Boson stars

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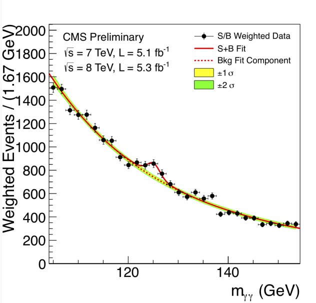
Generalities

What is a boson star ?

- Localized configuration of a self-gravitating complex scalar field.
- Introduced in the 1960s by Bonazolla, Pacini, Kaup and Ruffini.
- Corresponds to spin-0 particle $\Longrightarrow$ *boson*.
- At least one scalar field in nature :*Higgs boson*.



Why study boson star ?

- the simplest self-gravitating object with matter.
- scalar field appear in primordial cosmology (inflaton).
- Possible candidate for dark matter (rotation curves of galaxies).
- Alternative to black holes : can accommodate a big mass in a small radius without an event horizon.

Boson star model

- Scalar field has a $U(1)$ symmetry:

$$\Phi \longrightarrow \Phi \exp(i\alpha).$$

- The Lagrangian of the field is

$$\mathcal{L}_M = -\frac{1}{2}\left[g^{\mu\nu}\nabla_\mu\bar{\Phi}\nabla_\nu\Phi + V\left(|\Phi|^2\right)\right].$$

  V is a potential (for a free field $V = m^2/\hbar^2 \, |\Phi|^2$)
- The Lagrangian of gravity is

$$\mathcal{L}_g = \frac{1}{16\pi}R$$

The variation of the equation gives the Einstein-Klein-Gordon system.

Orders of magnitude

Free-field potential $V(|\Phi|^2) = \frac{m^2}{\hbar^2}|\Phi|^2$ **: mini-boson stars**

- Compton wavelength : $\frac{\hbar}{mc}$.
- Associated mass scale : $M = \frac{m_p^2}{m}$.
- Planck mass : $m_p = \sqrt{\hbar c/G}$.

Scalar field mass	m_{Higgs}	m_{proton}	m_{electron}
Boson star mass (solar mass)	$1 \cdot 10^{-21}$	$1 \cdot 10^{-19}$	$2 \cdot 10^{-16}$

Interaction potential $V(|\Phi|^2) = \frac{m^2}{\hbar^2}|\Phi|^2\left(1 + 2\pi\Lambda|\Phi|^2\right)$

- True coupling constant : $\lambda = 4\pi G m^2 \Lambda$.
- New mass scaling : $M \propto \lambda^{1/2}\frac{m_p^3}{m^2}$ (big for $\lambda \approx 1$).

Scalar field mass	m_{Higgs}	m_{proton}	m_{electron}
Boson star mass (solar mass)	$1 \cdot 10^{-4}$	2	$5 \cdot 10^{6}$

Numerical models

Ansatz for the field

$\Phi = \phi \exp\left[i\left(\omega t - k\varphi\right)\right]$

- $U(1) \Longrightarrow$ the action does not depend on (t, φ).
- ϕ and the metric fields depend only on (r, θ).
- k and ω appear as parameters in the equations.
- k is an integer and ω a real number.
- $k = 0$ corresponds to spherically symmetric configurations.

Asymptotic behavior

- Asymptotically, ϕ obeys

$$\Delta_3\phi - \frac{k^2}{r^2 \sin^2\theta}\phi - \left(1-\omega^2\right)\phi = 0$$

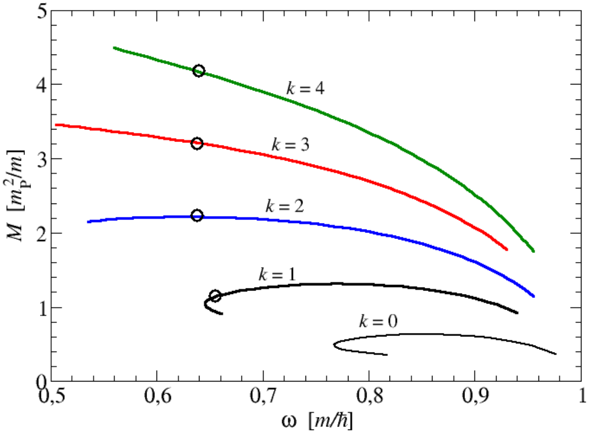
- It follows that the field is localized if and only if $\omega < 1$.
- When $\omega \to 1$, $\phi \to 0$ and its size tends to infinity.
- The most relativistic boson stars are obtained for smaller ω.

Numerical solver : KADATH library www.kadath.obspm.fr

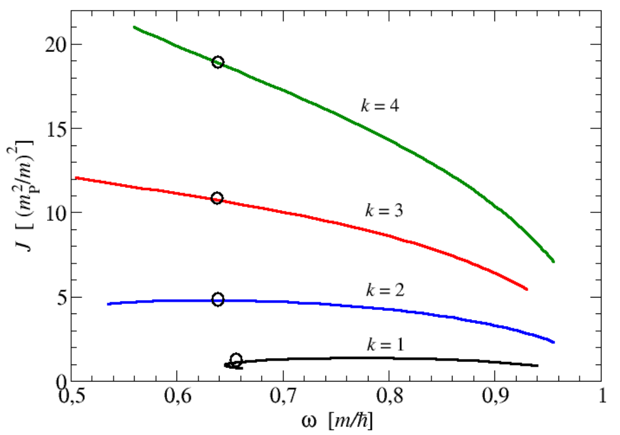
Properties

- 5 unknown fields.
- 5 coupled non-linear partial differential equations (elliptic-like).
- Kadath implements multi-domains *spectral methods*.
- Here uses several spherical domains.
- Chebyshev polynomials in r and trigonometrical functions in θ.
- Equations are discretized by the *weighted residual method*.
- The non-linear system is solved using a *Newton-Raphson iteration*.
- The first solution for each value of k is the most difficult to find.
- Once it is known, ω is slowly varied to construct a sequence.

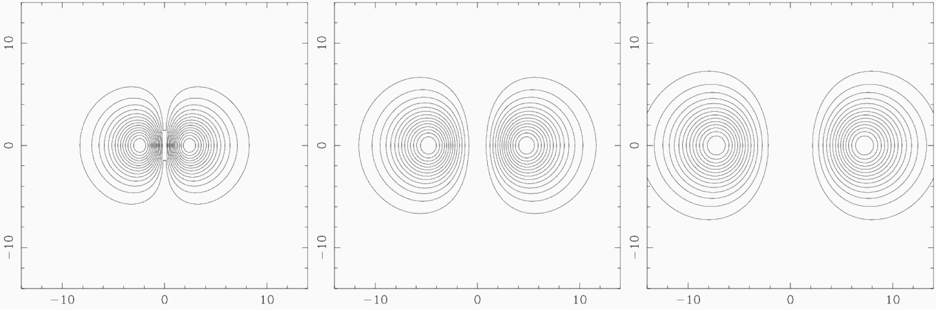
Mass



Momentum

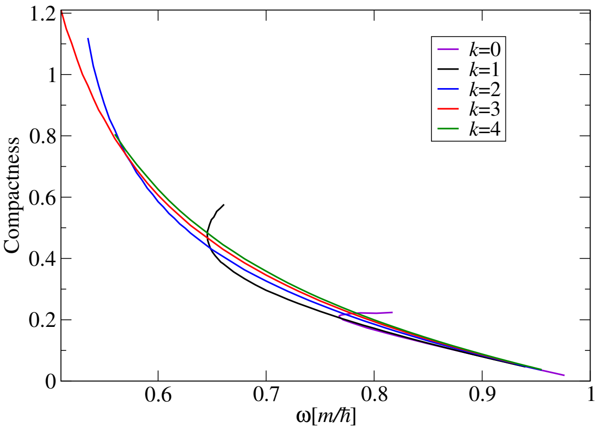


Field contours ; variation with k



Configuration of ϕ for $\omega = 0.8$ and $k = 1$, $k = 2$ and $k = 3$.

Compactness



Photon orbits

Effective potential method

- Metric in quasi-isotropic coordinates :

$$g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + A^2\left(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2\right) + B^2r^2\sin^2\theta\left(\mathrm{d}\varphi + \beta^\varphi\mathrm{d}t\right)^2$$

- Light rings : closed circular orbits of photons.

$$U^\alpha = \left(U^t, U^r, 0, U^\varphi\right)$$

- Two conserved quantities $U_\alpha\left(\partial_t\right)^\alpha = -E$ and $U_\alpha\left(\partial_\varphi\right)^\alpha = L$.
- Null geodesics $U_\alpha U^\alpha = 0$ leads to $\left(U^r\right)^2 + V_{\mathrm{eff}}\left(r\right) = 0$

$$V_{\mathrm{eff}} = \frac{1}{A^2}\left[-\frac{\left(\beta^\varphi L + E\right)^2}{N^2} + \frac{L^2}{B^2r^2}\right]$$

Circular orbits

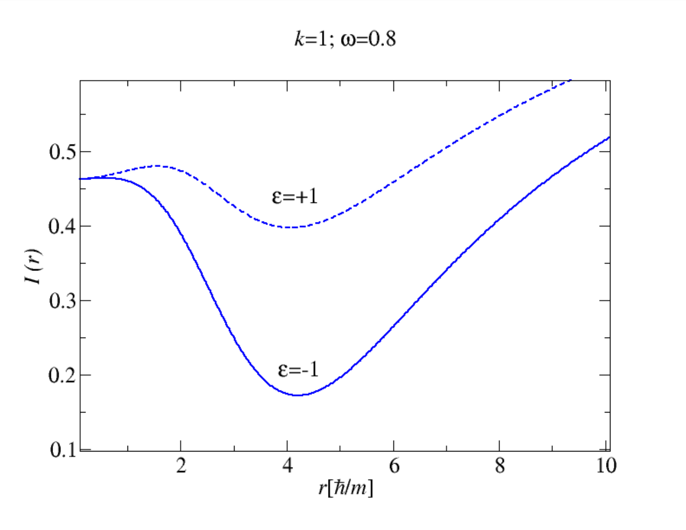
- Circular orbits require $V_{\text{eff}} = 0$ and $\partial_r V_{\text{eff}} = 0$
- First condition implies

$$\frac{E}{L} = -\beta^{\varphi} + \epsilon \frac{N}{Br} \quad ; \quad \epsilon = \pm 1$$

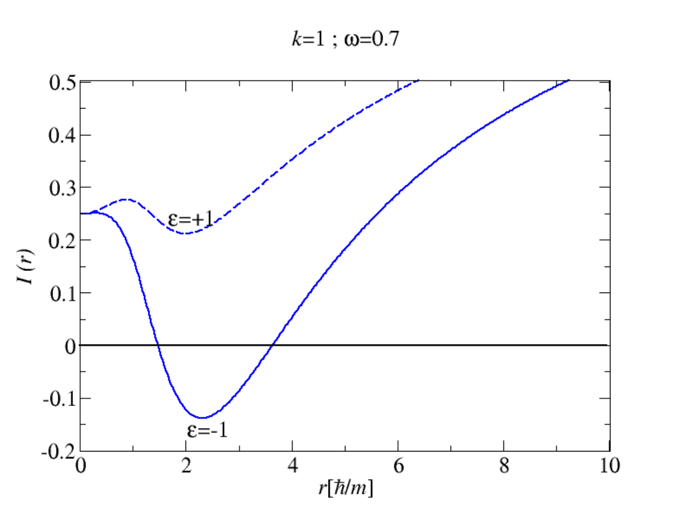
- Contrary to the massive case only the ratio E/L is constrained.
- Second condition reduces to finding the zeros of

$$I(r) = \left(\epsilon \frac{\partial_r \beta^{\varphi}}{NB}\right) r^2 + \left(\frac{\partial_r B}{B^3} - \frac{\partial_r N}{NB^2}\right) r + \frac{1}{B^2}$$

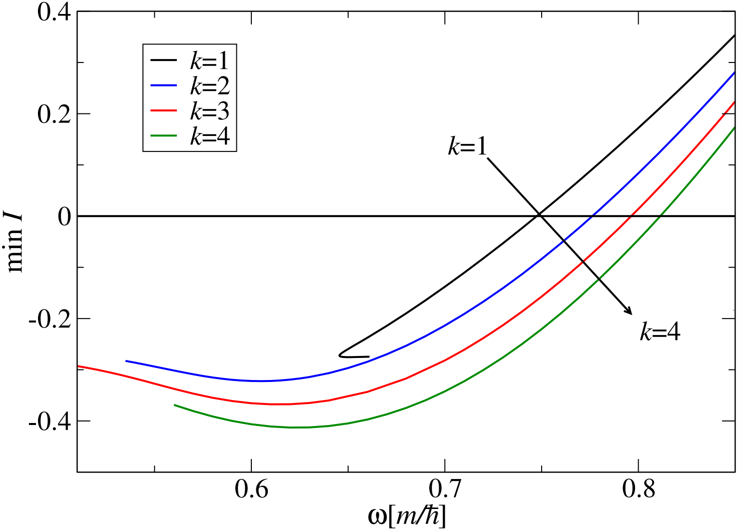
$I\,(r)$ without light rings



$I\,(r)$ with light rings



Existence of light rings



Properties

The orbital frequency is given by $\frac{\mathrm{d}\varphi}{\mathrm{d}t} = -\frac{N}{Br} - \beta^{\varphi}$.

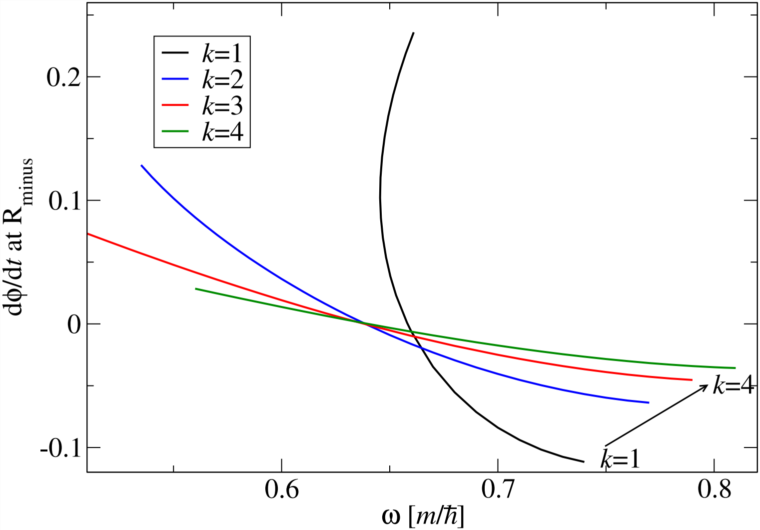
Black hole case

- Two light rings for Kerr black holes.
- One prograde and one retrograde.
- Both unstable.

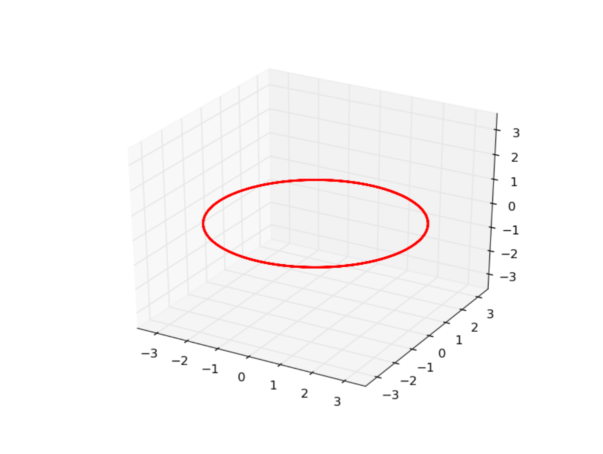
Boson star case

- Two light rights for relativistic enough configurations.
- The outer one is unstable and retrograde.
- The outer one is stable (and changes type).

Orbital frequency of the inner light ring

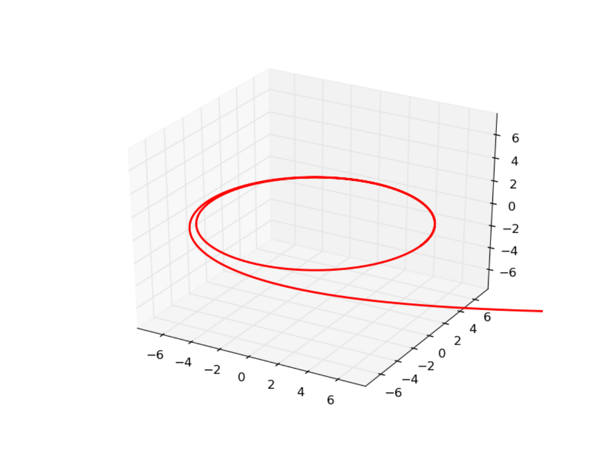


Inner light ring



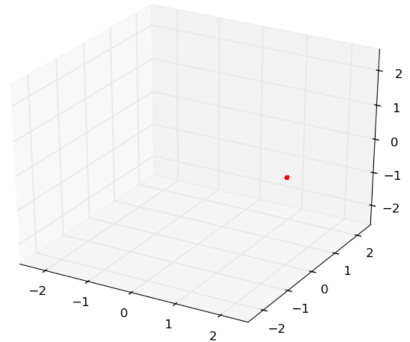
Direct integration with Gyoto ; $k = 2$ and $\omega = 0.7$.

Outer light ring



Direct integration with Gyoto ; $k = 2$ and $\omega = 0.7$.

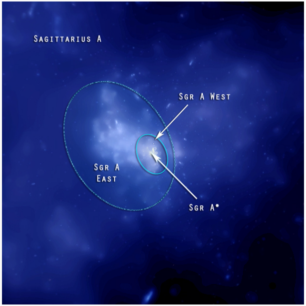
Light point

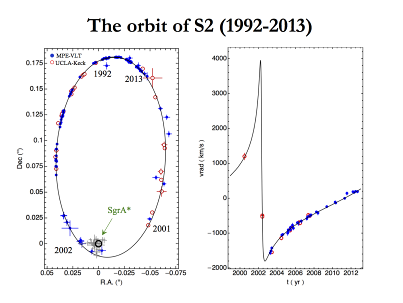


Direct integration with Gyoto ; $k = 2$ and $\omega = 0.6387$.

Observational perspectives

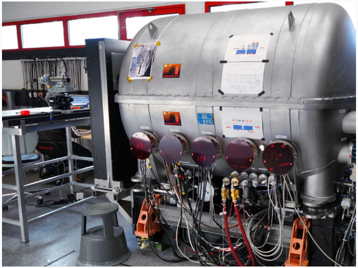
Galactic center





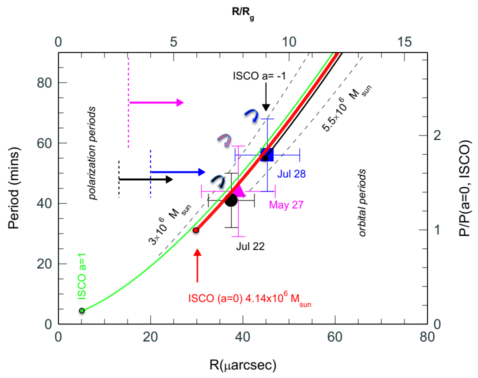
If it is a black hole, it is the one with the largest angular size on the sky.

Gravity instrument



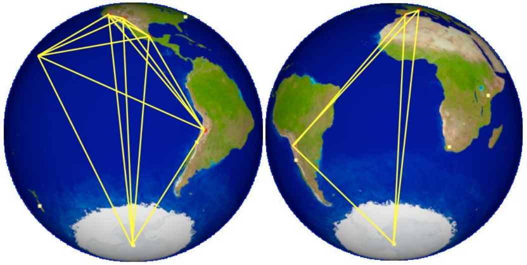
- Interferometer at VLT.
- Resolution $\approx 10\mu\text{as}$.

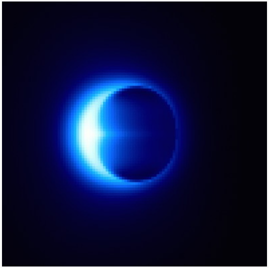
Hot spots close to the ISCO



- Material very close the the ISCO.
- But almost undistinguishible from the boson star case.

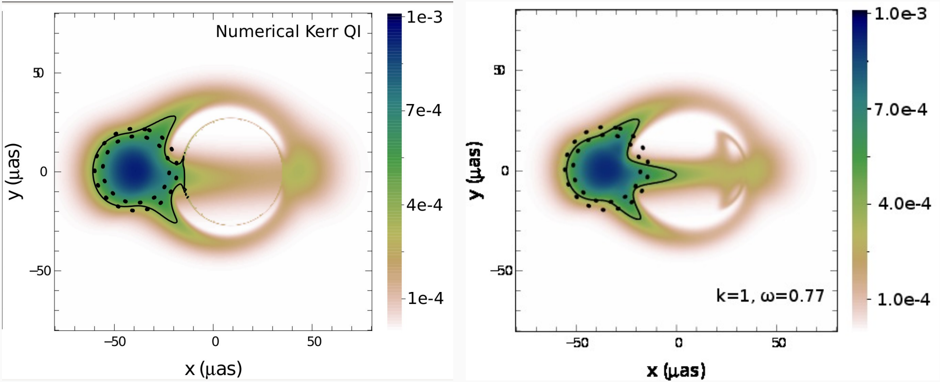
Event Horizon Telescope





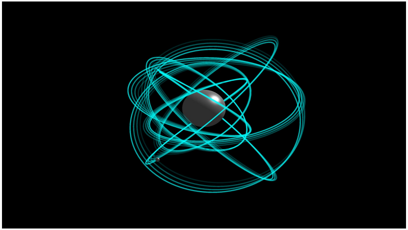
- Array of radio-telescopes.
- Interferometry with very long baseline
- Resolution about the size of Sgr A* $\approx 10\mu\mathrm{as}$.
- First observations consistent with disk models.

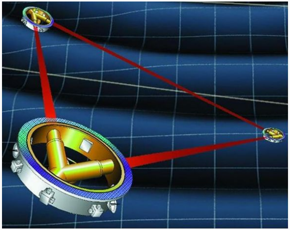
Accretion disks : BH vs BS



Almost no difference

Use of gravitational waves : EMRI





- Stellar mass object captured by a supermassive object.
- Emits GW in the LISA-band.
- Small object maps the spacetime around the massive one.

Kerr or not Kerr ?

Testing the uniqueness theorem

- For Kerr, only M and S are independent.
- For instance $Q = -S^2/M$
- Moments are encoded in the GW.
- *Test* : assume they are independent and check the Kerr-ness.
- For 1 year EMRI : $\Delta \left(Q/M^3\right) \approx 10^{-4} - 10^{-2}$.

More detailed tests

- Previous test assumes nothing about the supermassive object.
- Can try to detect features associated with boson stars.
- In the spherical case, *Kesden et al.* study the effect of the absence of horizon.
- No work with rotation.

Conclusions

- Boson stars are one of the simplest alternative to black holes.
- Formation scenario uncertain.
- Numerical models have been computed.
- Observational constraints from the galactic center are coming up but no strong constraints yet.
- Detailed tests will be provided by gravitational waves (LISA).