Quantum Mechanics as an Effective Theory

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A Recreational Talk

Two major events since the start of the LHC

The discovery of the Higgs boson

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or the Englert-Higgs boson

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or the Brout-Englert-Higgs-Kibble boson
or the Brout-Englert-Higgs-Guralnik-Hagen-Kibble boson
or the Brout-Englert-Higgs-Guralnik-Hagen-Kibble Bose particle

The Non-discovery of any of the long promised BSM physics

Hierarchy "problem", "Naturalness",..."Fine-tuning",...

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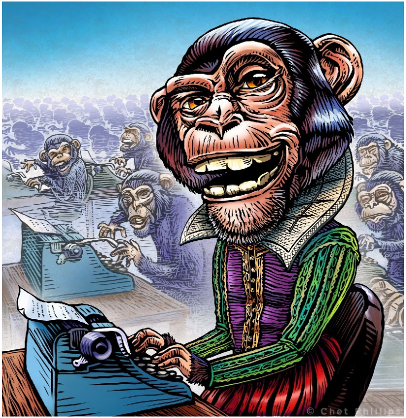
Emile Borel's Infinite Monkey Theorem



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what is restoring unitarity in massive gauge boson scattering processes at the TeV scale?

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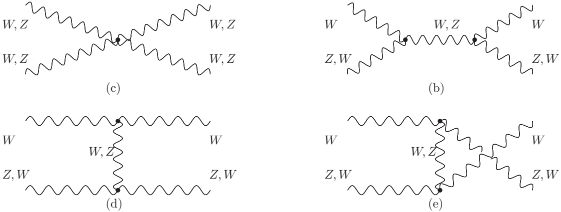

$$\mathcal{T}[VV' \to VV'] \sim (\dots)\frac{s^2}{M_V^4} + (\dots)\frac{s}{M_V^2} + (\dots)$$

Violation of unitarity $\Leftrightarrow$ total probability > 1 !!

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at $\lesssim$ 1 TeV

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all based on the postulate

$$|G^{(4)}(p_1, p_2, p_3, p_4)|^2 = \text{Transition Probability}$$

akin to $|\psi|^2 = P$ in the orthodox Quantum Mechanics.

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- quantum chemistry (reactive scattering, electronic structure, ...),
- atomic physics (photoionisation, atomtronics, ...),
- high-dimensional systems (rare-gas, ...),
- classical & quantum optics,
- nanoelectronics (fast nanometer devices,...),
- ...

see e.g. *Quantum Dynamics With Trajectories*, *R.E. Wyatt*, (*Springer* 2000)
Applied Bohmian Mechanics, *X.Orials*, *J.Mompart* (*ed.*),(*Pan Stanford Pub.* 2012)
Quantum Potential,*I. Licatti*, *D.Fiscaletti* (*Springer* 2014)

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This seems at odds with all what we learned at school about quantum mechanics!!

☹?

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James Cushing (1994)

...some references:

1) The undevided universe
D. Bohm & B.J. Hiley ed. Routledge

2) Quantum Mechanics Historical contingency and the Copenhagen hegemony
J.T. Cushing ed. The University of Chicago Press

3) Speakable and unspeakable in quantum mechanics
J.S. Bell ed. Cambridge University Press

4) The Quantum theory of motion
P.R. Holland ed. Cambridge University Press

...+ the original papers.

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- ...and uses the same formalism (so formalism manipulators need not worry, ...at least not yet)

Outline

Two radically opposite attitudes:

A There is NO physical motivation whatsoever:

- after all, ordinary Quantum Mechanics describes physics very well at the atomic and nuclear levels.
- going relativistic and infinite number of degrees of freedom $\rightarrow$ Quantum Field Theories describe accurately the subatomic world, quantum electrodynamics, hadron physics (strong interactions), electro-weak interactions, . . .

B There are severe conceptual problems in ordinary Quantum Mechanics

- Fuzzy definition of the 'classical' measuring apparatus
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- the Solvay congress of 1927 was a *decisive* event for the establishment of the Copenhagen (Bohr) interpretation of QM.
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- Bell even advocated it...(Bell's inequalities violated)

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$$i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = H\psi(\vec{x},t) = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2_{\vec{x}} + V(\vec{x})\right)\psi(\vec{x},t) \qquad (1)$$

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define (uniquely) two real valued functions $R(\vec{x},t)$ and $S(\vec{x},t)$ by

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define particle density $\rho \equiv R^2 = |\psi|^2$

$$\frac{\partial S}{\partial t}+\frac{(\vec{\nabla}S)^2}{2m}+V-\frac{\hbar^2}{2m}\frac{\vec{\nabla}^2 R}{R}=0\ \&\ \vec{v}=\frac{1}{m}\vec{\nabla}S$$

imply a *quantum mechanical* Newton's law:

$$m\frac{d^2\vec{x}}{dt^2}=-\vec{\nabla}(V+U)=\vec{F}$$

where $U=-\frac{\hbar^2}{2m}\frac{\vec{\nabla}^2 R}{R}$ is a *quantum mechanical potential*

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e.g. for a particle in a nonstationary state of two energy levels system (E_1, E_2) one finds

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$\rightarrow$ violent fluctuations of momentum $\vec{p}$ and energy E $\rightarrow$ in general very irregular and complicated trajectories resembling Brownian motion (Bohm '52)

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-a- the wave function ψ satisfies the Schrödinger equation

-b- the particle is guided by the wave through $\vec{v} \equiv \frac{d\vec{x}}{dt} = \frac{1}{m}\vec{\nabla}S_{|\vec{x}=\vec{x}(t)}$

-c- in practice, no control of the actual initial position of the particle $\rightarrow$ a statistical ensemble with probability density $P(\vec{x},t) = |\psi(\vec{x},t)|^2$

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$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$
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- what happens if $P(\vec{x},t_0) \neq |\psi(\vec{x},t_0)|^2$??

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- use conservation of probability and Bohmian dynamics (-a- & -b-) to show $\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \dot{X} \cdot \nabla f = 0$
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- If $\vec{X}(t)$ goes almost everywhere, (e.g. chaotic trajectories), then f becomes constant almost everyhwere; on the coarse-graned level $\bar{f} = cte$ everyhwere

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- If $\vec{X}(t)$ goes almost everywhere, (e.g. chaotic trajectories), then f becomes constant almost everyhwere; on the coarse-graned level $\bar{f} = cte$ everyhwere
- define a notion of entropy $H = \int d\Sigma |\psi|^2 f \, \log f$ and show that $\frac{\overline{dH}}{dt} \leq 0$

Notion of 'quantum equilibrium'

- consider a system of N particles, with space coords $(X_1, X_2, ..., X_N) \equiv \vec{X}$
- define $f(\vec{X}, t)$ by $P(\vec{X}, t) = f(\vec{X}, t) \times |\psi(\vec{X}, t)|^2$
- use conservation of probability and Bohmian dynamics (-a- & -b-) to show $\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \dot{X} \cdot \nabla f = 0$
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- define a notion of entropy $H = \int d\Sigma |\psi|^2 f \; \log f$ and show that $\frac{\overline{dH}}{dt} \leq 0$
- $\overline{H}$ reaches its minimum $\Leftrightarrow \; \overline{f} = 1 \; \Leftrightarrow \overline{P(\vec{X}, t)} = \overline{|\psi|^2}$

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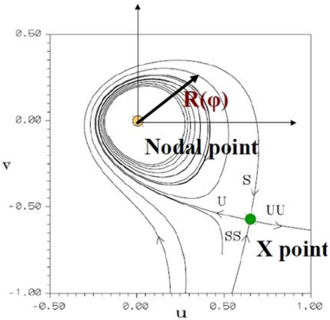
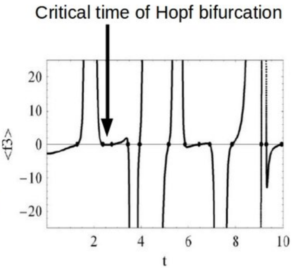
$\rightarrow$ interesting consequences

Outline

- Are Bohmian trajectories chaotic?
- An increased interest in recent years in 1-, 2-, or 3-particle systems in 2D and 3D boxes.
- The role of nodes (space-time points where $\psi(x, t) = 0$) and X-points (where the particle velocity in the frame of nodes $= 0$.)

review: *C.Efthymiopoulos et al*, *Annales de la Fondation de Broglie*, *Volume* 42 (2017)


0.50
0.00
-0.50
-1.00
-0.50
0.00
0.50
1.00
u
v
R(φ)
Nodal point
S
U
UU
SS
X point


Critical time of Hopf bifurcation
20
10
0
-10
-20
2
4
6
8
10
t
<f3>

$$\Psi[x_1,x_2,t]=\frac{2}{\pi}\sum_{m,n=1}^{2}\sin(mx_1)\sin(nx_2)e^{i(\theta_{mn}-E_{mn}t)},\ \ E_{mn}=\frac{1}{2}(m^2+n^2)$$

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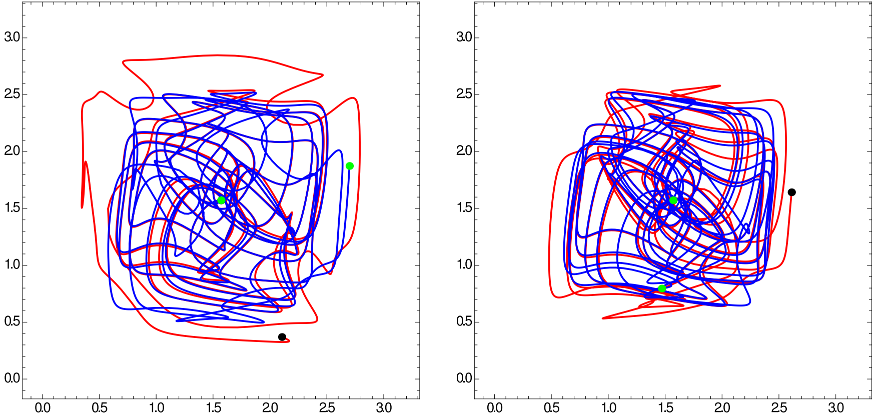
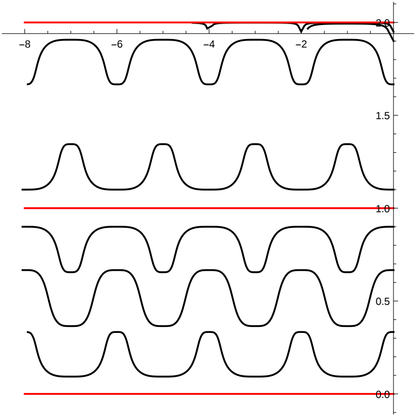


$\theta_{11}=1.1525988926093297,\ \theta_{12}=4.2775762116024665$ $\quad\theta_{11}=1.2,\ \theta_{12}=4.3$

$\theta_{21}=2.1660329888555025,\ \theta_{22}=2.8960554218806349$ $\quad\theta_{21}=2.2,\ \theta_{22}=2.9$



$$\left(-\hbar^2\frac{\nabla^2}{2m}+\lambda\delta(x_1-x_2)\right)\Psi^{\delta}_{n_1,n_2}(x_1,x_2)=E_{n_1,n_2}\Psi^{\delta}_{n_1,n_2}(x_1,x_2)$$

Outline

Valentini '91:

(1) P relaxes to $|\psi|^2$ at equilibrium in a gas of Bohmian particles

(2) proof not fully convincing: $\overline{H(t)} < \overline{H(0)}$ rather than $\frac{\overline{dH}}{dt} \leq 0$ (H-theorem)

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- the problem in (2) is typical in stat. phys. whenever a specific kinetic equation is lacking (e.g. Boltzmann equation, Vlasov/Landau equation, etc.)
- A Bohmian gas kinetic equation is still missing: difficult to derive, due to the non-local potential! Boltzmann's 'molecular chaos' hyp. cannot be used straightforwardly. A careful BBGKY hierarchy approach is needed.

Such an equation would be very important $\rightarrow$ quantitative information about relaxation to ’quantum equilibrium’
AND quantitative information about *statistical fluctuations*, i.e. about possible Lorentz violation in relativistic (field) theories...

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- an increaslingly large community (chem., phys.-chem., nonascience,...)is relying on its practical power to study quantum systems
- some communities still ignore it, as "useless or probably wrong".
- one can regard ordinary QM as a kind of effective 'low-energy' theory of an underlying physics where indeterminism is NOT a fundamental feature!
- Outlook
  - can it be tested?
  - can it be an alternative road to new physics, in particular in high energy physics beyond the standard model?

The Quantum Pandora Box?