Dark Matter on small, subgalactic, scales

Julien Lavalle
CNRS – LUPM
Based on work with **Gaétan Facchinetti**, **Thomas Lacroix**, and **Martin Stref**
(1610.02233, 1805.02402 + work in prep)

Rencontres de Physique des Particules

LPC – Clermont-Ferrand – January 24, 2019

LABORATOIRE MONTPELLIER
UNIVERS & PARTICULES

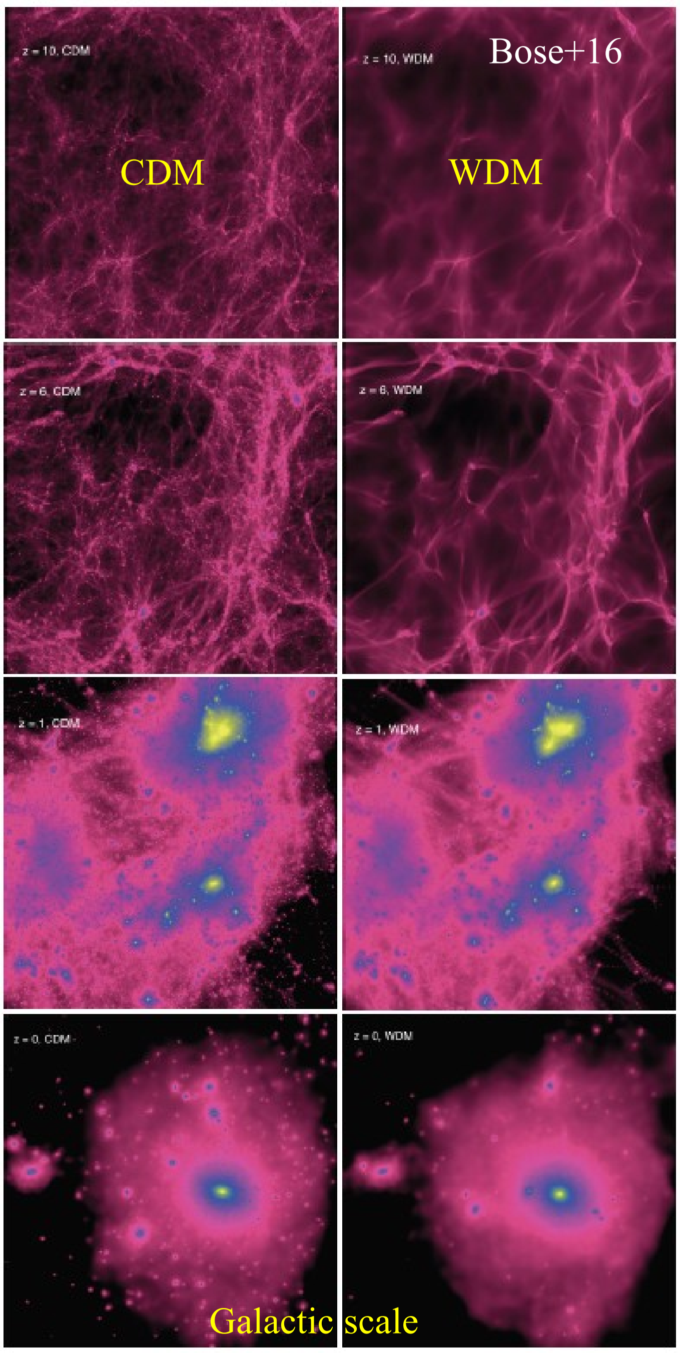
Outline

*** Why are small scales interesting?**

*** How to build a Milky Way halo for your favorite DM model?**

*** Summary**

The cold Dark Matter (CDM) paradigm



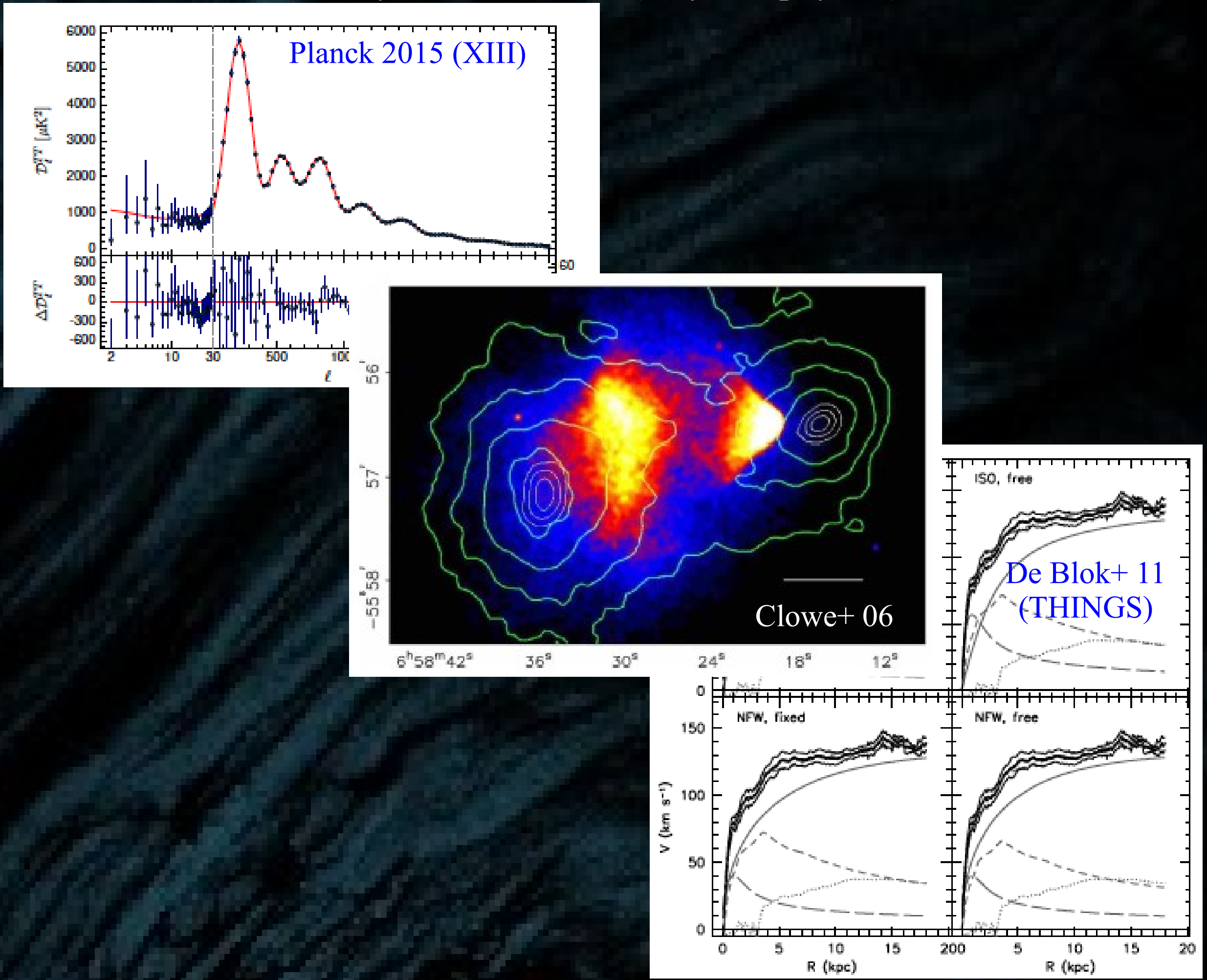
<u>So far, only gravitational evidence for DM</u>
(cosmological structures+CMB)

<u>CDM successes:</u>

- CMB peaks
- Successful structure formation (from CMB perturbations)

=> CDM seeds galaxies, galaxies embedded in DM halos

- Lensing in clusters + rotation curves of galaxies
- Also consistent with Tully-Fisher relation (baryonic physics)



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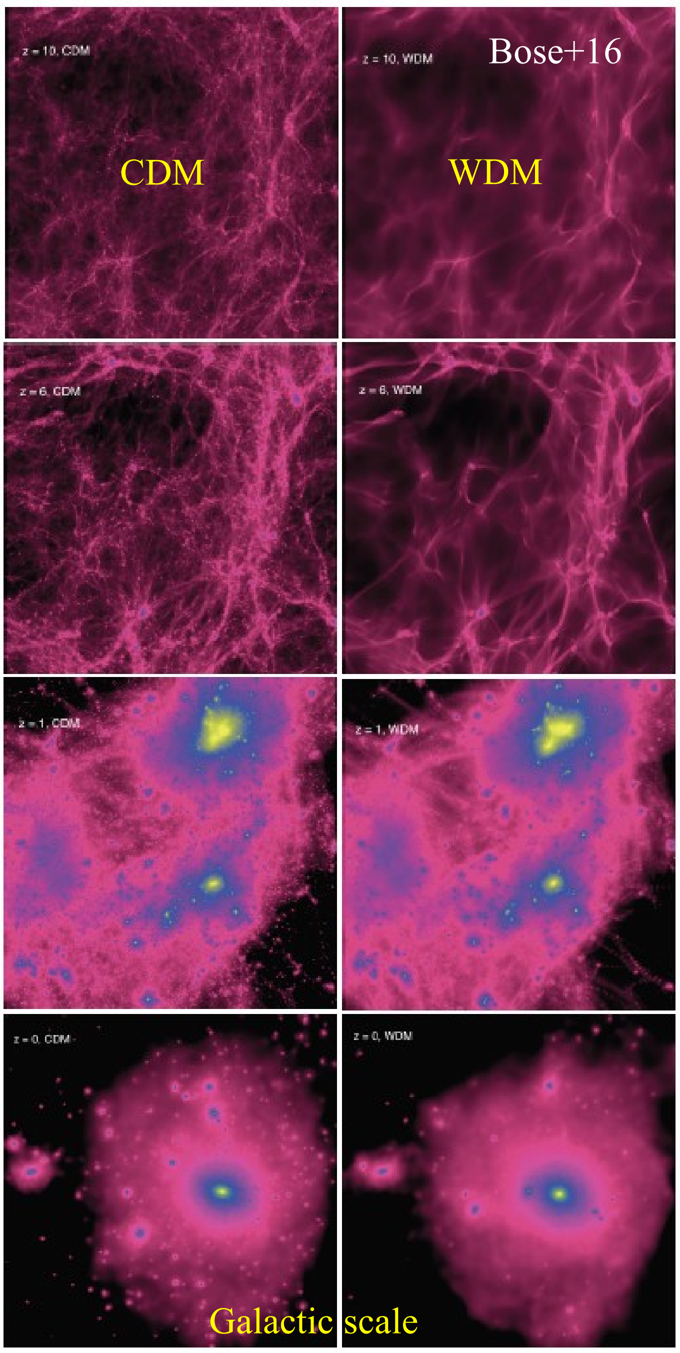

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Remaining issues:

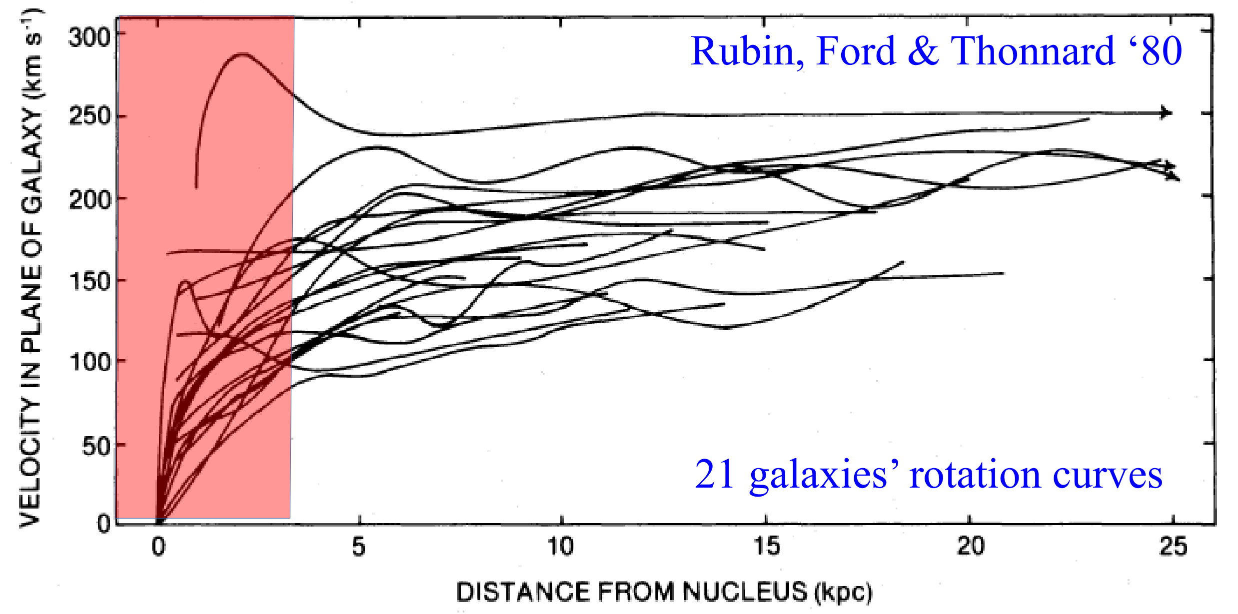
* Nature/origin of CDM – new particle/s?

* Some issues on **subgalactic** scales

NB: subhalos no longer a problem!
(faint objects continuously discovered + structure formation theory improved with baryonic physics)

Dark Matter on galactic scales

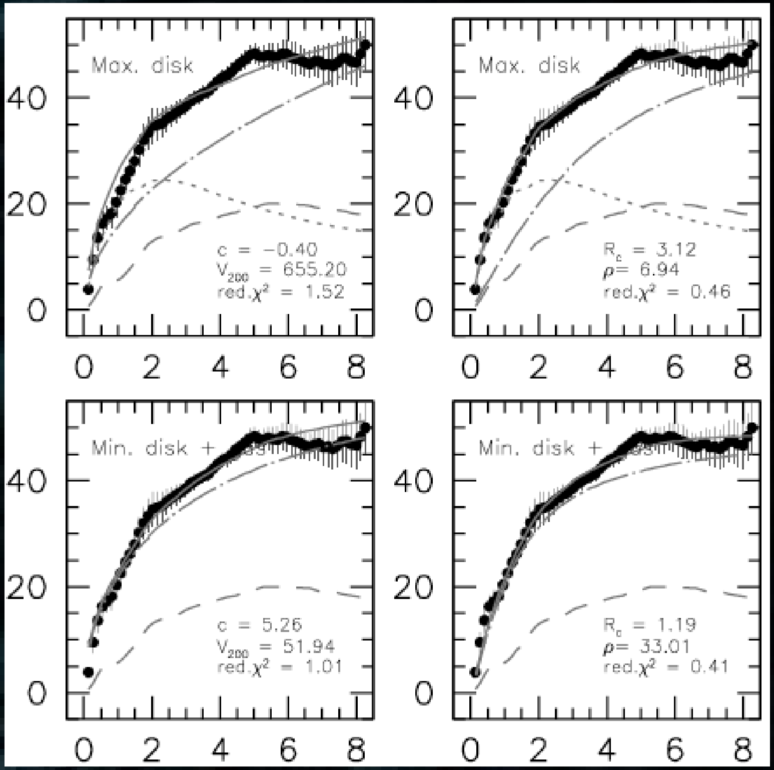
Bulk of luminous matter



Rubin, Ford & Thonnard '80

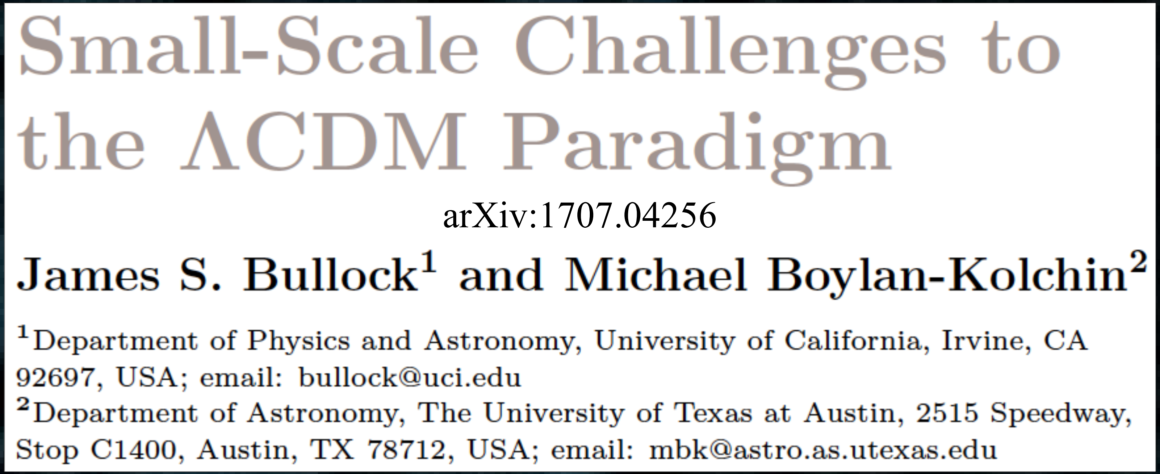
21 galaxies' rotation curves

Oh+11



* Keplerian decrease of rotation velocity not observed
* Stars and gas not bounded to the object unless invisible mass there
=> Spherical dark matter halo could explain this + natural stabilizer

CDM issues on small (subgalactic) scales



Small-Scale Challenges to the ΛCDM Paradigm

arXiv:1707.04256

James S. Bullock[1] and Michael Boylan-Kolchin[2]

[1]Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA; email: bullock@uci.edu
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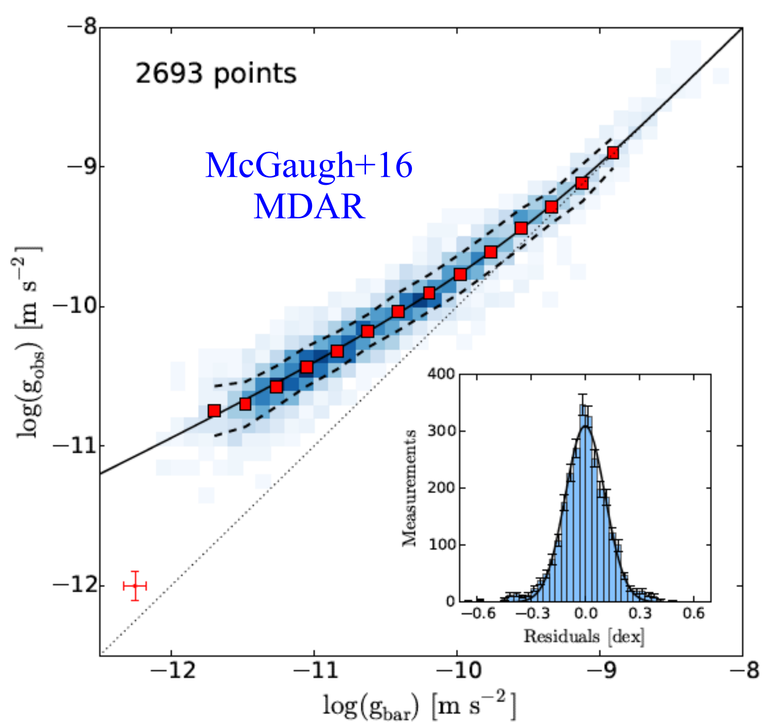
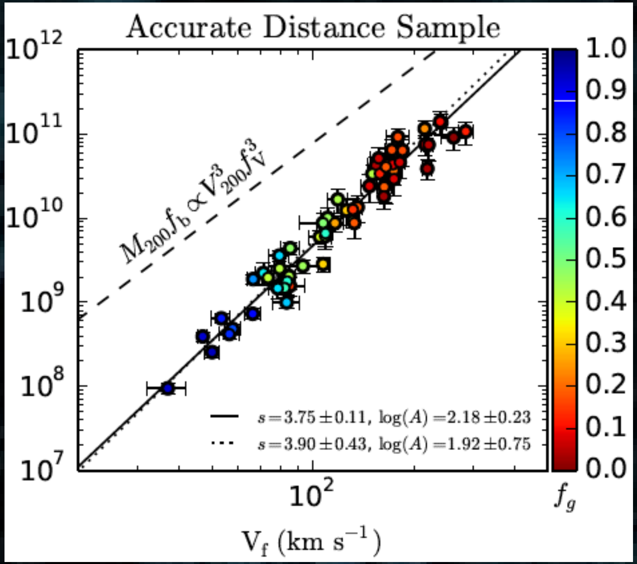
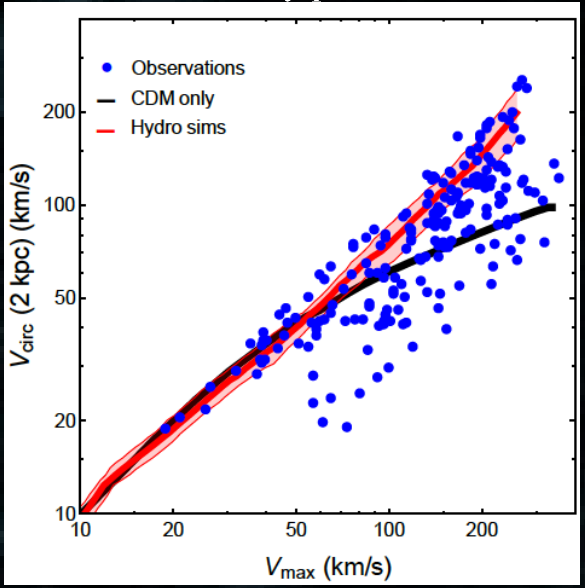


Tulin+18 after Oman+15
Diversity problem

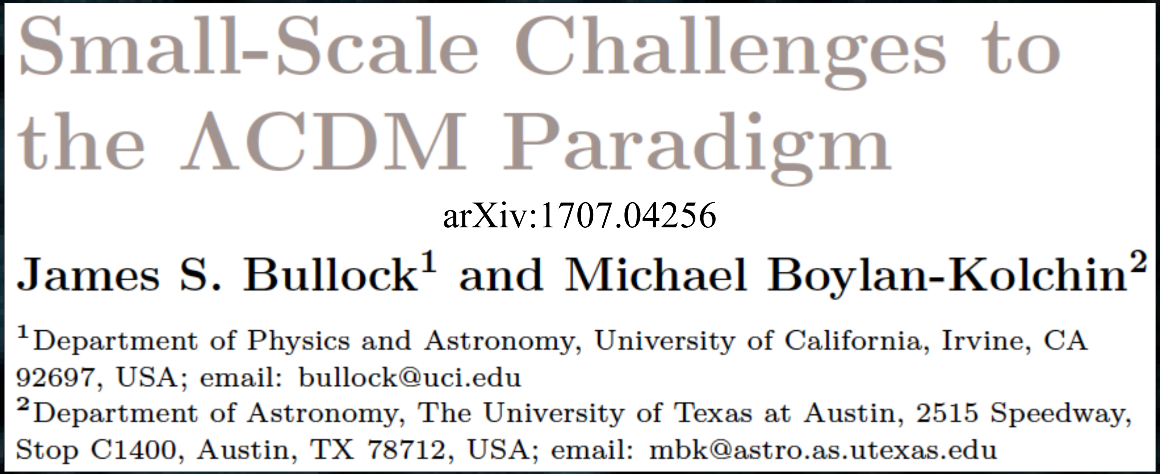


Lelli+15, BTFR



Core/cusp+diversity problems or regularity vs. diversity problems.
Maybe baryonic effects, but clear statistical answer needed.
Does same feedback recipe solve all problems at once?

CDM issues on small (subgalactic) scales



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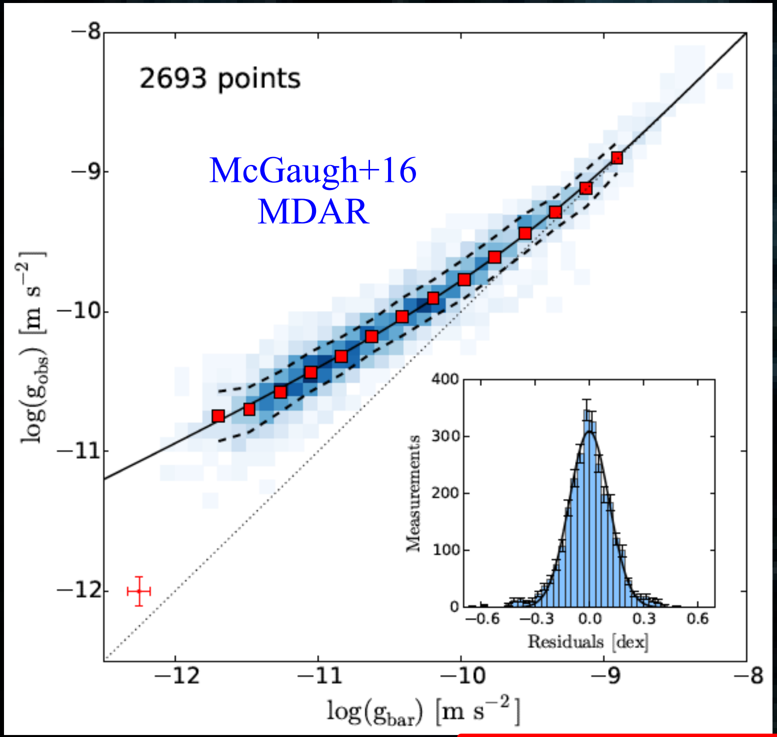
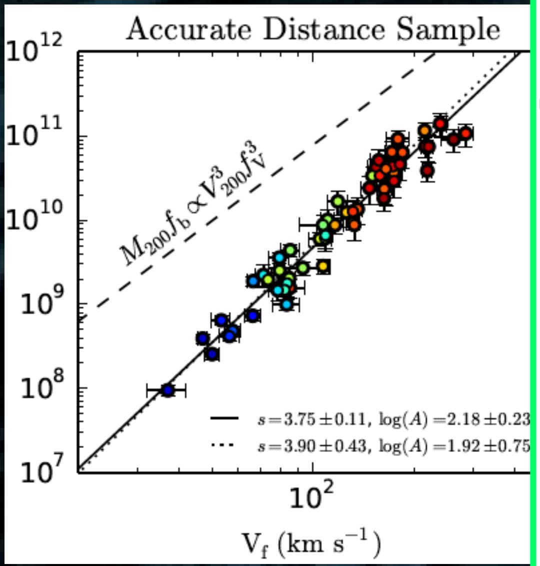
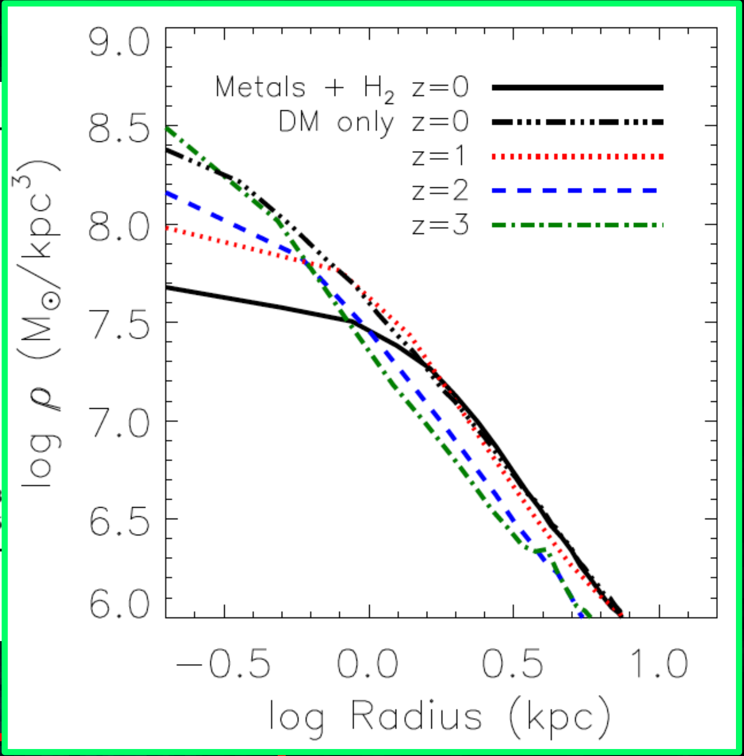




Lelli+15, BTFR

Governato+12
Cusps→cores



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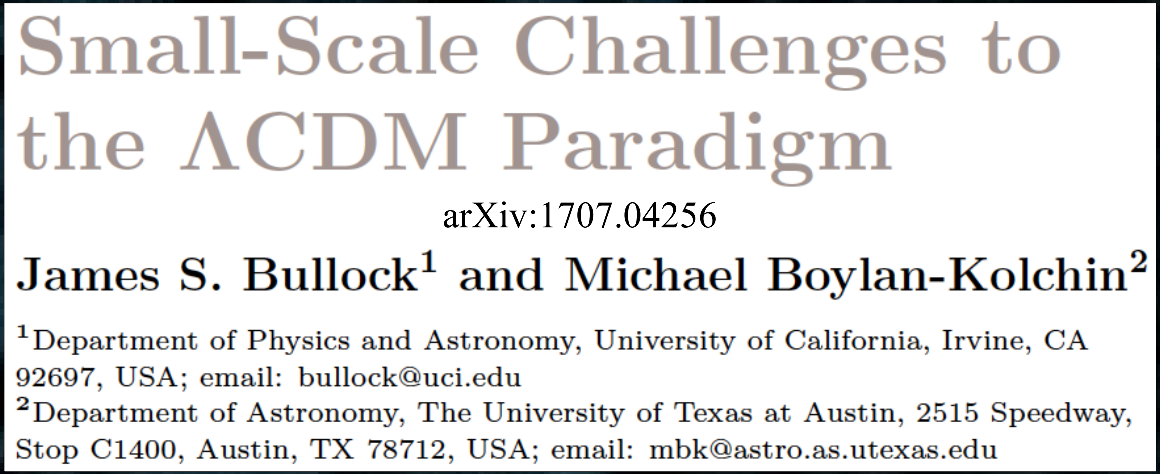

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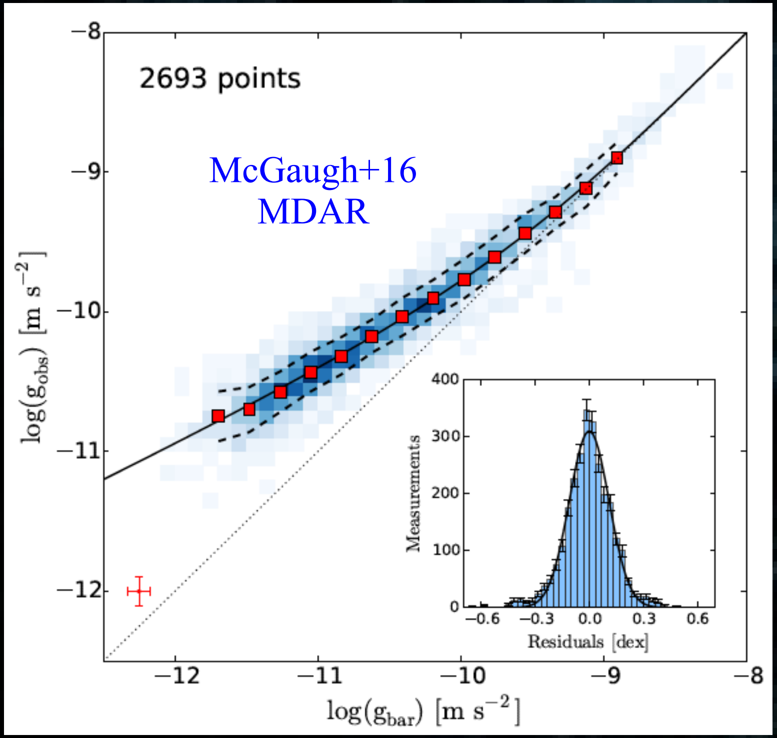
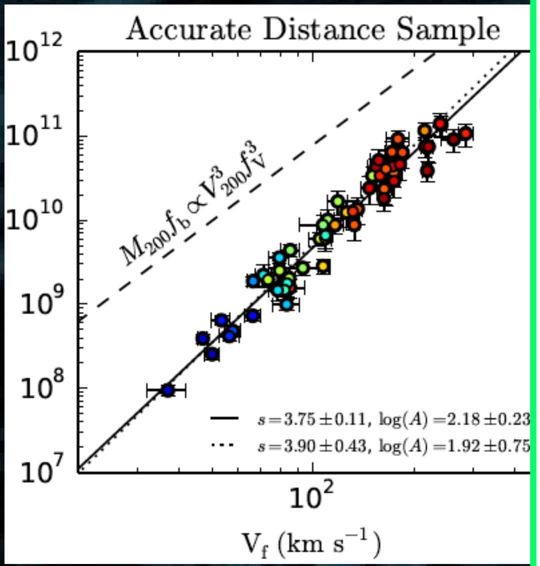


McGaugh+16
MDAR



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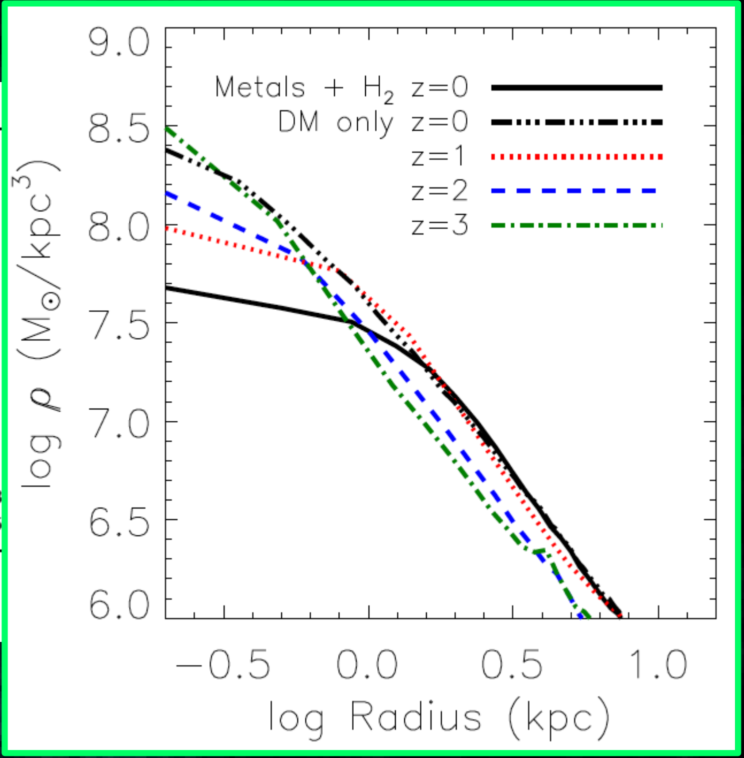

This has also motivated pure DM solutions: eg ULA, SIDM
→ probes on small scales important tests for all DM scenarios

Probing dark matter on small scales
1. Gravitational searches

Gravity (VLTI) @ ESO

Gaia satellite @ ESA

++ astrometry + microlensing + pulsar timing arrays + others (SKA, etc.)

are probing / will probe DM in the Milky Way:

* Dark matter at the Galactic center (e.g. S2 orbit – Lacroix ‘18, Abuter+’18)

* Global spatial distribution of dark matter [e.g. McMillan’17, Eilers+’18]

* (Coarse-grained) phase-space distribution of DM (more difficult)

* DM subhalos / compact objects

[see e.g. Van Tilburg+’18, Dror+’19, etc.]

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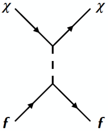
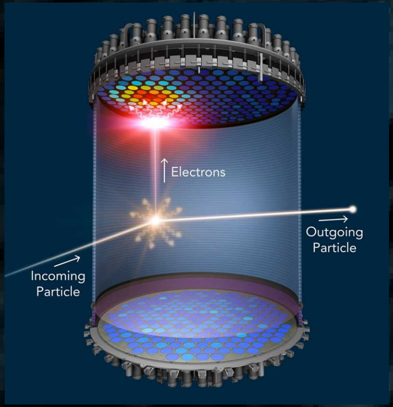
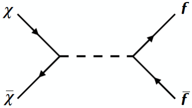
Different scenarios predict different structuring properties on small scales
→ additional test for DM candidates

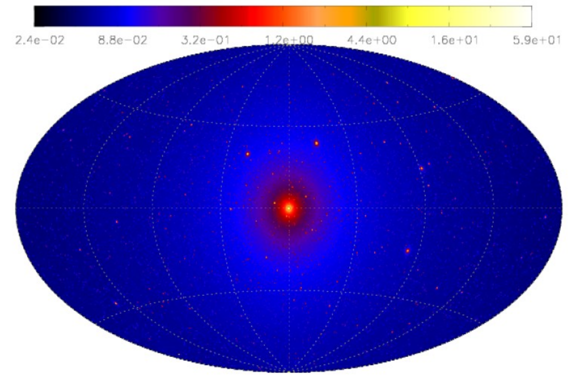
Probing dark matter on small scales
2. Particle searches



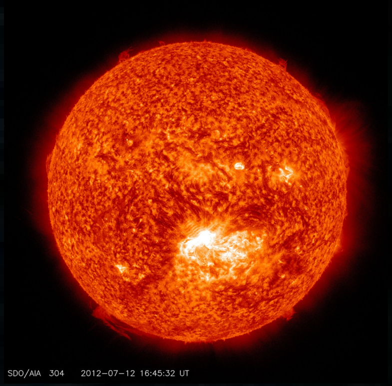


@KIPAC

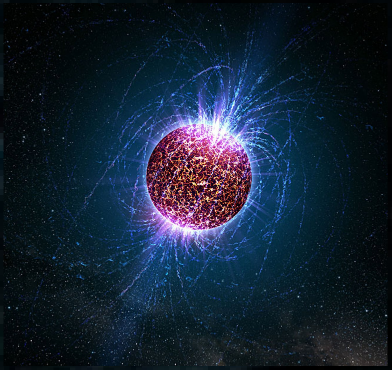




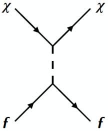
Pieri, JL+'11



@SDO/NASA



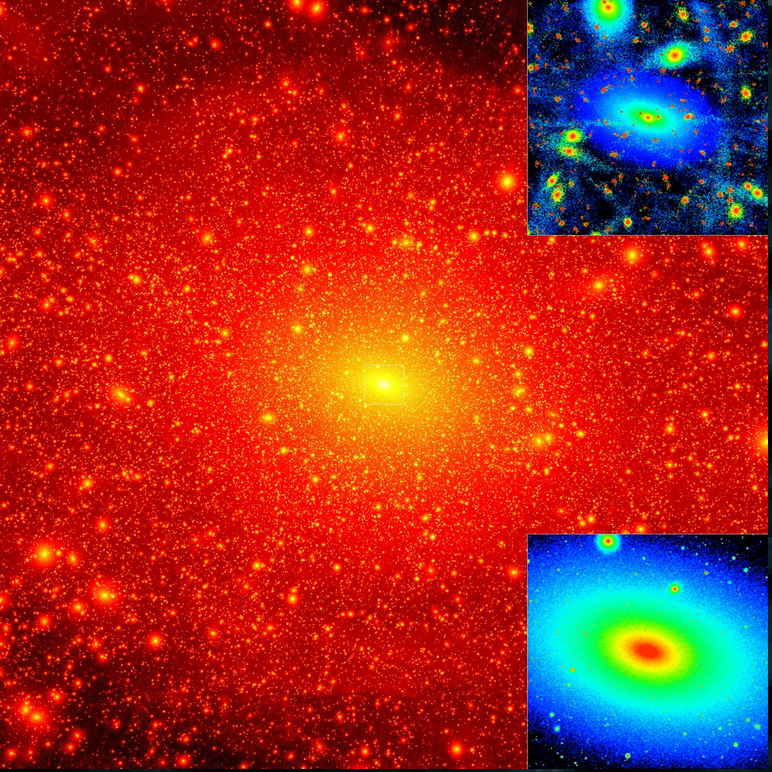
@Casey Reed/Penn State University



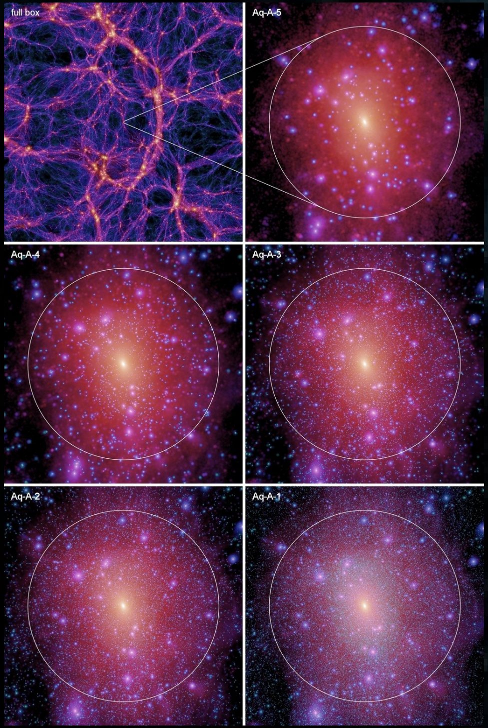
Direct searches (WIMPs or axions) + solar neutrinos:
→ local density
→ local velocity distribution
→ subhalos (???)

Indirect searches (Francesca Calore's talk):
→ detailed DM distribution (over the Milky Way)
→ subhalos (annihilation boost + individual sources)
→ position-dependent velocity distribution (e.g. Sommerfeld, p-wave) (Mathieu Boudaud's talk)

Interaction with stars:
→ global density + velocity distributions
→ encounters with subhalos



Via Lactea II, Diemand+08



Aquarius, Springel+08
[see also Molitor+’15]

Theoretical framework well defined:

* Inflation model → primordial power spectrum (model dependent)
* Transfer function (modes entering bef/aft eq)
* DM-plasma coupling properties (model dependent)
* Matter power spectrum (model-dependent cutoff)
* Press-Shechter and extensions → halo mass function (z)

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* Fully non-linear regime with cosmological simulations
=> Statistical properties of sub/halos + links with cosmology

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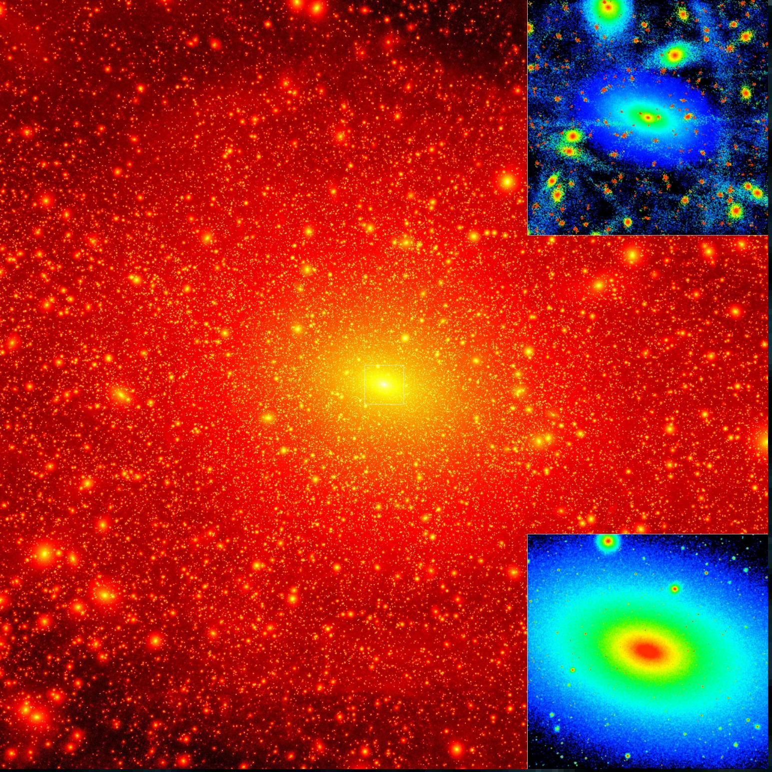

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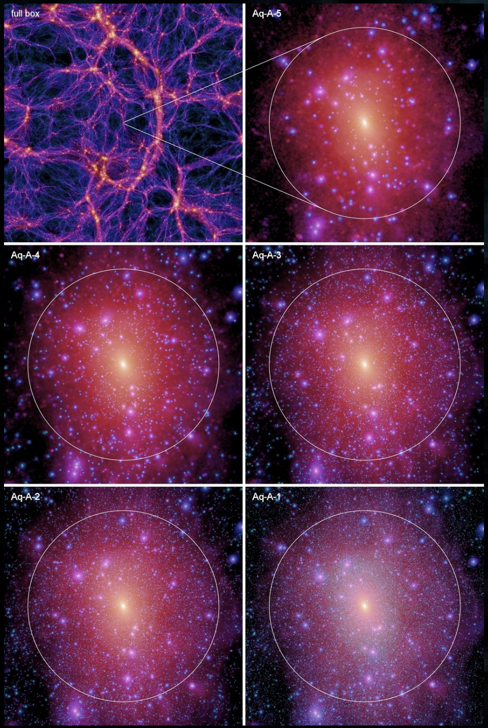

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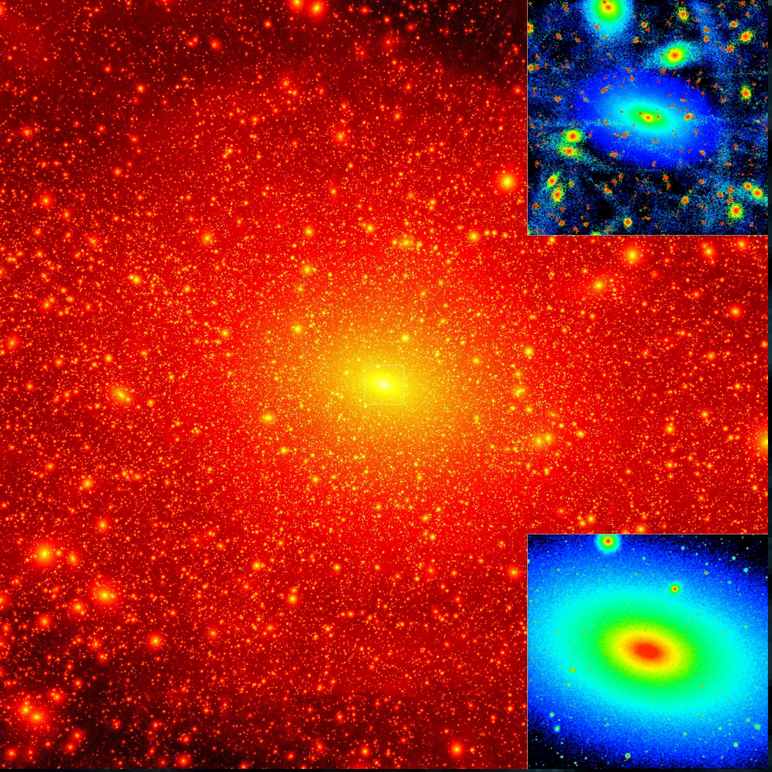

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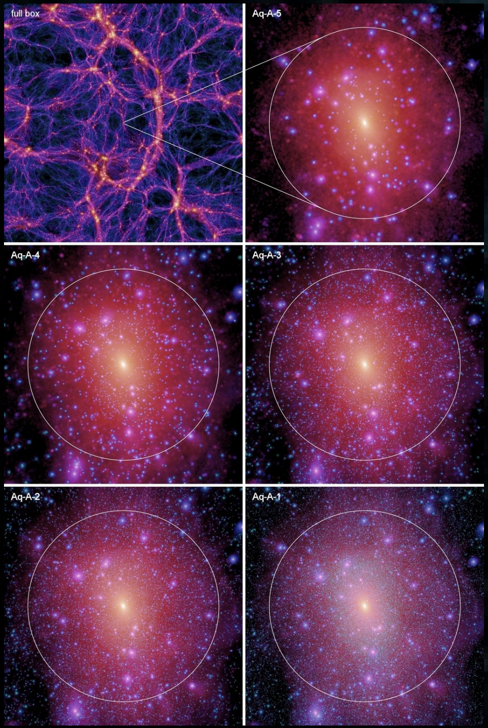
* Impact of baryons from hydro-runs / adiabatic growth of disks



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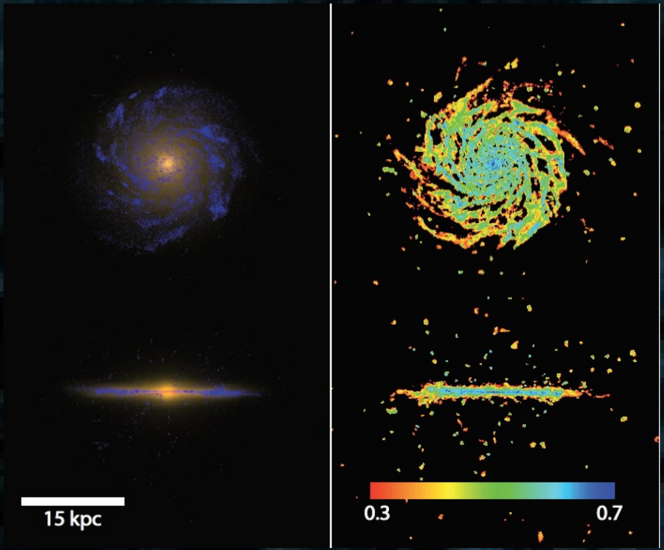
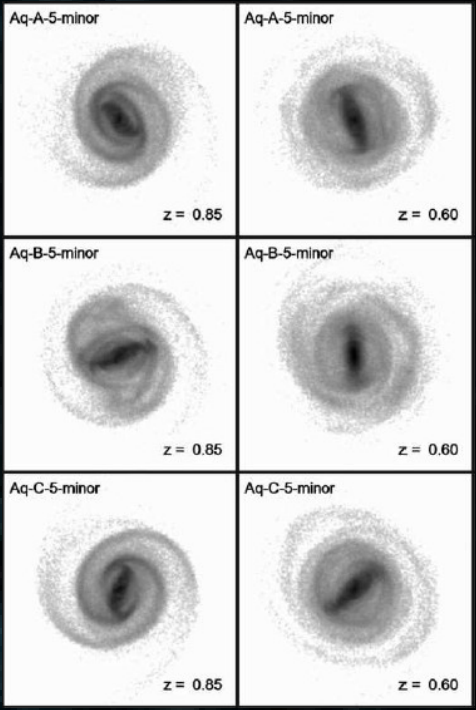
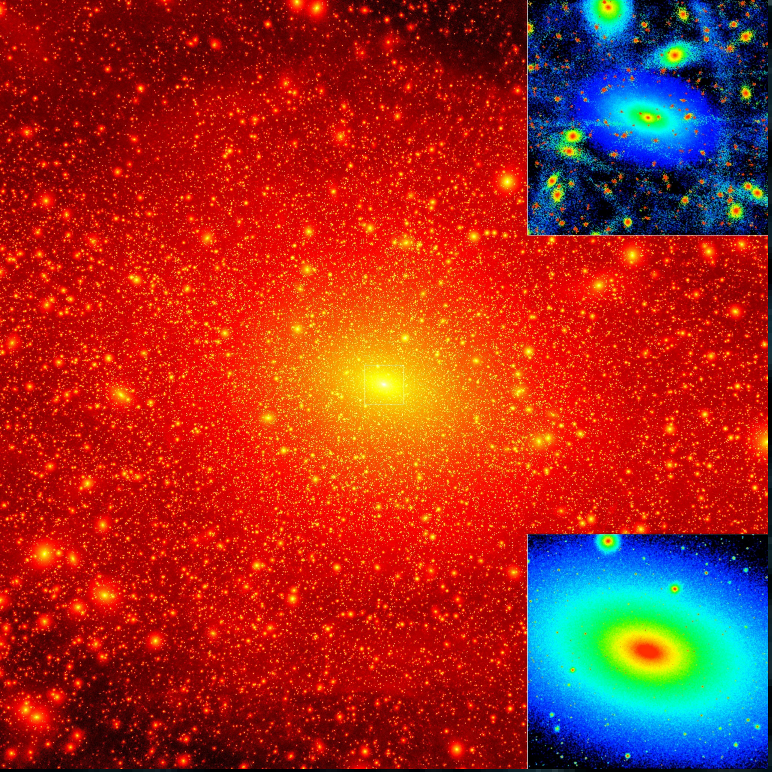


Eris, Guedes+11



Aquarius + baryons, Yurin+15



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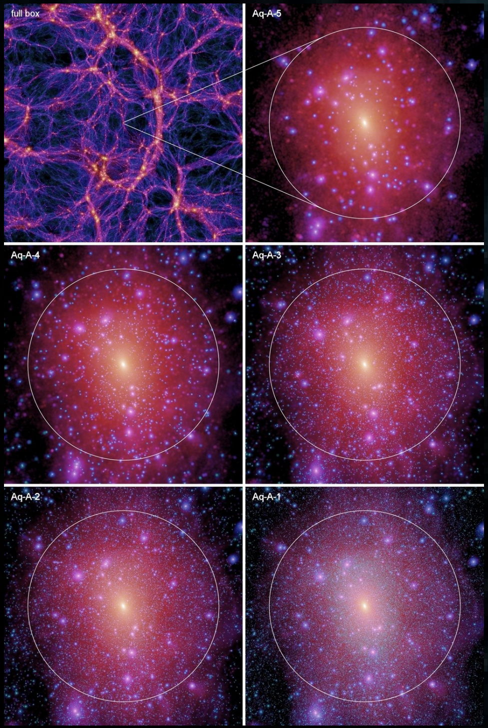


Aquarius, Springel+08
[see also Molitor+'15]

Problems are ...

* Resolution limit: compare $10^{5}\,M_{sun}$ with $10^{-10}\,M_{sun}$ (in DM-only)
* … getting worst in hydro-runs

…

* (Large uncertainties in baryonic physics)

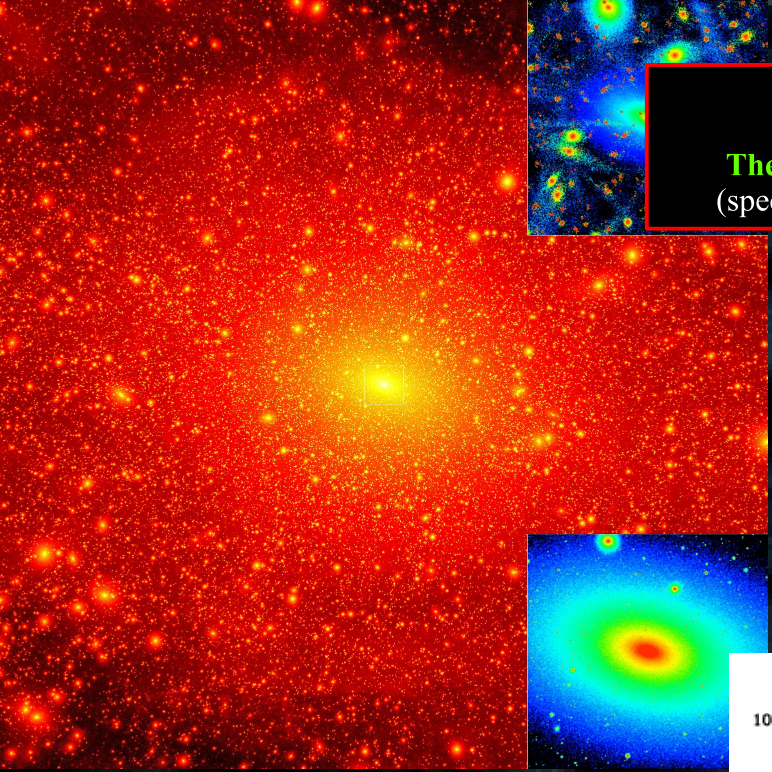
…

* How is "Milky Way-like" defined?
* What's special with "8 kpc" in a cosmological simulation?

...

Making predictions for DM searches?

The Milky Way a strongly constrained system!
(specific history + properties + observational data)



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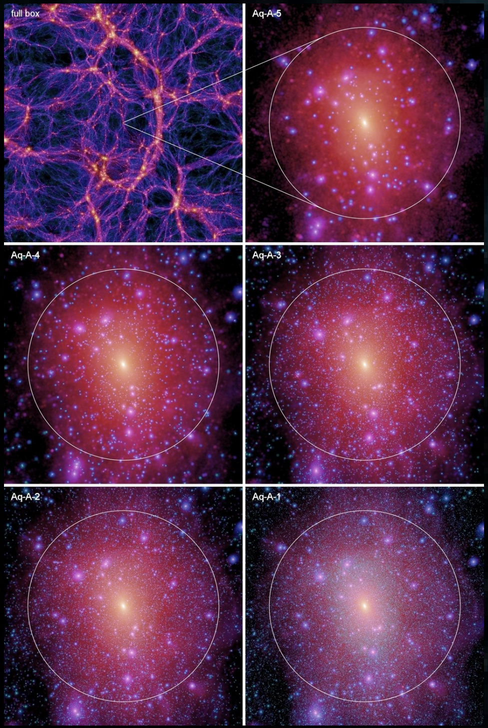

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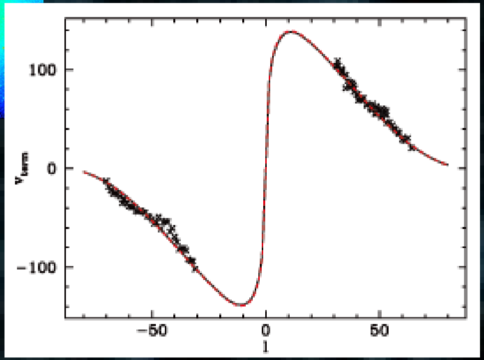
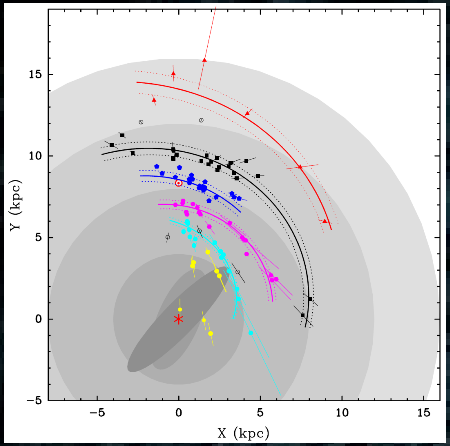
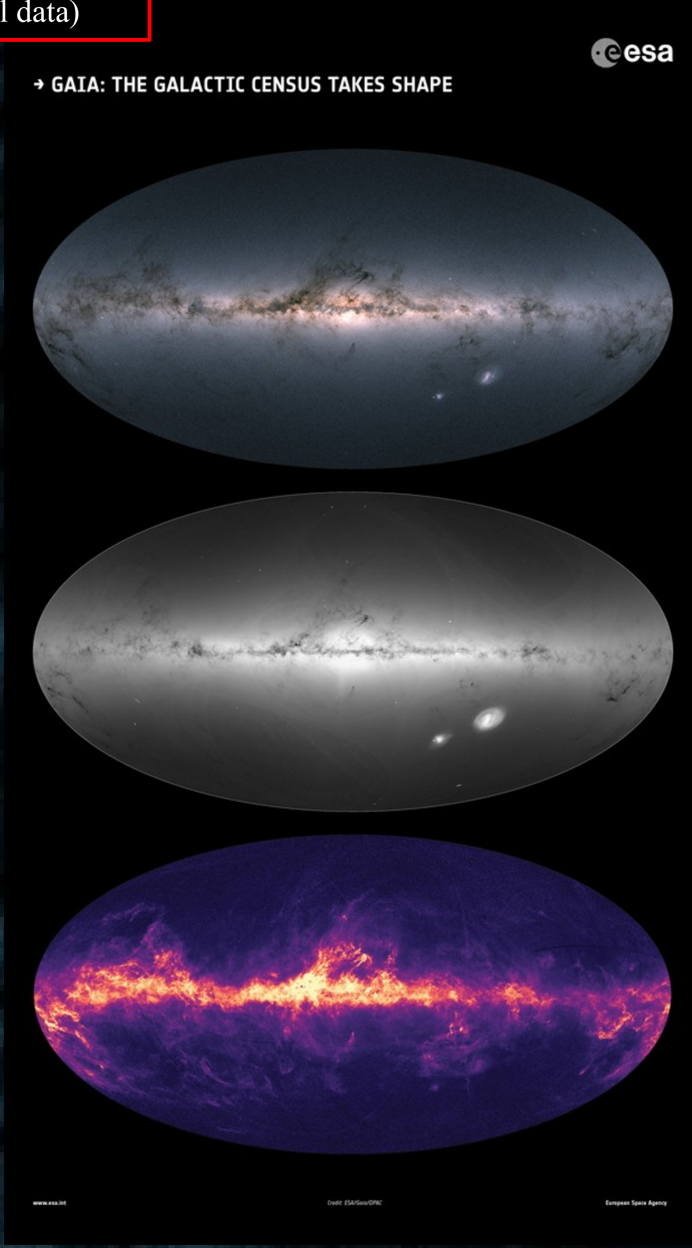


MW terminal velocities, McMillan ‘11



MW masers, Reid+14



Gaia: Data Release 2 (DR2) @ESA

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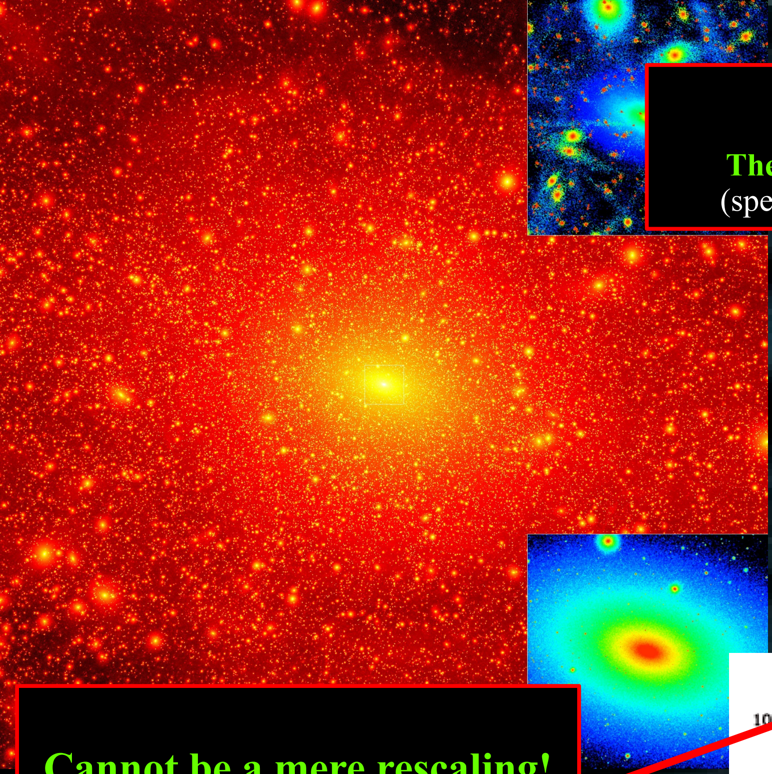
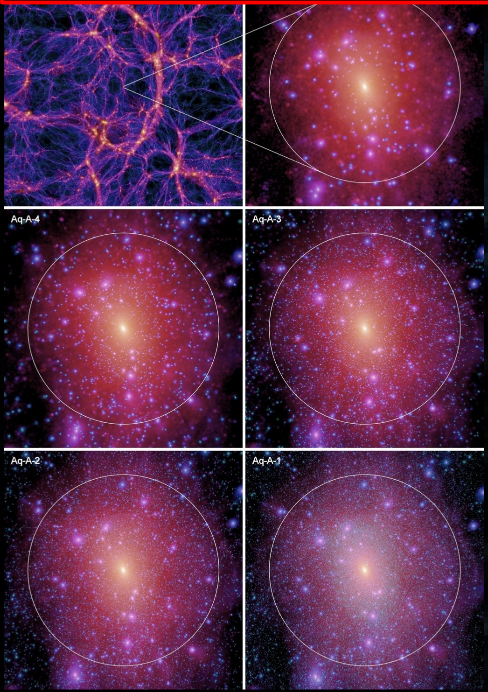


Cannot be a mere rescaling!



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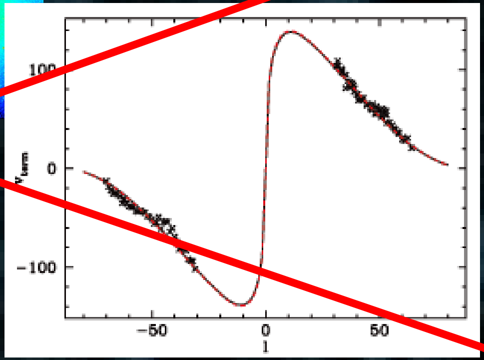

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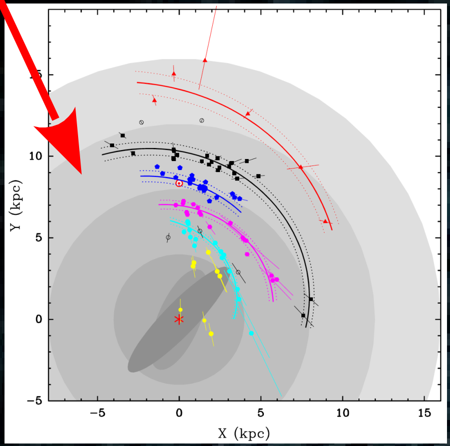
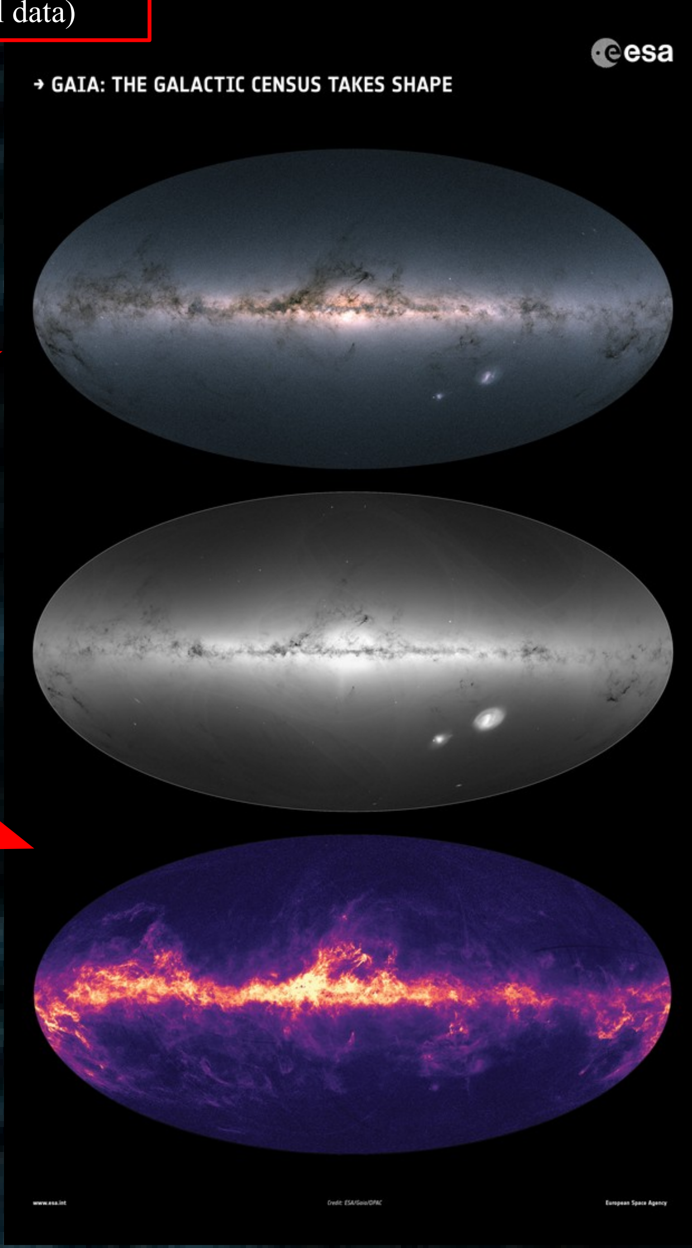


MW masers, Reid+14



Gaia: Data Release 2 (DR2) @ESA

Granularity of the Galactic DM halo:
How to build a theoretically+observationally constrained model?

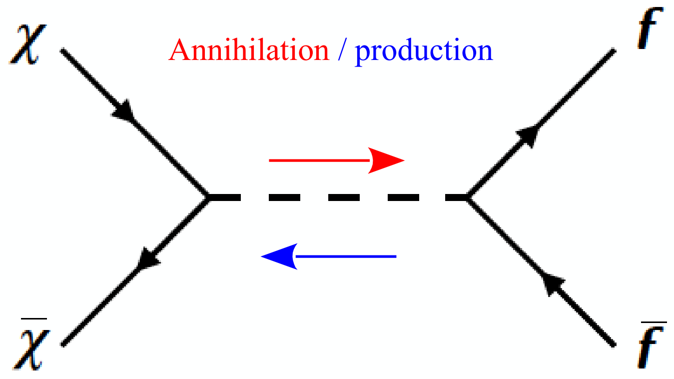
*** Particle physics input: the minimal clustering scale**

*** Structure formation: statistical properties of subhalos**

*** Dynamical/kinematic constraints + tidal effects**

Thermal relics from the early Universe

Production/annihilation => chemical+thermal equilibrium
→ Chemical decoupling => freeze out ($x_f=m/T_f\sim 20$)
→ Relic abundance fixed
NB: links with indirect searches



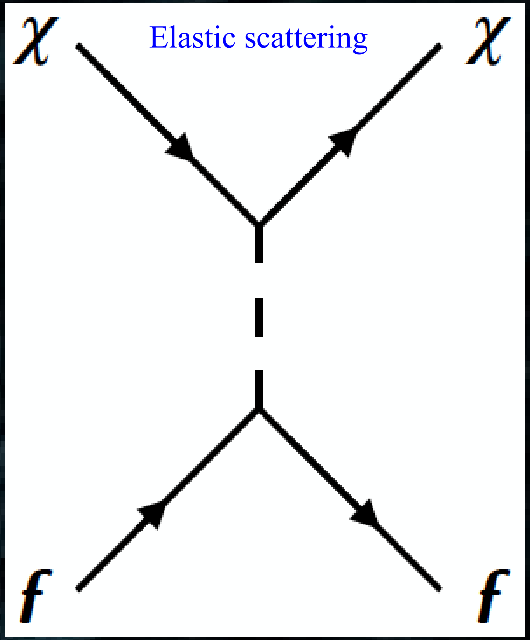
$$\Gamma_{\rm ann} = n_\chi \langle \sigma_{\rm ann} v \rangle$$

Elastic collisions can ensure thermal contact long after freeze out (plasma very dense) => DM particles still belong to the plasma (same temperature).

Thermal contact ceases
→ kinetic decoupling => free streaming ($x_k=m/T_k\sim 10^2$-10^4)

At matter-radiation eq., DM can only grow density fluctuations larger than path run after kinetic decoupling.

=> Sets the minimal scale of DM halo
NB: links with direct searches / interaction with stars

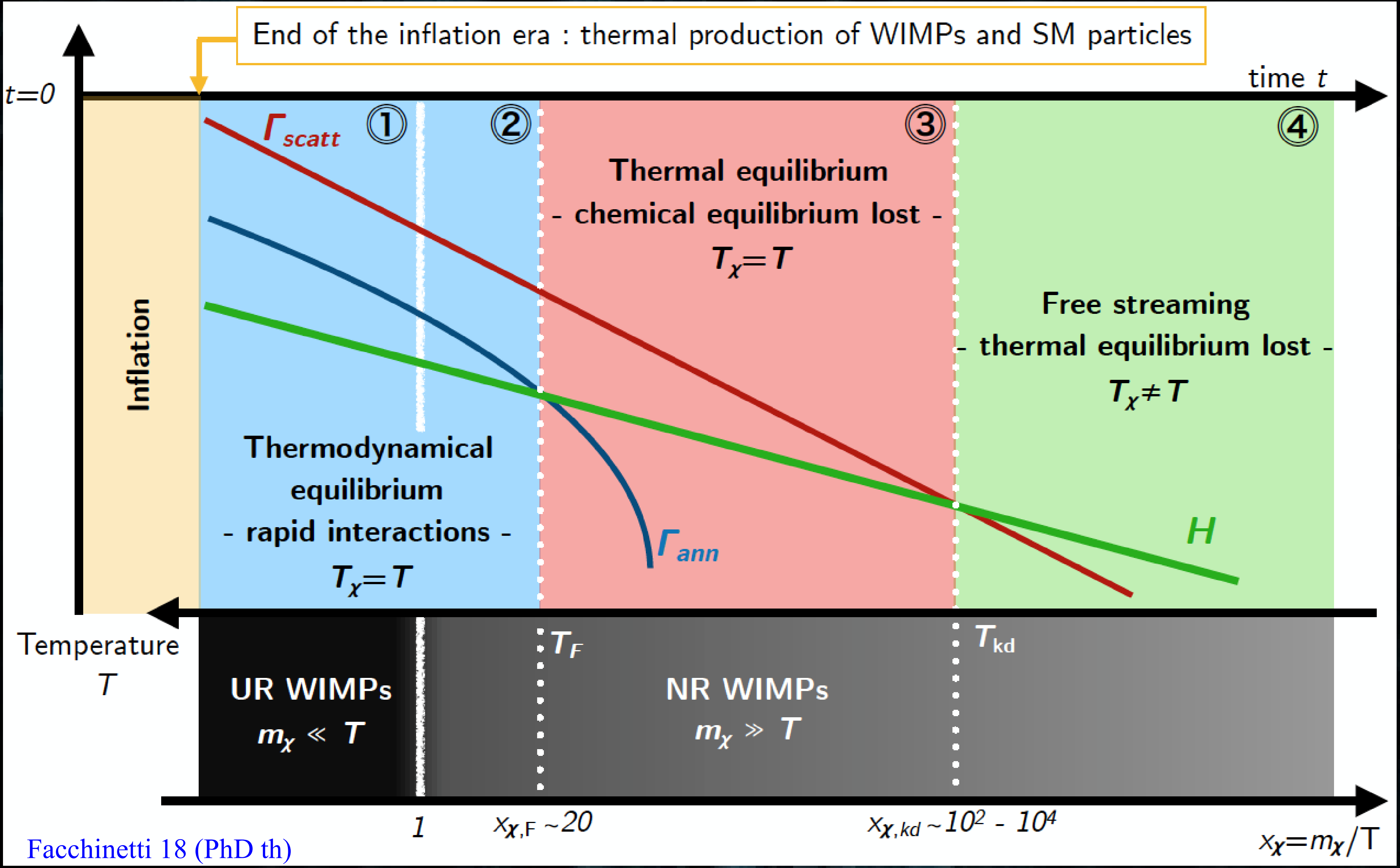


$$\Gamma_{\rm scatt.} = n_{\rm f} \langle \sigma_{\rm scatt} v \rangle$$

Solve moments of Liouville-Boltzmann equation for coupled species

$$\frac{d\, f(x^\mu, p^\mu)}{d\lambda} = \hat{C}[f]$$

Thermal relics from the early Universe



Facchinetti 18 (PhD th)

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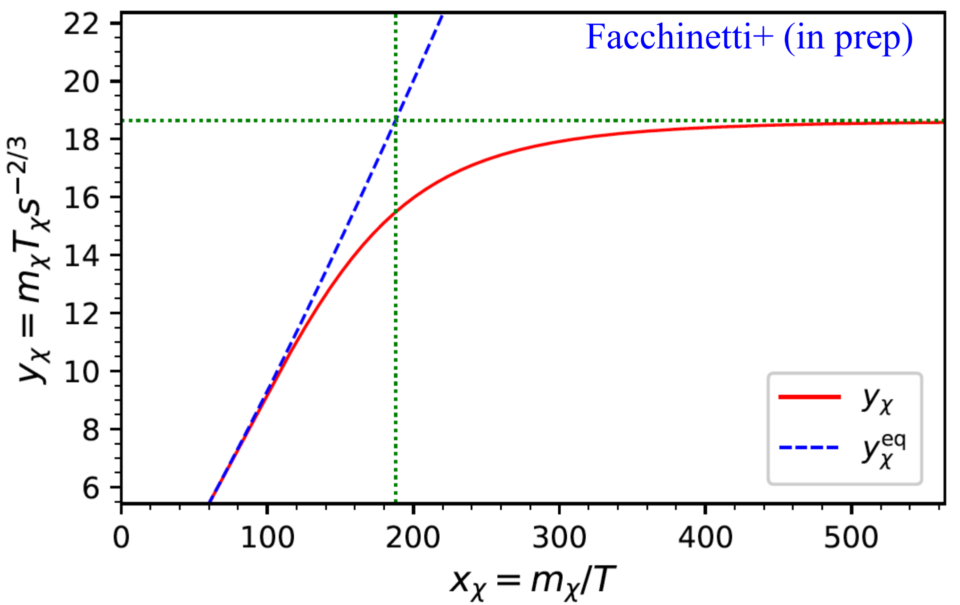
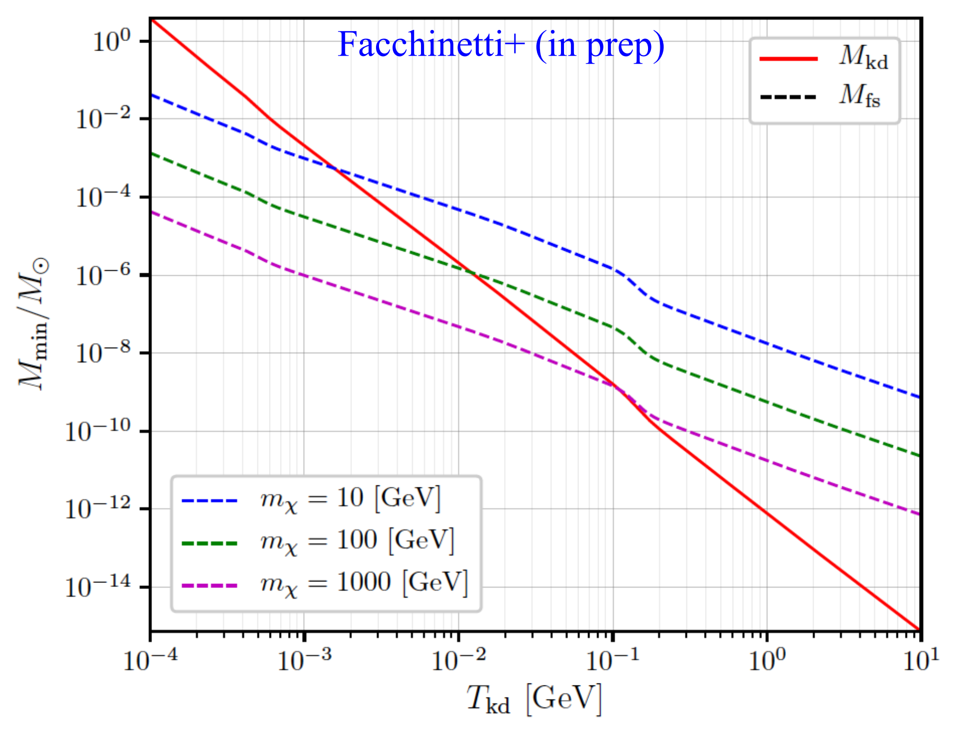




$$\frac{\mathrm{d}\ln(y_\chi)}{\mathrm{d}\ln(x_\chi)} = -\left(1 + \frac{\mathrm{d}\ln\left(h_{\mathrm{eff}}(T)\right)}{3\,\mathrm{d}\ln\left(T\right)}\right)\frac{\gamma(T)}{H}\left(1 - \frac{y_\chi^{\mathrm{eq}}}{y_\chi}\right)$$

$$\lambda_{\mathrm{fs}}(t) = a(t)\int_{t_{\mathrm{kd}}}^{t}\frac{v(t')}{a(t')}\mathrm{d}t'$$

Minimal halo mass from ~$10^{-12} M_{sun}$ (>1 TeV WIMPs) to ~$10^{-3} M_{sun}$ (<10 GeV WIMPs)
Like relic abundance, fixed by interaction properties of DM particles!
[see also Schwartz+, Hofmann+, Green+, Bringmann+, Boehm+, etc.]

Initial statistical/cosmological properties

The initial mass function
(linear + ~non-linear)

From primordial spectrum to mass function (ext. Press-Schechter)

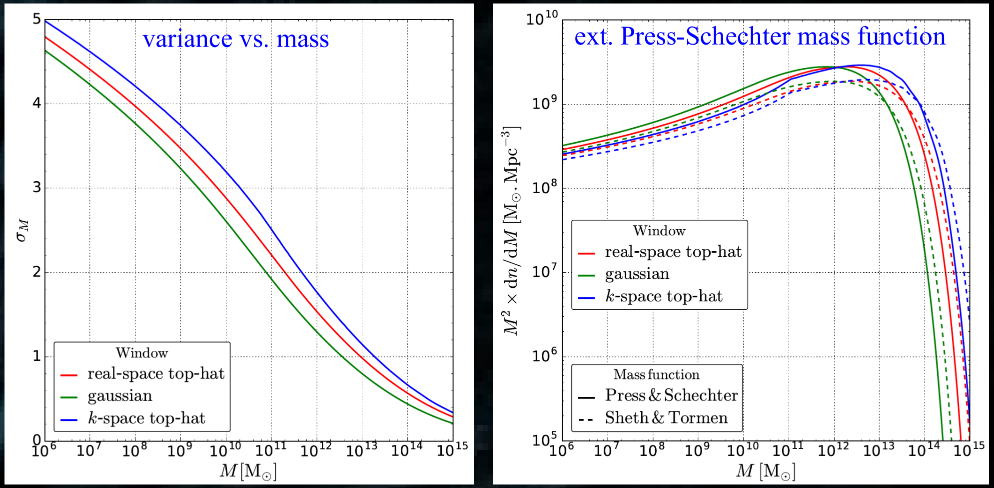
$$P(k,t) = D_+^2(t)\,\left\{P(k) \equiv A_0\,T^2(k)\,P_{\rm prim}(k)\right\}$$

$$\sigma^2(R) \equiv \varepsilon_R(|\vec{r}| \to 0) = \int d\ln k\,\Delta^2(k)\,|\tilde{W}(k,R)|^2$$

$$\frac{dn}{dM} = \left\{V_M^{-1} \equiv \frac{\rho_M}{M}\right\}\left|\frac{dF(\delta > \delta_c)}{dM}\right| = \frac{\rho_M}{M^2}\left|\frac{d\ln\sigma}{d\ln M}\right|\nu\,f(\nu)$$

$$\boxed{\frac{d\mathcal{P}(m_{200})}{dm_{200}} \overset{\sim}{\propto} m_{200}^{-\alpha_m}\left\{1 - e^{-\left[\frac{m_{200}}{m_{\rm cut}}\right]^n}\right\}}$$

Typically a power law with a cutoff (minimal) mass.



Stref, PhD th. '18

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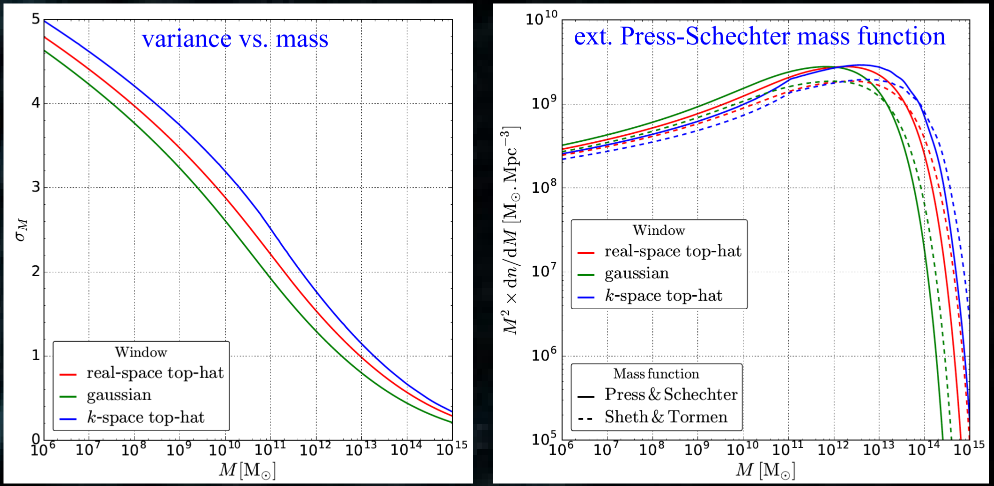
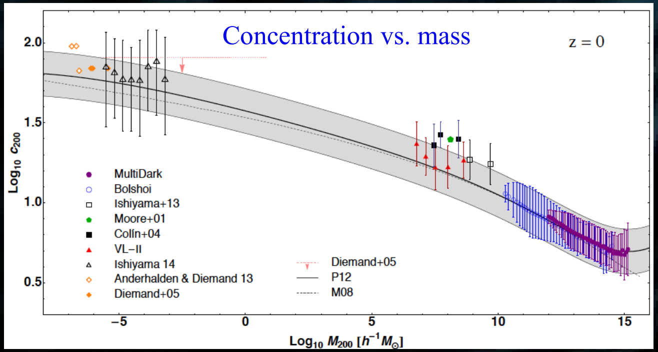
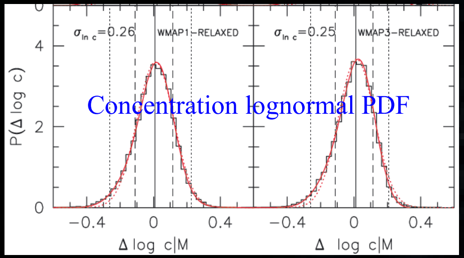


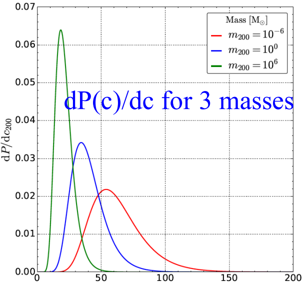
Stref, PhD th. '18



Sanchez-Conde+13



Maccio+08



Stref, PhD th. '18

Concentration function

Traces the density at collapse time.
Modeling based on 2-parameter fit
(Bullock+01, Maccio+08, Prada+12, Dutton+14, etc.)

$$m_{200}(z)=G_z\,m_*(z_c) \qquad \sigma(m_*(z_c))=\sigma_c^0\,D_+(z)$$

$$\bar{c}_{200}(m_{200},z)=K_{200}\left[\frac{\rho_c(z_c)}{\rho_c(z)}\right]^{1/3}$$

Fitting formula from Sanchez-Conde+13 + lognormal DF

$$\boxed{\frac{d\mathcal{P}_c}{dc}(c,m)=\frac{1}{K_c}\,\frac{\exp\left\{-\frac{(c-\bar{c}(m))^2}{2\,\sigma_c^2}\right\}}{c\,\sqrt{2\,\pi\,\sigma_c^2}}}$$

Defining the whole subhalo "phase space"

At MW formation, all (cosmological) properties factorize out

$$\frac{d^n N^0}{d\omega^n} = N_0 \, \frac{d\mathcal{P}_V^0(\vec{x})}{dV} \times \frac{d\mathcal{P}_m^0(m)}{dm} \times \frac{d\mathcal{P}_c^0(c,m)}{dc}$$

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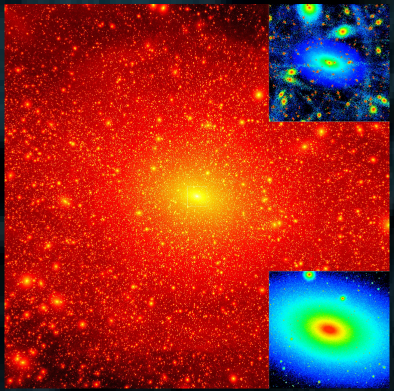

1st step: compute tides induced by final MW halo
=> parameter space becomes intricate!

=> generic enough to be calibrated from simulations
=> subhalo mass fraction ~10% in range (10^{-5},10^{-2}) M_h
(eg Diemand+08) fixes N_{tot}

$$\frac{d^n \bar{N}}{d\omega^n} = \frac{\bar{N}_{\rm tot}}{\bar{K}_w} \, \frac{d\bar{\mathcal{P}}_V(\vec{x})}{dV} \times \frac{d\bar{\mathcal{P}}_m(m,\vec{x})}{dm} \times \frac{d\bar{\mathcal{P}}_c(c,m,\vec{x})}{dc}$$

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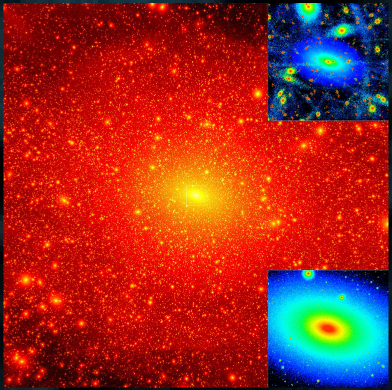

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2nd step: compute tides induced by MW baryons
=> parameter space even more intricate

=> CANNOT be calibrated from simulations

$$\boxed{\frac{d^n N}{d\omega^n} = \frac{N_{\rm tot}}{K_w} \, \frac{d\mathcal{P}_V(\vec{x})}{dV} \times \frac{d\mathcal{P}_m(m,\vec{x})}{dm} \times \frac{d\mathcal{P}_c(c,m,\vec{x})}{dc}}$$

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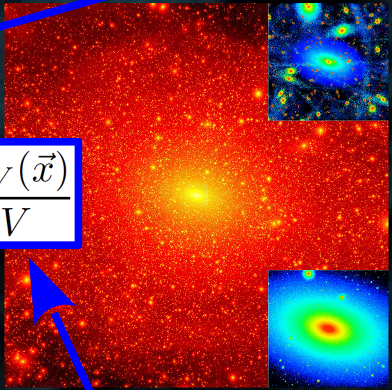
$$\frac{d^n N^0}{d\omega^n} = N_0 \frac{d\mathcal{P}^0_V(\vec{x})}{dV} \times \frac{d\mathcal{P}^0_m(m)}{dm} \times \frac{d\mathcal{P}^0_c(c,m)}{dc}$$

$$\frac{d\mathcal{P}^0_V(\vec{x})}{dV} = \frac{\rho_{\rm tot}(\vec{x})}{M_{\rm h}} = \frac{d\mathcal{P}_V(\vec{x})}{dV}$$

Hard sphere approx: subhalos track the evolving DM distribution, even after disruption.
=> redistribution of DM from subhalos to the smooth component.



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The dark halo: smooth vs subhalo component

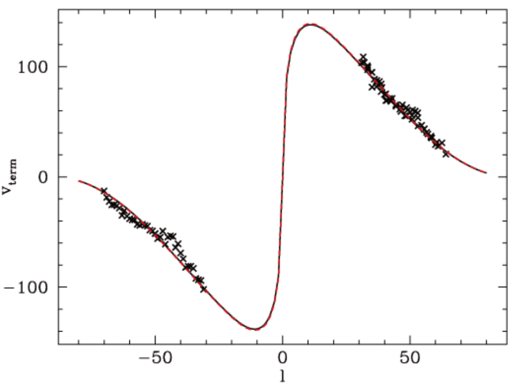
$$\rho_{\rm tot}(R) = \rho_{\rm sm}(R) + \rho_{\rm sub}(R)$$

Overall profile constrained by non-linear theory: NFW, Einasto +/- cores

++++

****** Strongly constrained by MW kinematic data ******

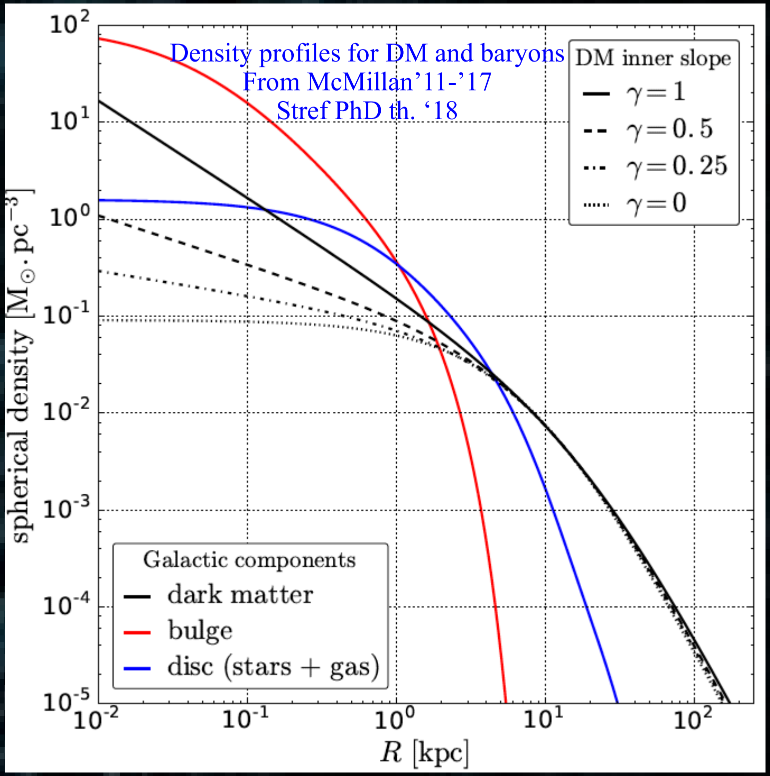
$$\rho_{\rm sub}(R) = \frac{N_{\rm sub}}{K_w}\,\frac{d\mathcal{P}_V(R)}{dV}\int_{m_{\rm min}}^{m_{\rm max}} dm \int_{c_{\rm min}(R)}^{c_{\rm max}} dc\, m_t(r_t(c,m,R),m,c)\,\frac{d\mathcal{P}_m}{dm}\,\frac{d\mathcal{P}_c}{dc}$$



McMillan'11

Series of kinematic constraints on baryons+DM mass models

++ will improve with Gaia ++


Density profiles for DM and baryons
From McMillan'11-'17
Stref PhD th. '18

Global tidal effects

Competition between global MW potential and internal subhalo potential → tidal radius

Solve EOM for vanishing test mass orbiting m and M (m<<M) in co-rotating frame of frequency ω (King '62, Spitzer '87).
=> Demand force to vanish (Lagrange points L2, L3)

$$\ddot{x} = \frac{G\,m}{x^2} - \frac{G\,M}{(R-x)^2} - \omega^2\left\{(\mu/m)R - x\right\} = 0$$

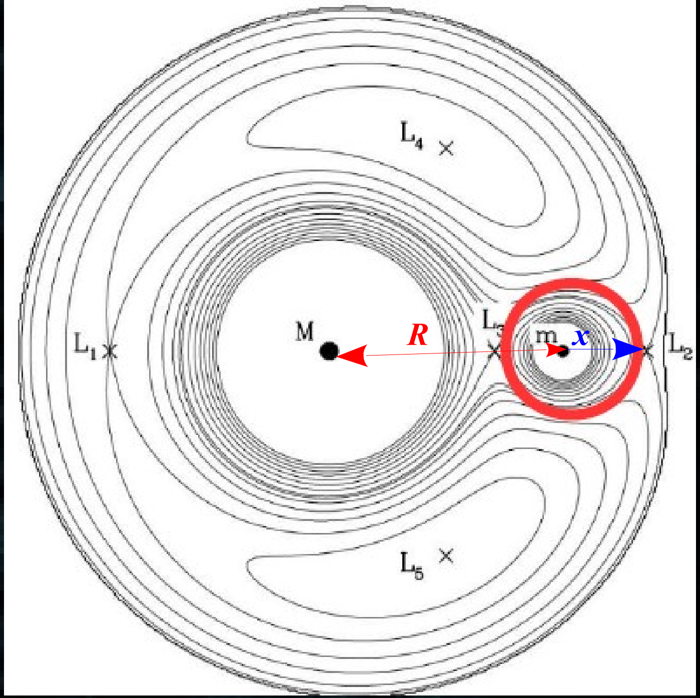
Point-like Jacobi tidal radius

$$r_{t\bullet} = r_{t\bullet}(R,m,M) = \left\{\frac{m_t}{3\,M}\right\}^{1/3} R$$

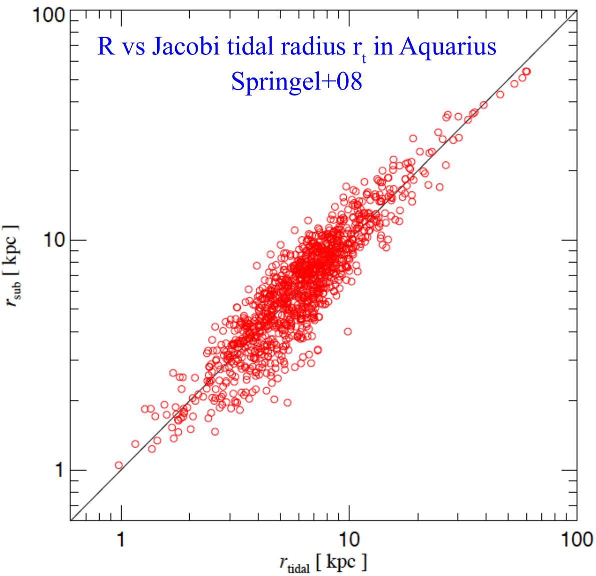
Extension to smooth systems

$$r_t = \left\{\frac{m(r_t)}{3\,M(R)\left(1-\frac{1}{3}\frac{d\ln M(R)}{d\ln R}\right)}\right\}^{1/3} R$$

Smooth Jacobi tidal radius



Binney&Tremaine '87, '08



Tides from stellar encounters and disk shocking

Encounters with stars:
(Ostriker+,Weinberg+, Gnedin+,80-00, Berezinsky+03)
*** impulse approximation during fly-by**
=> negligible wrt disk shocking

$$\Delta E = \frac{1}{2}\int \mathrm{d}^3\vec{r}\,\rho_{\rm int}(r)(\delta v_x - \delta\tilde{v}_x)^2$$

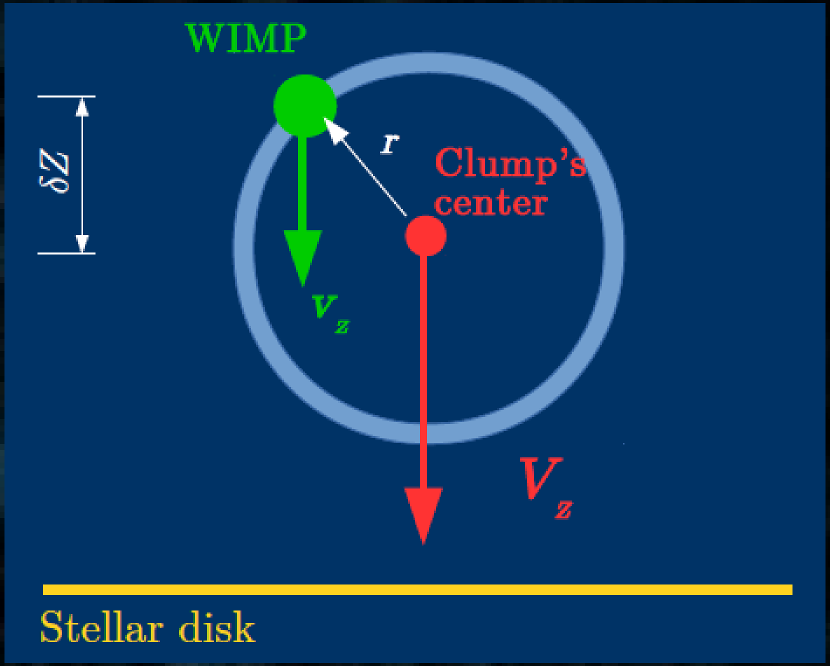
$$\Delta E = \frac{2\pi}{3}\left(\frac{2G_{\rm N}M_*}{v_{\rm rel}l^2}\right)^2\int_0^R \mathrm{d}r\, r^4\,\rho_{\rm int}(r)$$

Disk shocking:
*** impulse approximation during crossing**
*** adiabatic invariance correction**
=> the dominant effect

$$\begin{aligned}\frac{\mathrm{d}v_z}{\mathrm{d}t} &= g_{\rm d}(R,z_{\rm p}) - g_{\rm d}(R,z_{\rm c})\\ &\simeq \Delta z\,\frac{\partial g_{\rm d}}{\partial z}(z_{\rm c})\ ,\end{aligned}$$

$$\Delta v_z = \int \mathrm{d}t\,\Delta z(t)\,\frac{\partial g_{\rm d}}{\partial z}\left[z_{\rm c}(t)\right]$$

$$\boxed{\epsilon_k(z) \equiv \frac{2\,g^2_{z,\rm disk}(z=0)\,z^2}{V_z^2}\,A(\eta)}$$



Associated tidal radius

Differential definition (default)

$$\boxed{r_{t,i}\ \text{such that}\ \langle\epsilon_k\rangle(r_{t,i}) = -\tilde{\phi}(r_{t,i}, r_{t,i-1})}$$

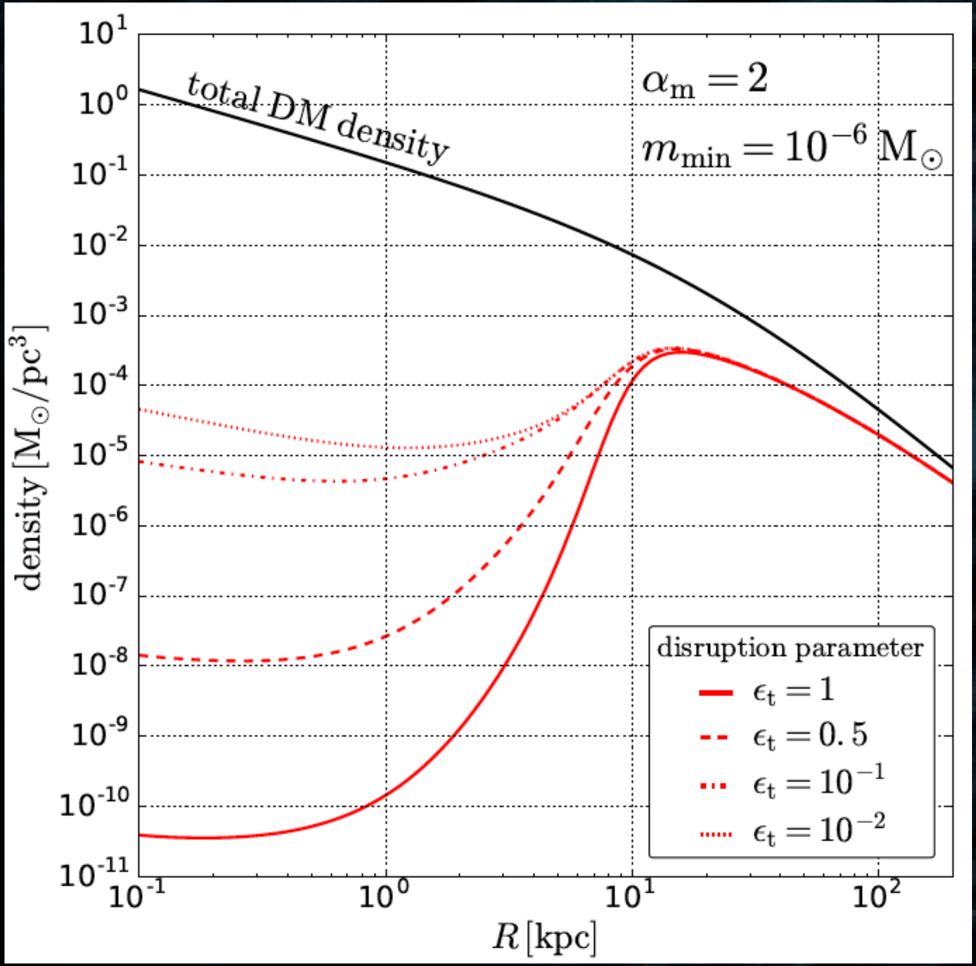
Integrated definition

$$r_t\ \text{such that}\ N_{\rm cross}\,E_k(r_t,R) = E_b(r_t)$$

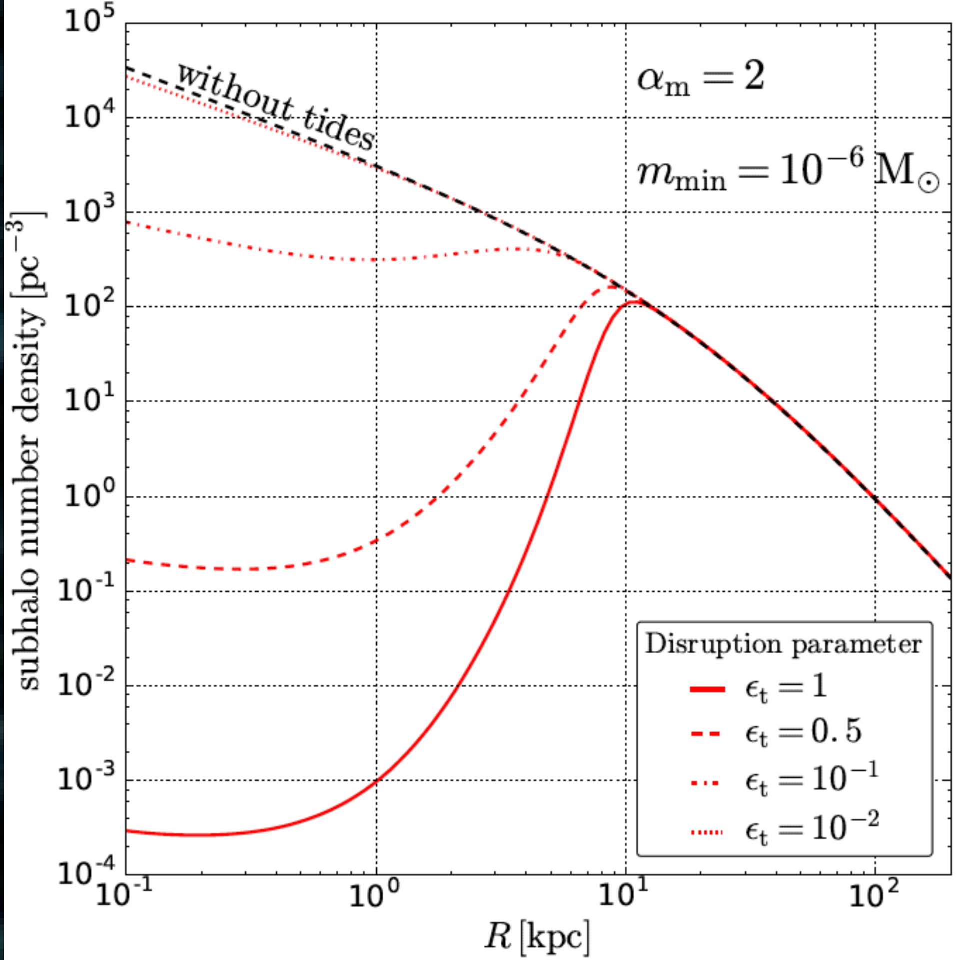
Fit from D'Onghia+10

$$\frac{\tilde{E}_k(r_t,R)}{E_b(r_t)} = \frac{(1.84\,r_{1/2})^2\,g^2_{z,\rm disk}}{3\,\tilde{\sigma}_v^2\,V_z^2}$$

Impact of tidal disruption on mass/number profiles



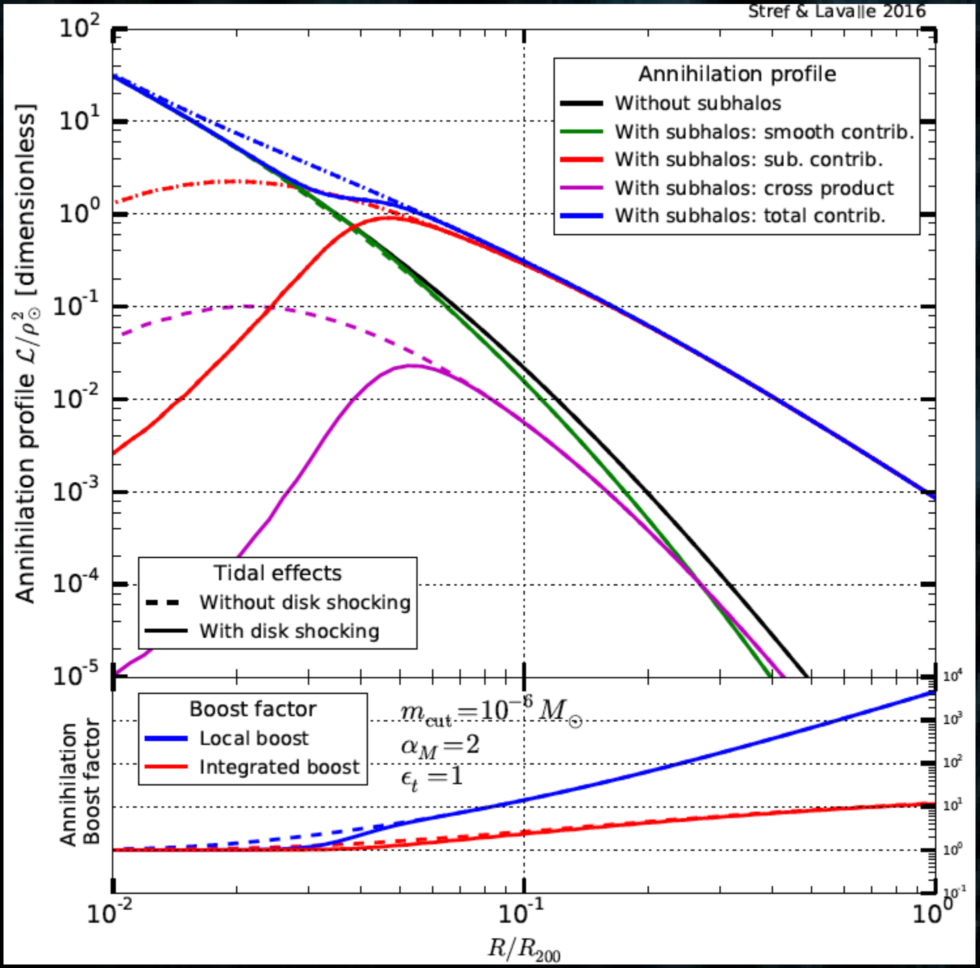
Global subhalo mass density profile, Stref PhD th. '18



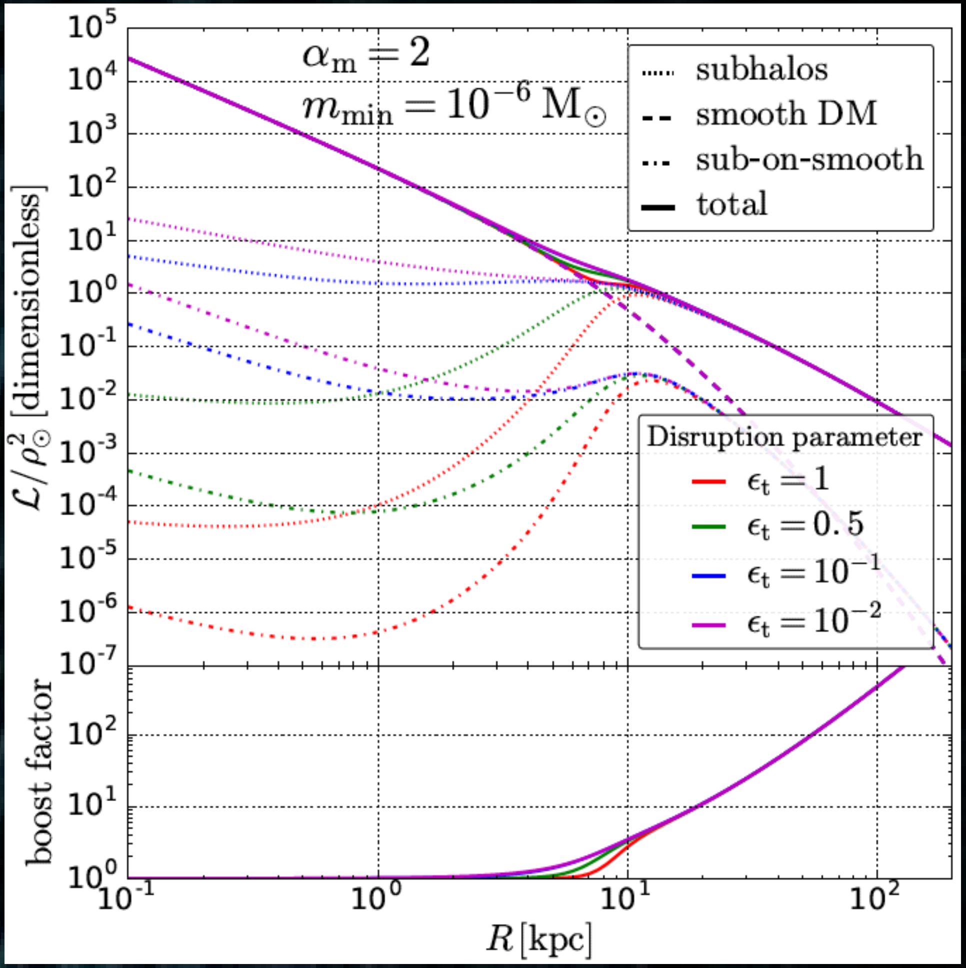
Subhalo number density profile, Stref PhD th. '18

Very large number density of tiny clumps expected locally!
(for >1 TeV WIMPs, ~0.5/star gravitationally captured!)

Amplification of annihilation rate in the Milky Way



Annihilation profile + local/integrated boost, Stref+17



Annihilation profiles and local boosts, varying ε_t, Stref PhD. th ‘18

Minimal mass has impact for α >1.9
(always in the central regions due to effective mass index => local fluctuations suppressed)
[see also Silk&Stebbins’93, Bergström+’98, JL+07, etc.]

Summary

* Milky Way a perfect place to probe DM properties on small scales!
⟶ complementarity of gravitational + non-gravitational effects/searches
⟶ very interesting test of DM scenarios (even feebly-interacting DM)!

* Theoretical + observational self-consistence of DM distribution very important: global + granularity + phase-space properties

* Generic semi-analytic method to build a Milky Way halo (Stref+'17-19), which includes information from:
- the primordial power spectrum (can be tuned to preferred inflation model)
- structure formation (Press-Schechter theory + concentration model)
- current observational constraints (to be updated with Gaia)

=> can be compared with cosmological simulations on relevant scales + probe arbitrarily small scales + self-consistently tuneable to the real Milky Way

* Room for improvement (ongoing)

* Can be applied to all CDM candidates: WIMPs, axions, PBHs, etc.

* Predictions for / constraints from WIMP searches being revised:
⟶ gamma rays + antimatter cosmic rays (Facchinetti, Lacroix, Stref+ in prep)
⟶ capture of mini-halos by stars! (new)

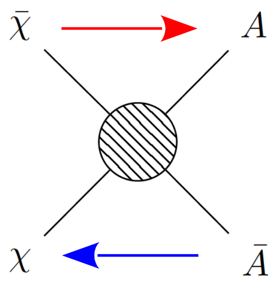
* Application to e.g. PBH microlensing (Clesse+ in prep)

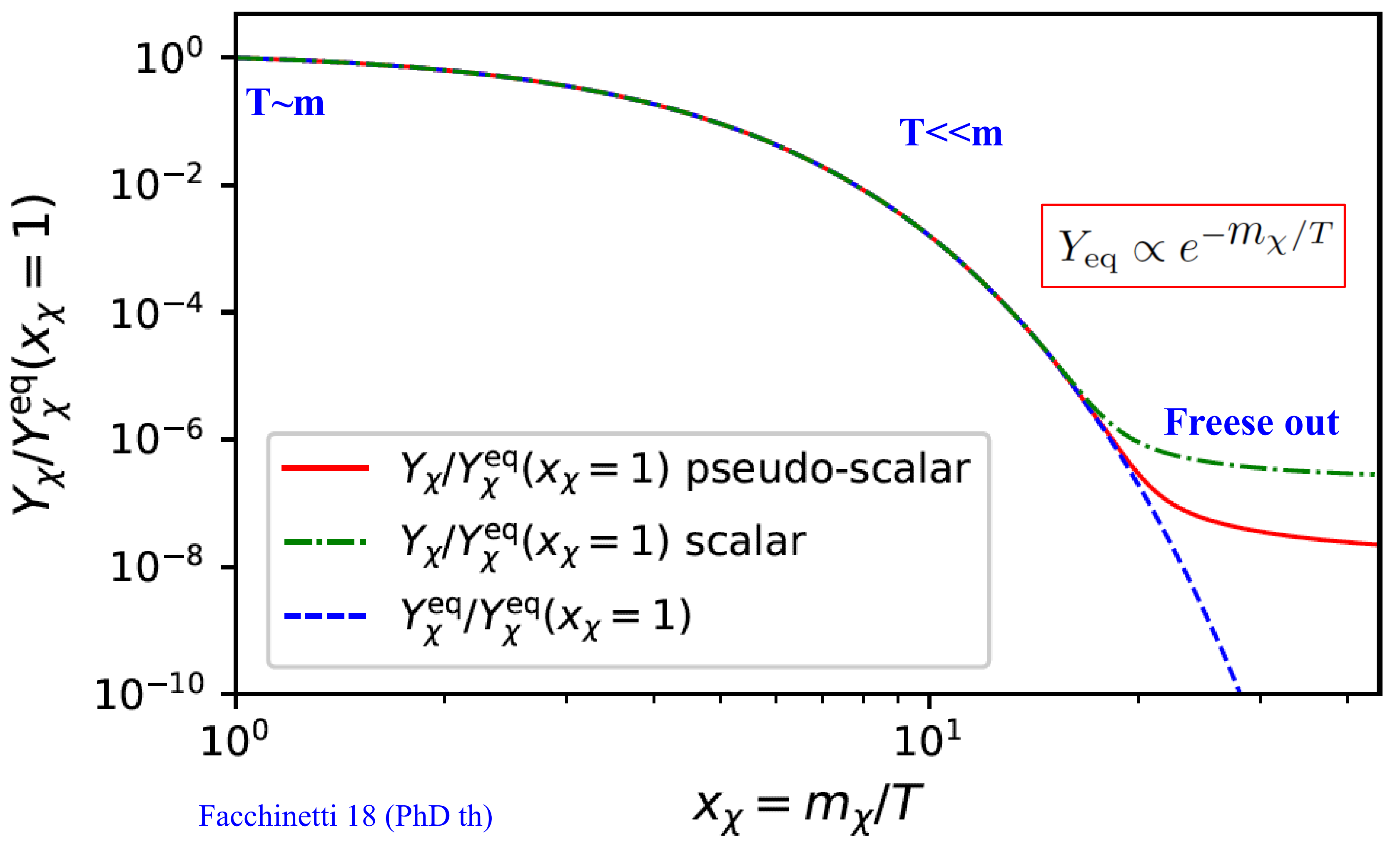
Backup

Thermal production in the early Universe

Master equation: Boltzmann equation (e.g. Lee & Weinberg '77, Bernstein+'85-88)

$$\frac{d\,f(x^\mu,p^\mu)}{d\lambda} = \widehat{C}[f] \longrightarrow \frac{dY_\chi}{dx} \propto \frac{g_\star^{1/2}(x)}{x^2}\,\langle\sigma v\rangle\,\{Y^2_{\chi,\mathrm{eq}} - Y^2_\chi\}$$





Facchinetti 18 (PhD th)

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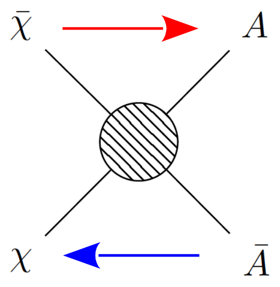
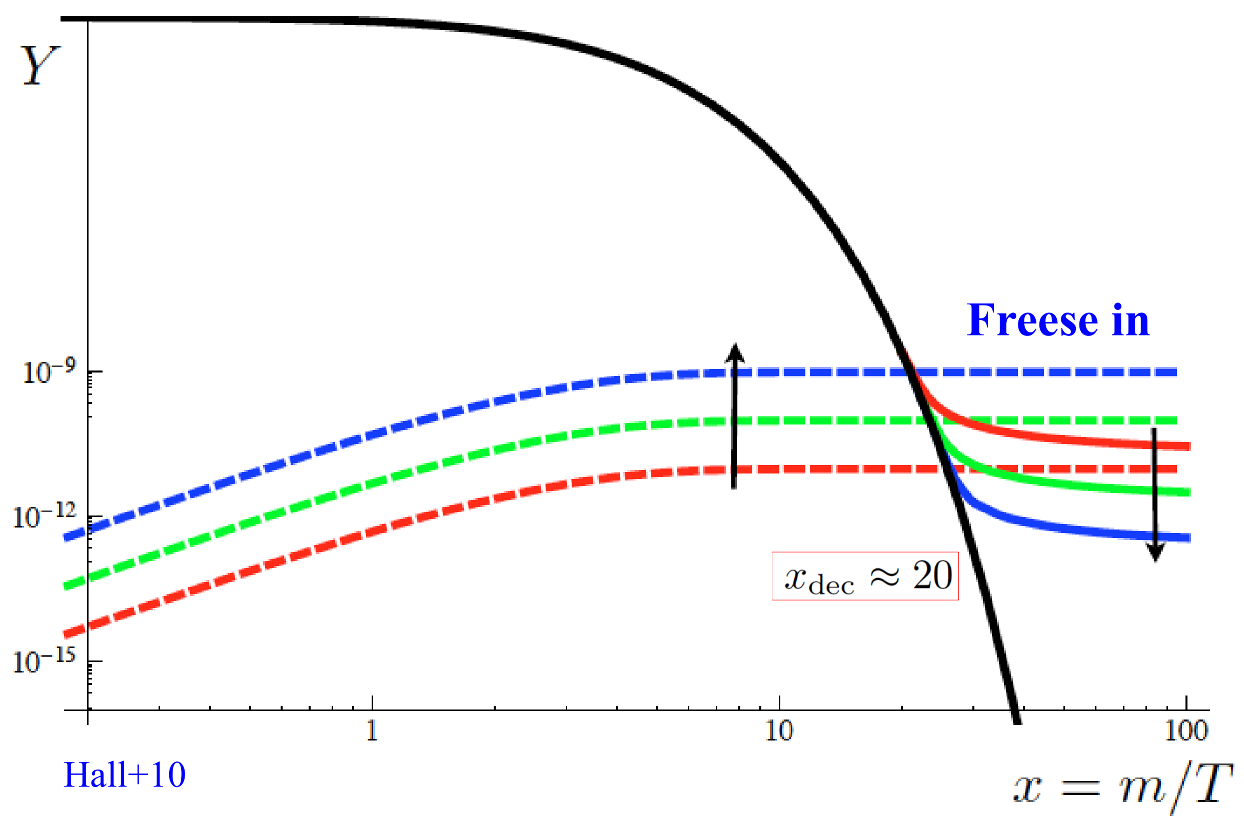




Freeze-in mechanism:
Dodelson & Widrow '94
McDonald '02
Hall+ 10

$$\Omega_{\mathrm{WIMP}} \overset{\sim}{\propto} \frac{1}{g_\star^{1/2}(x_{\mathrm{dec}})\,\langle\sigma v\rangle}$$

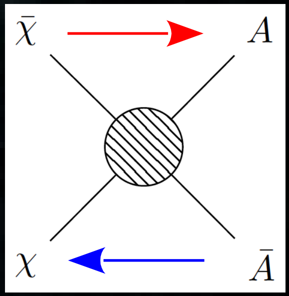
$$\Omega_{\mathrm{FIMP}} \overset{\sim}{\propto} g_\star^{1/2}(x_{\mathrm{dec}})\,\langle\sigma v\rangle$$

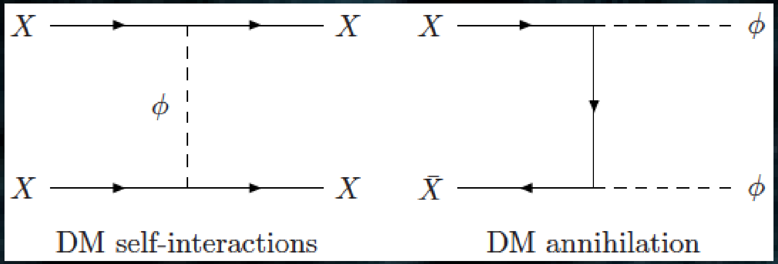
$$\langle\sigma_{\chi\bar{\chi}}v\rangle_{\mathrm{FIMP}} \ll \langle\sigma_{\chi\bar{\chi}}v\rangle_{\mathrm{WIMP}}$$

Thermal production in the early Universe

Master equation: Boltzmann equation (e.g. Lee & Weinberg '77, Bernstein+'85-88)

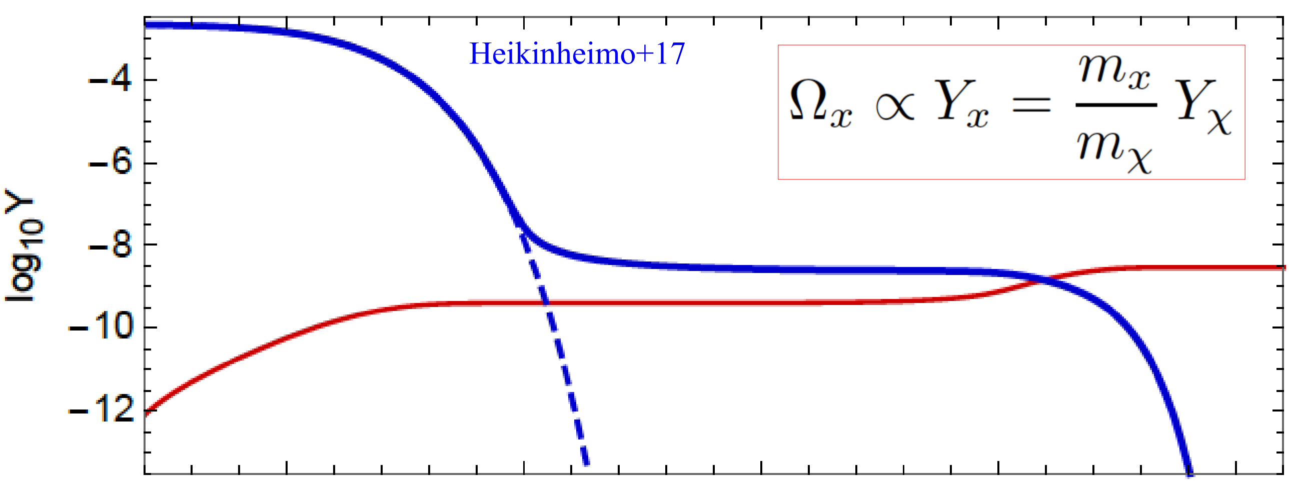
$$\frac{d\, f(x^\mu, p^\mu)}{d\lambda} = \widehat{C}[f] \quad \longrightarrow \quad \frac{dY_\chi}{dx} \propto \frac{g_\star^{1/2}(x)}{x^2} \langle \sigma v \rangle \, \{ Y^2_{\chi,\mathrm{eq}} - Y^2_\chi \}$$





DM self-interactions DM annihilation

Self-interacting Dark Matter (SIDM – core/cusp pb)
- * Light DM (eg Heikinheimo+16, Chu+16)
- * Strong dark sector (eg Kaplan+10, Hochberg+14, Kamada+16)
- * Light mediators (eg Feng+09, Buckley+09, Bringmann+15)



$$\Omega_x \propto Y_x = \frac{m_x}{m_\chi} Y_\chi$$

$$\Omega_{\mathrm{WIMP}} \tilde{\propto} \frac{1}{g_\star^{1/2}(x_{\mathrm{dec}}) \langle \sigma v \rangle}$$

$$\Omega_{\mathrm{FIMP}} \tilde{\propto} g_\star^{1/2}(x_{\mathrm{dec}}) \langle \sigma v \rangle$$

Tidal disruption criterion (criteria?)

Subhalo tidal mass

$$m_t = m(r_t) = 4\,\pi\,r_s^3 \int_0^{x_t} dx\,x^2\,\rho(x\,r_s)\,\zeta(x_t)$$

$dm = m_{200} - m_t$ given back to the smooth component

Disruption function

$$\zeta\left(x_t \equiv \frac{r_t}{r_s}\right) \equiv \theta\left(x_t - \varepsilon_t\right)$$

Disruption free parameter ε_t

$$\boxed{x_t = \frac{r_t}{r_s} \geq \varepsilon_t \Longleftrightarrow c_{200} \geq c_{\min}(R)}$$

Minimal concentration independent from mass!

Tidal disruption criterion (criteria?)

Subhalo tidal mass

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Minimal concentration independent from mass!

How much is ε_t???

Tidal disruption criterion (criteria?)

Subhalo tidal mass

$$m_t = m(r_t) = 4\pi r_s^3 \int_0^{x_t} dx\, x^2 \rho(x\, r_s)\, \zeta(x_t)$$

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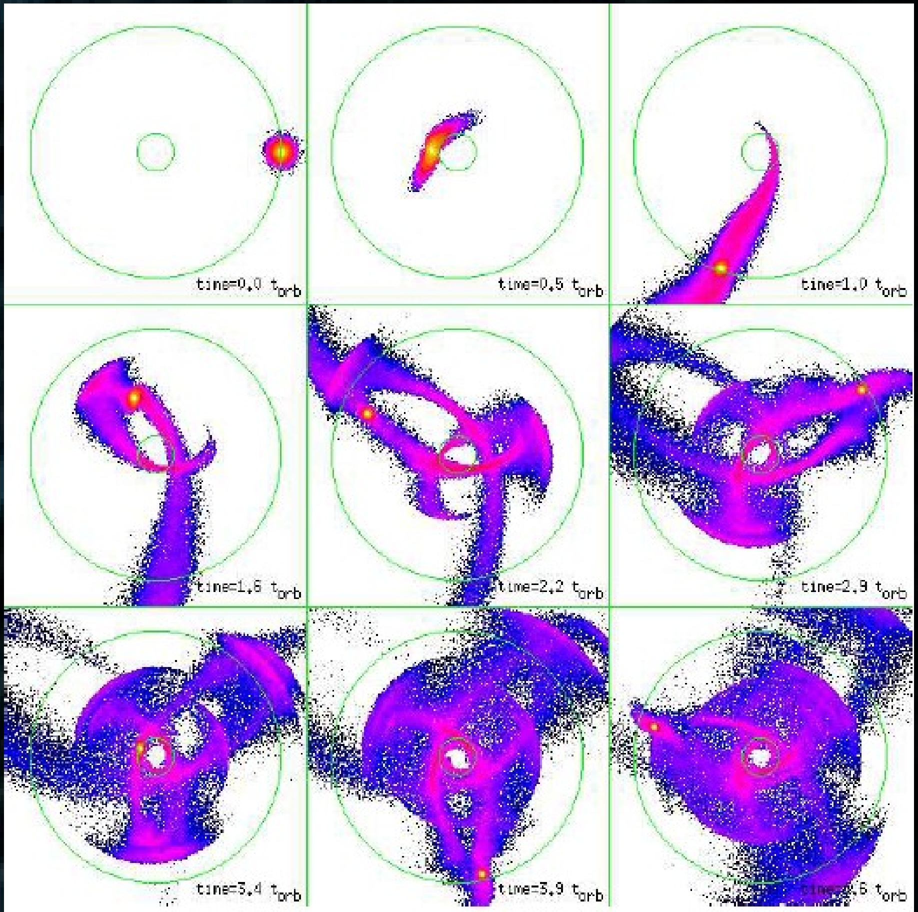

Hayashi+03

From past numerical studies

$\varepsilon_t \sim 1$

Tidal disruption criterion (criteria?)

Subhalo tidal mass

$$m_t = m(r_t) = 4\,\pi\,r_s^3 \int_0^{x_t} dx\,x^2\,\rho(x\,r_s)\,\zeta(x_t)$$

$dm = m_{200} - m_t$ given back to the smooth component

Disruption function

$$\zeta\left(x_t \equiv \frac{r_t}{r_s}\right) \equiv \theta\left(x_t - \varepsilon_t\right)$$

Disruption free parameter ε_t

$$\boxed{x_t = \frac{r_t}{r_s} \geq \varepsilon_t \Longleftrightarrow c_{200} \geq c_{\min}(R)}$$

Minimal concentration independent from mass!

But …

What about adiabatic invariants?
→ If mini-cores dense enough, fast orbits should be resilient down to x_t<<1 …

Recent work by van den Bosh+’17’18 suggests tidal disruption strongly overestimated in simulations. See also Errani+17.

NB: again a resolution issue → analytical arguments may catch on.

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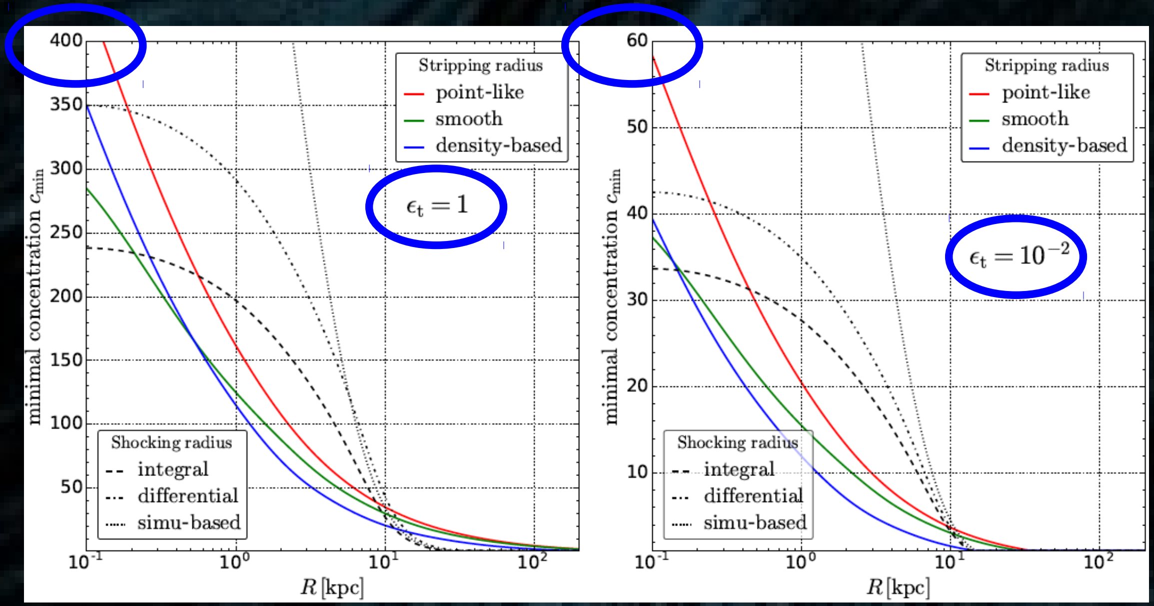
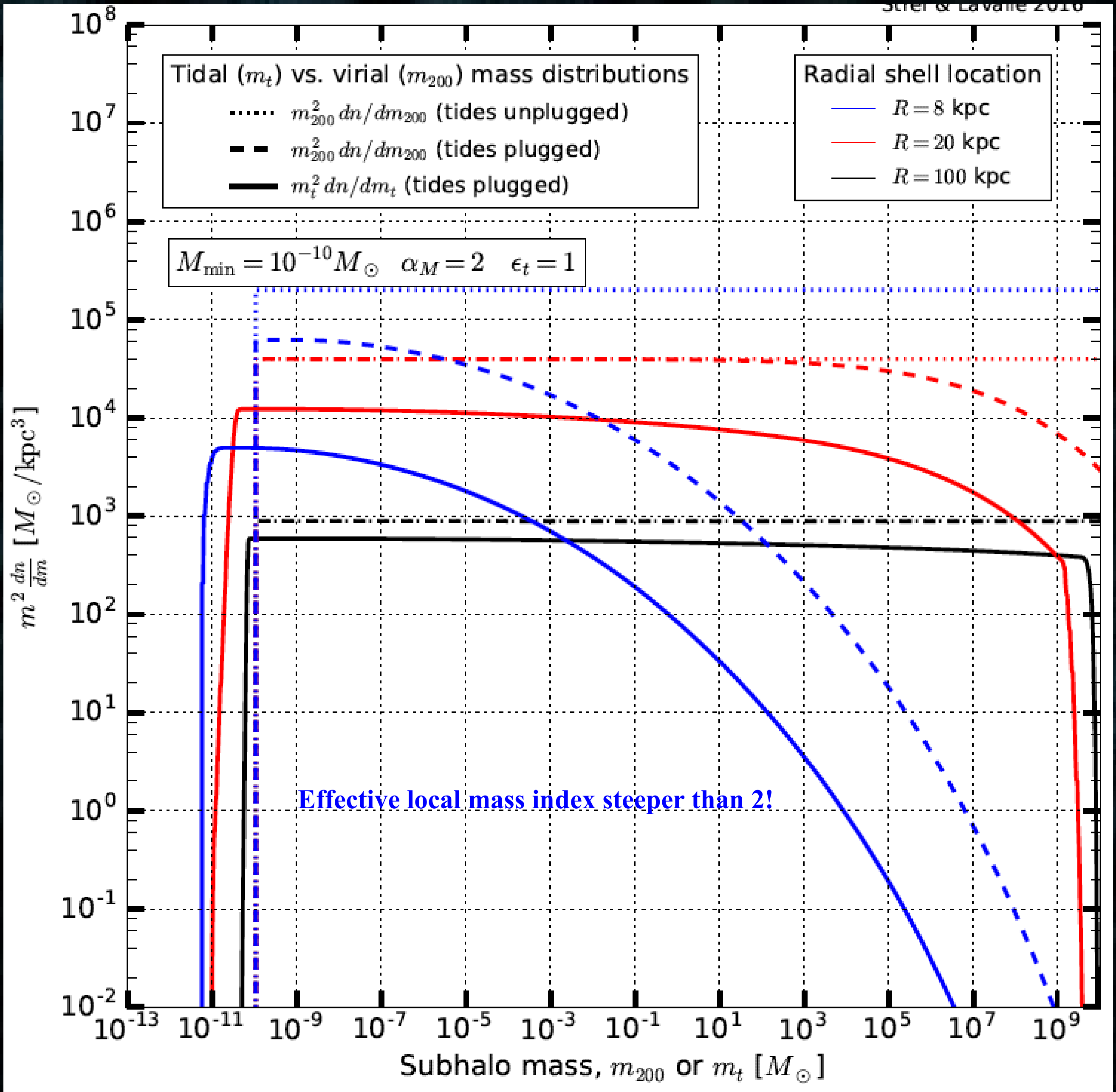


Minimal concentration vs position, Stref PhD th. '18 => mean concentration gets spatial-dependent
(see also Pieri+11, Moline+15)

Post-tides properties

Concentration function cut from the left => spatial-dependent mass index!



Effective local mass index steeper than 2!

Modified local mass function, Stref+17

Evolution of species in the Early Universe

$$\frac{d\,f(x^{\mu},p^{\mu})}{d\lambda}=\widehat{C}[f]$$

$$\frac{dY_{\chi}}{dx}\propto-\frac{g_{\star}^{1/2}(x)}{x^{2}}\,\langle\sigma v\rangle\,\{Y_{\chi}^{2}-Y_{\rm eq}^{2}\}$$

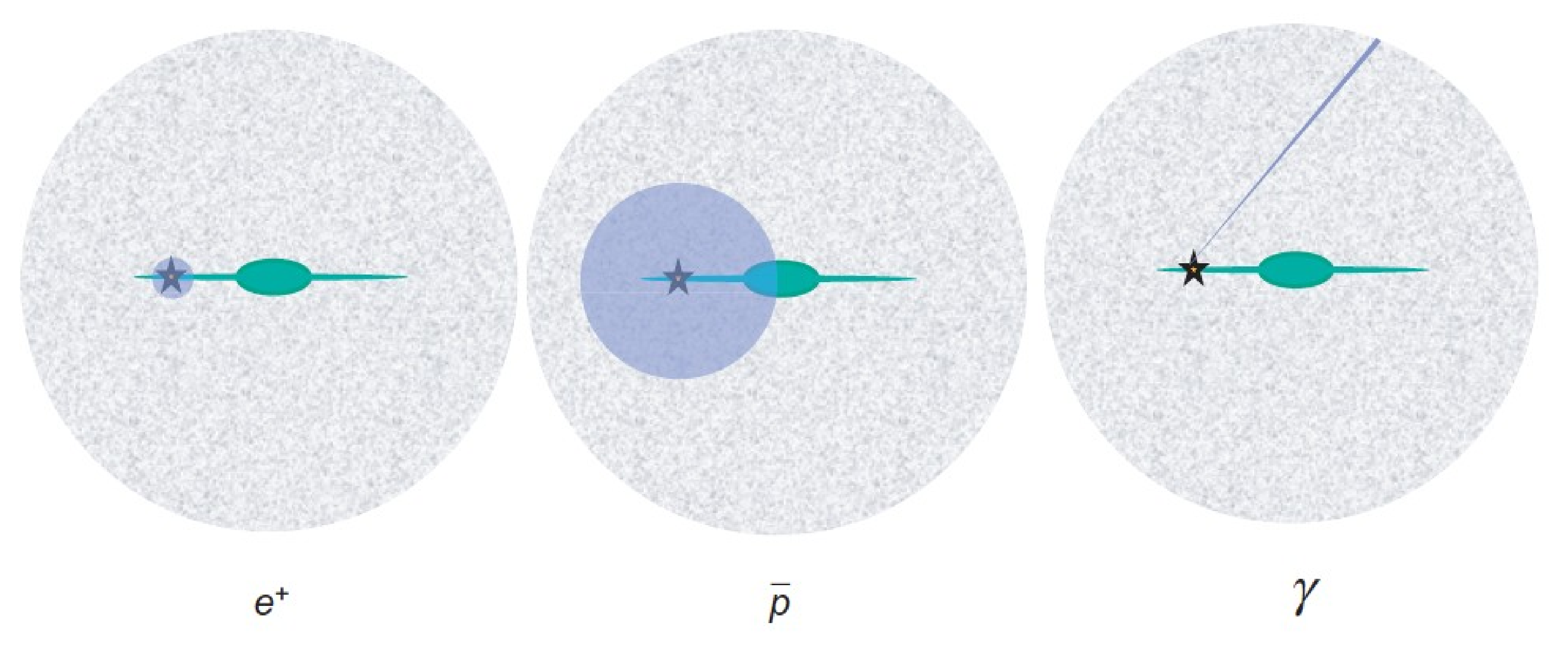
$$T_{\chi}\equiv\left\langle\frac{p^{2}}{3m_{\chi}}\right\rangle=\frac{g_{\chi}}{3m_{\chi}n_{\chi}}\int p^{2}f_{\chi}(p,t)\frac{\mathrm{d}^{3}\mathbf{p}}{(2\pi)^{3}}.$$

$$\frac{\partial T_{\chi}}{\partial t}+2HT_{\chi}=\gamma(T)(T-T_{\chi})$$

$$\gamma(T)=\frac{1}{48g_{\chi}m_{\chi}^{3}\pi^{3}}\sum_{\mathrm{species}\,i}\int_{m_i}^{\infty}\mathrm{d}\omega\,f_i^{\rm eq}(\omega,t)\frac{\partial}{\partial\omega}\left(\int_{-4p_{\rm cm}^{2}}^{0}(-t)\widetilde{|\mathcal{M}_i|^{2}}\mathrm{d}t\right)$$

$$\frac{\mathrm{d}\ln(y_{\chi})}{\mathrm{d}\ln(x_{\chi})}=-\left(1+\frac{\mathrm{d}\ln\left(h_{\rm eff}(T)\right)}{3\,\mathrm{d}\ln\left(T\right)}\right)\frac{\gamma(T)}{H}\left(1-\frac{y_{\chi}^{\rm eq}}{y_{\chi}}\right)$$

Boost factors in context

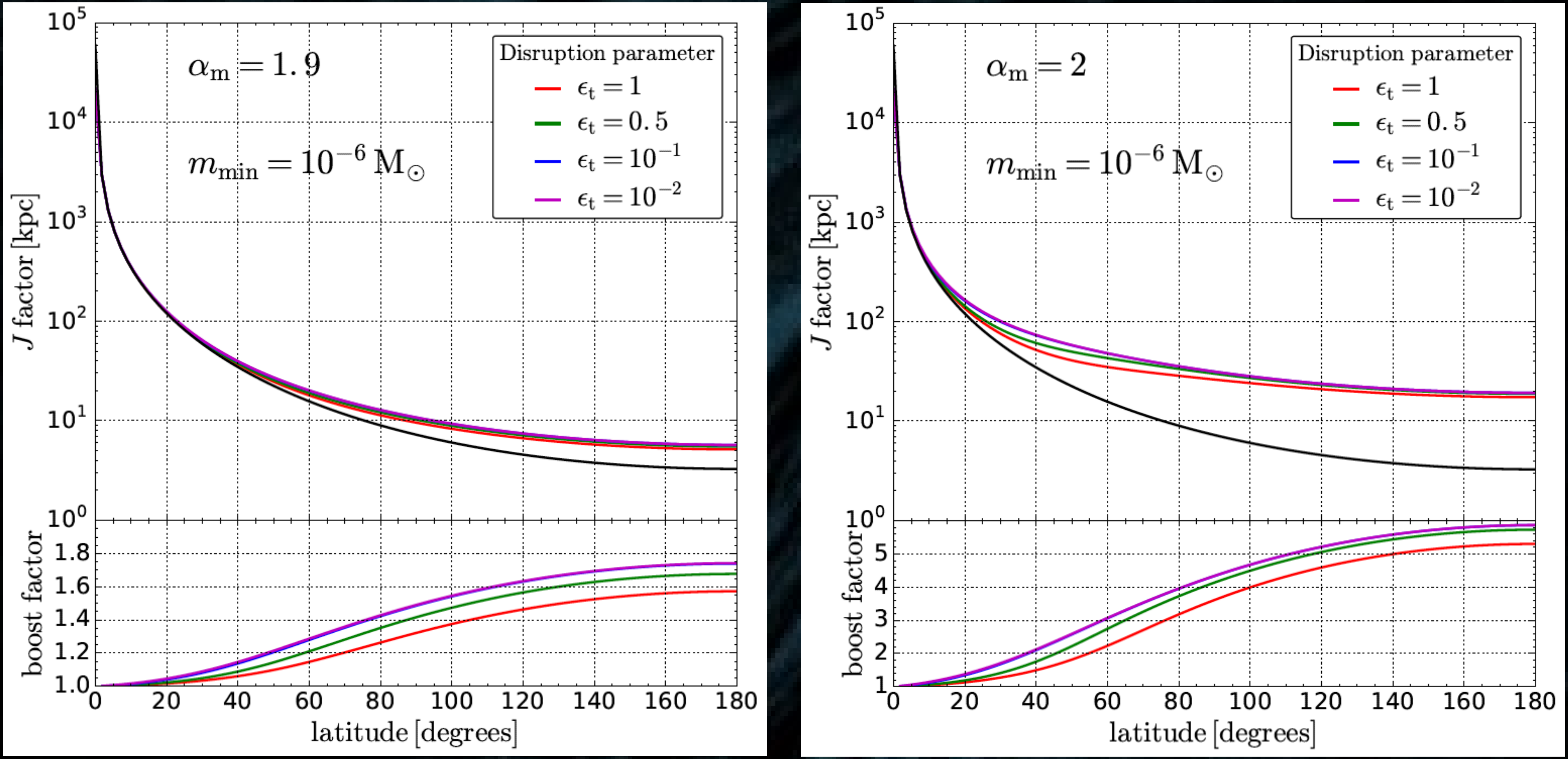


Bergström'09

Boost factor depends on integration volume!

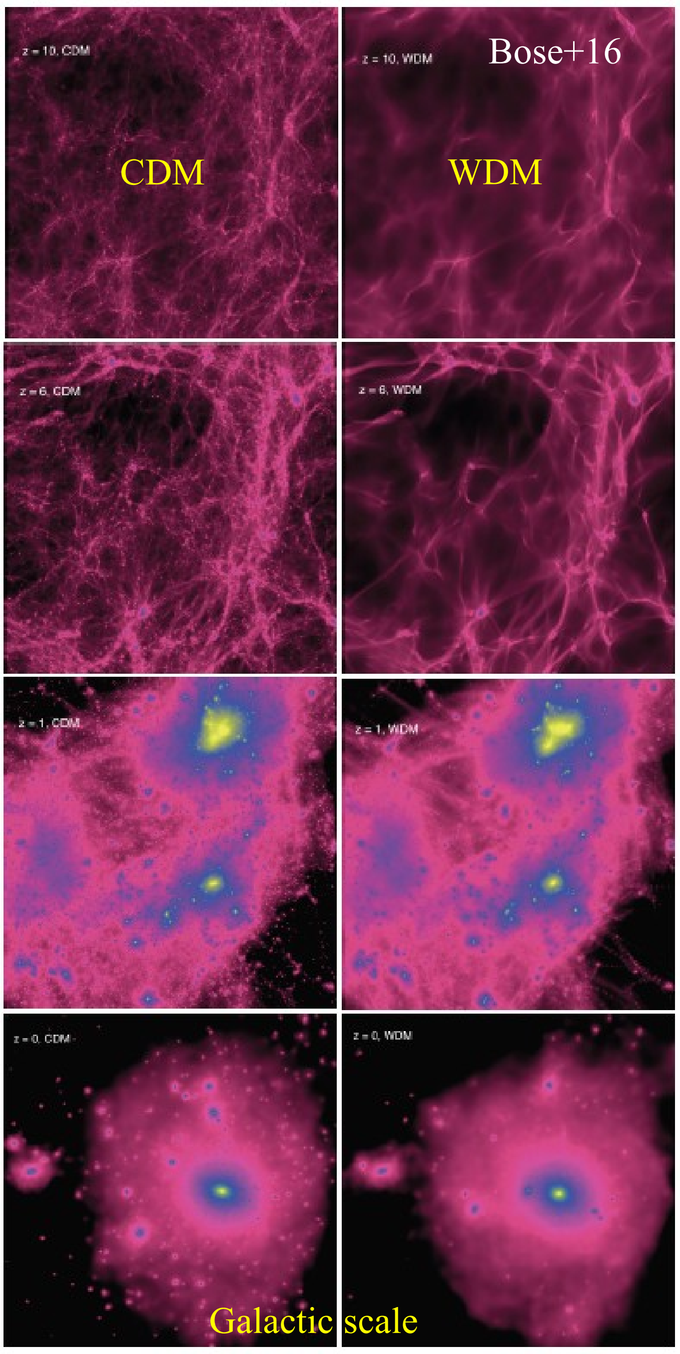
See also Silk & Stebbins'93, Begström+99,
Lavalle+07-08

J factors! (at last)



Stref PhD th ‘18

Kinetic decoupling, free streaming scale, and small-scale structures



$$\boxed{\lambda_{\rm fs} = a_{\rm eq} \int_{t_{\rm kd}}^{t_{\rm eq}} dt \frac{v(t)}{a(t)} \approx v_{\rm kd}(a_{\rm kd}/a_{\rm eq})/H_{\rm eq}}$$

* Density perturbations grow efficiently after matter-radiation equivalence

* Kinetic decoupling time sets free-streaming scale

* Other competing effects (collisional damping)

=> Minimal size of structures have impact on DM searches
=> Depends on DM interaction properties

[e.g. Hofmann+01, Berezinsky+03, Green+04-05, Bertschinger 06, Bringmann+07]

Kinetic decoupling, free streaming scale, and small-scale structures

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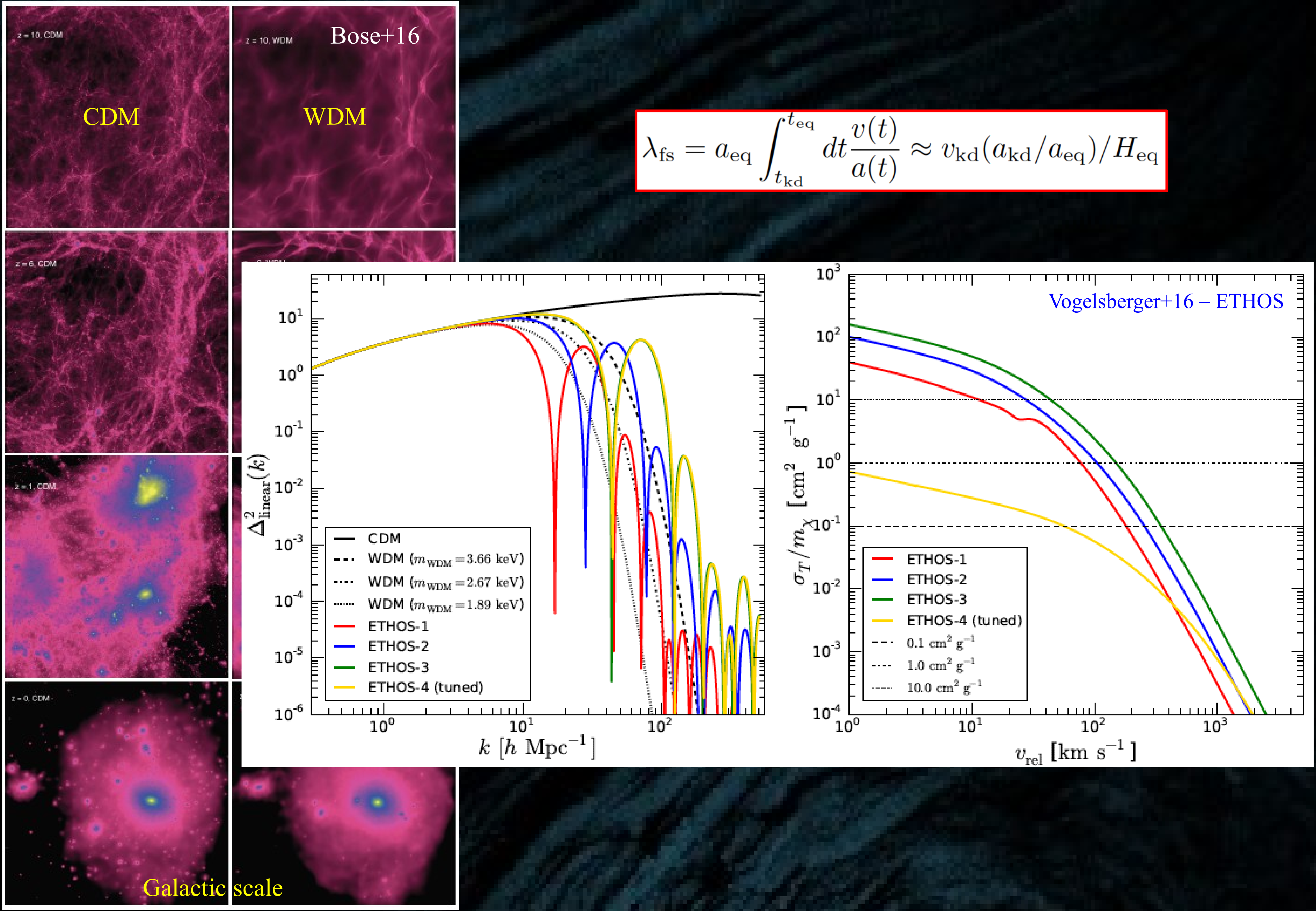
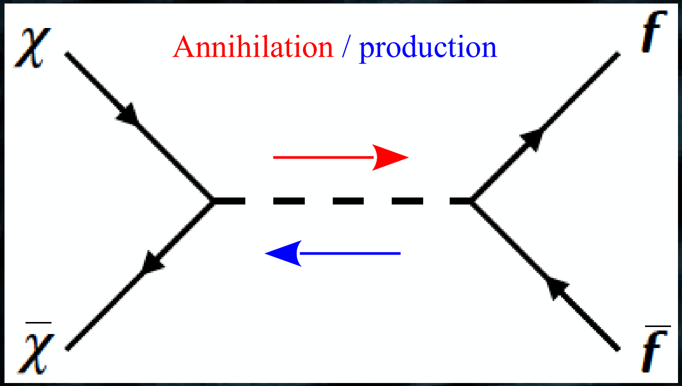


Searches for thermal dark matter

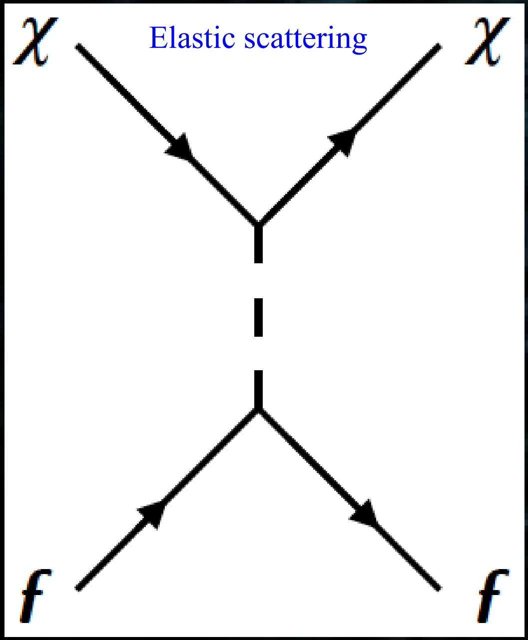


$$\Gamma_{\rm ann} = n_\chi \left\langle \sigma_{\rm ann} v \right\rangle$$

* Production at colliders (model dependent)
=> collider searches

* Annihilation/decay rate potentially large in dense DM regions: centers of halos + CMB
=> indirect searches

* Beware velocity dependence
(scalar exchange between fermions v-suppressed; pseudo-scalar exchange is not)



$$\Gamma_{\rm scatt.} = n_{\rm f} \left\langle \sigma_{\rm scatt} v \right\rangle$$

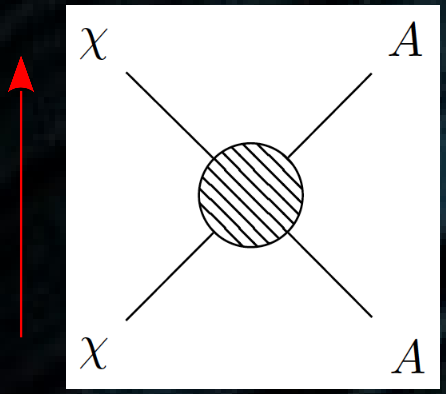
* elastic or inelastic scattering
→ nuclear recoils at underground experiments
=> direct searches

→ scattering with astrophysical objects
=> stellar physics
=> neutrinos from capture+annihilation in stars
=> indirect searches

* Beware velocity dependence
(pseudo-scalar exchange v-suppressed;
scalar exchange is not)

e.g. Goodmann & Witten '84, Drukier+ '85

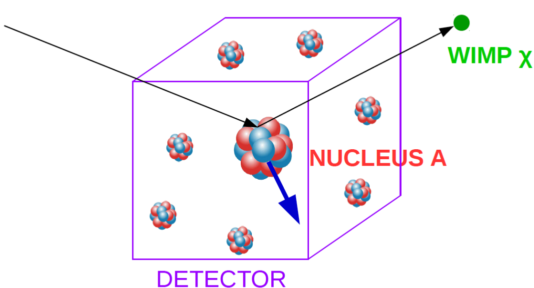
Only terms not velocity suppressed (v ~ 0.001 c in MW halo)



$$\mathcal{L}_{\text{int}} \supset \begin{cases} \displaystyle\sum_{\text{quarks}} \left\{ \underbrace{\alpha_{\text{v}}\, \bar\chi\, \gamma_\mu\, \chi \bar q\, \gamma^\mu\, q}_{\text{Dirac WIMPs}} + \alpha_{\text{scal.}}\, \bar\chi\, \chi \bar q\, q \right\} \\ \displaystyle\sum_{\text{quarks}} \{\alpha_{\text{a}}\, \bar\chi\, \gamma_\mu\, \gamma_5\, \chi\, \bar q\, \gamma^\mu\, \gamma_5\, q\} \end{cases}$$

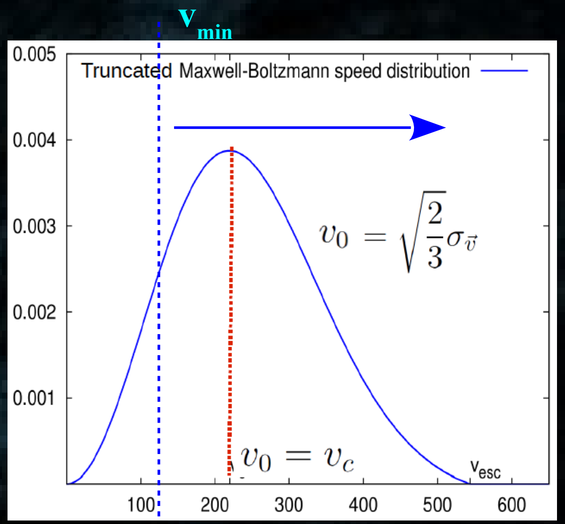
$$f_n \propto \langle n | \sum_{\text{quarks}} \alpha_q \frac{m_q}{m_n} \bar q q | n \rangle \longrightarrow$$

Quark mass content of nucleons:
Lattice QCD calculations
(ongoing)



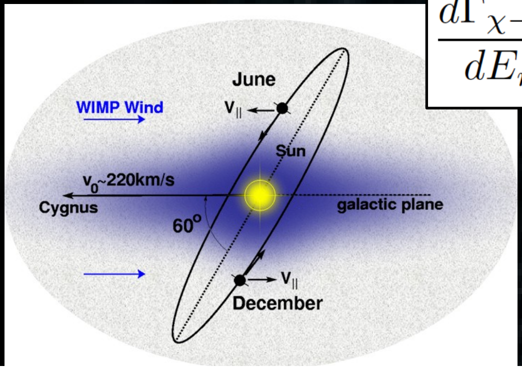
$$\sigma_{\chi-\text{N}} \propto \begin{cases} (Z\, f_p + (A-Z)\, f_n)^2 \approx A^2\, f_p^2 & \text{(spin-independent)} \\ (a_p \langle S_p \rangle + a_n \langle S_n \rangle)^2 & \text{(spin-dependent)} \end{cases}$$

$$\frac{d\Gamma_{\chi-\text{N}}}{dE_r}(E_r, t) = \frac{\sigma_{\chi-\text{N}}\, F^2(E_r)}{2\, \mu_r^2} \frac{\rho_\odot}{m_\chi} \int_{v > v_{\text{min}}} d^3\vec v \frac{f(\vec v, t)}{v}$$



Astro uncertainties:
*** local WIMP phase space**
*** local DM density**

$$v_{\text{min}}(E_r) = \sqrt{\frac{m_N\, E_r}{2\, \mu}}$$



$$\vec v_{\text{obs}} = \vec v_{\text{halo}} - \{\vec v_\oplus(t) \equiv \vec v_\odot + \vec v_{\oplus/\odot}(t)\}$$