Gluon Content of the Pion and Kaon at an EIC

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U.S. DEPARTMENT OF ENERGY | Office of Science

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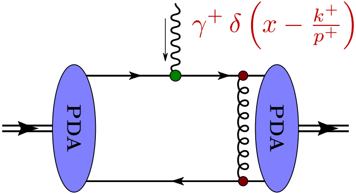
Much Ado About Nothing

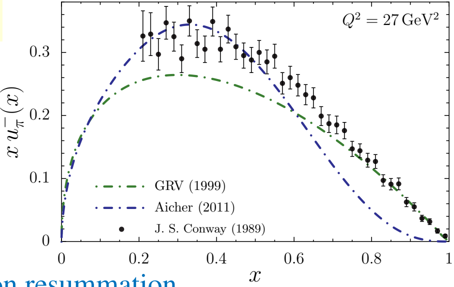


- Longstanding perturbative QCD prediction that pion PDF near $x = 1$ behaves as

$$q_\pi(x) \simeq (1-x)^{2+\gamma_q(\mu^2)}$$

- Since large-x quarks are the source of large-x gluons, near $x = 1$ expect

$$g_\pi(x) \simeq (1-x)^{3+\gamma_g(\mu^2)}$$



- However, pion-induced DY data, and a resent re-analysis that also included leading-neutron data, seem to prefer $q(x) \sim (1-x)^1$ near $x = 1$
- Potential resolution to this "puzzle" provided by Aicher *et al.* $\Longrightarrow$ soft-gluon resummation
- *However, pQCD predictions need only set in very near $x = 1$, the observed $q(x) \simeq (1-x)^1$ behavior could be real where data exists*

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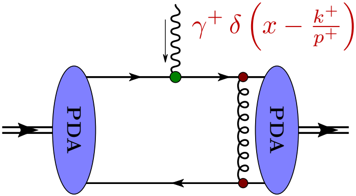

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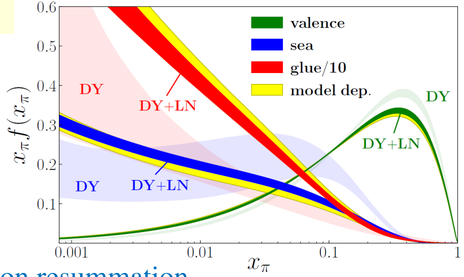
[Barry, Sato, Melnitchouk, and Ji, Phys. Rev. Lett., 152001 (2018)]



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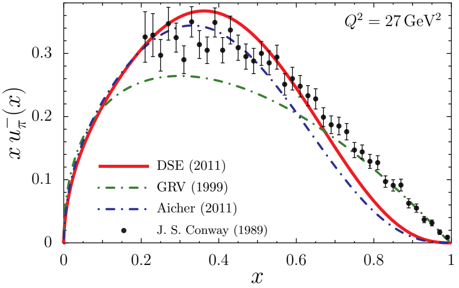
DSEs + Pion PDFs

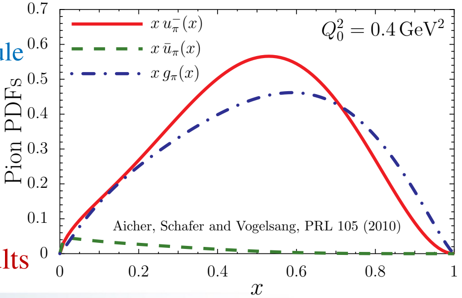
- DSE prediction [Hecht (2001); etc] that $q_\pi(x) \simeq (1-x)^2$ as $x \to 1$
  - related to $1/k^2$ dependence of BSE kernel at large relative momentum



- DSEs PDFs calculations have used Ward-Identity ansatz (WIA)

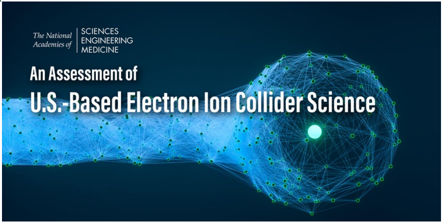
$$\Lambda_q(z,p,n) \to \Lambda_q^{\mathrm{WIA}}(z,p,n) = \delta\,(1-z)\;n^\mu\,\frac{\partial}{\partial p^\mu}\,S_q^{-1}(p)$$

  - WIA respects baryon sum rule but not higher moments e.g. momentum sum rule
  - momentum is not distributed correctly between quarks and gluons

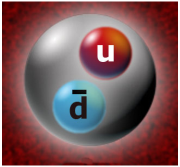


- Aicher: $g_\pi(x) \sim (1-x)^{1.3}$ as $x \to 1$
  - pQCD predicts: $g_\pi(x) \sim (1-x)^3$
- Inconsistencies in Aicher & DSE results

Pion + High-priority science questions



- The NAS *Assessment of a U.S. based Electron Ion Collider* identified three high-priority science questions
  - How does the mass of the nucleon arise?
  - How does the spin of the nucleon arise?
  - What are the emergent properties of dense systems of gluons?



- The pion and kaon can be understood as a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT, but are also Goldstone modes associated with DCSB in QCD
- The dynamical breaking of chiral symmetry (DCSB) in QCD gives rise to $\sim 500\,\text{MeV}$ mass splittings in hadron spectrum & massless Goldstone bosons in chiral limit $(\pi,\ K,\ \eta)$
- Therefore, understanding the nucleon mass is not sufficient
  - must also understand the mass of the pion $(u\bar{d},\ldots)$ and kaon $(u\bar{s},\ldots)$

Hadron Masses in QCD

- Quark/gluon contributions to masses (& angular momentum) are accessed via matrix elements of QCD's (symmetric) energy-momentum tensor

$$T^{\mu\nu} = T^{\nu\mu}, \qquad \partial_\mu T^{\mu\nu} = \partial_\mu T_q^{\mu\nu} + \partial_\mu T_g^{\mu\nu} = 0, \qquad T^{\mu\nu} = \underset{\text{[traceless]}}{\overline{T}^{\mu\nu}} + \underset{\text{[trace]}}{\widehat{T}^{\mu\nu}}$$

- The trace piece of $T^{\mu\nu}$ takes the form (un-renormalized)

$$T^\mu_\mu = \sum_{q=u,d,s} \underbrace{m_q\,(1+\gamma_m)\,\overline{\psi}_q\psi_q}_{\text{quark mass term}} + \underbrace{\frac{\tilde{\beta}(g)}{2\,g}\,F^{\mu\nu,a}F^a_{\mu\nu}}_{\text{trace anomaly}}$$

- At zero momentum transfer

$$\langle p\,|T^{\mu\nu}|\,p\rangle = 2\,p^\mu p^\nu \quad \Longrightarrow \quad \left\langle p\,\middle|T^\mu_\mu\middle|\,p\right\rangle = 2\,m^2$$

  - in chiral limit entire hadron mass from gluons!
  - Dmitri Kharzeev – Proton Mass workshops at Temple University and ECT*

- Understanding difference in pion and proton is key to hadron masses:

$$\left\langle \pi\,\middle|T^\mu_\mu\middle|\,\pi\right\rangle = 2\,m_\pi^2 \overset{\text{chiral limit}}{\rightarrow} 0, \qquad \left\langle N\,\middle|T^\mu_\mu\middle|\,N\right\rangle = 2\,m_N^2$$

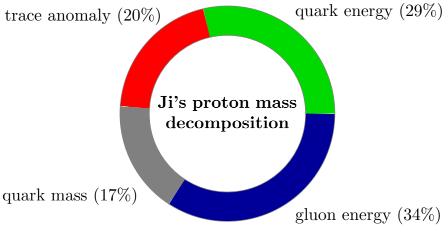
Rest Frame Hadron Mass Decompositions

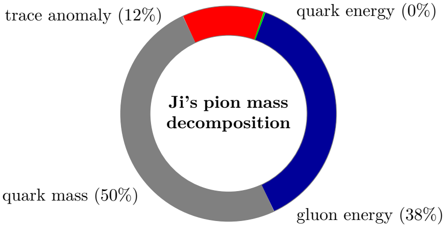
- Xiangdong Ji proposed hadron mass decomposition [PRL **74**, 1071 (1995); PRD **52**, 271 (1995)]

$$m_p = \frac{\langle p \,|\int d^3x\, T^{00}(0,\vec{x})|\, p\rangle}{\langle p|p\rangle}\Bigg|_{\text{at rest}} = \underbrace{M_q + M_g}_{\text{quark and gluon energies}} + \underbrace{M_m}_{\text{quark mass}} + \underbrace{M_a}_{\text{trace anomaly}}$$

$$M_q = \tfrac{3}{4}\,(a-b)\,m_p,\quad M_g = \tfrac{3}{4}\,(1-a)\,m_p,\quad M_m = b\,m_p,\quad M_a = \tfrac{1}{4}\,(1-b)\,m_p,$$

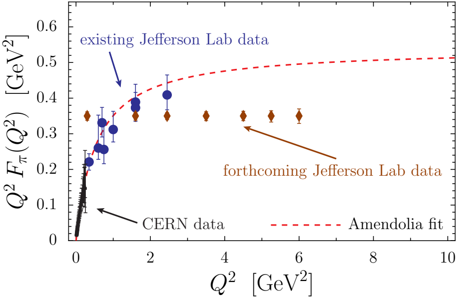
  - a = quark momentum fraction, b related to sigma-term or anomaly contribution
  - [See Cédric Lorcé, EPJC **78**, (2018) for decomposition with pressure effects]

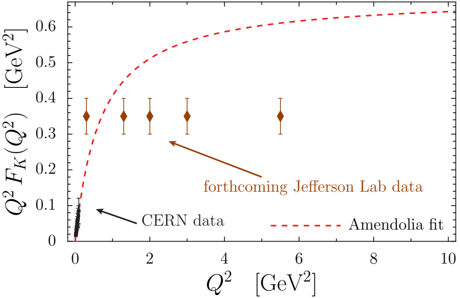




- In chiral limit ($m_q \to 0$) pion has no rest frame ($m_\pi = 0$) – how to interpret Ji's pion mass decomposition?

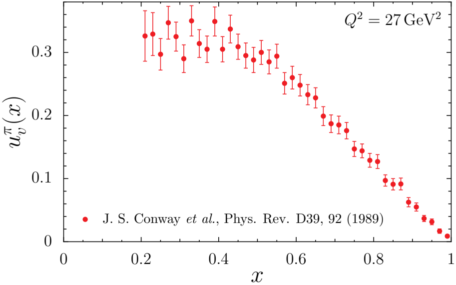
What we know about the Pion and Kaon

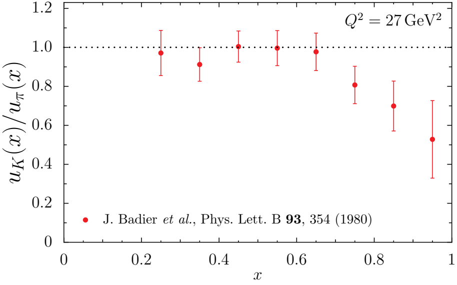




- Pion and kaon structure is slowly being revealed using: π^-/K^- beams at CERN; Sullivan type experiments at Jefferson Lab; π^- beam at Fermilab; and $e^+e^- \to \pi^+\pi^-,\ K^+K^-$ in the time-like region
- 40 years of experiments has revealed, e.g.
  - $r_{\pi^+} = 0.672 \pm 0.008, \qquad r_{K^+} = 0.560 \pm 0.031, \qquad r_{K^0} = -0.277 \pm 0.018$
- Still a lot more to learn about pion and kaon structure:
  - quark and gluon PDFs; TMDs including Boer-Mulders function; $q, g \to \pi/K$ fragmentation functions, quark and gluon GPDs; gravitational form factors

What we know about the Pion and Kaon





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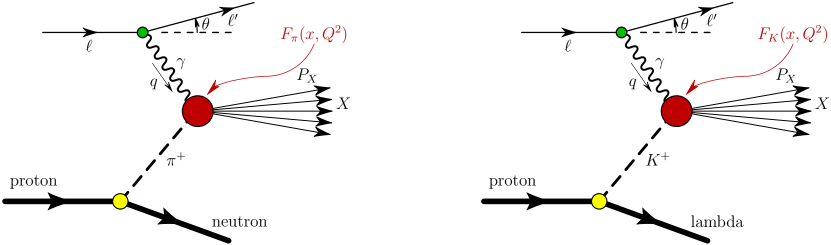
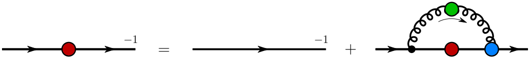
Pion & Kaon Structure at JLab and an EIC



- At Jefferson Lab and an EIC pion and kaon structure can be accessed via the so-called *Sullivan* processes
  - initial pion/kaon is off mass-shell – need extrapolation to pole
  - proven results for form factors – what about quark and gluon PDFs, TMDs, GPDs, *etc*, at an EIC?
- Explored this ideal at a series of workshops on "*Pion and Kaon Structure at an Electron–Ion Collider*" (PIEIC)
  - 1−2 June 2017, Argonne National Laboratory www.phy.anl.gov/theory/pieic2017/
  - 24−25 May 2018, The Catholic University of America www.jlab.org/conferences/pieic18/

QCD's Dyson-Schwinger Equations

- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - tractability $\Longrightarrow$ must implement a symmetry preserving truncation
- The most important DSE is QCD's gap equation $\Longrightarrow$ quark propagator

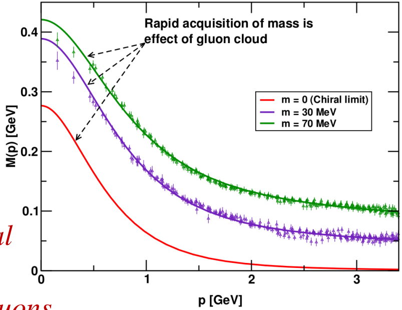


  - ingredients – dressed gluon propagator & dressed quark-gluon vertex

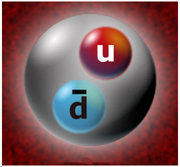
$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

- Mass function, $M(p^2)$, exhibits dynamical mass generation, even in chiral limit
  - *mass function is gauge dependent and therefore NOT an observable!*
- *Hadron masses are generated by dynamical chiral symmetry breaking – caused by a cloud of gluons dressing the quarks and gluons*

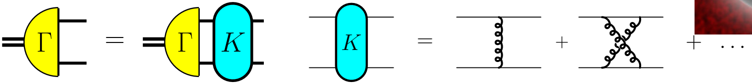
[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



Calculating and Predicting Pion Structure



- In QFT a two-body bound state (*e.g.* a pion, kaon, *etc*) is described by the Bethe-Salpeter equation (BSE):

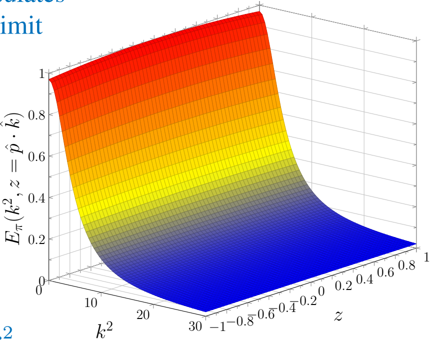


  - the kernel must yield a solution that encapsulates the consequences of DCSB, *e.g.*, in chiral limit $m_\pi = 0$ & $m_\pi^2 \propto m_u + m_d$

- Pion Bethe-Salpeter vertex

$$\Gamma_\pi(p,k) = \gamma_5 \Big[E_\pi(p,k) + \not{p}\, F_\pi(p,k) + \not{k}\, k\cdot p\, G_\pi(p,k) + i\sigma^{\mu\nu} k_\mu p_\nu\, H_\pi(p,k) \Big]$$

  - $\chi_{\text{BSE}} = S(k+\tfrac{1}{2}p)\,\Gamma_\pi(p,k)\,S(k-\tfrac{1}{2}p)$
  - large relative momentum: $E_\pi \sim F_\pi \sim 1/k^2$

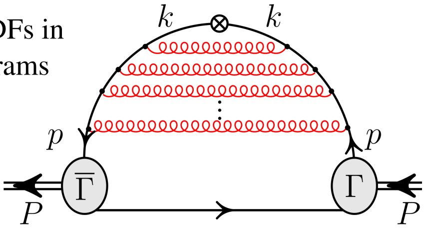


- Challenging to go beyond rainbow-ladder trunction and maintain symmetries

Pion PDFs – Self-Consistent DSE Calculations

- To self-consistently determine hadron PDFs in rainbow-ladder must sum all planar diagrams

$$q(x) \propto \text{Tr} \int \frac{d^4p}{(2\pi)^4}\, \overline{\Gamma}_M(p,P)\, S(p)$$
$$\times\, \Gamma_q(x,p,n)\, S(p)\, \Gamma_M(p,P)\, S(p-P)$$



- DSEs are formulated in Euclidean space – evaluate $q(x)$ by taking moments
- The *hadron dependent* vertex $\Gamma_q(x,p,n)$ satisfies an inhomogeneous BSE
- However can define a *hadron independent* vertex $\Lambda_q(x,p,n)$

$$\Gamma_q(x,p,n) = \iint dy\, dz\, \delta(x - yz)\, \delta\Big(y - \frac{p\cdot n}{P\cdot n}\Big)\, \Lambda_q(z,p,n)$$

- $\Lambda_q(x,p,n)$ satisfies the inhomogeneous BSE

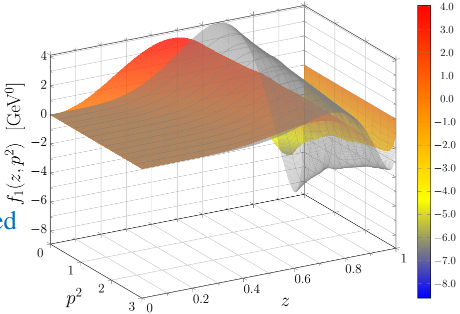
$$\Lambda_q(z,p,n) = iZ_2\, \not{n}\, \delta(1-z) - \iint du\, dw\, \delta(z - uw) \int \frac{d^4\ell}{(2\pi)^4} \delta\Big(w - \frac{\ell\cdot n}{p\cdot n}\Big)$$
$$\times\, \gamma_\mu\, \mathcal{K}_{\mu\nu}(p-\ell)\, S(\ell)\, \Lambda_q(u,\ell,n)\, S(\ell)\, \gamma_\nu$$

PDFs of a Dressed Quark

- *Hadron independent* vertex has form

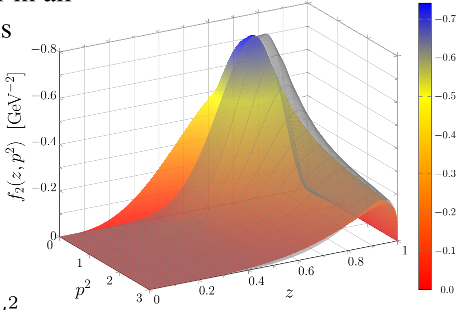
$$\Lambda_q(z,p,n) = i\not{n}\,\delta(1-z) + i\not{n}\,f_1^q(z,p^2) + n\cdot p\left[i\not{p}\,f_2^q(z,p^2) + f_3^q(z,p^2)\right]$$

  - the functions $f_i^q(z,p^2)$ can be interpreted as unpolarized PDFs in a dressed quark of virtuality p^2



- These functions are universal – appear in all RL-DSE unpolarzied PDF calculations

- Distributed support in z is immediate indication gluons carry significant momentum
  - heavier s quark support nearer $z=1$
  - WIA $\implies$ $\Lambda_q(z,p,n) \propto \delta(1-z)$



- Renormalization condition means dressing functions vanish when $p^2=\mu^2$

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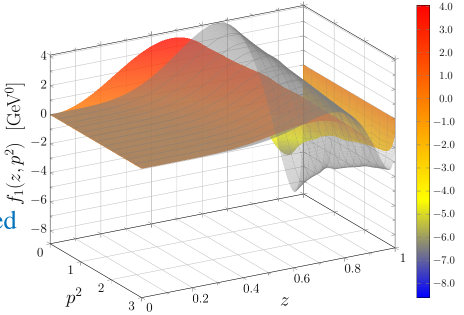

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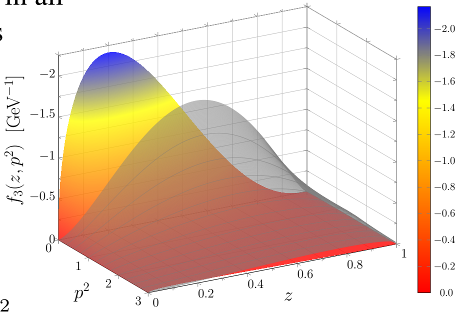

Self-Consistent DSE Results

- For pion and kaon PDFs included for first time gluons self-consistently
  - correct RL-DSE pion PDFs in excellent argeement with Conway *et al.* data and recent JAM analysis
  - agrees with $x \to 1$ pQCD prediction
- Treating non-perturbative gluon contributions correctly pushes support of $q_\pi(x)$ to larger x
  - gluons remove strength from $q_\pi(x)$ at low to intermediate x – baryon number then demands increased support at large x
  - cannot be replicated by DGLAP – DSE splitting functions are dressed
- *Immediate consequence of gluon dressing is that gluons carry 35% of pion's and 30% of kaon's momentum*

[K. Bednar, ICC, P. Tandy, *et al.*, arXiv:1811.12310 [nucl-th]]