Effective approach to lepton observables: the seesaw case

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What will this talk be about?

- The type-I seesaw mechanism (hereon referred to just as the 'seesaw') is a very well-known model, a leading theoretical explanation of neutrino masses
- It has already been studied in great detail
- Two main purposes for our work:
  A. Demonstrate the advantages of an effective field theory approach (EFT)
  B. Update/improve previous phenomenological analyses

The seesaw mechanism in one slide

- Introduce n_s right-handed gauge singlets:

$$\mathcal{L} = \overline{L}_{SM} + i\overline{N_R}\not{\partial}N_R - \left(\frac{1}{2}\overline{N_R}MN_R^c + \overline{N_R}Y\tilde{H}^\dagger l_L + h.c.\right) \quad (1)$$

- M is $n_s \times n_s$ symmetric mass matrix, can be taken real, positive and diagonal by appropriate rotations
- Y is $n_s \times 3$ sterile neutrino Yukawa couplings
- Neutrino masses given by

$$m_\nu = -\frac{v^2}{2}Y^T M^{-1} Y \quad (2)$$

Effective field theory

- EFT already discussed this morning, here we work in SMEFT
- Only dim-5 operator is Weinberg operator,

$$Q_{W,\alpha\beta} = (\overline{l^c_{L\alpha}}\tilde{H}^*)(\tilde{H}^\dagger l_{L\beta}) \tag{3}$$

- The Warsaw basis[1] is one (complete, non-redundant) SMEFT basis at dim-6, has 59 operators and 2499 parameters
- We consider operators with dim ≤ 6
- Match at tree-level (with one exception), run at 1-loop: aim to find all WCs at leading order

[1] JHEP **10** (2010) 085

Matching at sterile neutrino mass scale, M

- Integrating out sterile neutrinos at scale M at tree-level gives

$$\mathcal{L}_M^{tree} = \frac{(Y^T M^{-1} Y)_{\alpha\beta}}{2}(Q_{W,\alpha\beta} + h.c.) + \frac{S_{\alpha\beta}}{4}\left(Q^{(1)}_{Hl,\alpha\beta} - Q^{(3)}_{Hl,\alpha\beta}\right), \tag{4}$$

where the dim-6 operators are

$$Q^{(1)}_{Hl,\alpha\beta} \equiv (\overline{l_{L\alpha}}\gamma_\mu l_{L\beta})(H^\dagger i\overleftrightarrow{D^\mu} H); \quad Q^{(3)}_{Hl,\alpha\beta} \equiv (\overline{l_{L\alpha}}\gamma_\mu \sigma^A l_{L\beta})(H^\dagger i\overleftrightarrow{D^\mu}\sigma^A H).$$

and where we define the important quantity

$$S_{\alpha\beta} \equiv \left(Y^\dagger M^{-1*} M^{-1} Y\right)_{\alpha\beta}, \tag{5}$$

- S gives non-unitarity of PMNS,

$$U_{PMNS} = \left(\mathbb{1} - \frac{v^2}{4} S\right) U_\nu \tag{6}$$

for unitary U_ν, while active-sterile mixing is $\theta = \frac{v}{\sqrt{2}} Y M^{-1}$

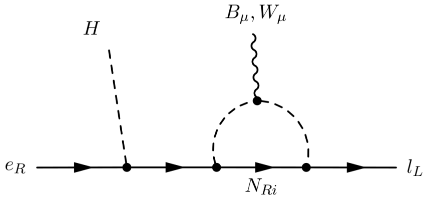
1-loop matching at scale M

- Tree-level matching, 1-loop running doesn't give dipole WCs
- Integrate out sterile neutrino at 1-loop with diagram below
- Contribution to Lagrangian at scale M is

$$\mathcal{L}_M^{loop} = \frac{(SY_e^\dagger)_{\alpha\beta}}{192\pi^2}\left(g_2 Q_{eW,\alpha\beta} - g_1 Q_{eB,\alpha\beta}\right) + h.c., \qquad (7)$$

where dipole operators are defined as

$$Q_{eW,\alpha\beta} = (\overline{l_{L\alpha}}\sigma_{\mu\nu}e_{R\beta})\sigma^A H W^{A\mu\nu}; \quad Q_{eB,\alpha\beta} = (\overline{l_{L\alpha}}\sigma_{\mu\nu}e_{R\beta})HB^{\mu\nu}.$$



Running and mixing below M

- We've done the matching at M, both at tree-level and 1-loop
- Three types of running/mixing: dim-5 into dim-5, dim-6 into dim-6, and dim-5 squared into dim-6
- WCs generated by mixing are depend on same parameters as those generated at tree-level: dim-6 into dim-6 gives WCs proportional to

$$R_{\alpha\beta} \equiv \sum_i S^i_{\alpha\beta} \log \frac{M_i}{m_W} = \sum_i Y^*_{i\alpha} Y_{i\beta} M_i^{-2} \log \frac{M_i}{m_W} \qquad (8)$$

- All seesaw observables at our level of precision depend on either $(Y^T M^{-1} Y)_{\alpha\beta}$, $S_{\alpha\beta} \equiv (Y^\dagger M^{-1*} M^{-1} Y)_{\alpha\beta}$, or their log-enhanced versions

Consequences of tiny neutrino masses

- Smallness of neutrino masses severely bounds $Y^T M^{-1} Y$
- Suppose $M \sim$ TeV, then $Y^T M^{-1} Y$ can be suppressed due to symmetry (e.g. $U(1)_L$ in inverse seesaw) or fine-tuning
- In the limit $Y^T M^{-1} Y \to 0$, for $n_s = 2, 3$ we find

$$S_{\alpha\beta}, R_{\alpha\beta} \propto \lambda_\alpha \lambda_\beta, \tag{9}$$

  where $\lambda_e, \lambda_\mu, \lambda_\tau \in \mathbb{R}^+$
- Hence S, R have only three independent entries
- Lepton flavour-conserving entries determine flavour-violating ones: $S_{\alpha\beta} = \sqrt{S_{\alpha\alpha} S_{\beta\beta}}$ (same for R)

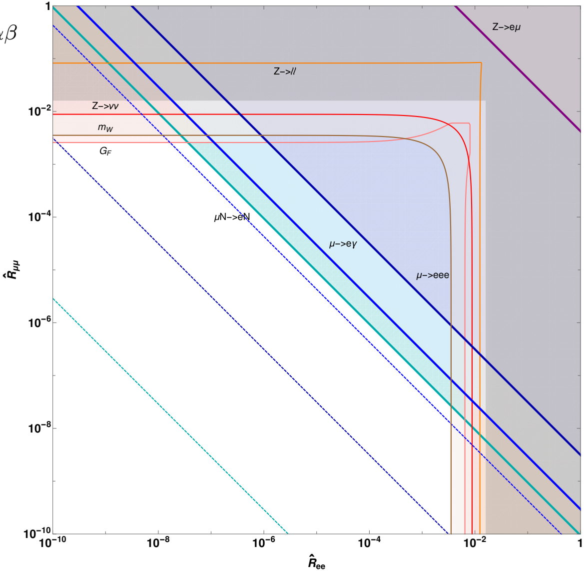
Consequences of tiny neutrino masses

- For $n_s > 3$, the picture becomes different:
  - In general, $S_{\alpha\beta}$ is no longer factorised
  - S obeys a Cauchy-Schwartz inequality, $|S_{\alpha\beta}| \leq \sqrt{S_{\alpha\alpha} S_{\beta\beta}}$
  - $S_{\alpha\alpha} > 0$ but $S_{\alpha\beta} \in \mathbb{C}$
- For $n_s = 4$, two of the three off-diagonal elements can vanish without diagonals vanishing

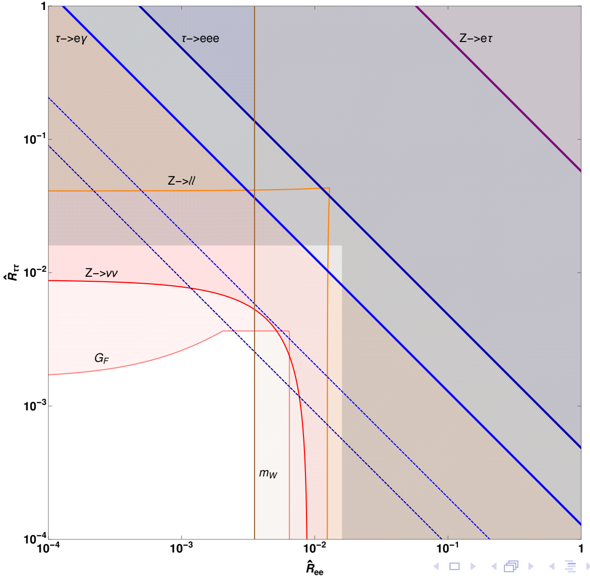
- In general, cancellation in $S_{\alpha\beta} \not\Leftrightarrow$ cancellation in $R_{\alpha\beta}$, so flavour-violation may be suppressed in observables depending on one but not the other

Summary plot: $\hat{R}_{\mu\mu}$ vs. $\hat{R}_{ee}$

$$\hat{R}_{\alpha\beta} = m_W^2 R_{\alpha\beta}$$



Summary plot: $\hat{R}_{\tau\tau}$ vs. $\hat{R}_{ee}$



Universality of charged lepton decays

- Shift in 4-fermion Lagrangian which describes $\ell_\alpha \to \ell_\gamma \overline{\nu_\beta} \nu_\delta$

$$\mathcal{L} \supset -\frac{4 G_F^{SM}}{\sqrt{2}} \left(\overline{\nu_\alpha} \gamma_\rho P_L \ell_\alpha\right) \left(\overline{\ell_\beta} \gamma^\rho P_L \nu_\beta\right) + \frac{C^{V,LL}_{\nu e,\alpha\beta\gamma\delta}}{\Lambda^2} \left(\overline{\nu_\alpha} \gamma_\rho P_L \ell_\delta\right) \left(\overline{\ell_\gamma} \gamma^\rho P_L \nu_\beta\right) , \quad (10)$$

with tree-level seesaw contribution,

$$\frac{C^{V,LL}_{\nu e,\alpha\beta\gamma\delta}}{\Lambda^2} \simeq \frac{1}{2} \left(S_{\alpha\delta} \delta_{\gamma\beta} + \delta_{\alpha\delta} S_{\gamma\beta}\right) . \quad (11)$$

- Hence measured quantity is not the SM value, but

$$G_F^2 \simeq \left| G_F^{SM} - \frac{1}{4\sqrt{2}} \left(S_{ee} + S_{\mu\mu}\right) \right|^2 + \frac{1}{32} \left(2|S_{e\mu}|^2 + |S_{e\tau}|^2 + |S_{\mu\tau}|^2\right) , \quad (12)$$

and similarly for $G_F^{e\tau}$ and $G_F^{\mu\tau}$.

Universality of charged lepton decays

- Bounds on universality of charged lepton decays gives

$$\frac{G_F^{\mu\tau}}{G_F^{e\tau}} - 1 \simeq \frac{S_{ee} - S_{\mu\mu}}{4\sqrt{2}G_F} = 0.0018 \pm 0.0014 \qquad (13)$$

$$\frac{G_F^{e\tau}}{G_F} - 1 \simeq \frac{S_{\mu\mu} - S_{\tau\tau}}{4\sqrt{2}G_F} = 0.0011 \pm 0.0015 \qquad (14)$$

$$\frac{G_F^{\mu\tau}}{G_F} - 1 \simeq \frac{S_{ee} - S_{\tau\tau}}{4\sqrt{2}G_F} = 0.0030 \pm 0.0015 \qquad (15)$$

- Secondly, prediction of m_W changes since it can be written in terms of G_F, α, m_Z:

$$m_W \simeq m_W^{SM}\left[1 + \frac{s_w^4}{8\pi\alpha(1-2s_w^2)}(\hat{S}_{ee} + \hat{S}_{\mu\mu})\right], \qquad (16)$$

with $\sin^2 2\theta_w \equiv (2\sqrt{2}\pi\alpha)/(G_F m_Z^2)$

- This gives the strong constraint, $\hat{S}_{ee} + \hat{S}_{\mu\mu} \lesssim 1.4 \times 10^{-3}$

Electric dipole moments

- The electric dipole moment (EDM) of charged leptons, d_α, is related to the electromagnetic dipole WC by

$$d_\alpha \equiv \frac{\sqrt{2}v}{\Lambda^2} \text{Im}\left(C_{e\gamma,\alpha\alpha}\right) \ . \tag{17}$$

- In the seesaw, the 1-loop contribution to $C_{e\gamma,\alpha\alpha}$ is real, therefore EDM vanishes at one loop
- Using flavour transformation properties[2], found parametric form so could estimate

$$\begin{aligned} |d_e| &\sim \frac{2e}{(16\pi^2)^2}\left(\frac{v}{\sqrt{2}}\right)^4 \text{Im}\left(\left[SY_e^\dagger Y_e S, S\right]_{ee}\right) m_e \\ &= \frac{2em_e v^2}{(16\pi^2)^2}(m_\tau^2 - m_\mu^2)\text{Im}\left(S_{e\tau}S_{\tau\mu}S_{\mu e}\right) \end{aligned} \tag{18}$$

- Leads to the mild constraint $|\text{Im}(\hat{S}_{e\mu}\hat{S}_{e\tau}\hat{S}_{\mu\tau})| \lesssim 0.02$

[2] Nucl. Phys. **B924** (2017) 417-452

Conclusions

- Full EFT analysis of the seesaw, including 1-loop effects
- Studied a wide range of pheno, all depending on S or R
- Showed how tiny neutrino masses affects the forms of S and R
- Found that $\mu \to e$ transitions impose strongest limit on seesaw, but flavour-conserving observables are more constraining than bounds on $\tau \to e, \mu$ transitions
- EFT approach validated by current and future constraints

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Back-up slides

Back-up slides

Running from M down to m_W

- RGEs for running/mixing of dim-5 and dim-6 WCs

$$\frac{dC^W}{d\log\mu} = \gamma_W C^W; \quad \frac{dC^i}{d\log\mu} = \gamma^i_j C^j + \gamma^i_W C^{W\dagger} C^W, \tag{19}$$

- A lot of hard work already done for us: γ_W [3], γ^i_W [4] and γ^i_j [5] all previously calculated (we find discrepancy with one term in γ^i_W)
- Recall $R_{\alpha\beta} \equiv \sum_i S^i_{\alpha\beta} \log\frac{M_i}{m_W} = \sum_i Y^*_{i\alpha} Y_{i\beta} M_i^{-2} \log\frac{M_i}{m_W}$
- Long list of WCs generated by mixing, e.g.

$$\frac{C^{He}_{\alpha\beta}(m_W)}{\Lambda^2} \simeq \frac{1}{16\pi^2}\left[\frac{1}{2} y_\alpha R_{\alpha\beta} y_\beta - \frac{g_1^2}{3}\mathrm{tr}(R)\delta_{\alpha\beta}\right] \tag{20}$$

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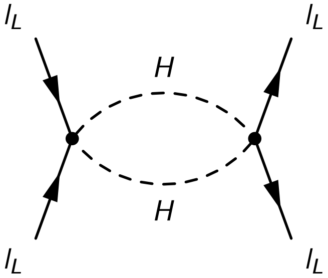
[3] Phys. Lett. **B519** (2001) 238242

[4] Nucl. Phys. **B705** (2005) 269-295; Phys. Rev. **D98** (2018) 095014

[5] JHEP **10** (2013) 087; JHEP **01** (2014) 035; JHEP **04** (2014) 159

Mixing C_W^2 into $C^{d=6}$

- As well as $C_i^{d=6} \to C_j^{d=6}$ mixing, have $C_W^2 \to C^{d=6}$, e.g.



- This contributes to the RGE of C^{ll} as

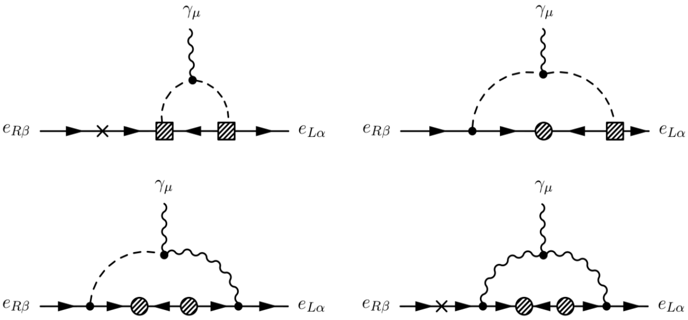
$$16\pi^2 \frac{dC^{ll}_{\alpha\beta\gamma\delta}}{d\log\mu} = -2\frac{C^\dagger_{W,\alpha\gamma}}{\Lambda}\frac{C_{W,\beta\delta}}{\Lambda} \qquad (21)$$

Matching dipoles at m_W

- 1-loop matching at m_W necessary for dipoles, gives contribution

$$\frac{C_{e\gamma,\alpha\beta}}{\Lambda^2}\frac{v}{\sqrt{2}} = -\frac{ev^2(C^{W\dagger}C^W Y_e^\dagger)_{\alpha\beta}}{64\pi^2 m_W^2\Lambda^2}\frac{v}{\sqrt{2}} = -\frac{e^3 U_{\alpha i}U^*_{\beta i}m_i^2 m_\beta}{256\pi^2 s_w^2 m_W^4} \quad (22)$$

for $\mathcal{O}_{e\gamma,\alpha\beta} = (\overline{\ell_\alpha}\sigma_{\mu\nu}P_R\ell_\beta F^{\mu\nu})v/\sqrt{2}$



EW-scale decays

- Most constraining decay channel is $Z \to \nu\nu$ since tree-level
- EFT computation gives seesaw prediction for effective number of light neutrinos,

$$N_\nu \simeq \sum_{\alpha=e,\mu,\tau} \left(1 - \frac{v^2}{2} S_{\alpha\alpha}\right)^2 + \frac{v^4}{2} \left(|S_{e\mu}|^2 + |S_{e\tau}|^2 + |S_{\mu\tau}|^2\right), \tag{23}$$

compared with LEP measurement $N_\nu = 2.9840 \pm 0.0082$

- Define dimensionless $\hat{S} = m_W^2 S$, then above equation at $\mathcal{O}(S)$ gives bound

$$\mathrm{tr}[\hat{S}] \lesssim 3.5 \times 10^{-3} \tag{24}$$

Flavour-violating charged lepton decays

- Many types of processes, including $\ell_\alpha \to \ell_\beta \gamma$, $\ell_\alpha \to \ell_\beta \ell_\beta \ell_\beta$, $\mu \to e$ conversion
- For $\ell_\alpha \to \ell_\beta \gamma$, the rate is (in agreement with old calculations[6])

$$BR(\ell_\alpha \to \ell_\beta \gamma) \simeq \frac{m_\alpha^3 v^2}{8\pi \Lambda^4 \Gamma_\alpha} \left(|C_{e\gamma,\alpha\beta}|^2 + |C_{e\gamma,\beta\alpha}|^2 \right) \simeq \frac{\alpha_{em} m_\alpha^5}{36(16\pi^2)^2 \Gamma_\alpha} |S_{\alpha\beta}|^2 \tag{25}$$

- This sets current (future) bounds of $|\hat{S}_{e\mu}| < 6.8(2.6) \times 10^{-6}$, $|\hat{S}_{e\tau}| < 4.5(1.8) \times 10^{-3}$, $|\hat{S}_{\mu\tau}| < 5.2(1.4) \times 10^{-3}$

[6] Nucl. Phys. **B125** (1977) 285; Phys. Rev. Lett. **45** (1980) 1908

The charged lepton electric dipole moment

- EDM proportional to imaginary part of $C_{e\gamma,\alpha\alpha}$
- What is shortest chain of Yukawas that a) combines with $(\overline{l_L} e_R)$ to form a G_L singlet, and b) is anti-Hermitian?
- $C_{e\gamma}^{1-loop} \propto S Y_e^\dagger$ fulfils first condition but not the second
- Can show that $[Y^\dagger Y Y_e^\dagger Y_e Y^\dagger Y, Y^\dagger Y] Y_e^\dagger$ is the smallest combination
- This gives EDM when neutrinos are Dirac $(M = 0)$, is analogous to EDM of down-type quarks in basis where Y_d is diagonal
- However, N_R are heavy, Majorana, so need insertions of M^{-1}
- Hence $d_\alpha \propto [S Y_e^\dagger Y_e S, S] Y_e^\dagger$

Effective field theory procedure

Schematically, there are several steps:

1. Match UV-complete theory onto EFT at the mass scale(s), Λ, of the heavy states which are integrated out: gives specific non-zero Wilson coefficients (WCs)

2. Run from Λ down to the electroweak scale, m_W

3. Match Standard Model EFT onto low-energy EFT (integrate out t, h, Z, W)

4. Run from m_W down to fermion mass scale, m_f, for low-energy observables