Black holes in the Cubic Galileon theory

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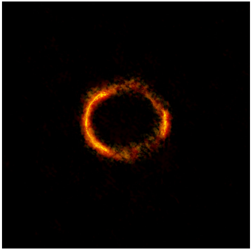
Work in collaboration with Eric Gourgoulhon, Philippe Grandclément, Christos Charmousis

Rencontre du GdR Ondes Gravitationnelles
31/01/19

Outline

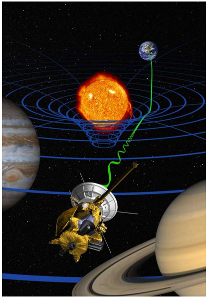
Experimental tests

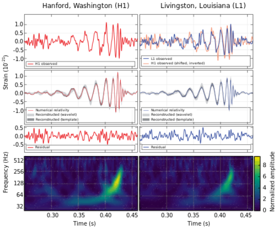
- Experimental successes of General Relativity:



Light deflection

Shapiro delay





Gravitational waves

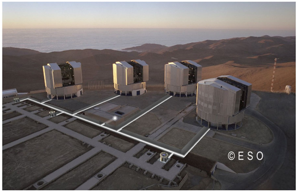
...

$\longrightarrow$ lack of strong field tests

Current observations

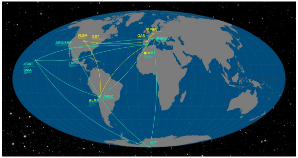
- Data from *Sgr A** ($4 \times 10^6 M_\odot$ black hole at the center of the galaxy)

GRAVITY



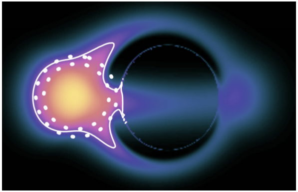
*High precision astrometry of the bodies orbitting Sgr A**

EVENT HORIZON TELESCOPE



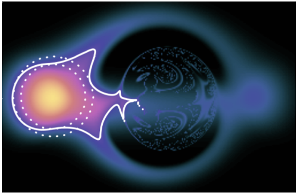
*Pictures of the surrondings of Sgr A**

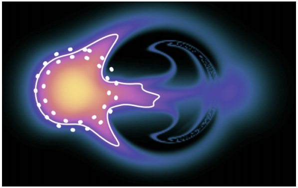
Predictions of GR



Kerr black hole

Black hole with scalar hair[1]





Boson star[2]

[1] Vincent, Gourgoulhon, Herdeiro & Radu, Phys. Rev. D 94, 084045 (2016) *using the libraries* LORENE *and* KADATH *and the ray-tracing code* GYOTO

[2] Vincent, Meliani, Grandclément, Gourgoulhon & Straub, Class. Quantum Grav. 33, 105015 (2016)

Modified gravity

- What if the predictions do not agree with the observations ?

  $\longrightarrow$ modify GR to account for observed deviations

[3]Hawking, Commun. Math. Phys. 25, 167 (1972)
[4]L. Hui and A. Nicolis, Phys. Rev. Lett. 110, 241104 (2013)
[5]T.P. Sotiriou and V. Faraoni, Phys. Rev. Lett. 108, 081103 (2012)

Modified gravity

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- Yet, no interest in modified theories with same BH as GR:

  - 1971: vacuum stationary BH of Brans-Dicke[3]

  - 2013: vacuum BH with static, spherically symmetric metric and scalar field of covariant Galileon[4]

  - 2014: asymptotically flat vacuum stationary BH of a large class of scalar-tensor theories[5]

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Distinctive features

- Admits static solutions different from Schwarzschild[6]

[6]Babichev, Charmousis, Lehébel & Moskalets, JCAP09(2016)011

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- Emerges in the decoupling limit of the "DGP" brane model (self-accelerating with screening)

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Distinctive features

- Admits static solutions different from Schwarzschild[6]

- Emerges in the decoupling limit of the "DGP" brane model (self-accelerating with screening)

- Consistent with $c_{GW} = c$ (GW170817 + GRB170817A)

[6] Babichev, Charmousis, Lehébel & Moskalets, JCAP09(2016)011

Dynamics

- Vacuum action (coupling constants ζ, η, γ):

$$S_{CG}\left[g,\phi\right] = \int \left[\zeta(R^{(g)} - 2\Lambda) - \eta\nabla_\mu\phi\nabla^\mu\phi + \gamma\nabla_\mu\phi\nabla^\mu\phi\Box\phi\right]\sqrt{|\det g|}d^4x$$

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- Metric equations (Einstein-like in spite of nonminimal coupling):

$$\frac{\delta S_{CG}}{\delta g_{\mu\nu}} = 0 \;\longrightarrow\; G_{\mu\nu} + \Lambda g_{\mu\nu} = T^{(\phi)}_{\mu\nu}$$

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- Scalar equation (current conservation from shift symmetry $\phi \to \phi + c$):

$$\frac{\delta S_{CG}}{\delta\phi} = 0 \longrightarrow \nabla_\mu J^\mu = 0$$

Hairy BH solutions

- Recall no-scalar-hair theorem: different theory $\nRightarrow$ different black holes:

  for the Cubic Galileon, $\phi = \phi(r) \Rightarrow g$ is Schwarzschild

[7] Babichev, Charmousis, Lehébel & Moskalets, JCAP09(2016)011

Hairy BH solutions

- Recall no-scalar-hair theorem: different theory $\not\Rightarrow$ different black holes:

for the Cubic Galileon, $\phi = \phi(r) \Rightarrow g$ is Schwarzschild

$\longrightarrow$ Introduce a linear time dependence[7]: $\phi = qt + \Psi(r)$

$\rightarrow$ Preserves spacetime symmetries

$\rightarrow$ Interesting cosmological dynamics

$\rightarrow$ Yields BH different from GR ones

[7] Babichev, Charmousis, Lehébel & Moskalets, JCAP09(2016)011

Rotating black hole

- (t, ϕ)-orthogonality ("circularity"): $g_{tr} = g_{t\theta} = g_{\phi r} = g_{\phi\theta} = g_{r\theta} = 0$

$$\implies g_{\mu\nu}(r,\theta) = \begin{pmatrix} -N^2 + B^2\omega^2 r^2 \sin^2\theta & 0 & 0 & -\omega B^2 r^2 \sin^2\theta \\ 0 & A^2 & 0 & 0 \\ 0 & 0 & A^2 r^2 & 0 \\ -\omega B^2 r^2 \sin^2\theta & 0 & 0 & B^2 r^2 \sin^2\theta \end{pmatrix}$$

[8] B. Kleihaus, J. Kunz, and E. Radu, Rotating Black Holes in Dilatonic Einstein-Gauss-Bonnet Theory, Phys. Rev. Lett. 106, 151104 (2011)

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$\longrightarrow$ Relevant for rotating stars with no meridional flow

$\longrightarrow$ Adapted to rotating BH, *e.g.* in dilatonic Einstein-Gauss-Bonnet[8]

[8] B. Kleihaus, J. Kunz, and E. Radu, Rotating Black Holes in Dilatonic Einstein-Gauss-Bonnet Theory, Phys. Rev. Lett. 106, 151104 (2011)

Equations and boundary conditions

- Inject circular metric and scalar ansatz into five (recombined) equations of motion:

$$\Delta_3 N = \mathcal{S}_N \; ; \qquad \Delta_2[NA] = \mathcal{S}_A \; ;$$

$$\Delta_2[NBr\sin\theta] = \mathcal{S}_B \; ; \qquad \Delta_3[\omega r \sin\theta] = \mathcal{S}_\omega \; ;$$

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- Set boundary conditions:
  - asymptotic flatness: $N_\infty = A_\infty = B_\infty = 1 \; ; \; \omega_\infty = 0 \; ; \; \partial_\theta \Psi_\infty = 0$
  - vanishing lapse: $N_\mathcal{H} = 0$ (causes degeneracy)
  - vanishing expansion: $\theta^{(l)}_\mathcal{H} = 0$
  - rotating horizon: $\omega_\mathcal{H} = \Omega_\mathcal{H}$
  - finite norm of scalar gradient $(\partial\phi)^2_\mathcal{H}$
  - no conical singularity: $A_{|_\Delta} = B_{|_\Delta}$

Spectral methods

- Discretization: consider the truncated decompositions onto standard basis functions

$$e.g.\quad A(r,\theta) = \sum_{i=0}^{N_r} \sum_{j=0}^{N_\theta} \tilde{A}_{ij}\, T_i(r) \cos(2j\theta)$$

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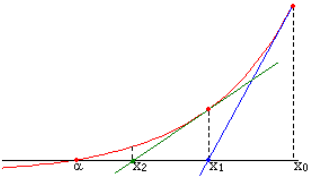
$\longrightarrow$ Exponential convergence of the series for smooth functions

$\longrightarrow$ Transforms any system of PDE's into a nonlinear *algebraic* system, solved with Newton-Raphson algorithm implemented in KADATH library[9]



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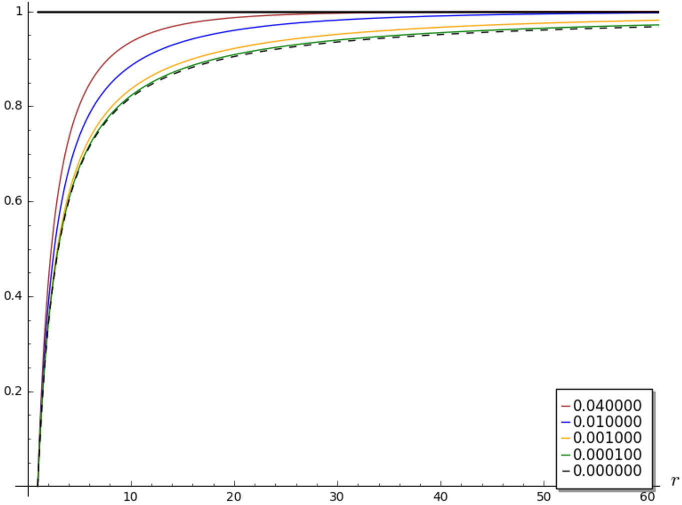
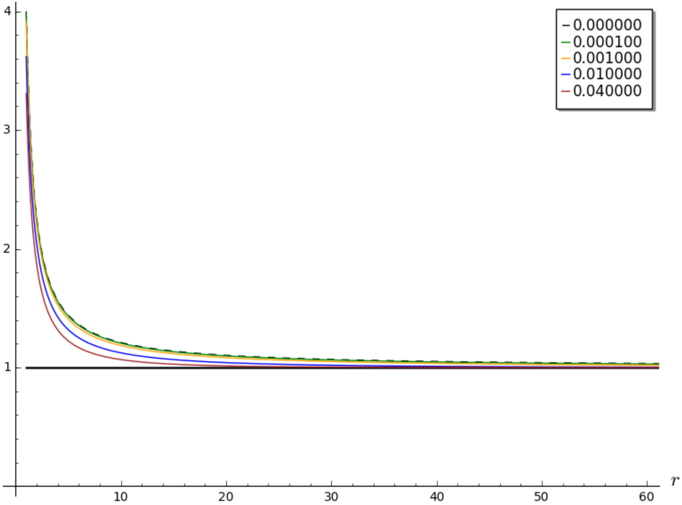
Static solutions with increasing nonminimal coupling



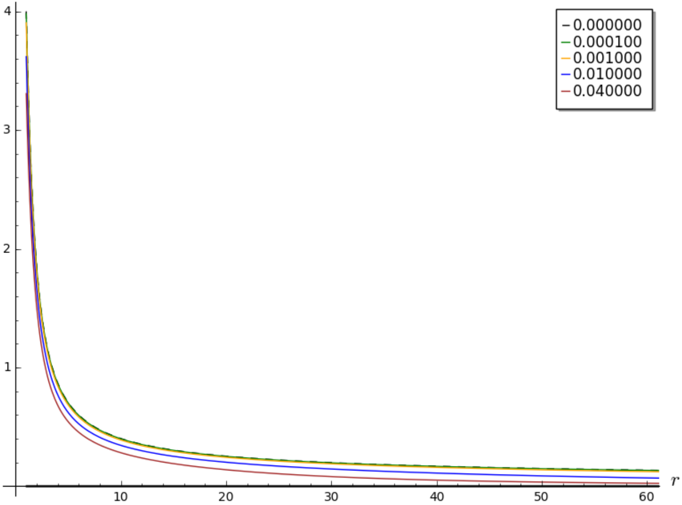
Radial profile of lapse N (varying $\alpha_1\alpha_2 = q^3 r_0 \frac{\gamma}{\zeta}$*)*

Static solutions with increasing nonminimal coupling



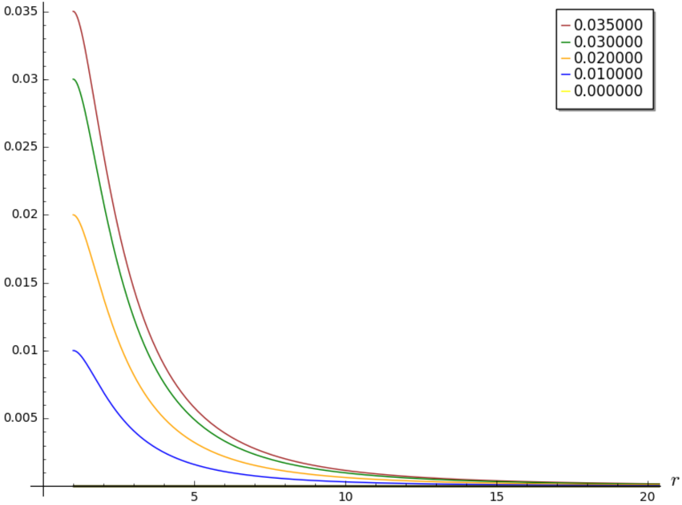
Radial profile of A (varying $\alpha_1\alpha_2 = q^3 r_0 \frac{\gamma}{\zeta}$*)*

Static solutions with increasing nonminimal coupling



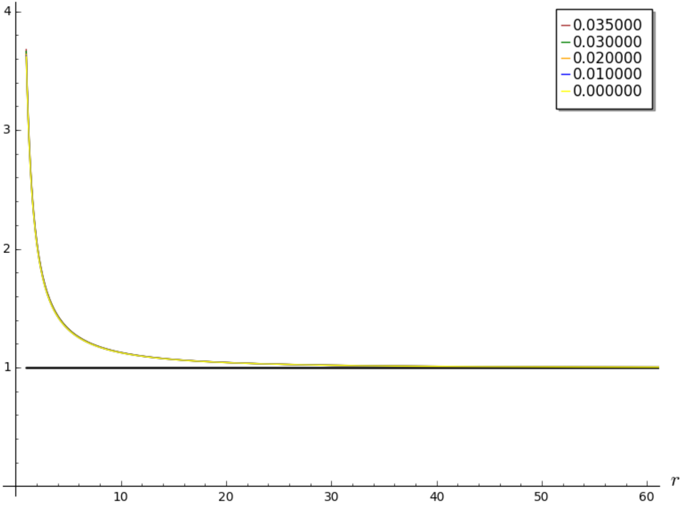
Radial profile of $N\partial_r\Psi$ (varying $\alpha_1\alpha_2 = q^3 r_0 \frac{\gamma}{\zeta}$)

Rotating solutions with constant coupling



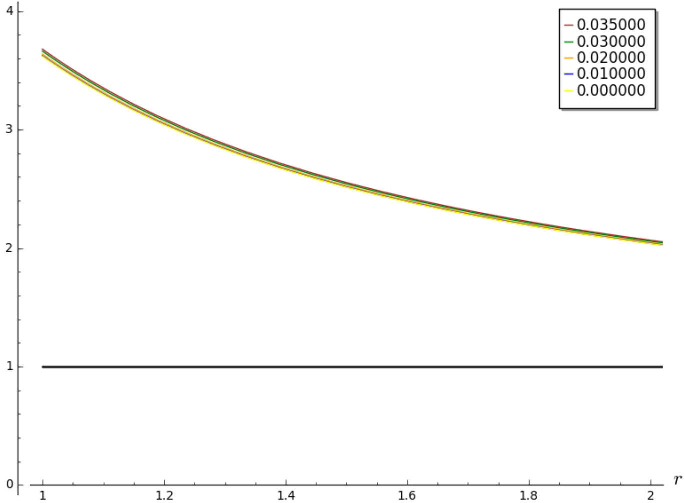
Radial profile of ω (varying $\bar{\Omega}_H = r_H \Omega_H$)

Rotating solutions with constant coupling



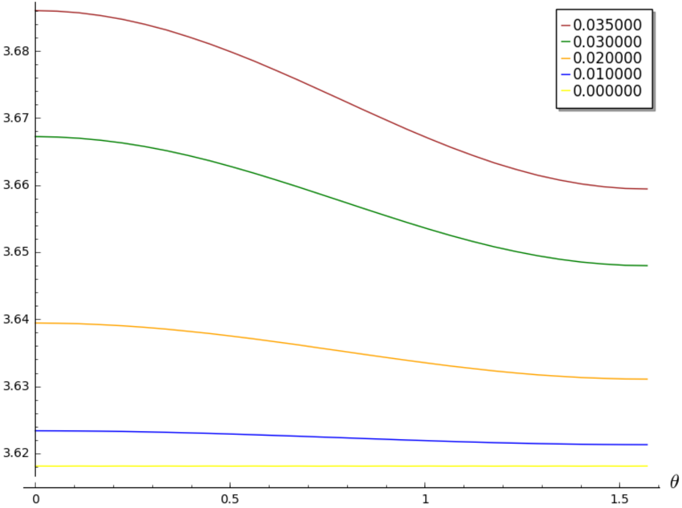
Radial profile of A (varying $\bar{\Omega}_H = r_H \Omega_H$)

Rotating solutions with constant coupling



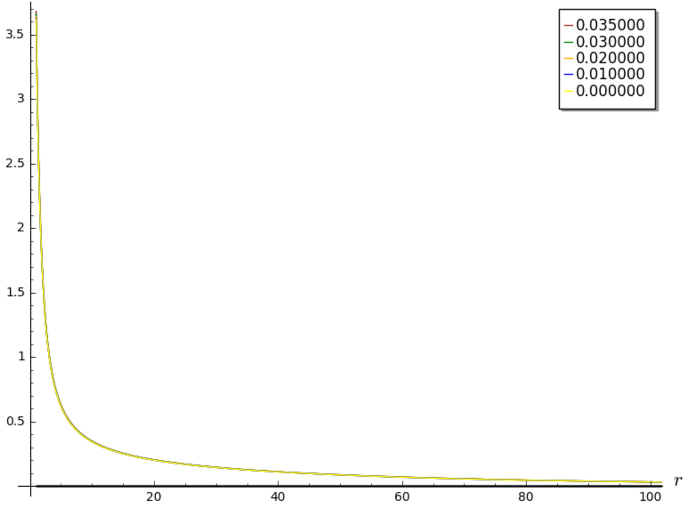
Zoom on radial profile of A (varying $\bar{\Omega}_H = r_H \Omega_H$*)*

Rotating solutions with constant coupling



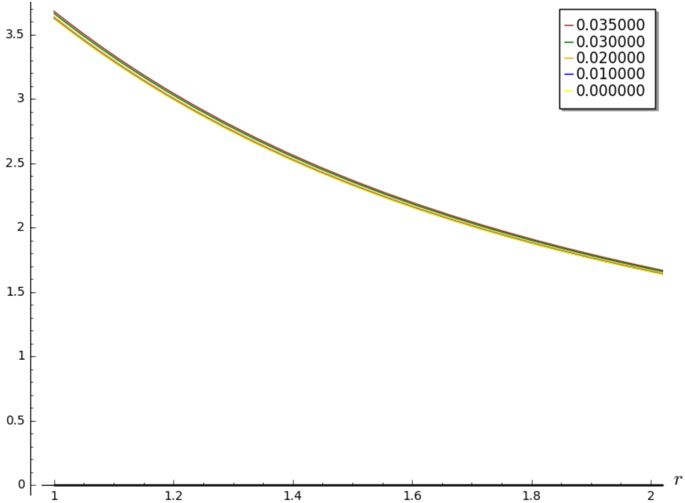
Angular profile of A at horizon (varying $\bar{\Omega}_H = r_H \Omega_H$)

Rotating solutions with constant coupling



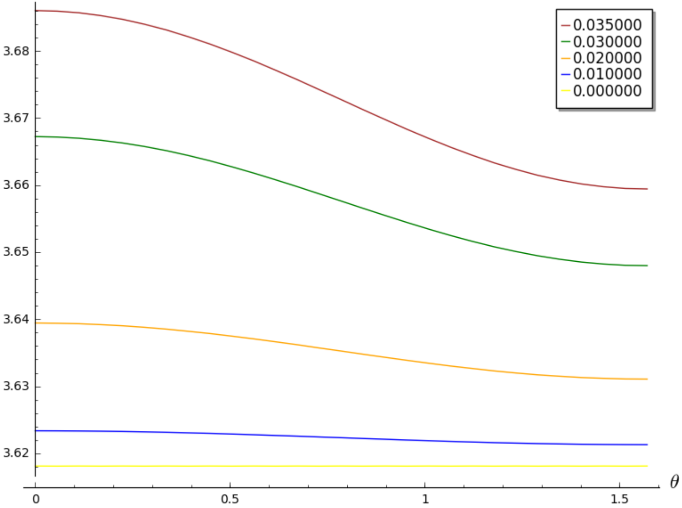
Radial profile of Z (varying $\bar{\Omega}_H = r_H \Omega_H$*)*

Rotating solutions with constant coupling



Zoom on radial profile of Z (varying $\bar{\Omega}_H = r_H \Omega_H$*)*

Rotating solutions with constant coupling



Angular profile of Z at horizon (varying $\bar{\Omega}_H = r_H \Omega_H$)

Next steps

- Determine the causal type of scalar gradient

- Search for an ergoregion

- Reach rapidly rotating branch

- Extract global quantities

- Try other boundary conditions for the scalar (on the norm of the current, or at infinity)

- Integrate null and timelike geodesics

Summary

- Observations of *Sgr A** provide new tests of GR in the strong field regime

- The Cubic Galileon is a well motivated modified theory that could account for deviations from GR

- Ongoing numerical computations of rotating BH should allow to predict the deviations from GR (*e.g.* from the integration of null geodesics)