The sound of DHOST

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Beyond Horndeski theory

Horndeski theory

Vainshtein mechanism
(weak gravity)
=>
Recover of General Relativity
=>
theory passes observational tests

beyond
Horndeski theory,
DHOST/EST

Breaking of the
Vainshtein
mechanism inside
matter

Beyond Horndeski theory

Breaking of the Vainshtein mechanism inside matter

Neutron stars are different compared to those in General Relativity

One can use this to resolve some problems

Vainshtein mechanism for Horndeski theory

Newtonian order (Linearised metric)

$$\mathrm{d}s^2 = (-1+2\Phi)\,\mathrm{d}t^2 + (1+2\Psi)\,\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

- Φ and Ψ are the Newtonian potentials ($\Phi = \Psi$ for GR)
- Corrections in Horndeski theory: $$\frac{d\Phi}{dr} = \frac{G_N M}{r}\left[1 + 2\alpha^2\left(\frac{r}{r_\mathrm{V}}\right)^n\right]$$
- α is the coupling between the scalar and matter, n is model dependent parameter
- For solar mass object $r_\mathrm{V} \sim \mathcal{O}(0.1\ \ \mathrm{kpc}) \ \Rightarrow$ for $r \ll r_\mathrm{V}$ GR is restored (solar system tests are passed)

n.b. special case $\alpha = 0$ in particular theories (not Vainshtein screening but rather special solution where the scalar field is stealth)

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Vainshtein mechanism for Horndeski theory

Newtonian order (Linearised metric)

$$\mathrm{d}s^2 = (-1+2\Phi)\,\mathrm{d}t^2 + (1+2\Psi)\,\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

- In Horndeski theory GR solution is valid also inside matter
- In beyond Horndeski theory:

$$\frac{d\Phi}{dr} = \frac{G_{\mathrm{N}}M(r)}{r^2} + \frac{\Upsilon_1 G_{\mathrm{N}}}{4}\frac{d^2M(r)}{dr^2}$$
$$\frac{d\Psi}{dr} = \frac{G_{\mathrm{N}}M(r)}{r^2} - \frac{5\Upsilon_1 G_{\mathrm{N}}}{4r}\frac{dM(r)}{dr}$$

(Breaking of GR)

[Kobayashi et al'15]

$$M(r) \equiv 4\pi\int_0^r r^2\rho(r)dr$$

- Υ_1 and Υ_2 are non-zero in beyond Horndeski theory

Some local constraints on beyond Horndeski parameters

- $\Upsilon_1 > -2/3$ for a sensible stellar profile
- $-0.22 < \Upsilon_1 < 0.027$

Chandrasekhar mass of dwarf stars

Consistency of the minimum mass for hydrogen burning

Static configurations

What happens in dynamics?

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Dynamics in DHOST

It is difficult to solve nonlinear PDEs

$$\cdots \quad + G_{4XX}\Big\{ -\frac{1}{2}[(\Box\phi)^2 - (\nabla_\alpha\nabla_\beta\phi)^2]\nabla_\mu\phi\nabla_\nu\phi + 2\nabla_\lambda X\nabla^\lambda\nabla_{(\mu}\phi\nabla_{\nu)}\phi$$
$$- \nabla_\lambda X\nabla^\lambda\phi\nabla_\mu\nabla_\nu\phi - 2\nabla_{(\mu}X\nabla_{\nu)}\phi\Box\phi - \nabla^\alpha\phi\nabla_\alpha\nabla_\mu\phi\nabla^\beta\phi\nabla_\beta\nabla_\nu\phi$$
$$- \frac{1}{2}g_{\mu\nu}(X[(\Box\phi)^2 - (\nabla_\alpha\nabla_\beta\phi)^2] + 2\nabla^\alpha X\nabla^\beta\phi\nabla_\alpha\nabla_\beta\phi - \nabla_\alpha X\nabla^\alpha\phi\Box\phi \quad \ldots = 0$$

Assuming quasi-staticity

so that some time-derivatives drops off the EOMs

Dynamics in DHOST

$$ds^2 = -[1 + 2\Phi(t,r)]dt^2 + [1 + 2\Psi(t,r)](dr^2 + r^2 d\Omega^2)$$

$$\Phi' = \frac{G_{\rm N} M}{r^2} + \frac{\Upsilon_1 G_{\rm N}}{4} M'' \qquad\qquad M = 4\pi \int r^2 \, dr \rho(t,r)$$

Assumptions:

$$\boxed{\text{the gravitational potentials are small, } (\Phi,\Psi) \ll 1\text{, and the gradient of the scalar field is small with respect to its time derivative, } (\varphi'/\dot{\varphi})^2 \ll 1,}$$

- quasi-staticity,
- Vainshtein regime, i.e., the canonical kinetic term — normally assumed to be present in the action — is subdominant compared to the non-linear terms.

Implied by a weak-field approximation, and should be satisfied in configurations where the curvature remains small, such as the Solar System.
Technical assumption to neglect terms $\mathcal{O}[(\varphi'/\dot{\varphi})^4]$ with respect to $\mathcal{O}[(\varphi'/\dot{\varphi})^2]$.

Dynamics in DHOST

$$ds^2 = -[1 + 2\Phi(t,r)]dt^2 + [1 + 2\Psi(t,r)](dr^2 + r^2 d\Omega^2)$$

$$\Phi' = \frac{G_\mathrm{N} M}{r^2} + \frac{\Upsilon_1 G_\mathrm{N}}{4} M'' \qquad M = 4\pi \int r^2 \, dr \rho(t,r)$$

Assumptions:

- the gravitational potentials are small, $(\Phi, \Psi) \ll 1$, and the gradient of the scalar field is small with respect to its time derivative, $(\varphi'/\dot{\varphi})^2 \ll 1$,
- quasi-staticity,
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Quasi-staticity condition is satisfied when the sound in matter propagates much slower than the spin-2 and spin-0 perturbations.

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Dynamics in DHOST

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Can be lifted, so that one does not assume that the Vainshtein regime is on.

$$\Phi' = \frac{G_\mathrm{N} M}{r^2} + \frac{\Upsilon_1 G_\mathrm{N}}{4} M'' + \alpha_1 \dot{\varphi}^2 r^2$$

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Hydrodynamics in DHOST

$$\Phi' = \frac{G_\mathrm{N} M}{r^2} + \frac{\Upsilon_1 G_\mathrm{N}}{4} M'' + \alpha_1 \dot{\varphi}^2 r^2 \quad \text{modified Newtonian potential}$$

$$\frac{\partial(\rho\vec{v})}{\partial t} + (\vec{v}\cdot\vec{\nabla})(\rho\vec{v}) = -\rho\vec{\nabla}\Phi - \vec{\nabla}P \quad \text{Euler equation}$$

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho\vec{v}) = 0 \quad \text{continuity equation}$$

ρ is density and P is pressure

Isentropic perturbations:

$$P = P_0 + \delta P,$$

$$\rho = \rho_0 + \delta\rho.$$

$$\frac{\delta P}{\delta \rho} = \left.\frac{\partial P}{\partial \rho}\right|_S$$

linearise in $\delta\rho$ and combine equations

The sound of DHOST

$$\boxed{\ddot{\delta\rho} - \left(\left.\frac{\partial P}{\partial \rho}\right|_S + 4\pi G_{\rm N} \frac{\Upsilon_1}{4} \rho_0 r^2 \right) \delta\rho'' = 0}$$

$$c^2_{\rm DHOST} = \left.\frac{\partial P}{\partial \rho}\right|_S + 4\pi G_{\rm N} \frac{\Upsilon_1}{4} \rho_0 r^2$$

Short-wavelength (or large distance) limit:

$$\frac{r}{\lambda} \gg 1, \quad \frac{r}{\lambda} \gg \frac{1}{\Upsilon_1} \frac{M}{\rho_0 r^3}.$$

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The sound of DHOST: implications

$$c^2_{\mathrm{DHOST}} = \left.\frac{\partial P}{\partial \rho}\right|_S + 4\pi G_{\mathrm{N}} \frac{\Upsilon_1}{4} \rho_0 r^2$$

- The sound speed strongly depends on the background (cosmological) solution
- The speed of sound in media depends on the distance to the center of the mass distribution (or to the source of the perturbation, if one has in mind a homogeneous medium).
- The absolute value of the sound speed grows as distance squared
- The planar limit of a spherical wave (when r goes to infinity) does not coincide with the result for a planar wave (discontinuity?).

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The sound of DHOST: constraints

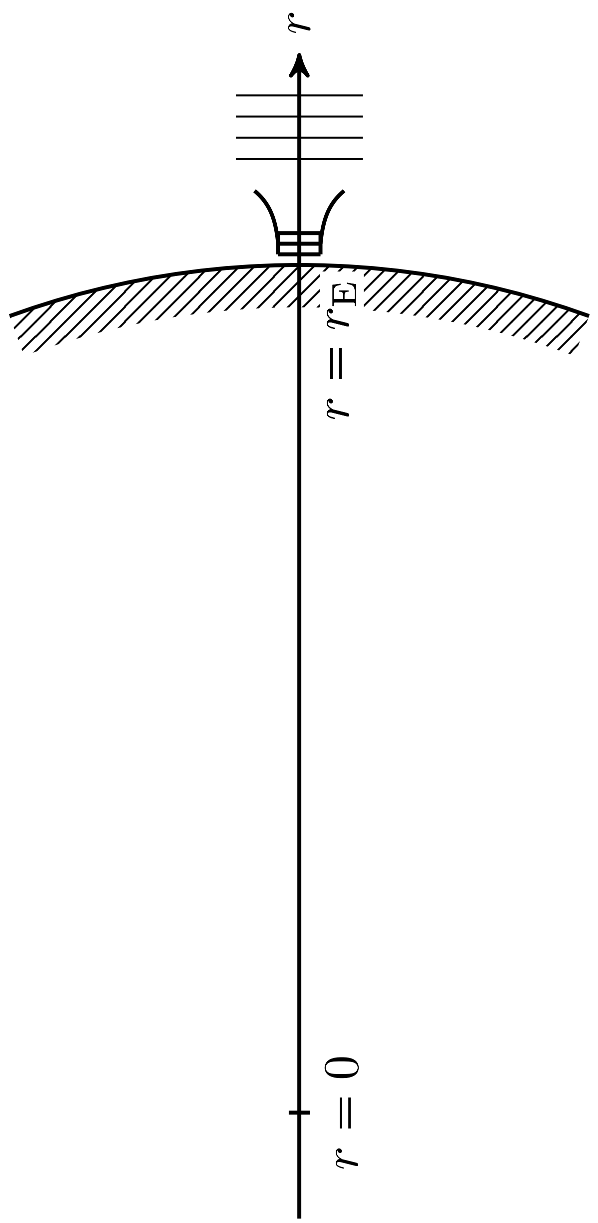
Dust in a homogeneous universe

$$c^2_{\rm DHOST} = 4\pi G_{\rm N}\frac{\Upsilon_1}{4}\rho_0 r^2$$

Negative Υ_1 are excluded.

The sound of DHOST: constraints

Sound waves in the air



$$c^2_{\mathrm{DHOST}} = \left.\frac{\partial P}{\partial \rho}\right|_S + 4\pi G_\mathrm{N} \frac{\Upsilon_1}{4} \rho_0 r^2$$

$$c^2_{\mathrm{GR}} = \frac{\gamma R T}{\mathcal{M}}$$

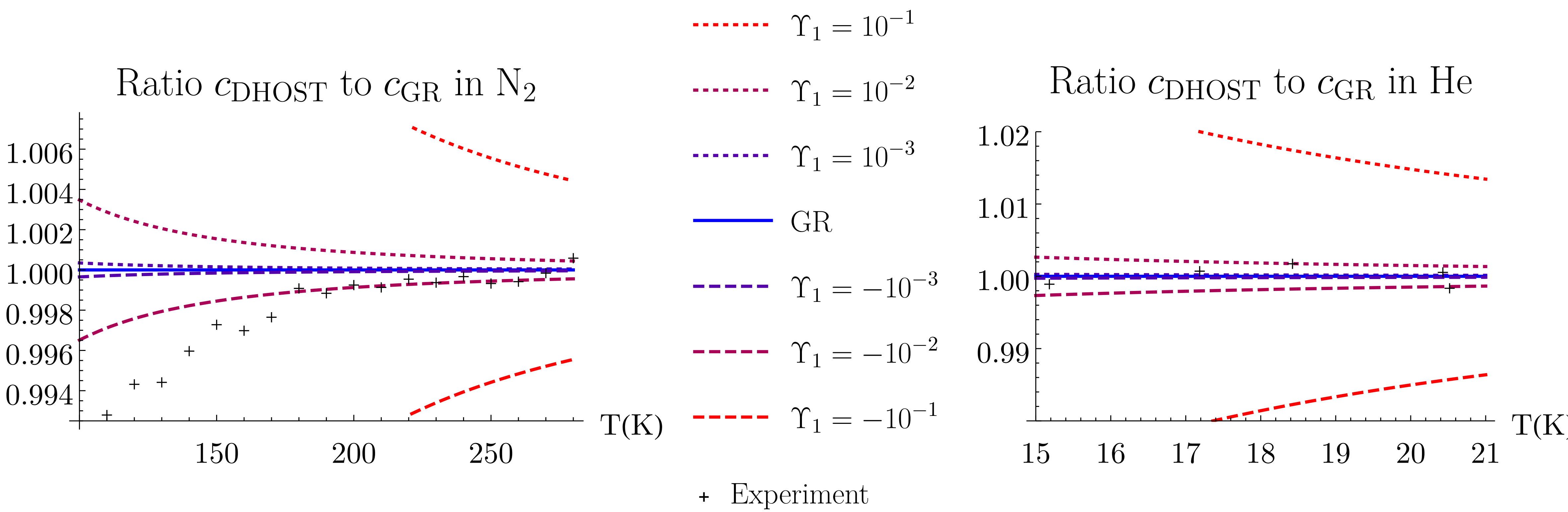
In the appropriate regime of temperature and pressure, the typical difference between the measured speed of sound and the prediction of the ideal gas model is of order 0.2 %, or even less.

The sound of DHOST: constraints

Sound waves in the air

$$c^2_{\mathrm{DHOST}} = \left.\frac{\partial P}{\partial \rho}\right|_S + 4\pi G_{\mathrm{N}} \frac{\Upsilon_1}{4} \rho_0 r^2$$

Contrasting to experiments $\quad |\Upsilon_1| \lesssim 10^{-2}$



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Conclusions

- DHOST/EST theories, considered as dark energy candidates, affect local physics.
- Speed of sound in media changes, altering even present day non-relativistic hydrodynamics.
- As a consequence, some parameters (negative Υ_1) are excluded.
- The speed of sound even in the atmosphere of the Earth, is significantly affected in a generic DHOST theory, leading to $|\Upsilon_1| \lesssim 10^{-2}$
- Only exact spherical symmetry
- Very low energy cutoff scale?

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