Effective field theory approach to scalar-tensor inspiraling

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January 31, 2019

Introduction

Conventional PN calculations :

$$\Box h^{\mu\nu} = -16\pi G T^{\mu\nu} \Rightarrow \text{Solve for } h^{\mu\nu} \Rightarrow \text{Plug back in the action}$$

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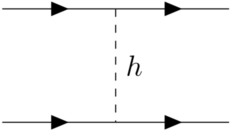
EFT approach :

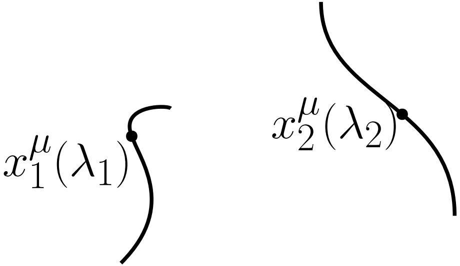
$$e^{iS_{\text{ef}}} = \int \mathcal{D}[h_{\mu\nu}] e^{iS}$$



Goldberger and Rothstein (2006)
Porto (2006)
+ many developments...

Outline

Invariances of the system



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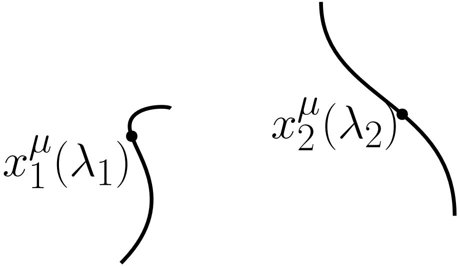

- $x^\mu \to x'^\mu(x) \Rightarrow$ Use R and $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

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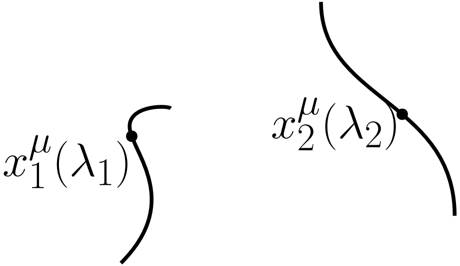
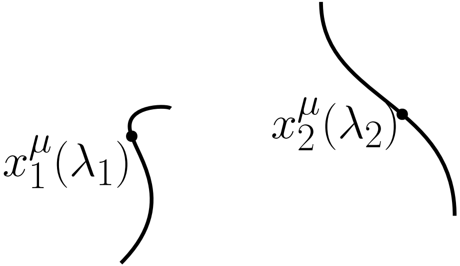


- $x^\mu \to x'^\mu(x) \Rightarrow$ Use R and $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
- $\lambda_a \to \lambda'_a(\lambda_a) \Rightarrow$ Use $d\tau = \sqrt{g_{\mu\nu}dx^\mu dx^\nu}$

Invariances of the system



Fluctuating field

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- $x^\mu \rightarrow x'^\mu(x) \Rightarrow$ Use R and $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
- $\lambda_a \rightarrow \lambda'_a(\lambda_a) \Rightarrow$ Use $d\tau = \sqrt{g_{\mu\nu}dx^\mu dx^\nu} = dt\sqrt{1 - v^2 - h_{\mu\nu}v^\mu v^\nu}$

Point particles action

$$S = S_{grav} + S_{pp,1} + S_{pp,2}$$

$$S_{grav} = \frac{m_P^2}{2} \int d^4x \sqrt{-g} R$$

$$S_{pp,a} = - m_a \int d\tau_a$$

Point particles action

$$S = S_{grav} + S_{pp,1} + S_{pp,2}$$

$$S_{grav} = \frac{m_P^2}{2}\int d^4x\sqrt{-g}R - \boxed{\frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}$$

$$S_{pp,a} = -\,m_a\int d\tau_a + \boxed{a\frac{m_a}{m_P}\int d\tau_a\phi + b\frac{m_a}{m_P^2}\int d\tau_a\phi^2} + \ldots$$

'Quantum' gravity

Integrate out fluctuating fields :

$$e^{iS_{\text{ef}}[\mathbf{x}_1(t),\mathbf{x}_2(t)]} = \int \mathcal{D}[h_{\mu\nu}]\mathcal{D}[\phi]e^{iS[\mathbf{x}_1(t),\mathbf{x}_2(t),h_{\mu\nu},\phi]}$$

S_{ef} contains the dynamics of the point-particles only

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S_{ef} contains the dynamics of the point-particles only

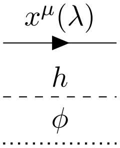
$$\Re(S_{\text{ef}}) = \int dt L[\mathbf{x}_a, \mathbf{v}_a]$$ Conservative dynamics

and

$$\Im(S_{ef}) = \frac{T}{2}\int dE d\Omega \frac{d^2\Gamma}{dE d\Omega}$$ Dissipative dynamics

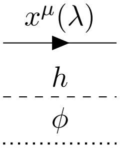
In practice

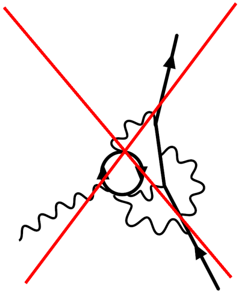
- Feynman expansion



In practice

- Feynman expansion

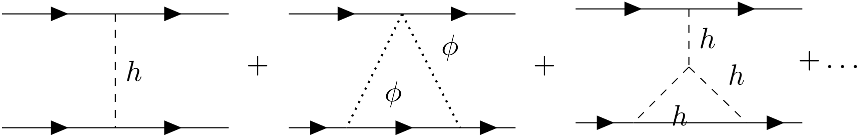




- No loops ! $\frac{\hbar}{L} \sim 10^{-76}$ where $L = mrv$

In pratice

$iS_{\text{ef}} =$



Power-counting rules

Small expansion parameter

$\frac{Gm}{r} \sim v^2$

Vertex

Expression

Weight

Power-counting rules

Small expansion parameter

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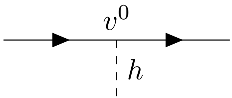
Vertex	
Expression	$\frac{m}{2} \int dt h_{00}$
Weight	v^0

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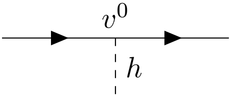
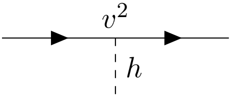
Vertex		
Expression	$\frac{m}{2}\int dt h_{00}$	$\frac{m}{2}\int dt h_{ij} v^i v^j$
Weight	v^0	v^2

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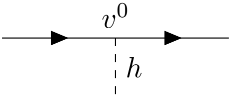
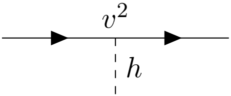
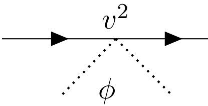
Vertex			
Expression	$\frac{m}{2}\int dt h_{00}$	$\frac{m}{2}\int dt h_{ij}v^i v^j$	$bm\int dt\phi^2$
Weight	v^0	v^2	v^2

Outline

v^0 (or Newtonian) Lagrangian

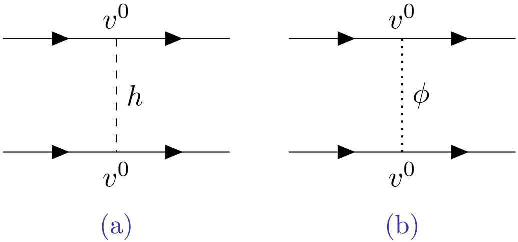


Figure: Feynman diagrams contributing to the Newtonian potential

v^0 (or Newtonian) Lagrangian

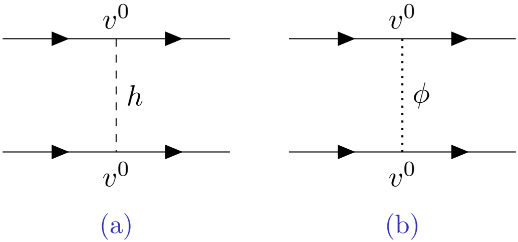
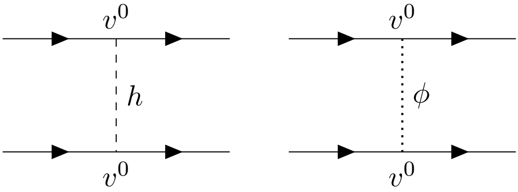


Figure: Feynman diagrams contributing to the Newtonian potential

$$L_{v^0} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{G_N m_1 m_2}{|\mathbf{x_1}(t) - \mathbf{x_2}(t)|}(1 + 2a^2)$$

v^0 (or Newtonian) Lagrangian



(a) (b)

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$$\tilde{G} = G_N(1 + 2a^2)$$

v^2 (or EIH) Lagrangian

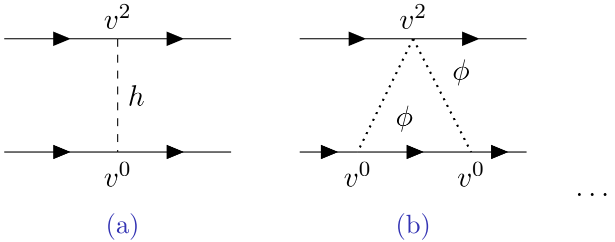
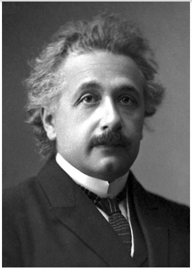


Figure: Some Feynman diagrams contributing to the v^2 Lagrangian

v^2 (or EIH) Lagrangian



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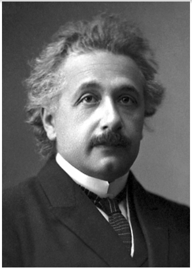

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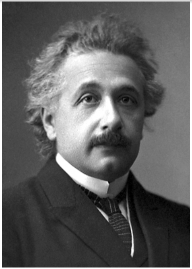


$$L_{EIH} = \frac{1}{8}\sum_a m_a v_a^4$$

$$+\frac{\tilde{G}m_1m_2}{2|\mathbf{x}_{12}|}\left[(v_1^2+v_2^2)-3\mathbf{v}_1\cdot\mathbf{v}_2-\frac{(\mathbf{v}_1\cdot\mathbf{x}_{12})(\mathbf{v}_2\cdot\mathbf{x}_{12})}{|\mathbf{x}_{12}|^2}+2\gamma(\mathbf{v}_1-\mathbf{v}_2)^2\right]$$

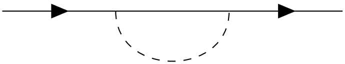
$$-\frac{\tilde{G}^2m_1m_2(m_1+m_2)}{2|\mathbf{x}_{12}|^2}(2\beta-1)$$

Renormalization of the mass

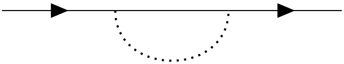
(a) (b)

Figure: Diagrams contributing to the mass renormalization.

Renormalization of the mass



(a)

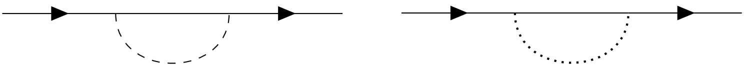


(b)

Figure: Diagrams contributing to the mass renormalization.

$$-m_{\text{bare}} \int dt \quad \rightarrow \quad -(m_{\text{bare}} + E(\Lambda)) \int dt$$

Renormalization of the mass



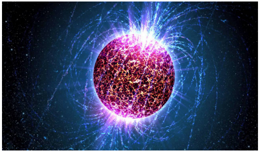
(a) (b)

Figure: Diagrams contributing to the mass renormalization.

$$-m_{\text{bare}} \int dt \quad \rightarrow \quad -(m_{\text{bare}} + E(\Lambda)) \int dt$$

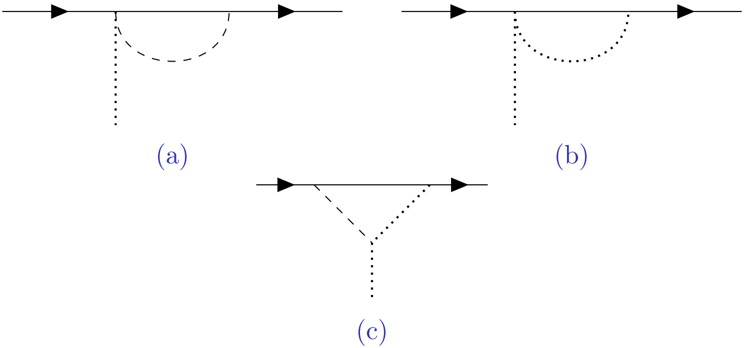
$$E(\Lambda) = -\frac{\tilde{G}}{2} \int d^3x d^3y \frac{\rho(\mathbf{x})\rho(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}$$

$E = 0$



$E \neq 0$

Renormalization of the charge



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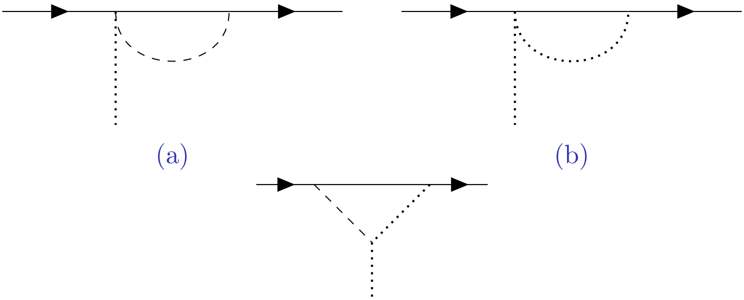

(a) (b) (c)

Figure: Diagrams contributing to the charge renormalization.

$$a_{\text{bare}} \frac{m_{\text{bare}}}{m_P} \int dt\, \phi \qquad \to \qquad a(\Lambda) \frac{m(\Lambda)}{m_P} \int dt\, \phi$$

Renormalization of the charge

$$\begin{cases} a_{\text{bare}} \to a(\Lambda) \\ \tilde{G} = G_N(1+2a^2) \end{cases}$$

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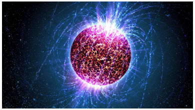
$$\Rightarrow \tilde{G}_{AB} = \tilde{G}\left[1 + (4\tilde{\beta} - \tilde{\gamma} - 3)\left(\frac{E_A}{m_A} + \frac{E_B}{m_B}\right)\right]$$

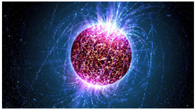
Renormalization of the charge

$$\frac{\tilde{G} m_1 m_2}{r}$$

$$\begin{cases} a_{\text{bare}} \to a(\Lambda) \\ \tilde{G} = G_N(1 + 2a^2) \end{cases}$$



$$\frac{\tilde{G}_{12} m_1 m_2}{r}$$



$$\Rightarrow \tilde{G}_{AB} = \tilde{G}\left[1 + (4\tilde{\beta} - \tilde{\gamma} - 3)\left(\frac{E_A}{m_A} + \frac{E_B}{m_B}\right)\right]$$

Nordtvedt (1968)

Outline

Multipole expansion

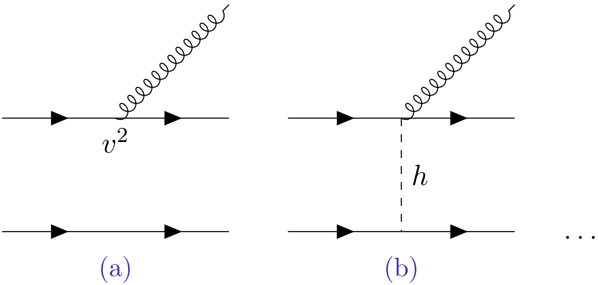


Figure: Some Feynman diagrams for the emission of one radiation scalar.

Multipole expansion

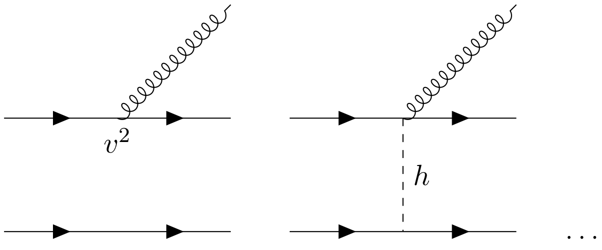


Figure: Some Feynman diagrams for the emission of one radiation scalar.

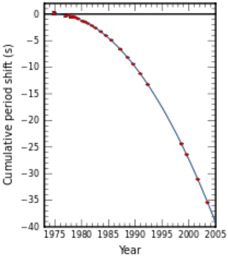
Net result :

$$S_{\text{int}} = \frac{1}{2}\int dt I_h^{ij} R_{0i0j} + \frac{1}{m_P}\int dt \left(I_\phi \bar{\phi} + I_\phi^i \partial_i \bar{\phi} + \frac{1}{2} I_\phi^{ij} \partial_i \partial_j \bar{\phi} \right) + \dots$$

Radiated power

Quadrupole formula :

$$P_h = \frac{G_N}{5} \left\langle \dddot{I}_h^{ij\,2} \right\rangle + \dots$$



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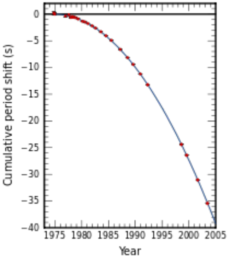

Monopole, dipole and quadrupole scalar radiation :

$$P_\phi = 2G_N \left(\left\langle \dot{I}_\phi^{\,2} \right\rangle + \frac{1}{3} \left\langle \ddot{I}_\phi^{i\,2} \right\rangle + \frac{1}{30} \left\langle \dddot{I}_\phi^{ij\,2} \right\rangle + \dots \right)$$

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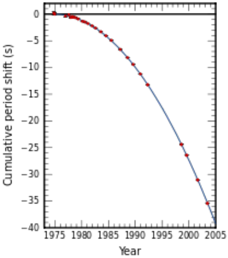

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$I_\phi = \text{Const} + \text{v}^2\text{correction},\ I_\phi^i \propto a_1 - a_2$

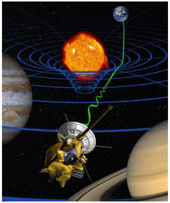
Outline

Conformal tranformation

$a\frac{m}{m_P}\int d\tau\phi$ and $b\frac{m}{m_P^2}\int d\tau\phi^2$ can be seen as

Conformal coupling

$$\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu}$$



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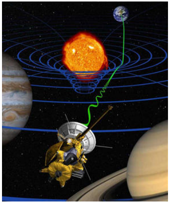
$$\begin{aligned}\Rightarrow -m\int d\tilde{\tau} &= -m\int d\tau\sqrt{A} \\ &= -m\int d\tau\left(1 - a\frac{\phi}{m_P} - b\frac{\phi^2}{m_P^2}\right)\end{aligned}$$



Disformal operator...

Other operator allowed :

$$m \int d\tau \partial_\mu \phi \frac{dx^\mu}{d\tau}$$

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$$m \int d\tau \partial_\mu \phi \frac{dx^\mu}{d\tau} = m \int d\tau \frac{d\phi}{d\tau}$$

is a total derivative !

Disformal operator...

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is a total derivative !

but

$$m \int d\tau \left(\frac{d\phi}{d\tau} \right)^2$$

is allowed...

...from disformal transformation

Disformal coupling

$$\tilde{g}_{\mu\nu} = A(\phi) g_{\mu\nu} + \frac{1}{M^2 m_P^2} \partial_\mu \phi \partial_\nu \phi$$

...from disformal transformation

Disformal coupling

$\tilde{g}_{\mu\nu} = A(\phi) g_{\mu\nu} + \frac{1}{M^2 m_P^2} \partial_\mu \phi \partial_\nu \phi$

$$\Rightarrow d\tilde{\tau}^2 = d\tau^2 \left(A + \frac{1}{M^2 m_P^2} \left(\partial_\mu \phi \frac{dx^\mu}{d\tau} \right)^2 \right)$$

The disformal energy and dissipated power

Disformal Lagrangian (conservative dynamics) :

$$L_{\text{dis}} = 4a^2 \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{M^2} \left(\frac{d}{dt} \frac{1}{r} \right)^2 , \quad r = |\mathbf{x}_1 - \mathbf{x}_2|$$

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Disformal monopole (dissipative dynamics) :

$$I_{\text{dis}} = 8a \frac{G_N m_1 m_2}{M^2} \frac{d^2}{dt^2} \frac{1}{r}$$

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Circular orbit : $L_{\mathrm{dis}} = I_{\mathrm{dis}} = 0$!

Outline

Conclusions

- We generalized NRGR to a scalar-tensor theory
- Effective theory point of view : operators allowed by symmetries
- Disformal couplings : no effect on circular orbits