

# $D$ -term of nucleon

Peter Schweitzer (UConn)

## Outline

- **Introduction**

energy momentum tensor  $T^{\mu\nu}$  (EMT)  
 $D$ -term last unknown fundamental global property

- **What do we know from theory & experiment?**

soft pion theorems, chiral theory, models, lattice  
first extractions of  $\pi^0$  and nucleon  $D$ -term from data

- **What do we learn?**

interpretation: internal forces, stability, mechanical properties  
applications: visualization of dynamics, hadrocharmonia, large- $N_c$  baryon

- **Outlook**

recent review: M.V.Polyakov, PS, Int. J. Mod. Phys. A **33**, 1830025 (2018) [arXiv:1805.06596]

supported by



# Introduction: Energy momentum tensor (EMT)

- $T^{\mu\nu}$ : conserved  $\partial^\mu \hat{T}_{\mu\nu} = 0$ , Noether current of translations, couples to gravity
- Poincaré group generators:  $\hat{P}^\mu = \int d^3x \hat{T}^{0\mu}$ ,  $\hat{M}^{\kappa\nu} = \int d^3x (x^\kappa \hat{T}^{0\nu} - x^\nu \hat{T}^{0\kappa})$
- Casimir operators:  $\hat{P}^\mu \hat{P}_\mu \rightarrow m^2$ ,  $\hat{W}^\mu \hat{W}_\mu \rightarrow m^2 J(J+1)$  where  $\hat{W}^\kappa = -\frac{1}{2} \varepsilon^{\kappa\mu\nu\sigma} \hat{M}_{\mu\nu} \hat{P}_\sigma \rightarrow$  **mass & spin**
- QCD: quark & gluon gauge-invariant, total  $\hat{T}_{\mu\nu} = \sum_q T_{\mu\nu}^q + \hat{T}_{\mu\nu}^g$  conserved, trace anomaly

## form factors nucleon (Kobzarev, Okun, Pagels 1960s)

- $$\langle p' | \hat{T}_{\mu\nu}^a | p \rangle = \bar{u}(p') \left[ \begin{aligned} & A^a(t, \mu^2) \frac{P_\mu P_\nu}{M} \\ & + J^a(t, \mu^2) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) \Delta^\rho}{2M} \\ & + D^a(t, \mu^2) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4M} + \bar{c}^a(t, \mu^2) g_{\mu\nu} \end{aligned} \right] u(p)$$

$2P = (p' + p), \Delta = (p' - p), t = \Delta^2$   
 total form factors scale invariant  
 $A(t) = \sum_a A^a(t, \mu^2), J(t), D(t);$   
 and  $\sum_a \bar{c}^a(t, \mu^2) = 0$

- constraints: **mass**  $\Leftrightarrow A(0) = 1 \Leftrightarrow$  quarks + gluons carry 100 % of nucleon momentum
- **spin**  $\Leftrightarrow J(0) = \frac{1}{2} \Leftrightarrow$  quarks + gluons carry 100 % of nucleon spin \*
- **D-term**  $\Leftrightarrow D(0) \equiv D \rightarrow$  unconstrained! **Last global unknown!**

Higher spin  $J = 0, \frac{1}{2}, 1, \dots \rightarrow$  more form factors. Each particle has a  $D$ -term.

Cosyn, Cotogno, Freese, Lorcé; Polyakov, Sun 2019; Peter Lowdon's poster yesterday on constraints

# $D$ on same footing as mass, spin, charge:

$|N\rangle =$  **strong**-interaction particle. Use other forces to probe it!

---

**em:**  $\partial_\mu J_{\text{em}}^\mu = 0$   $\langle N'|J_{\text{em}}^\mu|N\rangle \longrightarrow G_E(t), G_M(t) \longrightarrow Q, \mu, \dots$

---

**weak:** PCAC  $\langle N'|J_{\text{weak}}^\mu|N\rangle \longrightarrow G_A(t), G_P(t) \longrightarrow g_A, g_p, \dots$

---

**gravity:**  $\partial_\mu T_{\text{grav}}^{\mu\nu} = 0$   $\langle N'|T_{\text{grav}}^{\mu\nu}|N\rangle \longrightarrow A(t), J(t), D(t) \longrightarrow M, J, D, \dots$

---

*global properties:*

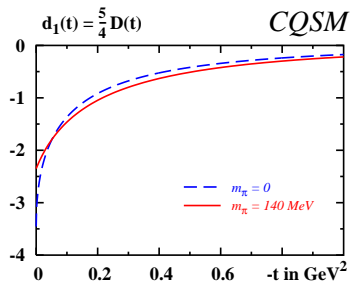
$Q_{\text{prot}}$	=	$1.602176487(40) \times 10^{-19} \text{C}$
$\mu_{\text{prot}}$	=	$2.792847356(23) \mu_N$
$g_A$	=	$1.2694(28)$
$g_p$	=	$8.06(0.55)$
$M$	=	$938.272013(23) \text{MeV}$
$J$	=	$\frac{1}{2}$
$D$	=	<b>?</b>

and more:  
 $t$ -dependence ... ..  
 parton structure, etc ... ..

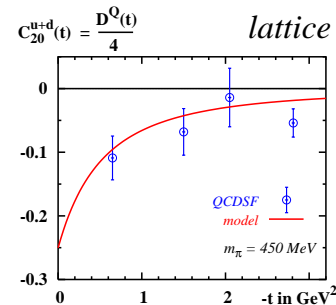
$\hookrightarrow D =$  "last" global unknown  
 which value does it have?  
 how can it be measured?  
 what does it mean?

# D-term in theory

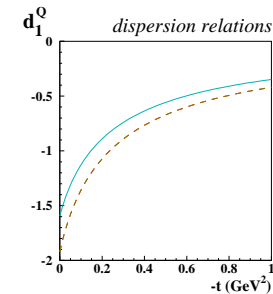
- **spin 0:** free Klein-Gordon field  $D = -1$   
(Pagels 1966; Hudson, PS 2017)
- **spin 0:** Goldstone bosons of chiral symmetry breaking  $D = -1$   
Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999); Donoghue, Leutwyler (1991)  
 $D_\pi \approx -0.97 \pm 0.01$ ,  $D_K \approx -0.77 \pm 0.15$  Hudson, PS (2017)
- **spin  $\frac{1}{2}$ :** free theory  $D = 0$  (analog to  $g = 2$  magnetic moment)  
implicit in Donoghue et al (2002), explicit in Hudson, PS PRD97 (2018) 056003
- **models** help understanding Hudson, PS PRD97 (2018)  
chiral quark-soliton model: "switch off strong chiral interaction"  $\Rightarrow D \rightarrow 0$   
bag model: free fermions  $\rightarrow$  "introduce boundary" (interaction!)  $\Rightarrow$  negative  $D \neq 0$  emerges!
- **distinction boson vs fermion:**  $\begin{cases} \text{non-interacting boson } D = -1 \\ \text{non-interacting fermion } D = 0 \end{cases}$  ...is there a deeper reason?
- **results:** (cf. Adam Freese in parallel session)



**chiral quark soliton**  
Goeke et al, PRD75 (2007)  
and PRC75 (2007)



**lattice QCD**  
QCDSF, PRL92 (2004)  
recently gluon  $D^g(t) < 0$   
Shanahan, Detmold, (2019)



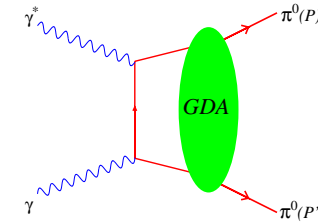
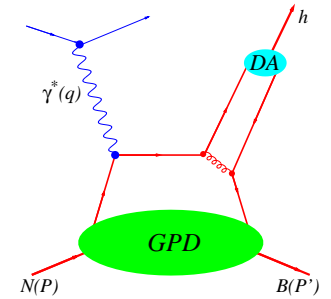
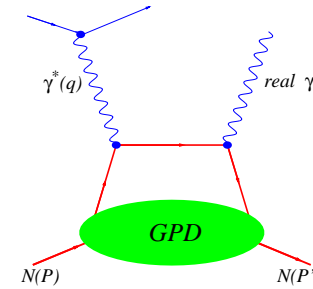
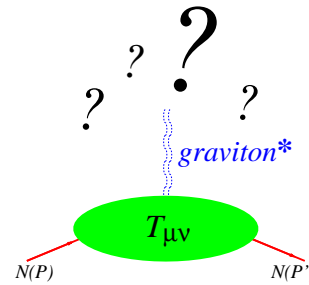
**dispersion relations**  
Pasquini et al (2014)  
pion PDFs input,  $4 \text{ GeV}^2$

# How to measure?

- **direct probe: graviton**  
(in principle only)
- **indirect probe: photon** → **GPDs**  
D.Müller et al, (1994)

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle N'(p') | \bar{\psi}_q(-\frac{\lambda n}{2}) n_\mu \gamma^\mu \mathcal{W} \psi_q(\frac{\lambda n}{2}) | N(p) \rangle$$

$$= \bar{u}(p') \left[ n_\mu \gamma^\mu H^q(x, \xi, t) + \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} E^q(x, \xi, t) \right] u(p)$$



- DVCS, hard meson production  
Ji, (1997); Radyushkin, (1996). Collins et al (1997)
- generalized distribution amplitudes  
 $t > 2m^2$ , study unstable particles e.g.  $\pi^0$

## polynomiality

$$\int dx x H^q(x, \xi, t) = A^q(t) + \xi^2 D^q(t)$$

$$\int dx x E^q(x, \xi, t) = B^q(t) - \xi^2 D^q(t)$$

$$B^q(t) = 2J^q(t) - A^q(t)$$

- GPDs convoluted in **Compton form factors** (e.g. DVCS, LO)

$$\mathcal{H}(\xi, t, \mu^2) = \sum_q e_q^2 \int dx \left[ \frac{1}{x - \xi - i\varepsilon} - \frac{1}{x + \xi - i\varepsilon} \right] H^q(x, \xi, t, \mu^2)$$

- **dispersion relation**

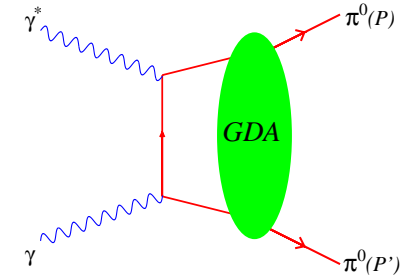
$$\Re \mathcal{H}(\xi, t, \mu^2) = \frac{1}{\pi} \text{PV} \int dx \left[ \frac{1}{\xi - x} - \frac{1}{\xi + x} \right] \Im \mathcal{H}(x, t, \mu^2) - \Delta(t, \mu^2)$$

**model-independent** extraction of  $A^q(t)$ ,  $J^q(t)$  (→ Ji sum rule)  
difficult in foreseeable future  
**for  $D(t)$  situation better!**

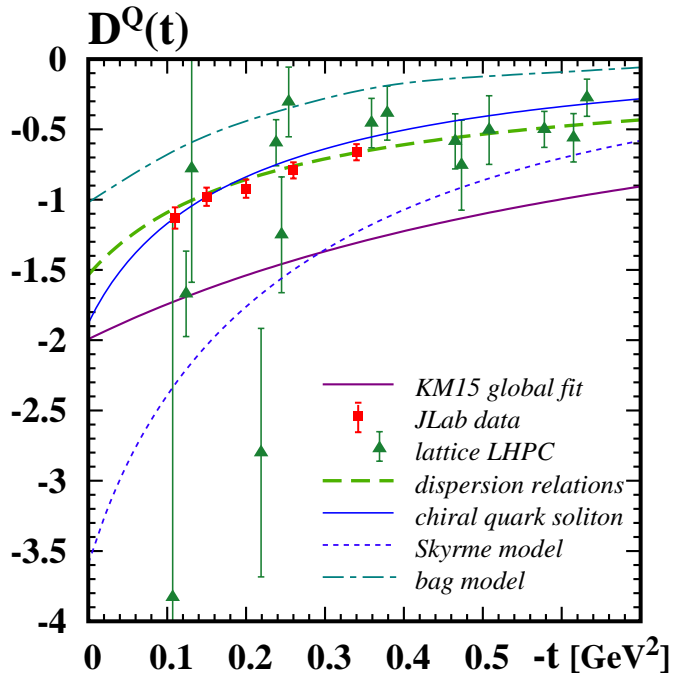
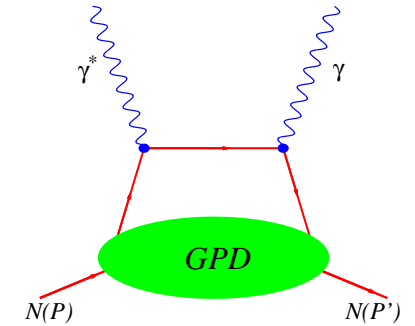
$\Re$  ↔ unpol.  $\sigma$ (DVCS)  
 $\Im$  ↔ beam-spin asymmetry  
 $\Delta(t, \mu^2) \rightarrow D(t)$  for  $\mu \rightarrow \infty$   
Teryaev; Diehl, Ivanov; Polyakov

# first insights from experiment

- **$D$ -term of  $\pi^0$**   $\gamma\gamma^* \rightarrow \pi^0\pi^0$  in  $e^+e^-$  Bell data: Masuda et al, PRD 93 (2016)  
 $D_{\pi^0}^Q \approx -0.7$  at  $\langle Q^2 \rangle = 16.6 \text{ GeV}^2$  Kumano, Song, Teryaev, PRD97 (2018)  
 chiral symmetry: total  $D_{\pi^0} \approx -1$  (gluons contribute the rest)



- **$D$ -term of nucleon** Burkert, Elouadrhiri, Girod, **Nature** 557, 396 (2018)  
 JLab data: PRL100 (2008) & PRL115 (2015)  
 beam-spin asym.  $\rightarrow \text{Im } \mathcal{H}$       unpol. cross sect.  $\rightarrow \text{Re } \mathcal{H}$



$\Delta(t, \mu^2) \rightarrow D^Q(t)$  under assumptions!

Model-dependent (very first attempt)  
 But proof of principle: method works  
 K. Kumerički, **Nature** 570, 7759 (2019)  
 cf. talk by Paweł Sznajder

explore scale dependence of  $\Delta(t, \mu^2) \rightarrow$  **EIC**

What will we learn from this  
 last unknown fundamental property?

**interpretation** as 3D-densities (okay for nuclei and nucleon) M.V.Polyakov, PLB 555 (2003) 57

Breit frame:  $\Delta^\mu = (0, \vec{\Delta})$ : static EMT  $\mathbf{T}_{\mu\nu}(\vec{r}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{r}} \langle P' | \hat{T}_{\mu\nu} | P \rangle$

$$\int d^3r T_{00}(\vec{r}) = M \quad \text{known}$$

$$\int d^3r \varepsilon^{ijk} s_i r_j T_{0k}(\vec{r}, \vec{s}) = \frac{1}{2} \quad \text{known}$$

$$-\frac{2}{5} M \int d^3r \left( r^i r^j - \frac{r^2}{3} \delta^{ij} \right) T_{ij}(\vec{r}) \equiv D \quad \text{new!}$$

with:  $T_{ij}(\vec{r}) = \mathbf{s}(\mathbf{r}) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + \mathbf{p}(\mathbf{r}) \delta_{ij}$  stress tensor

$\mathbf{s}(\mathbf{r})$  related to distribution of *shear forces*  
 $\mathbf{p}(\mathbf{r})$  distribution of *pressure* inside hadron }  $\rightarrow$  “mechanical properties”

• EMT conserved  $\Leftrightarrow \partial^\mu \hat{T}_{\mu\nu} = 0 \Leftrightarrow \nabla^i T_{ij}(\vec{r}) = 0$

$\hookrightarrow$  necessary condition for **stability**  $\int_0^\infty dr r^2 \mathbf{p}(r) = 0$  (von Laue, 1911)

$$D = -\frac{16\pi}{15} m \int_0^\infty dr r^4 s(r) = 4\pi m \int_0^\infty dr r^4 \mathbf{p}(r) \rightarrow \text{shows how internal forces balance}$$

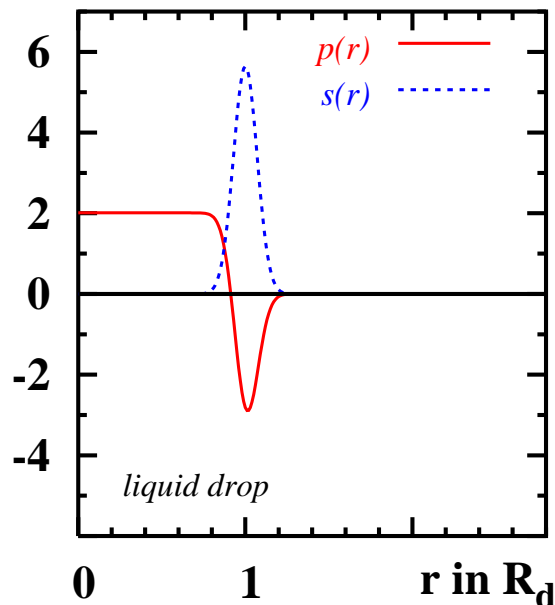
(already sign insightful! So far always negative)

**remark on 2D densities:** exact partonic probability densities [M. Burkardt \(2000\)](#)  
 applied to EMT form factors in works by Cédric Lorcé, Arek Trawiński, Harvey Moutarde  
 poster by Arek on Monday, talk by Cédric on Thursday

## intuition from models:

- liquid drop model of nucleus

$p(r)$  &  $s(r)$  in  $\gamma R_d^{-1}$  (c)



radius  $R_A = R_0 A^{1/3}$ ,  $m_A = m_0 A$

surface tension  $\gamma = \frac{1}{2} p_0 R_A$ ,  $s(r) = \gamma \delta(r - R_A)$

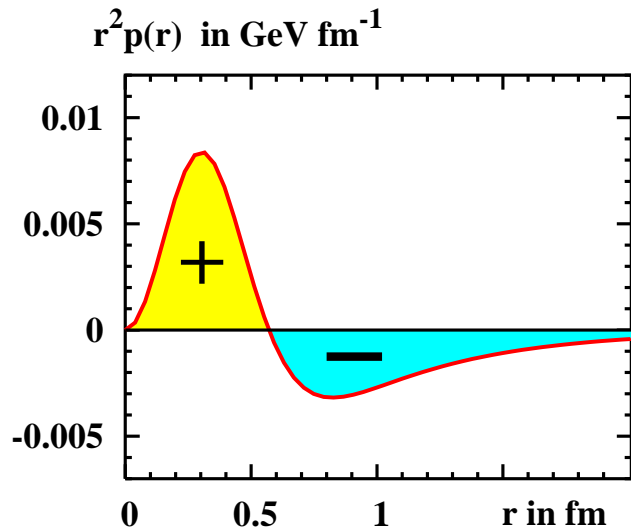
pressure  $p(r) = p_0 \Theta(R_A - r) - \frac{1}{3} p_0 R_A \delta(r - R_A)$

$D$ -term  $D = -\frac{4\pi}{3} m_A \gamma R_A^4 \approx -0.2 A^{7/3}$

[M.V.Polyakov PLB555 \(2003\)](#);  
 tested in Walecka model [Guzey, Siddikov \(2006\)](#)  
 alternative result in [Liuti, Taneja, PRC 72 \(2005\)](#)

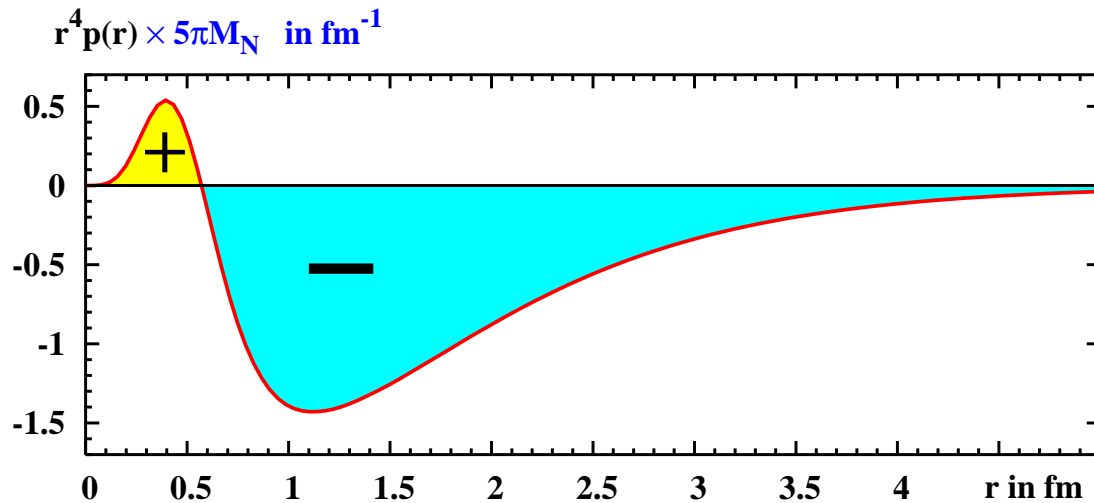


- chiral quark soliton model of nucleon



- $p(0) = 0.23 \text{ GeV}/\text{fm}^3 \approx 4 \times 10^{34} \text{ N}/\text{m}^2$   
 $\gtrsim 10\text{-}100 \times (\text{pressure in center of neutron star})$
- $p(r) = 0$  at  $r = 0.57 \text{ fm}$  change of sign in pressure
- $p(r) = - \left( \frac{3g_A^2}{8\pi f_\pi} \right)^2 \frac{1}{r^6}$  at large  $r$  in chiral limit  $m_\pi \rightarrow 0$

Goeke et al, PRD75 (2007) 094021



recall:  $\int_0^\infty dr r^2 p(r) = 0$

$D = 4\pi m \int_0^\infty dr r^4 p(r) < 0$

negative sign of  $D \Leftrightarrow$  stability (necessary condition)

# mechanical radius

- $T_{ij}(\vec{r}) = s(r) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij} =$  symmetric  $3 \times 3$  matrix  $\rightarrow$  diagonalize:

$$\frac{2}{3} s(r) + p(r) = \text{normal "force" (eigenvector } \vec{e}_r)$$

$$-\frac{1}{3} s(r) + p(r) = \text{tangential "force" } (\vec{e}_\theta, \vec{e}_\phi, \text{ degenerate for spin 0 and } \frac{1}{2})$$

- mechanical stability  $\Leftrightarrow$  normal force directed towards outside

$$\Leftrightarrow T^{ij} e_r^j dA = \underbrace{\left[ \frac{2}{3} s(r) + p(r) \right]}_{>0} e_r^i dA \quad \Rightarrow \quad D < 0 \text{ (proof!) Perevalova et al (2016)}$$

- define:  $\langle r^2 \rangle_{\text{mech}} = \frac{\int d^3r r^2 \left[ \frac{2}{3} s(r) + p(r) \right]}{\int d^3r \left[ \frac{2}{3} s(r) + p(r) \right]} = \frac{6D(0)}{\int_{-\infty}^0 dt D(t)}$  vs  $\langle r_{\text{ch}}^2 \rangle = \frac{6G'_E(0)}{G_E(0)}$  "anti-derivative"

$$\text{intuitive result for large nucleus } \frac{2}{3} s(r) + p(r) = p_0 \Theta(R_A - r) \rightarrow \langle r^2 \rangle_{\text{mech}} = \frac{3}{5} R_A^2$$

M.Polyakov, PS arXiv:1801.05858 (Kumano, Song, Teryaev PRD (2018) used  $D'(0)$  but inadequate)

- in chiral limit  $\langle r^2 \rangle_{\text{mech}}$  finite vs  $\langle r_{\text{ch}}^2 \rangle$  divergent (better concept)
- neutron  $\langle r^2 \rangle_{\text{mech}}$  same as proton(!)  $\langle r_{\text{ch}}^2 \rangle = -0.11 \text{ fm}^2$  inappropriate concept for neutron size (see also recent work Lorcé, Moutarde, Trawiński)
- proton:  $\langle r^2 \rangle_{\text{mech}} \approx 0.75 \langle r_{\text{ch}}^2 \rangle$  in chiral quark soliton model

# Summary & Outlook

- **GPDs, GDAs** →  
form factors of **energy momentum tensor**
- **D-term**: last unknown global property,  
related to forces, attractive and physically appealing → “motivation”
- **first results**(!) from experiment/phenomenology for proton,  $\pi^0$   
compatible with results from theory and models
- define **pressure, forces & mechanical radius**  
→ unique, appealing, complementary information!
- **applications:**  
**imaging** of nucleon structure  
**hadrocharmonia** pentaquarks & tetraquarks  
and more applications
- rich **potential**, new **predictions**, some work is done  
lots of work still ahead of us
- I hope this talk showed:  
**appealing, interesting topics, to be continued!**

# Summary & Outlook

- **GPDs, GDAs** →  
form factors of **energy momentum tensor**
- **D-term**: last unknown global property,  
related to forces, attractive and physically appealing → “motivation”
- **first results**(!) from experiment/phenomenology for proton,  $\pi^0$   
compatible with results from theory and models
- define **pressure, forces & mechanical radius**  
→ unique, appealing, complementary information!
- **applications:**  
**imaging** of nucleon structure  
**hadrocharmonia** pentaquarks & tetraquarks  
and more applications
- rich **potential**, new **predictions**, some work is done  
lots of work still ahead of us
- I hope this talk showed:  
**appealing, interesting topics, to be continued!**

Thank you!

**Support slides**

- dispersion relation

for  $D(t) \rightarrow$  situation better: dispersion relation. How this helps?

$$\Re \mathcal{H}(\xi, t, \mu^2) = \frac{1}{\pi} \text{PV} \int dx \left[ \frac{1}{\xi - x} - \frac{1}{\xi + x} \right] \Im \mathcal{H}(x, t, \mu^2) - \Delta(t, \mu^2)$$

$\Re \mathcal{H}(\xi, t, \mu^2) \rightsquigarrow$  unpolarized DVCS cross section

$\Im \mathcal{H}(\xi, t, \mu^2) \rightsquigarrow$  beam-spin asymmetry in DVCS

$$\Delta(t, \mu^2) = 4 \sum_q e_q^2 \left[ d_1^q(t, \mu^2) + d_3^q(t, \mu^2) + d_5^q(t, \mu^2) + \dots \right]$$

$$\lim_{\mu \rightarrow \infty} d_1^Q(t, \mu^2) = d_1(t) \frac{N_f}{N_f + 4C_F}$$

$$\lim_{\mu \rightarrow \infty} d_1^g(t, \mu^2) = d_1(t) \frac{4C_F}{N_f + 4C_F}$$

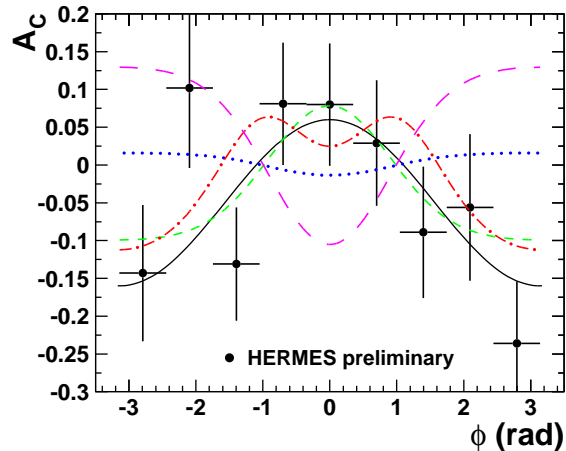
$$\lim_{\mu \rightarrow \infty} d_i^a(t, \mu^2) \rightarrow 0 \quad \text{for } i = 3, 5, \dots$$

$$\frac{4}{5} d_1(t) = D(t) \quad C_F = \frac{N_c^2 - 1}{2N_c}$$

Teryaev hep-ph/0510031  
 Anikin, Teryaev, PRD76 (2007)  
 Diehl and Ivanov, EPJC52 (2007)  
 Radyushkin, PRD83, 076006 (2011)  
 M.V.Polyakov, PLB 555 (2003) small  $x$

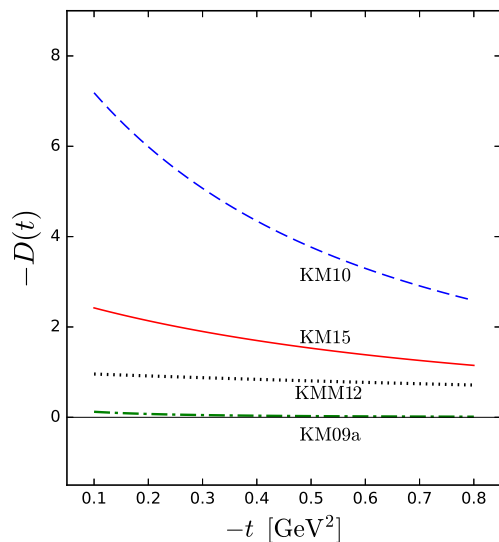
# Results from experiment & phenomenology

- HERMES proceeding NPA711, 171 (2002) (model-dependent)



beam charge asymmetry (DVCS  $e^+$  vs  $e^-$ )  
dotted line: VGG model without  $D$ -term (ruled out)  
dashed line: VGG model + positive  $D$ -term (ruled out)  
dashed-dotted: VGG model + **negative**  $D$ -term (yeah!)  
(cf. Belitsky, Müller, Kirchner, NPB 629 (2002) 323)

- fits by Kresimir Kumerički, Dieter Müller et al:  $D < 0$  needed! (model-independent)



DVCS parametrizations from:

Kumerički, Müller, NPB 841 (2010) 1

Kumerički, Müller, Murray, Phys. Part. Nucl. 45 (2012) 723

Kumerički, Müller, EPJ Web Conf. 112 (2016) 01012

Fig. 9 in ECT\* workshop proceeding 1712.04198

statistical uncertainty of  $D$  in KMM12:  $\sim 50\%$ ,

statistical uncertainty of  $D$  in KM15:  $\sim 20\%$ .

unestimated systematic uncertainty

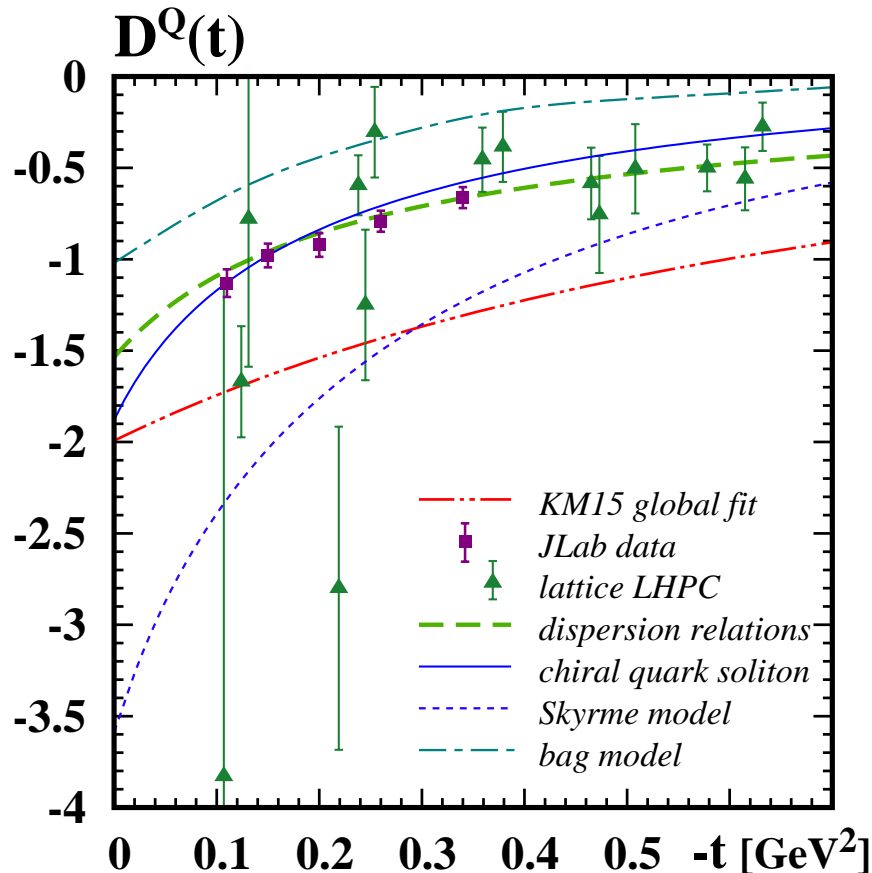
K.Kumerički private communication

## • $D$ -term of nucleon

Burkert, Elouadrhiri, Girod, *Nature* **557**, 396 (2018) based on:  
 Girod et al PRL 100 (2008) 162002 and Jo et al PRL 115 (2015) 212003

beam-spin asymmetry  $\rightarrow \text{Im } \mathcal{H}$

unpol. cross section  $\rightarrow \text{Re } \mathcal{H}$



$$\Delta(t, \mu^2) = 4 \sum_q e_q^2 \left[ d_1^q(t, \mu^2) + \dots \right]$$

### assumptions:

- neglect power corrections, NLO corrections at  $E_{\text{beam}} = 6 \text{ GeV}$  and  $\langle Q^2 \rangle = 1.5 \text{ GeV}^2$
- only  $\mathcal{H}$ , neglect  $\mathcal{E}$ , etc
- $\Delta(t, \mu^2) = 4 \sum_q e_q^2 d_1^q(t, \mu^2)$   
 with  $d_i^q(t, \mu^2)$  for  $i = 3, 5, \dots$  neglected  
 (in CQSM  $d_3^Q/d_1^Q \sim 0.3$ ,  $d_5^Q/d_1^Q \sim 0.1$   
 (Kivel, Polyakov, Vanderhaeghen (2001)))
- assume  $d_1^u \approx d_1^d$   
 (okay in CQSM, to be tested in experiment)  
 $\rightsquigarrow D^Q(t, \mu^2) \approx \frac{18}{25} \Delta(t, \mu^2)$
- how good are these approximations?  
 will see: JLab12, COMPASS, EIC, future experiments

$\Rightarrow$  CLAS, KM-fits, dispersion relations, models, lattice:  **$D$ -term negative**

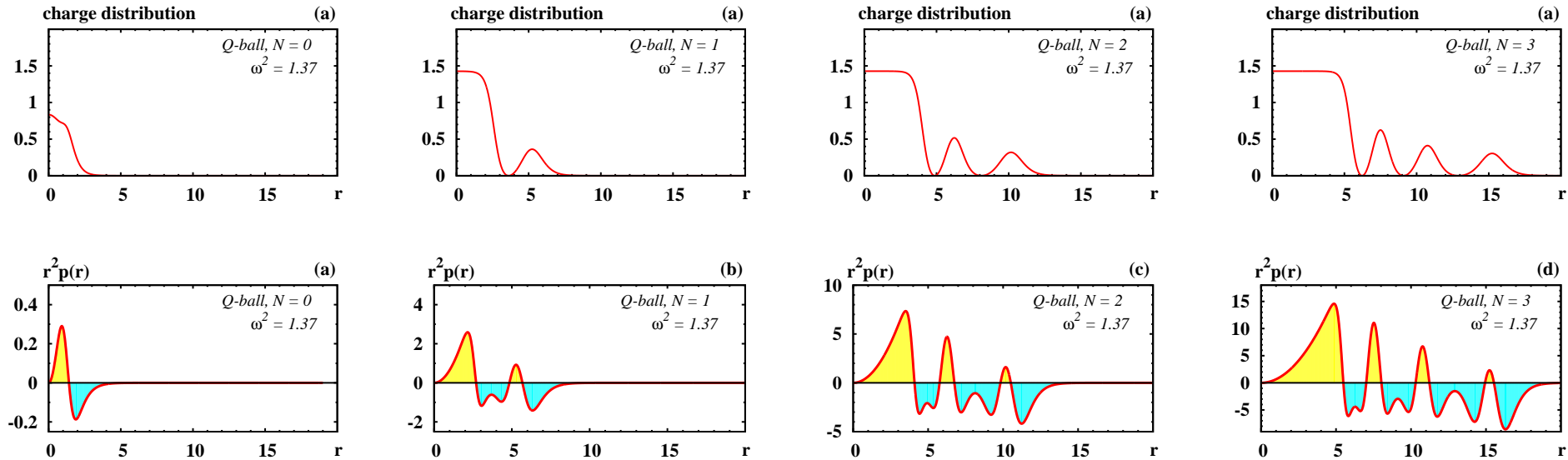


- more intuition from toy system:  $Q$ -ball

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \Phi^*) (\partial^\mu \Phi) - V \text{ with U(1) global symm., } V = A (\Phi^* \Phi) - B (\Phi^* \Phi)^2 + C (\Phi^* \Phi)^3, \quad \Phi(t, \vec{r}) = e^{i\omega t} \phi(r)$$

$N = 0$  ground state,  $N = 1$  first excited state, etc [Volkov & Wohnert \(2002\)](#), [Mai, PS PRD86 \(2012\)](#)

charge density exhibits  $N$  shells,  $p(r)$  exhibits  $(2N + 1)$  zeros



excited states unstable, but  $\int_0^\infty dr r^2 p(r) = 0$  always valid, and  $D$ -term always negative!

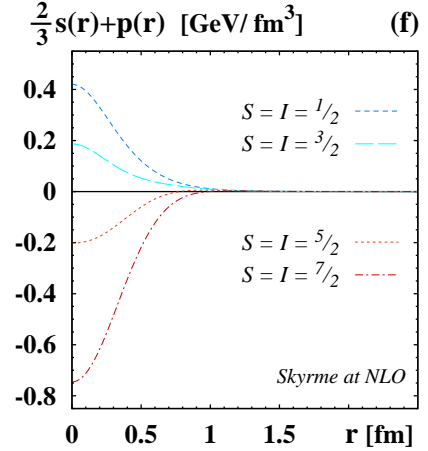
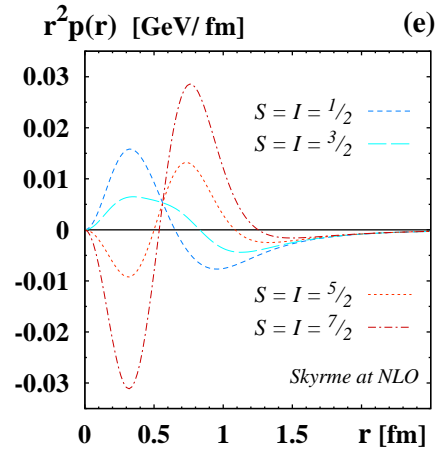
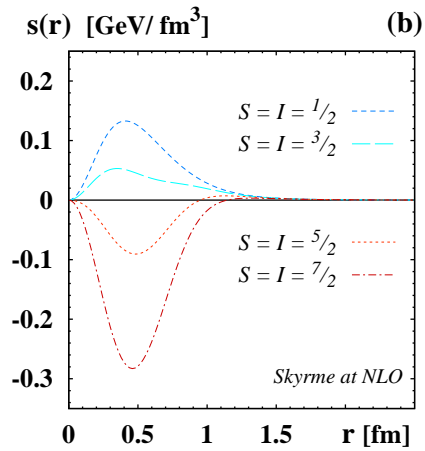
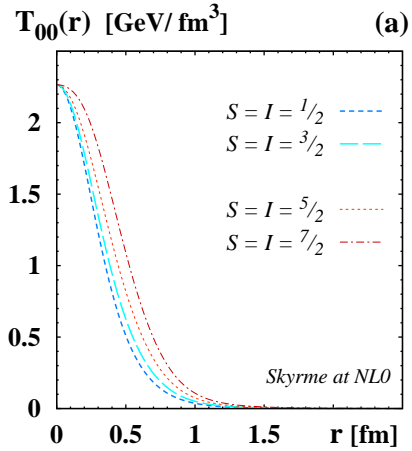
so far all  $D$ -terms negative: pions, nucleons, nuclei, nucleons in nuclear matter, photons,  $Q$ -balls,  $Q$ -clouds

could Roper resonance look like this? (possible to measure??) (transition GPDs???)  
 However e.g.  $\Delta$ -resonance, similar to nucleon! (lowest state for  $J = T = \frac{3}{2}$ , see below)

- side remark: you won't believe in how many models of the nucleon "the nucleon does not exist!!"  
 (explodes or implodes within  $t < 10^{-23}$ s vs  $t_{\text{proton}} > 10^{32}$  years ...)

# Application I: nucleon, $\Delta$ , large- $N_c$ artifacts Witten 1979

in large  $N_c$  baryons = rotational excitations of soliton with  $S = I = \underbrace{\frac{1}{2}, \frac{3}{2}}_{\text{observed}}, \underbrace{\frac{5}{2}, \dots}_{\text{artifacts}}$



$$M_B = M_{\text{sol}} + \frac{S(S+1)}{2\Theta}$$

nucleon  $s(r) \neq \gamma\delta(r-R)$   
 $\Delta$  much more diffuse

$\int_0^\infty dr r^2 p(r) = 0$   
 stability needs more:  
 $p(r) > 0$  in center,  
 negative outside  
 okay for nucleon,  $\Delta$   
 $\implies$  implies  $D < 0$

mechanical stability  
 $T^{ij} da^j \geq 0$   
 $\Leftrightarrow \frac{2}{3}s(r) + p(r) \geq 0$   
 artifacts do not satisfy!  
 $\implies$  **have positive  $D$ -term!!**  
**That's why they do not exist!**  
 EMT: dynamical understanding  
Perevalova et al (2016)

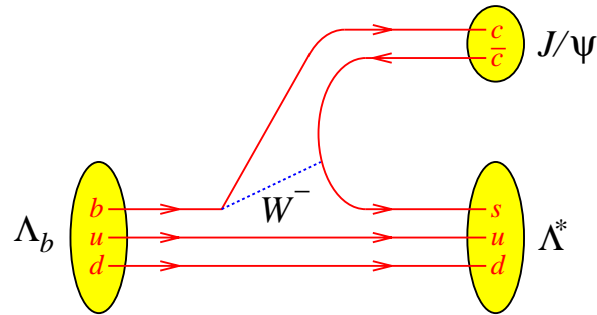
$\implies$  particles with positive  $D$  unphysical!!!

## Application II: hidden-charm pentaquarks as hadrocharmonia

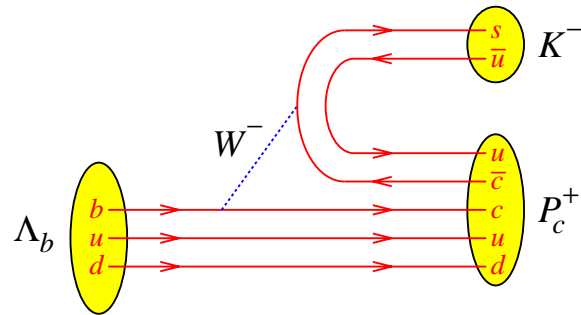
$\Lambda_b^0 \longrightarrow J/\Psi p K^-$  seen  
 Aaij *et al.* PRL 115 (2015)

$\Lambda_b^0$   $m = 5.6$  GeV,  $\tau = 1.5$  ps  
 $J/\Psi$   $m = 3.1$  GeV,  $\Gamma = 93$  keV,  $\Gamma_{\mu^+\mu^-} = 6\%$   
 $\Lambda^*$   $m = 1.4$  GeV or more,  $\Lambda^* \rightarrow K^- p$  in  $10^{-23}$ s

$\longrightarrow J/\Psi \Lambda^*$



$\longrightarrow P_c^+ K^-$



state	$m$ [MeV]	$\Gamma$ [MeV]	$\Gamma_{\text{rel}}$	mode	$J^P$
$P_c^+(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$(4.1 \pm 0.5 \pm 1.1)\%$	$J/\psi p$	$\frac{3}{2}^-$ or $\frac{5}{2}^+$
$P_c^+(4450)$	$4450 \pm 2 \pm 3$	$39 \pm 5 \pm 19$	$(8.4 \pm 0.7 \pm 4.2)\%$	$J/\psi p$	$\frac{5}{2}^\pm$ or $\frac{3}{2}^-$

## Hadrocharmonium approach Eides, Petrov, Polyakov, PRD93, 054039 (2016)

- **theoretical approach:** in heavy quark limit  $\Rightarrow V_{\text{eff}} = -\frac{1}{2} \alpha \vec{E}^2$  Voloshin, Yad. Fiz. **36**, 247 (1982)
- **chromoelectric polarizability** property of charmonium
  - $\alpha(1S) \approx (1.6 \pm 0.8) \text{ GeV}^{-3}$  Polyakov, PS PRD98 (2018); Sugiura et al, arXiv:1711.11219
  - $\alpha(2S) \approx 17 \text{ GeV}^{-3}$  Eides et al; Perevalova, Polyakov, PS, PRD 94, 054024 (2016)
  - $|\alpha(2S \rightarrow 1S)| \approx 2 \text{ GeV}^{-3}$  Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455
  - cf. quarkonia = Coulomb systems Peskin NPB 156 (1979) 365
- **chromoelectric field strength:**  $\vec{E}^2 \rightarrow T_{00}(r), p(r)$  from CQSM, Skyrme
- **compute quarkonium-nucleon bound state:** 
$$\left( -\frac{\vec{\nabla}^2}{2\mu} + V_{\text{eff}}(r) \right) \psi = E_{\text{bind}} \psi$$
- **results:**  $N$  and  $J/\psi$  form no bound state  
 $N$  and  $\psi(2S)$  form **two** bound states  
with nearly degenerate masses  $\sim 4450 \text{ MeV}$   
mass-splitting  $\mathcal{O}(10\text{--}20) \text{ MeV}$ ,  $J^P = \frac{1}{2}^-$  and  $\frac{3}{2}^-$ ,  
**important:** partial width  $\Gamma = |\alpha(2S \rightarrow 1S)|^2 \times \dots = \text{few tens of MeV}$
- **update:** 26 March 2019, Observation of new pentaquarks, Moriond conference:  
previous  $P_c^+(4450)$  resolved in:  
 $P_c^+(4440) \ m = (4440.3 \pm 1.3_{-4.7}^{+4.1}) \text{ MeV}$ ,  $\Gamma = (20.6 \pm 4.9_{-10.1}^{+8.7}) \text{ MeV}$   
 $P_c^+(4457) \ m = (4457.3 \pm 0.6_{-1.7}^{+4.1}) \text{ MeV}$ ,  $\Gamma = (6.4 \pm 2.0_{-1.9}^{+5.7}) \text{ MeV}$  **exciting!**
- **predictions:** bound states of  $\psi(2S)$  with  $\Delta$  and hyperons  $\leftarrow$  test approach

