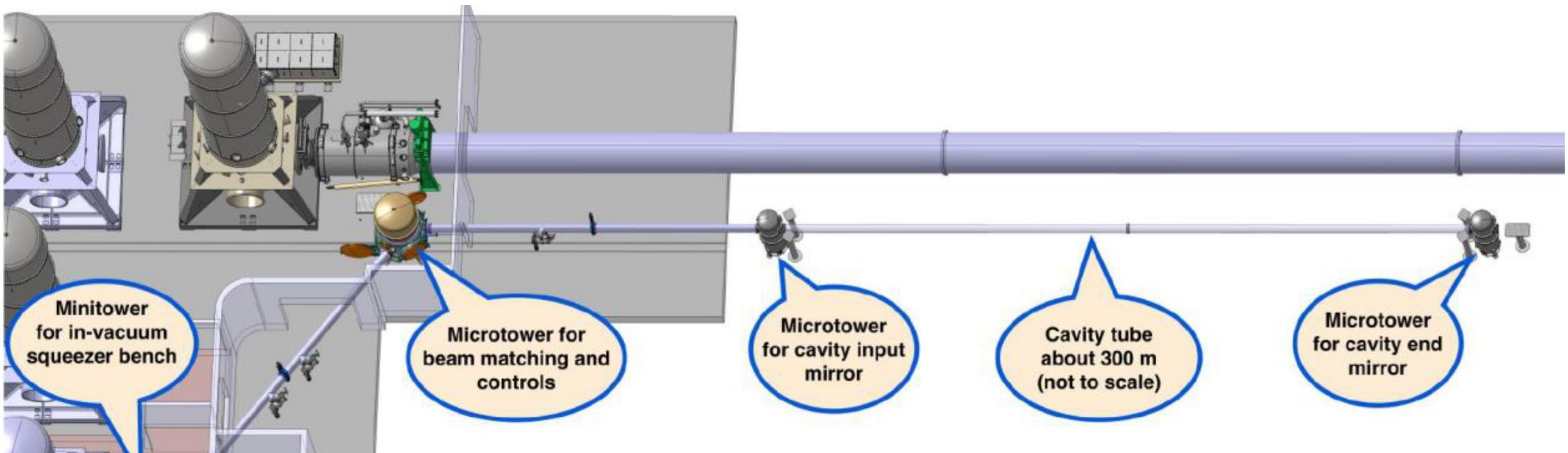


FDS Project Breakdown Structure

PBS follows the hardware components (not a WBS)

Responsibilities for construction of items for AdV+ have been allocated

a) Optical design and preparations for FDS ongoing
b) Smart infrastructure (HVAC) for NNC: under discussion

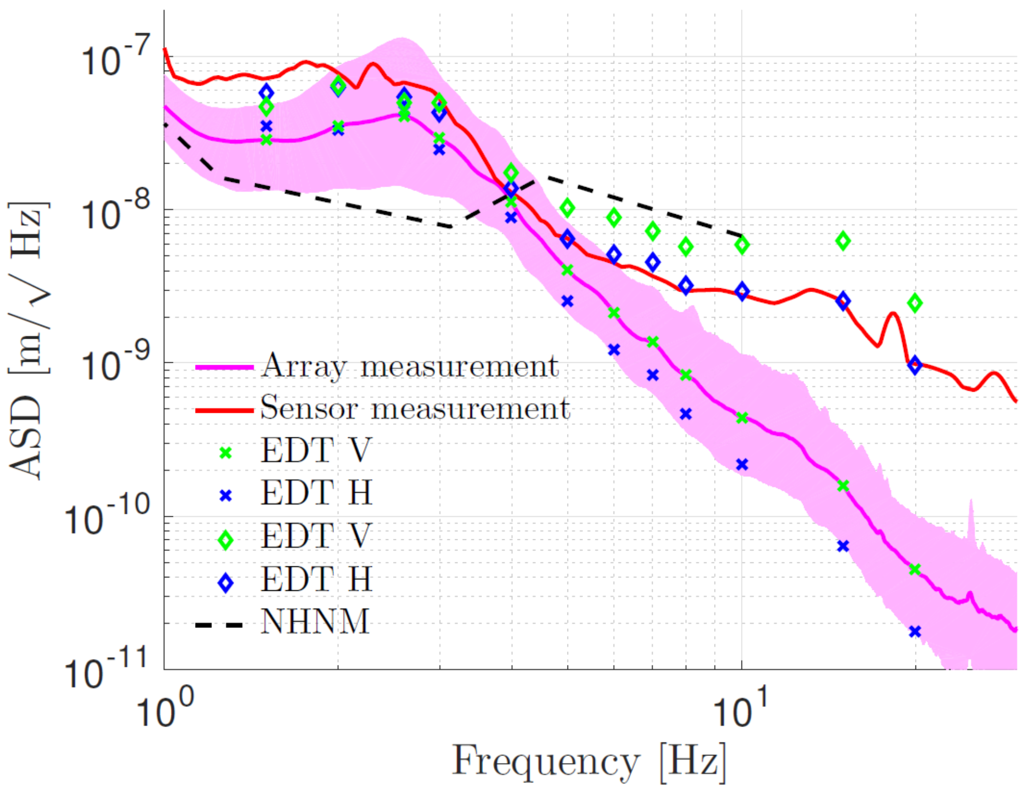


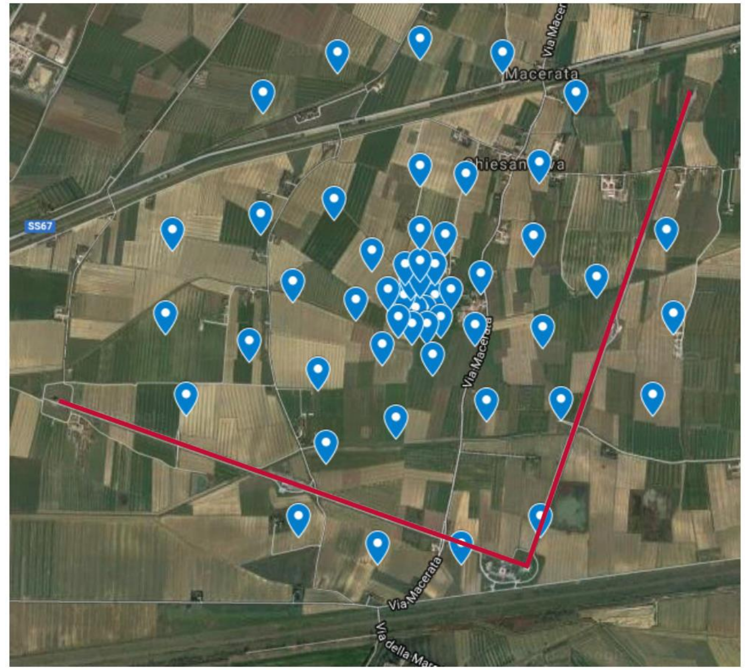
Virgo squeezer from AEI Hannover

Newtonian Noise Cancellation

Improvements to the infrastructure are expected to have a large impact

Noise at Central Building is about an order of magnitude higher than the noise in the vicinity of Virgo



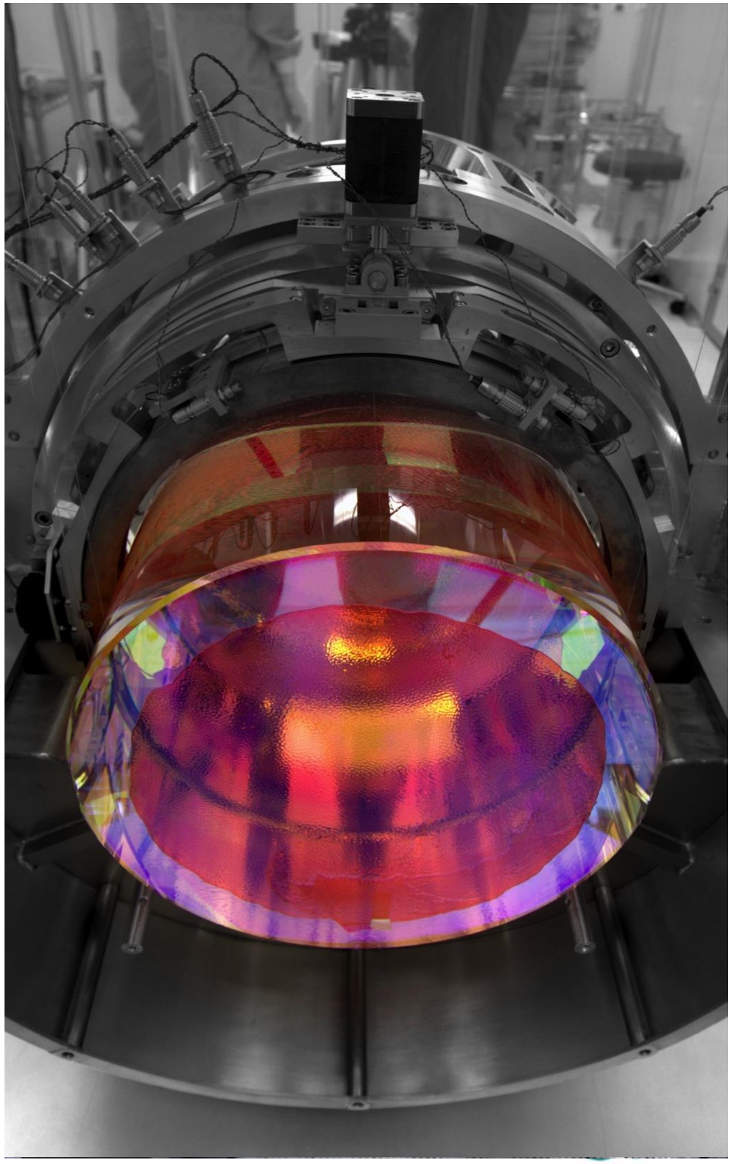


Need for emphasis on smart infrastructure for gravitational wave observatories

- Smart infrastructure design
- Newtonian noise modeling of infrastructure noise
- HVAC modification

AdV+ upgrade and extreme mirror technology

Laboratoire des Matériaux Avancés LMA at Lyon produced the coatings used on the main mirrors of the two working gravitational wave detectors: Advanced LIGO and Virgo. These coatings feature low losses, low absorption, and low scattering properties



Features

- Flatness < 0.5 nm rms over central 160 mm of mirrors by using ion beam polishing (robotic silica deposition was investigated)
- Ti:Ta_2O_5 and SiO_2 stacks with optical absorption about 0.3 ppm

Expand LMA capabilities for next generation

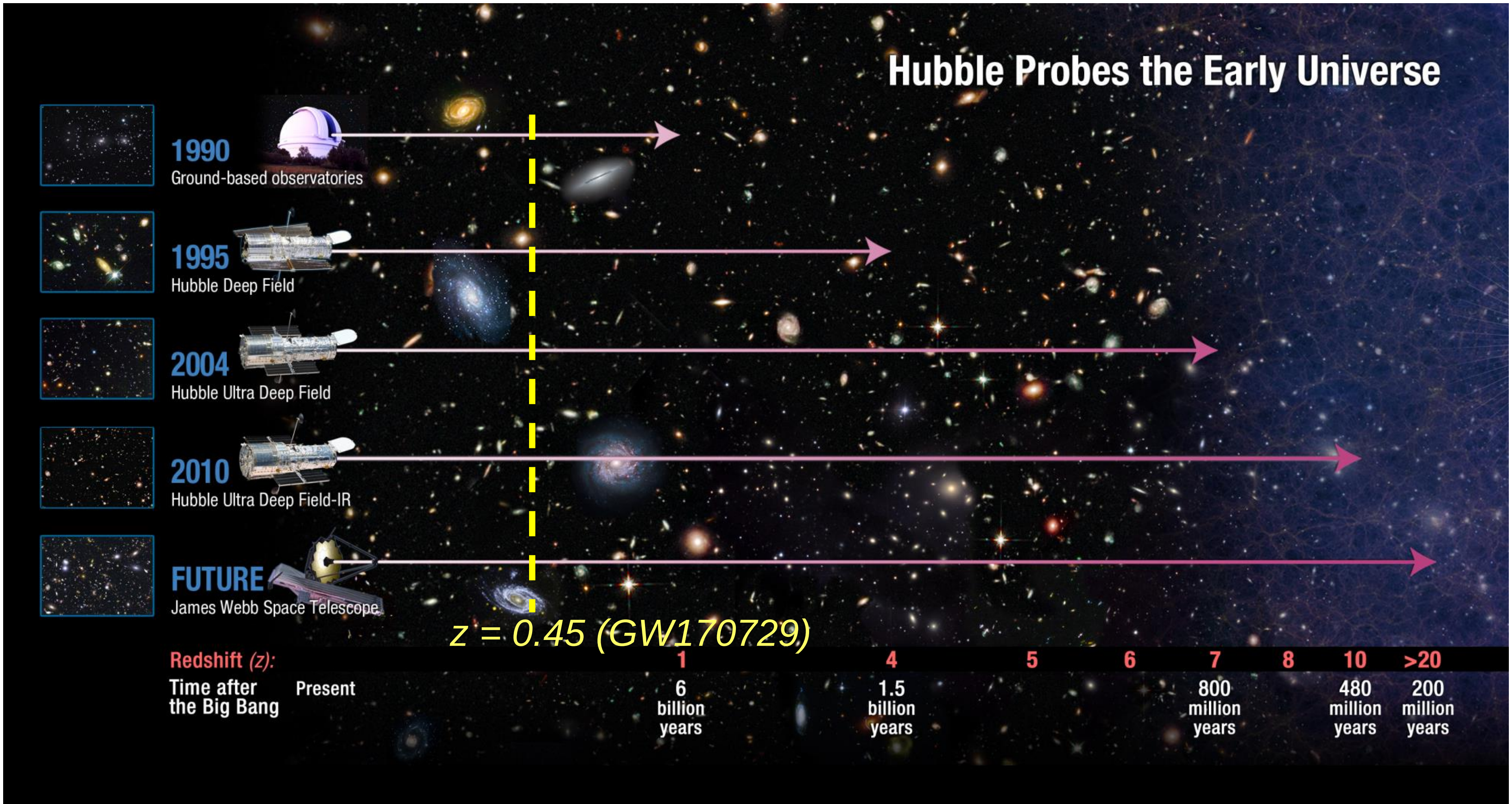
LMA is the only coating group known to be capable of scaling up

3G: observing all mergers in the Universe

This cannot be achieved with existing facilities and requires a new generation of GW observatories

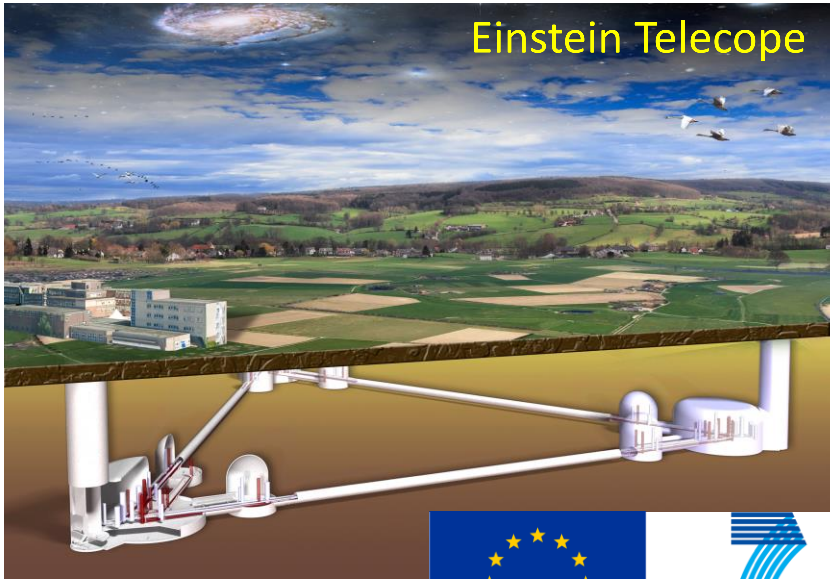
We want to collect high statistics (*e.g.* millions of BBH events), high SNR, distributed over a large z-range ($z < 20$)

This allows sorting data versus redshift, mass distributions, *etc*. Early warning, IMBH, early Universe, CW, …

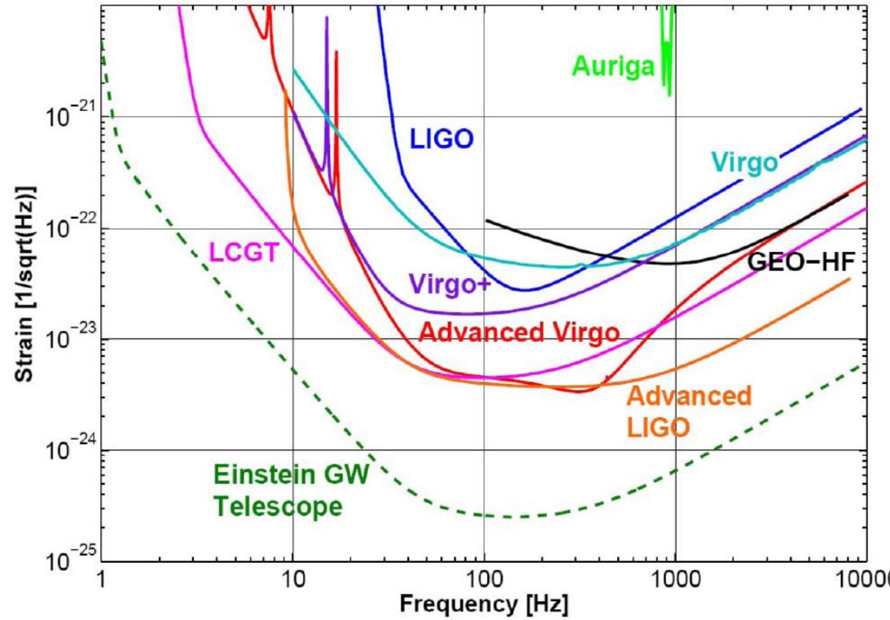


Einstein Telescope and Cosmic Explorer

Realizing the next gravitational wave observatories is a coordinated effort to create a worldwide 3G network

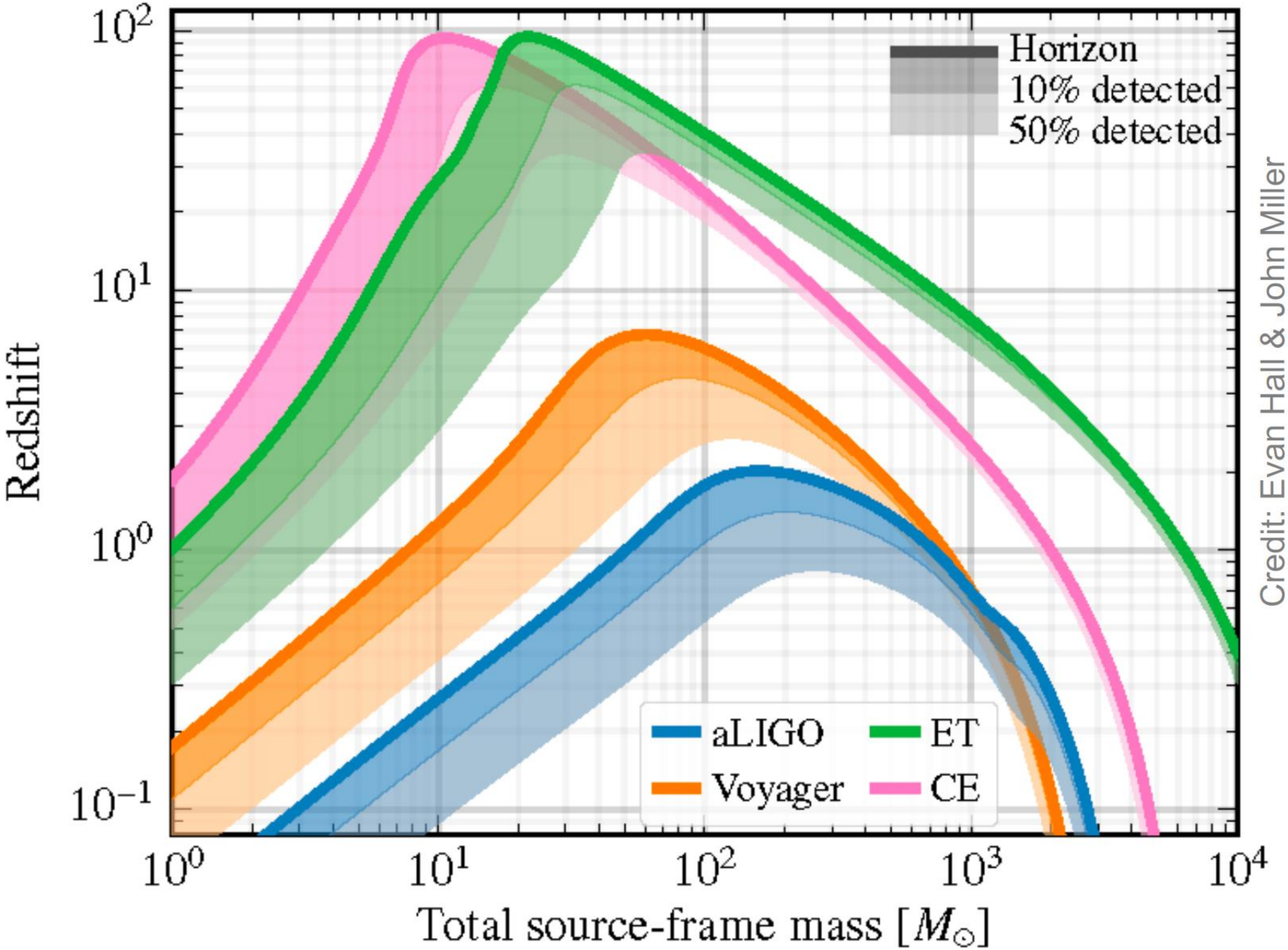


SEVENTH FRAMEWORK PROGRAMME



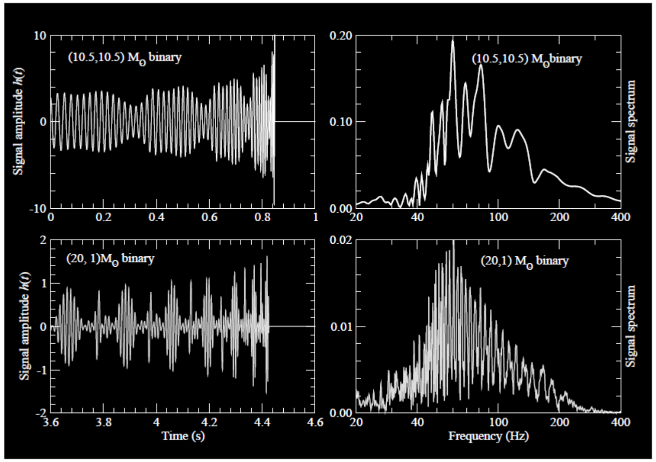
Einstein Telescope has excellent sensitivity

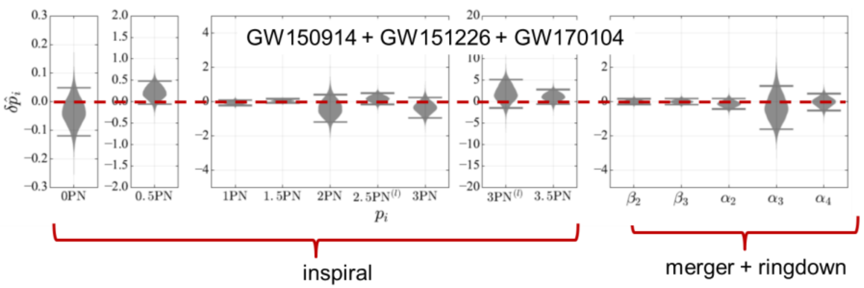
Einstein Telescope and Cosmic Explorer can observe the entire universe

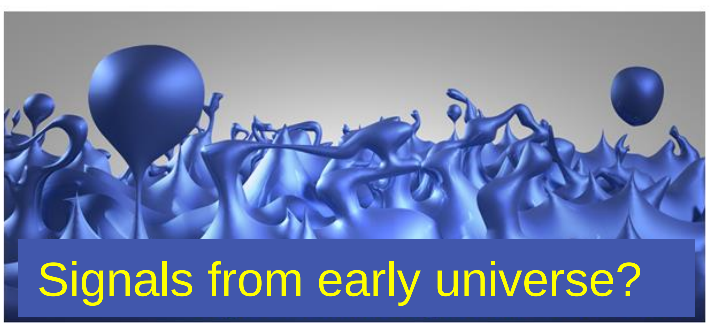


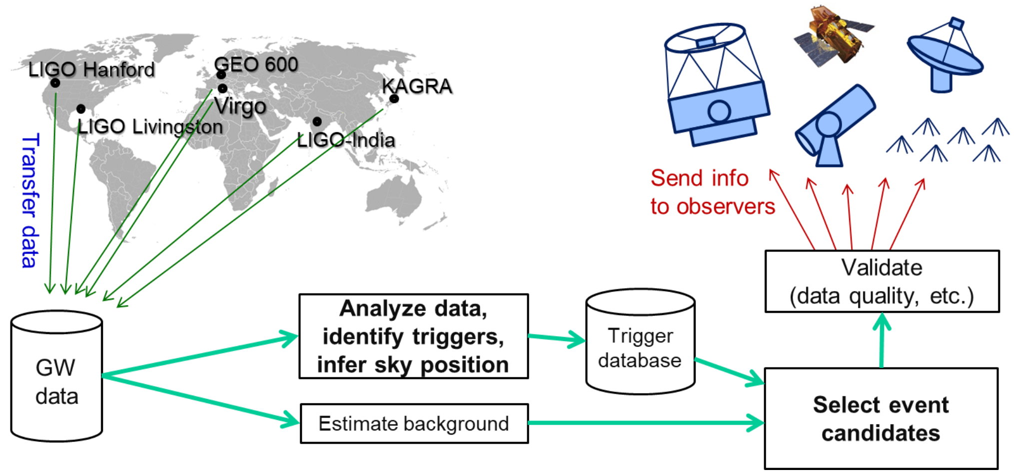
3G science

Detailed studies of gravity, near black holes. Early warning to EM follow-up community. Precision tests of detailed aspects of CBC. Cross correlation of the largest data sets. Access to early Universe







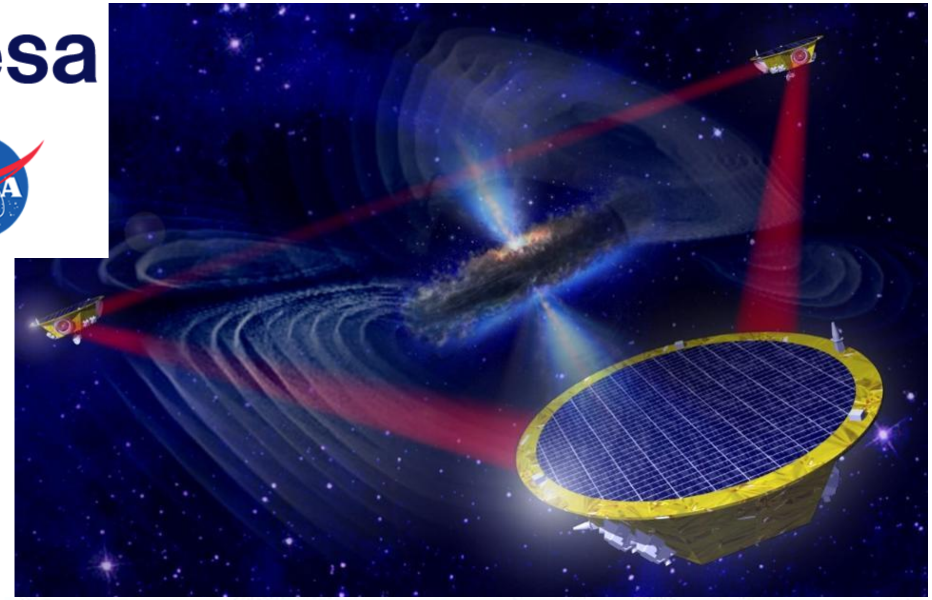


Bright future for gravitational wave research

LIGO and Virgo are operational. KAGRA in Japan joins this year, LIGO-India is under construction. ESA launches LISA in 2034. Einstein Telescope and CE CDRs financed, strong support by APPEC

Gravitational wave research

- LIGO and Virgo operational
- KAGRA to join next year
- LIGO-India under construction (2025)
- ESA selected LISA, NASA rejoins
- Pulsar Timing Arrays, such as EPTA and SKA
- Cosmic Microwave Background radiation



Einstein Telescope and Cosmic Explorer

- CDR ET financed by EU in FP7, CE by NSF
- APPEC gives GW a prominent place in the new Roadmap and especially the realization of ET

Next steps for 3G

- Organize the community and prepare a credible plan for EU funding agencies
- ESFRI Roadmap (2020)
- Support 3G: http://www.et-gw.eu/index.php/letter-of-intent


Thanks for your
attention !!

Laser interferometer detectors

Fundamental limits: shot noise

Determine the rate of arrival of photons $\bar{n}$ (in Hz) by making a set of measurements each lasting τ seconds

The mean number of photons in each measurement interval $\bar{N} = \bar{n}\tau$

Fractional precision of a single arrival time rate (or equivalent power) is $\frac{\sigma_{\bar{N}}}{\bar{N}} = \frac{\sqrt{\bar{n}\tau}}{\bar{n}\tau} = \frac{1}{\sqrt{\bar{n}\tau}}$

Each photon carries energy $\hbar\omega = 2\pi\hbar c/\lambda$

For power P_{out} at the output of the ITF, the mean photon flux is $\bar{n} = \frac{\lambda}{2\pi\hbar c} P_{\text{out}}$

Operating at half fringe we find $\frac{dP_{\text{out}}}{dL} = \frac{2\pi}{\lambda} P_{\text{in}}$. This is the sensitivity to the test mass *difference* δL

Number of photons per interval τ is $\bar{N} = \frac{\lambda}{4\pi\hbar c} P_{\text{in}}\tau$ with relative fluctuation $\sigma_{\bar{N}}/\bar{N} = \sqrt{4\pi\hbar c/\lambda P_{\text{in}}\tau}$

Equivalent to position difference fluctuations given by the fractional photon number fluctuation divided by the fractional output power change per unit position difference

$\sigma_{\delta L} = \frac{\sigma_N}{N} / \frac{1}{P_{\text{out}}} \frac{dP_{\text{out}}}{dL} = \sqrt{\frac{\hbar c\lambda}{4\pi P_{\text{in}}\tau}}$. The equivalent GW noise amounts to $\sigma_h = \frac{\sigma_{\delta L}}{L} = \frac{1}{L}\sqrt{\frac{\hbar c\lambda}{4\pi P_{\text{in}}\tau}}$

Shot noise decreases with integration time. Equivalently $h_{\text{shot}}(f) = \frac{1}{L}\sqrt{\frac{\hbar c\lambda}{2\pi P_{\text{in}}}}$

Thermal noise

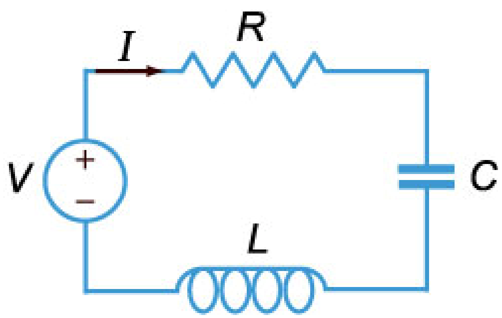
We cannot beat thermodynamics

Any mechanical DOF has a thermal motion of RMS amplitude

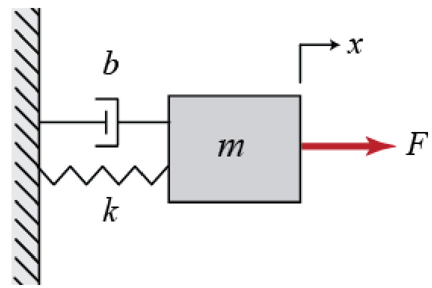
$$\sqrt{\langle y^2 \rangle} = \sqrt{\frac{K_B T}{m\omega_0^2}}$$

~ 3 pm for the fundamental pendulum mode in Virgo !!!

Nevertheless from statistical mechanics



$$Z_{el} = \frac{V}{I} \Longleftrightarrow Z_m = \frac{F}{v}$$



$$\langle F_{thermal} \rangle^2 = 4 K_B T \, \mathrm{Re}\left[Z_m(\omega)\right] \left[\frac{\mathrm{N}^2}{\mathrm{Hz}}\right]$$

Mechanical equivalent of Johnson noise