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Topics in QCD from DSEs

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2019 EIC User Group Meeting

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Motivation

Extract properties of hadrons from QCD

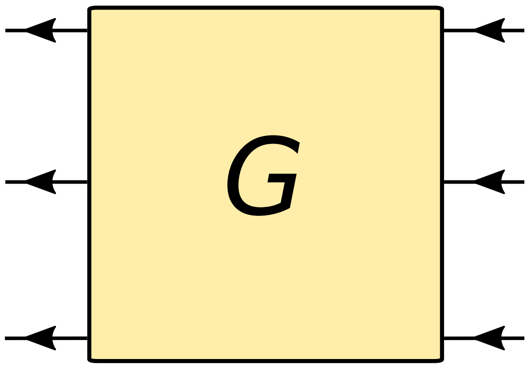
- Propagators and vertices
- Formulate description of bound-states in the continuum.

Test truncations against Hadronic Spectrum

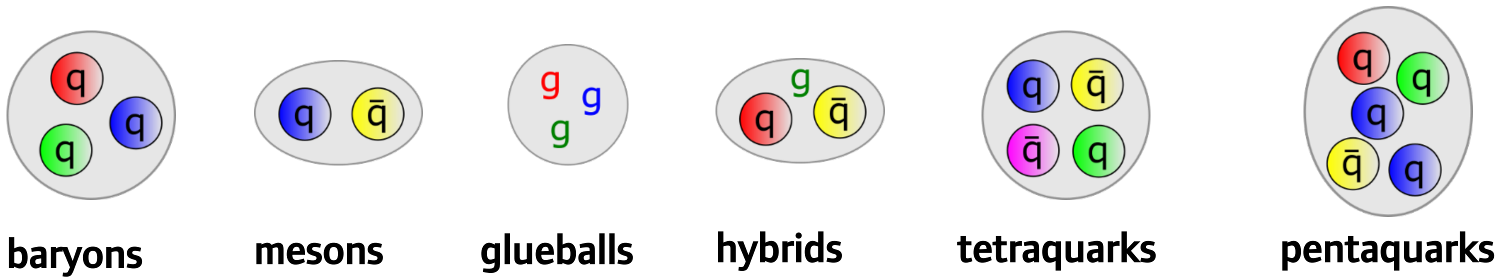
- Include/Exclude interaction terms

Interaction terms responsible for

- Binding quarks and (anti)quarks
- Unquenching effects
- Decay channels
- Splitting between parity partners …

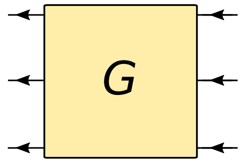


Extract from Green's functions



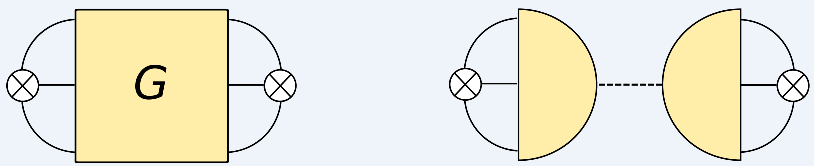
Hadronic States

Bound-states ↔ Poles



$$\mathbf{G}_{\alpha\beta\gamma;\alpha'\beta'\gamma'} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma\bar{\psi}_{\alpha'}\bar{\psi}_{\beta'}\bar{\psi}_{\gamma'}\,|0\rangle$$

Lattice:

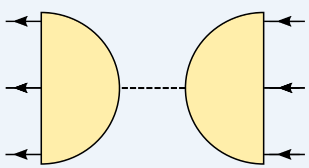


Construct gauge-invariant current correlators

$$e^{-mt} \quad \Longleftrightarrow \quad \frac{1}{p^2+m^2}$$

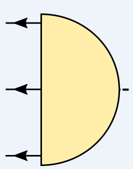
Exponential time-decay.

BSE:



$$\mathbf{G} \sim \sum_\lambda \frac{\Psi^\lambda \bar{\Psi}^\lambda}{p^2+m_\lambda^2}$$

Spectral decomposition.

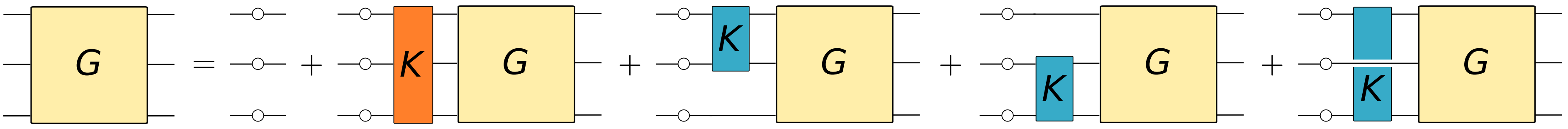


$$\mathbf{\Psi}^\lambda_{\alpha\beta\gamma} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma\,|\lambda\rangle$$

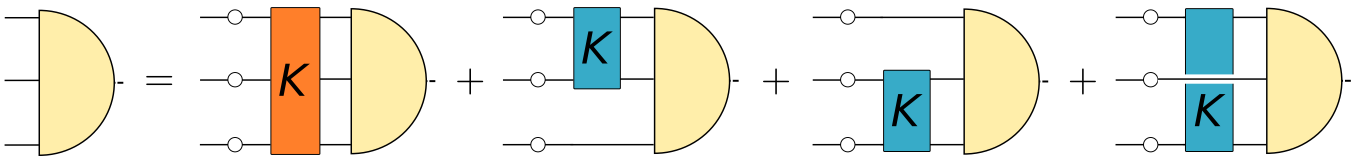
BS wavefunction

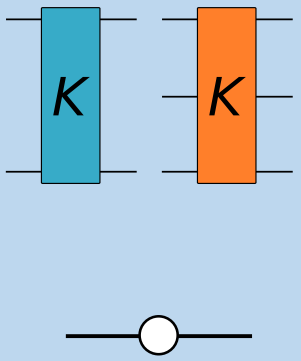
Hadronic States

Trade one unknown G, for another unknown K



Solution (on-shell) yields **Bethe-Salpeter equation**

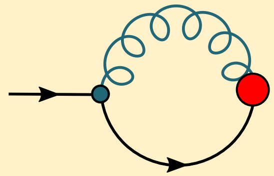


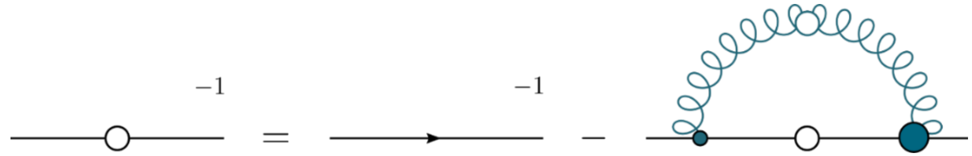


- Irreducible **2**-, **3**-, **n**-body kernels **define** equation
- **Dressed** particle constituents: Green's functions

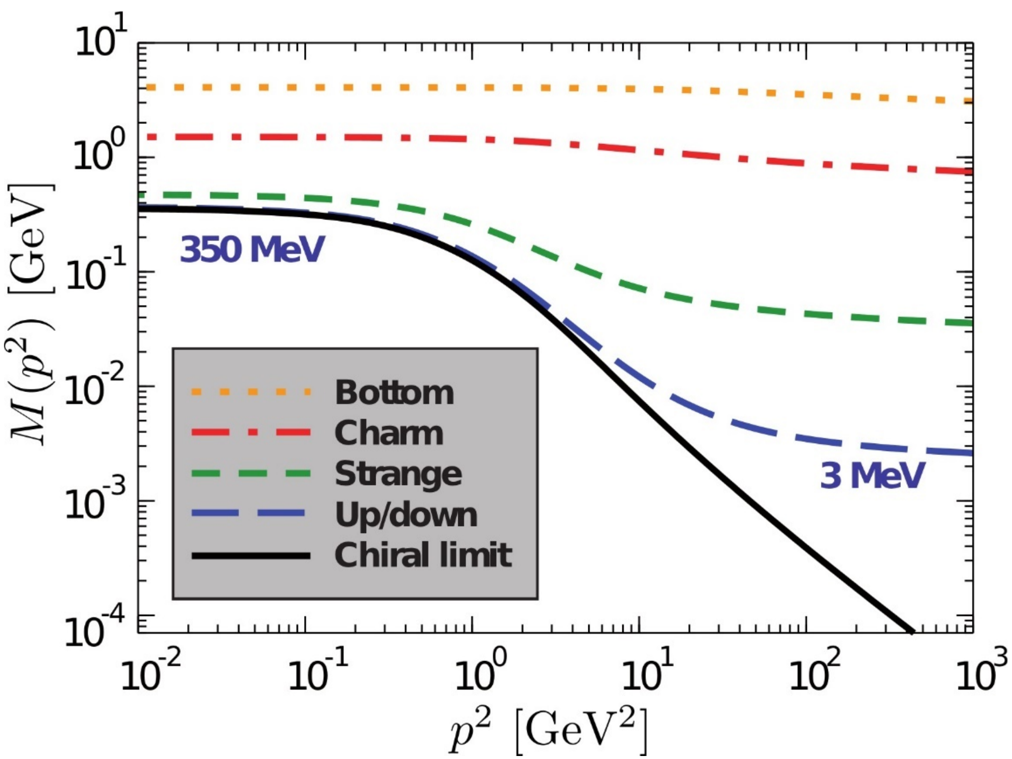
Dyson-Schwinger equations (DSE)

Provide dressed propagators (and in turn, vertices)

$$\frac{\delta\Gamma[\phi]}{\delta\psi} = \frac{\delta S[\phi]}{\delta\psi} +$$ 



$$S^{-1}(p) = A(p^2)\left(-i\not{p} + M(p^2)\right)$$



Dressed quantities means:

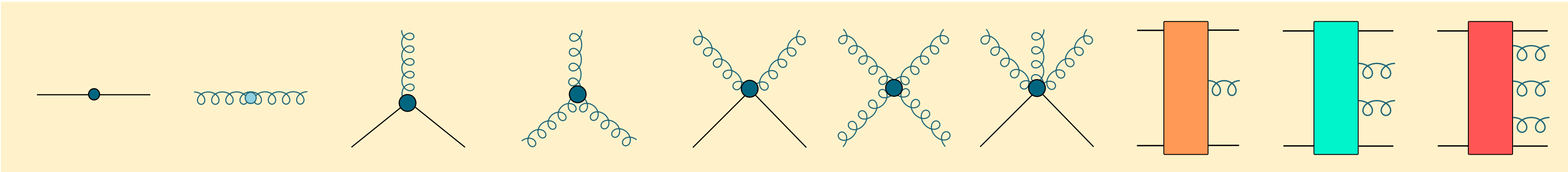
- Mass function runs
- Coupling runs
- Vertices run

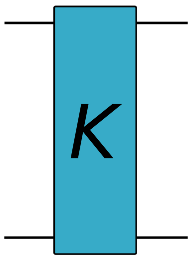
i.e. everything runs!

Difficult to disentangle in detail

Bethe-Salpeter Equations (BSE)

Infinite tower of coupled Green's functions to consider ... *truncation*

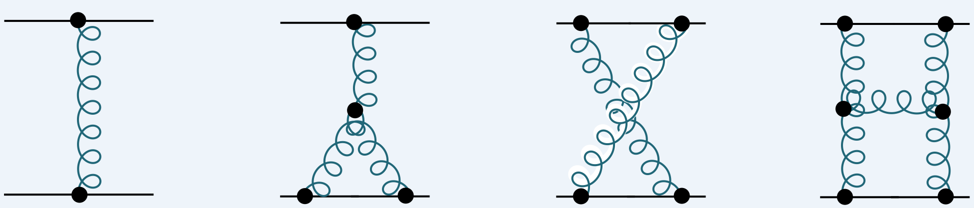




Expose corrections to the Bethe-Salpeter kernel

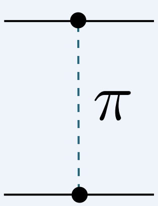
- Systematic and improvable
- Lead to meaningful inclusion of "physics"
- Preserve **axial-vector Ward-Takahashi identity**

Diagrammatic



[Fischer, RW PRL 103 (2009) 122001]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[Binosi, Chang, Papavassiliou, Qin, Roberts PRD 93 (2016) 096010]

Effective/Composite

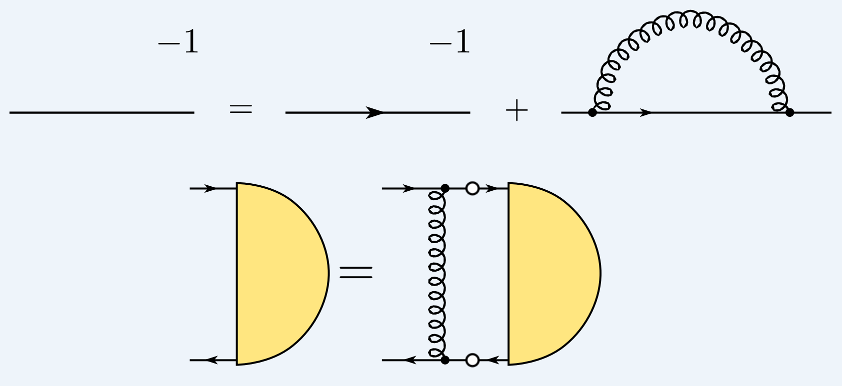


[Fischer, Nickel, Wambach ORD 76 (2007) 094009]
[Fischer, RW PRD 78 (2008) 074006]
[Sanchis-Alepuz, Fischer, Kubrak PLB 733 (2014) 151]

BSE – Rainbow-Ladder

Quark DSE

Meson BSE

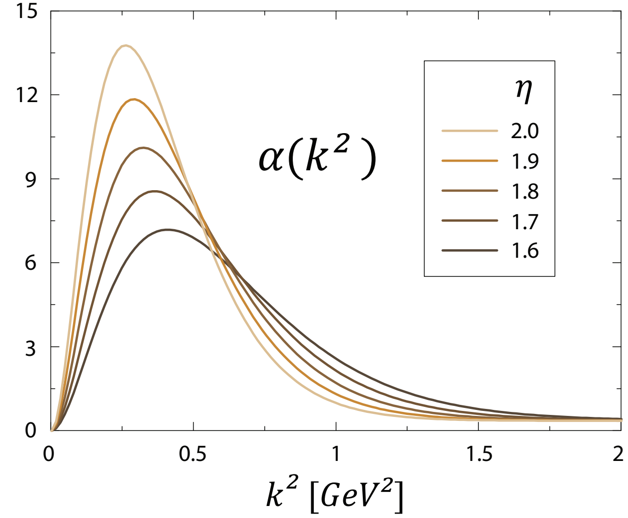


2PI 2-loop (rainbow-ladder)

Euclidean space:

- Time-like properties require analytic continuation of propagators/vertices into the complex plane.

- **Tree-level** vertices
- Gluon **model**



[Maris, Tandy PRC 60 (1999) 055214]

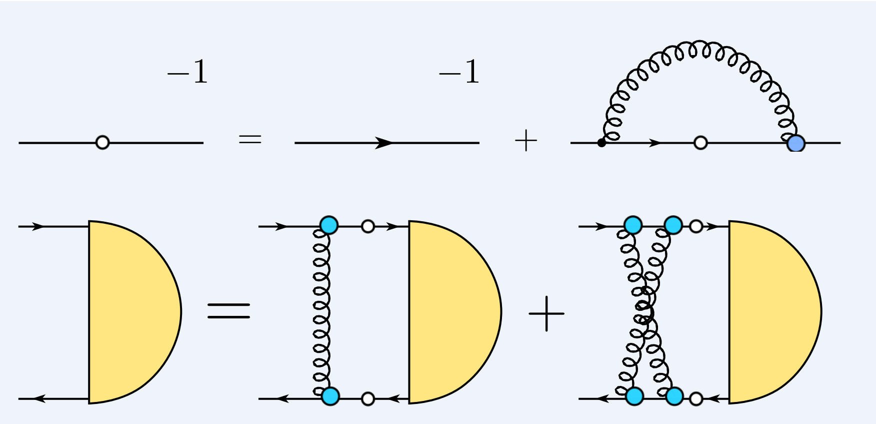
Routinely solved by "standard methods"

- Quark for complex momenta (Cauchy, shell-method, path deformation)
- One-loop BSE kernel independent of total momentum P

e.g. [Sanchis-Alepuz, RW, arXiv:1710.04903]

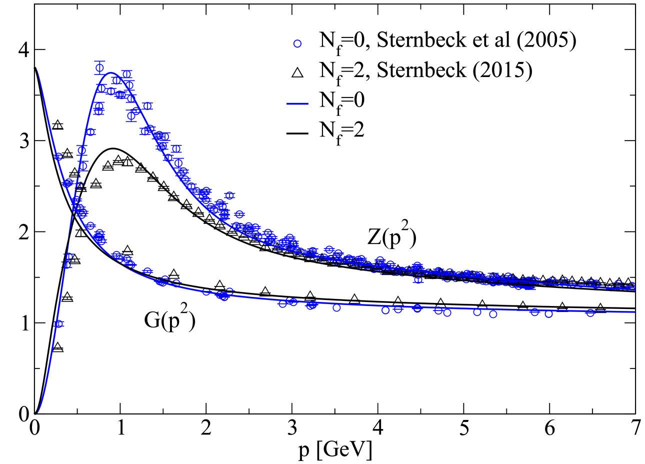
Quark DSE

Meson BSE



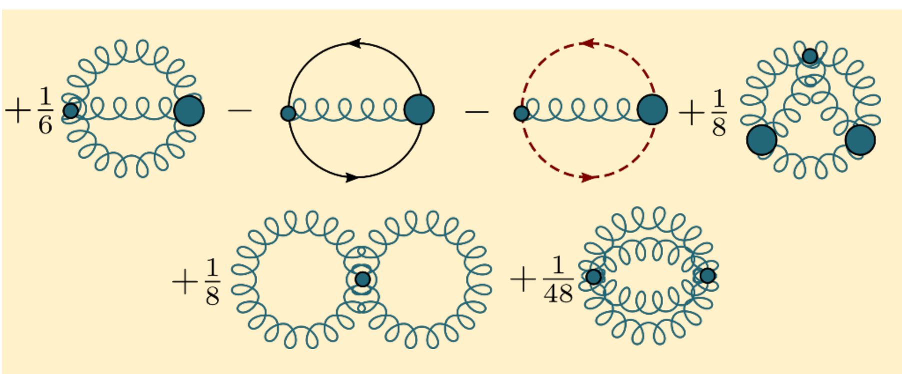
3PI 3-loop

- **Calculated** vertices
- Gluon from **lattice**

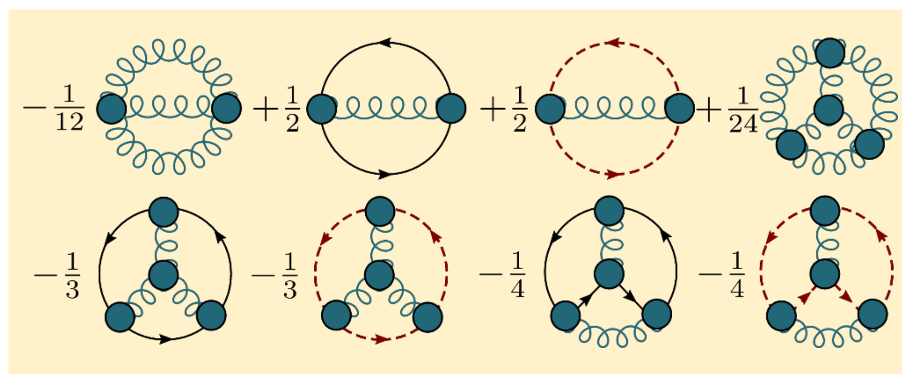


… truncate using e.g. nPI effective action

$\boldsymbol{\Phi^0}$: non-interacting part



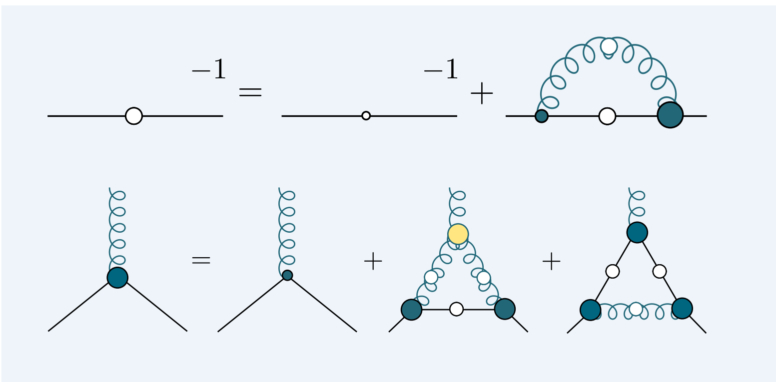
$\boldsymbol{\Phi^{int}}$: interacting part

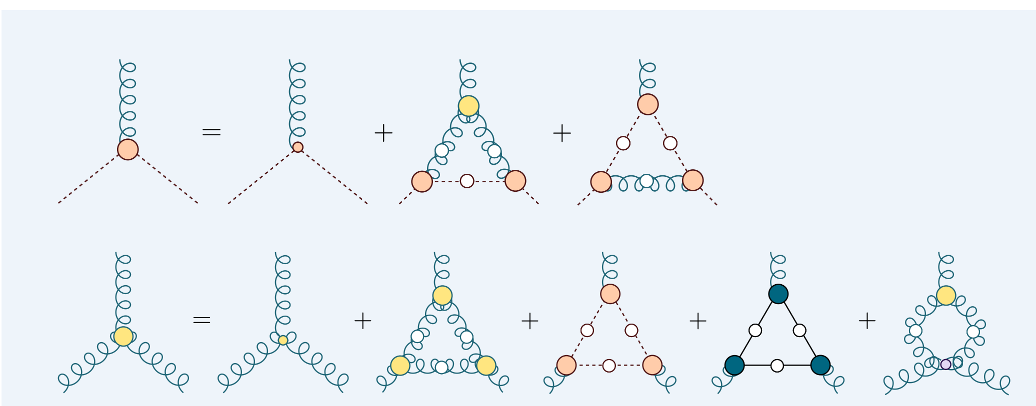


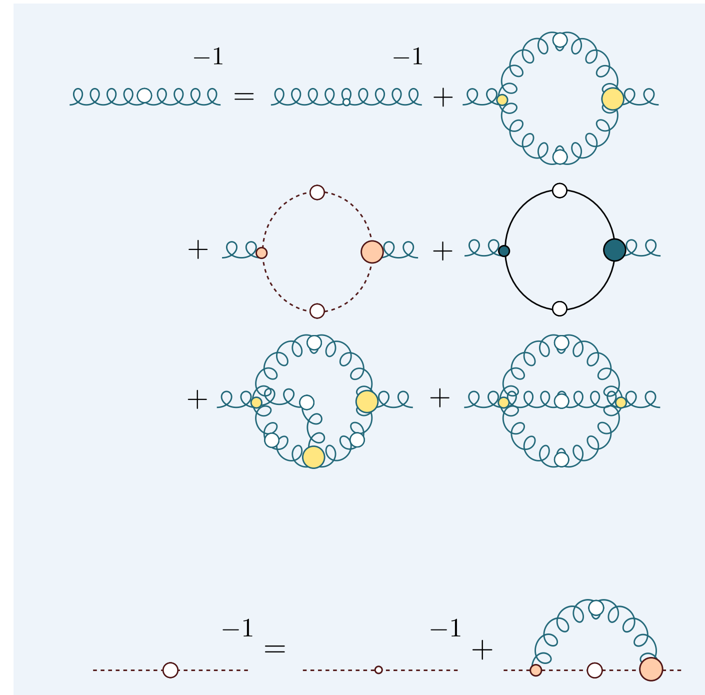
[RW, Fischer, Heupel, PRD93 (2016)]

BSE – Beyond Rainbow-Ladder

Yields **closed system of equations** for the dressed propagators and vertices that enter the Bethe-Salpeter kernel **K**.



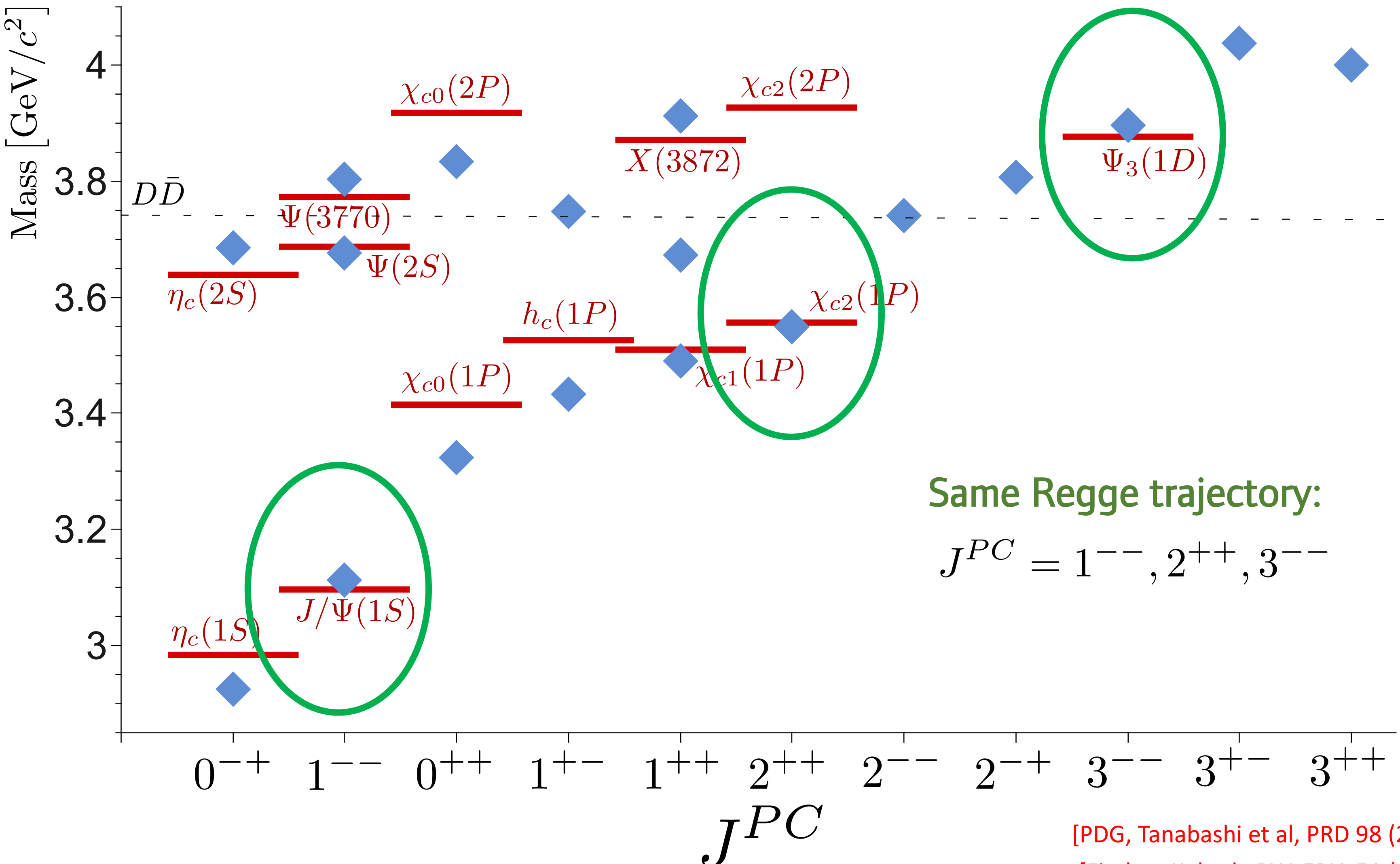




- **Seventeen** coupled integral equations.
- More for $N > 2$ quarks.

[RW, Fischer, Heupel, PRD93 (2016)]
[M. Q. Huber, EPJC77 (2017)]

Mesons: Charmonium, Rainbow-Ladder

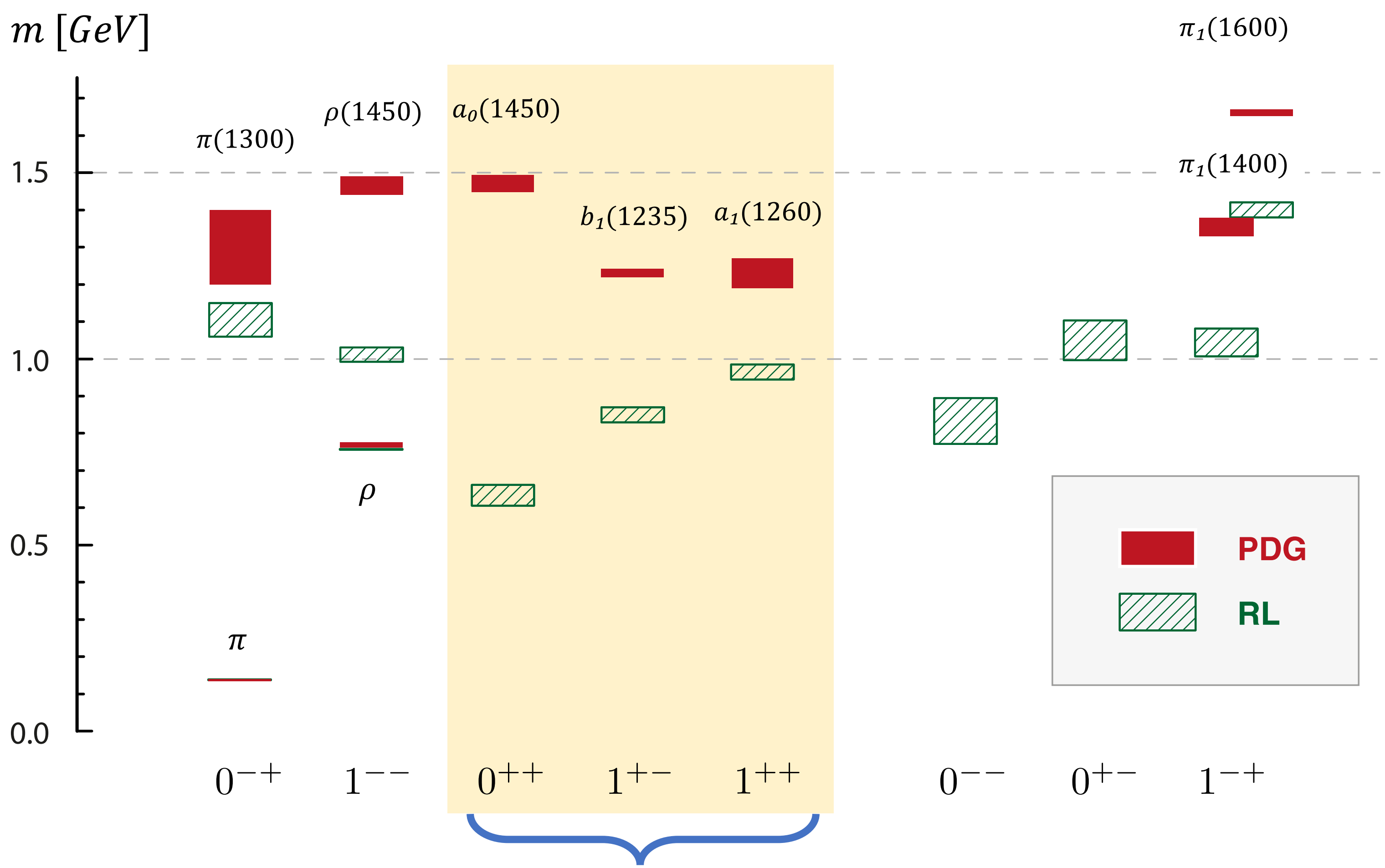


Same Regge trajectory:

$$J^{PC} = 1^{--}, 2^{++}, 3^{--}$$

[PDG, Tanabashi et al, PRD 98 (2018)]
[Fischer, Kubrak, RW, EPJA 51 (2015)]
[Hilger, Gomez-Rocha, Krassnigg, Lucha, EPJA 53 (2017)]
[LHCb Collaboration, Aaij et al, arXiv:1903.12240]

Mesons: Light, Rainbow-Ladder

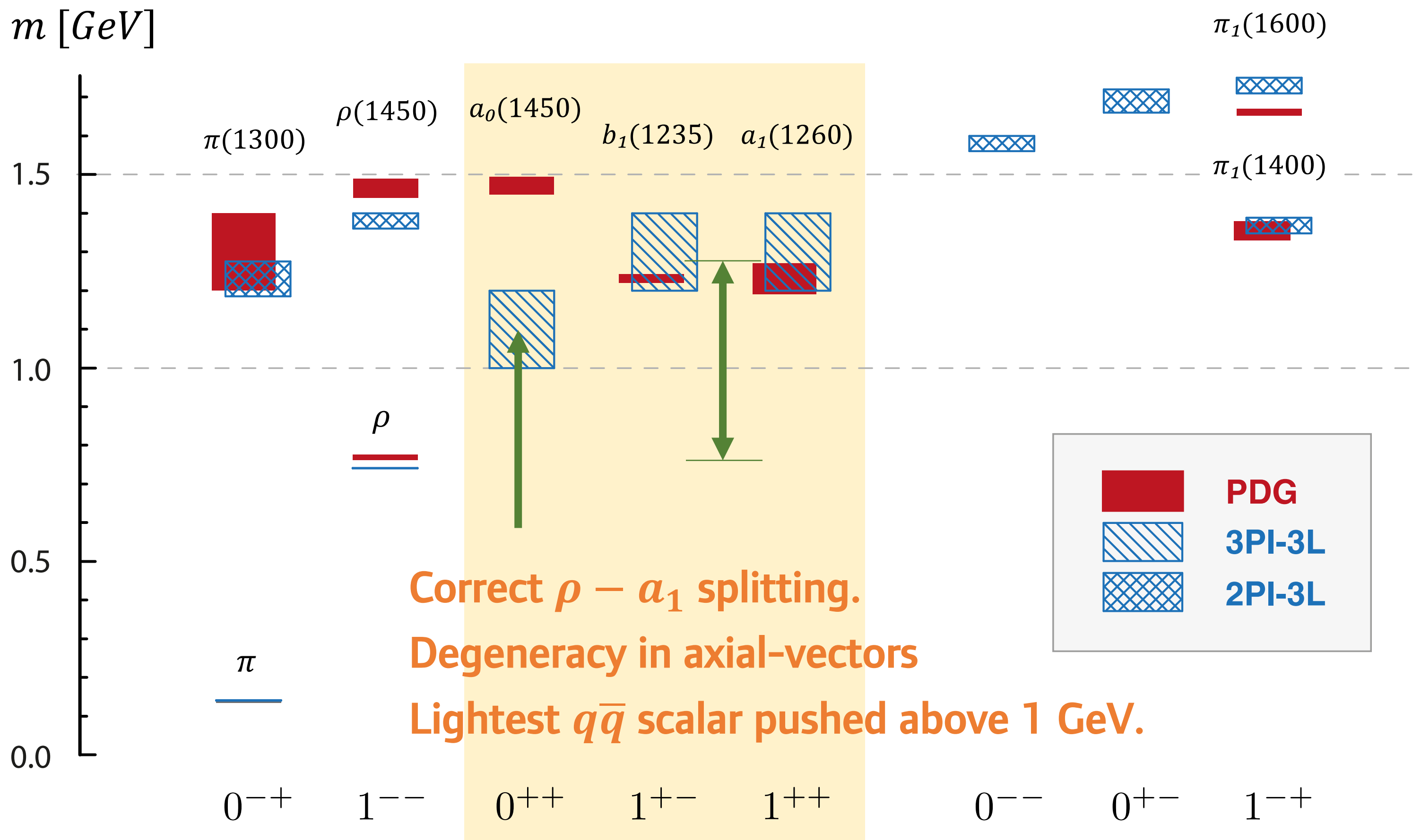


Deficient for scalars/axial-vectors

[Fischer, Kubrak, RW, EPJA 50 (2014)]

[Hilger, Gomez-Rocha, Krassnigg, Lucha, EPJA 53 (2017)]

Mesons: light, beyond Rainbow-Ladder



Correct $\rho - a_1$ splitting.

Degeneracy in axial-vectors

Lightest $q\bar{q}$ scalar pushed above 1 GeV.

But something is missing

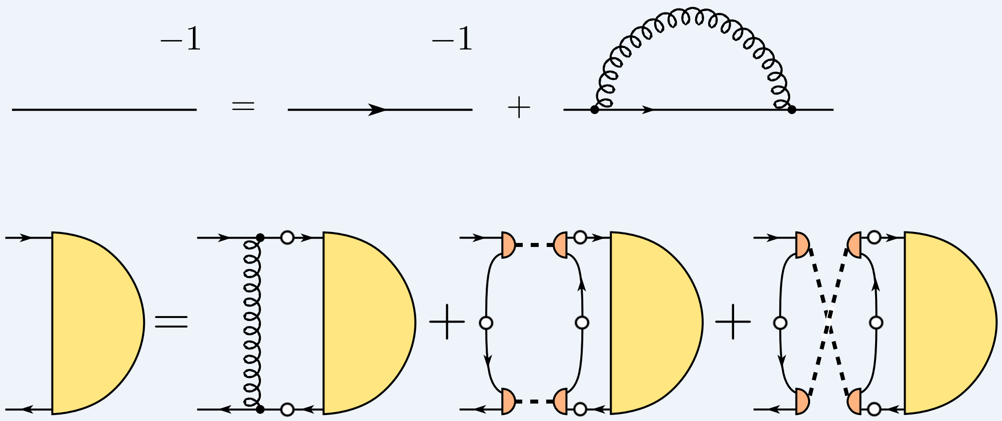
- Bound states **below** strong decay threshold: π, K, D, B
- Most hadrons lie **above** strong decay threshold

[RW, Fischer, Heupel, PRD93 (2016)]
[Chang, Roberts, PRL 103 (2009)]
[Chang, Roberts, PRC 85 (2012)]

Minimally: extend rainbow-ladder by including decay channel

Quark DSE

incl. decay



[RW, arXiv:1804.11161]

Specifically:

- Two-pion decay kernel
- **Couples** to *e.g.* vector and scalar mesons.
- Does **not couple** to pseudoscalar (CP and P): *maintains chiral symmetry*

Technical step:

- Integrating over intermediate states

[Watson, Cassing, FBS 35 (2004)]
[Fischer, Nickel, Wambach, PRD 76 (2007)]
[Fischer, RW, PRD 78 (2008)]

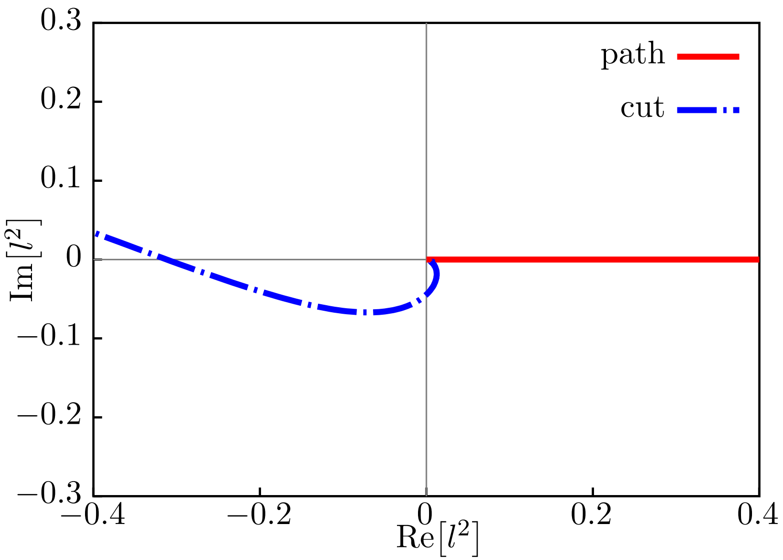
Resonances

Intermediate bound-state poles are integrated over; sweep out cuts in integration plane

Two-pion cuts

$$l_{\text{cut}}^2 = -z\sqrt{t} + \sqrt{t(z^2-1) - m_\pi^2}, \qquad t = P^2/4$$

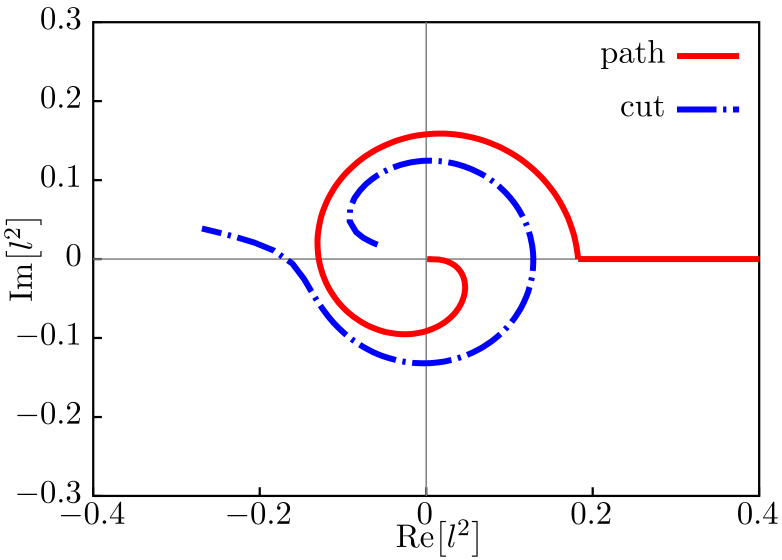
Below threshold



No obstruction

- Integrate along spacelike axis without problem.

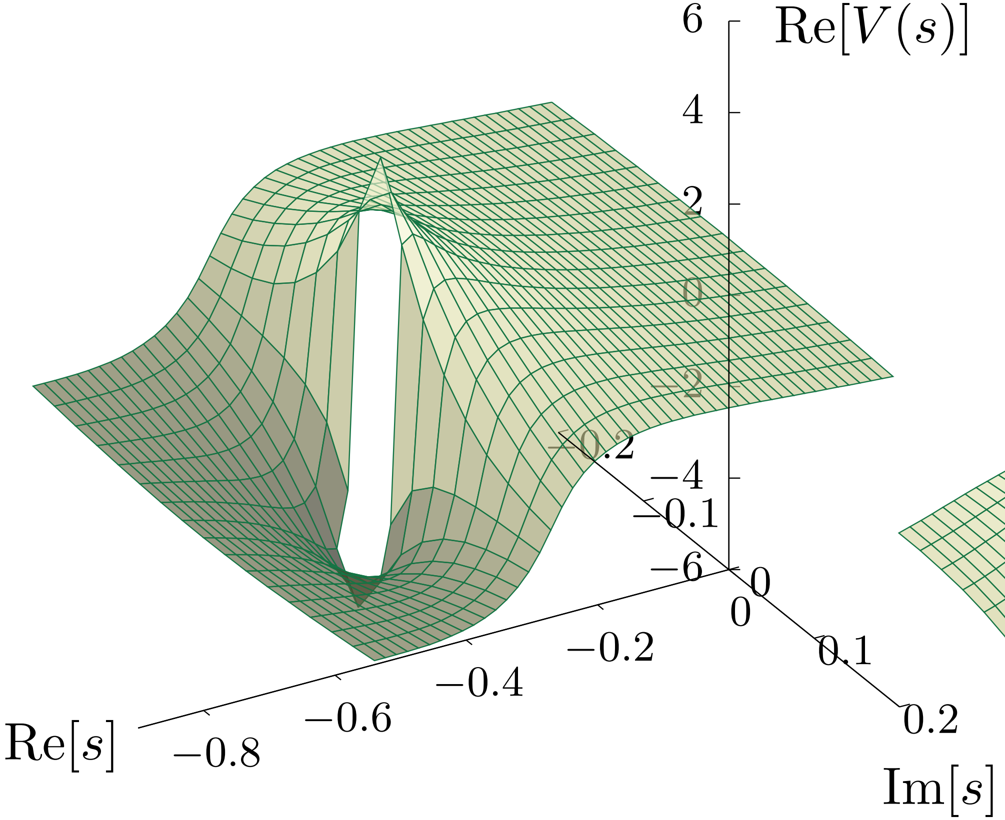
Above threshold



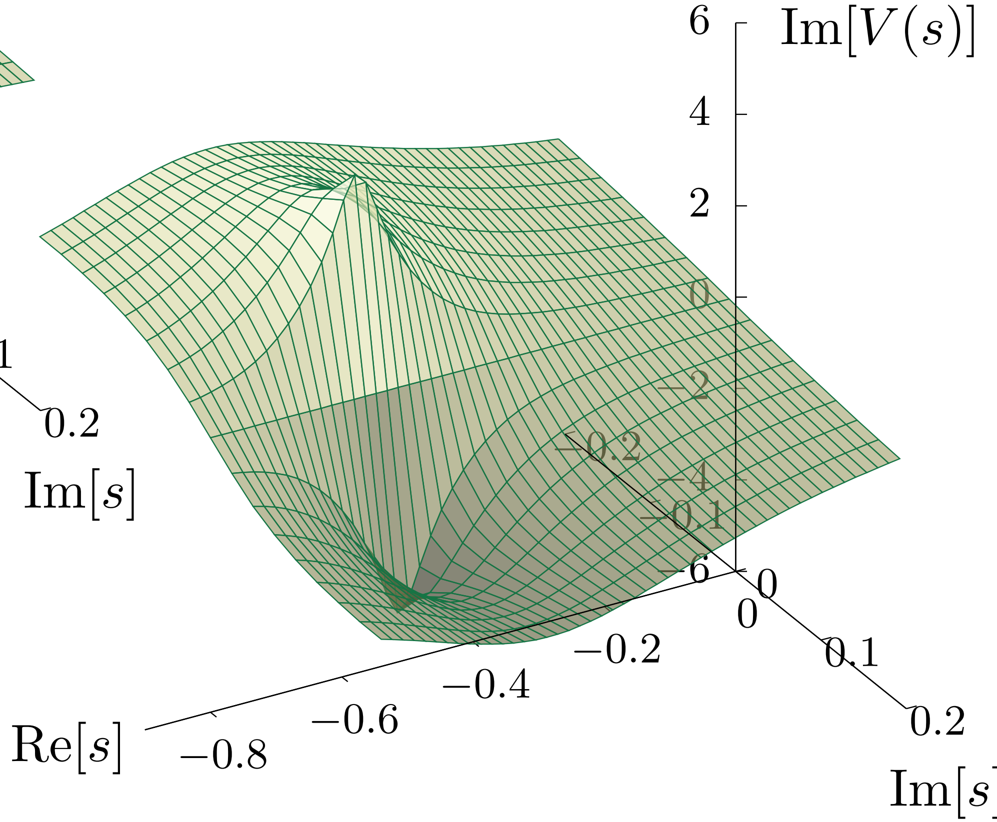
Path deformation required

- Avoid cut(s) during integration.

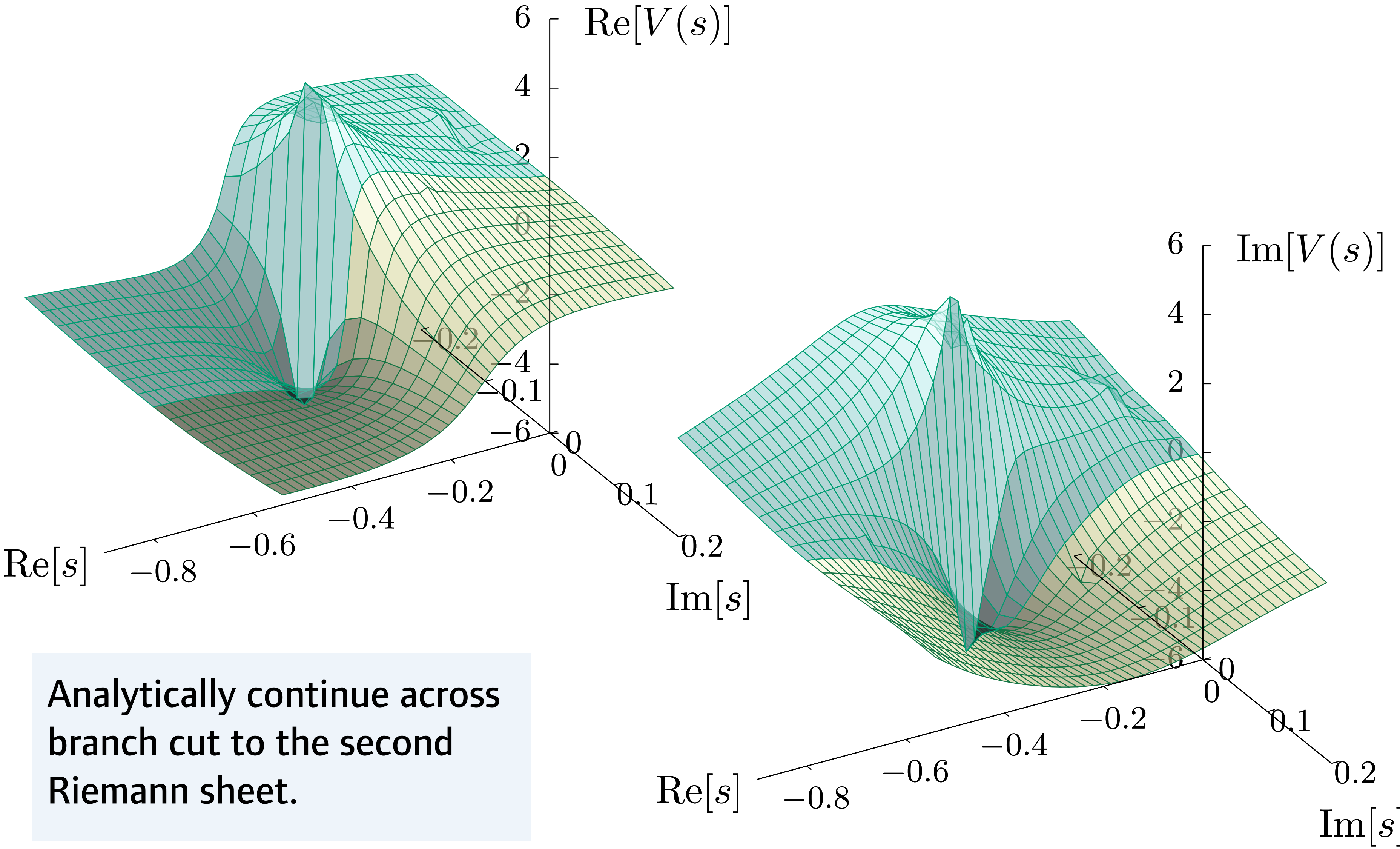
Resonances: without decay channel



(transformed z-axis with $\sinh^{-1}$)



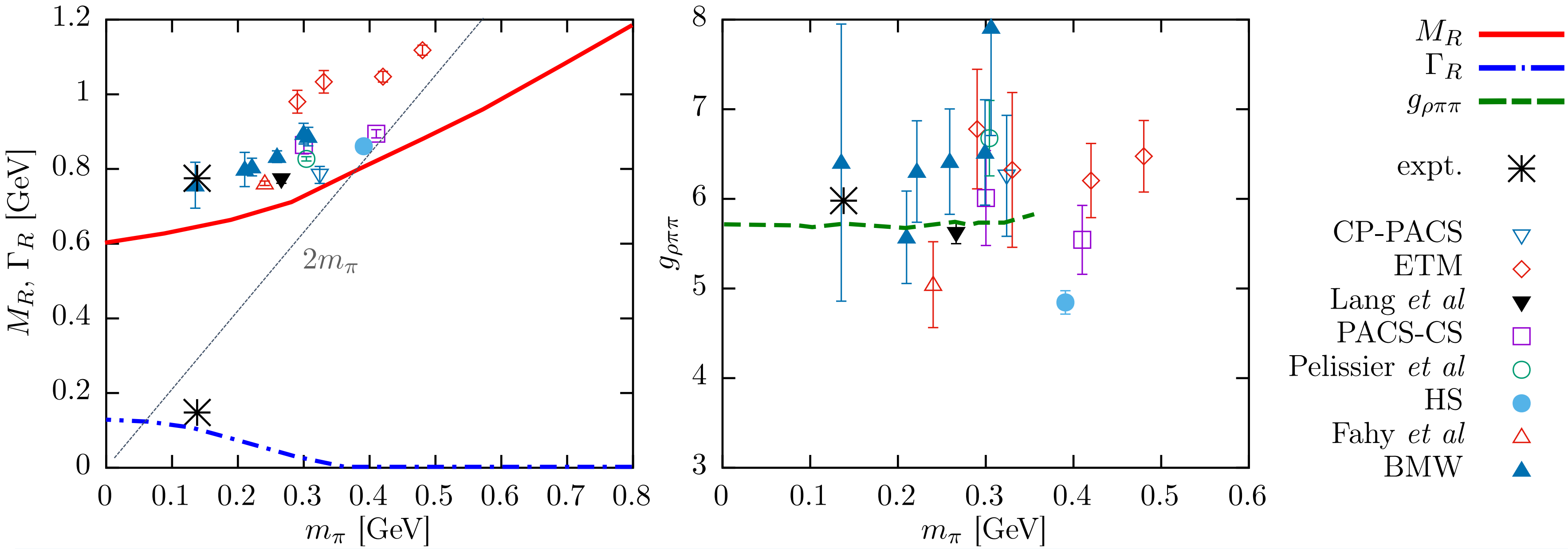
Pole readily apparent on the real-axis



Analytically continue across branch cut to the second Riemann sheet.

Poles on the "unphysical" sheet

Resonances



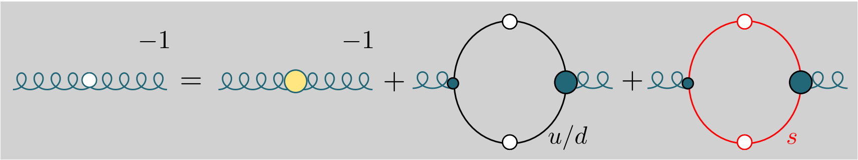
Here: strong coupling constant $g_{\rho\pi\pi} \sim 5.7$ **(experimental value $g_{\rho\pi\pi} \sim 6.0$)**

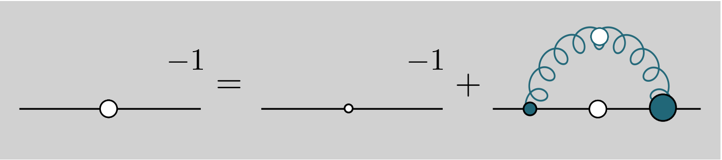
RL: (impulse approximation) $g_{\rho\pi\pi} \sim 5.2$

$$\Gamma_R = \frac{p^3}{M_R^2}\frac{g_{\rho\pi\pi}^2}{6\pi}, \quad p = \sqrt{M_R^2/4 - m_\pi^2}\,,$$

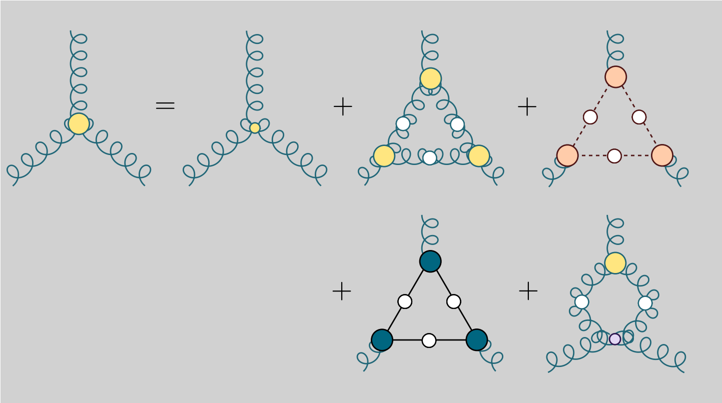
[Jarecke, Maris, Tandy, PRC67 (2003)]
[Mader, Eichmann, Blank, Krassnigg, PRD84 (2011)]

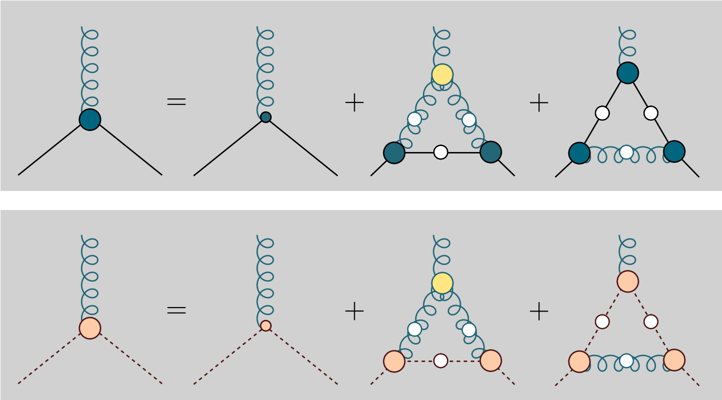
Quenched propagator from Lattice





Unquenching via 3PI

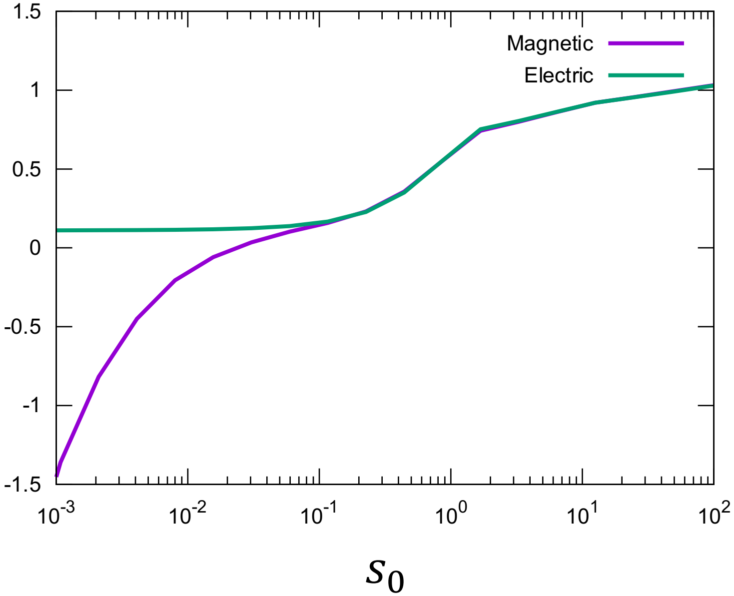




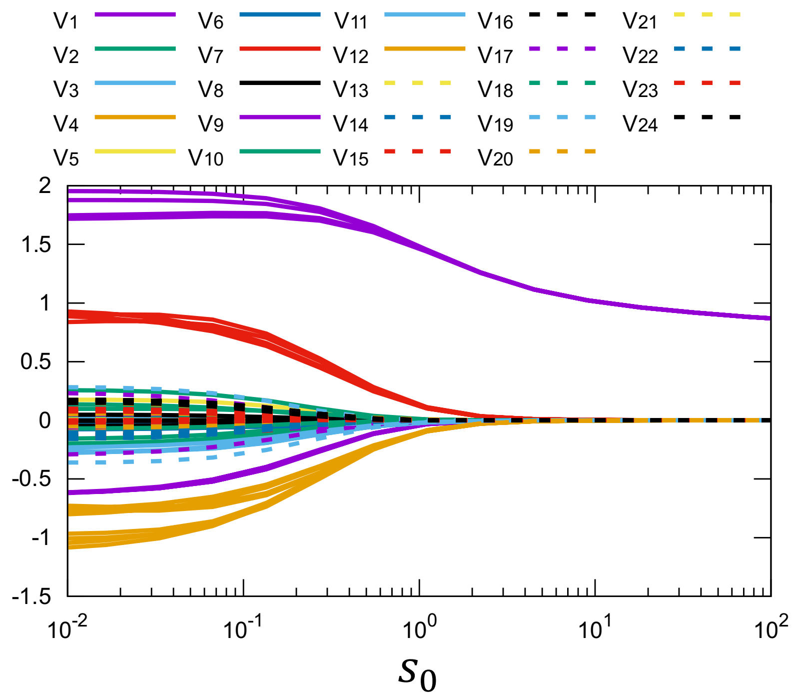
Depends explicitly on: T, μ, m

Vertices have been calculated

(T=100 MeV)



Three-gluon vertex



Quark-gluon vertex

Very much **work in progress**

- Ready to unify DSEs/BSEs with a single truncation (3PI) in both vacuum and in medium

Outlook: high-spin Baryon

First Rainbow-Ladder – then extend beyond:

nucleon

	s-wave	p-wave	d-wave
S=1/2	8	24	
S=3/2		12	20

64

delta

	s-wave	p-wave	d-wave	f-wave
S=1/2		24	40	
S=3/2	4	12	20	28

128

spin 5/2

	p-wave	d-wave	f-wave	g-wave
S=1/2		40	56	
S=3/2	12	20	28	36

192

[RW, H. Sanchis-Alepuz in prep.]

spin 7/2

	d-wave	f-wave	g-wave	h-wave
S=1/2		56	72	
S=3/2	20	28	36	44

256

spin 9/2

	f-wave	g-wave	h-wave	i-wave
S=1/2		72	88	
S=3/2	28	36	44	52

320

Summary

Main Goals

- one framework
  - Mesons, Baryons, Tetraquarks, Hybrids, Glueballs
  - Electromagnetic form factors, Anomalous magnetic moments
- DCSB, confinement

Main Challenges

- Control of systematic errors:
  - Intrinsic and by comparison to other methods

Main Results

- nPI is a powerful tool
- Not high precision, but quantitative and -- where not -- qualitative.
- Competitive contributions in many wide areas

Review

Eichmann, Sanchis-Alepuz, RW, Alkofer, Fischer 1606.9602 PPNP. 91 (2017) 1-100

Summary

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Thank you

Giessen

Review

Eichmann, Sanchis-Alepuz, RW, Alkofer, Fischer 1606.9602 PPNP. 91 (2017) 1-100