NLO computations for exclusive processes at small x

Renaud Boussarie

Brookhaven National Laboratory

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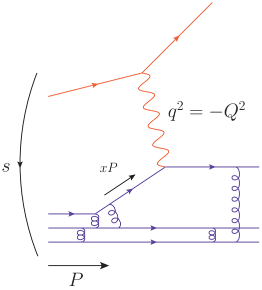
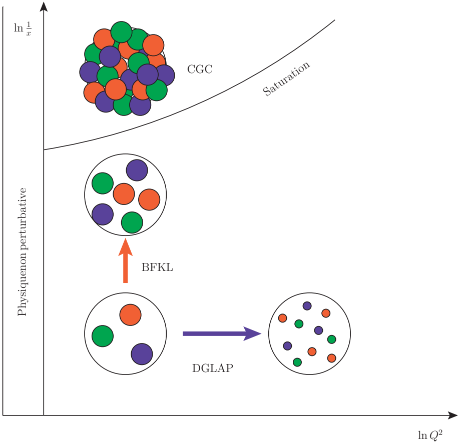
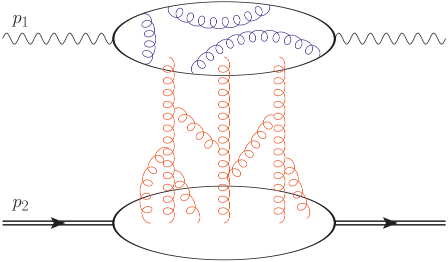
Accessing the partonic content of hadrons with an electromagnetic probe



Electron-proton collision (parton model)



Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2s} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone (Sudakov) vectors

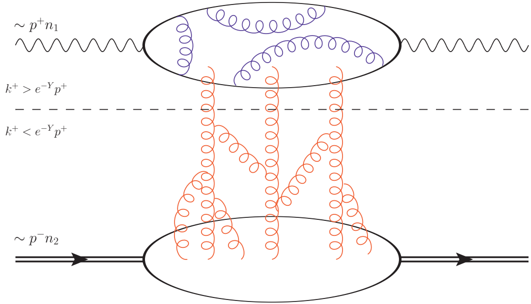
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \to (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation

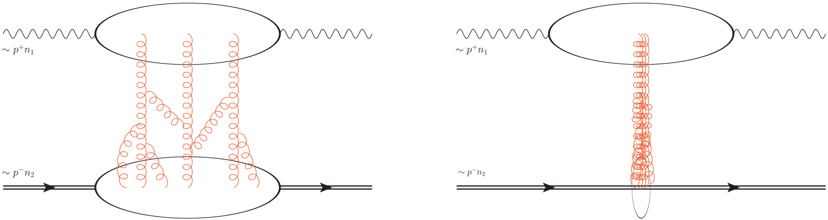


Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}) + b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k})$$

$e^\eta = e^{-Y} \ll 1$

Large longitudinal boost to the projectile frame



$$b^+(x^+, x^-, \vec{x}) \qquad\qquad \frac{1}{\Lambda} b^+\left(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}\right)$$

$$b^-(x^+, x^-, \vec{x}) \qquad \longrightarrow \qquad \Lambda b^-\left(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}\right)$$

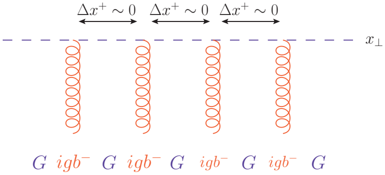
$$b^k(x^+, x^-, \vec{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad b^k\left(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}\right)$$

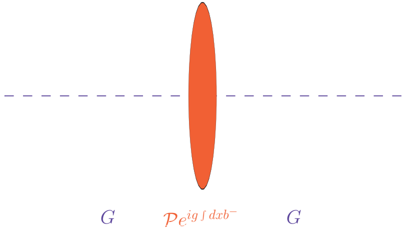
$$b^\mu(x) \to b^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O\left(\sqrt{\frac{m_t^2}{s}}\right)$$

Shockwave approximation

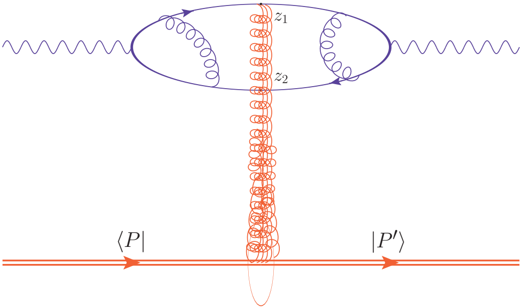
Effective Feynman rules in the slow background field

The interactions with the background field can be exponentiated





Factorized picture



Factorized amplitude

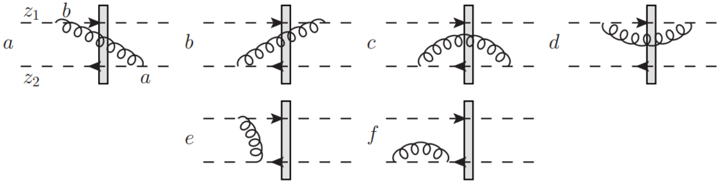
$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2 \, \Phi^\eta(\vec{z}_1, \vec{z}_2) \, \langle P' | [\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c] | P \rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Evolution for the dipole operator

$$\mathcal{U}_{12}^{\eta+\delta\eta} - \mathcal{U}_{12}^{\eta}$$



B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

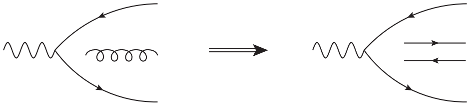
$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta} + \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}\right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}}{\partial \eta} = \ldots$$

Evolves a dipole into a double dipole

The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in the dipole B-JIMWLK equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

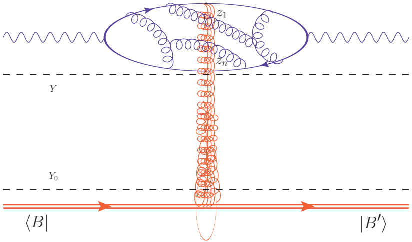
$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle \, \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

BFKL/BKP part Triple pomeron vertex

Non-linear term : saturation

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$.
- Evaluate the solution at a typical projectile rapidity $\eta = Y$, or at the rapidity of the slowest gluon
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n) \times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

Exclusive diffraction allows one to probe the $b_\perp$-dependence of the non-perturbative scattering amplitude

Exclusive diffractive production of a forward dijet

[R.B.,A.V.Grabovsky,L.Szymanowski,S.Wallon]

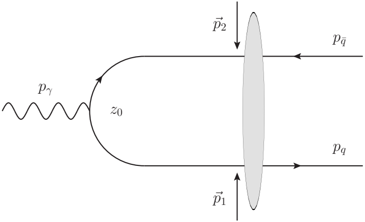
JHEP 1611 (2016) 149

Exclusive dijet production

- Regge-Gribov limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- Otherwise completely general kinematics
- Shockwave (CGC) Wilson line approach
- Transverse dimensional regularization $d = 2 + 2\varepsilon$, longitudinal cutoff

$$|p_g^+| > \alpha p_\gamma^+$$

LO diagram



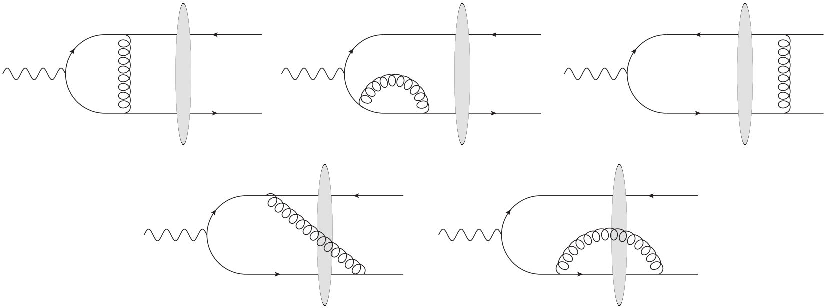
$$\mathcal{A} = \frac{\delta^{ik}}{\sqrt{N_c}} \int d^D z_0 [\bar{u}(p_q, z_0)]_{ij} (-ie_q) \hat{\varepsilon}_\gamma e^{-i(p_\gamma \cdot z_0)} [v(p_{\bar{q}}, z_0)]_{jk} \theta(-z_0^+)$$

$$= \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \, \Phi_0(\vec{p}_1, \vec{p}_2)$$

$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

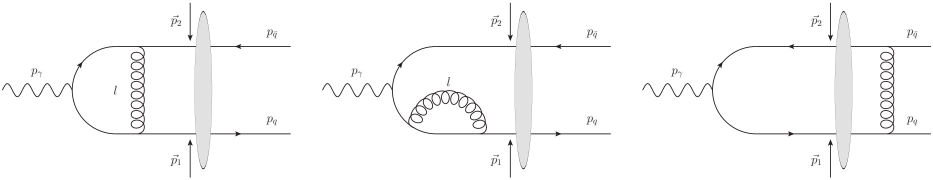
$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d \vec{z}_1 d^d \vec{z}_2 \, e^{-i(\vec{p}_1 \cdot \vec{z}_1) - i(\vec{p}_2 \cdot \vec{z}_2)} [\tfrac{1}{N_c} \mathrm{Tr}(U^\alpha_{\vec{z}_1} U^{\alpha\dagger}_{\vec{z}_2}) - 1]$$

NLO open $q\bar{q}$ production



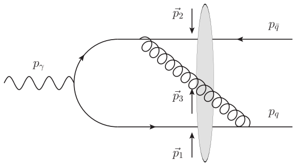
Diagrams contributing to the NLO correction

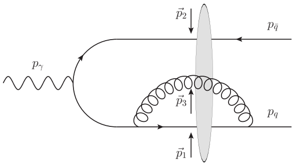
First kind of virtual corrections



$$\mathcal{A}_{NLO}^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_{V1}(\vec{p}_1, \vec{p}_2) \times C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle$$

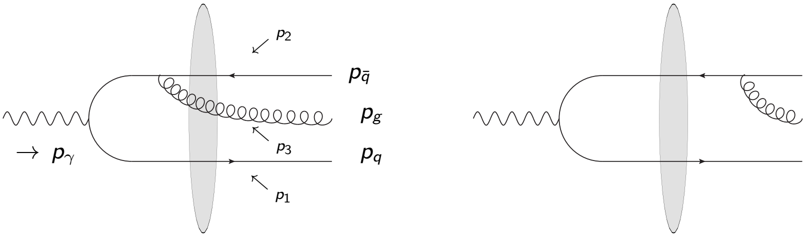
Second kind of virtual corrections





$$\mathcal{A}_{NLO}^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle$$
$$+ \Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \right| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left| P \right\rangle]$$

LO open $q\bar{q}g$ production



$$\mathcal{A}_R^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{R1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$
$$+ \Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \middle| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \middle| P \right\rangle]$$

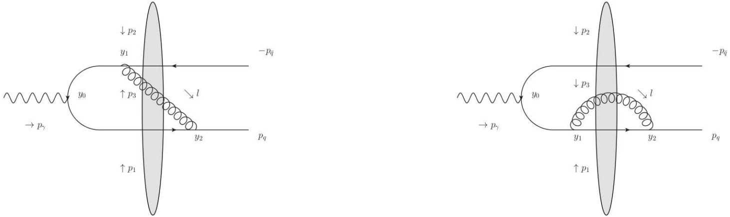
$$\mathcal{A}_R^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$
$$\times \Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

Divergences

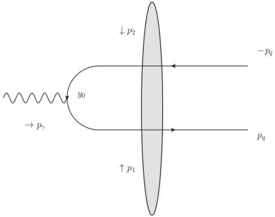
Divergences

- Rapidity divergence $p_g^+ \to 0$ — $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ — $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ — $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$, $\Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ — $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ — $\Phi_{R1}\Phi_{R1}^*$

Rapidity divergence



Double dipole virtual correction Φ_{V2}



B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Rapidity divergence

B-JIMWLK equation for the dipole operator

$$\frac{\partial \tilde{\mathcal{U}}_{12}^{\alpha}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{\mathcal{U}}_{13}^{\alpha} \tilde{\mathcal{U}}_{32}^{\alpha} + \tilde{\mathcal{U}}_{13}^{\alpha} + \tilde{\mathcal{U}}_{32}^{\alpha} - \tilde{\mathcal{U}}_{12}^{\alpha} \right)$$
$$\times \left[2 \frac{(\vec{k}_1 - \vec{p}_1) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{\left[(\vec{k}_1 - \vec{p}_1)^2 \right]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{\left[(\vec{k}_2 - \vec{p}_2)^2 \right]^{1-\frac{d}{2}}} \right) \right]$$

η rapidity divide, which separates the upper and the lower impact factors

$$\Phi_0 \tilde{\mathcal{U}}_{12}^{\alpha} \to \Phi_0 \tilde{\mathcal{U}}_{12}^{\eta} + 2 \log\left(\frac{e^{\eta}}{\alpha} \right) \mathcal{K}_{BK} \Phi_0 \tilde{\mathcal{W}}_{123}$$

Provides a counterterm to the $log(\alpha)$ divergence in the virtual double dipole impact factor:

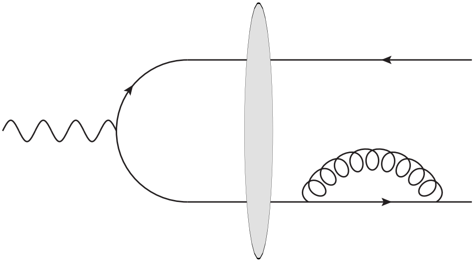
$\Phi_0 \tilde{\mathcal{U}}_{12}^{\alpha} + \Phi_{V2} \tilde{\mathcal{W}}_{123}^{\alpha}$ is finite and independent of α

Divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\quad\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\quad\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\quad\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+}p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+}p_{\bar{q}}$, $p_g^+ \to 0$ $\quad\Phi_{R1}\Phi_{R1}^*$

UV divergence

Dressing of the external lines



Some null diagrams just contribute to turning UV divergences into IR divergences

$$\Phi = 0 \propto \left(\frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}} \right)$$

Divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*,\ \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+}p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+}p_{\bar{q}},\ p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

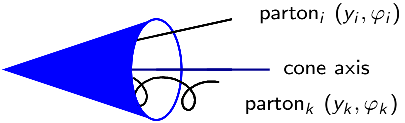
Soft and collinear divergence

Jet cone algorithm

We define a cone width for each pair of particles with momenta p_i and p_k, rapidity difference ΔY_{ik} and relative azimuthal angle $\Delta\varphi_{ik}$

$$(\Delta Y_{ik})^2 + (\Delta\varphi_{ik})^2 = R_{ik}^2$$

If $R_{ik}^2 < R^2$, then the two particles together define a single jet of momentum $p_i + p_k$.



Applying this in the small R^2 limit cancels our soft and collinear divergence.

Divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence

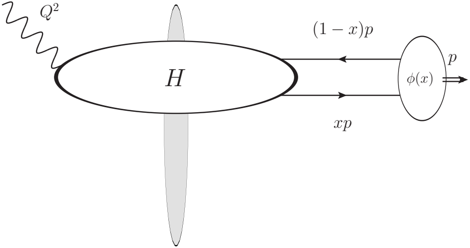
The remaining divergences cancel the standard way:
virtual corrections and real corrections cancel each other

Exclusive production of a forward light neutral vector meson

[RB, Grabovsky, Ivanov, Szymanowski, Wallon]
Phys.Rev.Lett. 119 (2017) ; arXiv:1612.08026

s-channel collinear factorization

Twist 2: ρ_L production



$$\int d^4 z \mathcal{H}^{ij}_{\alpha\beta}(z) \left\langle \rho(p) \left| \bar{\psi}^i_\alpha(z) \psi^j_\beta(0) \right| 0 \right\rangle$$

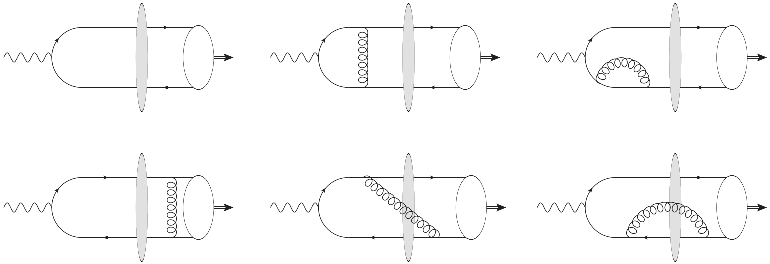
$$\to \frac{1}{4N_c} \int d^4 z \ \mathrm{tr}_{c,D} \left[\mathcal{H}(z) \Gamma^\lambda \right] \left\langle \rho(p) \left| \bar{\psi}(z) \Gamma_\lambda \psi(0) \right| 0 \right\rangle$$

Singlet transition $\Rightarrow$ only virtual diagrams contribute.

Leading twist matrix element:

$$\left\langle \rho_L(p) \left| \bar{\psi}(z) \gamma^\lambda \psi(0) \right| 0 \right\rangle \quad \to \quad f_\rho m_\rho p^\lambda \int_0^1 dx \ e^{-ixp\cdot z} \varphi_\parallel(x)$$

Exclusive diffractive production of a light neutral vector meson



$$\mathcal{A}_0 = -\frac{e_V f_V \varepsilon_\beta}{N_c}\int_0^1 dx\varphi_\parallel(x)\int\frac{d^d\vec{p}_1}{(2\pi)^d}\frac{d^d\vec{p}_2}{(2\pi)^d}$$
$$\times\ (2\pi)^{d+1}\delta\left(p_V^+ - p_\gamma^+\right)\delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2\right)$$
$$\times\ \Phi_0^\beta\left(x,\ \vec{p}_1,\ \vec{p}_2\right)\tilde{\mathcal{U}}_{12}^\eta.$$

Leading twist for a longitudinally polarized meson
Otherwise general kinematics, including transverse virtual photon (twist 3) contributions, and the photoproduction limit (for large $t-$channel momentum transfer)

ERBL evolution equation

Efremov, Radyushkin, Brodsky, Lepage evolution equation for a DA

Renormalization of the bilocal operator

$$\bar{\psi}(z)\gamma^{\mu}\psi(0)$$

$\Rightarrow$ Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial\varphi(x,\mu_F^2)}{\partial\ln\mu_F^2} = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{(4\pi)^{\epsilon}}\left(\frac{\mu_F^2}{\mu^2}\right)^{\epsilon}\int_0^1 dz\varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

$\mathcal{K}$ = ERBL kernel

ERBL evolution equation

Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial \varphi(x,\mu_F^2)}{\partial \ln \mu_F^2} = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\left(\frac{\mu_F^2}{\mu^2}\right)^\epsilon \int_0^1 dz \varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

where we parameterize the ERBL kernel for consistency as

$$\begin{aligned}\mathcal{K}(x,z) &= \frac{x}{z}\left[1+\frac{1}{z-x}\right]\theta(z-x-\alpha) \\ &+ \frac{1-x}{1-z}\left[1+\frac{1}{x-z}\right]\theta(x-z-\alpha) \\ &+ \left[\frac{3}{2}-\ln\left(\frac{x(1-x)}{\alpha^2}\right)\right]\delta(z-x).\end{aligned}$$

It is equivalent to the usual ERBL kernel

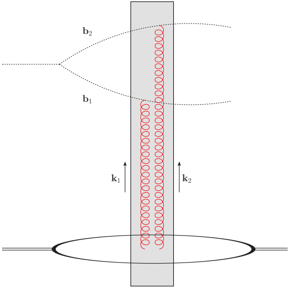
It provides the right counterterm to obtain a finite amplitude

Quick remarks on the equivalence
with standard parton distributions

Exclusive low x cross section

Exclusive low x amplitude = GTMD amplitude: LO

[Altinoluk, RB]



$$\mathcal{H}^{ij}\left(k_{1\perp},\, k_{2\perp}\right) \otimes \left\langle P' \left| F^{-i} W F^{-j} W \right| P \right\rangle$$

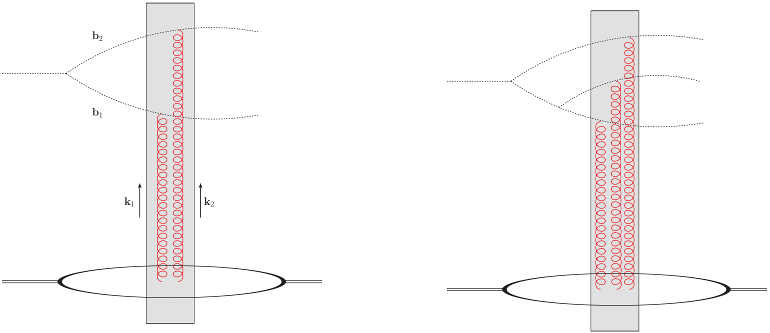
Every exclusive low x process probes a Wigner distribution!

Proven at leading twist by [Hatta, Xiao, Yuan] for DIS, extended to infinite power for any color flow

Exclusive low x cross section

Exclusive low x amplitude = GTMD amplitude: NLO?

[RB, Mehtar Tani?]



Common tools

From the low x/TMD equivalence:

- Target Sudakov log resummation for small x processes [Mueller, Xiao, Yuan], [Xiao, Yuan, Zhou]
- Phenomenological Sudakov log simulation [Kotko, Kutak, Sapeta, Stasto, Strikman], [Van Hameren, Kotko, Kutak, Sapeta]
- TMD evolution could be one way of fixing the negativity issues for low x cross sections at NLO
- A full reinterpretation of the multiple scattering effects in the so-called saturation regime [Altinoluk, RB]
- A better understanding of the importance of linearly polarized gluon distributions at small x [See D. Boër and V.Skokov's talk]
    - In particular, any small x observable has maximally polarized gluon distributions at large k_t [Altinoluk, RB]

Conclusion

- We provided the full computation of the impact factor for the exclusive diffractive production of a dijet and of a light neutral vector meson with NLO accuracy in the shockwave approach
- These processes both constrain gluon Wigner distributions
- Our results are thus perfectly suited for precision saturation physics and gluon tomography at the EIC

Backup

Remaining divergence

Soft real emission

$$\left(\Phi_{R1}\Phi_{R1}^*\right)_{soft} \propto \left(\Phi_0\Phi_0^*\right)\int_{\text{outside the cones}} \left|\frac{p_q^\mu}{(p_q.p_g)} - \frac{p_{\bar{q}}^\mu}{(p_{\bar{q}}.p_g)}\right|^2 \frac{dp_g^+}{p_g^+}\frac{d^d p_g}{(2\pi)^d}$$

Collinear real emission

$$\left(\Phi_{R1}\Phi_{R1}^*\right)_{col} \propto \left(\Phi_0\Phi_0^*\right)\left(\mathcal{N}_q + \mathcal{N}_{\bar{q}}\right)$$

Where $\mathcal{N}$ is the number of jets in the quark or the antiquark

$$\mathcal{N}_k = \frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2-\frac{d}{2})}\int_{\alpha p_\gamma^+}^{p_{jet}^+}\frac{dp_g^+ dp_k^+}{2p_g^+ 2p_k^+}\int_{\text{in cone k}}\frac{d^d\vec{p}_g d^d\vec{p}_k}{(2\pi)^d\,\mu^{d-2}}\frac{\mathrm{Tr}\left(\hat{p}_k\gamma^\mu\hat{p}_{jet}\gamma^\nu\right)d_{\mu\nu}(p_g)}{2p_{jet}^+\left(p_k^- + p_g^- - p_{jet}^-\right)^2}$$

Those two contributions cancel exactly the virtual divergences

Total divergence

$$(d\sigma_1)_{div} = \alpha_s \frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}} \left(\frac{N_c^2-1}{2N_c}\right) (S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2}) d\sigma_0$$

Virtual contribution

$$\begin{aligned} S_V &= \left[2\ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - 3\right]\left[\ln\left(\frac{x_j x_{\bar{j}} \mu^2}{(x_j \vec{p}_{\bar{j}} - x_{\bar{j}} \vec{p}_j)^2}\right) - \frac{1}{\epsilon}\right] \\ &+ 2i\pi \ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) + \ln^2\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - \frac{\pi^2}{3} + 6 \end{aligned}$$

Real contribution

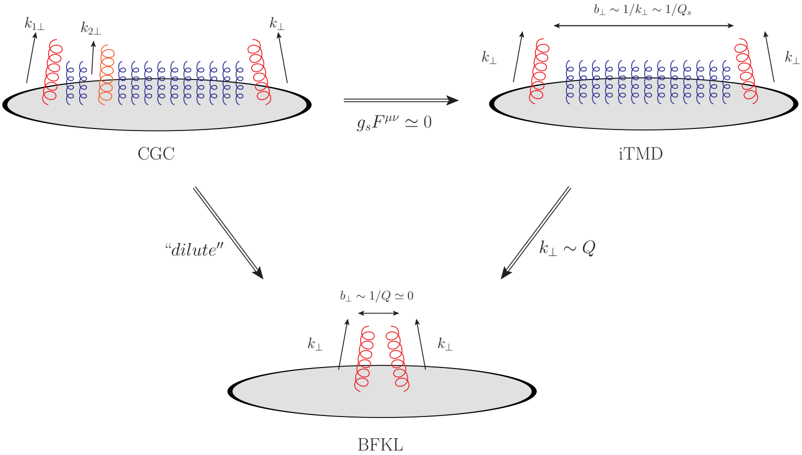
$$\begin{aligned} S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} &= 2\left[\ln\left(\frac{(x_{\bar{j}} \vec{p}_j - x_j \vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2} \vec{p}_j^{\,2}}\right) \ln\left(\frac{4E^2}{x_{\bar{j}} x_j (p_\gamma^+)^2}\right)\right. \\ &+ 2\ln\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right)\left(\frac{1}{\epsilon} - \ln\left(\frac{x_{\bar{j}} x_j \mu^2}{(x_{\bar{j}} \vec{p}_j - x_j \vec{p}_{\bar{j}})^2}\right)\right) - \ln^2\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right) \\ &+ \left.\frac{3}{2}\ln\left(\frac{16\mu^4}{R^4 \vec{p}_j^{\,2} \vec{p}_{\bar{j}}^{\,2}}\right) - \ln\left(\frac{x_j}{x_{\bar{j}}}\right) \ln\left(\frac{x_j \vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}} \vec{p}_j^{\,2}}\right) - \frac{3}{\epsilon} - \frac{2\pi^2}{3} + 7\right] \end{aligned}$$

Total "divergence"

$$\begin{aligned} div &= S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} \\ &= 4\left[\frac{1}{2}\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\left(\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right) + \frac{3}{2}\right)\right. \\ &+ \left.\ln(8) - \frac{1}{2}\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right) + \frac{13-\pi^2}{2}\right] \end{aligned}$$

Our cross section is thus finite

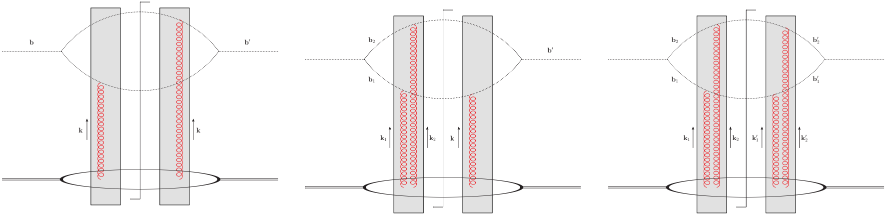
The dilute limit in terms of TMD distributions



Two kinds of multiple scattering effects: higher genuine twists and higher kinematic twists

Inclusive low x cross section = TMD cross section

[Altinoluk, RB, Kotko], [Altinoluk, RB]



$$
\begin{aligned}
\sigma = {} & \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \left| F^{-i} W F^{-j} W \right| P \right\rangle \\
& + \mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W F^{-k} W \right| P \right\rangle \\
& + \mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \right| P \right\rangle
\end{aligned}
$$