Rencontre de Physique des Particules 2019

Revisiting RGEs for general gauge theories

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Assumption of **a diagonal wave-function renormalization**

(not appropriate for models with mixing in the scalar sector)

We have studied both problems, corrected the expressions and provided detailed explanations.

I. Schienbein, F. Staub, T. Steudtner and K. S., Nuclear Physics B 939 (2019) 1–48 [arXiv:1809.06797 [hep-ph]]

RGEs in a general gauge theory

- ***Lagrangian depends on couplings***
- ***After renormalization, these couplings depend on the energy scale (running parameters)***
- ***This dependence is described by the β-function of the coupling***

The β-function of x_k:

$$\mu \frac{d x_k}{d \mu} \equiv \beta_{x_k}$$

– in $\overline{MS}$ scheme
(dimensional regularization with modified minimal subtraction)

μ - is an arbitrary mass scale parameter

RGEs in a general gauge theory

The Lagrangian for a general renormalizable gauge theory:

Gauge fields

$V_\mu^A(x) \;\; (A = 1,\ldots d)$

of a compact simple group $\boldsymbol{G}$ of dim. $\boldsymbol{d}$.

Real scalar fields

$\phi_a(x) \;\; (a = 1,\ldots N_\phi)$

transform under a reducible rep. of $\boldsymbol{G}$ with generators Θ_{ab}^A

Complex fermion fields

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where

$$\begin{aligned}\mathcal{L}_0 = &-\frac{1}{4}F_A^{\mu\nu}F^A_{\mu\nu} + \frac{1}{2}D^\mu\phi_a D_\mu\phi_a + i\psi_j^\dagger\sigma^\mu D_\mu\psi_j \\ &-\frac{1}{2}\left(Y^a_{jk}\psi_j\zeta\psi_k\phi_a + Y^{a*}_{jk}\psi_j^\dagger\zeta\psi_k^\dagger\phi_a\right) - \frac{1}{4!}\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d\,,\end{aligned}$$

– contains no dimensional parameters

and

$$\mathcal{L}_1 = -\frac{1}{2}\left[(m_f)_{jk}\psi_j\zeta\psi_k + (m_f)^*_{jk}\psi_j^\dagger\zeta\psi_k^\dagger\right] - \frac{m^2_{ab}}{2!}\phi_a\phi_b - \frac{h_{abc}}{3!}\phi_a\phi_b\phi_c\,.$$

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M.E. Machacek, M.T. Vaughn,
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M.-x. Luo, H.-w. Wang, Y. Xiao, Phys. Rev. D67 (2003) 065019

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– includes all terms with dimensional parameters.

The dummy field method [1]

The idea: **we introduced a scalar "dummy field" – non-propagating, with no gauge interactions, and rewrote the dimensionless part of the Lagrangian**

$$D_\mu \phi_{\hat{d}} = 0$$

Yukawa coupling; Quartic coupling; Quartic coupling

$$\mathcal{L}_0 \supset -\frac{1}{2}\left(Y^{\hat{d}}_{jk}\psi_j\zeta\psi_k\phi_{\hat{d}} + \text{h.c.}\right) - 6\sum_{a,b=1}^{N_\phi}\frac{1}{4!}\lambda_{ab\hat{d}\hat{d}}\phi_a\phi_b\phi_{\hat{d}}\phi_{\hat{d}} - 4\sum_{a,b,c=1}^{N_\phi}\frac{1}{4!}\lambda_{abc\hat{d}}\phi_a\phi_b\phi_c\phi_{\hat{d}}$$

a dummy field; dummy fields; a dummy field

$$Y^{\hat{d}}_{jk}\phi_{\hat{d}} = (m_f)_{jk}$$

$$\lambda_{ab\hat{d}\hat{d}}\phi_{\hat{d}}\phi_{\hat{d}} = 2m^2_{ab}$$

$$\lambda_{abc\hat{d}}\phi_{\hat{d}} = h_{abc}$$

Fermion mass

$$\mathcal{L}_1 = -\frac{1}{2}\left[(m_f)_{jk}\psi_j\zeta\psi_k + (m_f)^*_{jk}\psi^\dagger_j\zeta\psi^\dagger_k\right]$$

Scalar mass

$$-\frac{m^2_{ab}}{2!}\phi_a\phi_b$$

Trilinear coupling

$$-\frac{h_{abc}}{3!}\phi_a\phi_b\phi_c$$

[1] – the idea, to our knowledge, was first mentioned by S.P. Martin and M.T. Vaughn, in "Two loop renormalization group equations for soft supersymmetry breaking couplings", Phys. Rev. D 50 (1994) 2282, arXiv: hep-ph/9311340

The dummy field method

<u>Example.</u> **The β-function of the fermion mass term can be obtained from the expressions for the Yukawa couplings, using the following mappings**

$$a \to \hat{d}\,,\; Y^a \to Y^{\hat{d}} \to m_f\,,\; Y^{\dagger a} \to Y^{\dagger \hat{d}} \to m_f^{\dagger}\,,\; \lambda_{abcd} \to \lambda_{\hat{d}bcd} \to h_{bcd}$$

1-loop β-function for the Yukawa couplings:

$$\beta_a^I = \frac{1}{2}\left[Y_2^{+}(F)Y^a + Y^a Y_2(F)\right] + 2Y^b Y^{+a} Y^b + 2\kappa Y^b Y_2^{ab}(S) - 3g^2\{C_2(F), Y^a\}\,,$$

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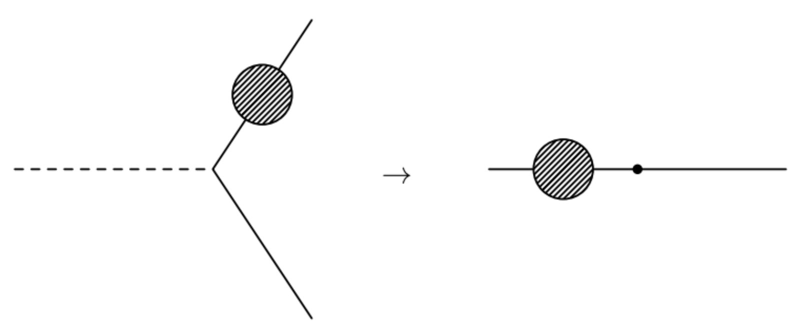

$$Y_2^{\dagger}(F)Y^a + Y^a Y_2(F) \to Y_2^{\dagger}(F)m_f + m_f Y_2(F)$$

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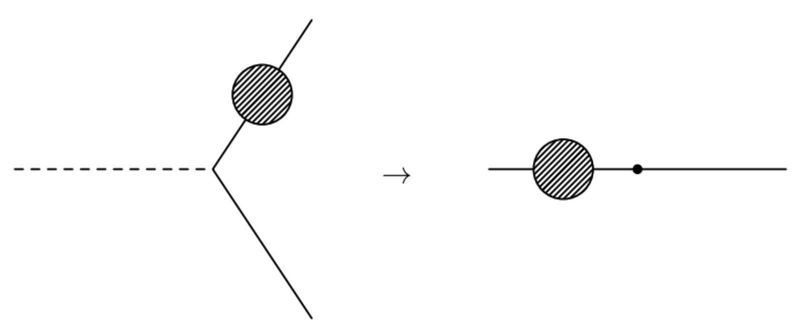

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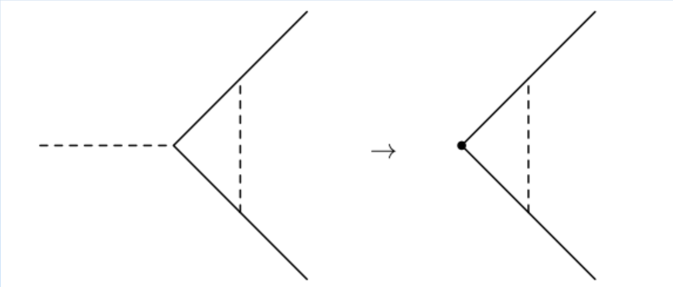

$$Y^b Y^{\dagger a} Y^b \quad \to \quad Y^b m_f^\dagger Y^b$$

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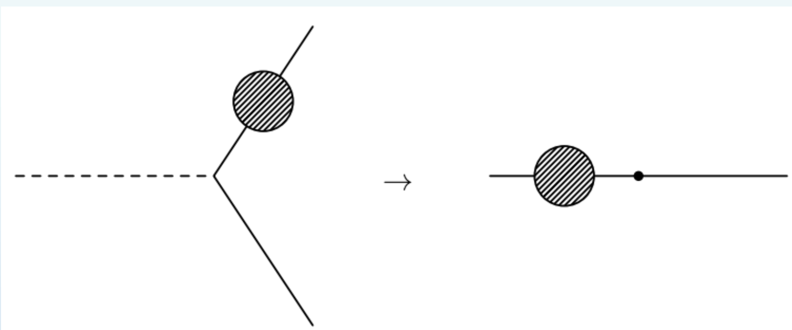

$$Y_2^\dagger(F)Y^a + Y^a Y_2(F) \to Y_2^\dagger(F)m_f + m_f Y_2(F)$$

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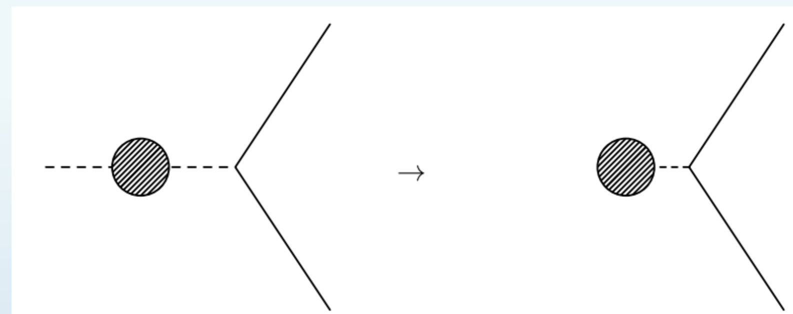
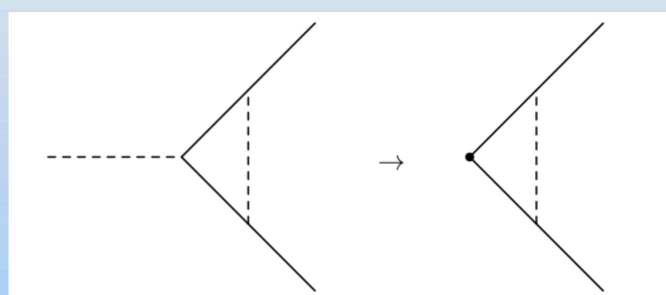


$$Y^b Y_2^{ab}(S) \quad \to \quad 0$$



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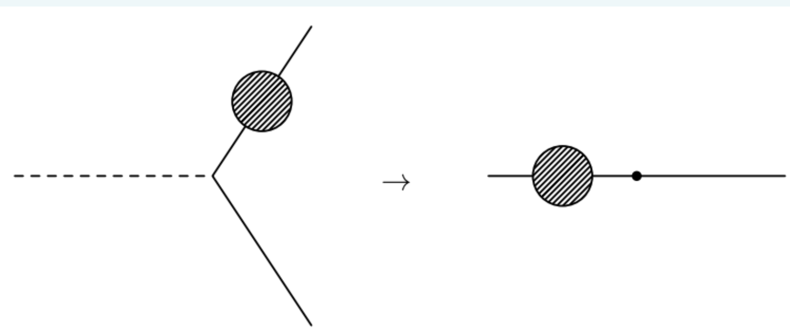

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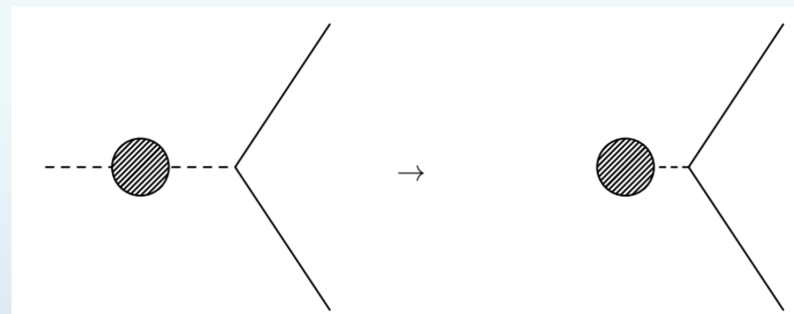
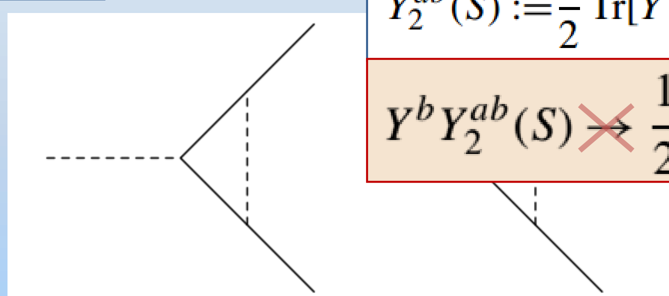


$$Y^b Y_2^{ab}(S) \quad \to \quad 0$$

$$Y_2^{ab}(S) := \frac{1}{2}\mathrm{Tr}[Y^{\dagger a}Y^b + Y^{\dagger b}Y^a]$$

$$Y^b Y_2^{ab}(S) \not\to \frac{1}{2}Y^b\,\mathrm{Tr}[m_f^\dagger Y^b + Y^{\dagger b} m_f]$$



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In this manner, the β-functions for the following parameters have been obtained:

Fermion mass: $\beta_{m_f}^{1-loop}, \beta_{m_f}^{2-loop}$ **out of** $\beta_a^{1-loop}, \beta_a^{2-loop}$ *(Yukawa c.)*

Trilinear sc.c.: $\beta_{h_{abc}}^{1-loop}, \beta_{h_{abc}}^{2-loop}$

Scalar mass sq.: $\beta_{m_{ab}^2}^{1-loop}, \beta_{m_{ab}^2}^{2-loop}$

(both) **out of** $\beta_{\lambda_{abcd}}^{1-loop}, \beta_{\lambda_{abcd}}^{2-loop}$ *(quartic sc.c.)*

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Scalar mass sq.:	$\beta_{m_{ab}^2}^{1-loop}, \beta_{m_{ab}^2}^{2-loop}$			

We've reconsidered <u>diagrammatically</u> and corrected

The dummy field method (summarized)

The dummy field method allows to derive the β-functions for <u>dimensionful</u> parameters out of those for the <u>dimensionless</u> parameters

1. **Consider the Lagrangian in the presence of the same particle content + 1 extra scalar dummy field**
2. **Write down the β-functions for the dimensionless parameters**
3. **Substitute:** $Y^{\hat{d}}_{jk} = (m_f)_{jk}\,, \quad \lambda_{ab\hat{d}\hat{d}} = 2m^2_{ab}\,, \quad \lambda_{abc\hat{d}} = h_{abc}$
4. **Keep in mind that the dummy field – is a real scalar, non-propagating, with no gauge interactions, i.e.**
   - ***Expressions with 2 identical internal indices (≡ a propagating dummy field) <u>must vanish</u>***
   - ***Vertices <gauge boson-dummy scalar> <u>must vanish</u>***
   - ***<u>Tadpole diagrams</u> (if appear) must be also <u>dropped out</u>***
5. **Enjoy the result: the β-functions for dimensionful parameters**

Numerical impact (I)

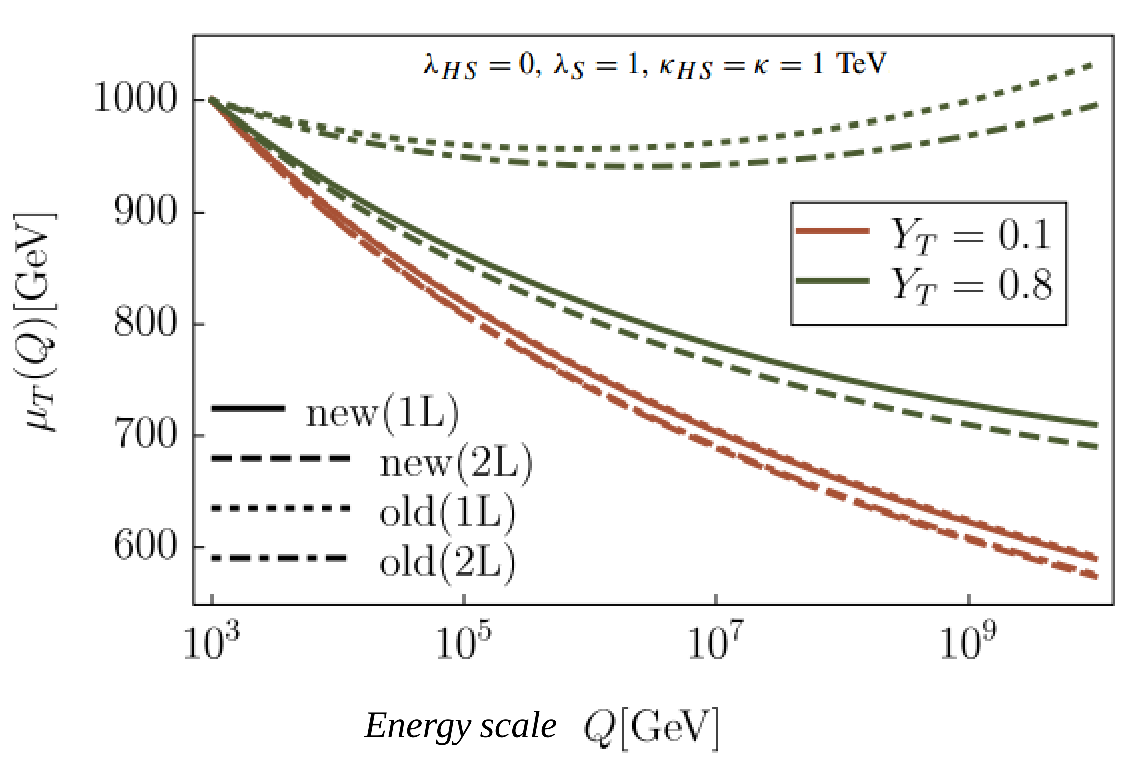
Running of fermion mass terms

$$\mathcal{L} \supset Y S f_1 f_2 + \mu f_1 f_2 + \text{h.c.}$$

For example: two heavy top-like states and a real singlet

$$T' : \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}} \,, \qquad \bar{T}' : \quad (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \,, \qquad S : \quad (\mathbf{1}, \mathbf{1})_0 \,,$$

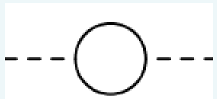
$$V = V_{SM} + \frac{1}{4}\lambda_S S^4 + \frac{1}{2}\lambda_{SH}|H|^2 S^2 + \kappa_{SH}|H|^2 S + \frac{1}{3}\kappa S^3 + \frac{1}{2} m_S^2 S^2 + \left(Y_T S \bar{T}' T' + \mu_T \bar{T}' T' + \text{h.c.}\right) .$$



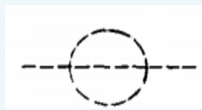
The discrepancy between the old and new results rapidly grows with increasing Y_T

The running mass μ_T of the vector-like top partners at one- and two-loop level for two different choices of the Yukawa coupling Y_T

Off-diagonal wave function renormalization



$$Y_2^{ab}(S) := \frac{1}{2}\,\mathrm{Tr}[Y^{\dagger a} Y^b + Y^{\dagger b} Y^a]\,,$$



$$\Lambda^2_{ab}(S) := \frac{1}{6} \sum_{c,d,e=1}^{N_\phi} \lambda_{acde}\lambda_{bcde}\,,$$

The assumption that

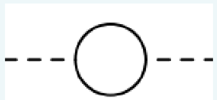
$$Y_2^{ab}(S) = Y_2(S)\delta_{ab} \quad \textbf{and} \quad \Lambda^2_{ab}(S) = \Lambda^2(S)\delta_{ab}$$

is reasonable only if the considered model <u>does not contain several scalar particles with identical quantum numbers</u>

thus, in general, contributions from off-diagonal wave-function corrections must be included

(affects the results for the dimensionless parameters (the quartic scalar couplings), and ⇨ the trilinear coupling, the scalar mass)

Off-diagonal wave function renormalization



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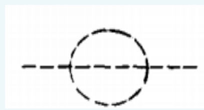

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Corrected

(affects the results for the dimensionless parameters (the quartic scalar couplings), and ⇨ the trilinear coupling, the scalar mass)

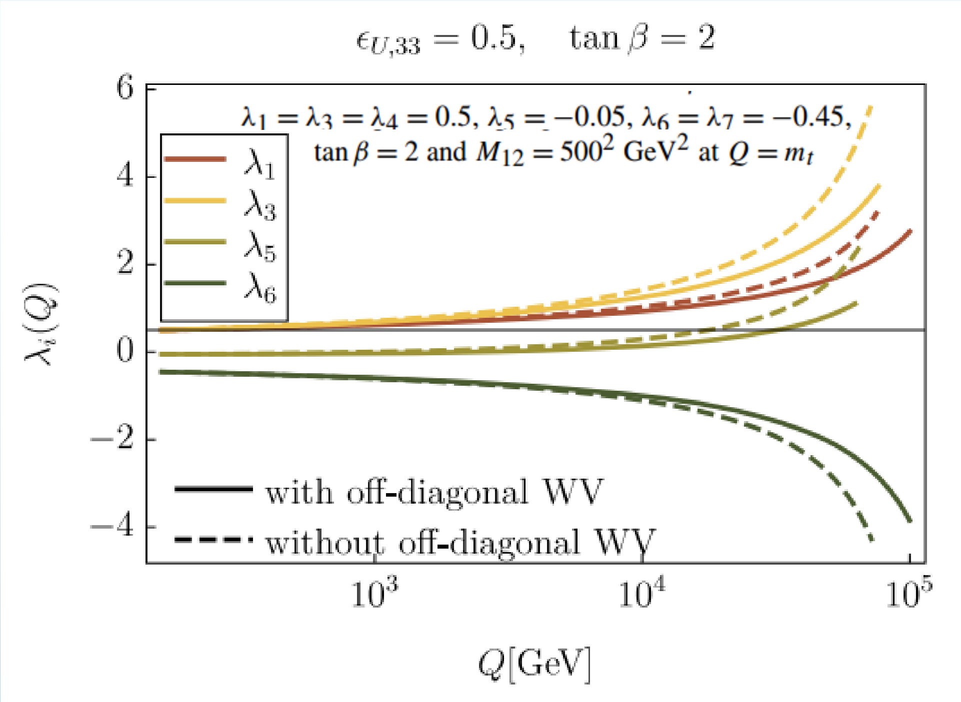
Numerical impact (II)

Example:
The general Two-Higgs-Doublet-Model type-III

$$V = m_1^2|H_1|^2 + m_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_2^\dagger H_1|^2 + \left(\frac{1}{2}\lambda_5(H_2^\dagger H_1) + \lambda_6|H_1|^2(H_1^\dagger H_2) + \lambda_7|H_2|^2(H_1^\dagger H_2) - M_{12}H_1^\dagger H_2 + \text{h.c.}\right)$$

$$\mathcal{L}_Y = -\left(Y_d H_1^\dagger dq + Y_e H_1^\dagger el - Y_u H_2 uq + \epsilon_d H_2^\dagger dq + \epsilon_e H_2^\dagger el - \epsilon_u H_1 uq + \text{h.c.}\right)$$

The additional one-loop contributions on the running of the quartic couplings lead to sizeable differences already for $\epsilon_{U,33}$= 0.5 and small $\tan\beta = 2$



The running of different quartic couplings in the THDM-III with and without the contributions of off-diagonal wave-function renormalisation

Conclusions

- We identified various mistakes in the literature for the β-functions of both dimensionless and dimensionful Lagrangian parameters
- The sources for these discrepancies: incorrect dummy field method application and assumption of a diagonal wave-function renormalization
- We obtained the correct expressions, cross-checked them and estimated the changes numerically
- We provided a detailed pedagogic discussion (of the dummy field method, in particular) and summarized all the correct expressions for the β-functions in one paper

I. Schienbein, F. Staub, T. Steudtner and K. S., Nuclear Physics B 939 (2019) 1–48 [arXiv:1809.06797 [hep-ph]]

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Thanks for your attention!