On the QCD phase diagram with light quarks from the Curci-Ferrari Model

Jan Maelger[1,2]

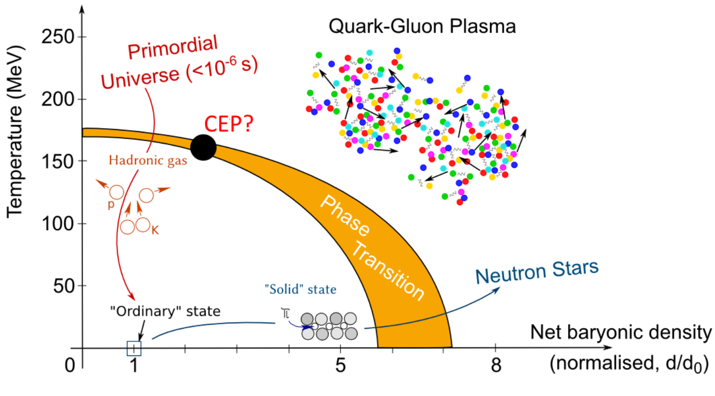
preliminary results

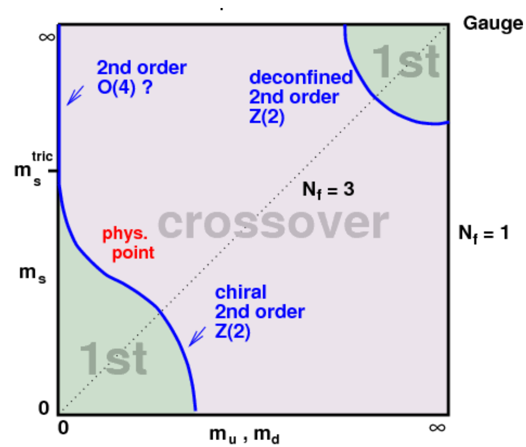
In collaboration with: U.Reinosa[1] and J.Serreau[2]

1. Centre de Physique Théorique, Ecole Polytechnique
2. AstroParticule et Cosmologie, Univ. Paris 7 Diderot

RPP 2019, January 25

Motivation - QCD Phase Diagram





Several other approaches on the market:

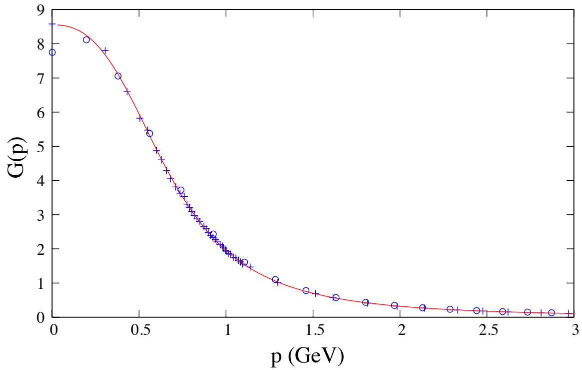
- Lattice QCD [de Forcrand, Philipsen, Rodriguez-Quintero, Mendes, ...]
- Dyson Schwinger Equations [Alkofer, Fischer, Huber, ...]
- Functional Renormalization Group [Pawlowski, Mitter, Schaefer...]
- Variational Approach [Reinhardt, Quandt, ...]
- Gribov-Zwanziger Action [Dudal, Oliveira, Zwanziger...]
- Matrix-, QM-, NJL-Model,... [Pisarski, Dumitru, Schaffner-B., Stiele, ...]
- Curci-Ferrari Model [Reinosa, Serreau, Tissier, Wschebor, ...]

Curci-Ferrari (CF) Model and gluon mass term

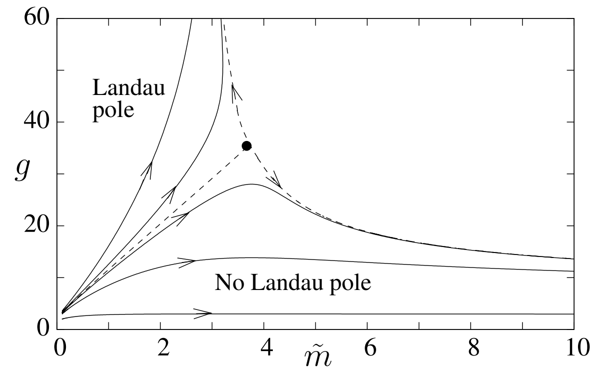
$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(\not{D} + M + \mu\gamma_0)\psi \right\} + S_{FP} + \int_x \left\{ \frac{1}{2} m^2 (A^a_\mu)^2 \right\}$$

This gluon mass term can be motivated in several ways

- phenomenologically from lattice data of the Landau gauge gluon propagator saturating in the IR
- Residual ambiguity after non-complete gauge-fixing in Fadeev-Popov procedure due to presence of Gribov copies



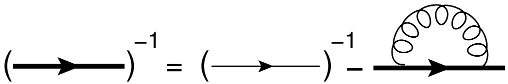
one-loop gluon propagator against lattice data, from [Tissier, Wschebor (2011)]
[Bogolubsky et al. (2009), Dudal, Oliveira, Vandersickel (2010)]

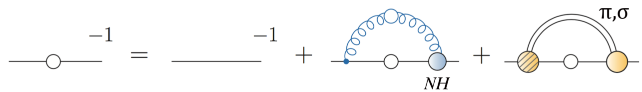


CF one-loop RG flow in YM,
from [Serreau, Tissier (2012)]

How to describe chiral symmetry breaking in CF?

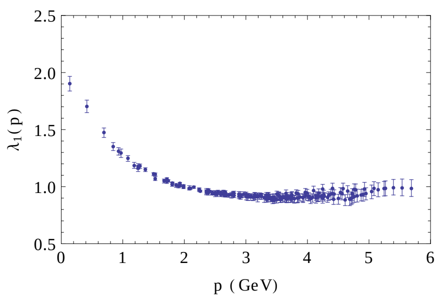
Rainbow-ladder equations have long been used to study the physics of chiral symmetry breaking





[Fischer (2018)]

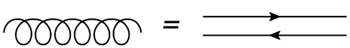
However, it can actually be derived as the leading order term in a systematic expansion scheme in CF [Peláez et al. (2018)] :



data [Skullerud et al. (2003)]

- resum in $g_q = \lambda_1 \, g_g$
- treat g_g perturbatively
- N_c large

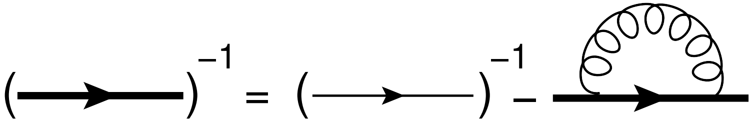
Expand at fixed t'Hooft coupling $\lambda = g_q^2 N_c$



- Extend to finite T and μ
- no mesonic fluctuations $\hat{\longrightarrow}$ mean-field treatment — enters at NLO
- no N_f dependence $\hat{\longrightarrow}$ flavor blind — enters at NLO
- no chiral anomaly $\hat{\longrightarrow}$ $U_A(1)$ unbroken — ??

  Easy in models like QM via $c\,\det(\Phi + \Phi^\dagger)$ — Hard in rainbow-ladder setting
- only sensible to compare to literature results with similar effects

CF rainbow-ladder eq$^{\text{n}}$ at finite T and μ



$$S^{-1}(P) = M_0 - (i\omega - \mu)\gamma_0 - \vec{p}\cdot\vec{\gamma} + 4g_0^2 \int_Q^T \gamma_\mu S(Q)\gamma_\nu\, G_{\mu\nu}(P-Q)$$

$$S(Q) = \langle q(Q)\bar{q}(0)\rangle$$

$$\int_Q^T = T\sum_{n\in\mathbb{Z}}\int \frac{d^3q}{(2\pi)^3}$$

$$G_{\mu\nu}(K) = P^\perp_{\mu\nu}(K)/(K^2 + m^2)$$

M_0 explicitly breaks chiral symmetry

Parity and rotation invariance imply that the tensor structure of the inverse quark propagator can be decomposed as

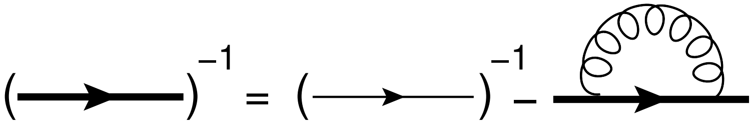
$$S^{-1}(P) = B - iA_0\gamma_0 - iA_v\vec{p}\cdot\vec{\gamma} - iC\gamma_0\vec{p}\cdot\vec{\gamma} \quad \text{with} \quad \underbrace{B(\omega, p = |\vec{p}\,|)}_{\text{order parameter}},\ A_0\ldots$$

set of coupled, non-linear integral equations:

$$B(P) = M_0 + g_0^2 \int_Q^T F_1(Q,P,B,A_0,A_v,C) \qquad A_v(P) = 1 + g_0^2 \int_Q^T F_3(Q,P,B,A_0,A_v,C)$$

$$A_0(P) = \omega + i\mu + g_0^2 \int_Q^T F_2(Q,P,B,A_0,A_v,C) \qquad C(P) = 0 + g_0^2 \int_Q^T F_4(Q,P,B,A_0,A_v,C)$$

CF rainbow-ladder eq^n at finite T and μ



$$S^{-1}(P) = M_0 - (i\omega - \mu)\gamma_0 - \vec{p}\cdot\vec{\gamma} + 4g_0^2 \int_Q^T \gamma_\mu S(Q)\gamma_\nu G_{\mu\nu}(P-Q)$$

$$S(Q) = \langle q(Q)\bar{q}(0)\rangle$$

$$\int_Q^T = T\sum_{n\in\mathbb{Z}}\int \frac{d^3q}{(2\pi)^3}$$

$$G_{\mu\nu}(K) = P^{\perp}_{\mu\nu}(K)/(K^2+m^2)$$

M_0 explicitly breaks chiral symmetry

Parity and rotation invariance imply that the tensor structure of the inverse quark propagator can be decomposed as

$$S^{-1}(P) = \mathrm{B} - iA_0\gamma_0 - iA_v\vec{p}\cdot\vec{\gamma} - iC\gamma_0\vec{p}\cdot\vec{\gamma} \quad \text{with} \quad \underbrace{\mathrm{B}(\omega, p = |\vec{p}\,|)}_{\text{order parameter}},\ A_0 \ldots$$

set of ~~coupled~~, non-linear integral equations:

$$B(P) = M_0 + g_0^2 \int_Q^T \tilde{F}_1(Q,P,B,\cancel{A_0,A_v,C})$$

$$A_0(P) = \omega + i\mu + \cancel{g_0^2 \int_Q^T F_3(Q,P,B,A_0,A_v,C)}$$

$$A_v(P) = 1 + \cancel{g_0^2 \int_Q^T F_3(Q,P,B,A_0,A_v,C)}$$

$$C(P) = 0 + \cancel{g_0^2 \int_Q^T F_3(Q,P,B,A_0,A_v,C)}$$

Localization

$$\boxed{B(P) = M_0 + 4g_0^2 \int_Q^T \frac{B(Q)}{Q_\mu^2 + B^2(Q)} \frac{1}{(P-Q)^2 + m^2}} \qquad Q_\mu \equiv (\omega_n + i\mu, \vec{q})$$

The quark mass function $B(Q)$ is an order paramter for chiral symmetry, i.e.

$$B(Q) \neq 0 \leftrightarrow \text{broken}\,\chi \qquad B(Q) = 0 \leftrightarrow \text{restored}\,\chi$$

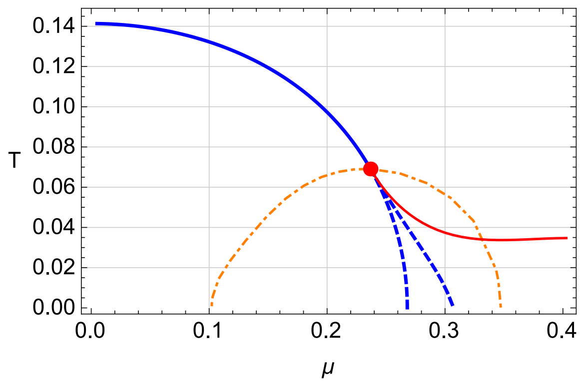
Numerically, it is possible to solve for the full momentum dependent $B(Q)$ iteratively.

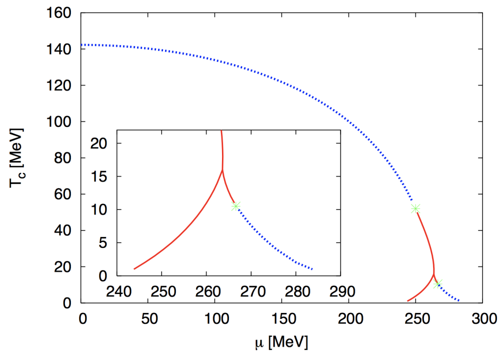
Instead, we shall follow a semi-analytic approach called localization: $B(Q) \hat{\rightarrow} B(0)$

Subtlety in $\hat{\rightarrow}$:
$Q \equiv (\omega_n, \vec{q})$ is fermionic, i.e. no zero Matsubara mode.

- $B(Q) \rightarrow B(\pm\pi T, 0)$ but $B^*(\omega, p) = B(-\omega, p)$, so we have $B \in \mathbb{C}$. $\rightarrow$ tested
- analytically continue $B(Q)$ to complex frequencies and consider the retarded component of the quark mass function via $B_R(q_0, q) = B(-i(q_0 + \mu) + 0^+, q)$. Then localize as $B_R(q_0, q) \rightarrow B_R(0,0) \equiv B \in \mathbb{R}$. $\rightarrow$ simpler

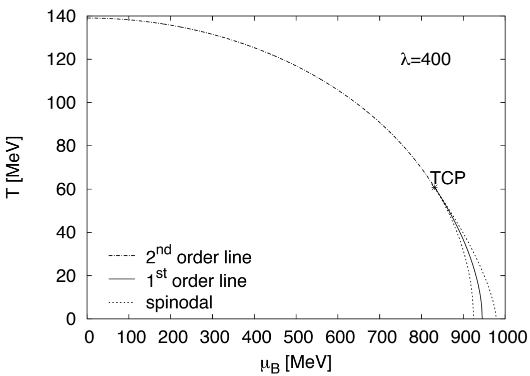
Results for the phase diagram





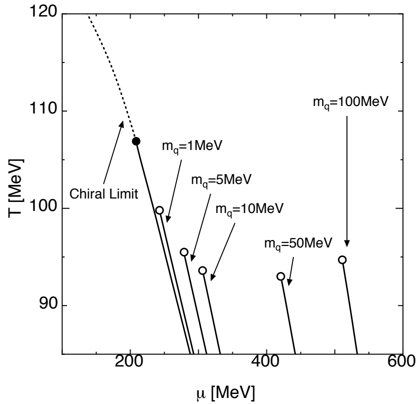
$N_f = 2$ Quark Meson Model + FRG

[Schaefer, Wambach (2005)]



Chiral Quark Model

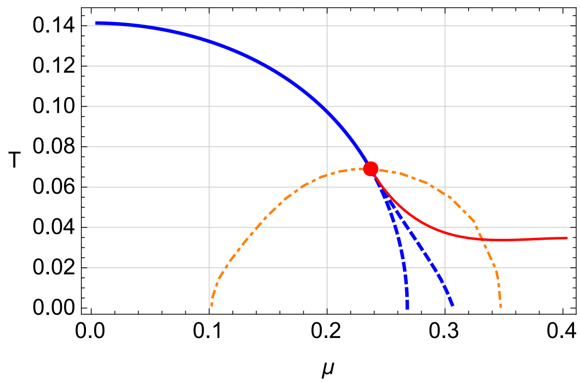
[Jakovac, Patkos, Szep, Szepfalusy (2003)]

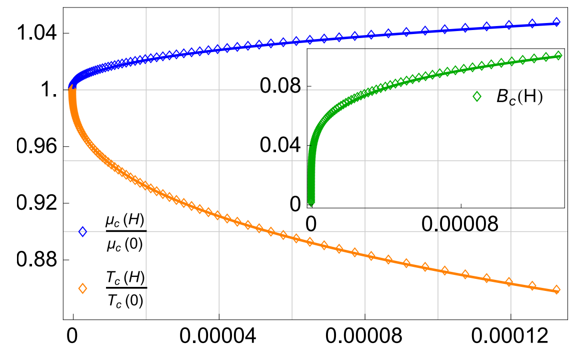


CJT Effective Potential for $N_f = 2$

[Hatta, Ikeda (2003)]

Approach to tricriticality and mean field scaling



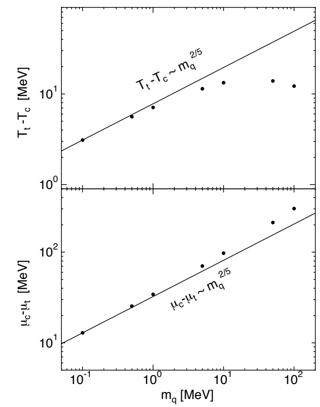


The approach of the critical quantities $\mu_c(H)$, $T_c(H)$, and $B_c(H)$, where $H = \pi^2 M_0/g_0^2$, towards the tricritical point in the chiral limit displays a scaling behavior with mean field exponents:

$$B_c(H) - B_{\text{tric}} \propto H^{1/5}$$

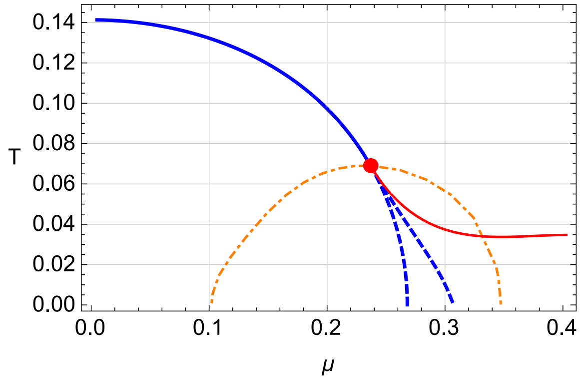
$$\mu_c(H) - \mu_{\text{tric}} \propto H^{2/5}$$

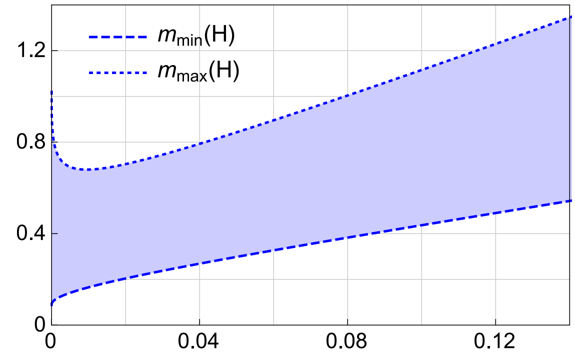
$$T_c(H) - T_{\text{tric}} \propto H^{2/5}$$



[Hatta, Ikeda (2003)]

Tricritical Point vanishing at $T = 0$





- Upon varying the gluon mass m, the tricritical point moves in the chiral phase diagram
- There exist two extremal values m_{min} and m_{max} for which the tric point vanishes on the $T = 0$ axis
- For any $m \in [m_{\text{min}}, m_{\text{max}}]$, the phase diagram exhibits a tricritical point
- Turning on a bare quark mass M_0, we can follow these extremal values as a function of $H = \pi^2 M_0 / g_0^2 : \quad m_{\text{min/max}}(H)$
- The blue area signifies the parameter region for m where a tric. point is allowed.

Conclusion & Outlook

CONCLUSION:

- The rainbow-ladder eq^n derives in the CF Model as the leading order term of a systematic expansion
- a simple localized version allows for a semi-analytic analysis
- The resulting phase diagram is in qualitative and quantitative agreement with literature results
- Tricritical point seems to depend on a non-zero gluon mass. Is this robust or an artifact of localization?

OUTLOOK:

- Roberge Weis symmetry at imaginary chemical potential - necessitates a non-trivial gluon background component associated to the Polyakov loop
- NLO in the expansion scheme: Mesonic fluctuations & Nf dependence, allows to study the Columbia plot.
- $U_A(1)$ anomaly?
- One common description for both heavy and light quark regimes
  $\longrightarrow$ study the phase diagram at the physical point
- ...

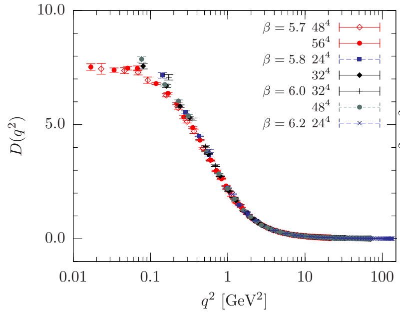
Backup slides

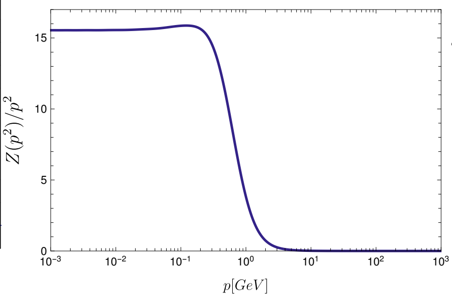
Call in the reinforcements!!

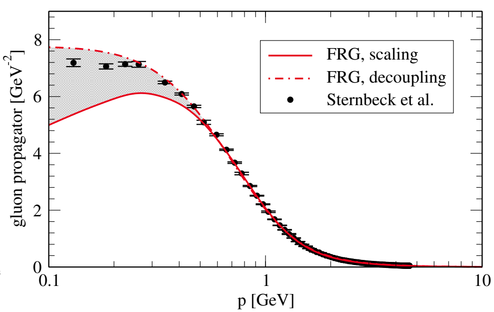
IR QCD not described by FP

1. In all covariant gauge fixings, the FP procedure is non-complete in the IR and leaves a residual ambiguity due to the presence of Gribov copies [Singer (1978)]
2. Landau gauge gluon propagator - decoupling behavior

[Sternbeck et al. (2006)] [Huber (2018)] [Cyrol et al. (2016)]







So clearly, in order to describe IR QCD, the FP Lagrangian is not enough and needs to be modified!