Revisiting RGEs for general gauge theories

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In collaboration with
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23-25 January 2019
LPC Clermont, France
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**Dummy field method**

for the $\beta$-functions for dimensionful parameters
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Nevertheless, we have revisited the derivation of the RGEs and identified mistakes in the literature, related to

- Inaccurate use of the **Dummy field method** for the $\beta$-functions for dimensionful parameters
- Assumption of a **diagonal wave-function renormalization** (not appropriate for models with mixing in the scalar sector)
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- Inaccurate use of the **Dummy field method** for the $\beta$-functions for dimensionful parameters
- Assumption of a diagonal wave-function renormalization (not appropriate for models with mixing in the scalar sector)

We have studied both problems, corrected the expressions and provided detailed explanations.

RGEs in a general gauge theory

- Lagrangian depends on couplings
- After renormalization, these couplings depend on the energy scale (running parameters)
- This dependence is described by the $\beta$-function of the coupling

The $\beta$-function of $x_k$:

\[
\mu \frac{dx_k}{d\mu} \equiv \beta_{x_k}
\]

- in $\overline{MS}$ scheme (dimensional regularization with modified minimal subtraction)

$\mu$ - is an arbitrary mass scale parameter
RGEs in a general gauge theory

The Lagrangian for a general renormalizable gauge theory:

**Gauge fields**

\[ V^A_\mu(x) \quad (A = 1, \ldots, d) \]

of a compact simple group \( G \) of dim. \( d \).

**Real scalar fields**

\[ \phi_a(x) \quad (a = 1, \ldots, N_\phi) \]

transform under a reducible rep. of \( G \) with generators \( \Theta^A_{ab} \).

**Complex fermion fields**

\[ \psi_j(x) \quad (j = 1, \ldots, N_\psi) \]

transform under a reducible rep. of \( G \) with generators \( t^A_{jk} \).

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + (\text{gauge fixing + ghost terms}), \]
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The Lagrangian for a general renormalizable gauge theory:

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \text{(gauge fixing + ghost terms)}, \]

where

\[ \mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu}^A F_{\mu\nu}^A + \frac{1}{2} D_{\mu} \phi_{a} D_{\mu} \phi_{a} + i \psi_{j}^{\dagger} \sigma_{\mu} D_{\mu} \psi_{j} - \frac{1}{2} \left( Y_{jk}^{a} \psi_{j} \phi_{a} + Y_{jk}^{a\dagger} \phi_{a} \psi_{j} \right) - \frac{1}{4!} \lambda_{abcd} \phi_{a} \phi_{b} \phi_{c} \phi_{d}, \]

– contains no dimensional parameters

and

\[ \mathcal{L}_1 = -\frac{1}{2} \left[ (m_{f})_{jk} \psi_{j} \phi_{k} + (m_{f})_{jk}^{*} \phi_{k} \psi_{j}^{\dagger} \right] - \frac{m_{ab}^{2}}{2!} \phi_{a} \phi_{b} - \frac{h_{abc}}{3!} \phi_{a} \phi_{b} \phi_{c}. \]

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RGEs in a general gauge theory

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\[ - \frac{1}{2} \left( Y^a_{jk} \psi_j \bar{\psi}_k \phi_a + Y^a_{jk} \bar{\psi}_j \psi_k \phi_a \right) - \frac{1}{4!} \lambda_{abcd} \phi_a \phi_b \phi_c \phi_d, \]

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Dimensionless parameters

Dimensionful parameters


The dummy field method

**The idea:** we introduced a scalar “dummy field” – non-propagating, with no gauge interactions, and rewrote the dimensionless part of the Lagrangian

\[ D_\mu \phi_d = 0 \]

\[ \mathcal{L}_0 \supset -\frac{1}{2} \left( Y_{jk} \psi_j \phi_k \phi_d + h.c. \right) - 6 \sum_{a,b=1}^{N_\phi} \frac{1}{4!} \lambda_{ab\hat{d}\hat{d}} \phi_a \phi_b \phi_d \phi_d - 4 \sum_{a,b,c=1}^{N_\phi} \frac{1}{4!} \lambda_{abcd} \phi_a \phi_b \phi_c \phi_d \]

**Fermion mass**

\[ Y_{jk} \phi_d = (m_f)_{jk} \]

**Scalar mass**

\[ m_{ab}^2 \phi_a \phi_b \]

**Trilinear coupling**

\[ h_{abc} \phi_a \phi_b \phi_c \]

**Yukawa coupling**

\[ \lambda_{ab\hat{d}\hat{d}} \phi_a \phi_b \phi_d \phi_d = 2m_{ab}^2 \]

**Quartic coupling**

\[ \lambda_{abcd} \phi_a \phi_b \phi_c \phi_d = h_{abc} \]

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The dummy field method

Example. The $\beta$-function of the fermion mass term can be obtained from the expressions for the Yukawa couplings, using the following mappings:

\[
a \rightarrow \hat{d}, \quad Y^a \rightarrow Y^\hat{d} \rightarrow m_f, \quad Y^{\dagger a} \rightarrow Y^{\dagger \hat{d}} \rightarrow m_f^\dagger, \quad \lambda_{abcd} \rightarrow \lambda_{\hat{d}bcd} \rightarrow h_{bcd}
\]

1-loop $\beta$-function for the Yukawa couplings:

\[
\beta^I_a = \frac{1}{2} \left[ Y_2^+(F)Y^a + Y^a Y_2(F) \right] + 2Y^b Y^{+a} Y^b + 2\kappa Y^b Y_2^{ab}(S) - 3g^2\{C_2(F), Y^a\},
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Example. The $\beta$-function of the fermion mass term can be obtained from the expressions for the Yukawa couplings, using the following mappings:

$$a \rightarrow \hat{d}, \ Y^a \rightarrow Y^d \rightarrow m_f, \ Y^{+a} \rightarrow Y^{+d} \rightarrow m_f^+, \ \lambda_{abcd} \rightarrow \lambda_{\hat{d}bcd} \rightarrow h_{bcd}$$

1-loop $\beta$-function for the Yukawa couplings:

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Diagram:

$$Y_2^+(F)Y^a + Y^aY_2(F) \rightarrow Y_2^+(F)m_f + m_fY_2(F)$$

$$\{C_2(F), Y^a\} \rightarrow \{C_2(F), m_f\}$$
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Example. The $\beta$-function of the fermion mass term can be obtained from the expressions for the Yukawa couplings, using the following mappings:

\[ a \rightarrow \hat{d}, \quad Y^a \rightarrow Y^{\hat{d}} \rightarrow m_f, \quad Y^{\dagger a} \rightarrow Y^{\dagger \hat{d}} \rightarrow m_f^{\dagger}, \quad \lambda_{abcd} \rightarrow \lambda_{\hat{d}bcd} \rightarrow h_{bcd} \]

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$$Y^bY_2^{ab}(S) \rightarrow 0$$

$$Y^bY^{\dagger a}Y^b \rightarrow Y^b m_f^{\dagger} Y^b$$
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1-loop $\beta$-function for the fermion mass:

\[ \beta^I_{m_f} = \frac{1}{2} \left[ Y_2^+(F)m_f + m_f Y_2(F) \right] + 2Y^b m_f^\dagger Y^b - 3g^2\{C_2(F), m_f\}. \]
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Example. The β-function of the fermion mass term can be obtained from the expressions for the Yukawa couplings, using the following mappings

\[ a \rightarrow \hat{a}, \ Y^{a} \rightarrow Y^{\hat{a}} \rightarrow m_{f}, \ Y^{+a} \rightarrow Y^{+\hat{a}} \rightarrow m_{f}^{+}, \ \lambda_{abcd} \rightarrow \lambda_{\hat{a}bcd} \rightarrow h_{bcd} \]

1-loop β-function for the Yukawa couplings:

\[ \beta_{a}^{I} = \frac{1}{2} \left[ Y_{2}^{+}(F)Y^{a} + Y^{a}Y_{2}(F) \right] + 2Y^{b}Y^{+a}Y^{b} + 2\kappa Y^{b}Y_{2}^{ab}(S) - 3g^{2}\{C_{2}(F), Y^{a}\}, \]

1-loop β-function for the fermion mass:

\[ \beta_{m_{f}}^{I} = \frac{1}{2} \left[ Y_{2}^{+}(F)m_{f} + m_{f}Y_{2}(F) \right] + 2Y^{b}m_{f}^{+}Y^{b} - 3g^{2}\{C_{2}(F), m_{f}\}. \]

In this manner, the β-functions for the following parameters have been obtained:

| Fermion mass: | \( \beta_{m_{f}}^{1\text{-loop}}, \beta_{m_{f}}^{2\text{-loop}} \) | out of | \( \beta_{a}^{1\text{-loop}}, \beta_{a}^{2\text{-loop}} \) (Yukawa c.) |
| Trilinear sc.c.: | \( \beta_{h_{abc}}^{1\text{-loop}}, \beta_{h_{abc}}^{2\text{-loop}} \) | out of | \( \beta_{\lambda_{abcd}}^{1\text{-loop}}, \beta_{\lambda_{abcd}}^{2\text{-loop}} \) (quartic sc.c.) |
| Scalar mass sq.: | \( \beta_{m_{ab}^{2}}^{1\text{-loop}}, \beta_{m_{ab}^{2}}^{2\text{-loop}} \) |
The dummy field method

Example. The $\beta$-function of the fermion mass term can be obtained from the expressions for the Yukawa couplings, using the following mappings:

$$a \rightarrow \hat{a}, \ Y^a \rightarrow Y^\hat{a} \rightarrow m_f, \ Y^{\hat{a}} \rightarrow Y^{\hat{d}} \rightarrow m^\dagger_f, \ \lambda_{abcd} \rightarrow \lambda_{\hat{d}bcd} \rightarrow h_{bcd}$$

1-loop $\beta$-function for the Yukawa couplings:

$$\beta^I_a = \frac{1}{2} \left[ Y^a_2(F) Y^a + Y^a Y_2(F) \right] + 2 Y^b Y^a Y^b + 2 \kappa Y^b Y_2^{ab}(S) - 3 g^2 \{ C_2(F), Y^a \},$$

1-loop $\beta$-function for the fermion mass:

$$\beta^I_{m_f} = \frac{1}{2} \left[ Y^a_2(F) m_f + m_f Y_2(F) \right] + 2 Y^b m_f Y^b - 3 g^2 \{ C_2(F), m_f \}.$$

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We’ve reconsidered diagrammatically and corrected
The dummy field method (summarized)

The dummy field method allows to derive the β-functions for dimensionful parameters out of those for the dimensionless parameters.

1. Consider the Lagrangian in the presence of the same particle content + 1 extra scalar dummy field.
2. Write down the β-functions for the dimensionless parameters.
3. Substitute:
   \[ Y^d_{jk} = (m_f)_{jk}, \quad \lambda_{abcd} = 2m^2_{ab}, \quad \lambda_{abcd} = h_{abc} \]
4. **Keep in mind** that the dummy field – is a real scalar, non-propagating, with no gauge interactions, i.e.
   - Expressions with 2 identical internal indices
     \( \equiv \text{a propagating dummy field} \) must vanish
   - Vertices \(<\text{gauge boson-dummy scalar}>\) must vanish
   - Tadpole diagrams (if appear) must be also dropped out
5. Enjoy the result: the β-functions for dimensionful parameters.
Numerical impact (I)

Running of fermion mass terms

For example: two heavy top-like states and a real singlet

The discrepancy between the old and new results rapidly grows with increasing $Y_T$

The running mass $\mu_T$ of the vector-like top partners at one- and two-loop level for two different choices of the Yukawa coupling $Y_T$
Off-diagonal wave function renormalization

The assumption that

$$Y_{2}^{ab}(S) := \frac{1}{2} \text{Tr}[Y^{+a}Y^{b} + Y^{+b}Y^{a}],$$

$$\Lambda_{ab}^{2}(S) := \frac{1}{6} \sum_{c,d,e=1}^{N_{\phi}} \lambda_{acde} \lambda_{bcde},$$

is reasonable only if the considered model does not contain several scalar particles with identical quantum numbers

thus, in general, contributions from off-diagonal wave-function corrections must be included

(affects the results for the dimensionless parameters (the quartic scalar couplings), and the trilinear coupling, the scalar mass)
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The assumption that

\[ Y_{2}^{ab}(S) = Y_{2}(S) \delta_{ab} \quad \text{and} \quad \Lambda_{ab}^{2}(S) = \Lambda^{2}(S) \delta_{ab} \]

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Numerical impact (II)

Example:
The general Two-Higgs-Doublet-Model type-III

\[ V = \sum_{i=1}^{2} m_i^2 |H_i|^2 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_2^\dagger H_1|^2 + \left( \frac{1}{2} \lambda_5 (H_2^\dagger H_1) + \lambda_6 |H_1|^2 (H_1^\dagger H_2) + \lambda_7 |H_2|^2 (H_1^\dagger H_2) - M_{12} H_2^\dagger H_1 + h.c. \right) \]

\[ \mathcal{L}_Y = - \left( Y_d H_1^\dagger dq + Y_e H_1^\dagger el - Y_u H_2 uq + \epsilon_d H_2^\dagger dq + \epsilon_e H_2^\dagger el - \epsilon_u H_1 uq + h.c. \right) \]

The additional one-loop contributions on the running of the quartic couplings lead to sizeable differences already for \( \epsilon_{U,33} = 0.5 \) and small \( \tan \beta = 2 \)

The running of different quartic couplings in the THDM-III with and without the contributions of off-diagonal wave-function renormalisation.
Conclusions

- We identified various mistakes in the literature for the $\beta$-functions of both dimensionless and dimensionful Lagrangian parameters.

- The sources for these discrepancies: incorrect dummy field method application and assumption of a diagonal wave-function renormalization.

- We obtained the correct expressions, cross-checked them and estimated the changes numerically.

- We provided a detailed pedagogic discussion (of the dummy field method, in particular) and summarized all the correct expressions for the $\beta$-functions in one paper.

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Thanks for your attention!