# MORATrap: geometric optimization

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**Workshop MORA**
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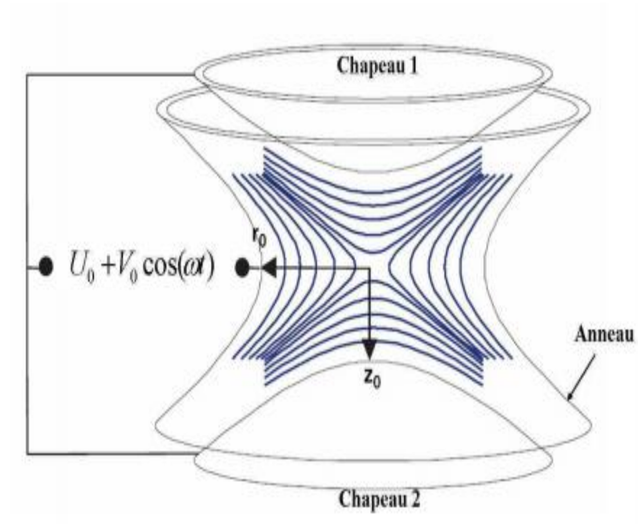
# Summary

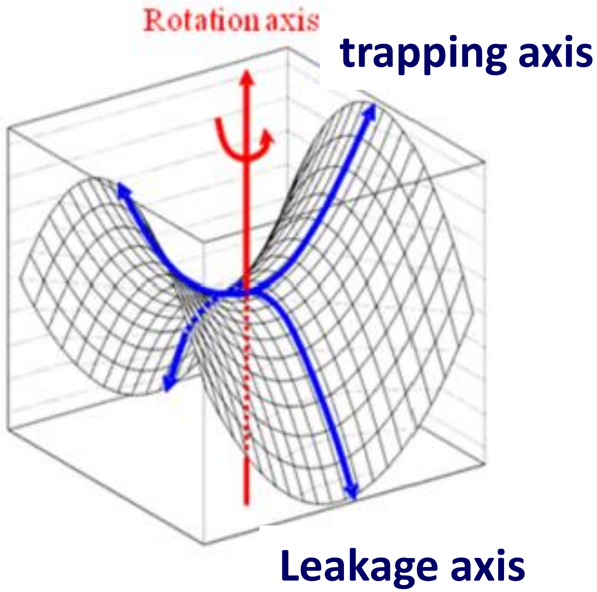
- Introduction
- Optimization steps & ELECTROBEM
- Result of MORATrap optimization
- MORATrap Vs LPCTrap
- Systematic Study
- Conclusion

# Introduction: Paul Trap

•**In Paul trap, ions are confined by a quadrupolar RadioFrequency (RF) potential**

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$




-A ring and 2 endcaps of hyperboloid shape

-Apply an oscillating voltage between the electrodes

-The resulting potential is :

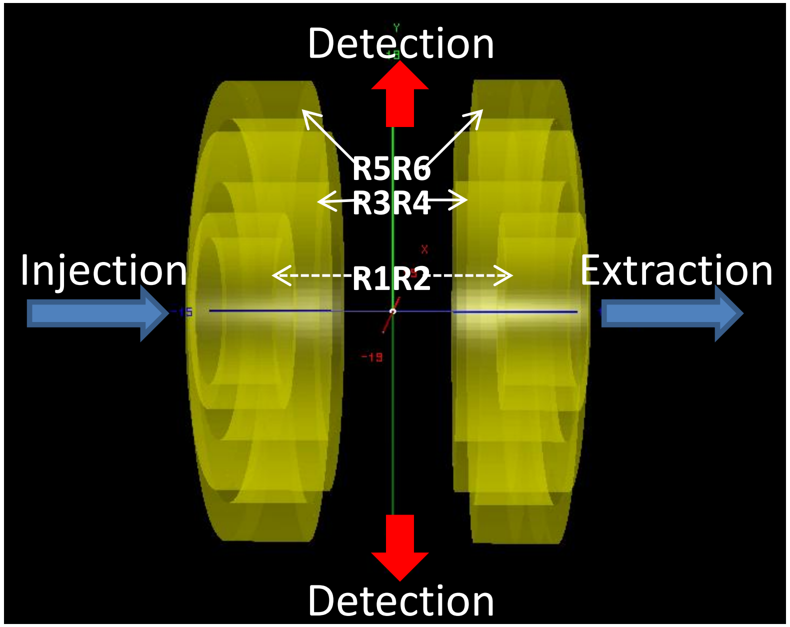
$$V(x,y,z,t) = (U_{DC} + V_{AC}\cos\omega t)\frac{x^2 + y^2 + 2z^2}{2r_0^2}$$

# Transparant Paul trap: LPCTrap

**LPCTrap:** precise measurement of the β-ν angular correlation coefficient in the β decay of radioactif ions

- transparency to β decay products

- efficient injection of the ion packets




- R1R2: RF voltage is applied to central rings (R1 and R2) to create a quadrupolar potential in trap center

- R3R4: polarized at a precise tension (during the injection and extraction phase)

- R5R6: polarized at a precise tension to correct the trap potential

# Optimization steps & ELECTROBEM

# Spherical harmonic

**Laplace's equation in spherical coordinate , in the absence of charge :**

$$\Delta\, V(\rho,\theta,\varphi) = 0$$

**The general solution of Laplace's equation in spherical coordinate :**

$$V(\rho,\theta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left(\frac{\rho}{\rho_0}\right)^n P_n^m(\cos\theta)\big(A_{nm}\cos(\mathrm{m}\varphi) - B_{nm}\sin(\mathrm{m}\varphi)\big)$$

**Potential due to :**

- **electrodes voltage**
- **geometry of electrodes**

# Spherical harmonic & ELECTROBEM

**In axi-symmetric systems (m = 0), the solution of Laplace's equation :**

$$V(\rho,\theta,\varphi) \xrightarrow[\text{(Z axis)}]{\text{Axial symmetry}} V(\rho,\theta) = \sum_{n=0}^{\infty} A_n \left(\frac{\rho}{\rho_0}\right)^n P_n(\cos\theta) \quad ; \ \rho \le \rho_0$$

**Optimize our trap in order to:**

- **reduce the harmonic of order higher than 2 (increase trapping time)**
- **maximize the ions cloud radius (maximize the number of trapped ions)**
- **optimize detector acceptance (increase the acceptance compared to LPCTrap)**

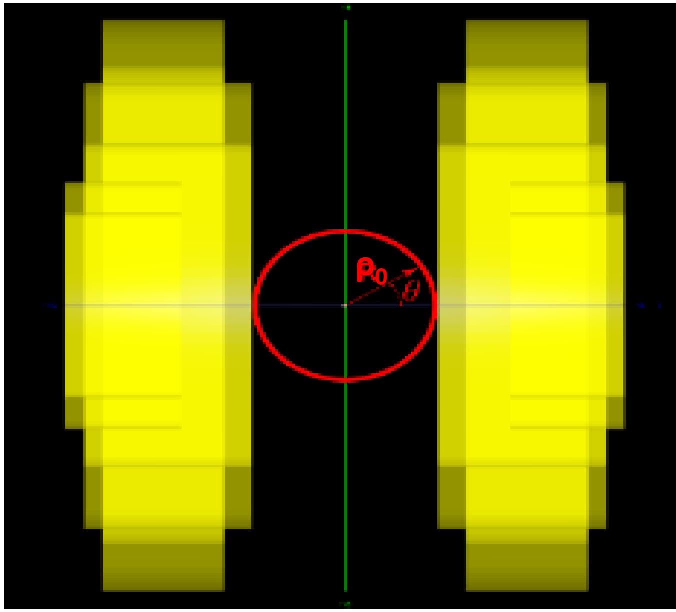
# Spherical harmonic & ELECTROBEM

In axi-symmetric systems (m = 0), the solution of Laplace's equation :

$$V(\rho,\theta) = \sum_{n=0}^{\infty} A_n \left(\frac{\rho}{\rho_0}\right)^n P_n(\cos\theta) \quad ; \quad \rho \leq \rho_0$$

To optimize the geometry of trap:

1. Calculate potential on a circle :
   - of center ( z=0, ρ=0)
   - and radius $\rho \leq \rho_0$



# Spherical harmonic & ELECTROBEM

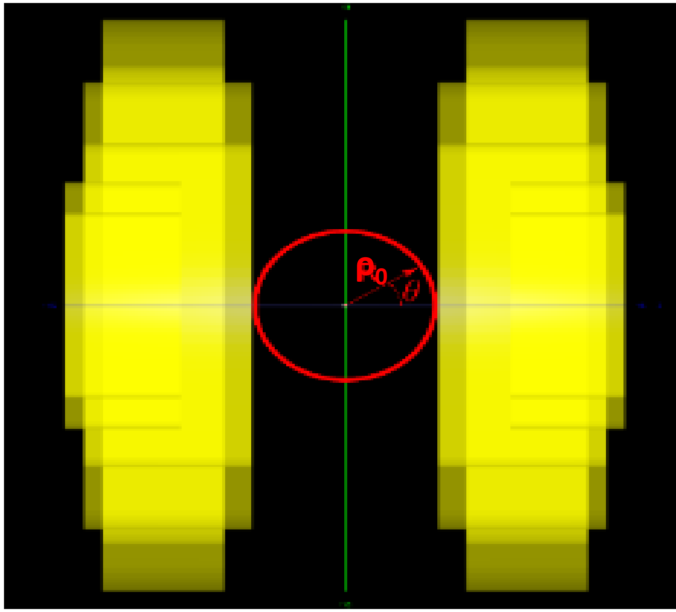
**In axi-symmetric systems (m = 0), the solution of Laplace's equation :**

$$V(\rho,\theta) = \sum_{n=0}^{\infty} A_n \left(\frac{\rho}{\rho_0}\right)^n P_n(\cos\theta) \quad ; \; \rho \le \rho_0$$

**To optimize our trap:**

2. **Extract real harmonic coefficient $A_n$ on a circle of maximum radius $\rho \le \rho_0$**

3. **Solve our equation** $$\frac{V^* - V_2}{V_2} = \frac{\sum_{n>2} \left(\frac{\rho}{\rho_0}\right)^n |\mathcal{A}_n|}{\left(\frac{\rho}{\rho_0}\right)^2 |\mathcal{A}_2|} \le 2\%$$




$$r' = \left(\frac{\rho}{\rho_0}\right)^2$$

**By minimizing (Minuit)**

**Our fitness function :** $$\frac{1}{|A_2|\, r'(\vec{p})}$$

maximize quadratic potential well

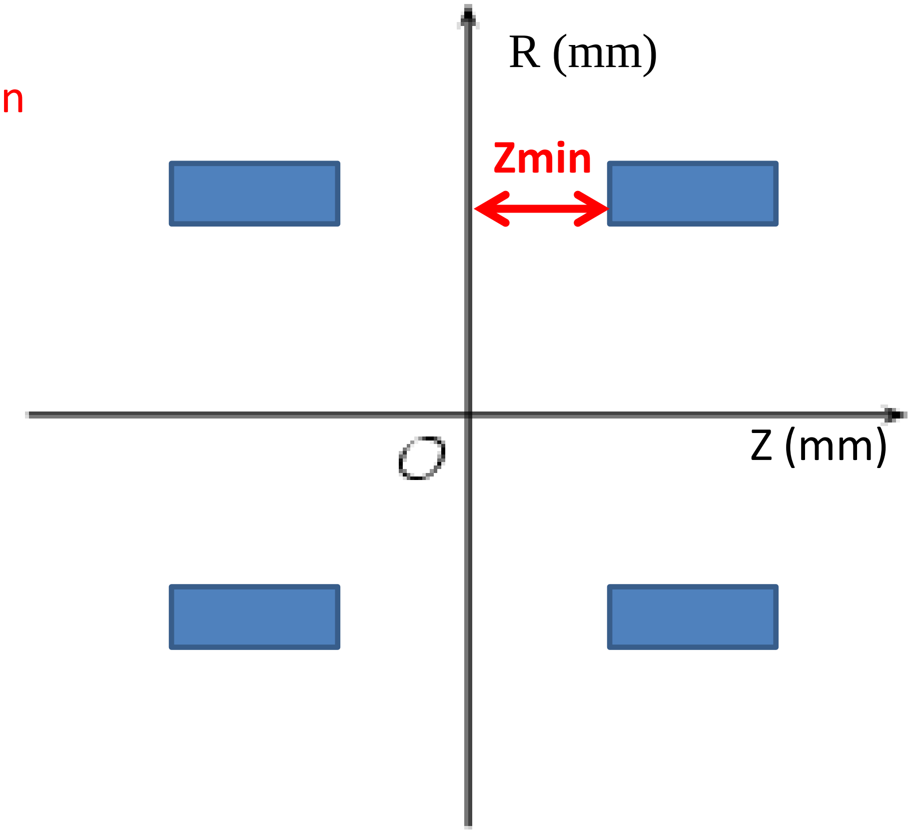
maximize radius of our cloud

$\vec{p} =$ optimization parameters

# Optimization Parameters

Several parameters can be used, depends on the geometry of electrodes:

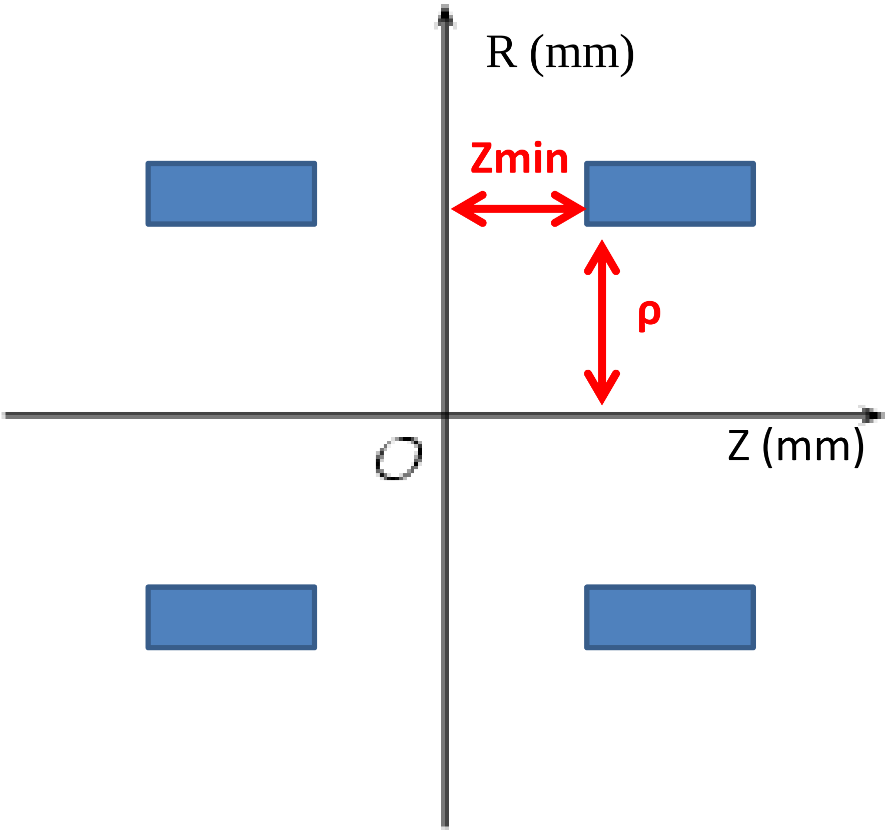
- Axial distance from the center of the trap Zmin



# Optimization Parameters

Several parameters can be used, depends on the geometry of electrodes:

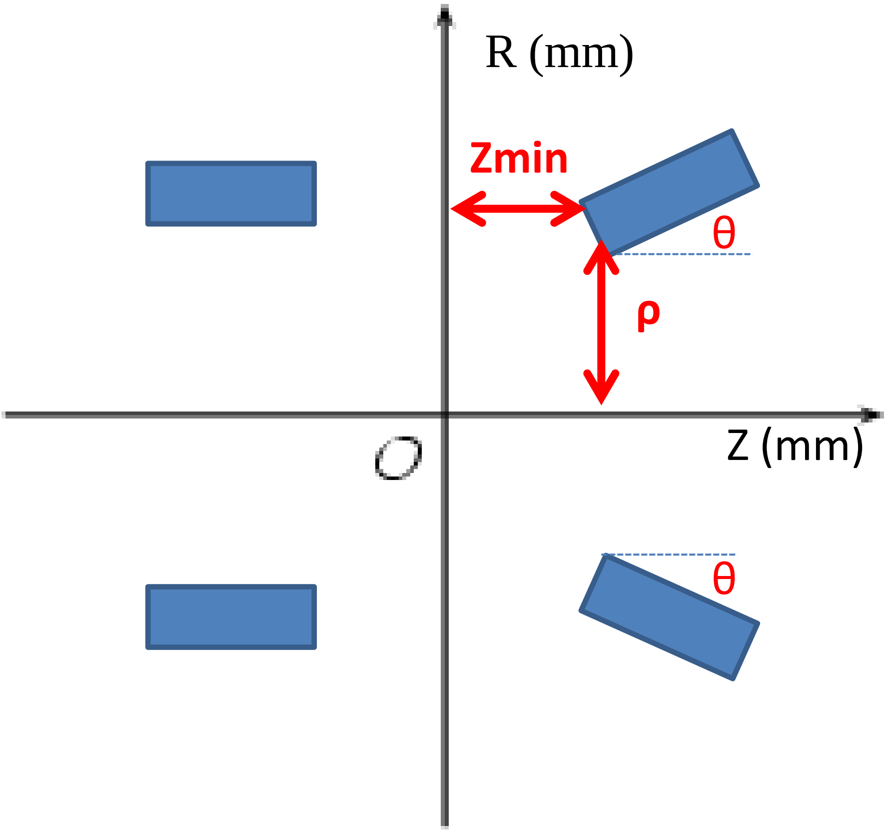
- Axial distance from the center of the trap Zmin
- Radial distance ρ from the center of the trap



# Optimization Parameters

Several parameters can be used, depends on the geometry of electrodes:

- Axial distance from the center of the trap Zmin
- Radial distance ρ from the center of the trap
- Electrode deviation angles θ



# Optimization Parameters

Several parameters can be used, depends on the geometry of electrodes:

- Axial distance from the center of the trap Zmin
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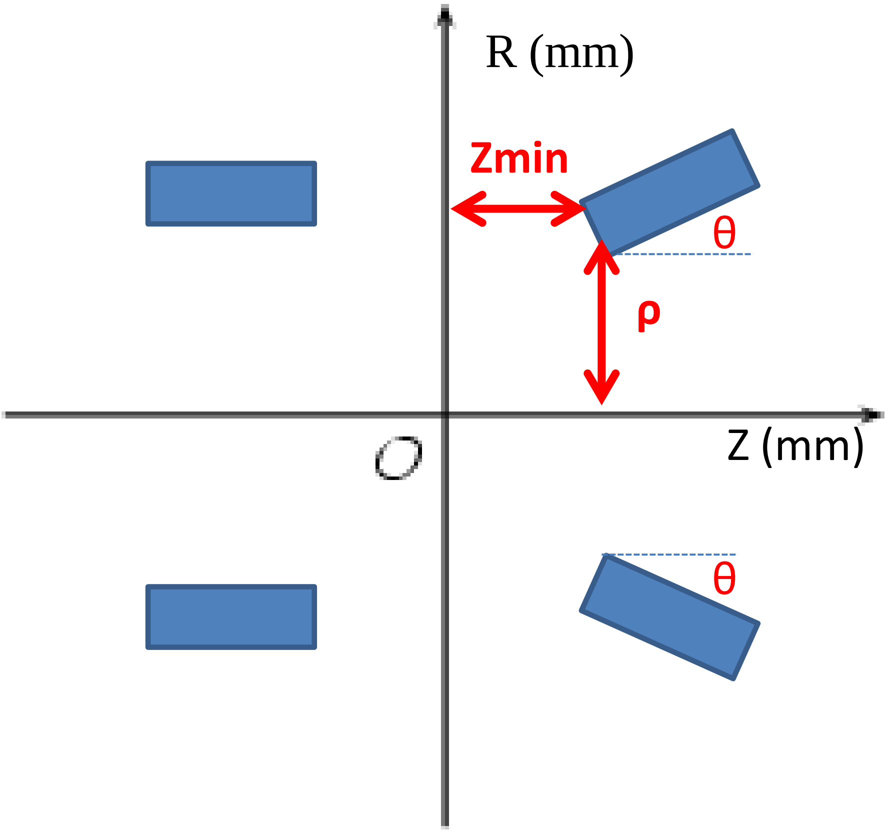
- Electrode deviation angles θ

Zmin → Radial acceptance

ρ & θ → Axial acceptance



# Optimization Parameters

Several parameters can be used, depends on the geometry of electrodes:

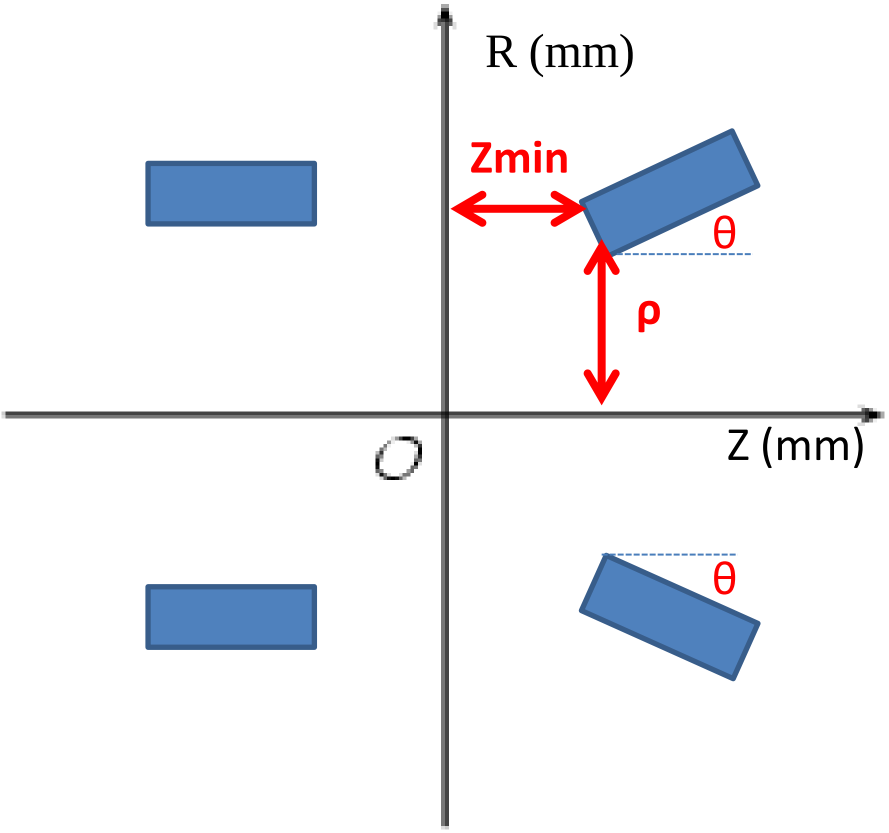
- Axial distance from the center of the trap Zmin
- Radial distance ρ from the center of the trap
- Electrode deviation angles θ

Zmin → Radial acceptance

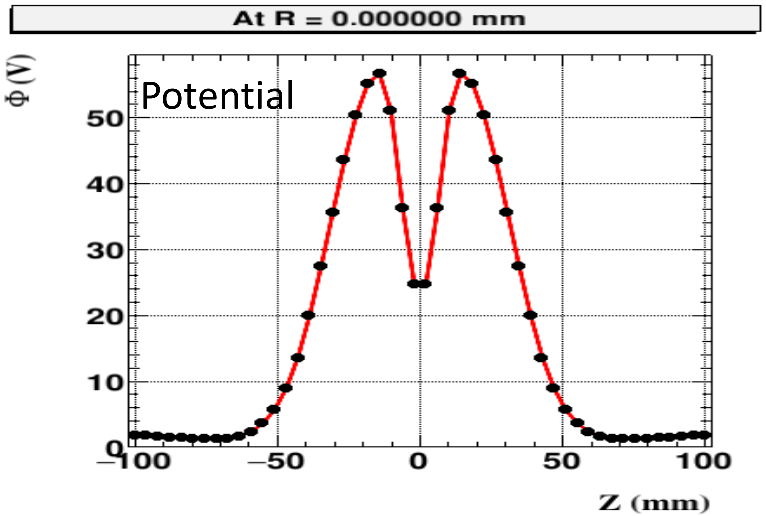
ρ & θ → Axial acceptance

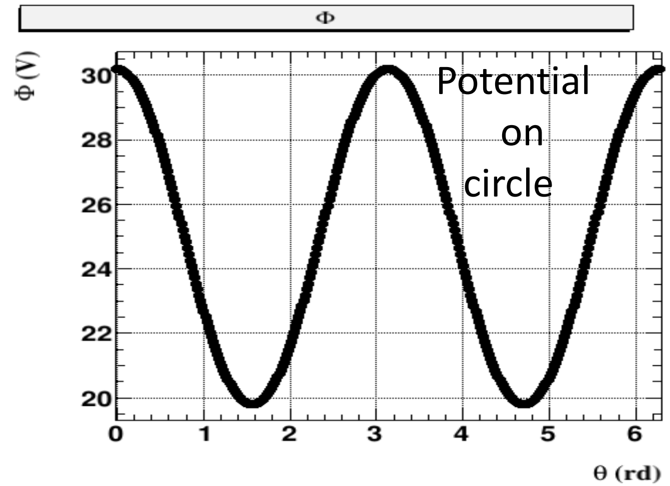
- Electrode thickness
- Gaps between the electrodes
- parameters related to section geometry

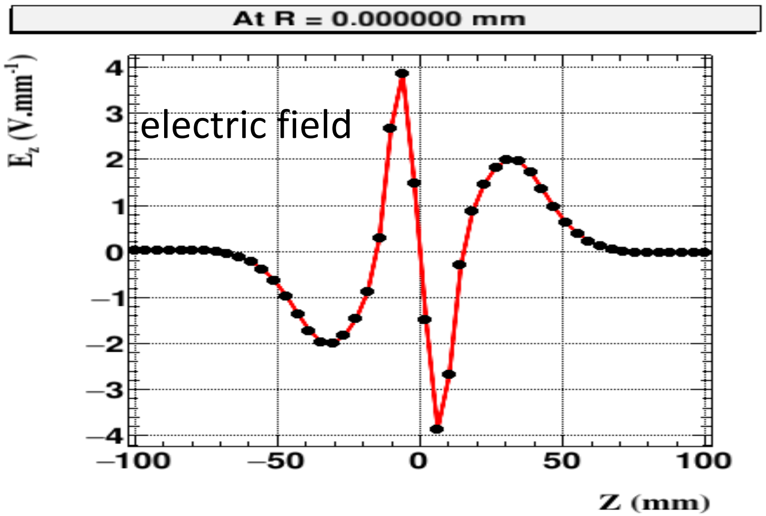
……

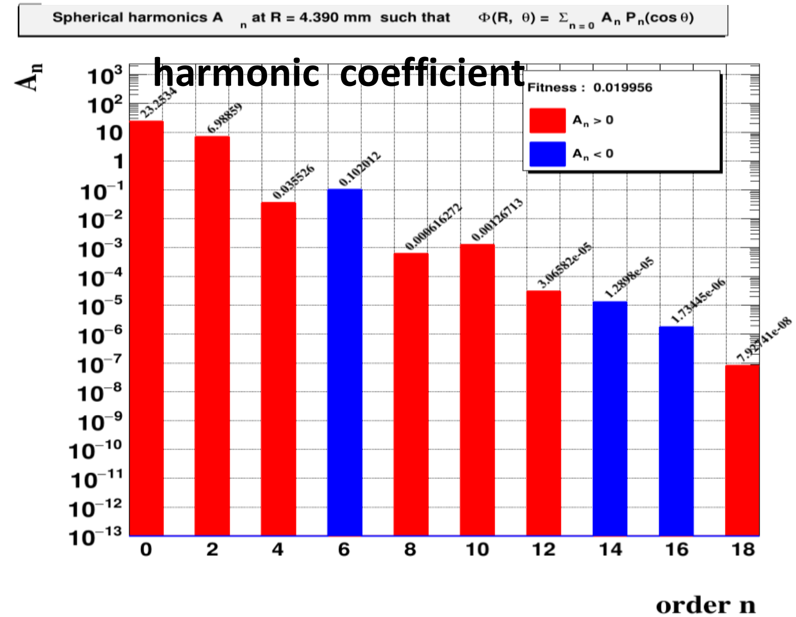


# ELECTROBEM simulation



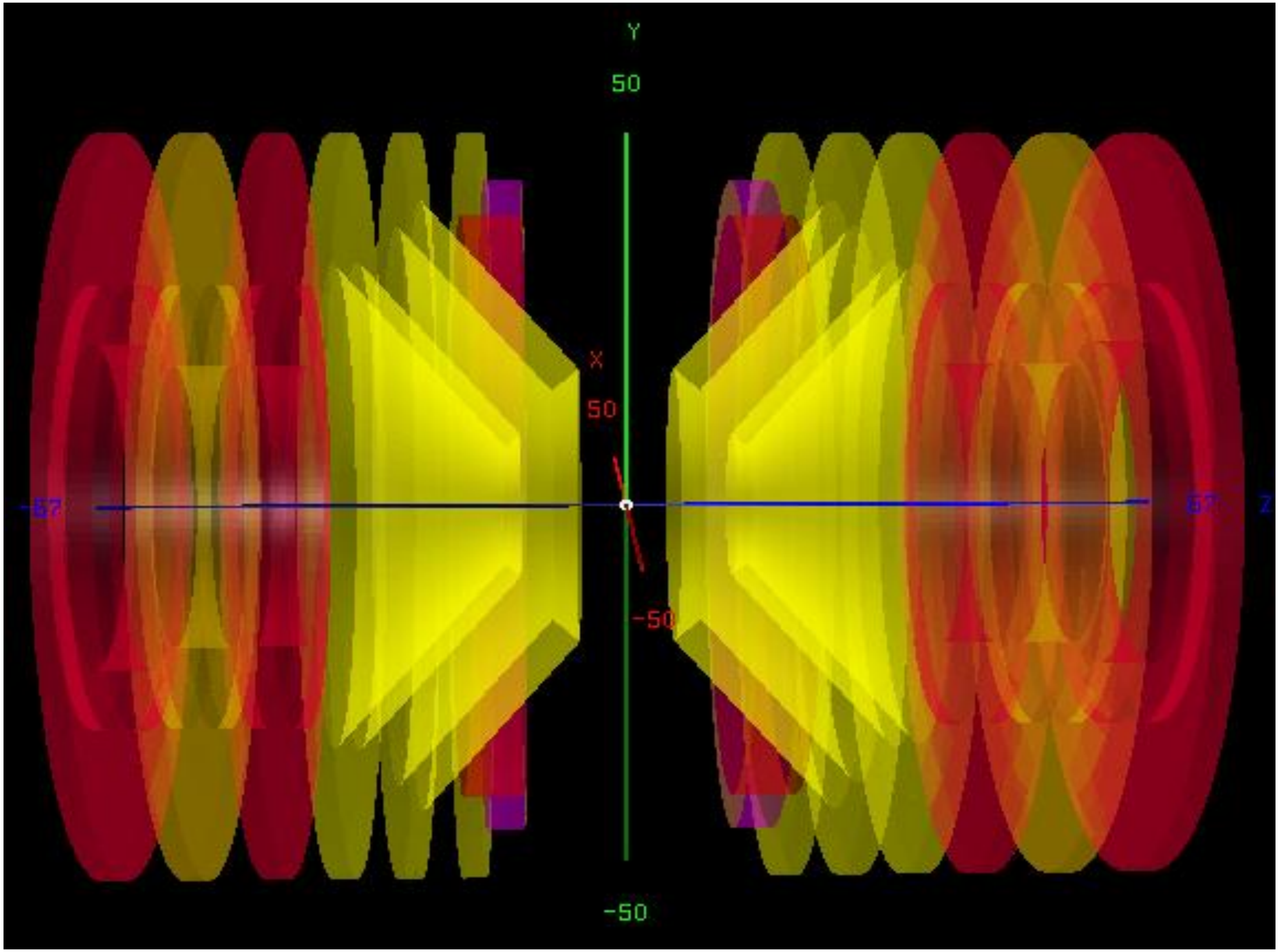



# Result of MORATrap optimization

# MORATrap: Optimization Result

**Optimized Electrodes + Einzel Lens**

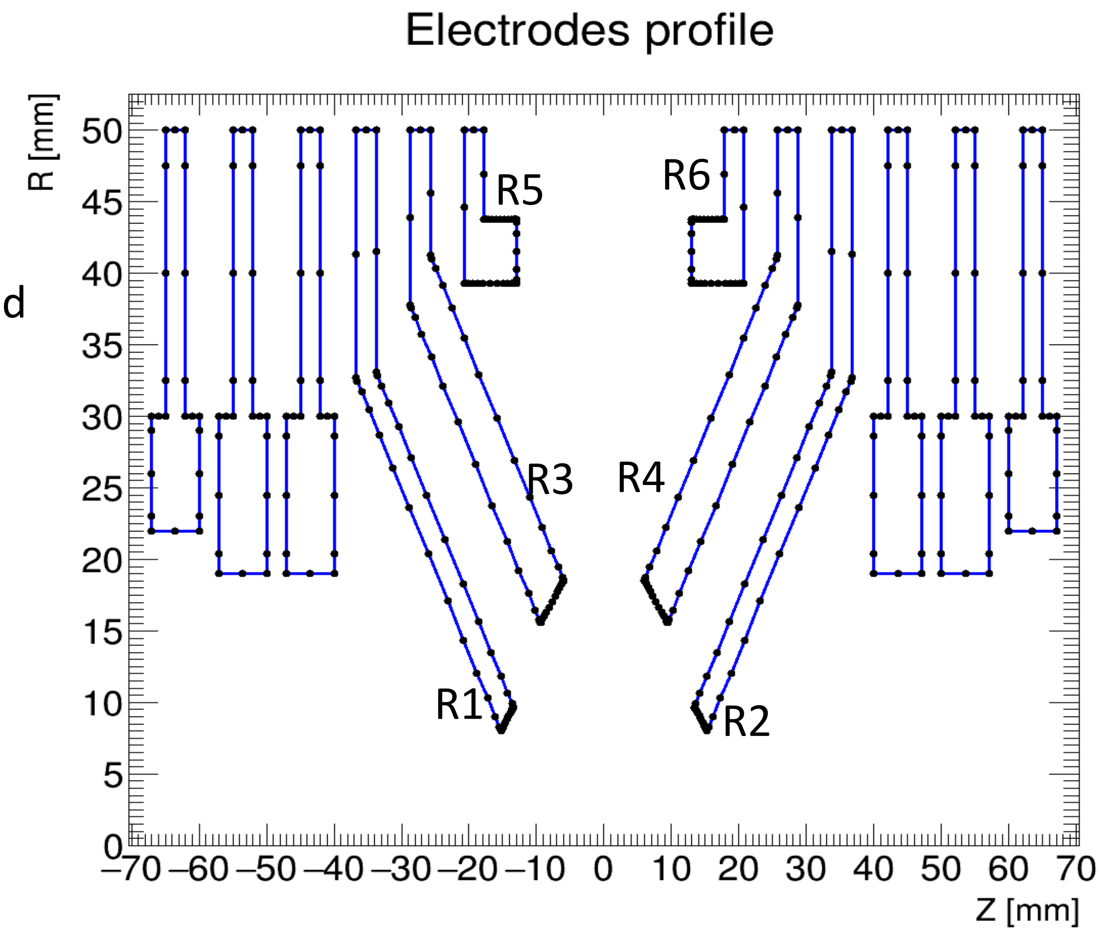


# MORATrap: Optimization Result

**Free parameters:**

- fixed the parameters related to the electrode R5R6

- Minimisation with 6 parameters related to electrodes :
  - -R3R4: Zmin, ρ, thickness
  - -R1R2: Zmin, thickness
  - - And deviation angle



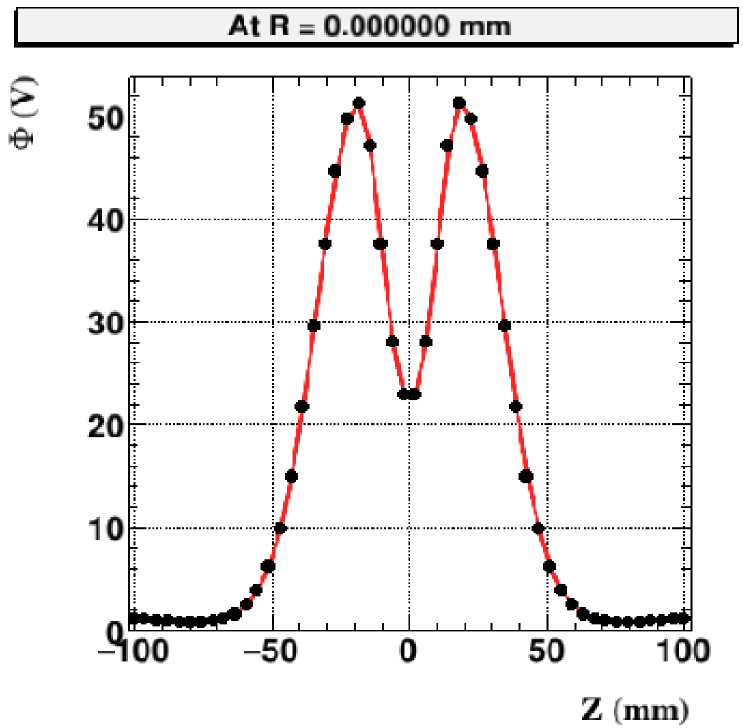
# MORATrap: Optimization Result (Harmonic & potential)

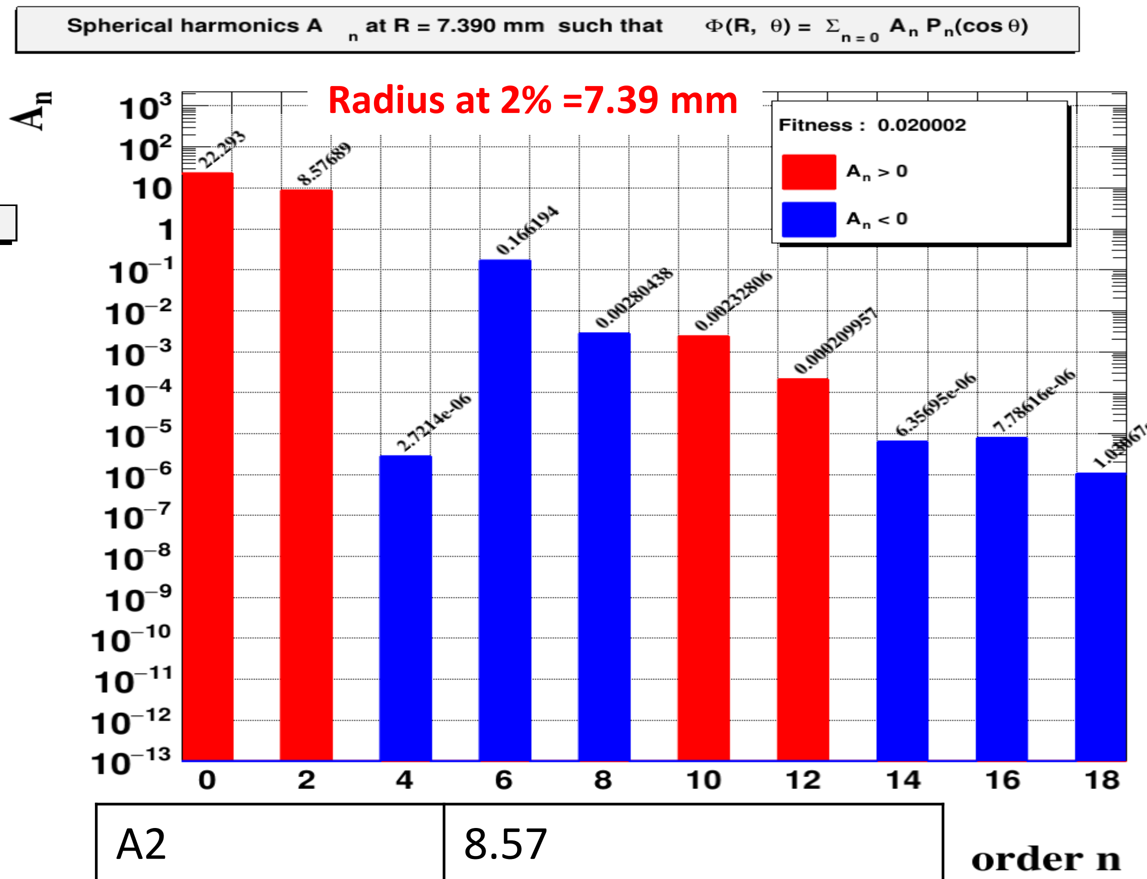
**Potential on ring pairs:**

**V12=60 V (RF)**

**V34=0 ; V 56=0 V**

**V (Einzel Lens) =0V**



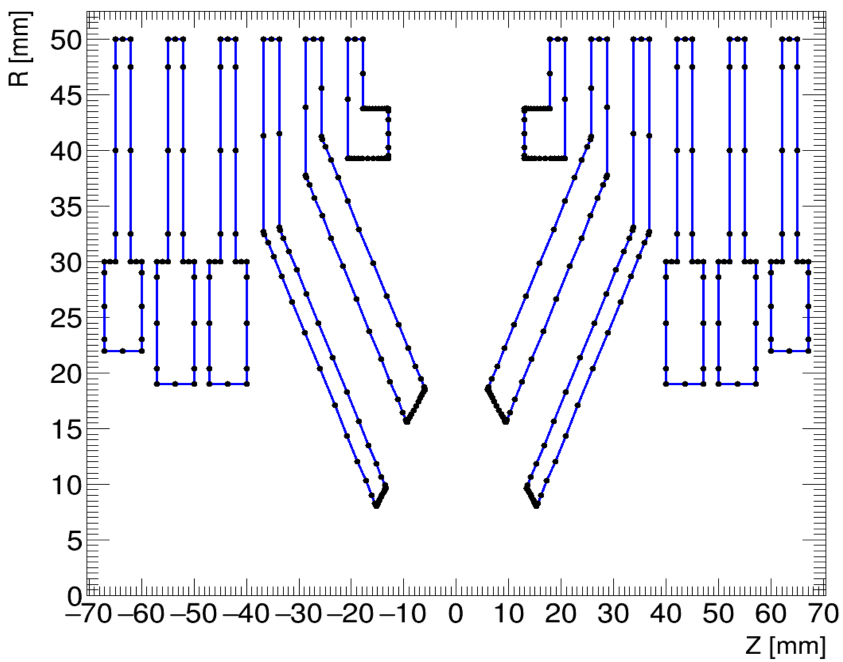

| | |
|---|---|
| A2 | 8.57 |
| A4 | 2.72 e-06 |
| A6 | 0.16 |
| A8 | 0.0028 |

# MORATrap: Optimization Result (dimensions)

**Electrodes dimensions**

| M | $R_{12}$ | $R_{34}$ | $R_{56}$ |
|---|---|---|---|
| **Zmin (mm)** | 13.41 | 6.0 | 13. |
| **Inner radius (mm)** | 8.0 | 15.56 | 39.29 |
| **Outer radius (mm)** | 9.67 | 18.53 | 43.79 |
| **thickness (mm)** | 2.17 | 3.94 | 4.5 |
| **Potentiels (V)** | +/-60 (RF) | 0. | 0 |

Electrodes profile

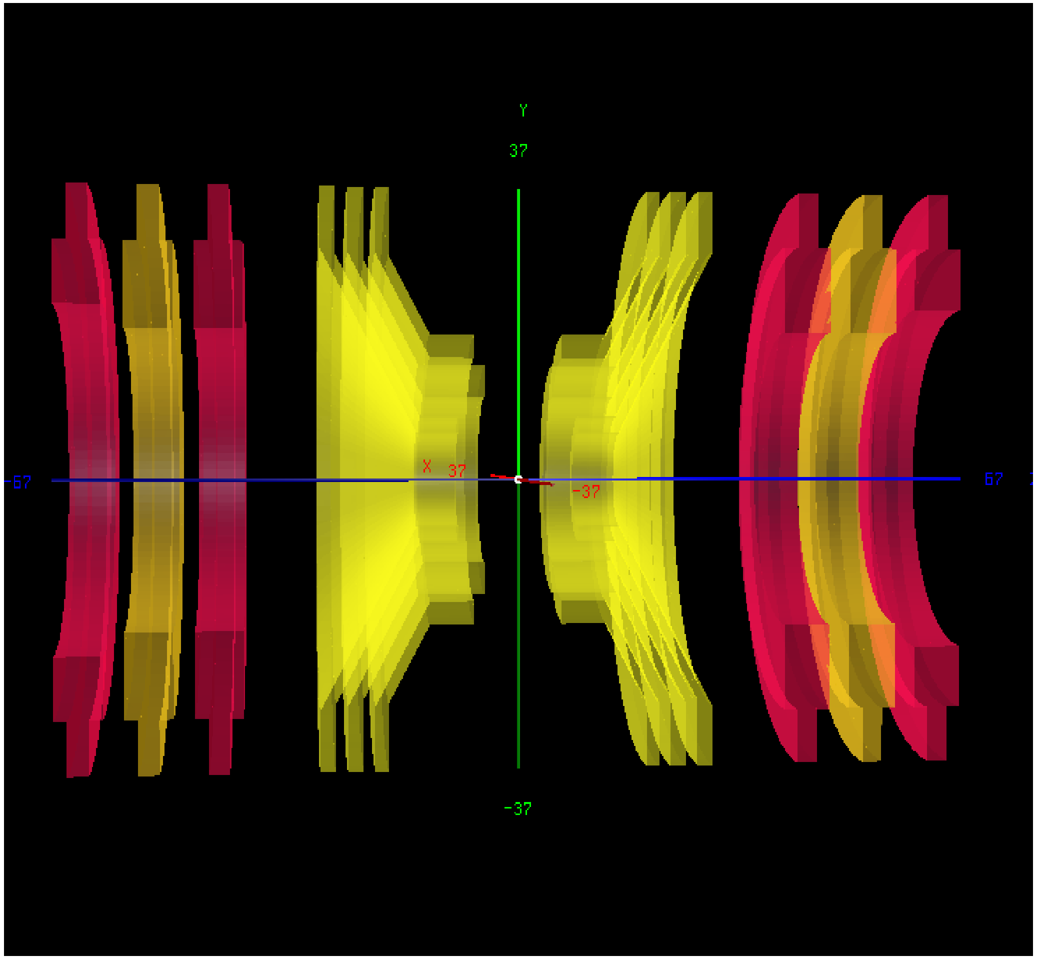


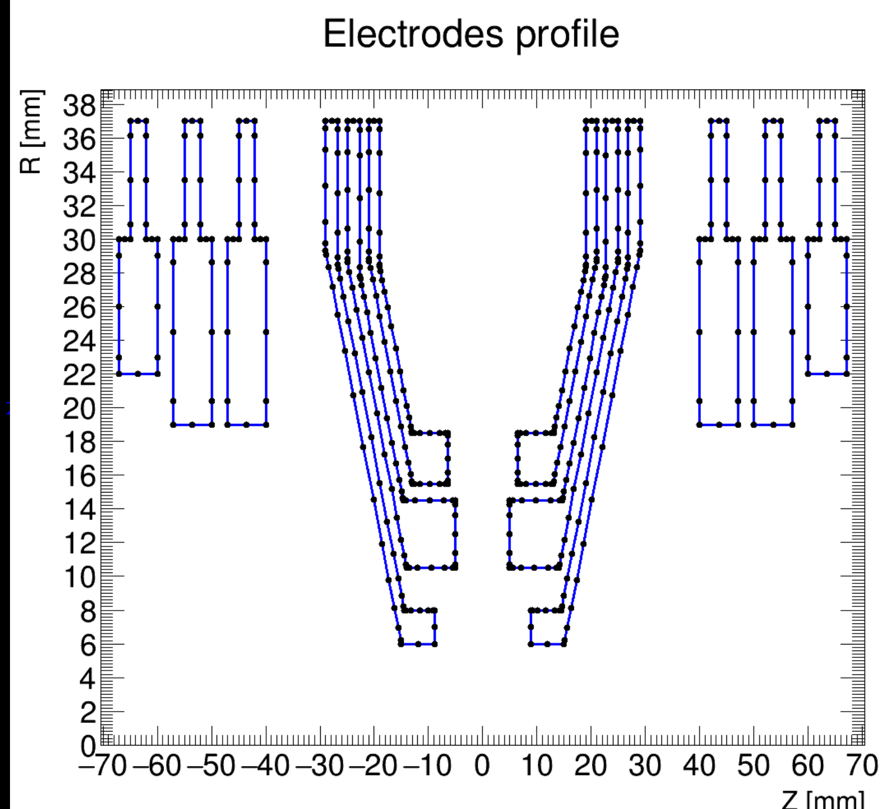

**Angles an redius at 2%**

| | |
|---|---|
| Radial angle | 18 deg |
| Axial angle | 27.69 deg |
| **Radius at 2%** | 7.39 mm |

# MORATrap Vs LPCTrap

# LPCTrap: ELECTROBEM simulation



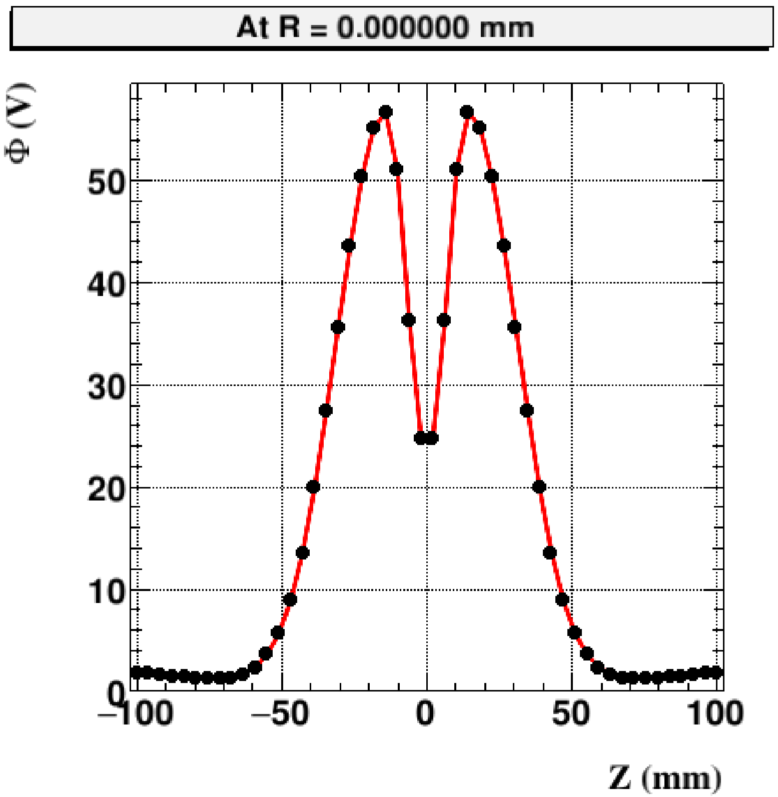



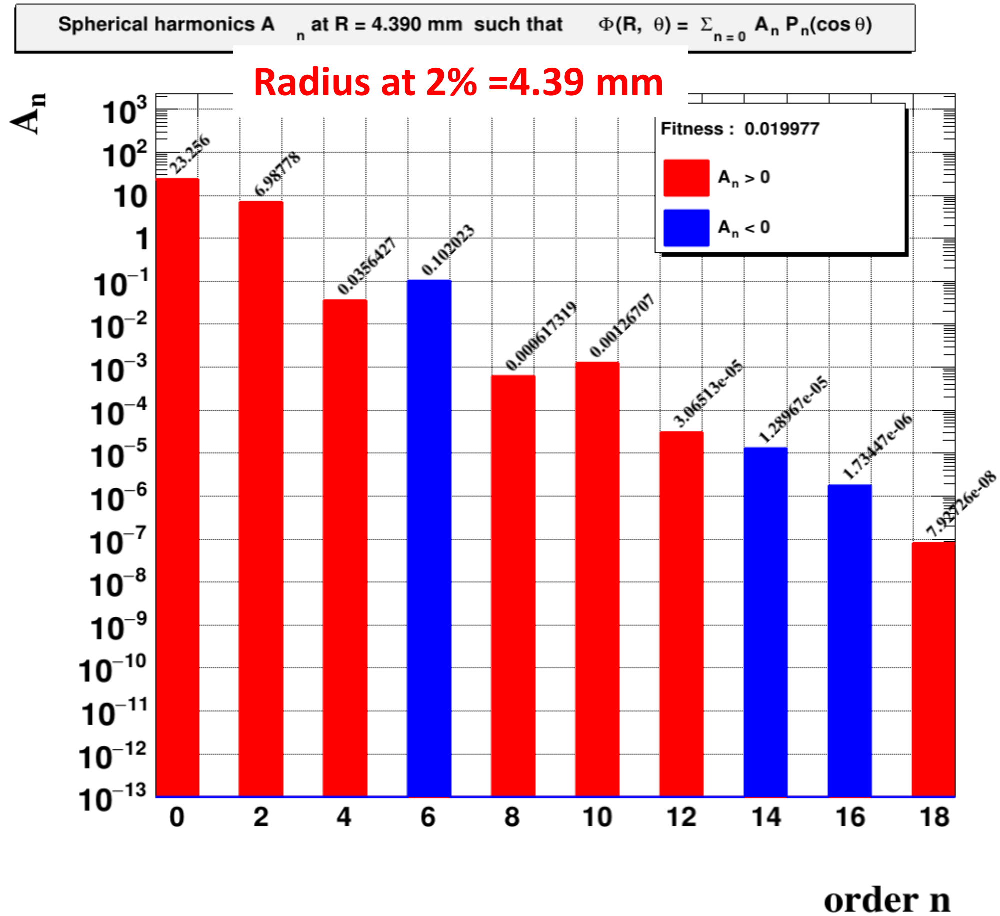

Axial angle = 21 deg
Radial angle = 18 deg
Radius at 2 % = 4.39 mm

# LPCTrap: ELECTROBEM simulation

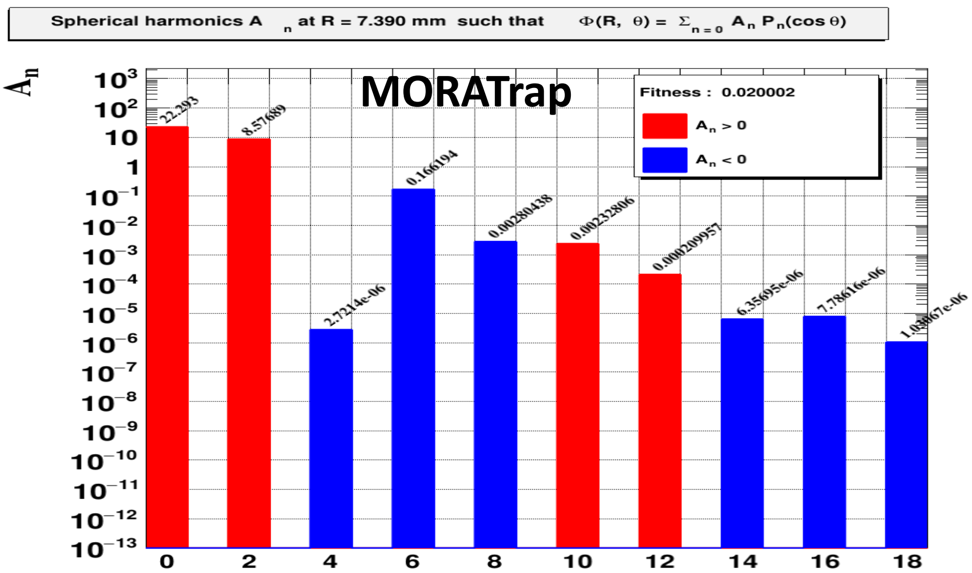
(Harmonics and potential)

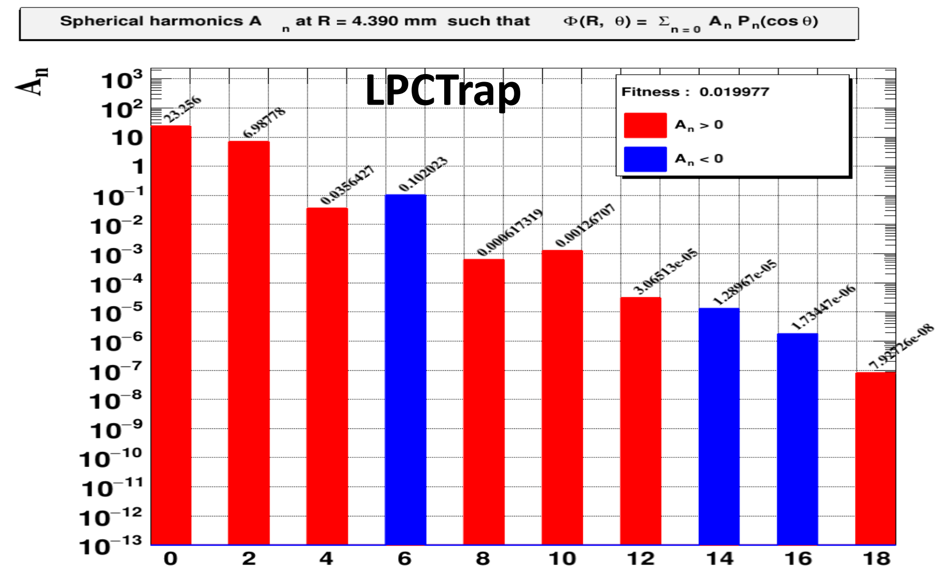
**Potentials on ring pairs:** **V12=60 V, V34=0 ; V 56=12 V; V (Einzel Lens) =0V**



# MORATrap Vs LPCTrap




**MORATrap**

| | |
|---|---|
| Radial angle | **18 deg** |
| Axial angle | **27.69 deg** |
| Radius at 2% | **7.39 mm** |
| A2 | **8.57** |
| A4 | **0.0000027** |
| **A6** | **0.16** |

**LPCTrap**

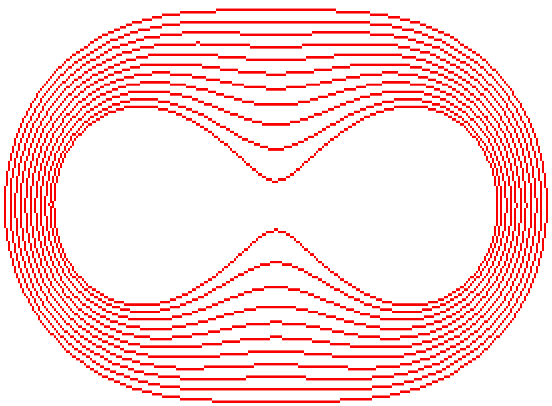
| | |
|---|---|
| Radial angle | 18 deg |
| Axial angle | 21 deg |
| Radius at 2% | 4. 39 mm |
| A2 | 6.98 |
| A4 | 0.37 |
| A6 | 0.11 |

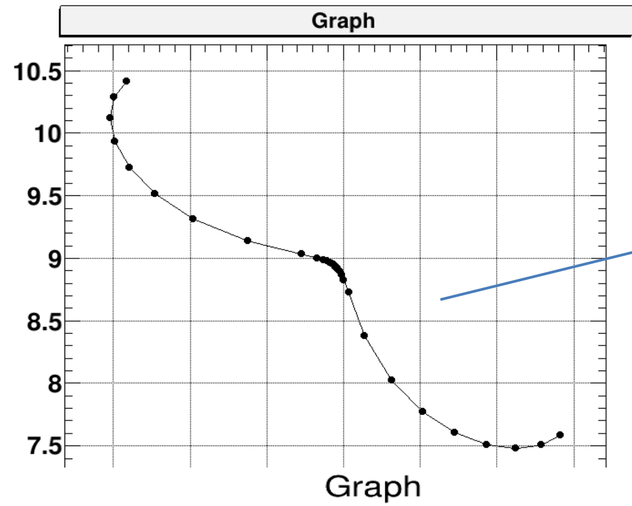
# Optimization Parameters

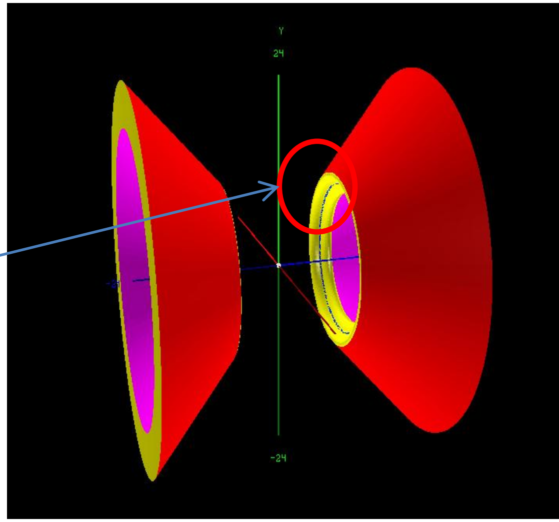
## (to reduce A6)

Many forms of conical, cylindrical electrodes and different shape of section in front of the ions cloud are tested to reduce the harmonic A6:

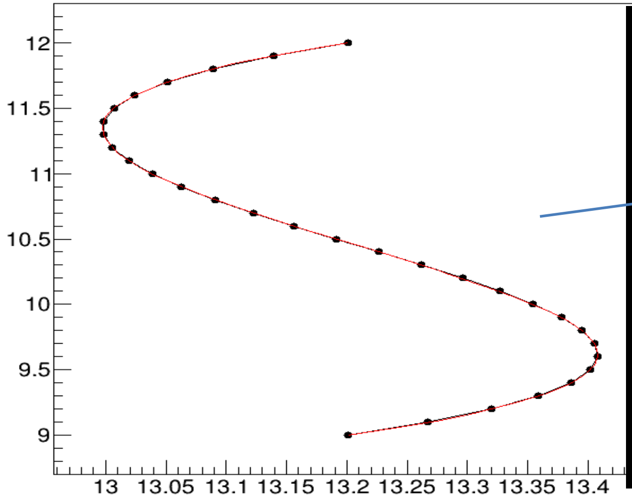
**1) Cassini Oval**

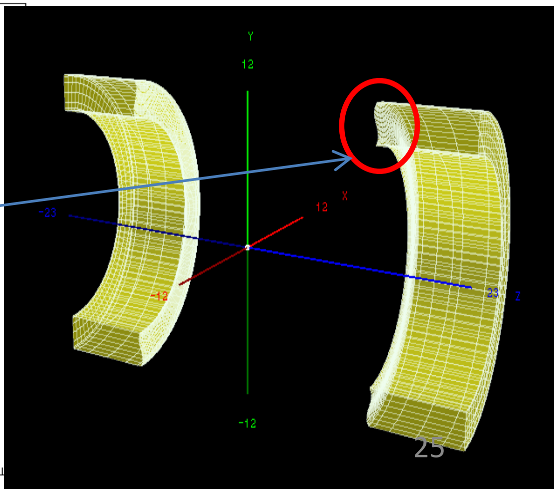




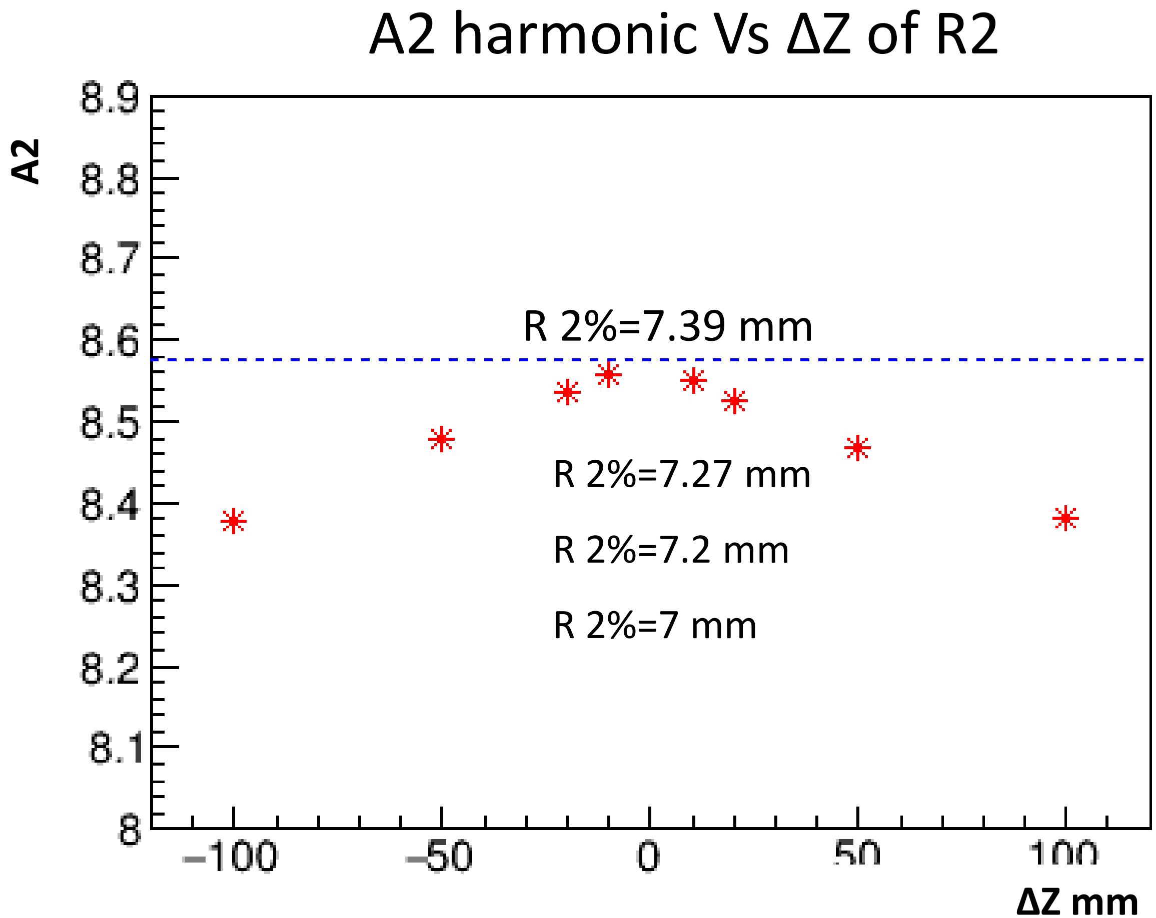
**2) Spline Curves**

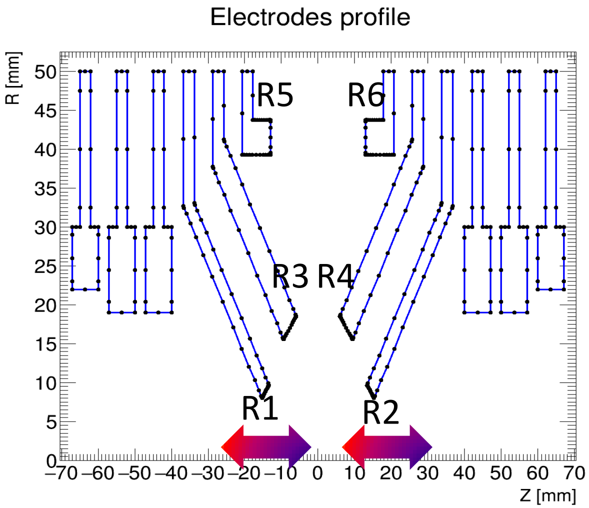


# MORATrap: Systematic study

Move the electrodes R2, R4, R6 separately, along the Z axis +/-100, +/-50,+/-20, +/-10 μm
(The blue line shows the reference value )



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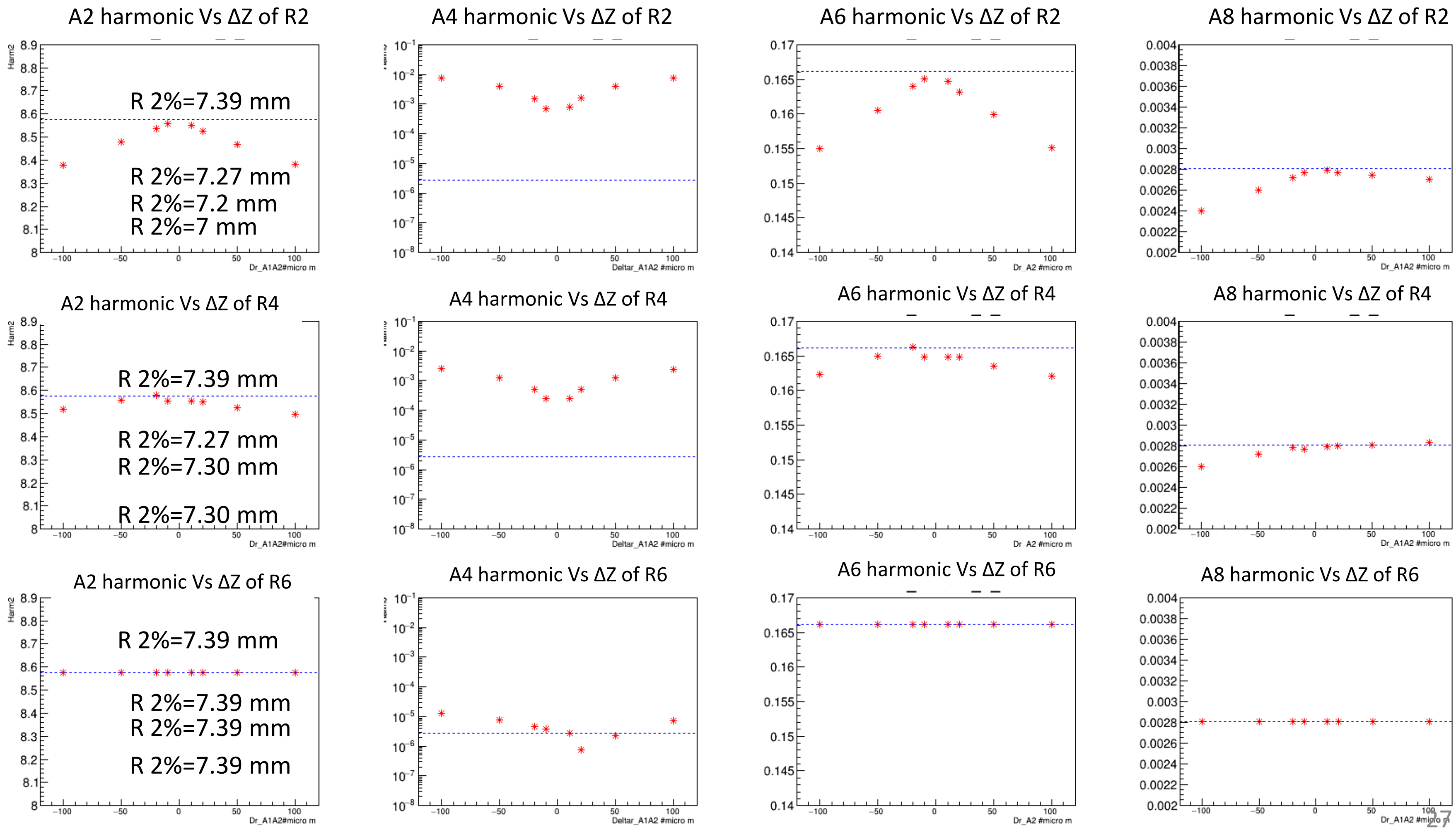
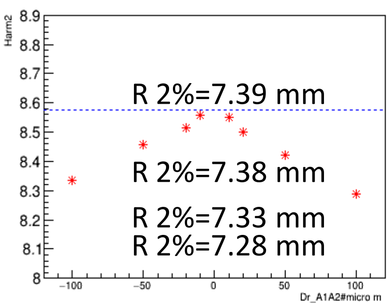


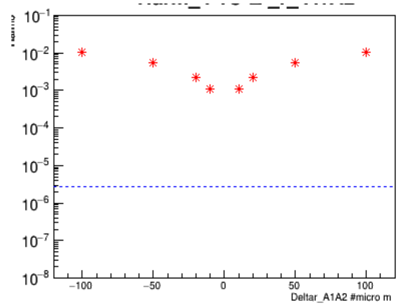
# MORATrap: Systematic study

Move the **radius** of the electrodes R1R2, R2R3 , R4R5: +/-100, +/-50,+/-20, +/-10 μ m
(The blue line shows the reference value )

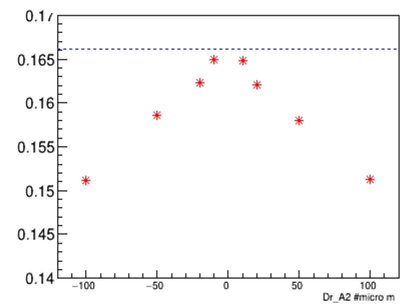
A2 harmonic Vs ΔR of R12



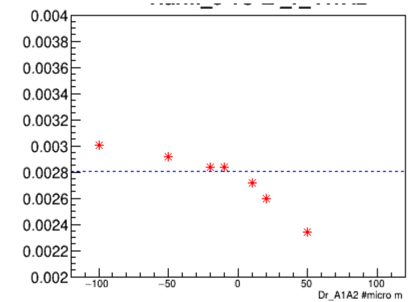

A4 harmonic Vs ΔR of R12



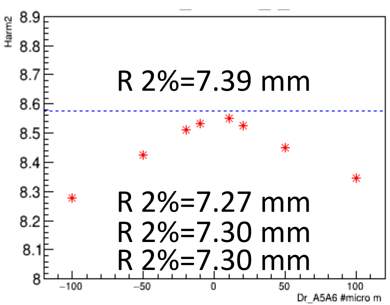

A6 harmonic Vs ΔR of R12



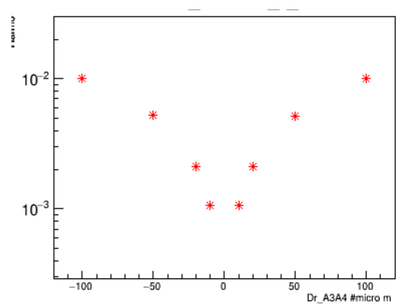

A6 harmonic Vs ΔR of R12



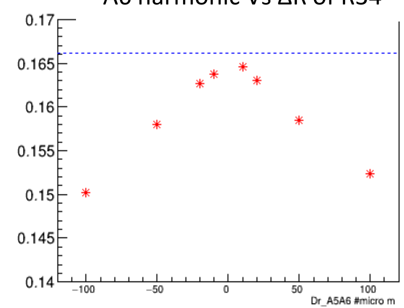

A2 harmonic Vs ΔR of R34



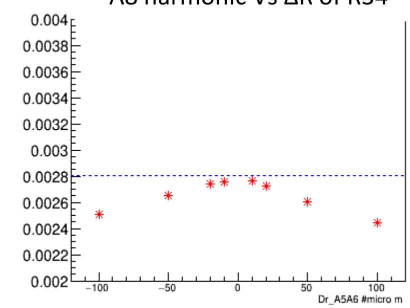

A4 harmonic Vs ΔR of R34



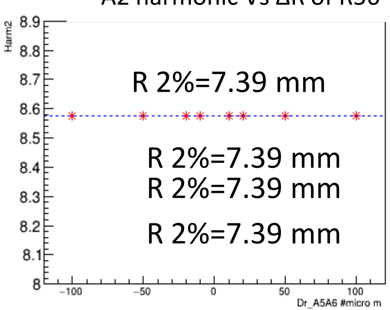

A6 harmonic Vs ΔR of R34



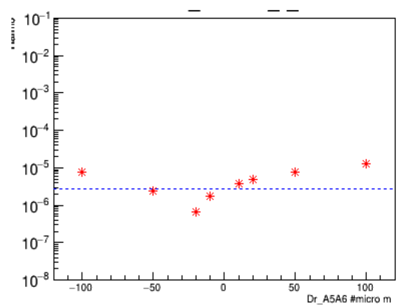

A8 harmonic Vs ΔR of R34



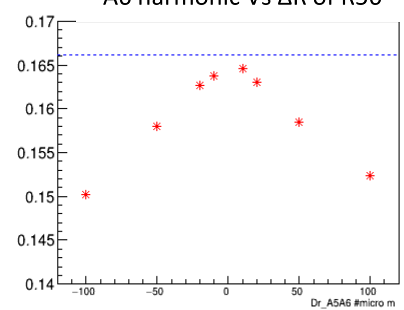

A2 harmonic Vs ΔR of R56



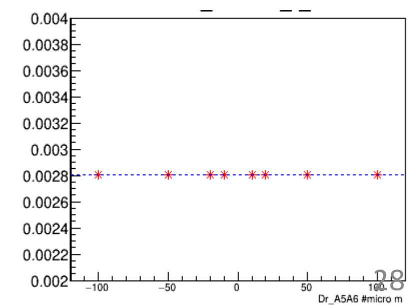

A4 harmonic Vs ΔR of R56




A6 harmonic Vs ΔR of R56

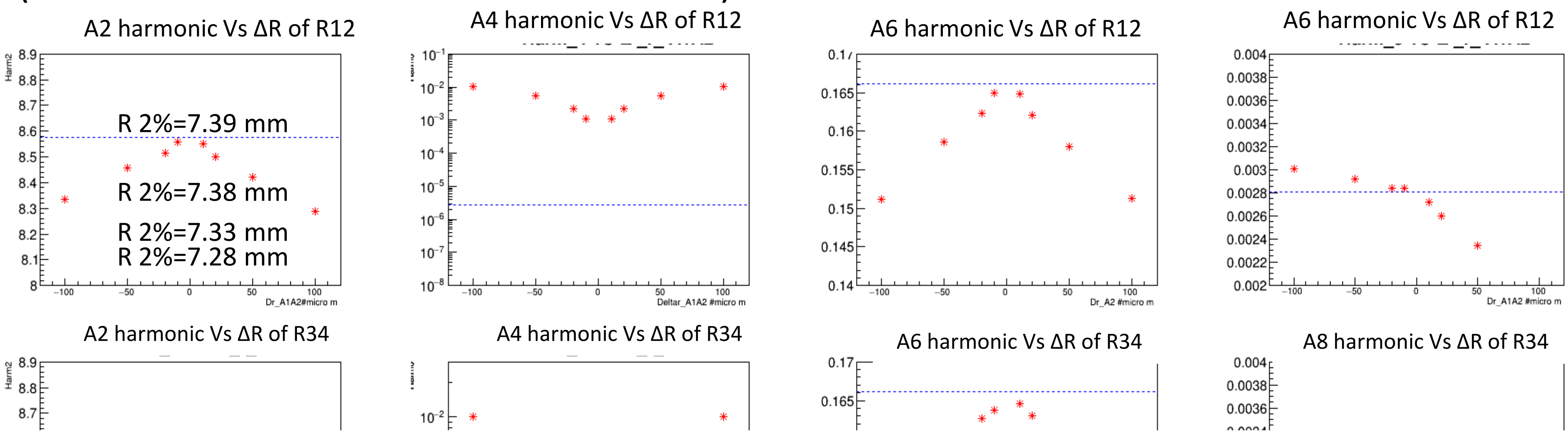



A8 harmonic Vs ΔR of R56

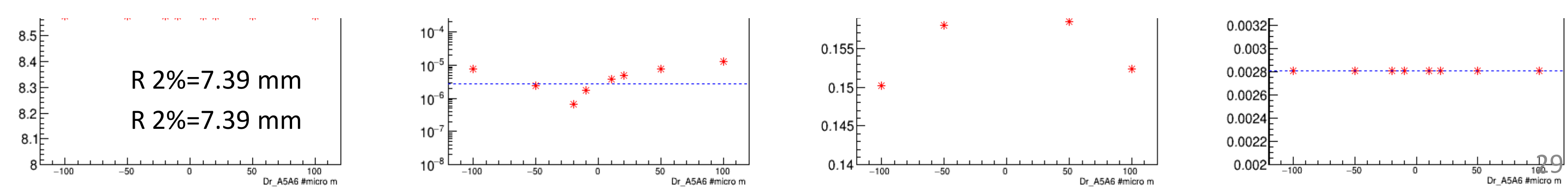


# MORATrap: Systematic study

Move the **radius** of the electrodes R1R2, R2R3 , R4R5: +/-100, +/-50,+/-20, +/-10 µ m
(The blue line shows the reference value )




- **Sensitivity is of the order of >50 µm for the electrode R1R2**
- **Sensitivity is much lower for the electrode R3R4 and R5R6 (100 µm)**



# Conclusion

**The optimized geometry of MORATrap (compared to LPCTrap):**

- ✓ **axial solid angle increased**
- ✓ **Important radius at 2% (7.39 mm)**
- ✓ **reduce better the harmonic of order higher than 2**
- ✓**trapping efficiency increased (>58% Vs 20% LPCTrap) : B.M. Retailleau Presentation**

# What's next ?

- **Switch to the 3D version of ELECTROBEM:**

**- calculate the potential on a sphere at the center of trap (has been done, and checked against the potential in AxyELECTROBEM)**

**-a systematic study will be done (electrode rotation, torsion)**

Thank you for your attention

# Fitness Function

$$\Phi(\rho,\theta)=\sum_{n=0}^{\infty}\mathcal{A}_n\left(\frac{\rho}{\rho_0}\right)^n P_n(\cos\theta)$$

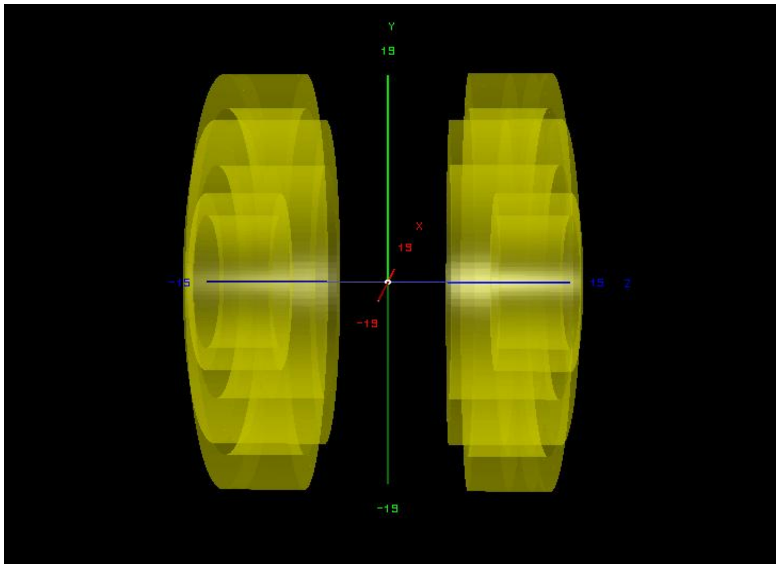
$$\frac{\sum_{n>2}\left(\frac{\rho}{\rho_0}\right)^n|\mathcal{A}_n|}{\left(\frac{\rho}{\rho_0}\right)^2|\mathcal{A}_2|}\leq 2\% \quad\Rightarrow\quad \left|\frac{\mathcal{A}_{10}}{\mathcal{A}_2}\right|r'^4+\ldots+\left|\frac{\mathcal{A}_4}{\mathcal{A}_2}\right|r'-0.02=0$$

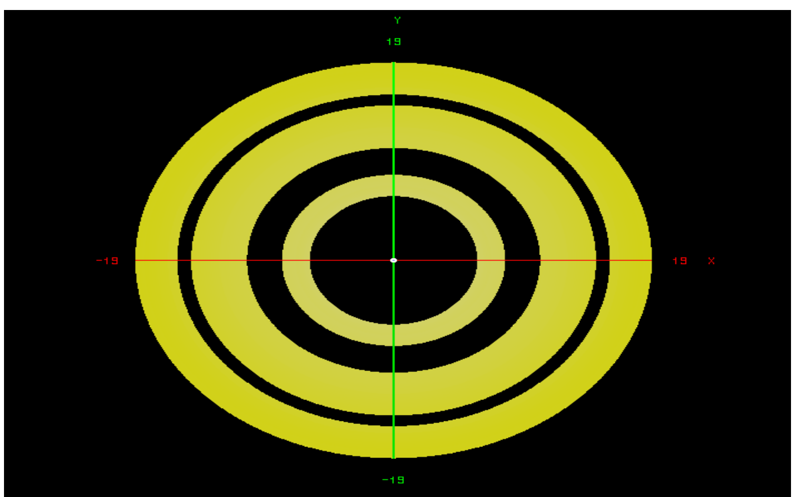
Où: $r'=\left(\frac{\rho}{\rho_0}\right)^2$

$$\frac{1}{|\mathcal{A}_2|\,r'(\vec{p})}$$

# LPCTrap

**The LPCTrap consists of three rings with dimensions:**




| Rings name | $A_{12}$ | $A_{34}$ | $A_{56}$ |
|---|---|---|---|
| Zmin (mm) | 8.8 | 5. | 6.5 |
| Inner radius (mm) | 6. | 10.5 | 15.5 |
| Outer radius (mm) | 8. | 14.5 | 18.5 |
| Length (mm) | 6.2 | 9 | 6.5 |
| Potentials (V) | +/-60 (RF) | 0. | 12. |