Beam-Beam Simulations with Crab Cavities

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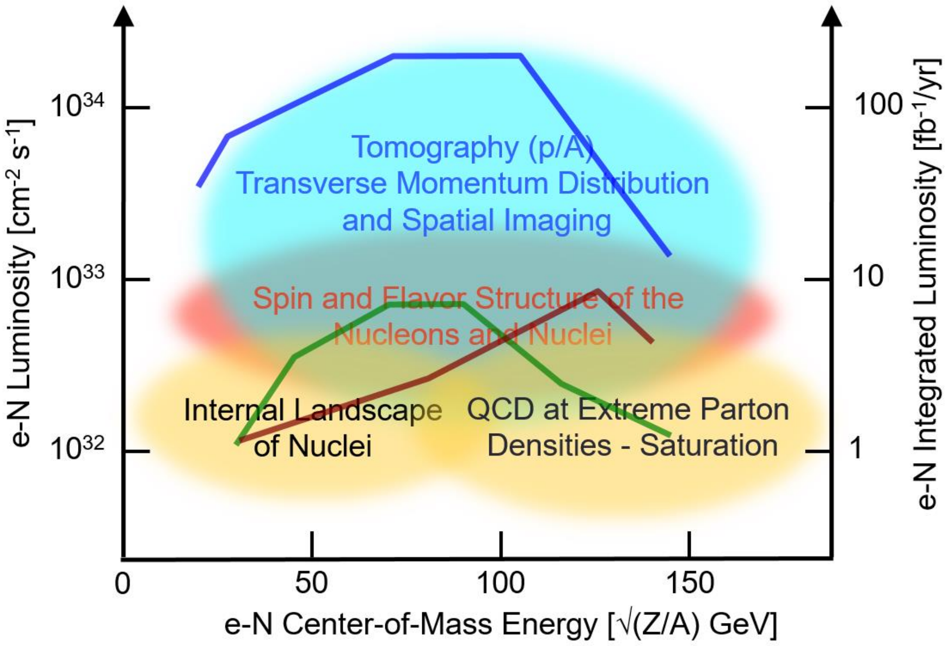
Outline

- Introduction
- Computational tool in beam-beam simulations
- Application to LHC HL upgrade study
- Application to eRHIC beam-beam study
- Summary

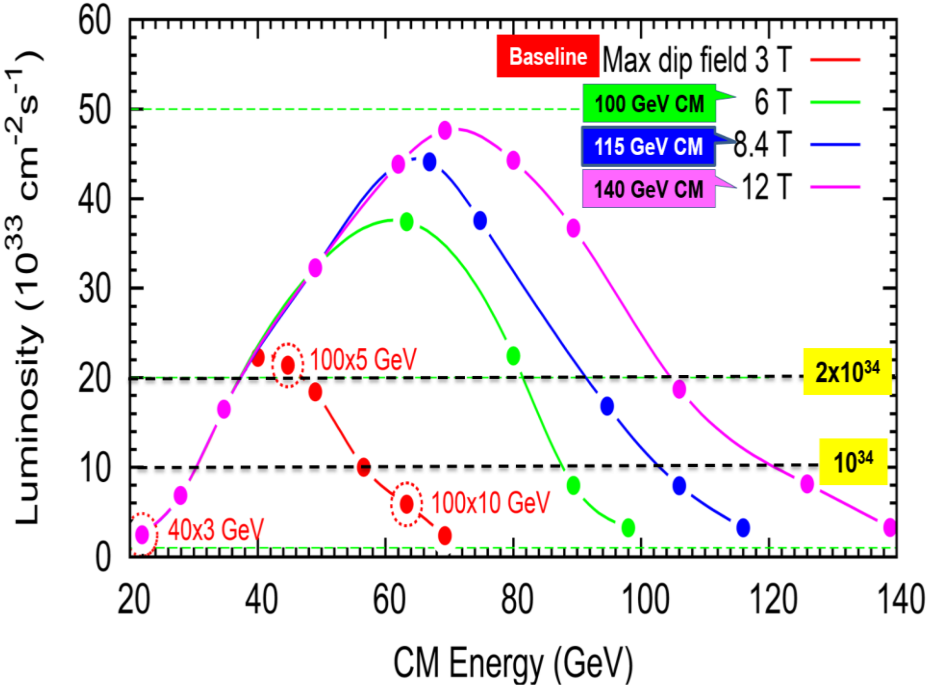
High Energy Collider Needs Large Luminosity for Physics Study

- The probability of event is proportional to the luminosity of colliding beams.
- Science case areas indicate the range of peak luminosities with which a statistically significant result can be achieved within a reasonable running time period.
- Both eRHIC and JLEIC designs require peak luminosity greater than $10^{33}/\text{cm}^2/\text{s}$.

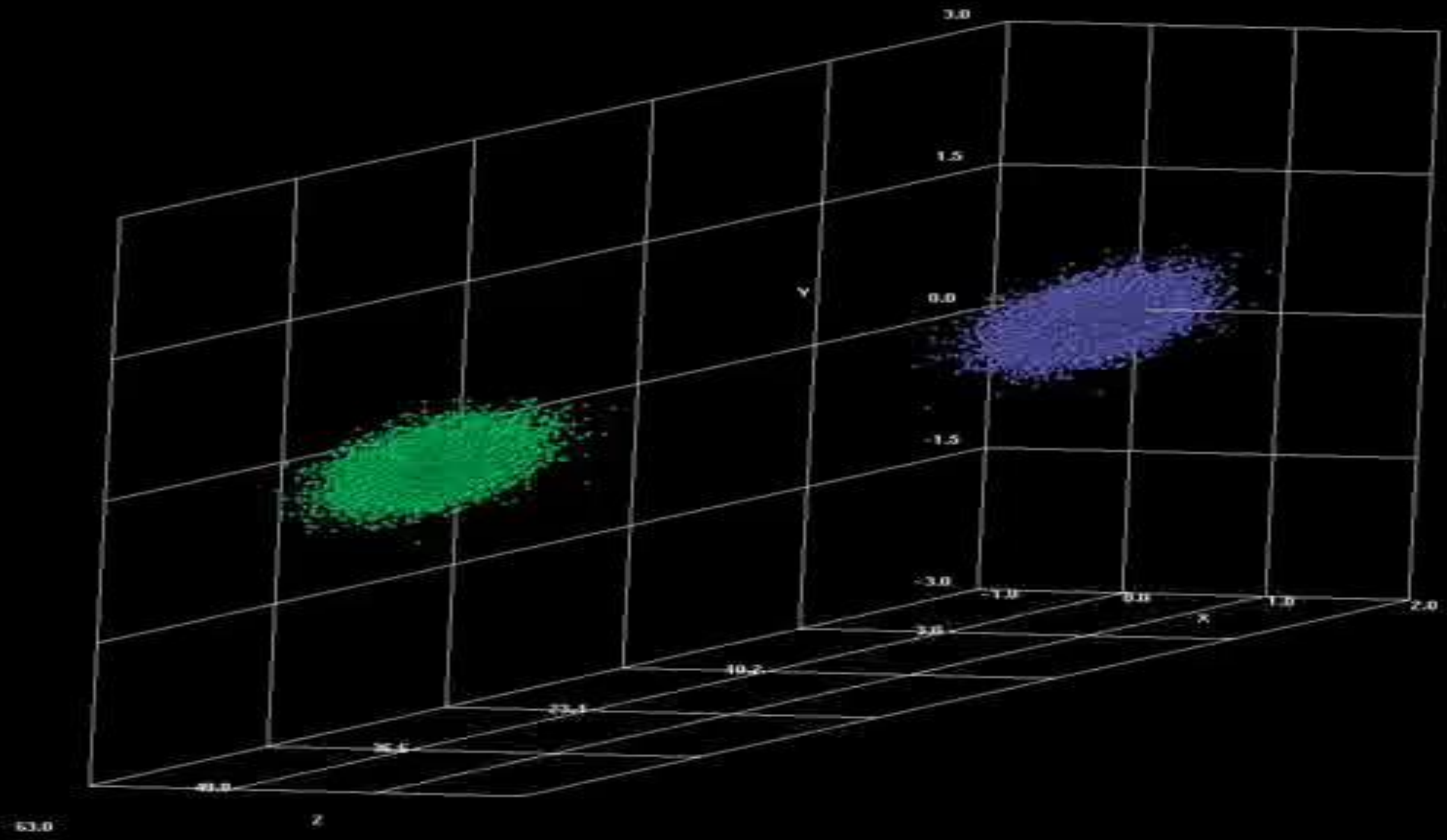
eRHIC



JLEIC



Electromagnetic Beam-Beam Effects Can Cause Beam Blowup Emittance Growth Limiting Final Luminosity



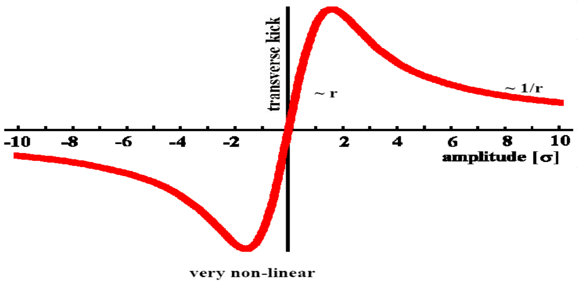
Electromagnetic Beam-Beam Interaction Limits the Luminosity

- Luminosity depends on:

$$L = f_b \left(\frac{4\pi\gamma_p\gamma_e}{r_p r_e} \right) (\xi_p \xi_e)(\sigma'_p \sigma'_e)$$

$$\xi_p = \frac{r_p \beta^*_p}{4\pi\gamma_p} \frac{N_e}{\sigma_e^2} \qquad \xi_e = \frac{r_e \beta^*_e}{4\pi\gamma_e} \frac{N_p}{\sigma_p^2}$$

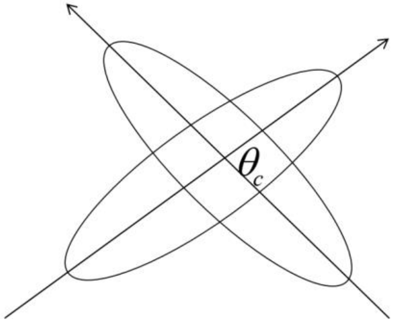
- Larger luminosity wants higher repetition rate, larger beam-beam parameters, smaller crossing angle and beam separation.
- However, beam-beam effects limit these factors and eventually luminosity:



- **Linear beam-beam force causes shift of machine work point**
- **Nonlinear beam-beam force causes beam instability, resonance diffusion, emittance growth, beam blow-up**

Crossing Angle Collision Is Used in High Luminosity Collider

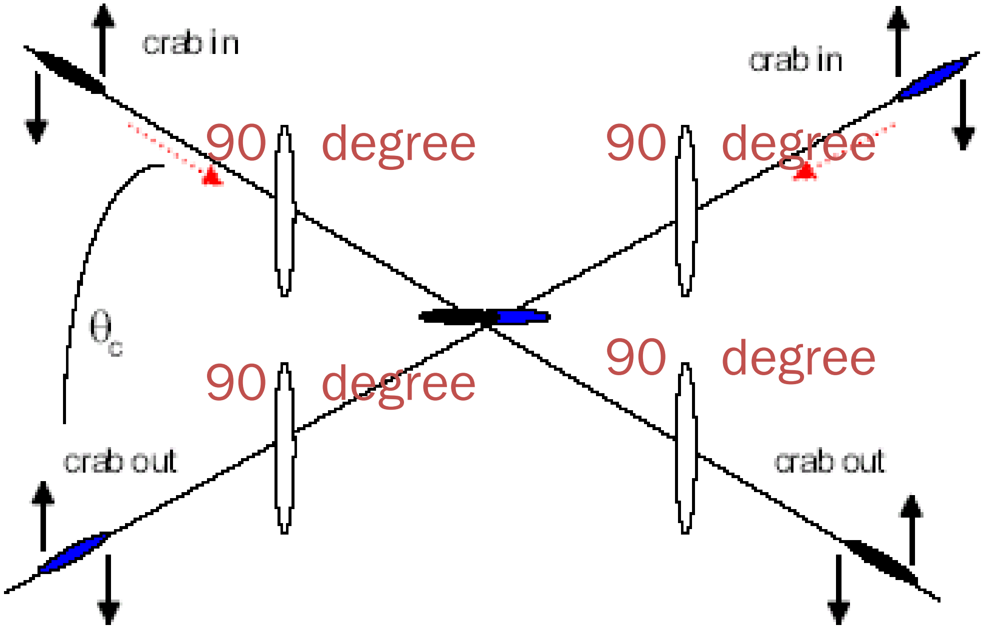
- reduces the parasitic long-range beam-beam effects that limit the beam lifetime
- reduces the background noise and improves the signal to noise ratio in the detector
- but also reduces luminosity and the probability of event



$$L = L_0 \frac{1}{\sqrt{1+\Theta^2}}; \quad \Theta \equiv \frac{\tan(\theta_c/2)\sigma_z}{\sigma_x}$$

Crab Cavity Recovers the Geometric Luminosity Loss

- Crab cavities located 90 degree phase from IP with right voltage recover luminosity loss



$$RF\ voltage:$$

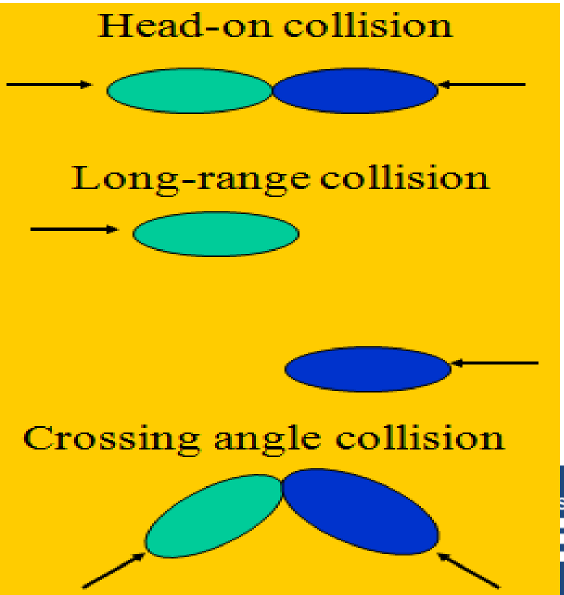
$$V = \frac{cE_s \tan(\frac{\theta_c}{2})}{\omega\sqrt{\beta_{x,crab}\beta_x^*}}$$

Computational Tool in Beam-Beam Simulation

Berkeley Lab Accelerator Simulation Toolkit

BeamBeam3D: A Parallel Colliding Beam Simulation Code

http://blast.lbl.gov



Some key features of the BeamBeam3D

- Multiple-slice model for finite bunch length
- New algorithm -- shifted Green function -- efficiently models long-range collisions
- Parallel particle-field based decomposition to achieve perfect load balance
- Lorentz boost to handle crossing angle
- Arbitrary closed-orbit separation
- Multiple bunches, multiple collision points
- Linear transfer matrix + one turn chromaticity
- Conducting wire, crab cavity, e-lens compensation model
- Feedback model
- Impedance model

Efficient Green's Function Method to the Poisson Equation for Beam-Beam Force Calculation (1)

$$\phi(r) = \int G(r, r')\rho(r')dr'$$

$$\phi(r_i) = h\sum_{i'=1}^{N} G(r_i - r_{i'})\rho(r_{i'})$$

$$\boxed{G(x, y) = -\frac{1}{2}\log(x^2 + y^2)}$$

Direct summation of the convolution scales as N^2 !!!!
N – total number of grid points

Efficient Green's Function Method to the Poisson Equation for Beam-Beam Force Calculation (2)

Hockney's Algorithm:- scales as (2N)log(2N)

- Ref: Hockney and Easwood, ***Computer Simulation using Particles***, McGraw-Hill Book Company, New York, 1985.

$$\phi_c(r_i) = h\sum_{i'=1}^{2N} G_c(r_i - r_{i'})\rho_c(r_{i'})$$

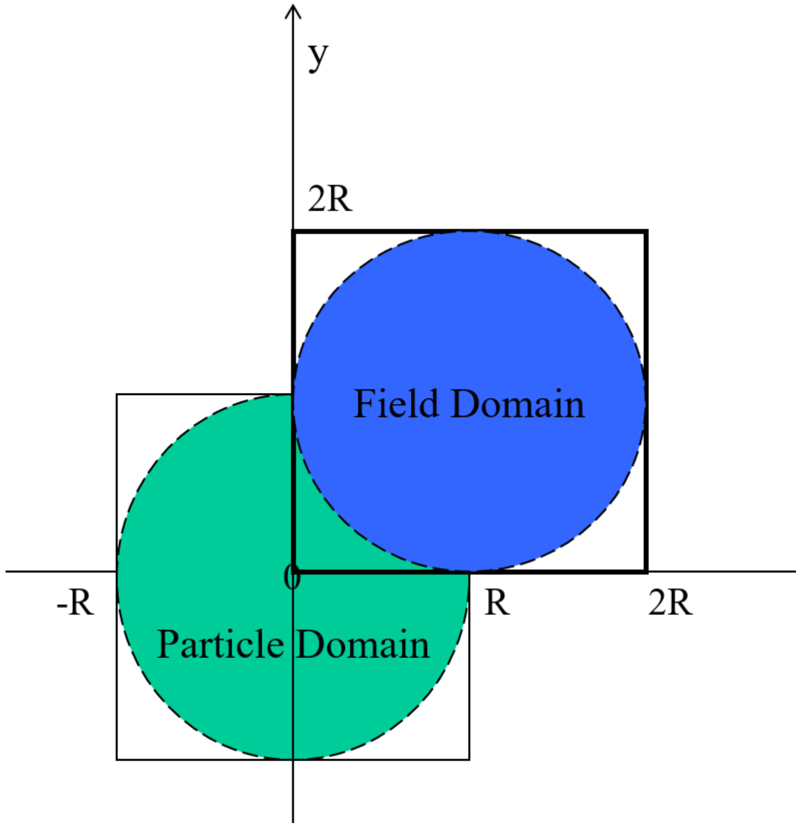
$$\phi(r_i) = \phi_c(r_i) \quad \text{for i} = 1, \text{N}$$

Shifted Green function Algorithm:

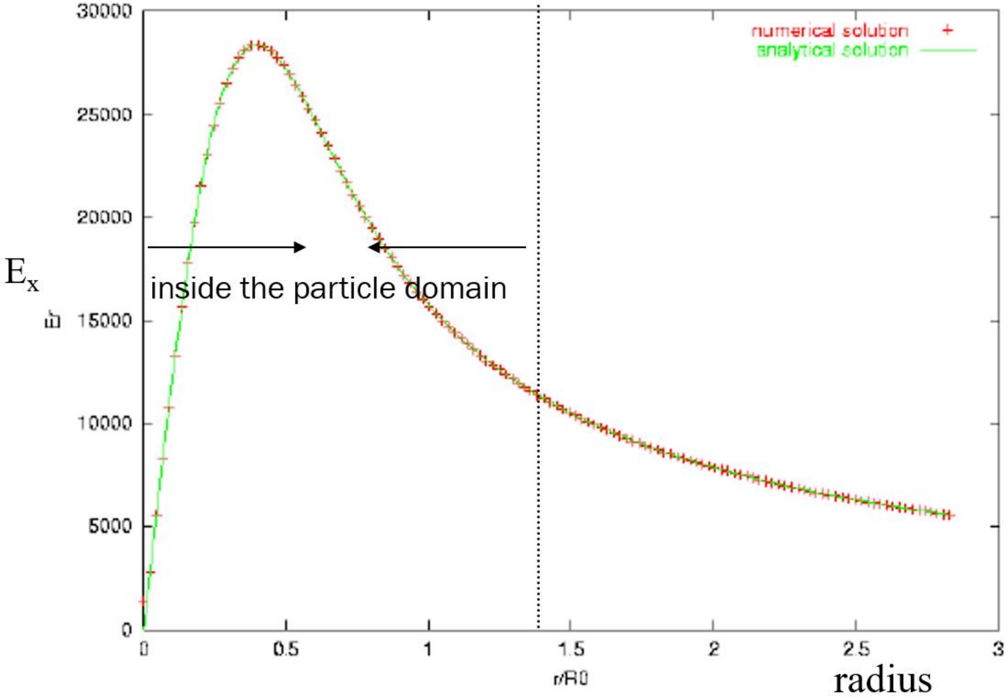
$$\phi_F(r) = \int G_s(r,r')\rho(r')dr'$$

$$G_s(r,r') = G(r + r_s, r')$$

Good Agreement between the Numerical Solution from the Shifted Green Function and the Analytical Solution



beam-beam field vs. radius

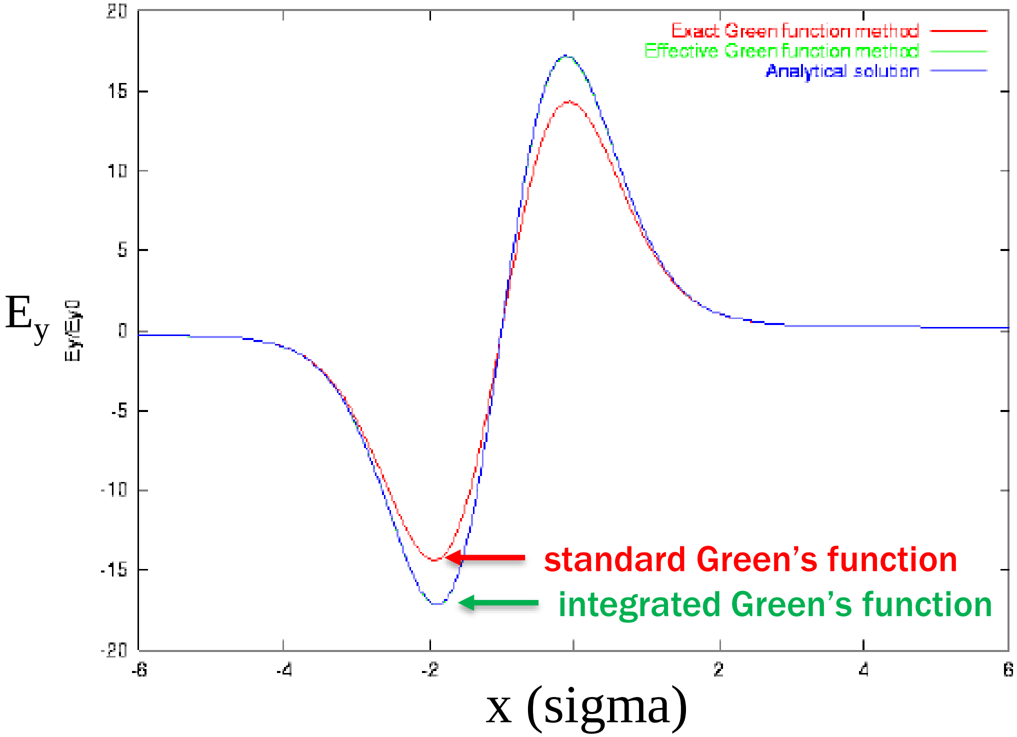


inside the particle domain

Efficient Green's Function Method to the Poisson Equation (3) (Integrated Green function algorithm for large aspect ratio)

$$\phi_c(r_i) = \sum_{i'=1}^{2N} G_i(r_i - r_{i'})\rho_c(r_{i'})$$

$$\boxed{G_i(r,r') = \oint G_s(r,r')\,dr'}$$

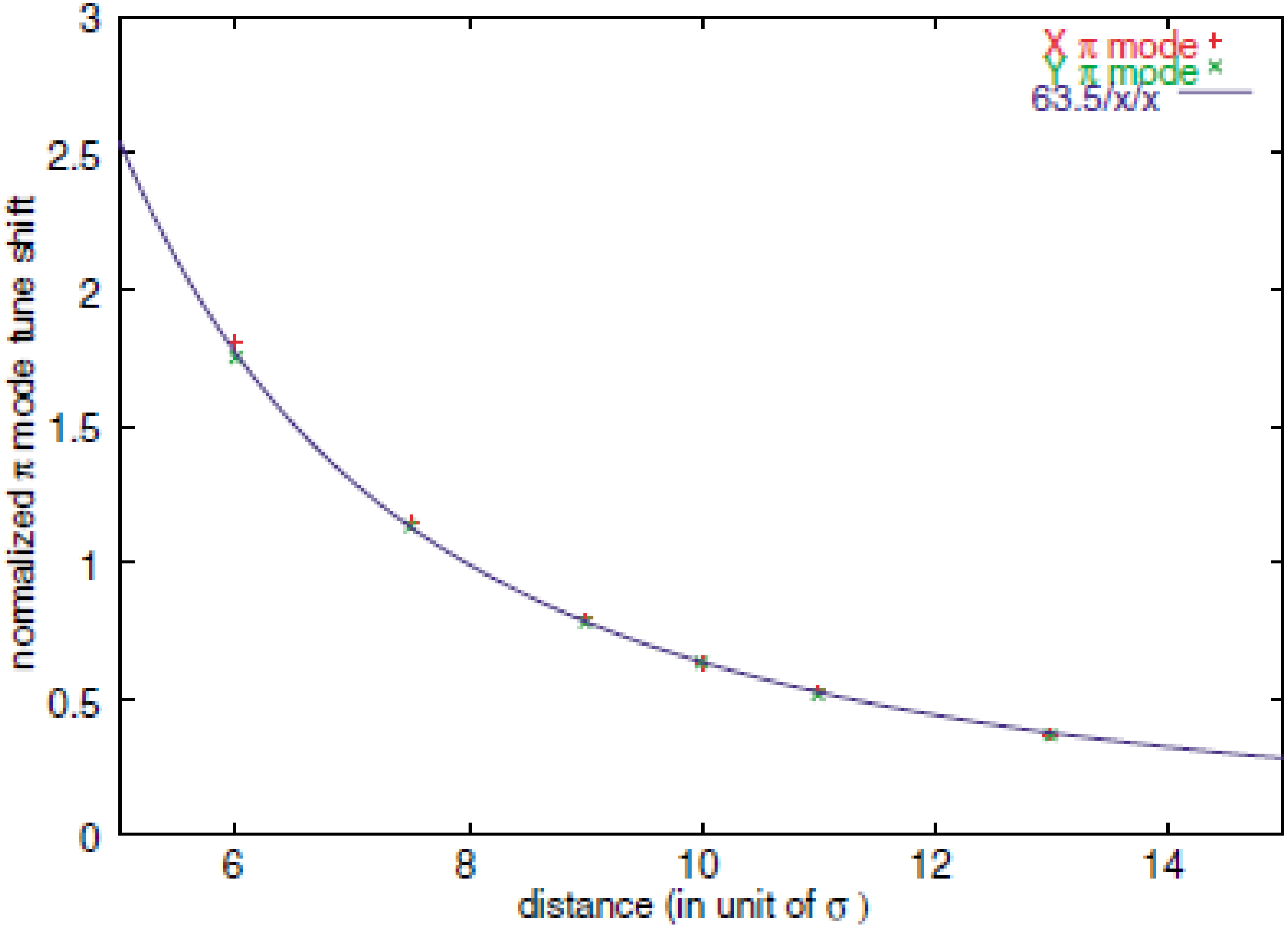


standard Green's function

integrated Green's function

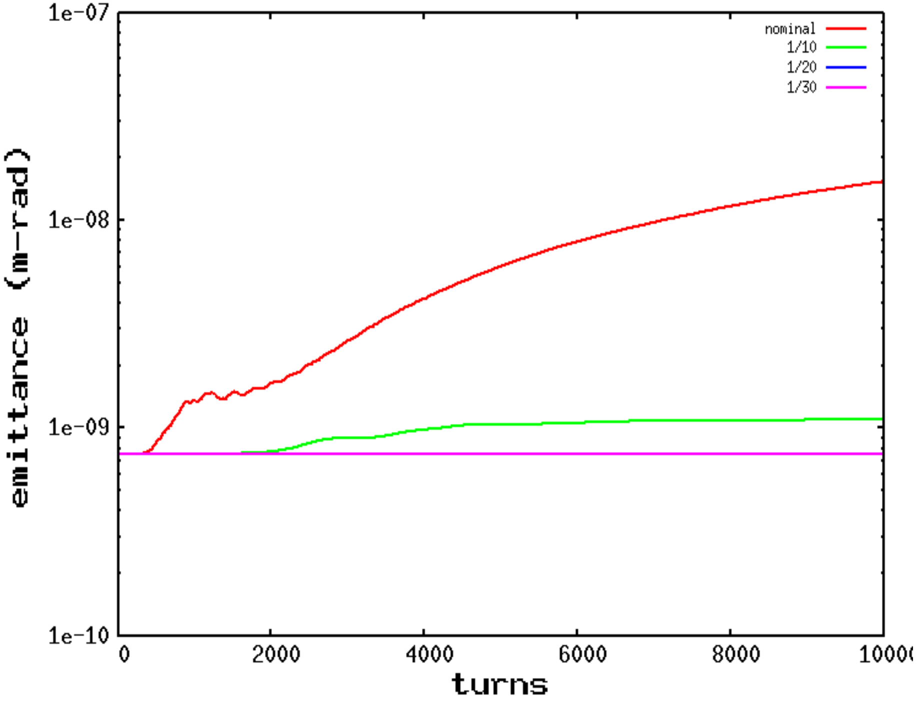
- Gaussian density distribution and horizontal-to-vertical aspect ratio 30

Pi Mode Tune Shift vs. Horizontal Separation (simulation vs. analytic model I)

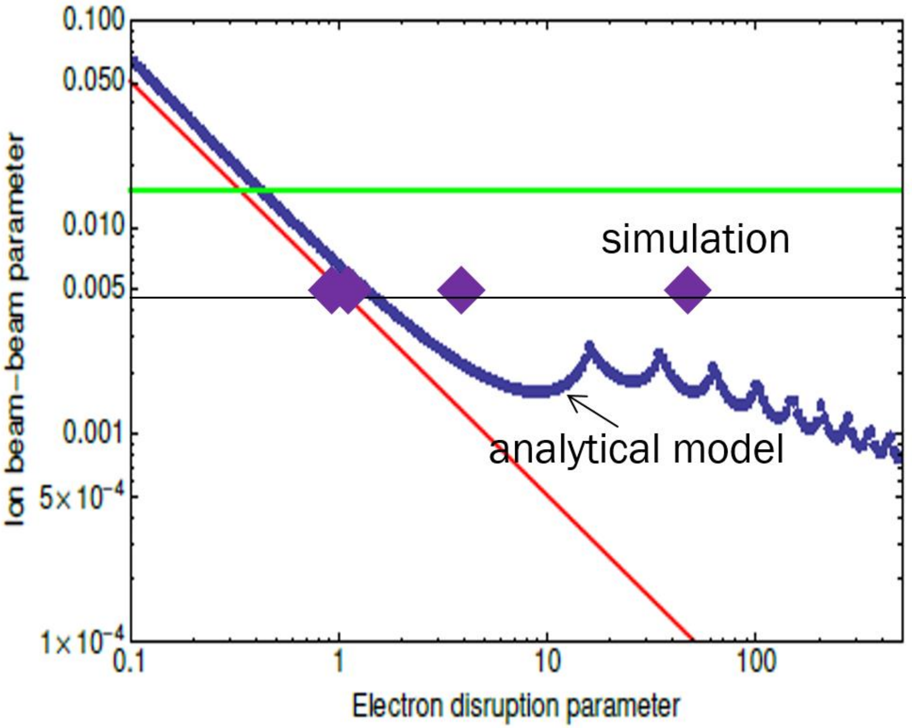


Threshold of Kink Instability vs. Electron Disruption Param. from 2-Particle Model, Multi-Particle Model, and Simulation

Emittance growth vs. ion intensity

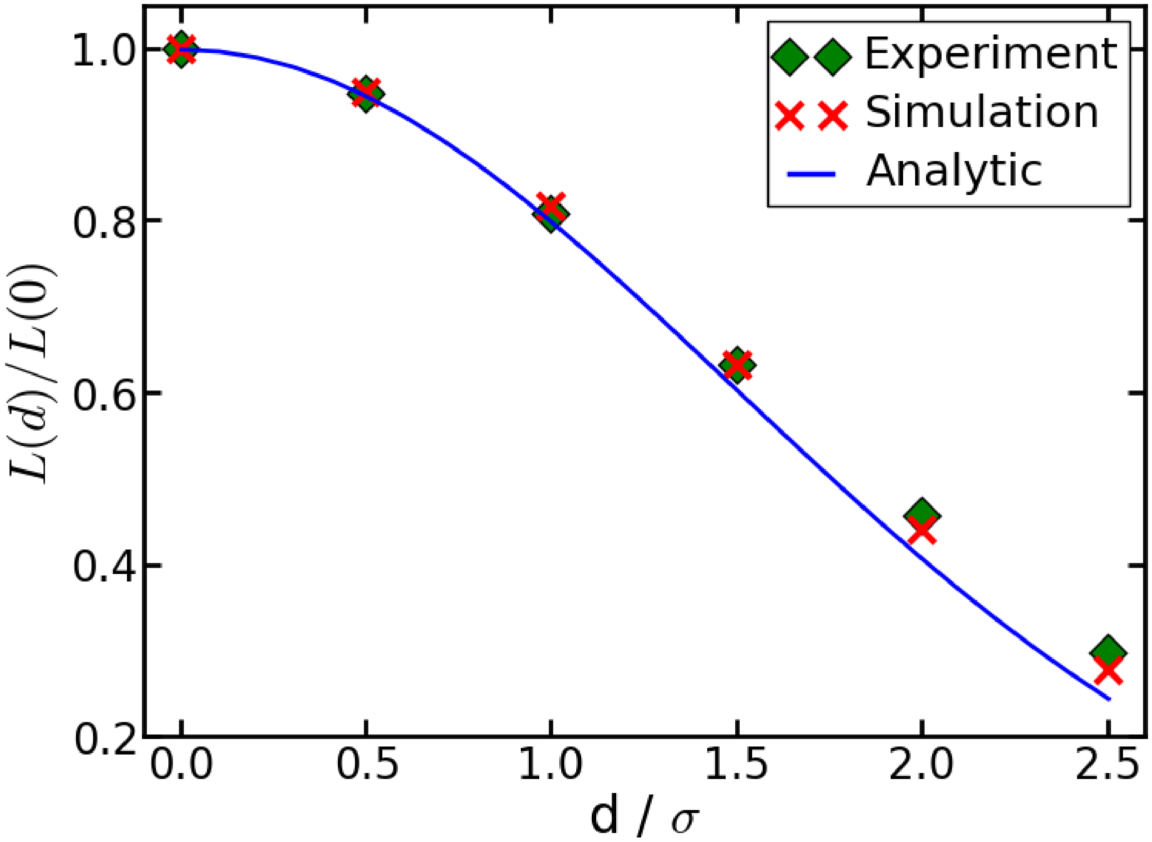


Analytical Model for Instability Threshold



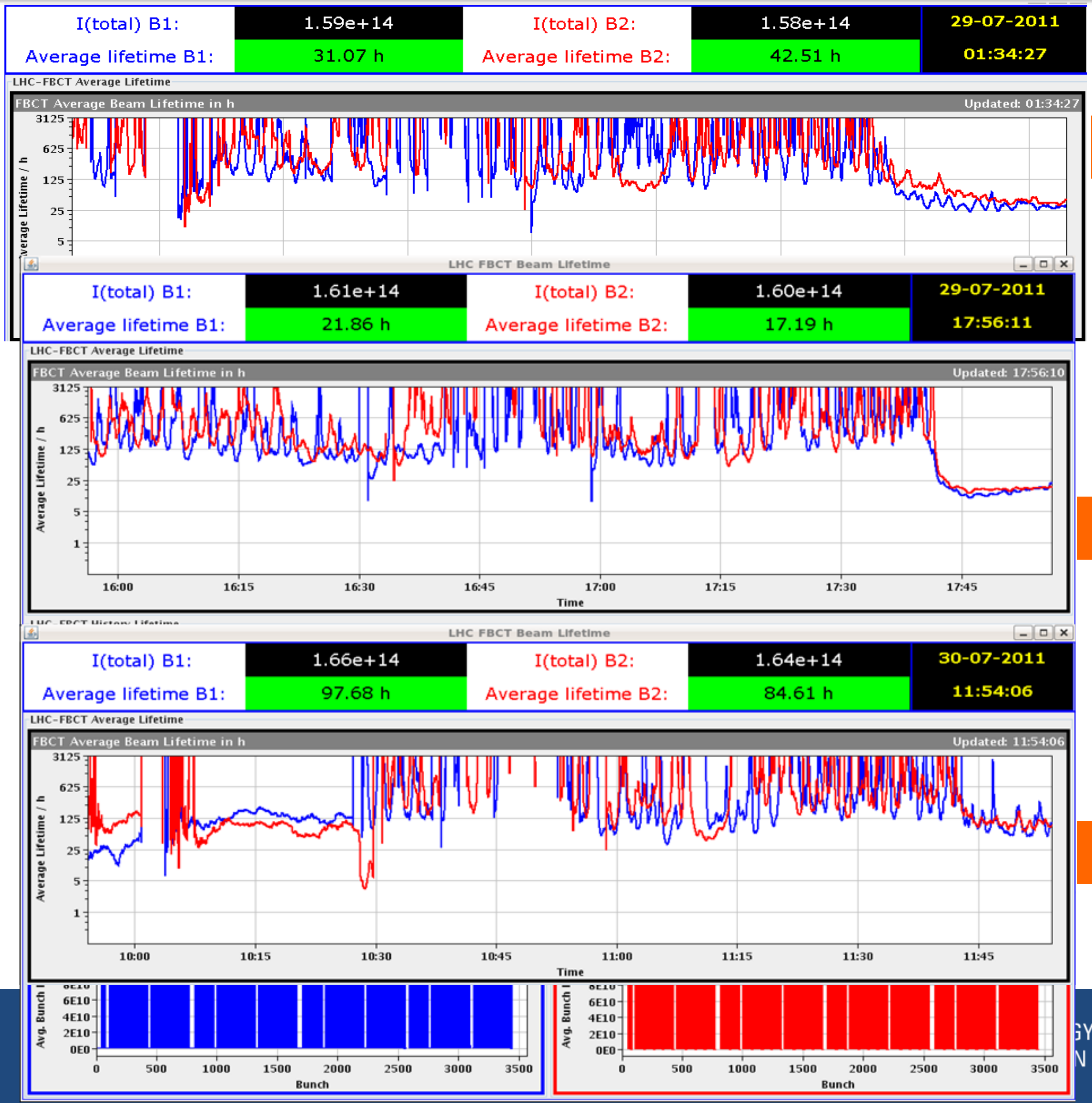
- Nominal working point will be unstable
- Reducing proton intensity moves the beam into stable regime

Simulation of LHC Observation (1) Luminosity vs. Offset



Very good agreement between experiment, simulation and theory

LHC Observation (II): Working Point Optimization



I(total) B1:	1.59e+14	I(total) B2:	1.58e+14	29-07-2011
Average lifetime B1:	31.07 h	Average lifetime B2:	42.51 h	01:34:27

LHC-FBCT Average Lifetime

FBCT Average Beam Lifetime in h Updated: 01:34:27

LHC FBCT Beam Lifetime

I(total) B1:	1.61e+14	I(total) B2:	1.60e+14	29-07-2011
Average lifetime B1:	21.86 h	Average lifetime B2:	17.19 h	17:56:11

LHC-FBCT Average Lifetime

FBCT Average Beam Lifetime in h Updated: 17:56:10

LHC FBCT Beam Lifetime

I(total) B1:	1.66e+14	I(total) B2:	1.64e+14	30-07-2011
Average lifetime B1:	97.68 h	Average lifetime B2:	84.61 h	11:54:06

LHC-FBCT Average Lifetime

FBCT Average Beam Lifetime in h Updated: 11:54:06

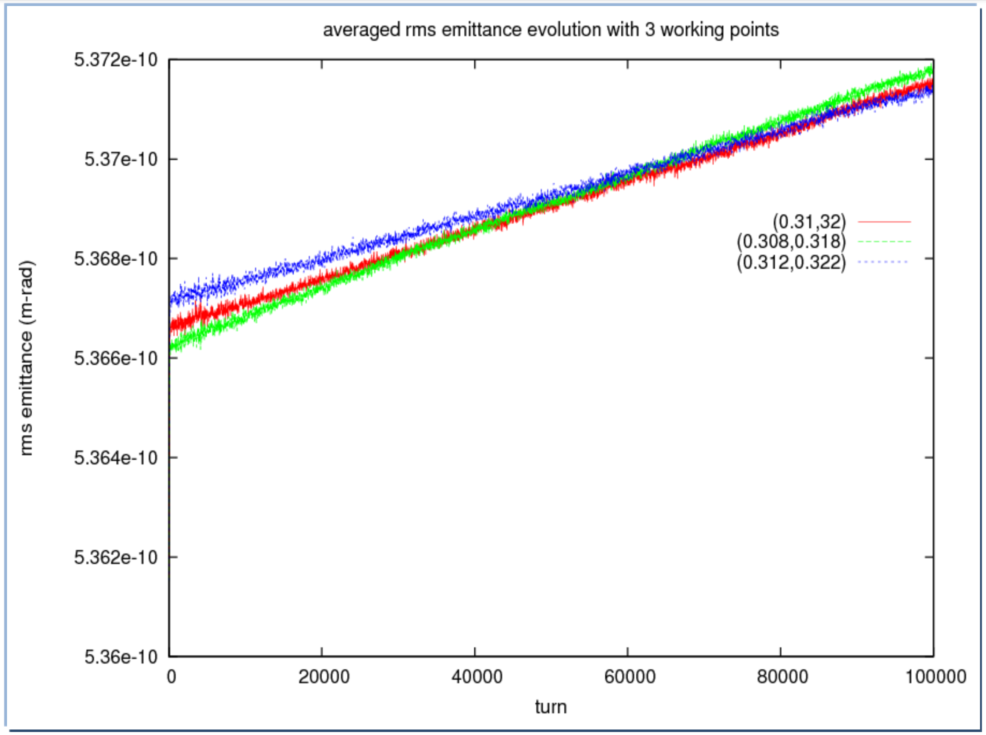
Fill 1990 – 0.31/0.32

Fill 1991 - 0.308/0.318

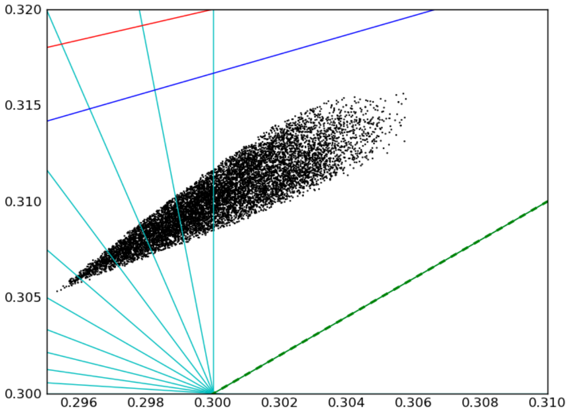
Fill 1992 - 0.312/0.322

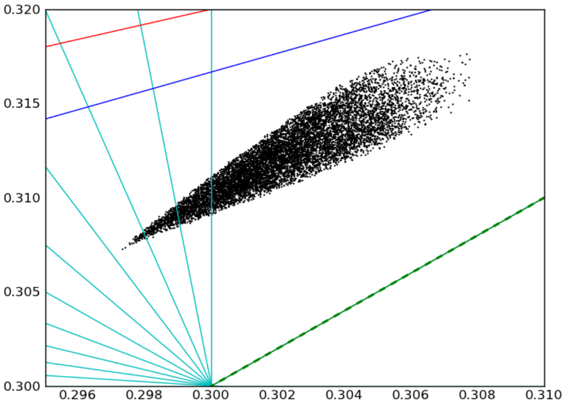
Courtesy of G. Arduini, J. Wenninger

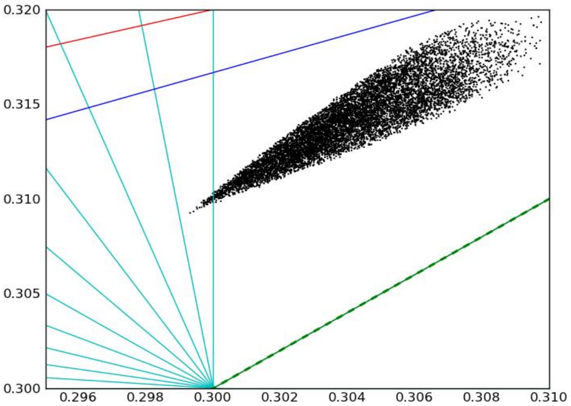
Simulations with Different Working Points Show Larger Emittance Growth with Lower Working Point



- larger emittance growth rate when the working point moves downward into 10th resonance



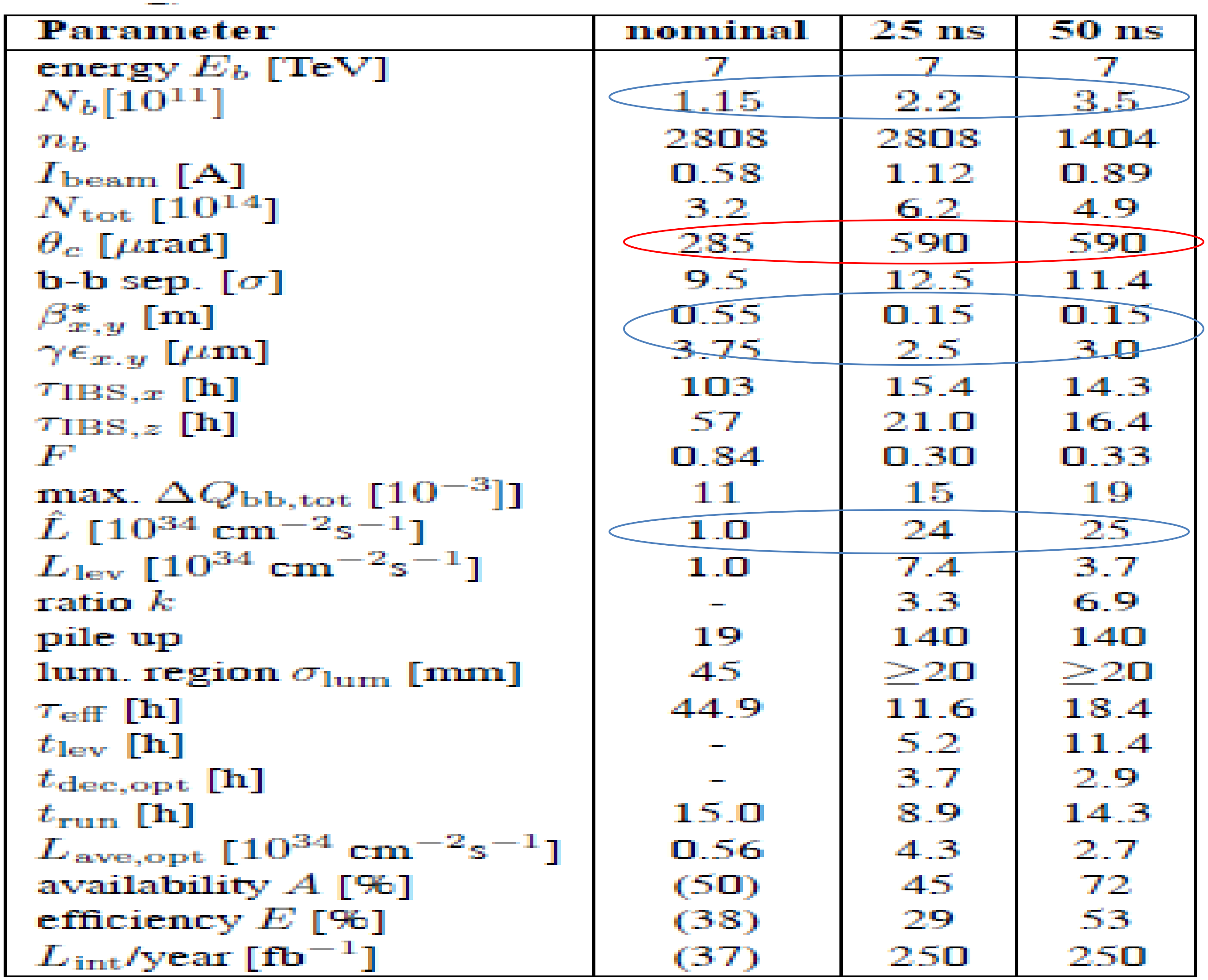




Application to LHC HL upgrade

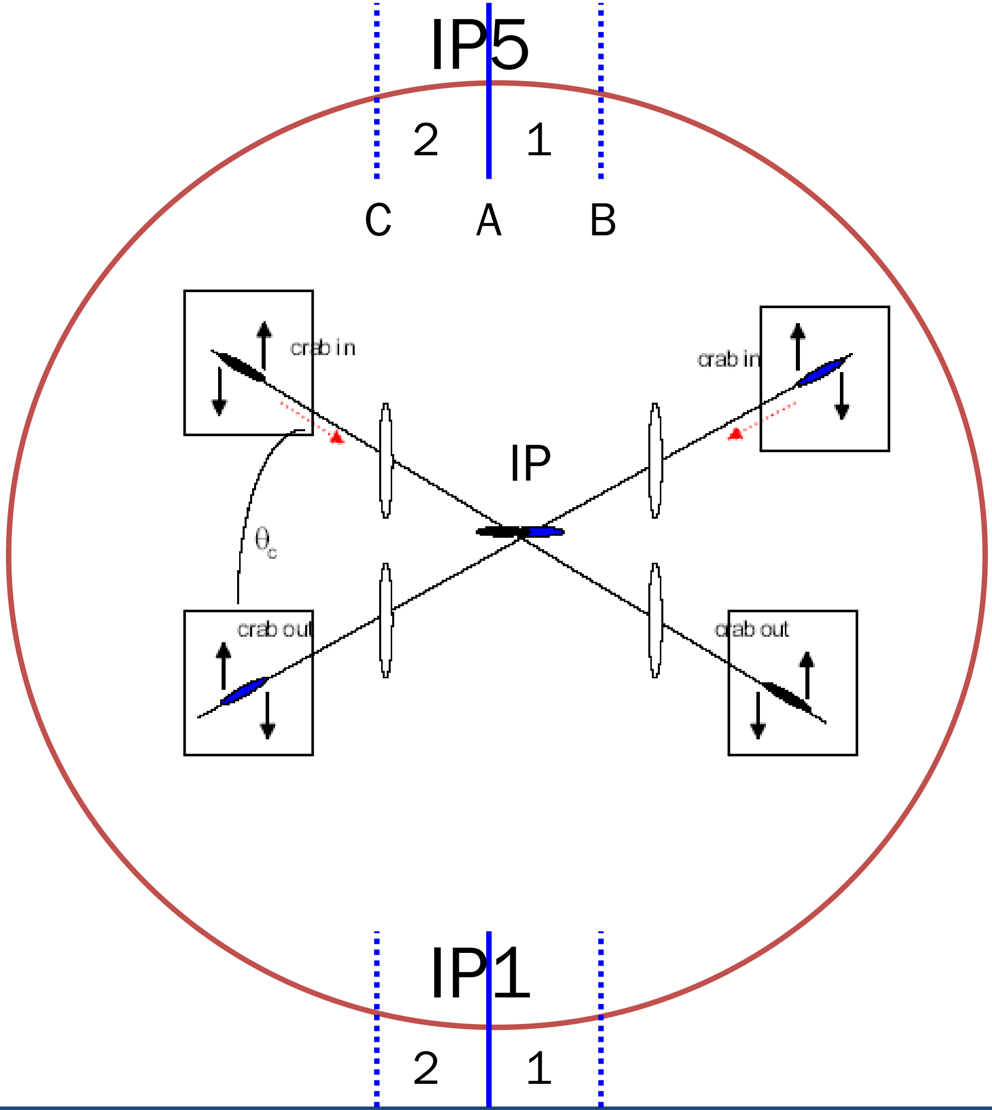
J. Qiang, S. Paret, A. Ratti, J. Barranco, T. Pieloni, G. Arduini, X. Buffat, Y. Papaphilippou, NIM-A 900, p. 53, 2018.

LHC Upgrade will Improve the LHC Luminosity by an Order of Magnitude, but with Larger Crossing Angle



Parameter	nominal	25 ns	50 ns
energy E_b [TeV]	7	7	7
$N_b [10^{11}]$	1.15	2.2	3.5
n_b	2808	2808	1404
I_{beam} [A]	0.58	1.12	0.89
N_{tot} [10^{14}]	3.2	6.2	4.9
θ_c [μrad]	285	590	590
b-b sep. [σ]	9.5	12.5	11.4
$\beta^*_{x,y}$ [m]	0.55	0.15	0.15
$\gamma\epsilon_{x,y}$ [μm]	3.75	2.5	3.0
$\tau_{\mathrm{IBS},x}$ [h]	103	15.4	14.3
$\tau_{\mathrm{IBS},z}$ [h]	57	21.0	16.4
F	0.84	0.30	0.33
max. $\Delta Q_{\mathrm{bb,tot}}$ [10^{-3}]	11	15	19
$\hat{L}$ [10^{34} cm^{-2}s^{-1}]	1.0	24	25
L_{lev} [10^{34} cm^{-2}s^{-1}]	1.0	7.4	3.7
ratio k	-	3.3	6.9
pile up	19	140	140
lum. region σ_{lum} [mm]	45	$\geq$20	$\geq$20
τ_{eff} [h]	44.9	11.6	18.4
t_{lev} [h]	-	5.2	11.4
$t_{\mathrm{dec,opt}}$ [h]	-	3.7	2.9
t_{run} [h]	15.0	8.9	14.3
$L_{\mathrm{ave,opt}}$ [10^{34} cm^{-2}s^{-1}]	0.56	4.3	2.7
availability A [%]	(50)	45	72
efficiency E [%]	(38)	29	53
L_{int}/year [fb^{-1}]	(37)	250	250

A Schematic of LHC Upgrade Using Crab Cavities



crabcavity RF voltage :

$$V = \frac{cE_s \tan(\frac{\theta_c}{2})}{\omega\sqrt{\beta_{x,crab}\beta_x^*}}$$

One Turn Transfer Map with Beam-Beam and Crab Cavity

$$M = Mb\ M1\ Mc_1\ M1^{-1}\ M\ M2^{-1}\ Mc_2\ M2$$

Mb: transfer map from head-on crossing angle beam-beam collision

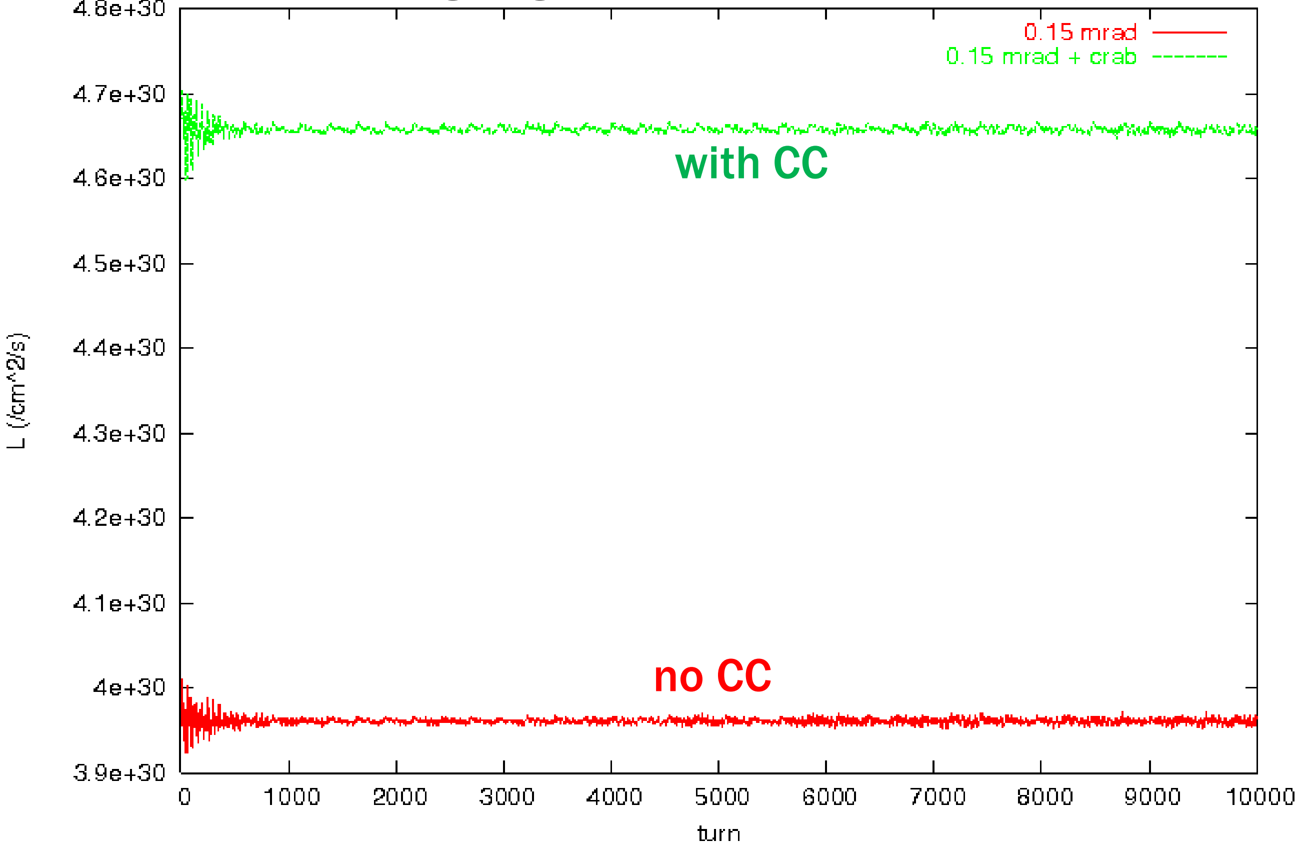
$Mc_{1,2}$: transfer maps from crab cavity deflection

M1-2: transfer maps between crab cavity and collision point

M: one turn transfer map of machine

Crab Cavity Helps Improve Luminosity (1)

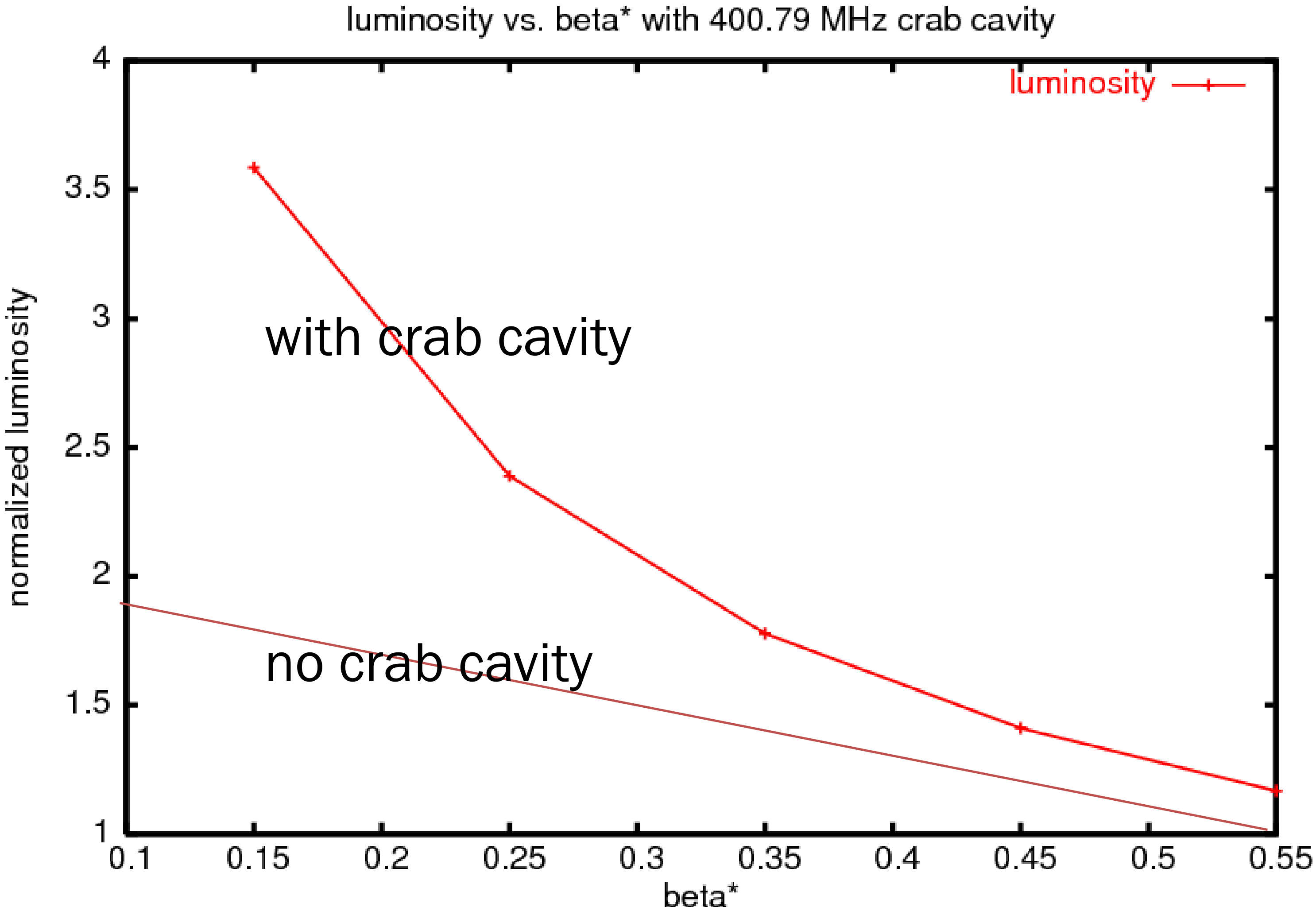
Luminoisty Evolution with 0.15 mrad Half Crossing Angle with/without Crab Cavity



turn

Crab Cavity Helps Improve Luminosity (2)

Luminosity vs. Beta* for LHC Crab Cavity Compensation



RF Noise in the Crab Cavity Causes Emittance Growth and Luminosity Degradation

$$x_i \propto V_{cc} \sin(kz_i + \delta\varphi)$$

0th order error (phase error):

$$\delta X = -\frac{c}{\omega_{cc}} \tan\left(\frac{\theta}{2}\right) \delta\varphi$$

1st order error (voltage error):

$$\delta x_i \propto \delta V_{cc} \sin(kz_i) \approx \delta V_{cc} kz_i$$

white noise offset collision drives emittance growth

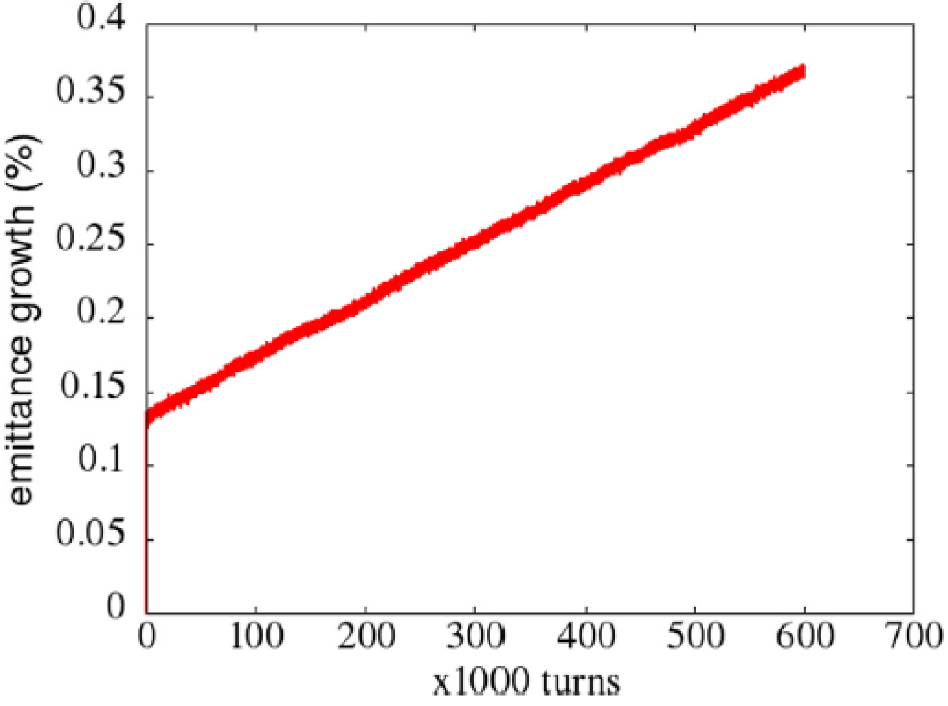
$$\frac{\delta\varepsilon}{\varepsilon} \approx \frac{K}{\left(1 + \frac{G}{2\pi|\xi|}\right)^2} \frac{\delta x^2}{\sigma_x^2}$$

Y. Alexahin, NIM A391,73 (1996)

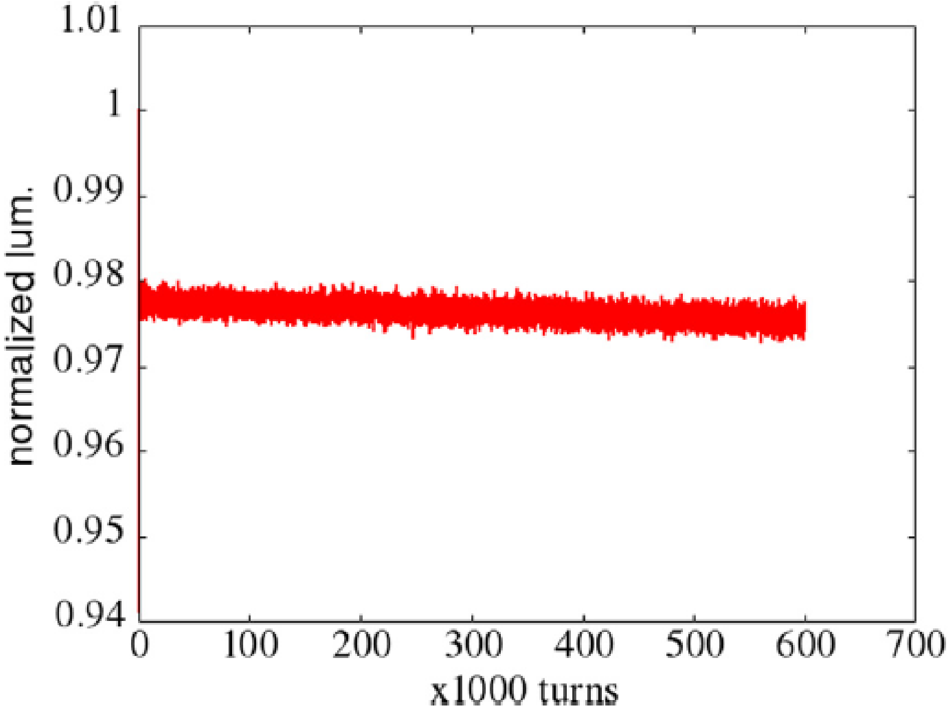
Crab Cavity White Phase Noise Causes Emittance Growth and Luminosity Degradation

Np = 2.2 x 10^{11}, beta* = 0.49 m

emittance growth

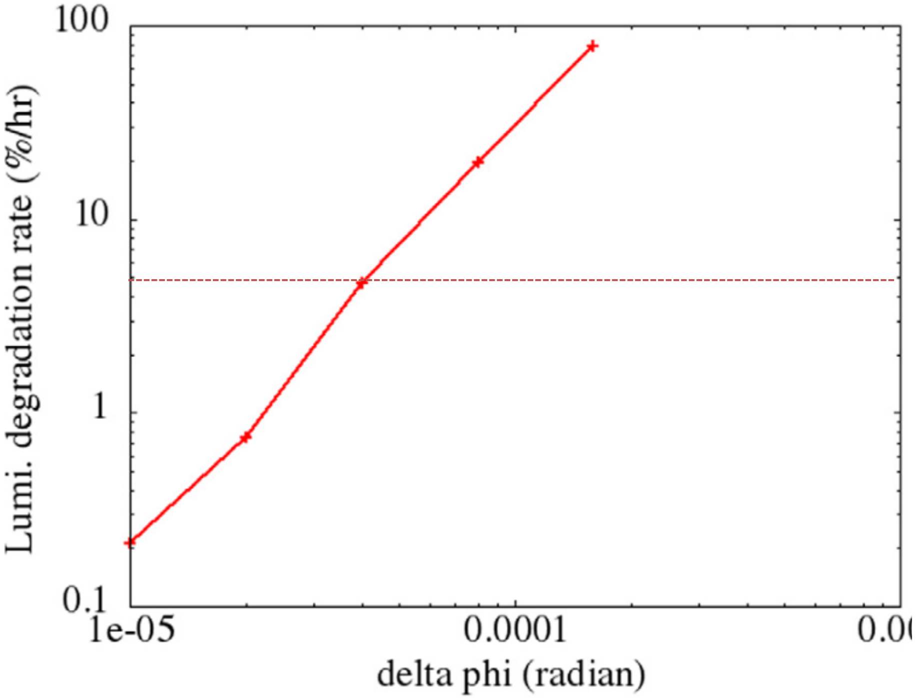


luminosity degradation

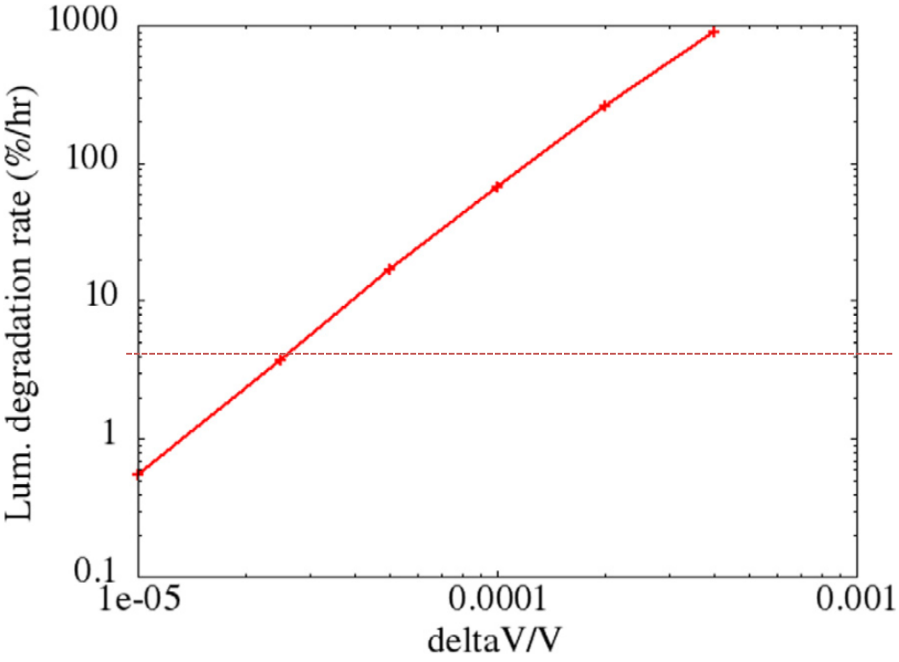


Luminosity Degradation due to Phase or Voltage White Noise

crab cavity white noise tolerance level ~10^{-5}



Lum. degradation rate vs. phase noise amp.



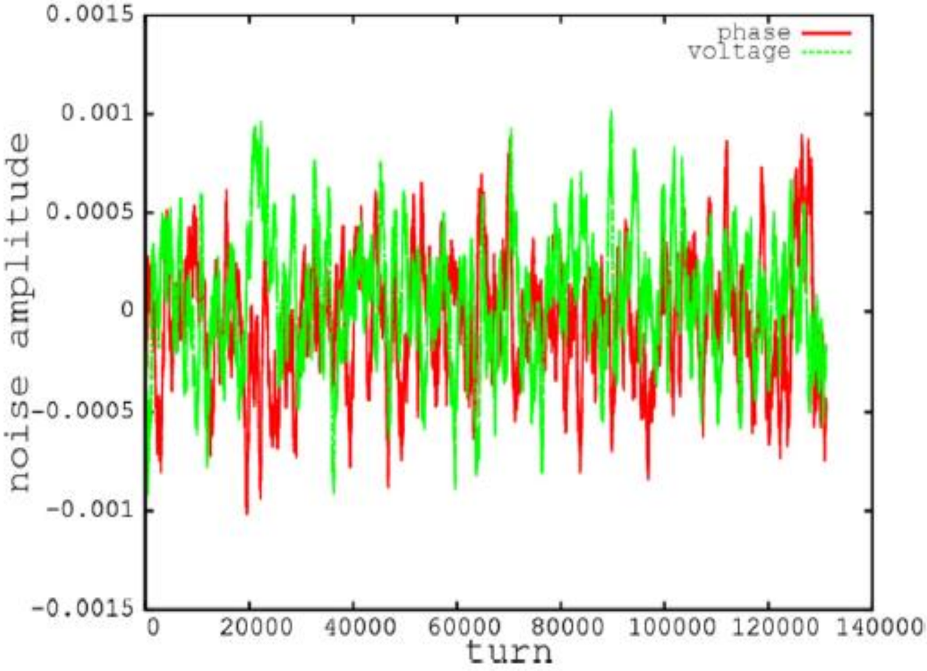
Lum. degradation rate vs. voltage noise amp

In order to have a good luminosity lifetime ~ 20 hours,
the white noise amplitude needs to be kept to the level of a few x10^{-5}.

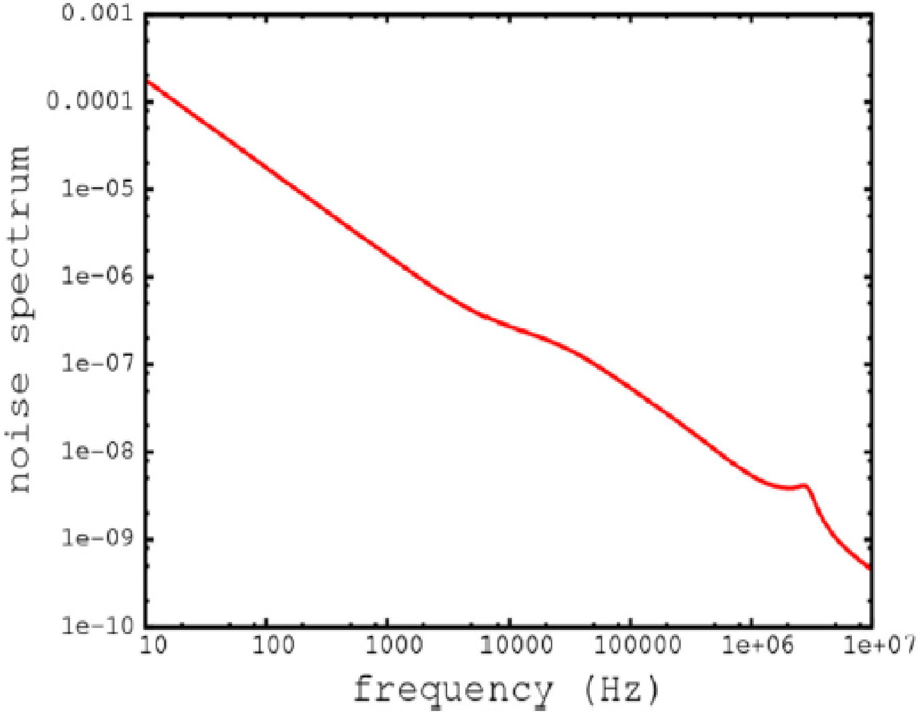
J. Qiang et al., in Proc. IPAC2015.

In Reality, the Noise in Crab Cavity Is not White Noise, but with Frequency Dependence

Phase and Amplitude Noise



Frequency-Dependent Crab Cavity Noise Power Spectrum

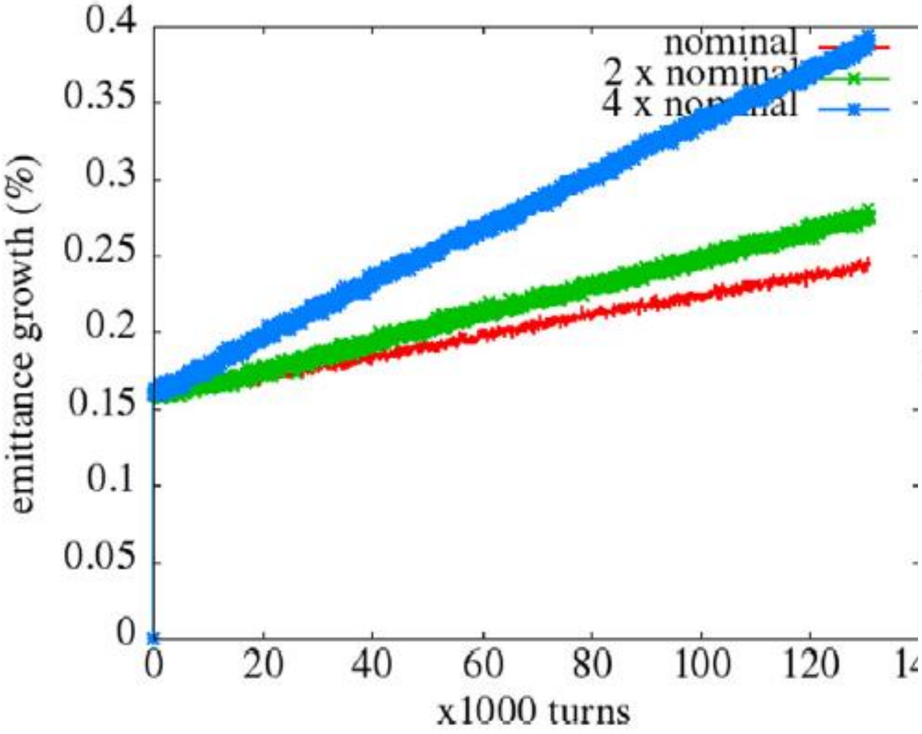


frequency (Hz)

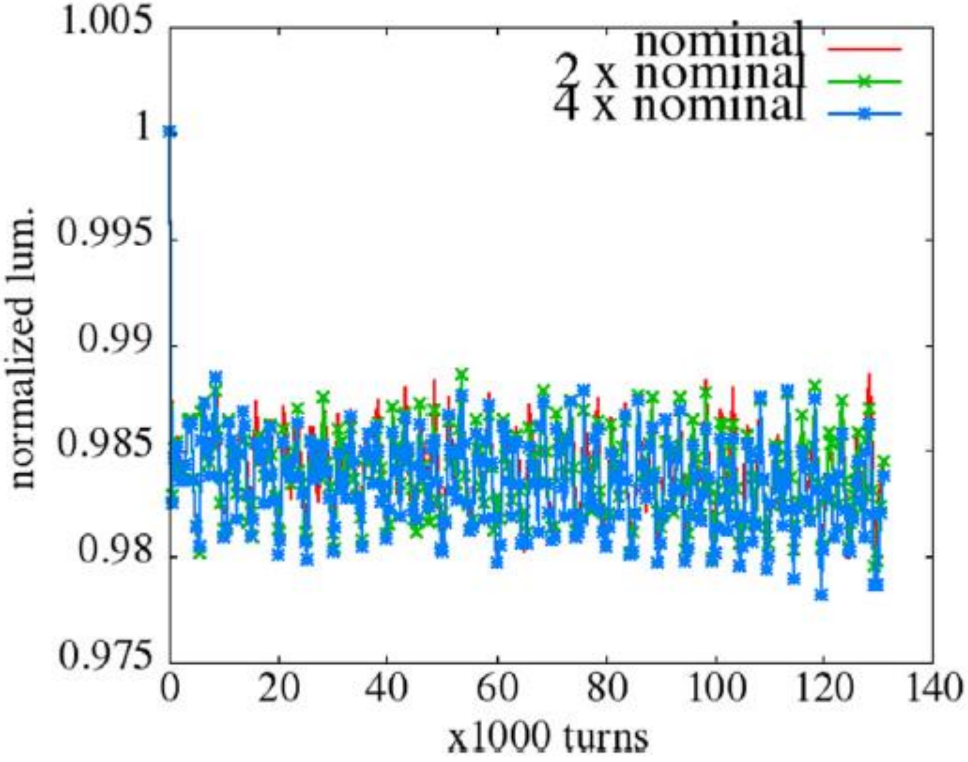
Courtesy of T. Mastori

Frequency Dependent Noise in Crab Cavity also Causes Emittance Growth and Luminosity Degradation

emittance growth

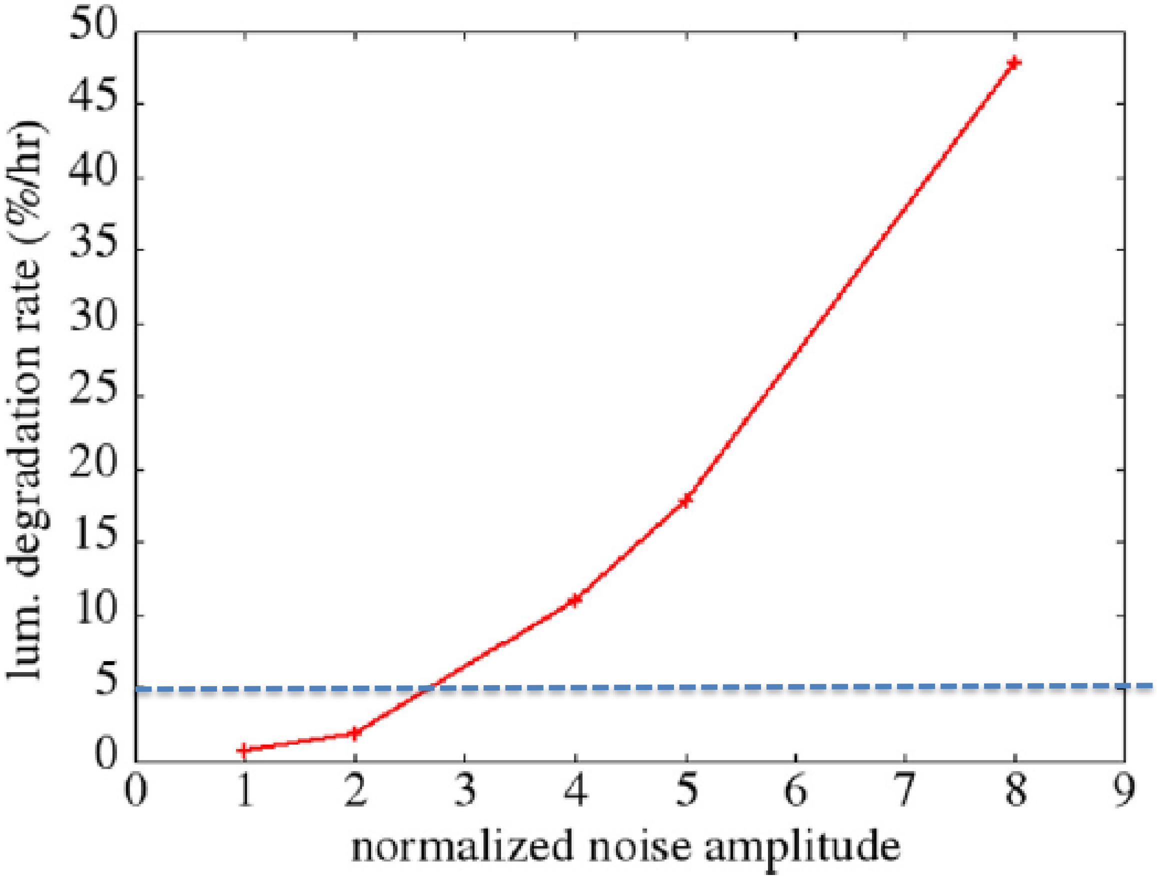


luminosity degradation



nominal noise amplitude = 3×10^{-4}

Luminosity Degradation Rate vs. Frequency Dependent Noise Amplitude



- To keep lum. degradation rate below 5%/hr, the noise amplitude should be kept to a few x10^{-4}.

Crab Cavity Has also RF Multipole Errors

$$
\begin{aligned}
P_{xi}^{n+1} &= P_{xi}^{n} + \frac{qc}{E}\sin(\omega z_i^n/c+\phi)\times \\
&\boxed{[b_2 x_i} + \boxed{b_3(x_i^2-y_i^2)} + \boxed{b_4(x_i^3-3x_i y_i^2)} + \\
&\boxed{b_5(x_i^4-6x_i^2y_i^2+y_i^4)}]
\end{aligned}
$$

$$
\begin{aligned}
P_{yi}^{n+1} &= P_{yi}^{n} - \frac{qc}{E}\sin(\omega z_i^n/c+\phi)\times \\
&[b_2 y_i + b_3 2x_i y_i + b_4(3x_i^2 y_i - y_i^3) + \\
&b_5(4x_i^3 y_i - 4x_i y_i^3)]
\end{aligned}
$$

$$
\begin{aligned}
\delta E_i^{n+1} &= \delta E_i^{n} + \frac{qc}{E}\cos(\omega z_i^n/c+\phi)\frac{\omega}{c}\times \\
&[b_2(x_i^2-y_i^2)/2 + b_3(x_i^3-3x_i y_i^2)/3 + \\
&b_4(x_i^4-6x_i^2y_i^2+y_i^4)/4 + \\
&b_5(x_i^5-10x_i^3y_i^2-3x_i y_i^4)/5]
\end{aligned}
$$

b_2 – quadrupole

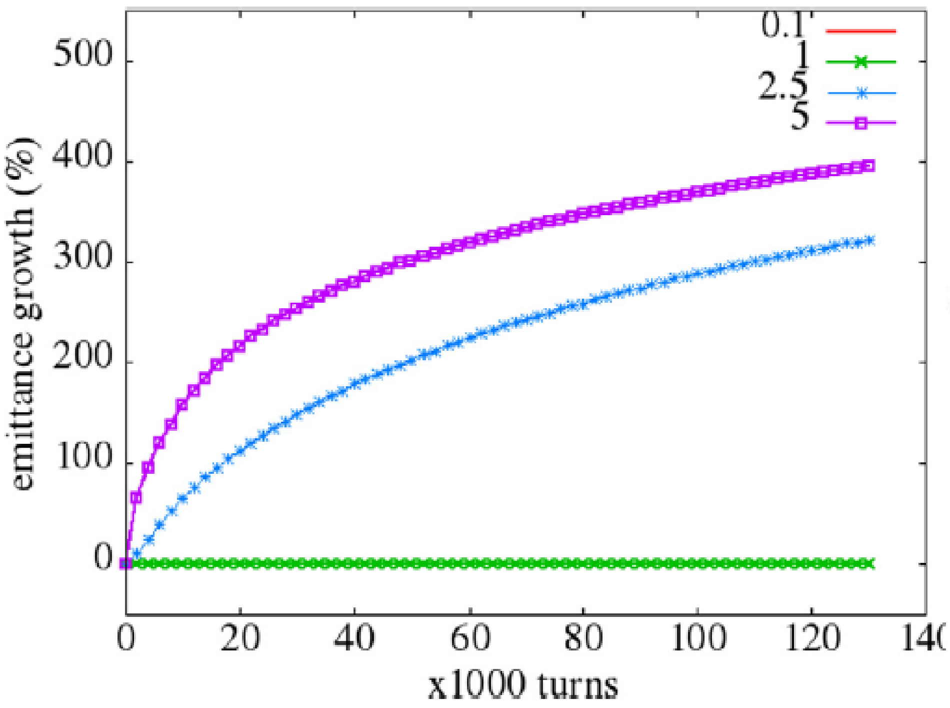
b_3 – sextupole

b_4 - octupole

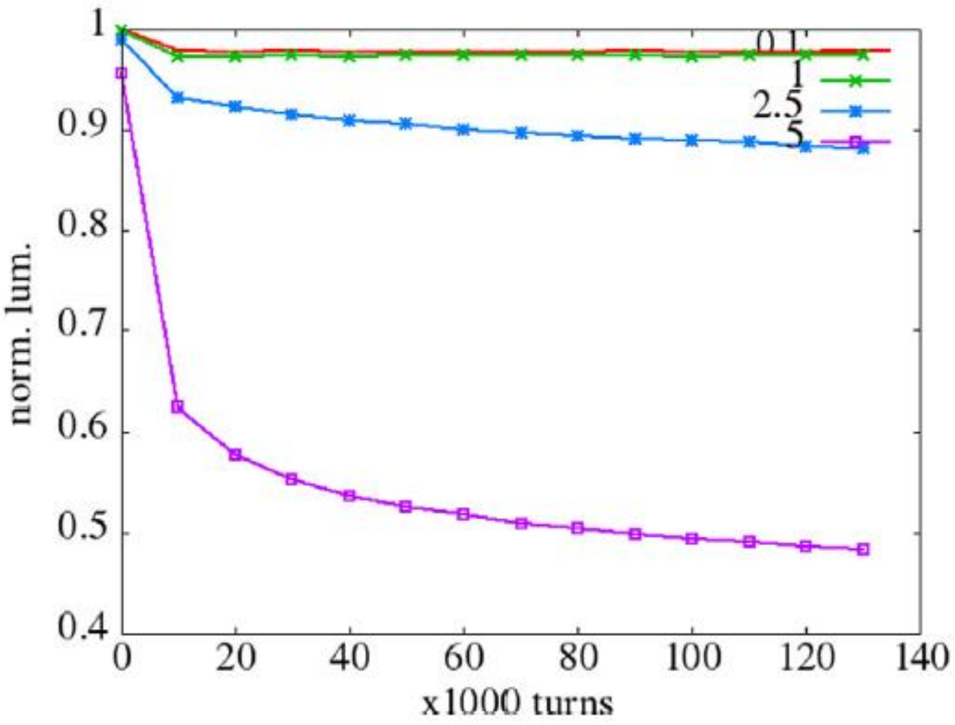
b_5 - decapole

Large RF Quadrupole Error Can Cause Emittance Growth and Luminosity Degradation

emittance growth

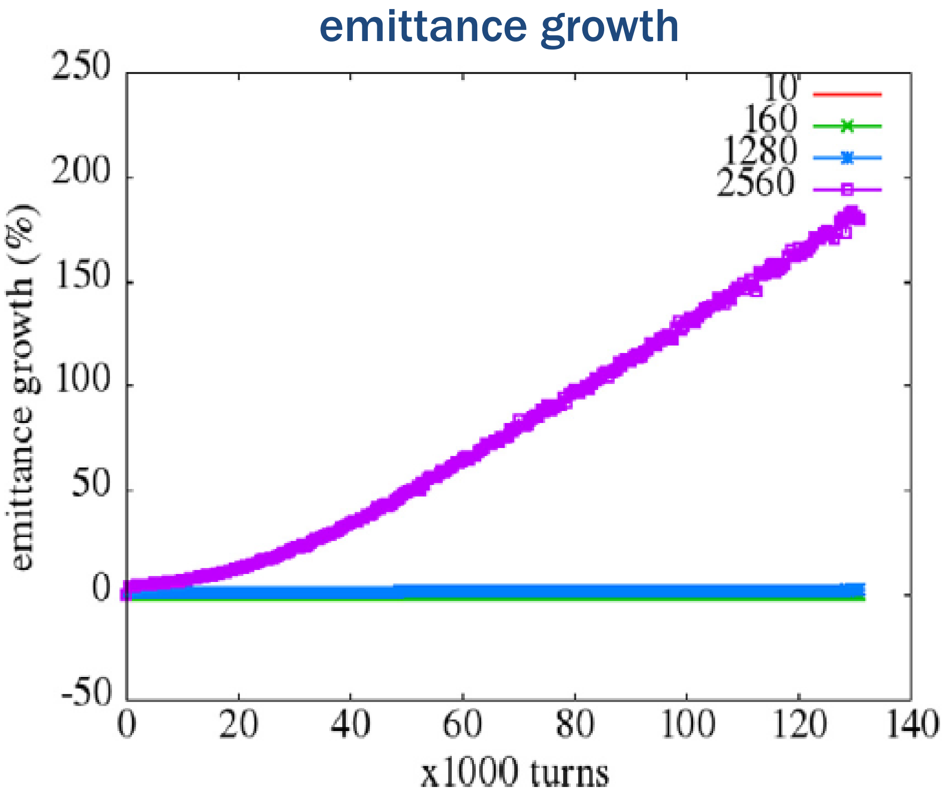


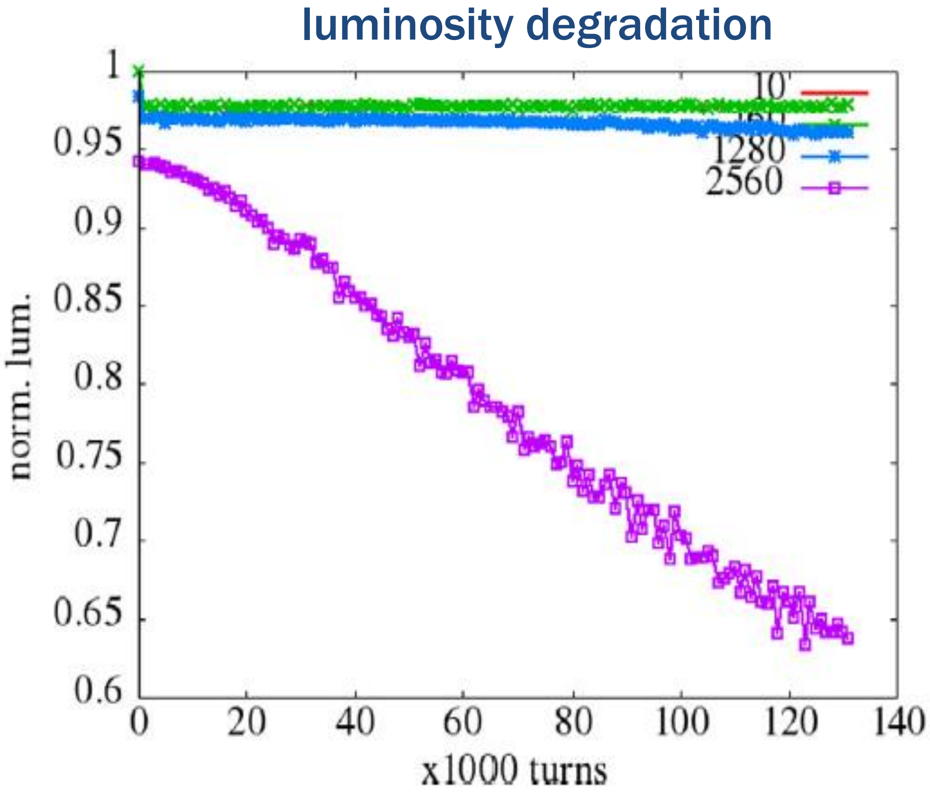
luminosity degradation



- significant lum. degradation with integrated RF quadrupole error beyond 2 T.
- current CC design quadupole error for HL LHC is below 0.2 T,

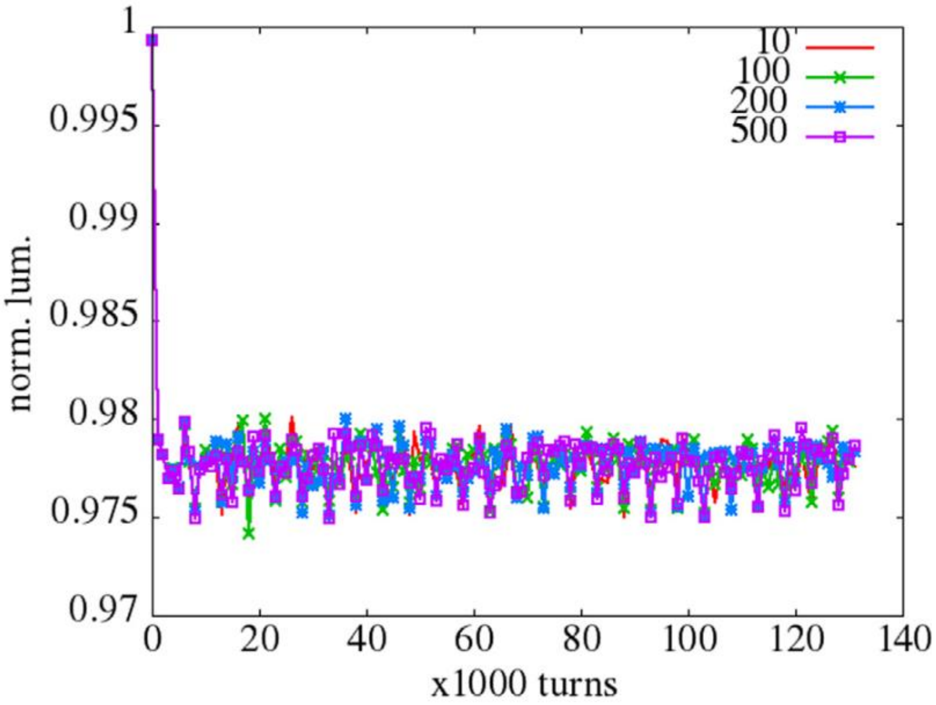
Large RF Sextupole Error Can Cause Emittance Growth and Luminosity Degradation

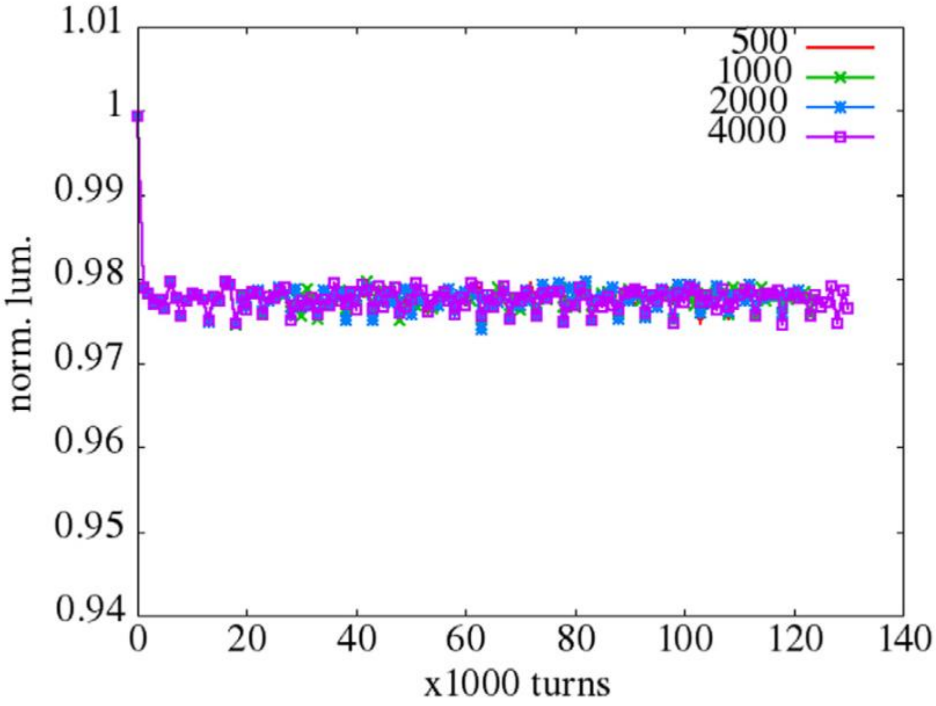

emittance growth


luminosity degradation

- significant lum. degradation with integrated sextupole error beyond 1280 T/m.
- current CC design sextupole error for HL LHC is below 10 T/m,

Luminosity Degradation not Sensitive to RF Octupole and Decapole Errors

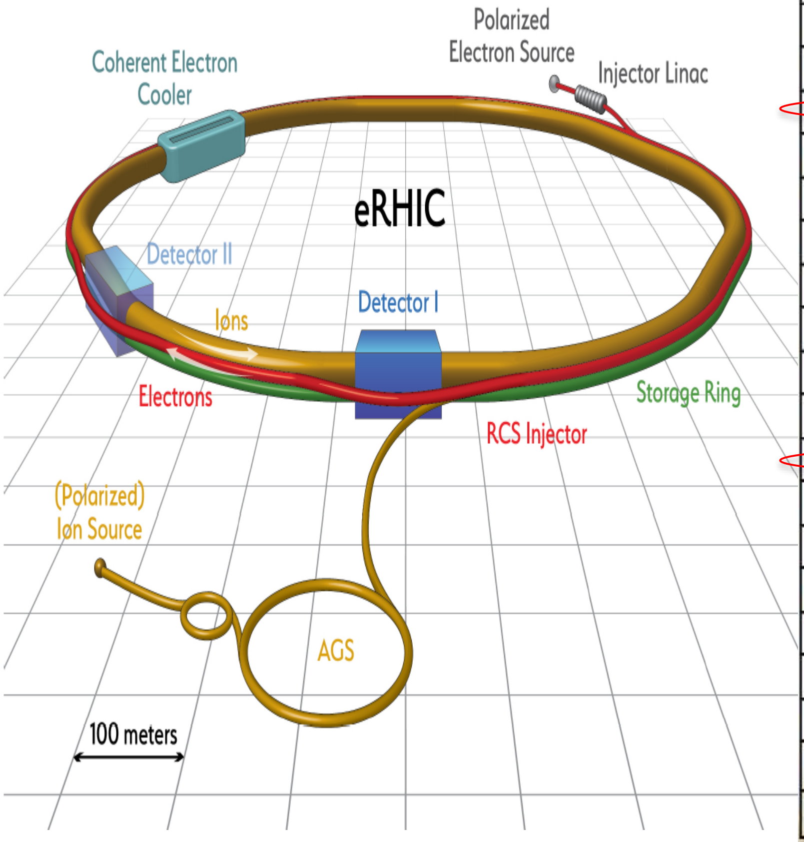

Octupole Error

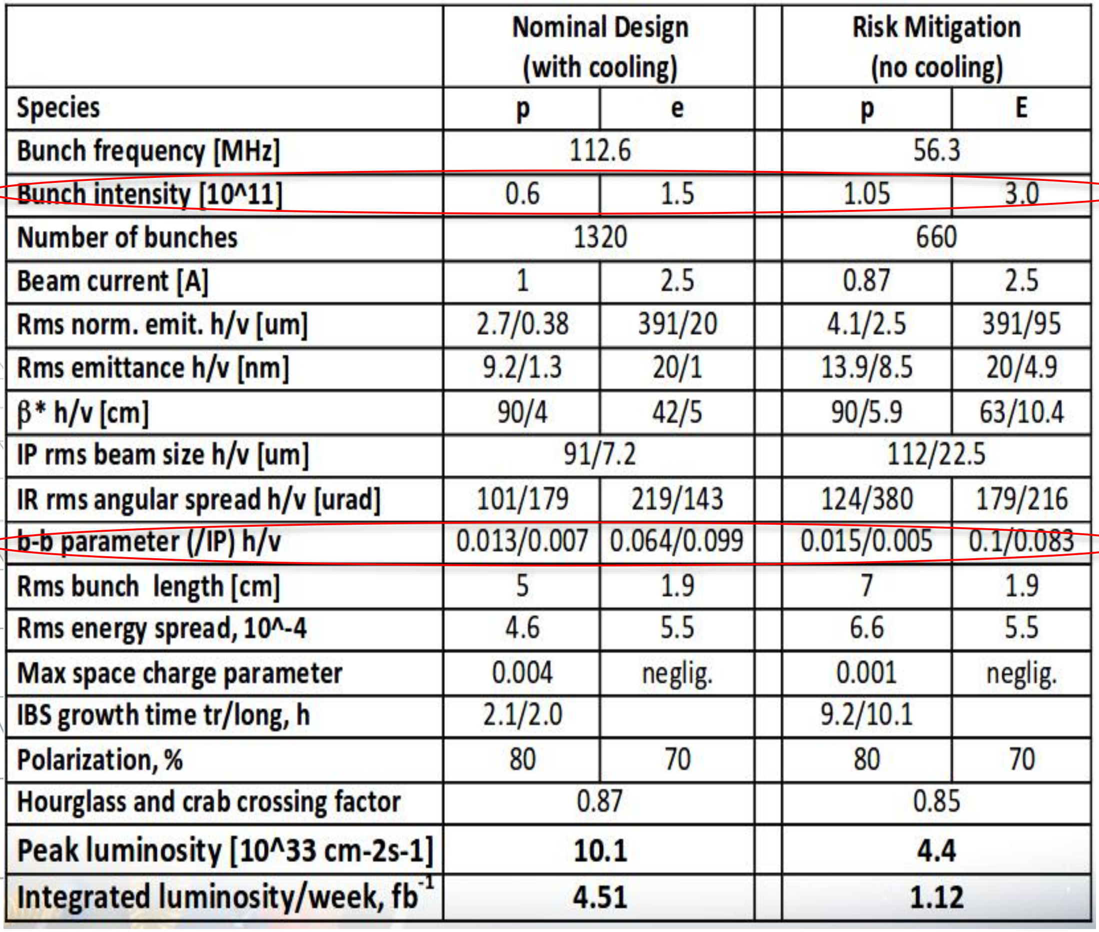

Decapole Error

- there are no noticeable luminosity degradation for integrated crab cavity RF octupole error up to 500 T/m/m and for decapole error up to 4000 T/m^3
- current CC design octupole error and decapole error for HL LHC are below 100 T/m/m and 3000 T/m^3.

Application to eRHIC beam-beam study

List of eRHIC Parameters





	Nominal Design (with cooling)		Risk Mitigation (no cooling)	
Species	p	e	p	E
Bunch frequency [MHz]	112.6		56.3	
Bunch intensity [10^11]	0.6	1.5	1.05	3.0
Number of bunches	1320		660	
Beam current [A]	1	2.5	0.87	2.5
Rms norm. emit. h/v [um]	2.7/0.38	391/20	4.1/2.5	391/95
Rms emittance h/v [nm]	9.2/1.3	20/1	13.9/8.5	20/4.9
β* h/v [cm]	90/4	42/5	90/5.9	63/10.4
IP rms beam size h/v [um]	91/7.2		112/22.5	
IR rms angular spread h/v [urad]	101/179	219/143	124/380	179/216
b-b parameter (/IP) h/v	0.013/0.007	0.064/0.099	0.015/0.005	0.1/0.083
Rms bunch length [cm]	5	1.9	7	1.9
Rms energy spread, 10^-4	4.6	5.5	6.6	5.5
Max space charge parameter	0.004	neglig.	0.001	neglig.
IBS growth time tr/long, h	2.1/2.0		9.2/10.1	
Polarization, %	80	70	80	70
Hourglass and crab crossing factor	0.87		0.85	
Peak luminosity [10^33 cm-2s-1]	**10.1**		**4.4**	
Integrated luminosity/week, fb^{-1}	**4.51**		**1.12**	

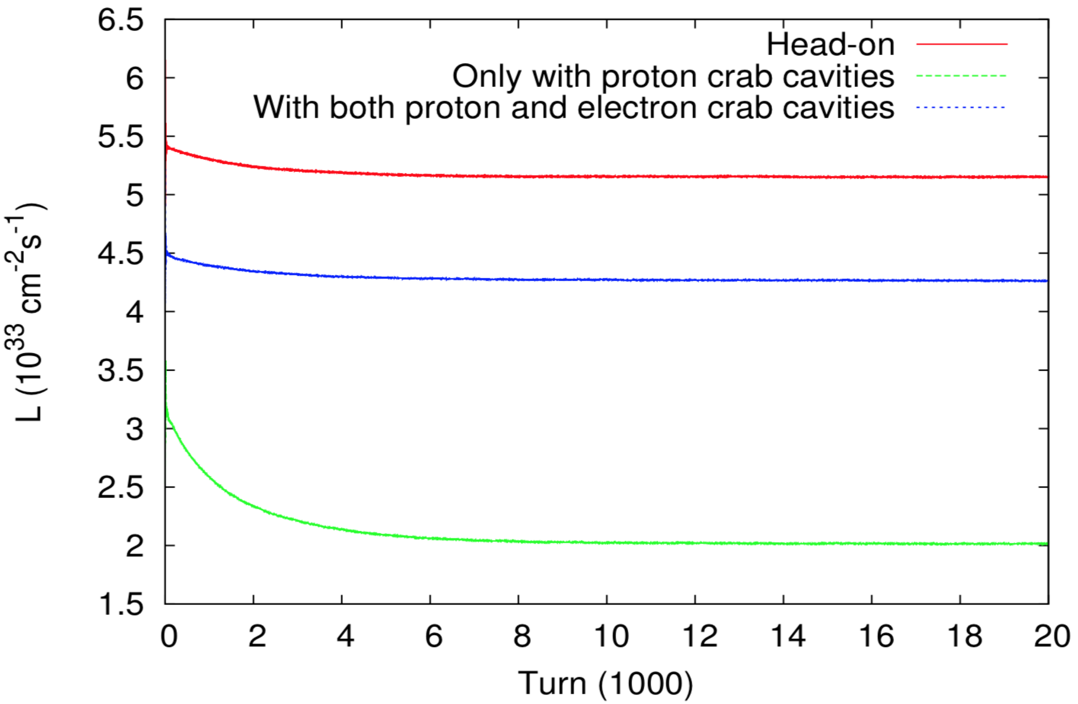
- 25 mrad crossing angle crab crossing, crab cavities 90° from IP

C. Montag, EIC collaboration meeting, 2018

Luminosity w/o Crab Cavities in eRHIC

☐ For current eRHIC design, crab cavities are needed for both proton and electron rings to obtain a high luminosity.

☐ Following plot shows luminosities w/o crab cavities. Here we assumed 338MHz for both proton and electron beams.

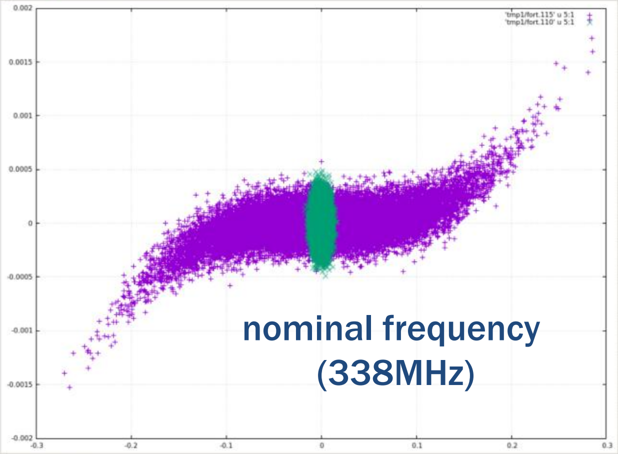


- With C.C. in both rings, luminosity is **83%** of that with head-on collision.

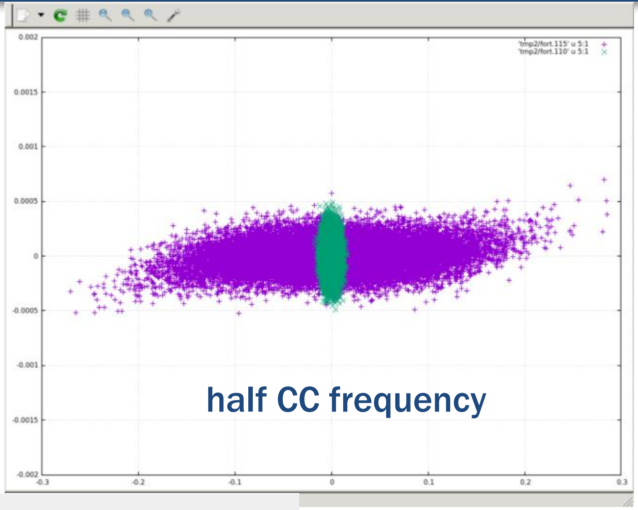
- Only C.C. in proton ring, luminosity is **47%** of that with C.C. in both rings.

Y. Luo, EIC collaboration meeting 2018

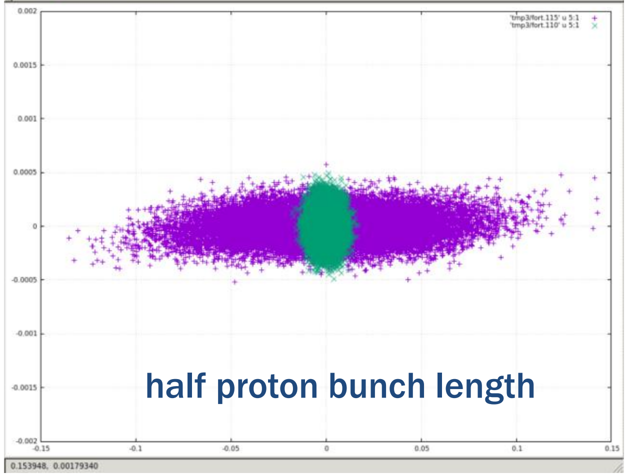
Beam-Beam Simulation Shows that Luminosity Degradation Depends on the Crab Cavity RF Frequency



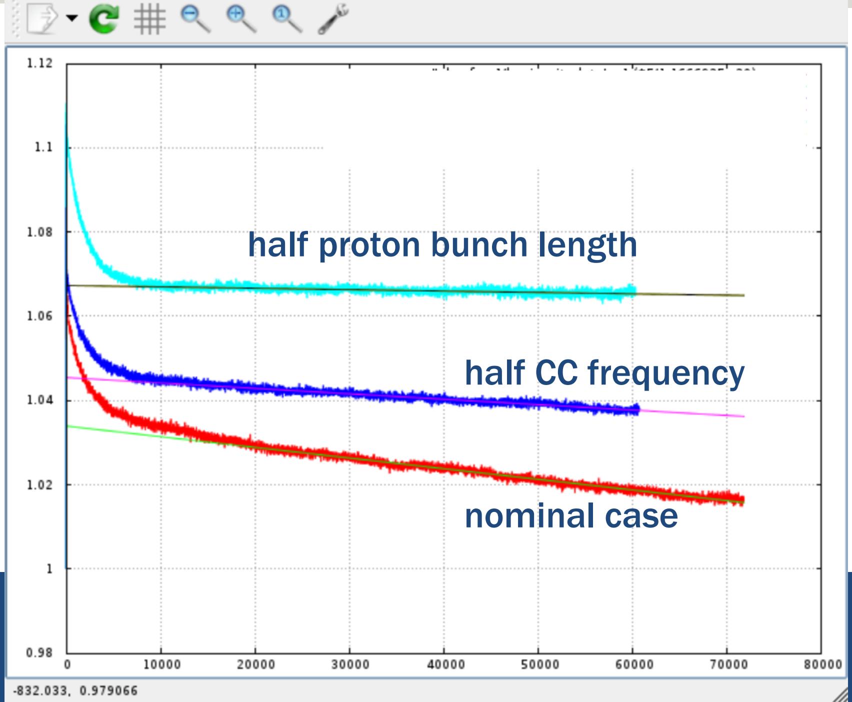
nominal frequency (338MHz)



half CC frequency



half proton bunch length



half proton bunch length

half CC frequency

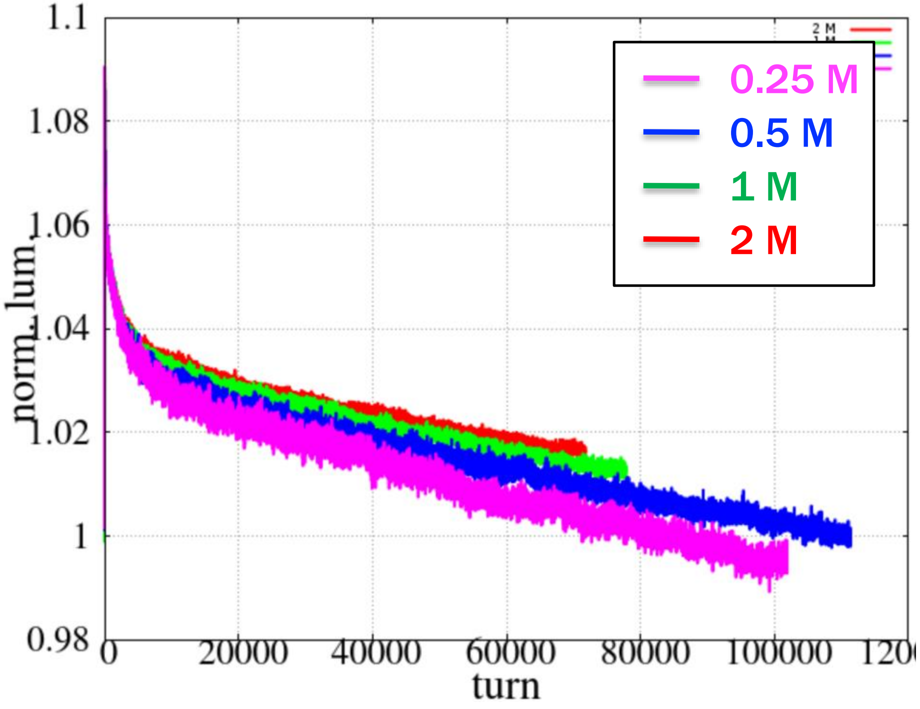
nominal case

- lower CC RF frequency or shorter proton bunch length helps improve luminosity lifetime

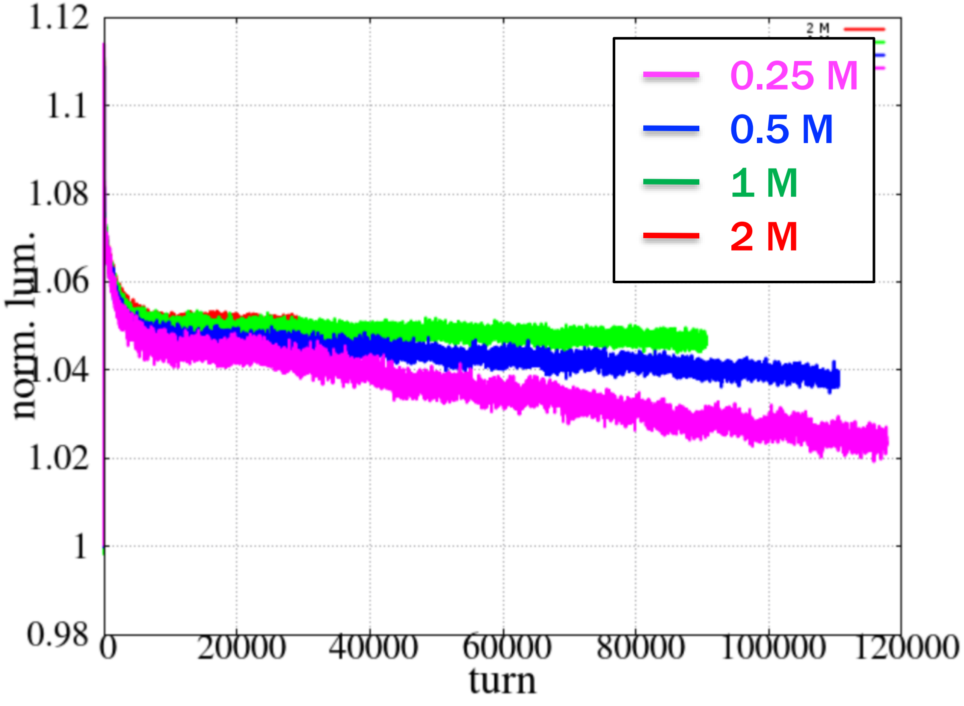
Predicted Luminosity Degradation from Beam-Beam Simulation Depends on the Number of Macroparticles

- Strong-strong beam-beam simulation subject to numerical noise driven emittance growth and luminosity degradation
- Increase of macroparticle number helps reduce numerical noise effects

with crossing angle/crab cavity



0 crossing angle/crab cavity



A Spectral Method Might Be Used to Mitigate the Numerical Noise Driven Emittance Growth

$$\rho(x,y) = \sum_{l=1}^{N_l}\sum_{m=1}^{N_m} \rho^{lm} \sin(\alpha_l x)\sin(\beta_m y)$$

$$\phi(x,y) = \sum_{l=1}^{N_l}\sum_{m=1}^{N_m} \phi^{lm} \sin(\alpha_l x)\sin(\beta_m y),$$

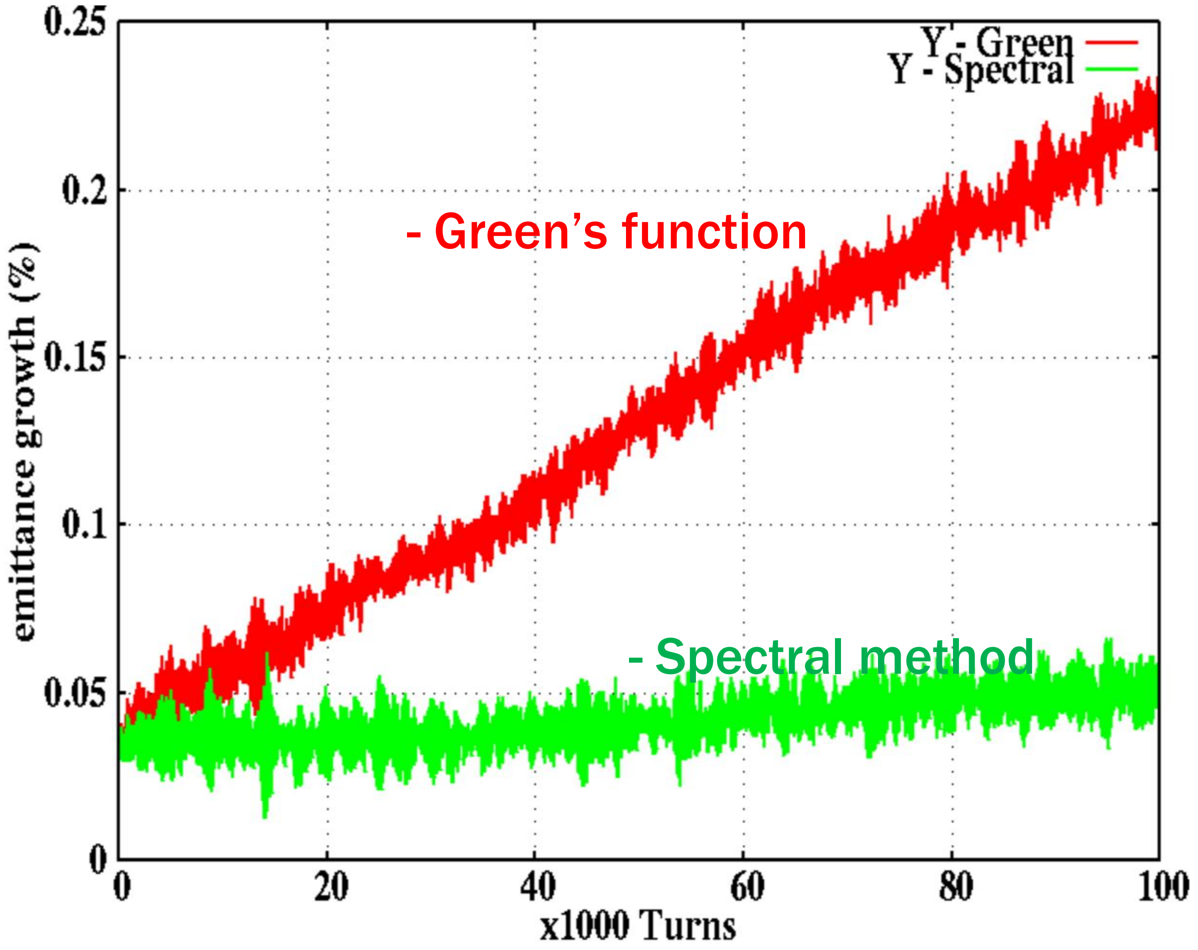
$$\rho^{lm} = \frac{4}{ab}\int_0^a\int_0^b \rho(x,y)\sin(\alpha_l x)\sin(\beta_m y)dxdy$$

$$\phi^{lm} = \frac{4}{ab}\int_0^a\int_0^b \phi(x,y)\sin(\alpha_l x)\sin(\beta_m y)dxdy,$$

where $\alpha_l = l\pi/a$ and $\beta_m = m\pi/b$.

$$\phi^{lm} = \frac{4\pi\rho^{lm}}{\gamma_{lm}^2}$$

where $\gamma_{lm}^2 = \alpha_l^2 + \beta_m^2$.



- Much smaller numerical noise driven emittance growth using the spectral method

Summary

- Crab cavity is an important accelerator component to improve luminosity in colliders
- Errors in the crab cavity such as RF phase and voltage errors, RF multipole errors need to be carefully controlled to minimize emittance growth and luminosity degradation
- Parameters of the crab cavity such as RF frequency can be chosen to optimize the performance of colliders
- Numerical noise associated with the beam-beam simulation causes artificial luminosity degradation and needs to be mitigated

Thank you for your attention!