Naked Singularities versus Scalar-Tensor Gravity in the Gravitational Wave Astronomy Area

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On singularities :

Considering:

– a physically motivated and tested theory Th1
– an astronomically relevant (in some sense) exact solution Sol1 of Th1

–1– a singularity entering a solution Sol1 of a theory Th1 is a spacetime location where Th1 fails;

–2– near the singularity location, Th1 has to be replaced by another theory Th2, that admits an exempt of singularity solution Sol2, but such that Sol2 asymptotically coincide with (or that can be reinterpreted as) Sol1 « far » from the Sol1 singularity.

This singularity: hidden or naked?

–3–a black hole (BH) like solution Sol1 of Th1 is a solution having a singularity that is hidden behind an horizon. Thence, an « external observer » does not belong to the causal future of the region where Th2 (significantly) differs from Th1;

–4– conversely, a naked singularity (NakS) entering a solution Sol1 of Th1 flags a region belonging to the causal past of an « external observer » where Th2 (significantly) differs from Th1.

With this in mind, it may be sound to conclude:

–5– ***the presence of a NakS*** in the solution Sol1 of Th1 should be considered as ***a chance, rather than a problem***, that would necessarily have to be cured by some mechanism (or conjecture/theorem) designed to prevent the NakS existence/formation;

–5'– more: this should be a motivation for seeking for solutions, otherwise physically motivated, that exhibit a NakS (instead of BH) structure;

–6– for such solutions, signatures of the NakS (versus BH having the same far parameters, like mass and angular momentum) in close to the NakS location (but nevertheless far enough for Th1 to be still relevant) are of the utmost observational interest, as ***ways to spot the existence of a close spacetime region where the theory Th2, instead of Th1, is expected to be at work.***

One could even spot that:

–7– incidentally, if two competing Th1 and Th1' theories having (resp.) Sol1 and Sol1' solutions that both are worth candidates to describe some astronomical setting, Sol1' exhibiting a NakS structure while Sol1 does not, the close to the NakS quantitative, or even qualitative, features of Sol1' w.r.t. Sol1 could result in ***original ways to discriminate between Th1 and Th1'***.

The ideas behind the previous discussion? Let the alluded theories be:

- Th1 = General Relativity
- Th1' = Brans-Dicke/Scalar-Tensor gravity

- (unknown) Th2 = gravity in its quantum regime

  ← … since it seems to be widely accepted any (naked or not) singularity entering any classical gravity theory should be naturally removed by a viable quantum description of the spacetime

- *the new born **gravitational wave astronomy** should be the most well dedicated tool to explore what happens in **the strongest local gravitational fields in our Universe**, that are probably the gravitational fields surrounding (hidden or naked) singularities*

NakS in GR

In spherical symmetry: Birkhoff theorem

→ Any spherical collapse ends up forming a BH

Nevertheless, during the **collapsing time**,
a NakS may (temporarily) appear (spoiling all its causal future)
← Lemaitre-Tolman-Bondi solutions

Keeping staticity, but ruling out spherical symmetry: NakSs are possible (γ-metric, bags of exact – seemingly unphysical – solutions)

Axial symmetry: NakSs are possible
The most known exemple: Kerr with $a > m$ – seemingly unphysical –

In **spherical symmetry**: no Birkhoff theorem …

The general spherically symmetric vacuum solution of the Brans-Dicke theory is Brans's Class I (excluding ω < – 3/2)

$$ds^2 = -\left(\frac{1-\frac{k}{r}}{1+\frac{k}{r}}\right)^{\mu-s} dt^2 + \left(1+\frac{k}{r}\right)^4 \left(\frac{1-\frac{k}{r}}{1+\frac{k}{r}}\right)^{2-\mu-s} \delta_{ij} dx^i dx^j \quad \& \quad \Phi(r) = \left(\frac{1-\frac{k}{r}}{1+\frac{k}{r}}\right)^{s}$$

with $\mu = \sqrt{4-(3+2\omega)s^2}$

The $s = 0$ case → Schwarzschild BH
Ohterwise: NakS or wormhole

In the large ω case, the solution asymptotically achieves the Fisher-Janis-Newman-Winicour form (that solves the scalar-Einstein equation $R_{ab} = \varphi_a \varphi_b$)

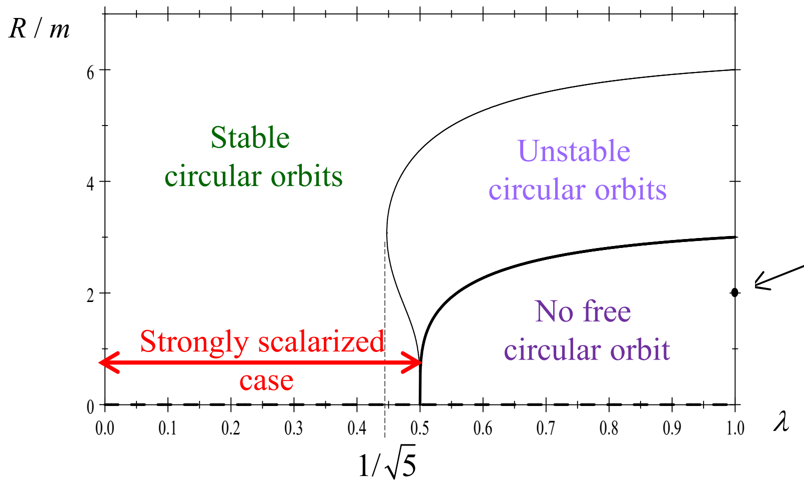
$$ds^2 = -\left(\frac{1-\frac{k}{r}}{1+\frac{k}{r}}\right)^{2\lambda} dt^2 + \left(1+\frac{k}{r}\right)^4 \left(\frac{1-\frac{k}{r}}{1+\frac{k}{r}}\right)^{2-2\lambda} \delta_{ij} dx^i dx^j$$

The λ = 1 case → Schwarzschild BH
Ohterwise: NakS (no longer wormhole cases …)

with $\lambda \in [0,1]$ & $\varphi(r) = \sqrt{2(1-\lambda^2)} \ln \frac{1-\frac{k}{r}}{1+\frac{k}{r}}$ ($R_{ab} = \partial_a \varphi \partial_b \varphi$)

$\sigma = 1-\lambda =$ scalar charge $\qquad m = 2\lambda k =$ mass

Circular orbits in FJNW (R = areal radius)



NakS at $r = k = \dfrac{m}{2\lambda} \quad \rightarrow \quad R = 0 \quad (\text{for } \lambda < 1)$

Chowdhury et al, PRD (2012)
BC, GRG (2017)

The far measured frequency
on a circular orbit is given by:

$$\nu_\infty(\lambda, r) = \frac{(2\lambda)^{3/2}}{2\pi m}\left(\frac{2\lambda r}{m}\right)^{3/2} \frac{\left(\frac{2\lambda r}{m} - 1\right)^{2\lambda - 1}\left(\frac{2\lambda r}{m} + 1\right)^{-2\lambda - 1}}{\sqrt{\left(\frac{2\lambda r}{m}\right)^2 - 2\lambda\frac{2\lambda r}{m} + 1}}$$

… that **diverges when $r \rightarrow$ NakS** in the strongly scalarized case BC, GRG (2017)

→ **It seems sound to conjecture that a low mass orbiting such a spherical NakS structure (EMRI approximation) should return orbital (and then gravitational radiation) frequencies that should be >> to the BH case (ICO, ISCO far frequencies)**

In **axial symmetry**

Of course, these spherical results raise a question: what if the solution is rotating?
→ Kerr like generalization of the FJNW solution

So far, only few rotating exact BD solutions are known

Sultana-Bose solution (PRD 2015): **not** asymptotically flat
→ may be useful to study some issues,
but its ultimate astrophysical relevance is debatable …

Some other BD exact non spherical vacuum BD solutions, or algorithm to derive some, have been proposed (from the middle 70'). But for almost all of them, there are things like:

- a PDE system remains to be solved to get an explicit metric
- non asymptotical flatness
- static (no rotation, despite not spherical)
- just one space (r) coordinate dependence
- a few ST exact solutions are know … but for $\omega(\Phi)$ functions specifically designed for obtaining solvable equations, otherwise not physically motivated …

A recent result (BC, 2018): For any (m,a), any static axial vacuum GR solution results in an **axially symmetric scalar-Einstein** solution, that reduces to the (m,a) Kerr solution when the scalar vanishes

More precisely, any solution of $R_{ab} = 0$ having the form

$$ds^2 = -e^{2U(\rho,z)}dt^2 + e^{-2U(\rho,z)}\left[e^{2\gamma(\rho,z)}\left(d\rho^2 + dz^2\right) + \rho^2 d\phi^2\right]$$

generates a solution of $R_{ab} = U_a U_b$ (partial derivatives), that reads

$$ds^2 = k_{00}dt^2 + 2k_{03}dt\,d\phi + k_{33}d\phi^2 + e^{\gamma(\alpha,\beta)}\left(k_{11}dr^2 + k_{22}d\theta^2\right) \quad \& \quad U(\alpha,\beta)$$

where $k_{ab}(r,\theta)$ is the (m,a) Kerr solution, and where $\alpha = \frac{\sqrt{r^2 - 2mr + a^2}}{\sqrt{m^2 - a^2}}\sin\theta$ & $\beta = \frac{r-m}{\sqrt{m^2 - a^2}}\cos\theta$

(The Sultana-Bose solution is a particular case)

These solutions:

- may serve to built BD vacuum solutions using the Wagoner conformal transformation
- but may also directly serve as $(\omega >> 1)$ BD solutions

As a consequence, an asymptotically flat solution of $R_{ab} = \varphi_a \varphi_b$ is:

$$\varphi(r,\theta) = \Lambda \frac{\sqrt{m^2 - a^2}}{\sqrt{r^2 - 2mr + a^2 + \left(m^2 - a^2\right)\cos^2\theta}}$$

$$ds^2 = k_{00}dt^2 + 2k_{03}dtd\phi + k_{33}d\phi^2 + \exp\left(-\Lambda^2 \frac{\left(m^2 - a^2\right)\left(r^2 - 2mr + a^2\right)\sin^2\theta}{2\left[r^2 - 2mr + a^2 + \left(m^2 - a^2\right)\cos^2\theta\right]^2}\right)\left(k_{11}dr^2 + k_{22}d\theta^2\right)$$

Some properties:

- reduces to Kerr for $\Lambda = 0$ (in this sense, a genuine BD generalization of the Kerr's GR solution)
- fails to return FJNW for $a = 0$ (Penney's metric instead …)
- ring like NakS (equator of the « Kerr's horizon »)
- equatorial circular orbits are « the same » as Kerr (same existence conditions in r terms) …
- … but radial distances between them are affected
- no longer orbital far frequency divergence …
- … but, considering the Penney's $a = 0$ case for convenience, no time freezing for radial infalls
  → a **qualitative difference** with vacuum GR (that also exists in FJNW)

Open issues & questions

1) Looking for exact (family of, isolated, …) solutions converging to FJNW for vanishing *a*
   → is the frequency divergence back?

2) Obtaining not only qualitative, but also quantitative properties on gravitational radiation in the EMRI case
   → in the FJNW spherical case, what is the GW amplitude when the orbital frequency achieves large values (strongly scalarized case)

3) NakS formation versus BD/ST matter collapses
   → numerical approach: only spherical cases have been simulated so far … favorizing BH like endstates …
   → … what happens in axisymmetric collapses?
   → mathematical approach: BD-LTB like solutions?

4) Any other suggestion welcome …

Thank you for your attention …