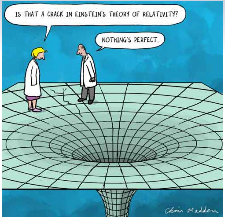

Scalar Tensor Theories for Dark Energy

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and also M. Crisostomi, D. Steer, F. Vernizzi (here)...

Motivations

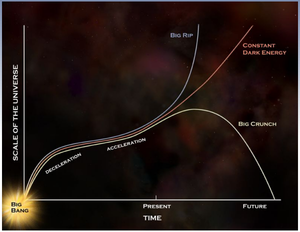
General Relativity is a beautiful story...

- Space-time is beautifully described in terms of Lorentz geometry in total agreements with observations...

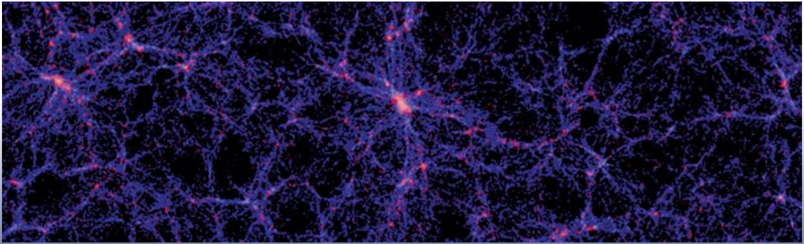
... But can it be the ultimate theory of space-time ?

- Very short (Planck) scale : singularities ?
  - Big bang singularity at the origin of the universe
  - Black hole singularity behind the horizon
  - $\Longrightarrow$ Breakdown of the theory ? Need of quantization ?
- Also, inflation is already a modification of general relativity...
- What about dark matter ? Is it a modification of gravity ?
- Very large (cosmological) scale : dark energy ?
  - Accelerated expansion of the universe leads to troubles
  - Signature of a modification of gravity laws ?

Overview



- Modification of gravity at large scale : account for dark energy
  - Construction of DHOST theories
  - Physical consequences and tests



Robustness of gravity

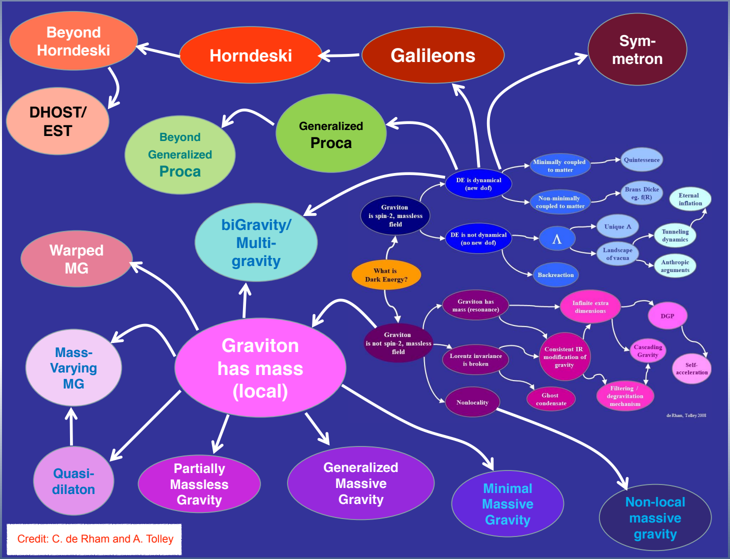
Uniqueness of gravity + cosmological constant :

- Hyp.1 : Space-time is of dimension 4 (+ symmetries)
- Hyp.2 : Gravity is described by a metric (spin 2) only
- Hyp.3 : Euler-Lagrange equations are second order
- Lovelock theorem : Einstein gravity + Cosmological constant

$$S[g_{\mu\nu}] = \frac{c^4}{16\pi G_N} \int d^4x \sqrt{|g|}\,(R - 2\Lambda)$$

No much room available...

Well... imagination has no limits !



Try to modify gravity

It is more likely that space-time is 4-dimensional

However, we assume that

- Gravity comes with a scalar field ϕ : a fifth force which is expected to be responsible for dark energy $\Longrightarrow$ Scalar-Tensor theories
- Equations of motion are not necessarily of second order

Motivations

- Adding a scalar is the simplest case, but there are more complicated scenarii (massive gravity, bi-gravity, vectors, extra-dimensions...)

  $\rightarrow$ Most of them contain a scalar mode...
- Higher order equations because the dynamics of gravity is governed by an action with second order derivatives : $\partial_\mu \partial_\nu g_{\rho\sigma} \rightarrow \partial_\mu \partial_\nu \phi$

How do we construct the most general viable scalar-tensor theories ?

$$S[g_{\mu\nu},\phi] = \int d^4x\sqrt{|g|}\Big[G_1(\phi,X) + G_2\Box\phi + G_3\,R$$
$$+C_2^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma} + C_3^{\mu\nu\rho\sigma\alpha\beta}\phi_{\mu\nu}\phi_{\rho\sigma}\phi_{\alpha\beta} + C_4^{\mu\nu\rho\sigma}\phi_{\mu\nu}R_{\rho\sigma} + \cdots\Big]$$

where any "free" tensor in this expansion depends on ϕ and $X \equiv \phi_\mu\phi^\mu$.

Let us see with a simple example from classical mechanics what happens when we have higher derivatives...

Toy-model : Higher order particle

Dynamics of a higher order point like particle $q(t)$

Action : $$S[q(t)] = \frac{1}{2}\int dt\,\left(\dot{q}^2 - \omega^2 q^2 + \alpha \ddot{q}^2\right)$$

EoM : $$\ddot{q} + \omega^2 q - \alpha \ddddot{q} = 0\,.$$

Degrees of Freedom

- One needs 4 initial conditions : $q(0)$, $\dot{q}(0)$, $\ddot{q}(0)$ and $\dddot{q}(0)$
- Hence, two degrees of freedom : two particles propagate !

Ostrogradski ghost

- The energy of the system is unbounded from below
- The extra DoF is called a ghost : there is an instability !

Toy-model : Degenerate higher order

Coupling two particles $q(t)$ and $X(t)$

$$S[q,X] = \frac{1}{2}\int dt\, \left(\dot{q}^2 - \omega^2 q^2 + \alpha \ddot{q}^2 + \dot{X}^2 - \omega^2 X^2 + 2\alpha\ddot{q}\dot{X}\right)$$

EoM : $\ddot{q} + \omega^2 q - \alpha \ddddot{q} - \alpha \dddot{X} = 0$ and $\ddot{X} + \omega^2 X + \alpha\dddot{q} = 0\,.$

How many Degrees of Freedom ?

- Not easy to see which initial conditions are necessary...
- In general, such theory propagates 3 DOF : X, q and the ghost !

Evading Ostrogradski instability

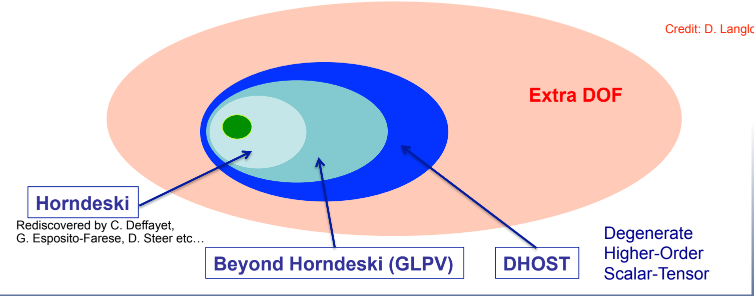
- Here, the theory is DEGENERATE $\Longrightarrow$ NO GHOST !

$$S[q,X] = \frac{1}{2}\int dt\, \left(\dot{Q}^2 + \dot{q}^2 - \omega^2 q^2 - \omega^2 X^2\right)\,, \quad Q = X + \alpha\dot{q}$$

Scalar-Tensor theories

Classification of scalar-tensor theories with no-ghost : DHOST Theories

$$S[g_{\mu\nu},\phi] = S_{EH}[g_{\mu\nu}] + \int d^4x\, L[\phi, \partial_\mu\phi, \partial_\mu\partial_\nu\phi, g_{\mu\nu}]$$



Explicit DHOST theories

The most general theory is

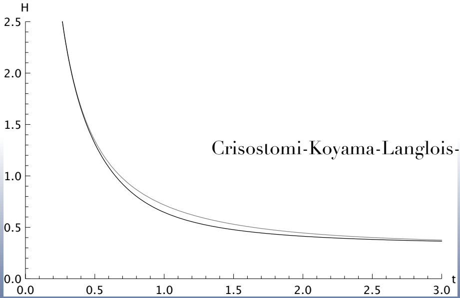
$$
\begin{aligned}
S \;=\; \int d^4x\sqrt{|g|}\,\big[&P(\phi,\phi_\mu\phi^\mu) + Q\Box\phi + G_N R \\
&+A_1\,(\Box\phi)^2 + A_2\,\phi_{\mu\nu}\phi^{\mu\nu} \\
&+A_3\,\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi + A_4\,\phi^\mu\phi_{\mu\nu}\phi_\rho\phi^{\rho\nu} + A_5\,(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2\big] \\
&+\text{Cubic terms}(\phi^3_{\mu\nu})\cdots
\end{aligned}
$$

- Needs a careful Hamiltonian analysis and the degeneracy does not reduce to field redefinitions. Langlois-Noui Deffayet-Esposito-Farese-Steer
- The functions G_N and A_α depend on ϕ and $X=\phi_\mu\phi^\mu$, and they are not independent (only three out of the six).
- It looks complicated... but Einstein action is also complicated ! We are missing a geometrical interpretation.

Cosmology

Cosmological background

- FRL geometry : $ds^2 = -dt^2 + a(t)^2 dx^2$
- Self-accelerating solutions preceded by a matter dominated era
- Modified equation of state for dark energy : $P = w(\phi)\rho$



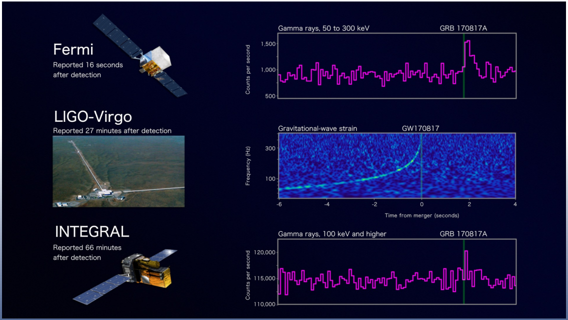
Crisostomi-Koyama-Langlois-Noui-Steer

Perturbations : GW waves

Linear perturbations about cosmological background

- Tensor perturbations : $ds^2 = -dt^2 + a(t)^2(\delta_{ij} + \gamma_{ij})dx^i dx^j$
- Gravity waves feel the fifth force and propagate in a medium

Langlois-Mancarella-Noui-Vernizzi $\quad c_T^2 = 1 + \alpha_T$



DHOST after GW170817

Assume $\alpha_T = 0$ holds exactly

$$S = \int d^4x\sqrt{|g|}\,[P(\phi,\phi_\mu\phi^\mu) + Q\Box\phi + G_N R$$
$$+\cancel{A_1(\Box\phi)^2 + A_2\phi_{\mu\nu}\phi^{\mu\nu}}$$
$$+A_3\,\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi + A_4\,\phi^\mu\phi_{\mu\nu}\phi_\rho\phi^{\rho\nu} + A_5\,(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2]$$
$$+\cancel{\cdots}$$

Creminelli-Vernizzi

GW propagate at the speed of light in any cosmological background !

Astrophysical tests

Quasi-static approximation : $r \ll H^{-1}$

$$ds^2 = -(1+2\Phi)dt^2 - (1-2\Psi)dx^2$$

Gravitational laws

$$\mathcal{M}(r) \equiv 4\pi \int_0^r \bar{r}^2 \rho(\bar{r}) \mathrm{d}\bar{r}$$

$$\frac{d\Phi}{dr} = \frac{G_\mathrm{N}\,\mathcal{M}(r)}{r^2} + \Xi_1\, G_\mathrm{N}\, \mathcal{M}''(r)\,,$$

$$\frac{d\Psi}{dr} = \frac{G_\mathrm{N}\,\mathcal{M}(r)}{r^2} + \Xi_2 \frac{G_\mathrm{N}\,\mathcal{M}'(r)}{r} + \Xi_3\, G_\mathrm{N}\, \mathcal{M}''(r)$$

with $(8\pi G_\mathrm{N})^{-1} \equiv 2f\,(1+\Xi_0)$

Crisostomi-Koyama
Langlois-Saito-Yamauchi-Noui
Dima-Vernizzi

Modifications of Newton laws can be constrained

- With non-relativistic stars : $-1/12 < \Xi_1 < 0.2$
- With Hulse-Taylor binary pulsar : $|\Xi_0| < 0.01$

Beltran-Jimenez-Piazza-Velten

Conclusion and beyond

Systematic study of large class of modified gravity theories

- Scalar-Tensor theories $S[g_{\mu\nu}, \phi]$ where ϕ responsible for dark force
- Theoretical classification of DHOST theories with NO GHOST
- Physical effects that could be measured in principle...

Are these models really relevant for dark energy ?

Existence of GW decay which could constraint even more DHOST theories and kills most of them ? Creminelli-Lewandowski-Tambalo-Vernizzi

Going further : modification at very strong gravity regime

- Parametrization of quantum gravity effects ?
- Find new interesting black hole solutions ?
- Compute new Gravity Waveforms ?