Gravitational waveforms in scalar-tensor theories

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Rencontre du groupe de travail

"Tests de la relativité générale et théories alternatives"

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Laboratoire de l'Univers et de ses Théories

How to modify General Relativity ?

The scalar-tensor action

$$\boxed{S_{\mathrm{ST}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi} g^{\alpha\beta}\partial_\alpha\partial_\beta\phi\right] + S_m\left(\mathfrak{m}, g_{\alpha\beta}\right)}$$

- **Simple**: add one massless scalar field
- **Minimal**: scalar field coupled only to gravity, not to the matter
  - Weak-field tests: Solar System, binary pulsar tests
  - Strong constraints on the parameters of the theory

ST action in Einstein frame

Conformal transformation

$$\tilde{g}_{\mu\nu} = \varphi\, g_{\mu\nu}\,, \qquad \varphi = \frac{\phi}{\phi_0} \qquad \text{with } \phi_0 = \phi(\infty) = \text{cst}$$

$$\boxed{S_{\mathrm{ST}} = \frac{c^3\,\phi_0}{16\pi G}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\tilde{R} - \frac{3+2\omega(\phi)}{2\varphi^2}\tilde{g}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi\right] + S_m\left(\mathfrak{m}, \frac{\tilde{g}_{\alpha\beta}}{\varphi}\right)}$$

- ▷ Stationary BHs indistinguishable from GR
- ▷ **Still interesting**: Strong deviations from GR for neutron stars
- ▷ **Well-posed** initial value problem

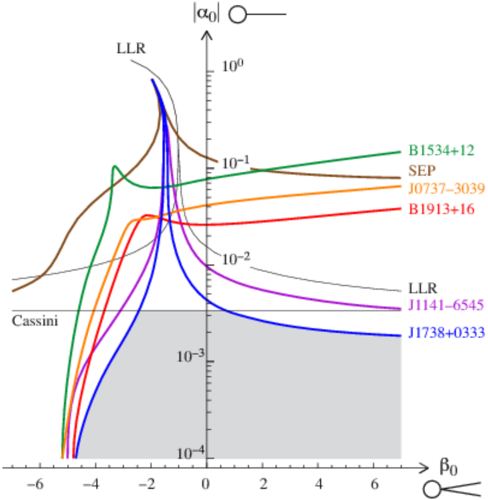
Violation of the Strong Equivalence Principle

- Incorporate the internal structure of compact, self-gravitating bodies
- **Eardley's approach**: masses depend on the scalar field $m_A(\phi)$

$$S_\mathrm{m} = -\sum_A \int \mathrm{d}t\, m_A(\phi)\, c^2 \sqrt{-g_{\alpha\beta}\frac{v_A^\alpha v_A^\beta}{c^2}}$$

- **Sensitivities**: $s_A = \left.\frac{\mathrm{d}\ln m_A(\phi)}{\mathrm{d}\ln\phi}\right|_0$
  - Neutron stars: $s_A \sim 0.2$ (depends on the equation of states)
  - Black holes: $s_A = 0.5$ (compacity M/R)
  - related to the scalar charge $\alpha_A \propto 1 - 2s_A$
- Higher order: $\tilde{\beta} \propto \left.\frac{\mathrm{d}^2\ln m_A(\phi)}{\mathrm{d}\ln\phi^2}\right|_0$

Binary pulsar constraints

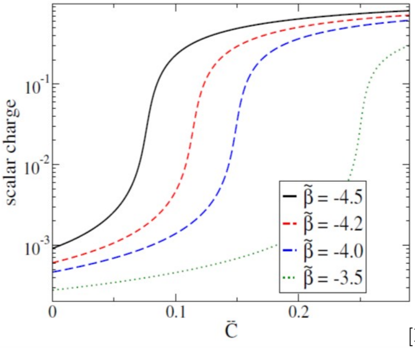


[Freire et al, 2012]

Binary coalescence in ST theories

Late inspiral - merger

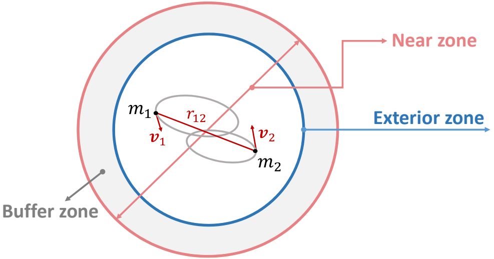
- ▷ Spontaneous scalarisation
- ▷ Dynamical scalarisation
  - Numerical results
  - Analytical approach using resummation tecniques



[Palenzuela et al, 2014]

Inspiral: post-Newtonian formalism

Isolated, compact, **slowly moving** and **weakly stressed** source



$$\boxed{\epsilon \equiv \frac{v_{12}^2}{c^2} \sim \frac{Gm}{r_{12}c^2} \ll 1}$$

Method of asymptotic matching

- Near zone : post-Newtonian expansion, $n\text{PN} = \mathcal{O}\left(\frac{1}{c^{2n}}\right)$
- Exterior zone : multipolar expansion in power of $\frac{r_{12}}{R}$

▶ **radiative moments** $\underset{\text{exp. in } 1/R}{\longleftarrow}$ **source moments** $\underset{\text{matching}}{\longrightarrow}$ **source**

INSPIRAL - TOWARDS 2PN WAVEFORMS

- Equations of motion at 3PN
- Tensor gravitational waveform to 2PN
- Scalar waveform to 1.5PN: **starts at −0.5PN**
- Energy flux to 1PN: **starts at −1PN**

$$\frac{\mathrm{d}E_{\text{dipole}}}{\mathrm{d}t} = \frac{4m\nu^2}{3rc^3}\left(\frac{G_{\text{eff}}m}{r}\right)^3(\alpha_2-\alpha_1)^2$$

- Phase evolution and amplitude to 2PN
  - Dipolar-driven regime : large separations or large scalar dipole
  - Quadrupolar-driven regime : most cases

Binary coalescence in ST theories

Inspiral - towards 2PN waveforms

- Equations of motion at 3PN (mPN formalism)
- Tensor gravitational waveform to 2PN (DIRE formalism)
- Scalar waveform to 1.5PN: **starts at −0.5PN** (DIRE)
- Energy flux to 1PN: **starts at −1PN** (DIRE)

$$\frac{\mathrm{d}E_{\text{dipole}}}{\mathrm{d}t} = \frac{4m\nu^2}{3rc^3}\left(\frac{G_{\text{eff}}\,m}{r}\right)^3 (\alpha_2-\alpha_1)^2$$

- Phase evolution and amplitude to 2PN
  - Dipolar-driven regime : large separations or large scalar dipole
  - Quadrupolar-driven regime : most cases

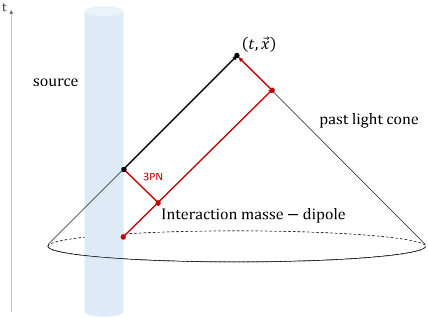
Potential issues when matching

- **Dimensional regularisation** to be implemented in both cases ?

3PN equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\text{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\text{PN}}}{c^5}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}^{\text{inst}}_{3\text{PN}}}{c^6}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}^{\text{tail}}_{3\text{PN}}}{c^6}}_{\text{cons, nonloc.}}$$

▷ **A scalar tail term: new ST effect at 3PN**



$$\boxed{L_{\text{tail}} \propto \frac{1}{c^6} I_i^{(2)}(t) \int_0^{+\infty} \mathrm{d}t\ \ln\left(\frac{\tau}{\tau_0}\right) I_i^{(3)}(t-\tau)}$$

3PN equations of motion

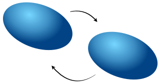
$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\text{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\text{PN}}}{c^5}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}^{\text{inst}}_{3\text{PN}}}{c^6}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}^{\text{tail}}_{3\text{PN}}}{c^6}}_{\text{cons, nonloc.}}$$

Dimensional regularisation

- **Ultraviolet divergences**: point-particle approximation
- **Infrared divergences**: PN solution at infinity
- **Nonlocal tail** term

Results

- BHs still indistinguishable from GR
- Conserved integrals of motion

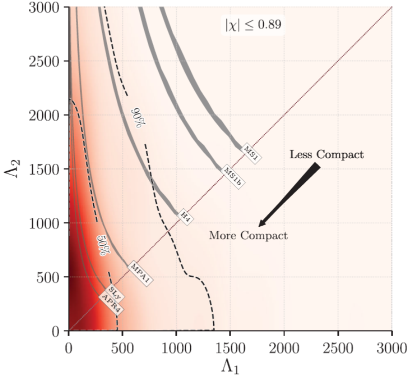


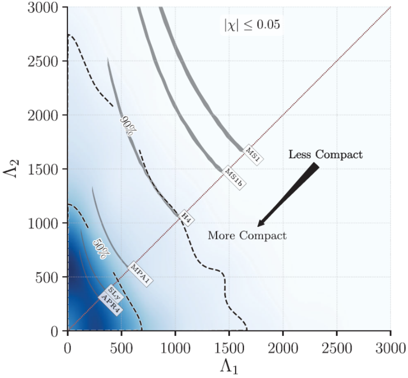
Reminder - General relativity

$$S_{\text{extended bodies}} = S_{\text{p.p.}} + \int \left(k_2 C^2_{\mu\nu\rho\sigma} + k_4 C^2 u^2 + \cdots\right) c\,\mathrm{d}s$$

- $\mathcal{Q}_{\mu\nu} = -\lambda_2\,\mathcal{E}_{\mu\nu}$
- starts at 5PN.
- Electric and magnetic-type tidal Love number: k_l^{E} and k_l^{B},

Constraints on the NS equation of state





The scalar tidal effect

$$\Delta S_{(fs)} = -\frac{1}{2}\sum_A \lambda_A^{(s)} \int \mathrm{d}s_A \, \left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)_A$$

- ▷ Time-varying scalar dipole moment $\propto$ external tidal field

$$\mathcal{Q}_\mu^{(s)} = -\lambda_{(s)}\mathcal{E}_\mu^{(s)}$$

- ▷ Scalar-type Love number $\lambda_{(s)}$

Consequence on scalar radiation

- ▷ 3PN order in the dynamics $\Delta\mathbf{a}_{(fs)} \sim \mathbf{a}_{(\mathrm{N})} \cdot \tilde{\lambda}^{(s)} x^3$ $\quad x = (m\omega)^{2/3}$
- ▷ small ST parameters but scales as $\left(\frac{R}{M}\right)^3$
- ▷ $\Delta\psi_{(fs)} \sim -\frac{1}{4\zeta(\alpha_1-\alpha_2)^2\eta x^{3/2}} \tilde{\Lambda}_{(s)} x^3 \Longrightarrow$ 2PN effect in the phase

Towards a full IMR waveform

- ▷ PN inspiral modelling
- ▷ Tidal effects - scalar-type Love number
- ▷ Effective one body formalism
- ▷ Late-inspiral effects
- ▷ Matching to numerical relativity