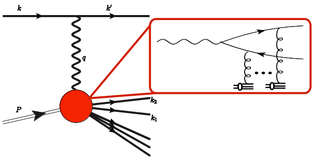
Measuring Weizsäcker-Williams distribution of linearly polarized gluons at EIC through dijet azimuthal asymmetries

Vladimir Skokov (NC State University)

A. Dumitru, V. S., T. Ullrich, Phys.Rev. C99 (2019), 015204, arXiv:1809.02615

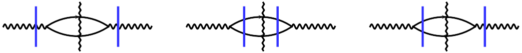
Dijet production in DIS at small x



- DIS dijet production: $\gamma^* \; A \;\; \rightarrow \;\;\; q \; \bar{q} \;\; X$
- Multiple scatterings of (anti) quark are accounted for by ressumation:

$$U(\mathbf{x}) = \mathbb{P} \exp \left\{ ig \int dx^- A^+(x^-, \mathbf{x}_\perp) \right\}$$

- In color dipole model this process corresponds to



$$\frac{d\sigma^{\gamma^* A \to q\bar{q}X}}{d^3k_1 d^3k_2} \propto$$

$$N_c \alpha_{em} e_q^2 \int \frac{d^2x_1}{(2\pi)^2} \cdots \; \exp\left(-i\mathbf{k}_1(\mathbf{x}_1 - \mathbf{y}_1) - i\mathbf{k}_2(\mathbf{x}_2 - \mathbf{y}_2)\right)$$

$$\sum_{\gamma\alpha\beta} \psi^{\mathrm{T,L}\gamma}_{\alpha\beta}(\mathbf{x}_1 - \mathbf{x}_2) \psi^{\mathrm{T,L}\gamma *}_{\alpha\beta}(\mathbf{y}_1 - \mathbf{y}_2) \left[1 + \frac{1}{N_c} \Big(\langle \mathrm{Tr}\; U(\mathbf{x}_1) U^\dagger(\mathbf{y}_1) U(\mathbf{y}_2) U^\dagger(\mathbf{x}_2) \rangle \right.$$

$$\left. - \langle \mathrm{Tr}\; U(\mathbf{x}_1) U^\dagger(\mathbf{x}_2) \rangle - \langle \mathrm{Tr}\; U(\mathbf{y}_1) U^\dagger(\mathbf{y}_2) \rangle \Big) \right]$$

↑ Quadrupole contribution

- Splitting wave function of $\gamma^\star$ with longitudinal momentum p^+ and virtuality Q^2

Dijet production in DIS

- Back-to-back jets ("correlation" limit):
  Total momentum $\mathbf{P} = (\mathbf{k}_1 - \mathbf{k}_2)/2 \gg$ momentum imbalance $\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2$;
- Expansion about $\mathbf{x}_1 \approx \mathbf{x}_2$ and $\mathbf{y}_1 \approx \mathbf{y}_2$ results in gradients of Wilson lines
- Allows to reduce quadrupole to 2-point functions

$$xG^{ij}_{WW}(\mathbf{q}) = \frac{8\pi}{S_\perp} \int \frac{d^2x}{(2\pi)^2} \frac{d^2y}{(2\pi)^2} e^{-\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})} \langle A^i_a(\mathbf{x}) A^j_a(\mathbf{y}) \rangle, \quad A^i(\mathbf{x}) = \frac{1}{ig} U^\dagger(\mathbf{x}) \partial_i U(\mathbf{x})$$

Weizsacker-Williams Color Electric field ↑

- Decomposition to conventional (unpolarized) and traceless (linearly polarized) contributions

$$xG^{ij}_{WW}(\mathbf{q}) = \frac{1}{2}\delta^{ij} x\, G^{(1)}(\mathbf{q}) - \frac{1}{2}\left(\delta^{ij} - 2\frac{q^i q^j}{q^2}\right) x\, h^{(1)}(\mathbf{q})$$

Talk by D. Boer ...

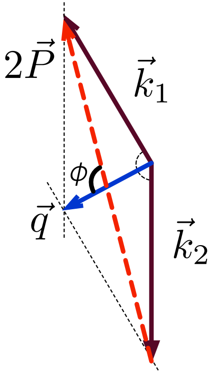
Correlations limit results for $\gamma^*_{\parallel,\perp}$

$$E_1 E_2 \frac{d\sigma^{\gamma_L^* A \to q\bar{q}X}}{d^3k_1 d^3k_2 d^2b} = \alpha_{em} e_q^2 \alpha_s \delta\left(x_{\gamma^*} - 1\right) z^2 (1-z)^2 \frac{8{\epsilon_f}^2 P_\perp^2}{(P_\perp^2 + {\epsilon_f}^2)^4} \times \left[\underbrace{xG^{(1)}(x, q_\perp)}_{\text{func of } q_\perp} + \underline{\cos(2\phi)}\; xh_\perp^{(1)}(x, q_\perp) \right]$$

z is long. momentum fraction of photon carried by quark $\quad \epsilon_f^2 = z(1-z)Q^2$

- **Azimuthal anisotropy is in angle between P and q, denoted by ϕ**

- Is $h_\perp^{(1)}$ important at small x?



$$2\vec{P} = \vec{k}_1 - \vec{k}_2$$
$$\vec{q} = \vec{k}_1 + \vec{k}_2$$

$\mathbf{k}_{1,2} = \mathbf{P} \pm \frac{1}{2}\mathbf{q}$

MV model results

- Analytical result in Gaussian approximation for $G^{(1)}$ and $h_\perp^{(1)}$
- In particular, using McLerran-Venugopalan model

$$xh^{(1)}(x,q^2) = \frac{N_c S_\perp}{2\pi^3\alpha_s}\int d|r|\,|r|J_2(|r|\,|q|)\left[1-\exp\left(-\frac{Q_s^2 r^2}{4}\log\frac{1}{r^2\Lambda_{\rm IR}^2}\right)\right]\frac{1}{r^2\log\frac{1}{r^2\Lambda_{\rm IR}^2}}$$

$$xG^{(1)}(x,q^2) = \frac{N_c S_\perp}{2\pi^3\alpha_s}\int d|r|\,|r|J_0(|r|\,|q|)\left[1-\exp\left(-\frac{Q_s^2 r^2}{4}\log\frac{1}{r^2\Lambda_{\rm IR}^2}\right)\right]\frac{1}{r^2}$$

A. Metz and J. Zhou Phys.Rev. D84 (2011) 051503
F. Dominguez, J.-W. Qiu , B.-W. Xiao, F. Yuan Phys.Rev. D85 (2012) 045003

- Limiting cases:

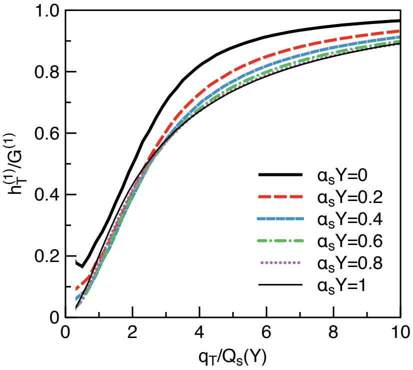
$\Lambda_{IR} \ll q \ll Q_s$, $xh^{(1)} \propto q^0$ and $xG^{(1)} \propto \ln\frac{Q_s^2}{q^2} \rightsquigarrow$ suppression of polarization $\frac{xh^{(1)}}{xG^{(1)}}$
$q \gg Q_s$, $xh^{(1)} \approx xG^{(1)} \propto 1/q^2 \rightsquigarrow$ maximal polarization

Numerics: small x evolution

- McLerran-Venugopalan initial conditions at $Y = \ln x_0/x = 0$

- Quantum evolution towards $x < x_0$ by solving JIMWLK
  using Langevin method

Small x evolution



- Fast departure from MV ($\alpha_s Y = 0$)
- Slow evolution towards smaller x
- Emission of small x gluons reduces degree of polarization.
  $q_\perp$ is scaled by exponentially growing $Q_s(Y)$: ratio at fixed $q_\perp$ decreases with rapidity.
- Approximate scaling at small x

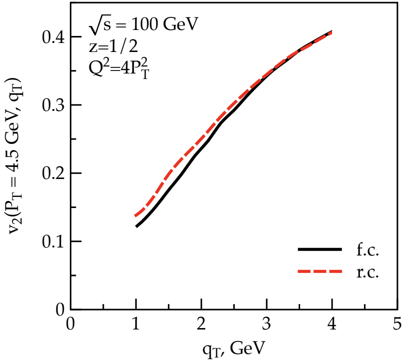
A. Dumitru, T. Lappi & V. S. Phys.Rev.Lett. 115 (2015) 25, 252301

Second harmonics of azimuthal anisotropy: $q_\perp$-dependence

- Azimuthal anisotropy

$$v_2(P_\perp, q_\perp) = \langle \cos 2\phi \rangle$$

- Fixed coupling results ("f.c."): $\alpha_s = 0.15$
- At a fixed $P_\perp$ no significant dependence on prescription for α_s



A. Dumitru, T. Lappi & V. S. Phys.Rev.Lett. 115 (2015) 25, 252301

Can it be measured at an EIC?

- Signal on partonic level; does it survive after jet reconstruction?
- Kinematic range at a future EIC is limited
- How significant signal compared to background?

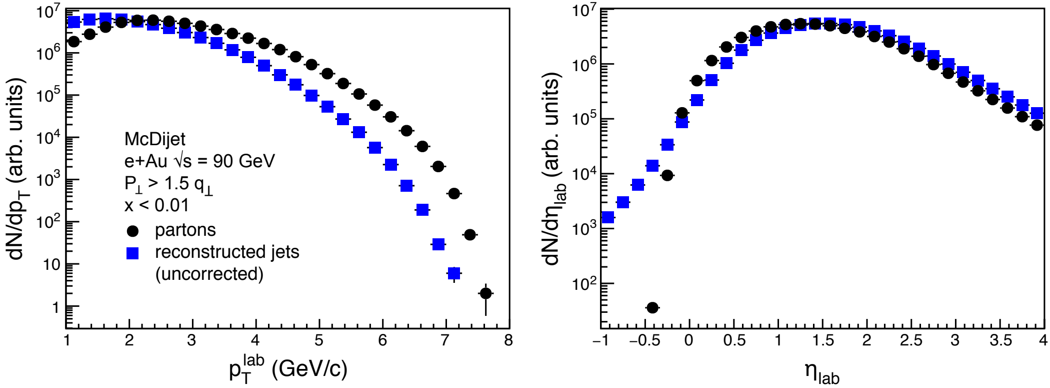
MCDijet: Monte-Carlo generator

To answer these questions: MCDijet

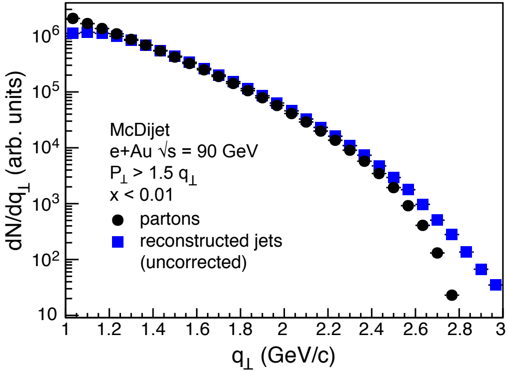
https://github.com/vskokov/McDijet
A. Dumitru, V. S., T. Ullrich, Phys.Rev. C99 (2019), 015204, arXiv:1809.02615

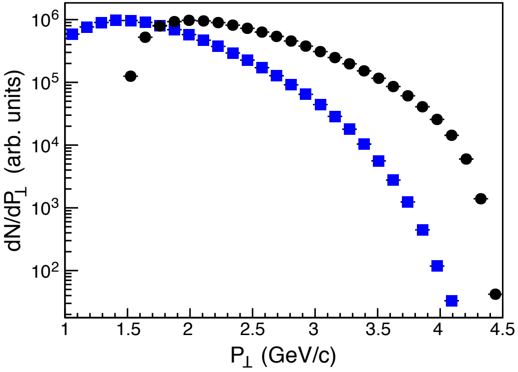
- Input: collision energy $\sqrt{s}$ and atomic number A
- Q_s and target area are adjusted according to A
- Output: partons' 4-momentum etc
- Pythia afterburner: partons $\rightarrow$ particles
- Jet reconstruction
- $\sqrt{s} = 90$ GeV only;
  $\sqrt{s} = 40$ GeV does not provide sufficient kinemaic range to extract signal

- Partons from MCDijet → parton shower algorithm from Pythia 8.2 → jets
- kt-algorithm from FastJet package with cone radius $R = 1$



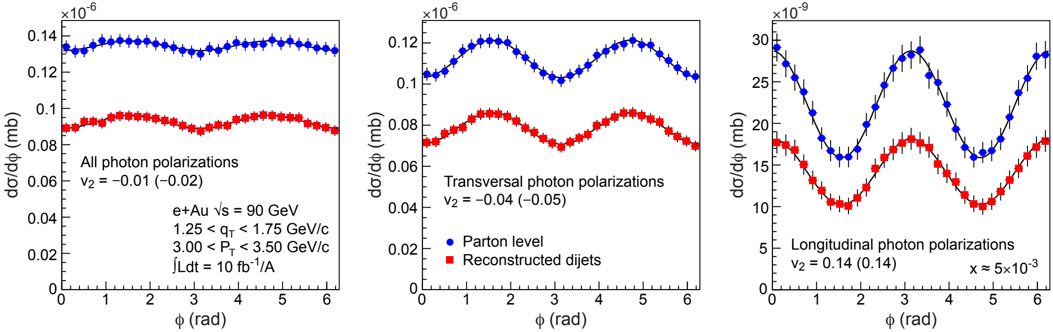
Reconstructed jets vs partons





- Momentum imbalance $q_\perp$ is well reproduced (important to extract distr. funct.)
- Significant distortion for total momentum of dijet $P_\perp$

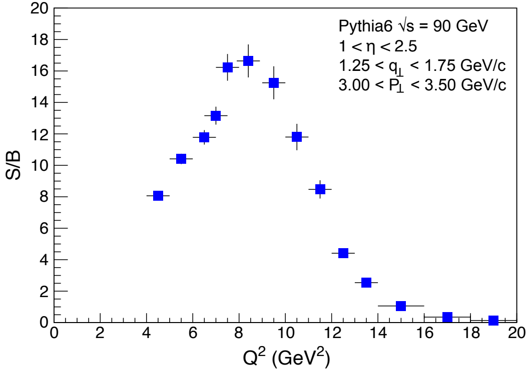
Reconstructed jets vs partons



- $\pi/2$ phase shift between transversal and longitudinal photon polarizations
- Reconstructed jets well reproduce original anisotropy
- Loss of dijet yield $\approx 25\%$ due to low-$p_\perp$ particles

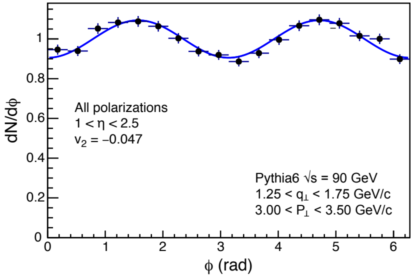
Background

- MCDijet does not generate complete event
- PYTHIA6 to study underlying activity
- Count $f_i + \gamma^* \to f_i + g$ and $g + \gamma^* \to f_i + \bar{f_i}$ as signal; rest as background
- $1 < \eta < 2.5$ to minimize background from beam remnants



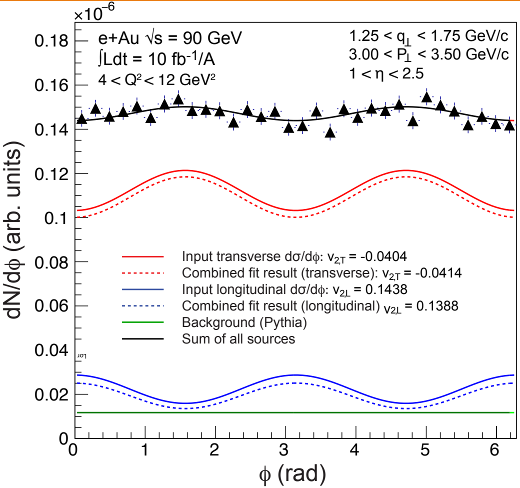
Acceptance: PYTHIA

- PYTHIA generates negative v_2 due to limited η acceptance



- The origin is due to trivial kinematics. Large values of $z \to 1$ are biased toward $\phi \approx \pi$
  Small values of $z \to 0$ are biased toward $\phi \approx 0$
- Finite rapidity acceptance generate a positive and finite v_2
- Measurements at an EIC will need to be corrected for this finite acceptance effect
- We subtract this modulation in our analysis

Extracted signal



- Lines: Combined fit based on LO dijet production cross section
- Proper measurement will require integrated luminosities $> 20\ \text{fb}^{-1}/A$

Outlook

- Ratio $q_\perp/P_\perp$ is not very small; thus corrections to correlation limit can be important
- Corrections come in two forms:

  contributing to higher order harmonics $\cos 4\phi$; they are suppressed by $q_\perp^2/P_\perp^2$

  contributing to isotropic and $\cos 2\phi$; they are suppressed by $Q_s^2/P_\perp^2 \log P_\perp/\Lambda$
- Estimates show that these corrections may modify signal by 25%

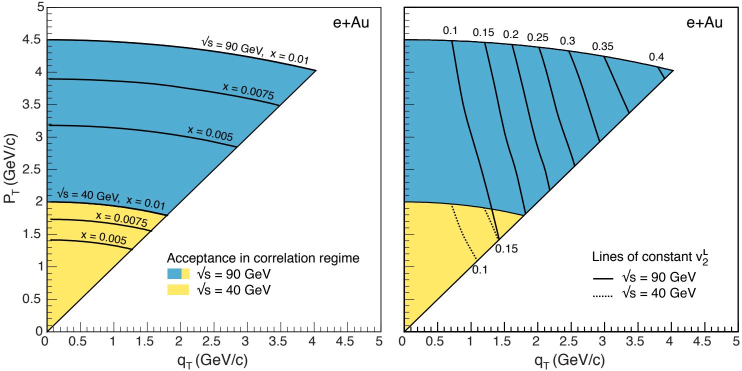
A. Dumitry & V. Skokov, arXiv:1605.02739

Conclusions

- 5-10% azimuthal anisotropy can be expected for EIC kinematics
- Reconstructed dijets reflect original partonic anisotropy remarkably well
- Unavoidable finite acceptance range leads to kinematic bias and non-zero "background" azimuthal anisotropy
  It was subtracted in our analysis
  Measurements will need to be corrected for this acceptance effect
- Based on estimate of background from Pythia: to extract anisotropy, and thus $xh_1^{(g)}/xG_1^{(g)}$ would require integrated luminosity $20 \text{ fb}^{-1}/A$


BACKUP

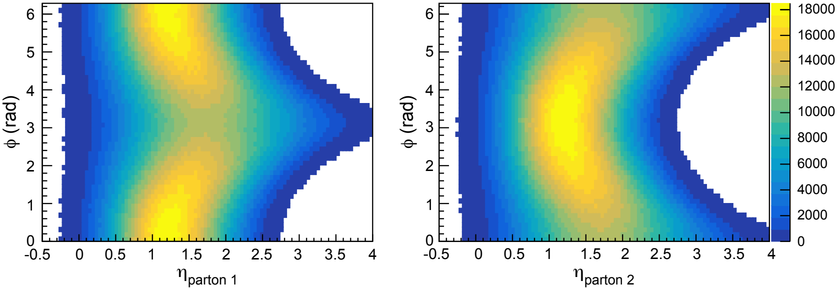
Kinematic range for EIC



- Substantial effect can only be observed at largest energy.
- Magnitude of $P_\perp$ must be sufficiently large to allow jet reconstruction.
- To probe $h^{(1)}$ wide range of $q_\perp$ and $P_\perp$ is required.

Acceptance

- This is all valid for a wide acceptance range in rapidity
- There is momentous correlations of the angle with η



Introduction

At small x, there are two different unintegrated gluon distributions (UGD):

- Dipole gluon distribution ($G^{(2)}$) + linearly polarized partner ($h^{(2)}$).
  Appears in many processes. Small x evolution is well understood.
  Maximal polarization $xh^{(2)} = xG^{(2)}$
- Weizsäcker-Williams (WW) gluon distribution ($G^{(1)}$) + linearly polarized partner ($h^{(1)}$).
  Degree of polarization is x- and transverse momentum dependent

	DIS	DY	SIDIS	$pA \to \gamma \text{ jet } X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$	$pA \to j_1 j_2 X$
$G^{(1)}$ (WW)	×	×	×	×	√	√	√	√
$G^{(2)}$ (DP)	√	√	√	√	×	×	×	√

	$pp \to \gamma\,\gamma\,X$	$pA \to \gamma^* \text{ jet } X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$h^{(1)}$ (WW)	√	×	√	√	√
$h^{(2)}$ (DP)	×	√	×	×	×

Daniël Boer, arXiv:1611.06089

First correction to correlation limit at small x I

- General small x expression for dijet cross section

$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^3k_1 d^3k_2} =$$

$$N_c\alpha_{em}e_q^2\delta(p^+ - k_1^+ - k_2^+)\int \frac{d^2x_1}{(2\pi)^2}\frac{d^2x_2}{(2\pi)^2}\frac{d^2y_1}{(2\pi)^2}\frac{d^2y_2}{(2\pi)^2}\exp\left(-i\mathbf{k}_1(\mathbf{x}_1-\mathbf{y}_1)-i\mathbf{k}_2(\mathbf{x}_2-\mathbf{y}_2)\right)$$

$$\sum_{\gamma\alpha\beta}\psi_{\alpha\beta}^{\mathrm{T,L}\gamma}(\mathbf{x}_1-\mathbf{x}_2)\psi_{\alpha\beta}^{\mathrm{T,L}\gamma*}(\mathbf{y}_1-\mathbf{y}_2)\Big[1+\frac{1}{N_c}\Big(\langle \mathrm{Tr}\ U(\mathbf{x}_1)U^\dagger(\mathbf{y}_1)U(\mathbf{y}_2)U^\dagger(\mathbf{x}_2)\rangle$$

$$-\langle \mathrm{Tr}\ U(\mathbf{x}_1)U^\dagger(\mathbf{x}_2)\rangle - \langle \mathrm{Tr}\ U(\mathbf{y}_1)U^\dagger(\mathbf{y}_2)\rangle\Big)\Big]$$

↑ Quadrupole contribution

- For arbitrary $\mathbf{k}_1$ and $\mathbf{k}_2$, one expects presence of non-trivial $\langle\cos 2n\phi\rangle, n\in Z$
- First correction to correlation limit (suppressed by $1/P^2$) includes terms $\propto (\mathbf{q}\cdot\mathbf{P})^4$ and thus results in $\langle\cos 4\phi\rangle \neq 0$

First correction to correlation limit at small x II

- Derivation is tedious but straight forward (see details in 1605.02739)
- Expectation of Wilson lines

$$\mathcal{Q}(\mathbf{x}_1, \mathbf{x}_2; \mathbf{x}_2', \mathbf{x}_1') = 1 + \frac{\langle \text{Tr } U(\mathbf{x}_1)U^\dagger(\mathbf{x}_1')U(\mathbf{x}_2')U^\dagger(\mathbf{x}_2)\rangle - \langle \text{Tr } U(\mathbf{x}_1)U^\dagger(\mathbf{x}_2)\rangle - \langle \text{Tr } U(\mathbf{x}_1')U^\dagger(\mathbf{x}_2')\rangle}{N_c}$$

is expanded in series wrt $\mathbf{u} = \mathbf{x}_1 - \mathbf{x}_2$ and $u' = \mathbf{x}_1' - \mathbf{x}_2'$:

$$\mathcal{Q} = u_i u_j' \mathcal{G}^{i,j}(v, v') + u_i u_j' u_k' u_l' \mathcal{G}^{i,jkl}(v, v') + u_i u_j u_k u_l' \mathcal{G}^{ijk,l}(v, v') + u_i u_j u_k' u_l' \mathcal{G}^{ij,kl}(v, v') + \cdots$$

- Following combination is relevant (momentum space)

$$\mathcal{G}^{ijmn}(x, q^2) = \mathcal{G}^{i,jmn}(x, q^2) + \mathcal{G}^{ijm,n}(x, q^2) - \frac{2}{3}\mathcal{G}^{ij,mn}(x, q^2)$$

- $\mathcal{G}^{ijmn}(x, q^2)$ results in corrections to isotropic and $\langle \cos 2\phi \rangle$, as well as non-trivial $\langle cos 4\phi \rangle$. I will focus on $\langle \cos 4\phi \rangle$.

First correction to correlation limit at small x III

- The amplitude of $\cos 4\phi$ is determined by

$$\Phi_2(x,q^2) = -\frac{2N_c}{\alpha_s}\mathfrak{P}_3^{ijmn}\mathcal{G}^{ijmn}(x,q^2)\ .$$

where $\mathfrak{P}_3^{ijmn}$ is projector extracting $\propto \cos 4\phi$

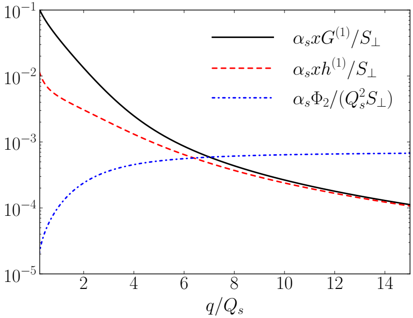
$$\mathfrak{P}_3^{ijmn} = -\frac{1}{6\sqrt{2}}\left(\delta_{ij}\delta_{mn}+\delta_{im}\delta_{jn}+\delta_{jm}\delta_{in}-2(\Pi_{ij}\Pi_{mn}+\Pi_{im}\Pi_{jn}+\Pi_{jm}\Pi_{in})\right); \quad \Pi_{ij}=\delta_{ij}-\frac{2q_iq_j}{q^2}$$

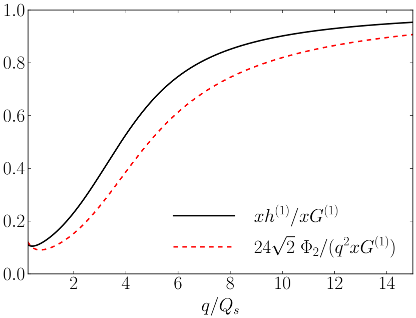
- For MV model

$$\Phi_2(q^2) = \frac{N_c}{\sqrt{2}\,3\pi\alpha_s}\frac{S_\perp}{(2\pi)^2}\int\frac{d|r|}{|r|^3}J_4(|r|\,|q|)\left[\frac{2}{\ln\frac{1}{r^2\Lambda_{\rm IR}^2}}\left\{1-\exp\left(-\frac{Q_s^2r^2}{4}\log\frac{1}{r^2\Lambda_{\rm IR}^2}\right)\right\}\right.$$
$$\left.+\frac{5}{\ln^2\frac{1}{r^2\Lambda_{\rm IR}^2}}\left\{1-\exp\left(-\frac{Q_s^2r^2}{4}\log\frac{1}{r^2\Lambda_{\rm IR}^2}\right)\left[1+\frac{Q_s^2r^2}{4}\log\frac{1}{r^2\Lambda_{\rm IR}^2}\right]\right\}\right]$$

- $\Phi_2(q^2)$ is positive-definite function
- Limiting cases: $\Lambda_{\rm IR}\ll q\ll Q_s$ $\quad \Phi_2(q^2)\sim(N_c/\alpha_s\log Q_s^2/\Lambda_{\rm IR}^2)\,S_\perp q^2$; $q\gg Q_s$, $\quad \Phi_2(q^2)\to(N_c/\sqrt{2}\,24\pi\alpha_s)\,(S_\perp/4\pi^2)\,Q_s^2$

MV results





These functions determine amplitudes of $\cos 2n\phi$ contributions to dijet angular distributions for $n = 0$, 1, 2, respectively.

A. Dumitru and V. S., arXiv:1605.02739

Dijet cross section

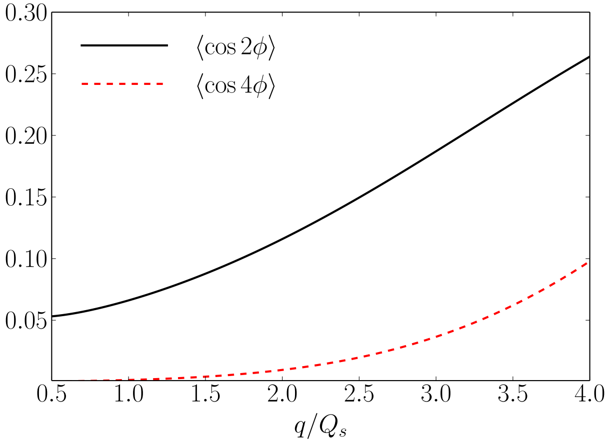
Dijet cross section to this order

$$
\begin{aligned}
&\frac{d\sigma^{\gamma_T^* A\to q\bar{q}X}}{d^2k_1dz_1d^2k_2dz_2} \\
&\quad = \alpha_s\alpha_{em}e_q^2\left(z_1^2+z_2^2\right)\left[\frac{P^4+\epsilon_f^4}{(P^2+\epsilon_f^2)^4}\left(xG^{(1)}(x,q^2)-\frac{2\epsilon_f^2P^2}{P^4+\epsilon_f^4}xh^{(1)}(x,q^2)\cos 2\phi+\mathcal{O}\left(\frac{1}{P^2}\right)\right)\right. \\
&\qquad\qquad \left.-\frac{48\epsilon_f^2P^4}{\sqrt{2}\,(P^2+\epsilon_f^2)^6}\Phi_2(x,q^2)\cos 4\phi\right]
\end{aligned}
$$

$$
\begin{aligned}
&\frac{d\sigma^{\gamma_L^* A\to q\bar{q}X}}{d^2k_1dz_1d^2k_2dz_2} \\
&\quad = 8\alpha_s\alpha_{em}e_q^2z_1z_2\epsilon_f^2\left[\frac{P^2}{(P^2+\epsilon_f^2)^4}\left(xG^{(1)}(x,q^2)+xh^{(1)}(x,q^2)\cos 2\phi+\mathcal{O}\left(\frac{1}{P^2}\right)\right)\right. \\
&\qquad\qquad \left.+\frac{48P^4}{\sqrt{2}\,(P^2+\epsilon_f^2)^6}\Phi_2(x,q^2)\cos 4\phi\right] .
\end{aligned}
$$

MV results

$\langle \cos 2\phi \rangle$ and $\langle \cos 4\phi \rangle$ in $\gamma_L^* + A \to q + \bar{q}$ dijet production from MV model:



$z = 1/2, \ P = 4.5 Q_s$

$\langle \cos 4\phi \rangle$ can be safely neglected in first approximation

Monte-Carlo Event generator

- McDijet: Dijet in DIS event generator https://github.com/vskokov/McDijet
- Input: collision energy $\sqrt{s}$ and atomic number A
- Q_s and target area are adjusted according to A
- Output: partons' 4-momentum etc
- Pythia afterburner: partons $\to$ particles
- Jet reconstruction

Goal is to study feasibility of extracting signal and its dependence on atomic number, A, and collision energy, $\sqrt{s}$