New modelling techniques for Generalised Parton Distributions

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In collaboration with:
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Phys.Lett. B780 (2018) 287-293

Definition of GPDs

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$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u + E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0}$$
$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5 u + \tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

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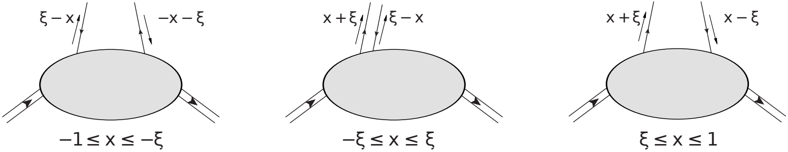
  - depend on three variables (x, ξ, t) and a scale μ_F,



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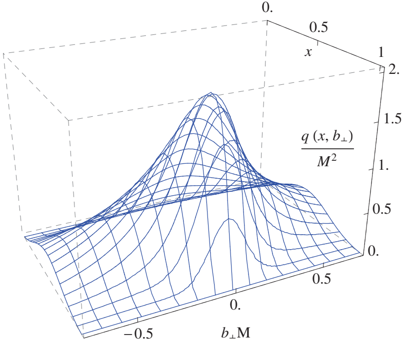
M. Burkardt, Phys. Rev. **D62**, 071503 (2000)



Pion GPD in Impact parameter space from:
C. Mezrag *et al.*, Phys. Lett. **B741**, 190-196 (2015)

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  - can be related to the Energy-Momentum tensor (GFF) through their $n = 1$ Mellin moments

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X. Ji, J. Phys. G24, 1181–1205 (1998)

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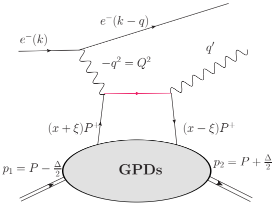
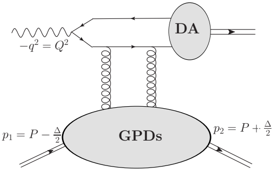
  - can be related to the Energy-Momentum tensor (GFF) through their $n = 1$ Mellin moments
  - are univeral, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \; C(x, \xi) H(x, \xi, t)$$





GPDs theoretical constraints

- Polynomiality:

$$\int_{-1}^{1} \mathrm{d}x \; x^m H^q(x,\xi,t) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C^q_{2j}(t) + mod(m,2)\xi^{m+1} C^q_{m+1}(t)$$

Lorentz Covariance

GPDs theoretical constraints

- Polynomiality:

Lorentz Covariance

- Positivity:

$$\left| H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norms

GPDs theoretical constraints

- Polynomiality:

Lorentz Covariance

- Positivity:

Positivity of Hilbert space norms

- Support:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic Quantum Mechanic

GPDs theoretical constraints

- Polynomiality:

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- Positivity:

Positivity of Hilbert space norms

- Support:

Relativistic Quantum Mechanic

- Soft Pion theorem

M.V. Polyakov, Nucl. Phys. **B555**, 231 (1999)
C. Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)

PCAC and Axial-Vector WTI

GPDs theoretical constraints

- Polynomiality:

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- Positivity:

  Positivity of Hilbert space norms

- Support:

  Relativistic Quantum Mechanic

- Soft Pion theorem

  PCAC and Axial-Vector WTI

Problems

There is no model (until now) fulfilling a priori all these constraints

- GPDs are related to Double Distributions (DDs) through:

$$H(x,\xi,t) = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t))\,\delta\,(x - \beta - \xi\alpha)$$

The Dirac δ insures that the polynomiality is fulfilled, independently of our choice of F and G

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- They also appear naturally in covariant modelling attempts

Classical modelling techniques I
Double Distributions

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Positivity property is not guaranteed, and may be violated.

- On the light front, hadronic states can be expanded on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Phi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Phi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \ldots$$

$$|P,N\rangle \propto \sum_\beta \Phi_\beta^{qqq}|qqq\rangle + \sum_\beta \Phi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \ldots$$

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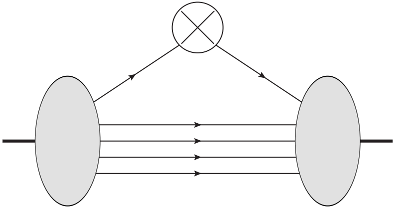
- Non-perturbative physics is contained within the N-particle LFWFs Φ^N
- This formalism allows to recover the probabilistic picture of non-relativistic quantum mechanics

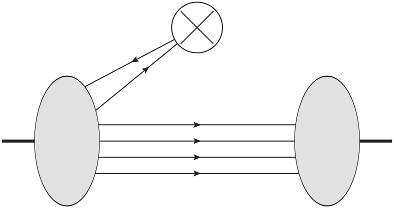
- On the light front, hadronic states can be expanded on a Fock basis

DGLAP: $|x| > |\xi|$



- Same N LFWFs
- No ambiguity

ERBL: $|x| < |\xi|$



- N and $N+2$ partons LFWFs
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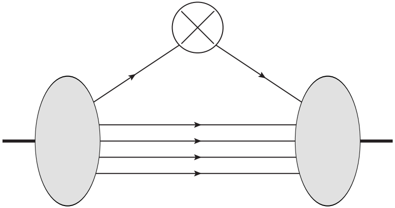

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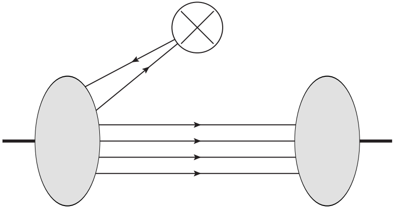

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LFWFs formalism has the positivity property inbuilt but polynomiality is lost by truncating both in DGLAP and ERBL sectors.

Radon Inverse transform

- Mathematical properties of GPDs:

$$H(x,\xi,t) - D(x/\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1-|\beta|}^{1-|\beta|} \mathrm{d}\alpha F(\beta,\alpha,t)\delta(x-\beta-\alpha\xi)$$

D. Müller *et al.*, Fortsch. Phys. 1994, 42, 101
A. Radyushkin, Phys.Rev., 1997, D56, 5524-5557
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J. Boman and E. T. Quinto, Duke Math. J. 55, 943 (1987).
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- Relation between DGLAP and ERBL region allowing the fulfilment of polynomiality
- Additional constraints are needed to fix the D-term

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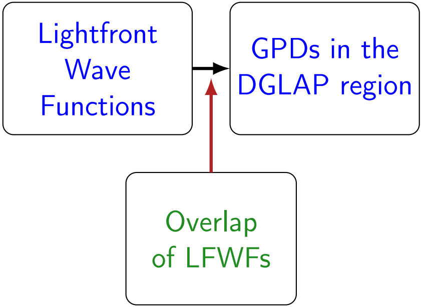
A new modelling procedure

Lightfront
Wave
Functions

A new modelling procedure



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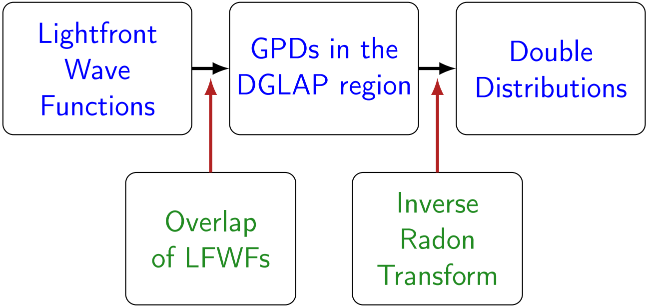

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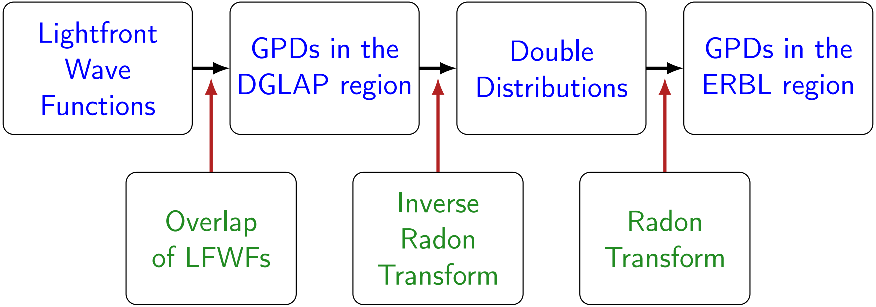

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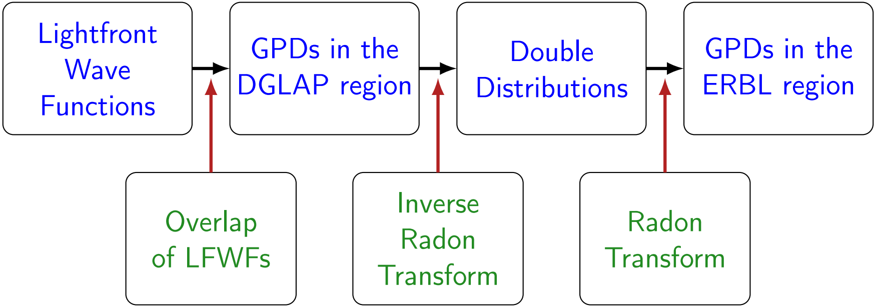

Reshuffling of the series in the ERBL region
$\rightarrow$ Polynomiality is fulfilled at every order in N.

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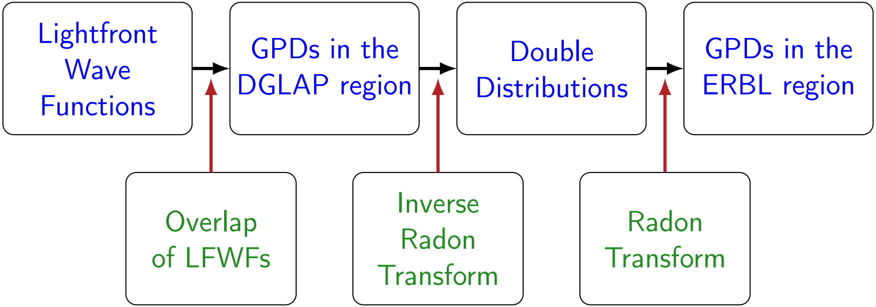

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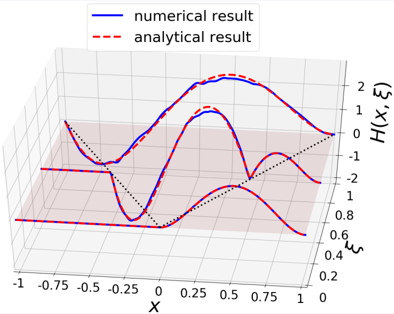
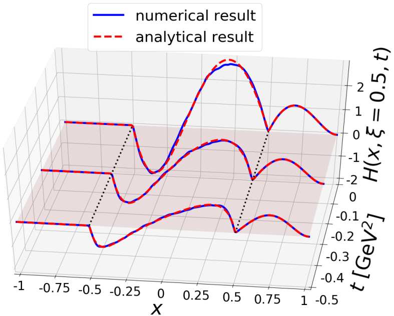
Applied and tested in various cases

- The inverse Radon transform is an ill-posed problem
- Numerical implementation can be challenging due to noise

$H(x, \xi, 0)$

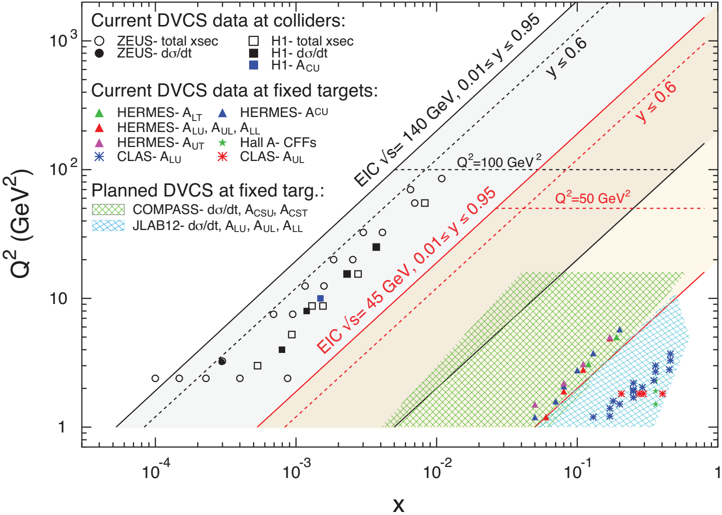


$H(x, 0.5, t)$

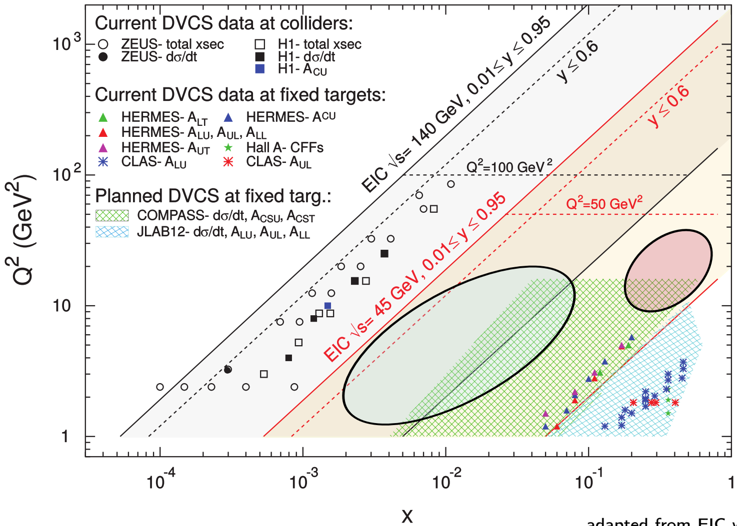


DVCS at EIC

Kinematical range



adapted from EIC white paper



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Motivations

- Small range of Q^2 available today in the valence region
- LO/LT not a good approximation at JLab Kinematics

M. Defurne *et al.*, Phys.Rev. C92 (2015) no.5, 055202
M. Defurne *et al.*, Nature Commun. 8 (2017) no.1, 1408

Valence-x DVCS at EIC

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Challenges

- Unclear whether the EIC will be able to measure high-x_B DVCS

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Work in Progress

- Recently started working with Jinlong Zhang (Stony Brook U.)
- Use PARTONS together with Stony Brook's EIC DVCS event generator
- Under which conditions could we obtain relevant valence-x DVCS data?

Conclusion

Modelling GPDs

- New formalism to model GPDs which **fulfils by construction all theoretical constraints for the first time**
- Based on Lightfront, from whichever non-perturbative approach you prefer (Bethe-Salpeter, Holographic QCD, Hamiltonian formulation...)
- Use for phenomenological applications?

EIC valence-x DVCS measurements

- Valence-x DVCS measurement may be possible (need to be checked)
- A welcome high-Q^2 complementary measurement to fix target experiments

Addendum: Mesons Structure at EIC

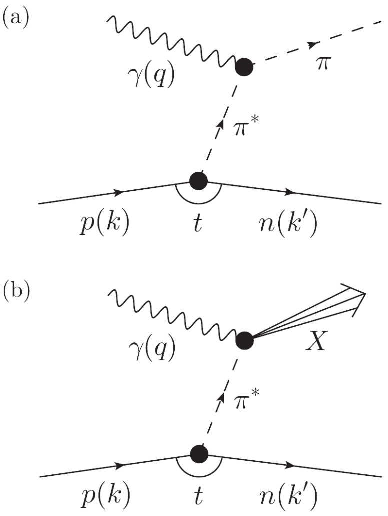

figure from arXiv:1907.08218

- "White paper" for Pion and Kaon physics at an EIC

  arXiv:1907.08218

- Two key point to study the internal structure of pions :
  - internal structure of Goldstone bosons, and the dynamic behind it
  - meson distribution amplitude play a significant role in DVMP

Thank you for your attention