GPDs and gravitational form factors of the proton

Adam Freese

Argonne National Laboratory

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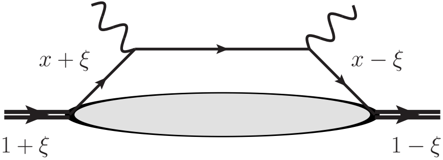
Why GPDs at an EIC?

- GPDs are the soft (non-perturbative) part of DVCS, DVMP, *etc.*



- **Hard exclusive reactions** are easier at an EIC!
  - Recoil proton/ion more easily measured than in fixed target experiments.
  - High luminosity helpful for events with low cross sections.
  - Hermetic coverage (central timing-optimized silicon, RICH for forward scattering) great for exclusive reactions.
- GPDs are related to literal spatial distributions by 2D Fourier transforms

$$\int \frac{\mathrm{d}^2\mathbf{k}_\perp}{(2\pi)^2} H^q(x, \xi = 0, t = -\mathbf{k}_\perp^2) e^{-i(\mathbf{k}_\perp \cdot \mathbf{b}_\perp)} = \rho^q(\mathbf{b}_\perp)$$

- GPDs encode information about decomposition & distribution of **mass**, **angular momentum**, and **forces** through polynomiality relations.

Gravitational form factors and the EMT

- **Gravitational form factors** encode information in the energy-momentum tensor (EMT):

$$\langle p'\lambda' \mid T^q_{\mu\nu}(0) \mid p\lambda\rangle = \bar{u}^{\lambda'}(p')\Bigg[\gamma_{\{\mu}P_{\nu\}}A^q(t) + \frac{iP_{\{\mu}\sigma_{\nu\}\Delta}}{2m_N}B^q(t) + \frac{\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu}}{4m_N}C^q(t) + m_N g_{\mu\nu}\bar{c}^q(t) + \frac{iP_{[\mu}\sigma_{\nu]\Delta}}{2m_N}D^q(t)\Bigg]u^\lambda(p)$$

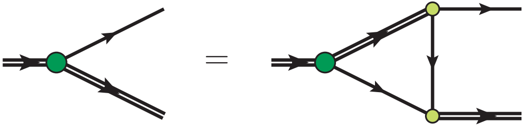
- Three of the GFFs accessible through GPDs:

$$\int \mathrm{d}x\, xH^q(x,\xi,t) = A^q(t) + \xi^2 C^q(t)\,, \qquad \int \mathrm{d}x\, xE^q(x,\xi,t) = B^q(t) - \xi^2 C^q(t)$$

- GFFs encode information about distribution of energy, angular momentum, and forces.
- **Cannot do graviton-exchange experiments**—but GPDs allow GFFs to be experimentally accessed! (DVCS, DVMP, ...)

Quark-diquark model

- **Quark-diquark model**: proton is dominated by configurations where two quarks are in a **diquark correlation**.
- Faddeev equation replaced by two-body equation with quark exchange kernel.



- Two species of diquarks dominate.
  1. Scalar, isoscalar ($S = 0$, $T = 0$)
  2. Axial vector, isovector ($S = 1$, $T = 1$)
- Evidence for diquarks in flavor-separated form factors

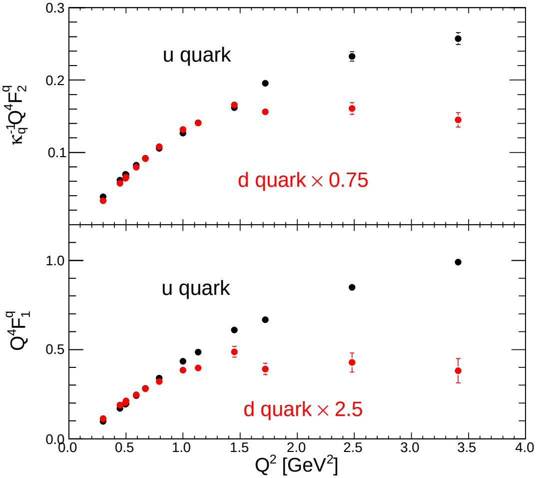
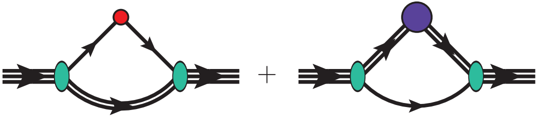
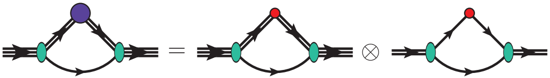


Figure from Cates *et al.*, PRL106 (2011) 252003

Diagrams and convolution



Proton GPD diagrams



Convolution equation

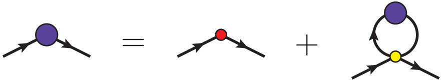
- GPD gets contributions from the lonely quark and from the diquark.
- Diquark contribution given by a convolution formula

$$H_{X,i}(x,\xi,t) = \sum_j \int \frac{\mathrm{d}y}{|y|} h_{Y/X,ij}(y,\xi,t) H_{Y,j}\left(\frac{x}{y},\frac{\xi}{y},t\right)$$

- Diquark GPDs include scalar and axial species, along with a transition GPD
- The dressed quark will also have a GPD, to be folded in through a second layer of convolution
- Practically, do model calculation in NJL model

Dressed quark GPDs

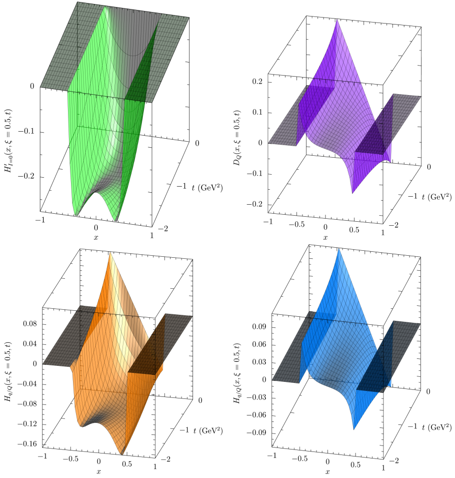
- GPD defined through **current quark fields**; proton made of **three dressed quarks**.
- Bethe-Salpeter equation gives GPD of dressed quarks in terms of current quarks:



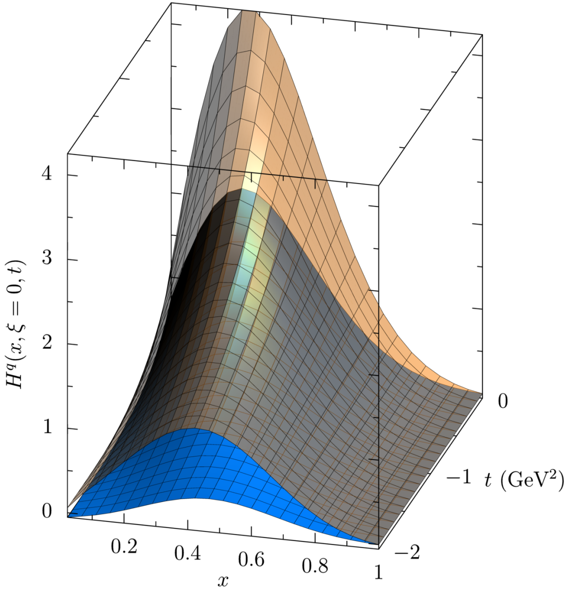
- Fold quark GPDs into proton GPD using another layer of convolution.
- Isospin mixture solutions:

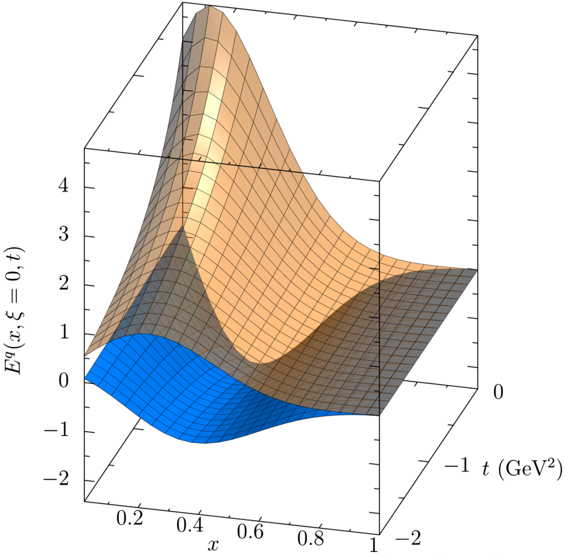
$$H_I(x,\xi,t) = \delta(1-x) + H'_I(x,\xi,t) + \delta_{I,0} D_Q(x,\xi,t)$$
$$E_I(x,\xi,t) = -\delta_{I,0} D_Q(x,\xi,t)$$

- Orange for u/U or d/D; blue for d/U or u/D.

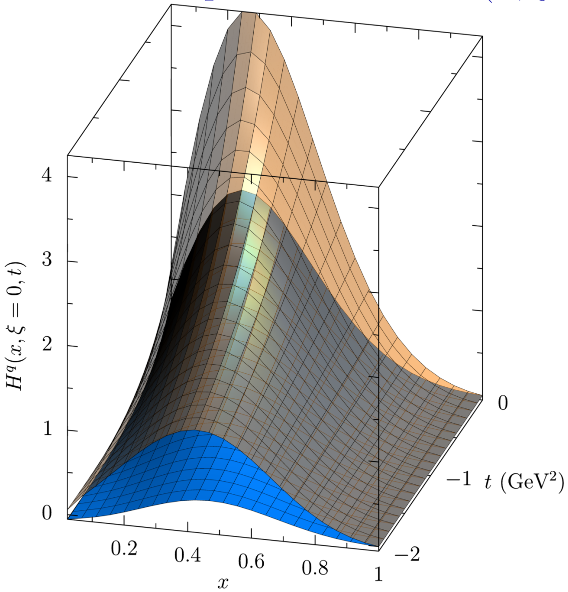


Non-skewed proton GPD results





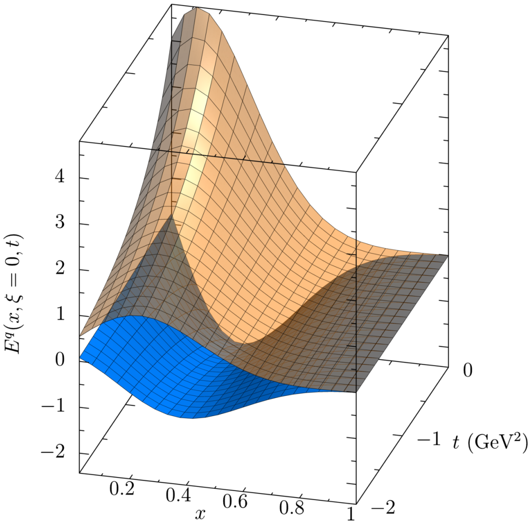
Non-skewed proton GPD: $H^q(x, \xi = 0, t)$



Orange is up; blue is down.

- GPD not dressed in DGLAP region
- **Forward limit**: $H^q(x, 0, 0) = q(x)$
- up-down differences due to diquark correlations
- **Scalar diquarks** dominate at low $-t$; up quarks carry more x than down
  - At $t = 0$, $\langle\langle x_u \rangle\rangle = 0.34$
  - At $t = 0$, $\langle\langle x_d \rangle\rangle = 0.32$
- **Axial diquarks** dominate at high $-t$; down quark carries more x
  - At $t = -2$ GeV^2, $\langle\langle x_u \rangle\rangle = 0.45$
  - At $t = -2$ GeV^2, $\langle\langle x_d \rangle\rangle = 0.50$

Non-skewed proton GPD: $E^q(x, \xi = 0, t)$



Orange is up; blue is down.

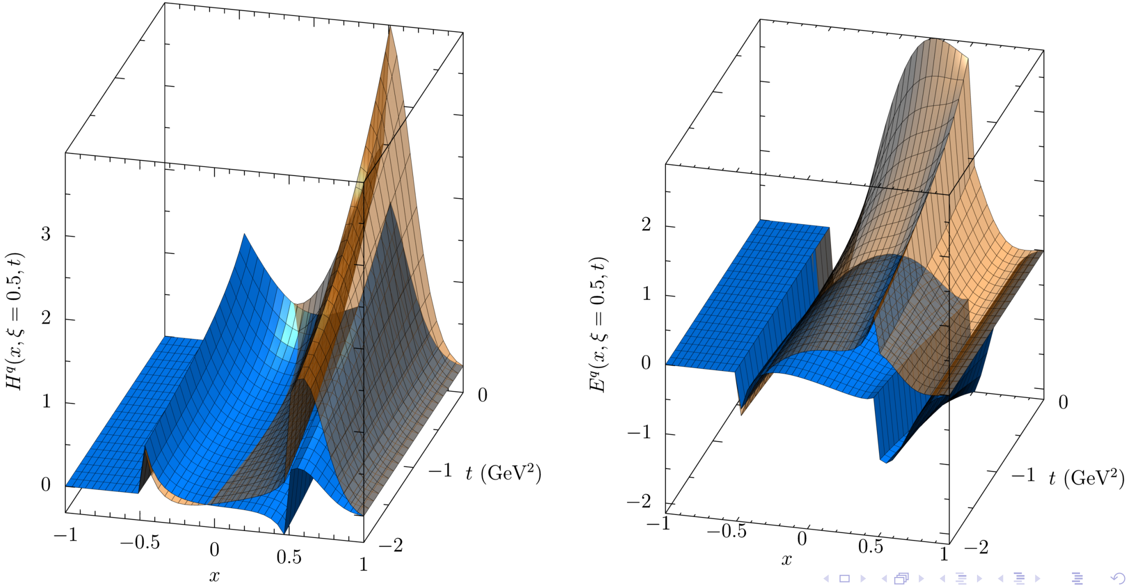
- **Axial diquarks** dominate at all $-t$; down quarks carry more x
- No forward limit; understand by moments

$$\int \mathrm{d}x\, E^q(x, 0, t) = F_2^q(t)$$

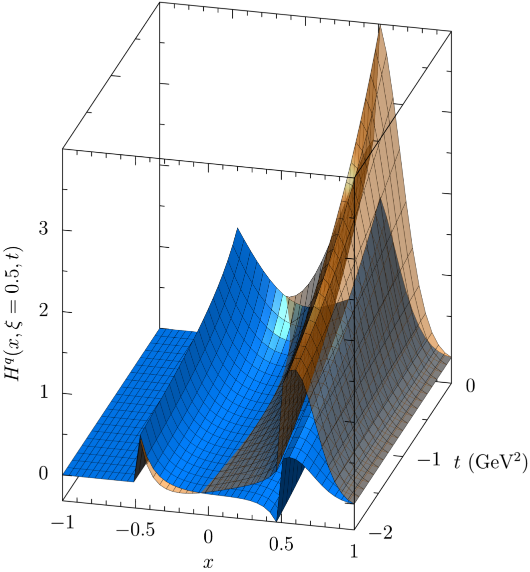
$$\int \mathrm{d}x\, x E^q(x, 0, t) = B^q(t) = 2J^q(t) - A^q(t)$$

- J^u and J^d **have opposite signs**
  - $J^u(0) = 0.52$ (close to total)
  - $J^d(0) = -0.02$
- κ^u and κ^d **have same sign**
  - $e_u F_2^u(0) = 1.15$
  - $e_d F_2^d(0) = 0.34$

Skewed proton GPD results



Skewed proton GPD: $H^q(x, \xi = 0.5, t)$



Orange is up; blue is down.

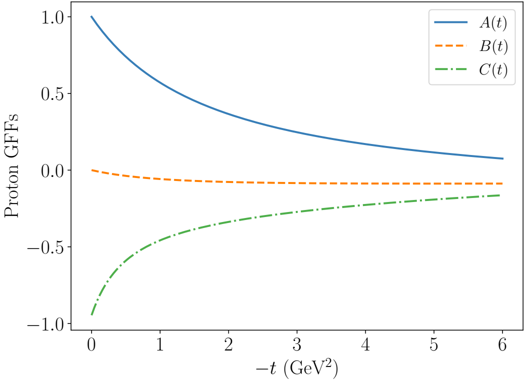
- GPD **dressed in ERBL region**: $|x| < |\xi|$
- up-down difference in ERBL region due to isospin-dependent dressing
- Near ρ-ω mass degeneracy: $H_{I=0} \approx H_{I=1}$

$$H_{u/U}(x,\xi,t) \approx H_{I=0}(x,\xi,t) + \frac{1}{2} D_Q(x,\xi,t)$$

$$H_{d/U}(x,\xi,t) \approx \frac{1}{2} D_Q(x,\xi,t)$$

- Down quark GPD dominated by $D_Q(x,\xi,t)$ in ERBL region

Gravitational form factors of the proton



Can extract **gravitational form factors** from GPDs; model is Lorentz covariant

$$\int \mathrm{d}x\, x H^q(x,\xi,t) = A^q(t) + \xi^2 C^q(t)$$

$$\int \mathrm{d}x\, x E^q(x,\xi,t) = B^q(t) - \xi^2 C^q(t)$$

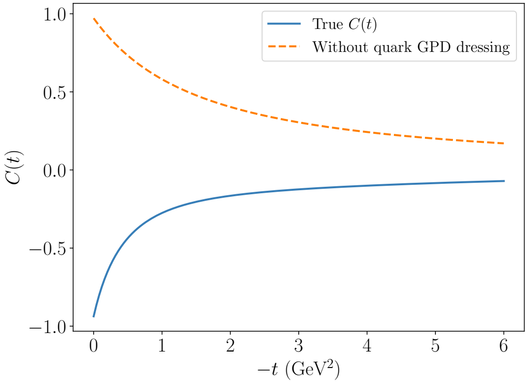
Sum rules: $A(0) = 1,\ B(0) = 0$

- $A(t)$ gives a mass form factor
- $J(t) = \frac{1}{2}\left(A(t) + B(t)\right)$ gives an angular momentum form factor; *n.b.*, $J(0) = \frac{1}{2}$
- The sum rules are guaranteed by Poincaré invariance of the model

D-term of the proton

$C(t)$ encodes **pressure distribution** (see Peter Schweitzer's talk)

- $C(0)$ not constrained by conservation laws
- $C(0) < 0$ is a stability condition
- $C(t)$ is affected by quark GPD dressing, but not $A(t)$ or $B(t)$



- $C(0) = -0.94 < 0$ in the NJL model
- Ignoring quark GPD dressing gives $C(0) = 0.97 > 0$
- Proton stability appears to **require** dressing the quark GPD!

Conclusions & outlook

Conclusions

- We've calculated leading-twist, helicity-independent proton GPDs in the NJL model.
- The GPDs show manifestations of diquark correlations.
- Can extract predictions for gravitational form factors, since model is covariant.
- Dressing of light cone correlator needed for correct description of ERBL region.
- Dressing required for proton stability.

Outlook

- Calculations for helicity-dependent and helicity-flip GPDs also possible.
- Plans to extend model to include intrinsic (non-perturbative) glue.

See also

- Talks by Peter Schweitzer (parallel session), Cédric Lorcé (Thursday)
- Posters by Arkadiusz Trawinski, David Arturo Amor Quiroz, & Peter Lowdon
- arXiv:1907.08256