

Probing New Physics in Lepton Flavour Violating Tau decays

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*In collaboration with A. Celis (LMU, Munich),
and V. Cirigliano (LANL)*

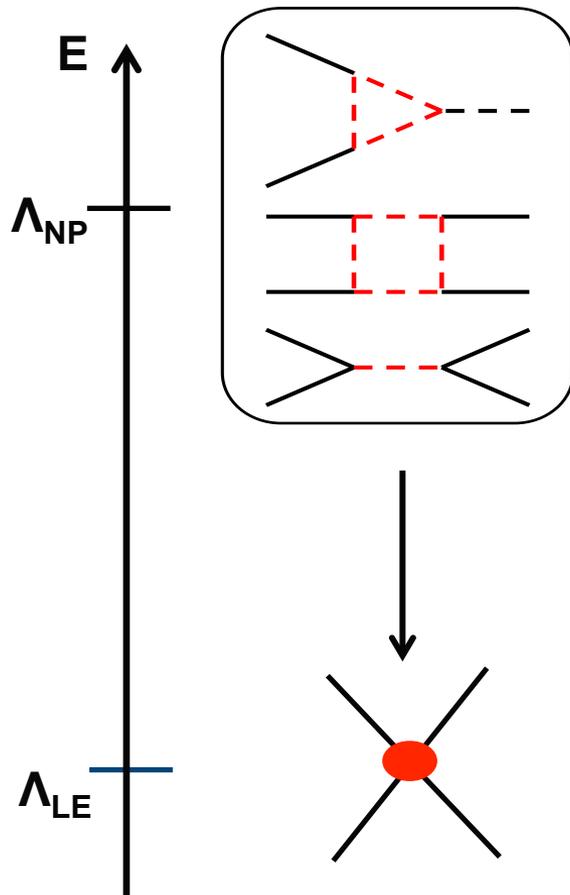
PRD 89 (2014) 013008, 095014

Outline

1. Introduction and Motivation
2. Charged Lepton-Flavour Violation from tau decays
3. Special Role of $\tau \rightarrow \mu\pi\pi$: hadronic form factors
4. Results
5. Conclusion and Outlook

1. Introduction and Motivation

1.1 Why study charged leptons?



- In the quest of New Physics, can be sensitive to very high scale:

- Kaon physics: $\frac{s\bar{d}s\bar{d}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^5 \text{ TeV}$
 $[\epsilon_K]$

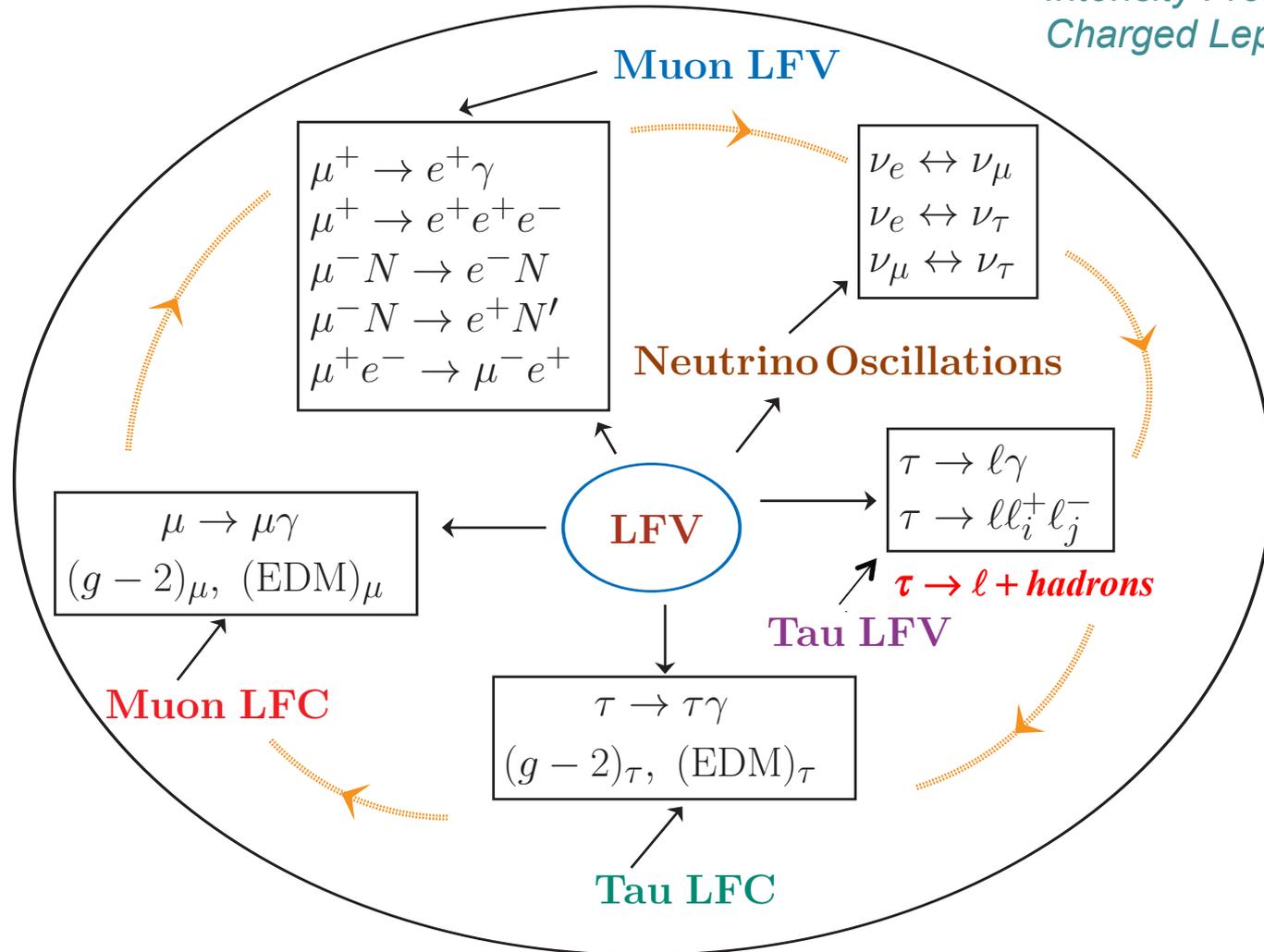
- Charged Leptons: $\frac{\mu\bar{e}f\bar{f}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^4 \text{ TeV}$
 $[\mu \rightarrow e\gamma]$

- At low energy: lots of experiments e.g., *MEG, COMET, Mu2e, E-969, BaBar, Belle-II, BESIII, LHCb* → huge improvements on measurements and bounds obtained and more expected
- In many cases no SM background: e.g., LFV, EDMs
- For some modes accurate calculations of hadronic uncertainties essential

→ Charged leptons very important to look for *New Physics!*

1.2 The Program

Intensity Frontier
Charged Lepton WG'13



2. Charged Lepton-Flavour Violation

2.1 Introduction and Motivation

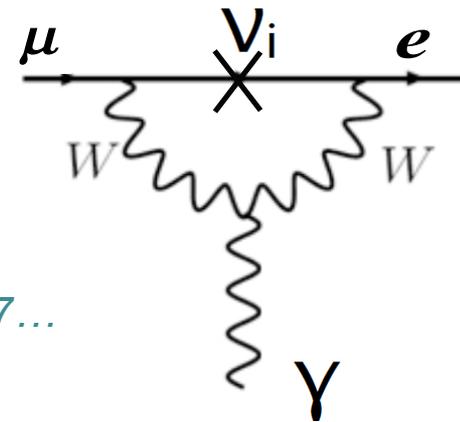
- Lepton Flavour Violation is an « accidental » symmetry of the SM ($m_\nu=0$)
- In the **SM** with massive neutrinos effective CLFV vertices are tiny due to GIM suppression \Rightarrow **unobservably small rates!**

e.g.: $\mu \rightarrow e\gamma$

$$Br(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U_{\mu i}^* U_{ei} \frac{\Delta m_{1i}^2}{M_W^2} \right|^2 < \mathbf{10^{-54}}$$

Petcov'77, Marciano & Sanda'77, Lee & Shrock'77...

$$[Br(\tau \rightarrow \mu\gamma) < \mathbf{10^{-40}}]$$



- Extremely **clean probe of beyond SM physics**

2.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

Talk by D. Hitlin @ CLFV2013

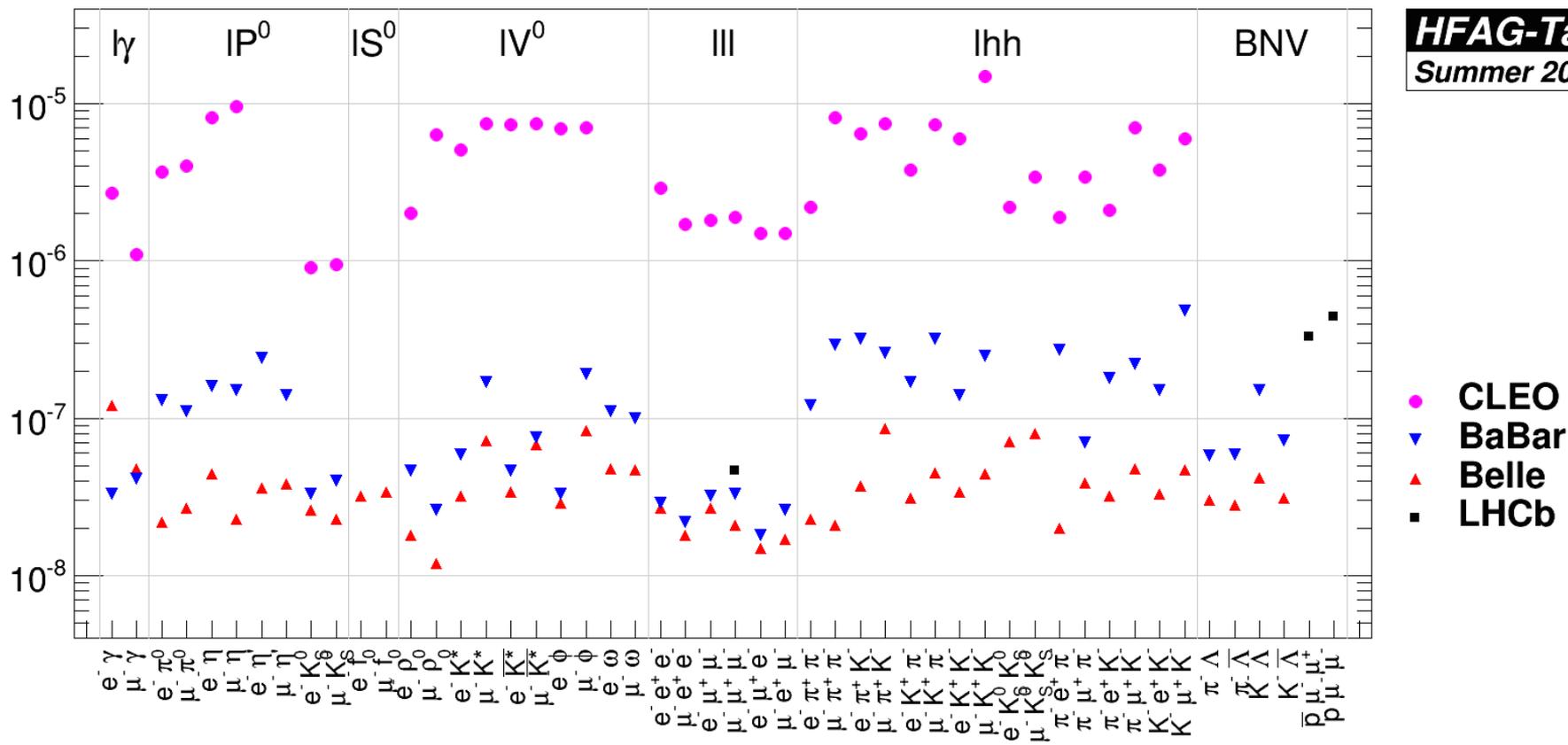
		$\tau \rightarrow \mu\gamma$ $\tau \rightarrow lll$	
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim, PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.2 CLFV processes: tau decays

- Several processes: $\tau \rightarrow l\gamma$, $\tau \rightarrow l_\alpha \bar{l}_\beta l_\beta$, $\tau \rightarrow lY$ $\leftarrow P, S, V, P\bar{P}, \dots$

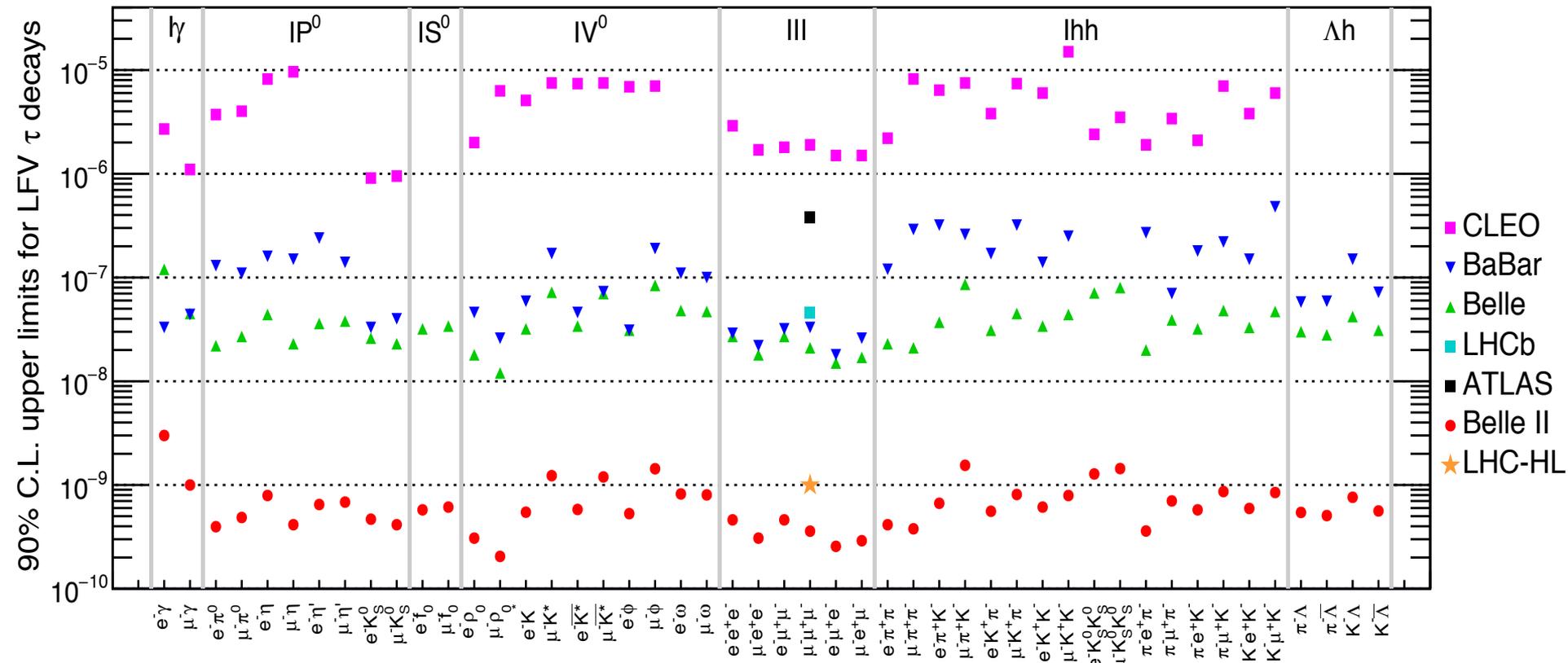
90% C.L. upper limits for LFV τ decays



- 48 LFV modes studied at Belle and BaBar

2.2 CLFV processes: tau decays

- Several processes: $\tau \rightarrow l\gamma$, $\tau \rightarrow l_\alpha \bar{l}_\beta l_\beta$, $\tau \rightarrow lY$ $\leftarrow P, S, V, P\bar{P}, \dots$



- Expected sensitivity 10^{-9} or better at *LHCb, Belle II, HL-LHC?*

2.3 Effective Field Theory approach

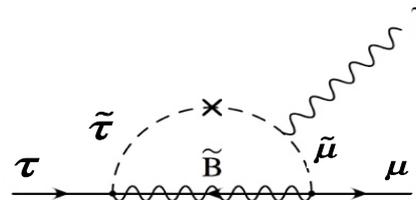
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \dots$$

- Build all $D > 5$ LFV operators:

➤ Dipole:

$$\mathcal{L}_{eff}^D \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$$

e.g.



See e.g.

Black, Han, He, Sher'02

Brignole & Rossi'04

Dassinger, Feldmann, Mannel, Turczyk'07

Matsuzaki & Sanda'08

Giffels et al.'08

Crivellin, Najjari, Rosiek'13

Petrov & Zhuridov'14

Cirigliano, Celis, E.P.'14

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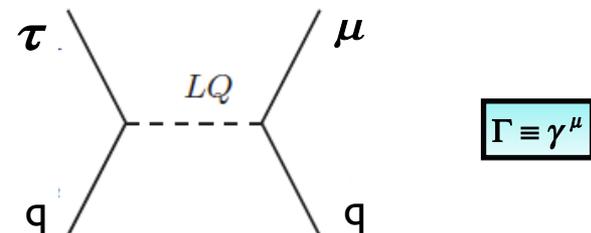
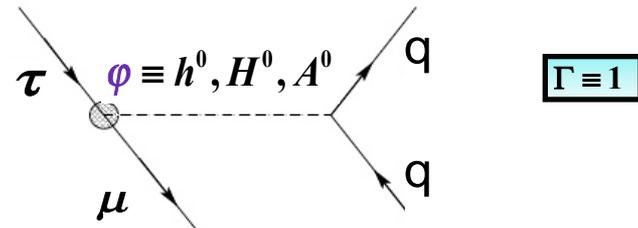
- Build all D>5 LFV operators:

➤ Dipole: $\mathcal{L}_{eff}^D \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$

- Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\mathcal{L}_{eff}^{S,V} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q$$

e.g.



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$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \dots$$

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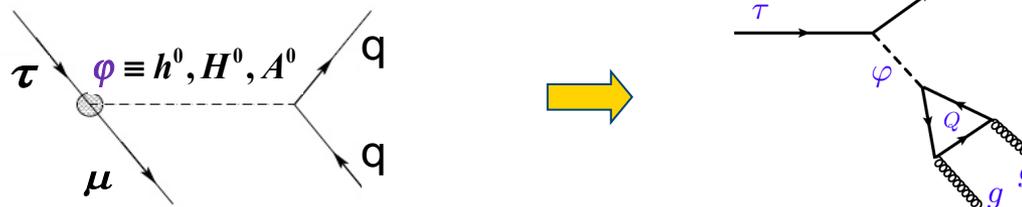
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- Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\mathcal{L}_{eff}^S \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q$$

- Integrating out heavy quarks generates *gluonic operator*

$$\frac{1}{\Lambda^2} \bar{\mu} P_{L,R} \tau Q \bar{Q} \rightarrow \mathcal{L}_{eff}^G \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau G_{\mu\nu}^a G_a^{\mu\nu}$$



2.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \dots$$

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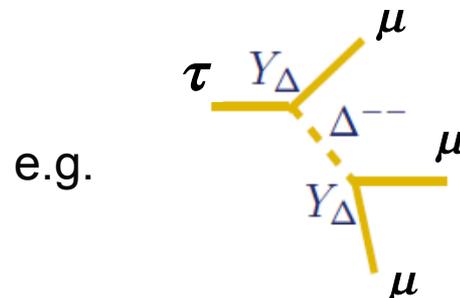
- Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\mathcal{L}_{eff}^S \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q$$

- 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu} \Gamma P_{L,R} \tau \bar{\mu} \Gamma P_{L,R} \mu$$

$$\Gamma \equiv 1, \gamma^\mu$$



See e.g.

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- Build all D>5 LFV operators:

- Dipole: $\mathcal{L}_{eff}^D \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$

- Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\mathcal{L}_{eff}^S \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q$$

- Lepton-gluon (Scalar, Pseudo-scalar):

$$\mathcal{L}_{eff}^G \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau G_{\mu\nu}^a G_a^{\mu\nu}$$

- 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu} \Gamma P_{L,R} \tau \bar{\mu} \Gamma P_{L,R} \mu$$

$$\Gamma \equiv 1, \gamma^\mu$$

- Each UV model generates a *specific pattern* of them

2.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \rightarrow 3\mu$	$\tau \rightarrow \mu\gamma$	$\tau \rightarrow \mu\pi^+\pi^-$	$\tau \rightarrow \mu K\bar{K}$	$\tau \rightarrow \mu\pi$	$\tau \rightarrow \mu\eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	—	—	—	—	—
O_D	✓	✓	✓	✓	—	—
O_V^q	—	—	✓ (I=1)	✓ (I=0,1)	—	—
O_S^q	—	—	✓ (I=0)	✓ (I=0,1)	—	—
O_{GG}	—	—	✓	✓	—	—
O_A^q	—	—	—	—	✓ (I=1)	✓ (I=0)
O_P^q	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	—	—	—	—	—	✓

- The notion of “*best probe*” (process with largest decay rate) is *model dependent*
- If observed, compare rate of processes  key handle on *relative strength* between operators and hence on the *underlying mechanism*

2.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

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$O_{S,V}^{4\ell}$	✓	—	—	—	—	—
O_D	✓	✓	✓	✓	—	—
O_V^q	—	—	✓ (I=1)	✓ (I=0,1)	—	—
O_S^q	—	—	✓ (I=0)	✓ (I=0,1)	—	—
O_{GG}	—	—	✓	✓	—	—
O_A^q	—	—	—	—	✓ (I=1)	✓ (I=0)
O_P^q	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	—	—	—	—	—	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important → sensitive to large number of operators!
- But need reliable determinations of the hadronic part: *form factors* and *decay constants* (e.g. $f_\eta, f_{\eta'}$)

2.5 Ex: Non standard LFV Higgs coupling

- $$\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2} (\bar{f}_L^i f_R^j H) H^\dagger H \quad \Rightarrow \quad -Y_{ij} (\bar{f}_L^i f_R^j) h$$

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v} \delta_{ij}$

Goudelis, Lebedev, Park'11
 Davidson, Grenier'10
 Harnik, Kopp, Zupan'12
 Blankenburg, Ellis, Isidori'12
 McKeen, Pospelov, Ritz'12
 Arhrib, Cheng, Kong'12

$$\mathcal{L}_Y = -m_i \bar{f}_L^i f_R^i - h \left(Y_{e\mu} \bar{e}_L \mu_R + Y_{e\tau} \bar{e}_L \tau_R + Y_{\mu\tau} \bar{\mu}_L \tau_R \right) + \dots$$

- Arise in several models *Cheng, Sher'97, Goudelis, Lebedev, Park'11*
Davidson, Grenier'10

Cheng, Sher'97

- Order of magnitude expected \Rightarrow No tuning: $|Y_{\tau\mu} Y_{\mu\tau}| \lesssim \frac{m_\mu m_\tau}{v^2}$

- In concrete models, in general further parametrically suppressed

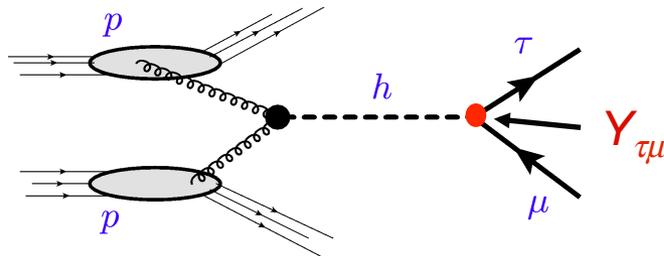
2.5 Ex: Non standard LFV Higgs coupling

- $$\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2} (\bar{f}_L^i f_R^j H) H^\dagger H \quad \Rightarrow \quad -Y_{ij} (\bar{f}_L^i f_R^j) h$$

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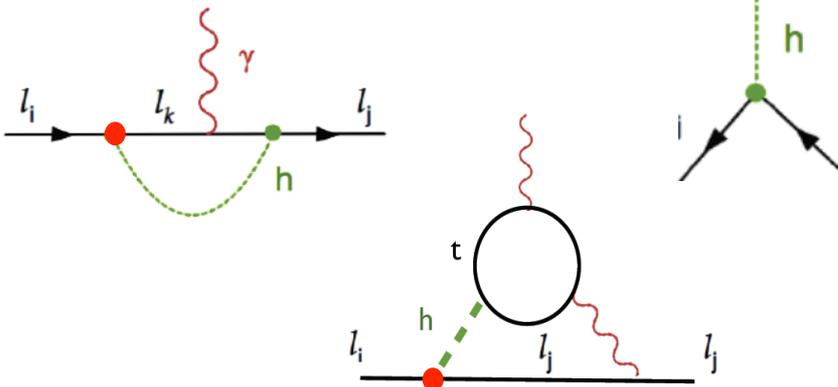
- High energy : LHC

In the SM: $Y_{ij}^{hSM} = \frac{m_i}{v} \delta_{ij}$



Hadronic part treated with perturbative QCD

- Low energy : D, S operators

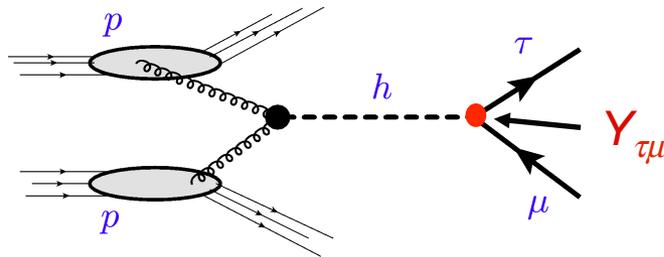


2.5 Ex: Non standard LFV Higgs coupling

- $$\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2} (\bar{f}_L^i f_R^j H) H^\dagger H \quad \Rightarrow \quad -Y_{ij} (\bar{f}_L^i f_R^j) h$$

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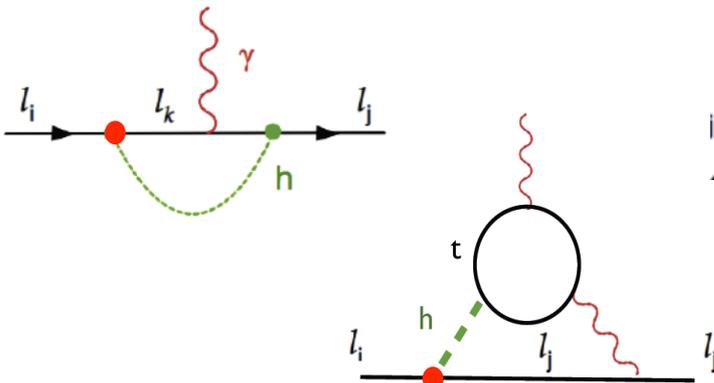
- High energy : LHC



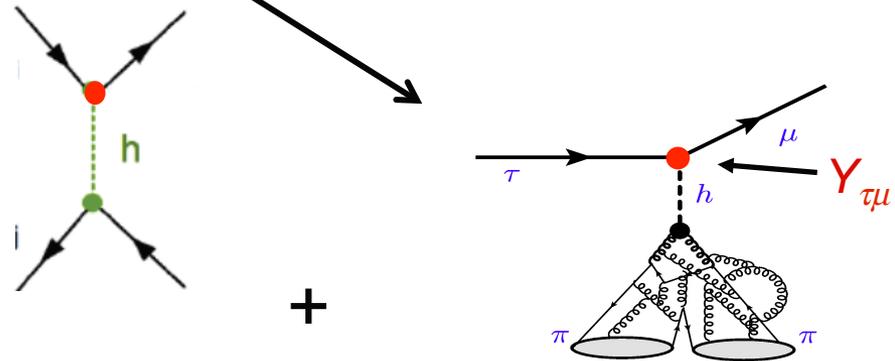
In the SM: $Y_{ij}^{hSM} = \frac{m_i}{v} \delta_{ij}$

Hadronic part treated with perturbative QCD

- Low energy : D, S, G operators



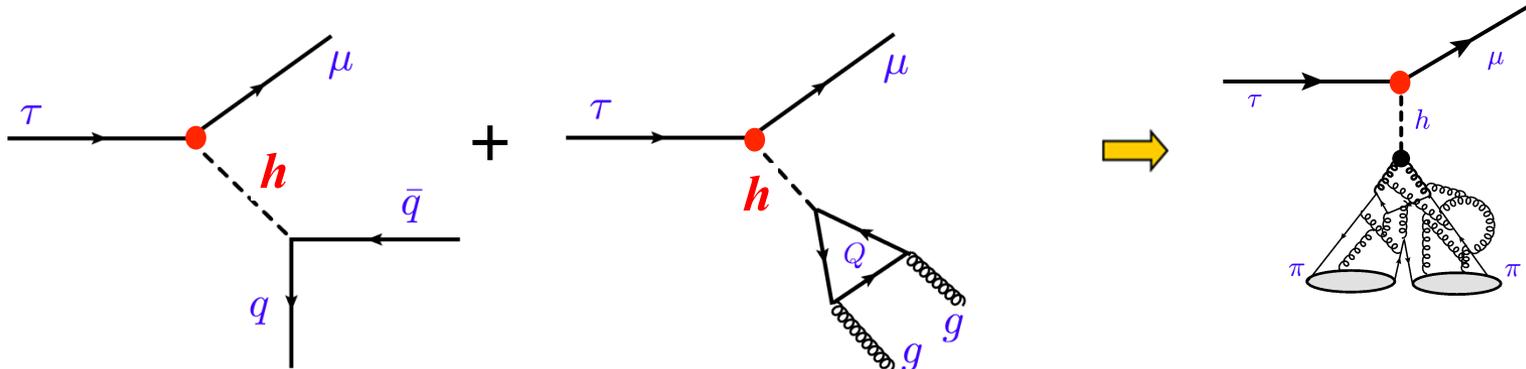
Reverse the process



Hadronic part treated with non-perturbative QCD

2.6 Constraints from $\tau \rightarrow \mu\pi\pi$

- Tree level Higgs exchange



- Problem : Have the hadronic part under control, ChPT not valid at these energies!

➡ Use *form factors* determined with *dispersion relations* matched at low energy to *CHPT*

Daub, Dreiner, Hanhart, Kubis, Meissner'13

Celis, Cirigliano, E.P.'14

- Dispersion relations: based on *unitarity*, *analyticity* and *crossing symmetry*

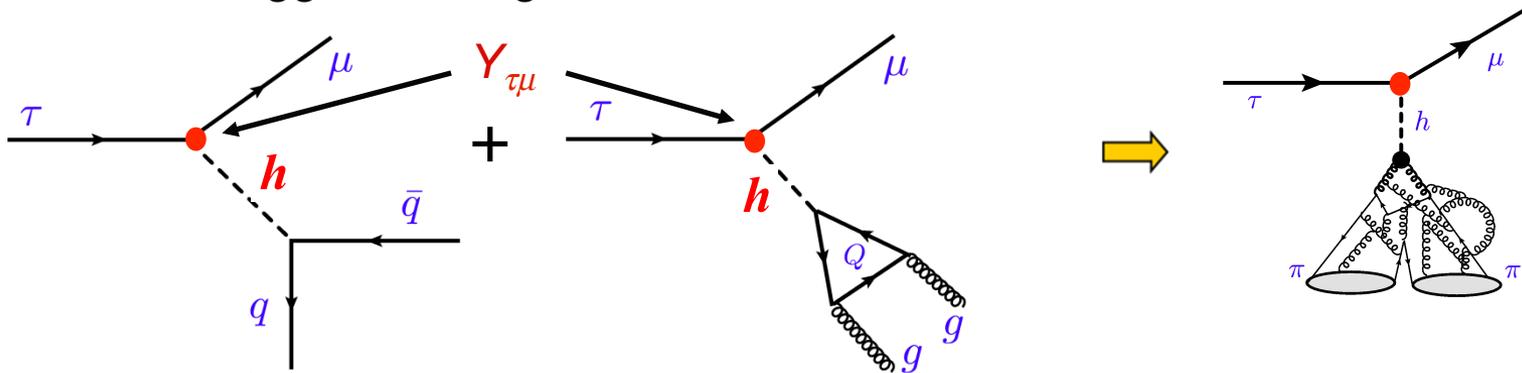
➡ Take *all rescattering* effects into account

$\pi\pi$ final state interactions important

3. Description of the hadronic form factors

3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

- Tree level Higgs exchange



$$\langle \pi^+ \pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$$

$$\langle \pi^+ \pi^- | \theta_\mu^\mu | 0 \rangle \equiv \theta_\pi(s)$$

$$\langle \pi^+ \pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$$

$$s = (p_{\pi^+} + p_{\pi^-})^2$$

$$\theta_\mu^\mu = -9 \frac{\alpha_s}{8\pi} G_{\mu\nu}^a G_a^{\mu\nu} + \sum_{q=u,d,s} m_q \bar{q}q$$

$$\frac{d\Gamma(\tau \rightarrow \mu \pi^+ \pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2 \sqrt{s - 4m_\pi^2}}{256\pi^3 m_\tau^3} \frac{(|Y_{\tau\mu}^h|^2 + |Y_{\mu\tau}^h|^2)}{M_h^4 v^2} |\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)|^2$$

$f(y_q^h)$

3.2 How to describe the form factors?

$$\langle \pi^+ \pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$$

$$\langle \pi^+ \pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$$

$$\langle \pi^+ \pi^- | \theta_\mu^\mu | 0 \rangle \equiv \theta_\pi(s)$$

$$\theta_\mu^\mu = -9 \frac{\alpha_s}{8\pi} G_{\mu\nu}^a G_a^{\mu\nu} + \sum_{q=u,d,s} m_q \bar{q}q$$

- Using LO ChPT:

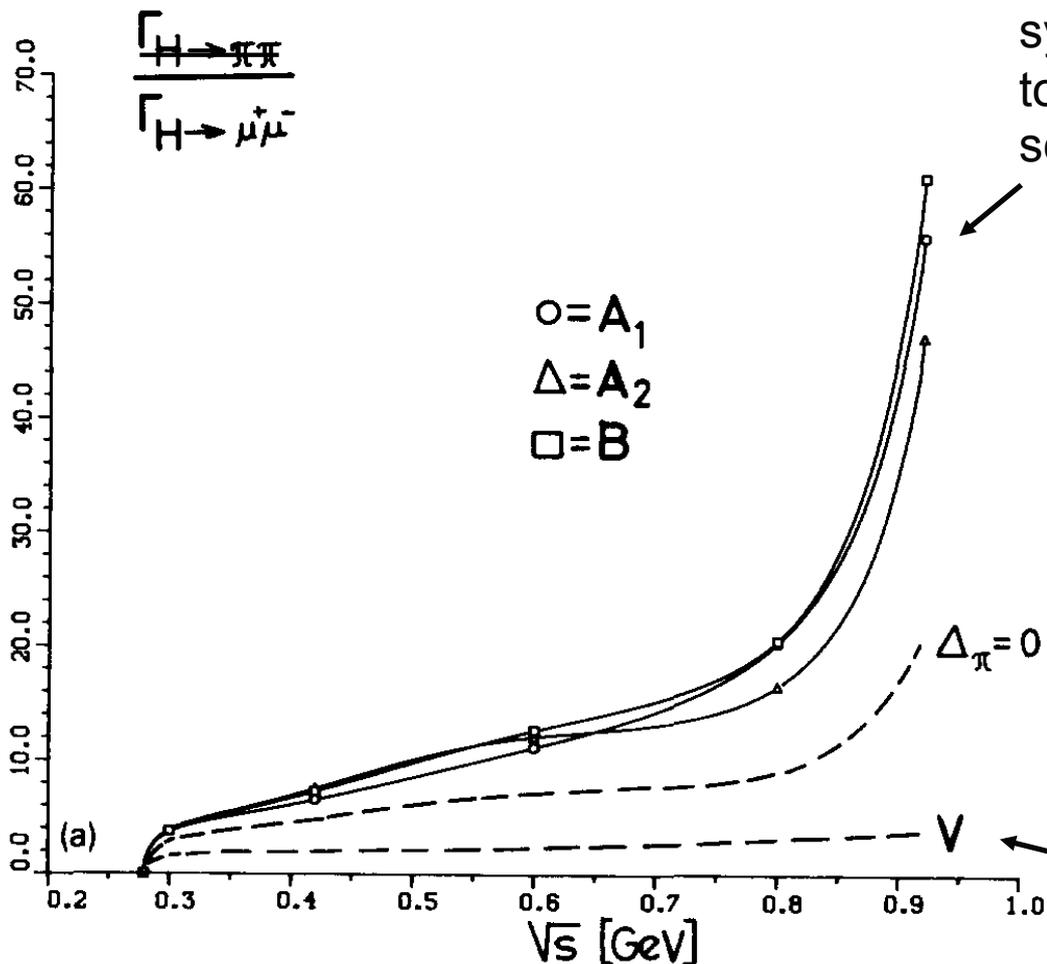
$$\Gamma_\pi(s) = m_\pi^2 \quad \theta_\pi(s) = s + 2m_\pi^2 + \mathcal{O}(p^4)$$

Voloshin'85

3.2 How to describe the form factors?

Donoghue, Gasser, Leutwyler'90

J.F. Donoghue et al. / Decay of a light Higgs boson



Using the triple constraints of chiral symmetry, analyticity, and unitarity, together with exp. input from pion scattering

very far from the naive expectation

$$\frac{\Gamma(h \rightarrow \mu^+\mu^-)}{\Gamma(h \rightarrow \pi^+\pi^-)} \sim \frac{m_\mu^2}{m_\pi^2}$$

Voloshin'85

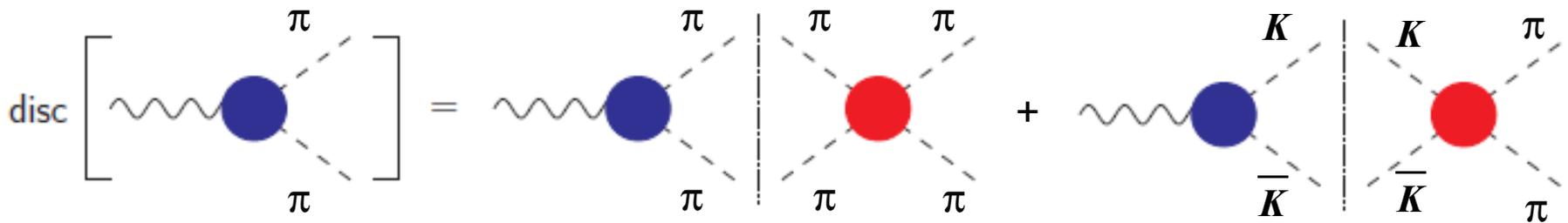
3.3 Unitarity

- Elastic approximation breaks down for the $\pi\pi$ S-wave at $K\bar{K}$ threshold due to the strong inelastic coupling involved in the region of $f_0(980)$

➔ Need to *solve a Coupled Channel Mushkhelishvili-Omnès* problem

Donoghue, Gasser, Leutwyler'90
Osset & Oller'98
Moussallam'99

- Unitarity ➔ the discontinuity of the form factor is known



$$\text{Im}F_n(s) = \sum_{m=1}^2 T_{nm}^*(s)\sigma_m(s)F_m(s)$$

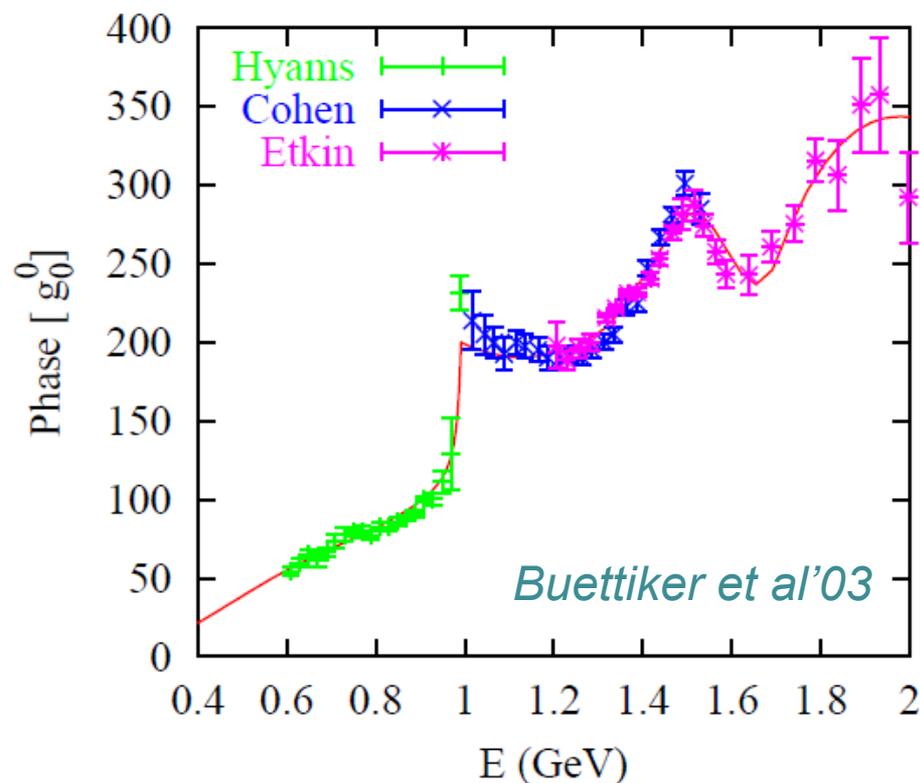
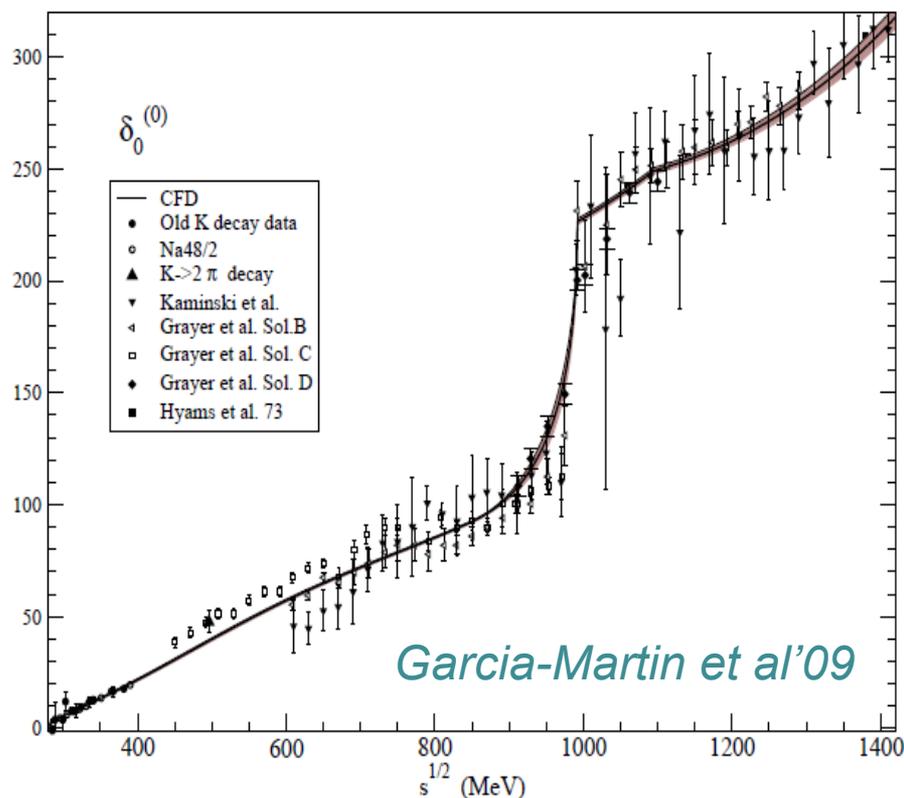
$n = \pi\pi, K\bar{K}$

Scattering matrix:

$$\begin{pmatrix} \pi\pi \rightarrow \pi\pi, \pi\pi \rightarrow K\bar{K} \\ K\bar{K} \rightarrow \pi\pi, K\bar{K} \rightarrow K\bar{K} \end{pmatrix}$$

3.4 Inputs for the coupled channel analysis

- Inputs : $\pi\pi \rightarrow \pi\pi, K\bar{K}$



- A large number of theoretical analyses *Descotes-Genon et al'01*, *Kaminsky et al'01*, *Buettiker et al'03*, *Garcia-Martin et al'09*, *Colangelo et al.'11* and all agree
- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* \Rightarrow reconstruct T matrix

3.5 Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

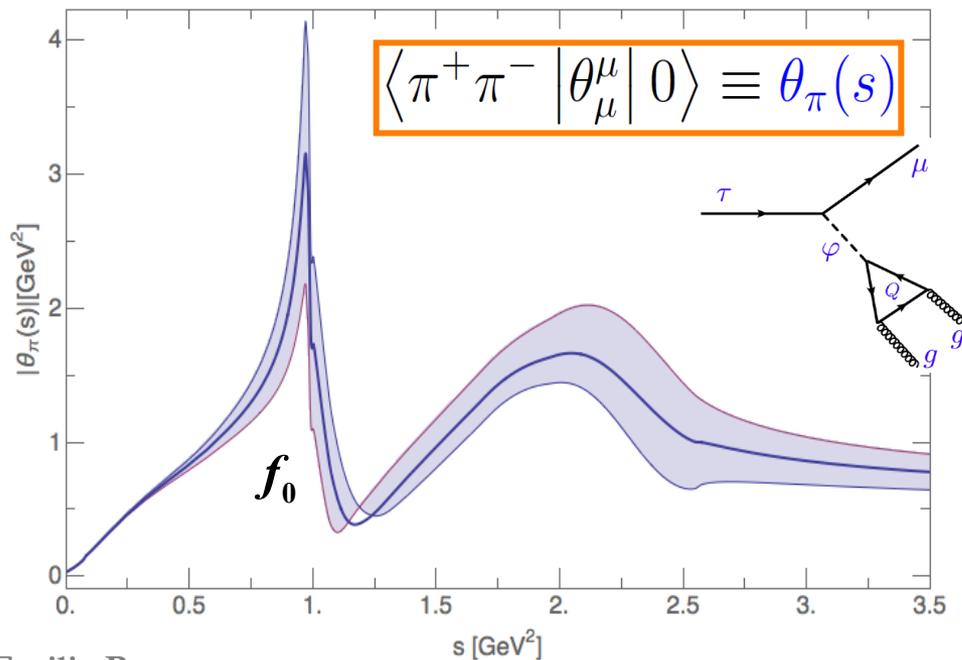
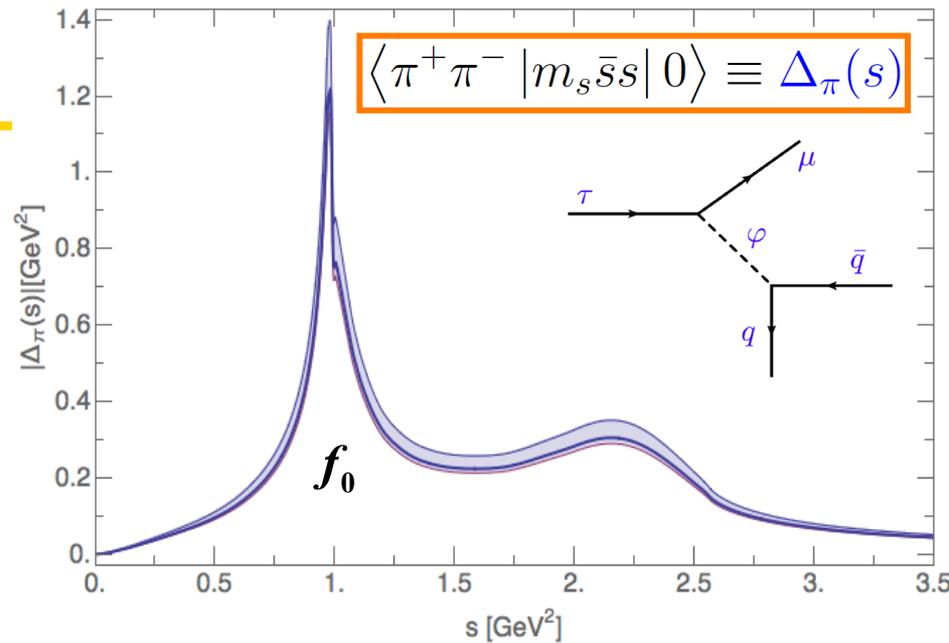
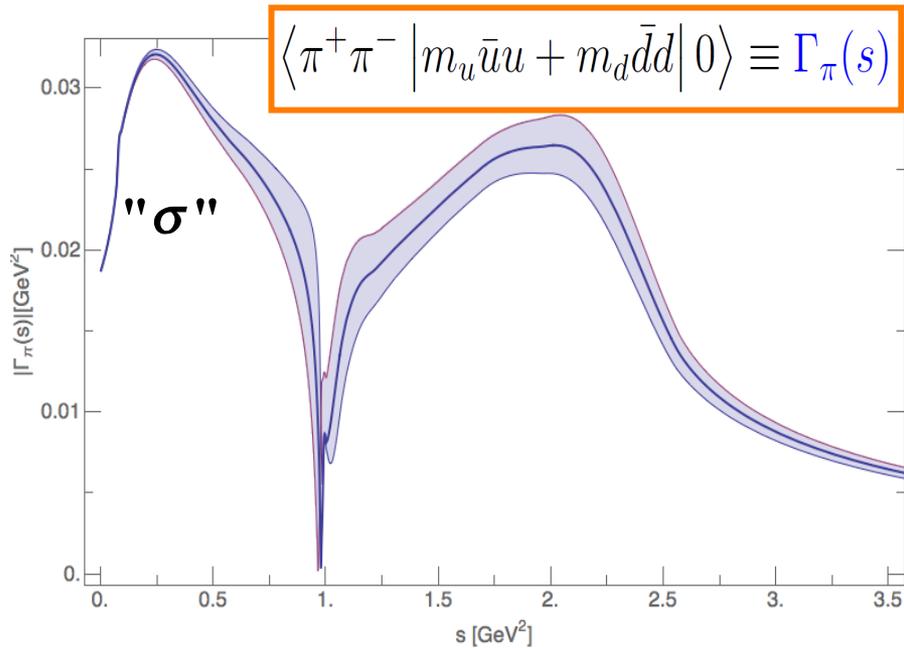
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}}F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution falling as $1/s$ for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving dispersive integral equations iteratively starting with *Omnès functions* that are solutions of the one-channel unitary condition

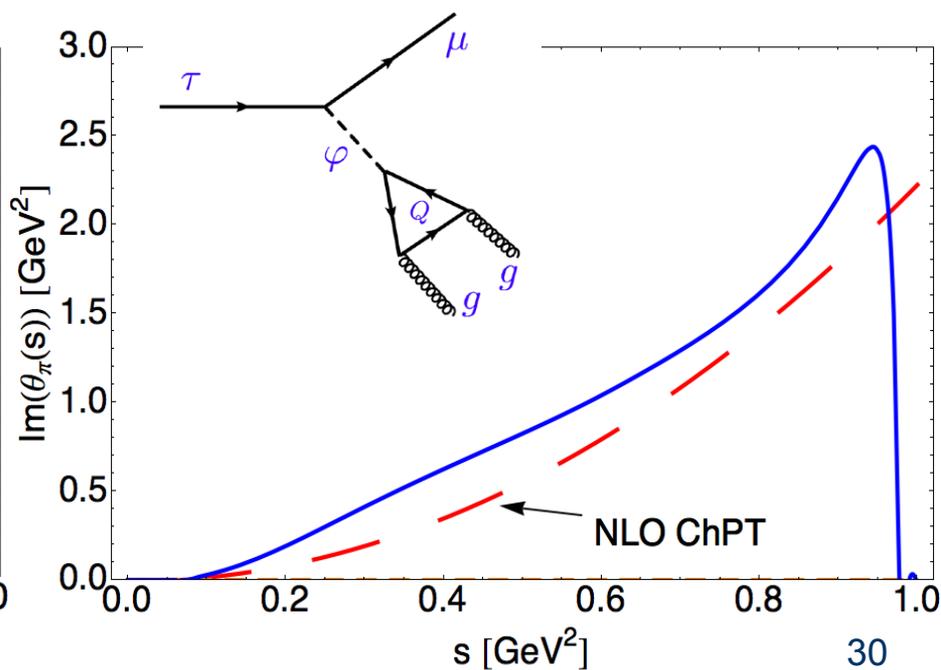
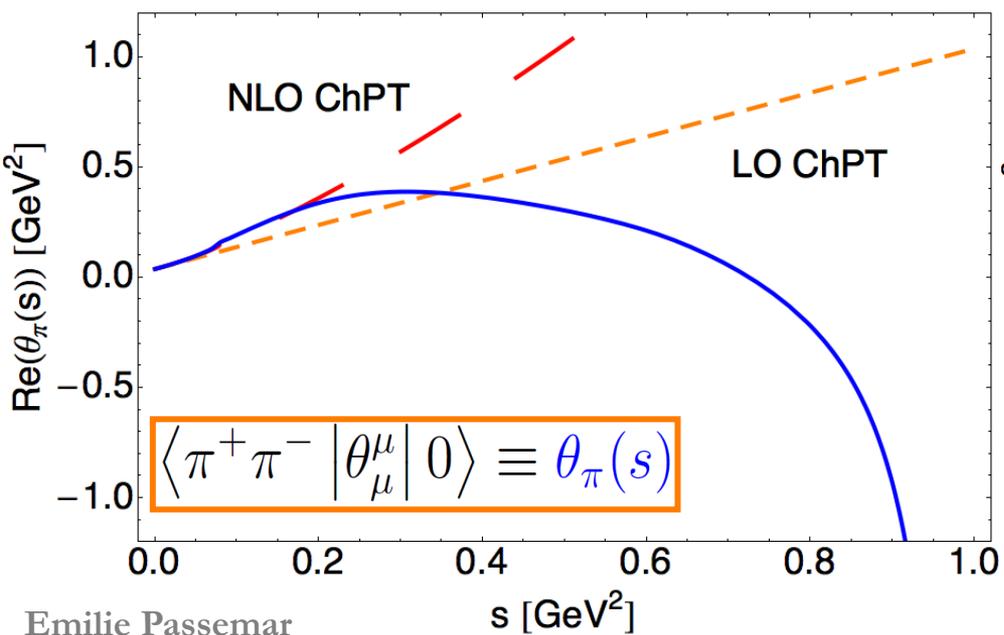
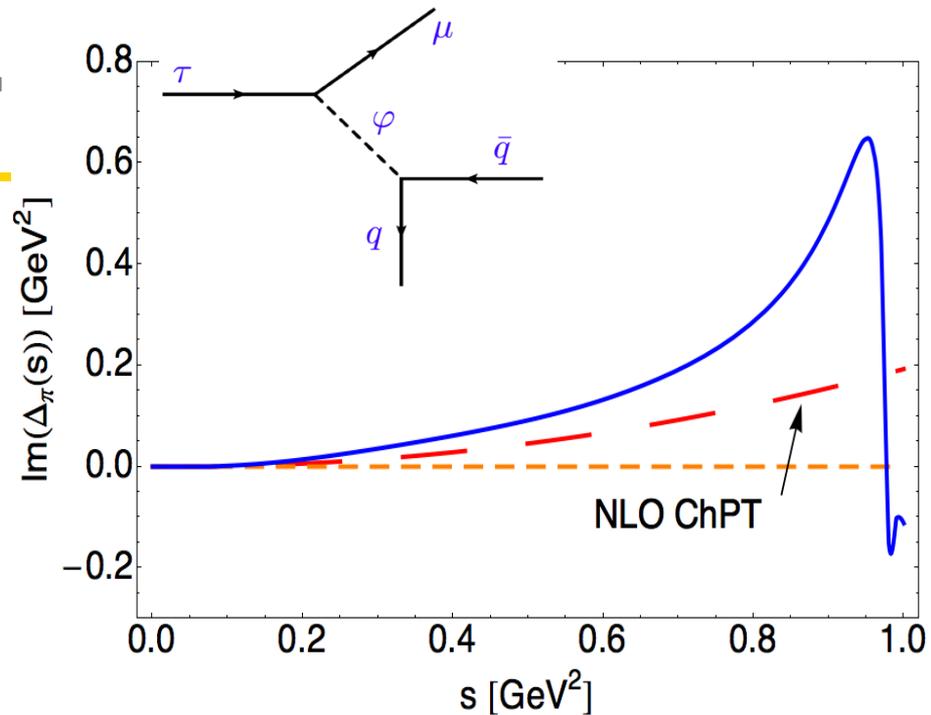
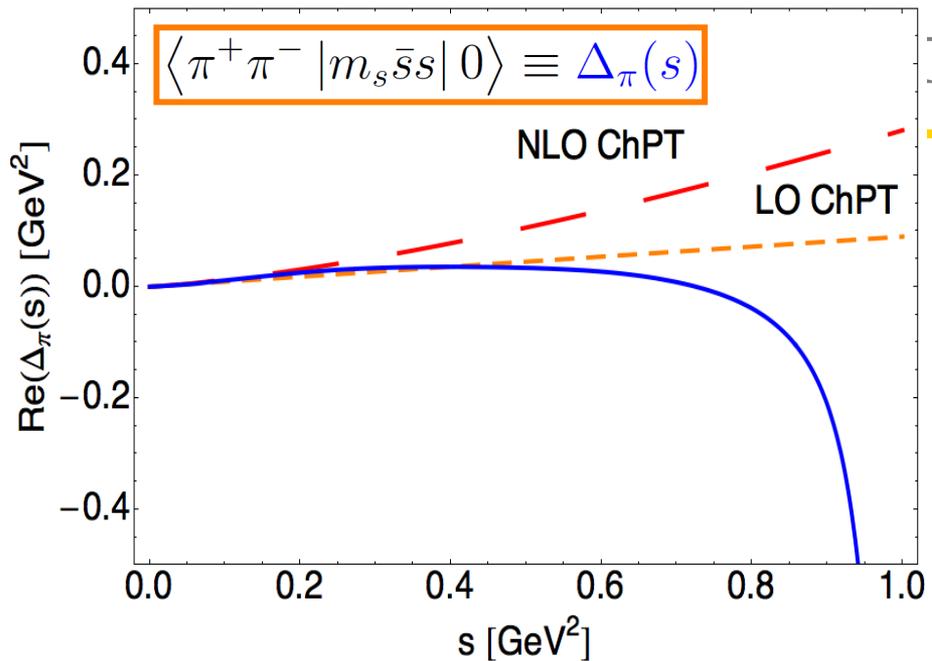
$$\Omega_{\pi,K}(s) \equiv \exp \left[\frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dt}{t} \frac{\delta_{\pi,K}(t)}{(t-s)} \right]$$



- Uncertainties:

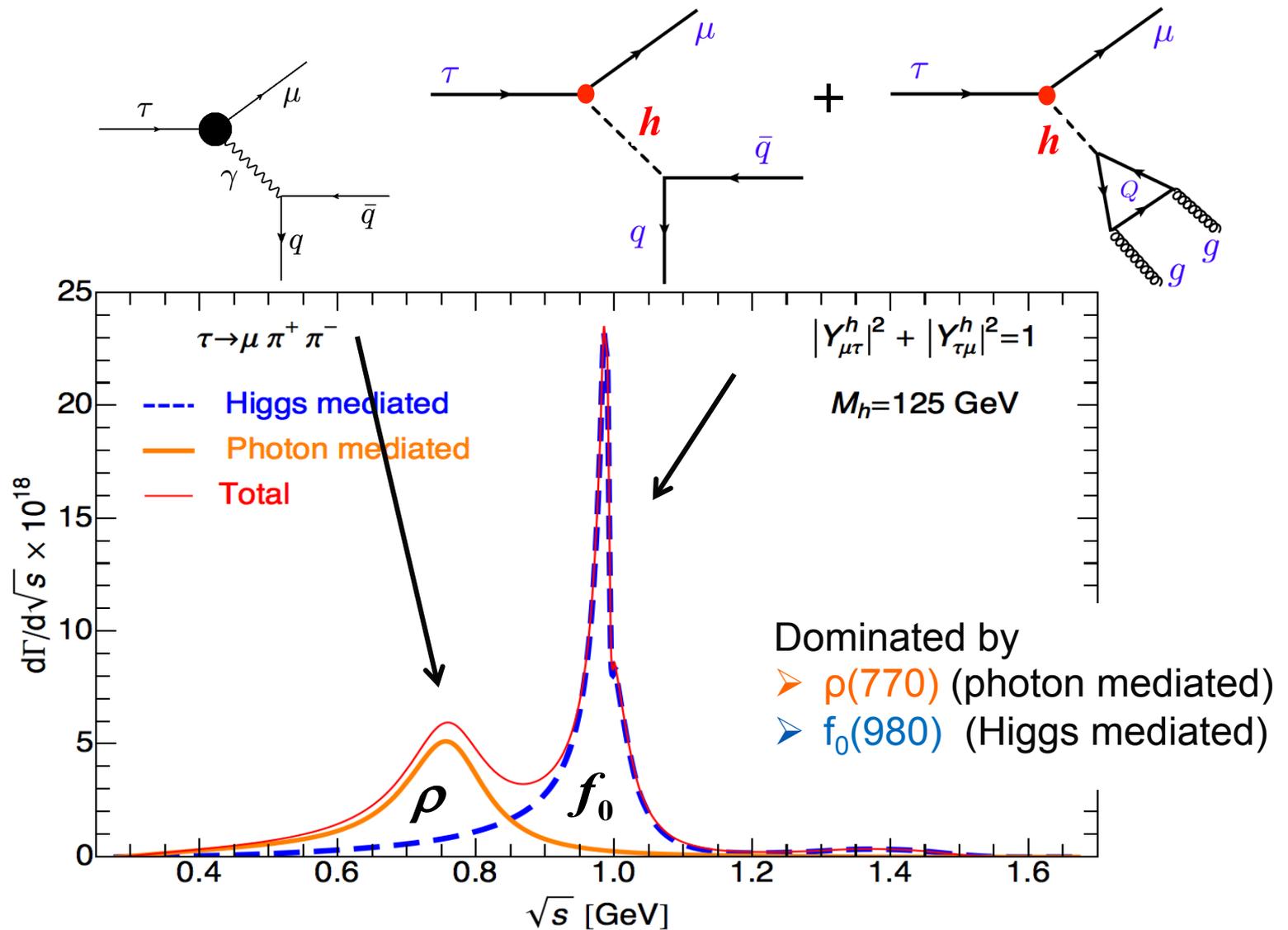
- Varying s_{cut} (1.4 GeV² - 1.8 GeV²)
- Varying the matching conditions
- T matrix inputs

See also *Daub et al.'13*



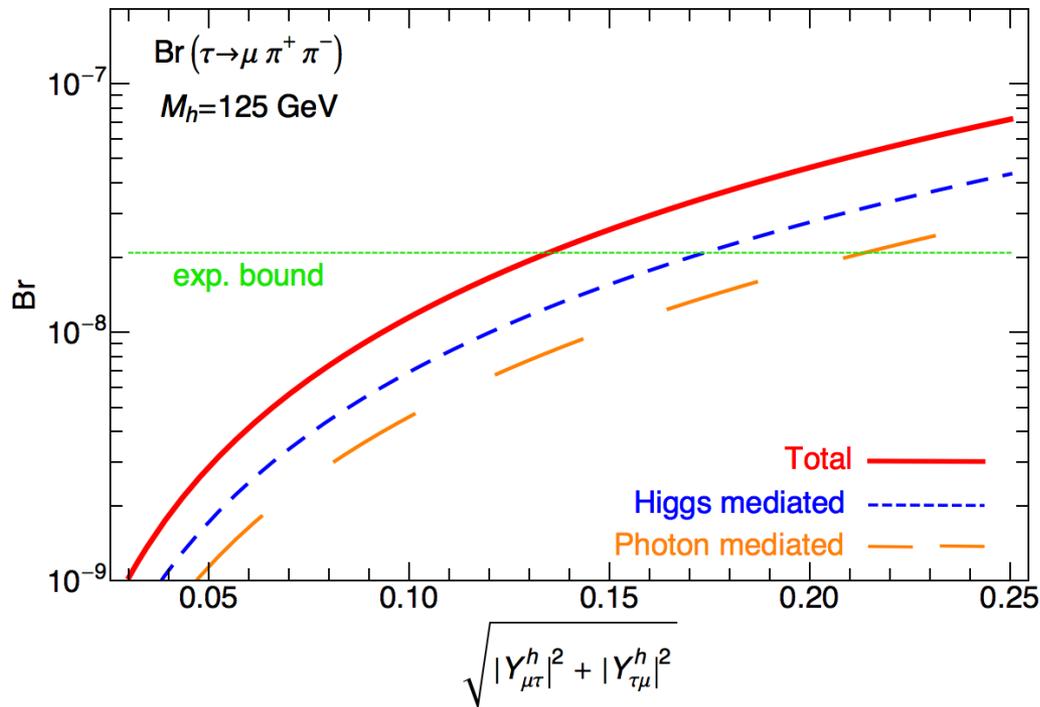
4. Results

4.1 Spectrum



4.2 Bounds

Celis, Cirigliano, E.P.'14



Bound:

$$\sqrt{|Y_{\mu\tau}^h|^2 + |Y_{\tau\mu}^h|^2} \leq 0.13$$

Process	(BR × 10 ⁸) 90% CL	$\sqrt{ Y_{\mu\tau}^h ^2 + Y_{\tau\mu}^h ^2}$	Operator(s)
$\tau \rightarrow \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \rightarrow \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \rightarrow \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \rightarrow \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \rightarrow \mu\pi^0\pi^0$	< 1.4 × 10 ³ [87]	< 6.3	Scalar, Gluon

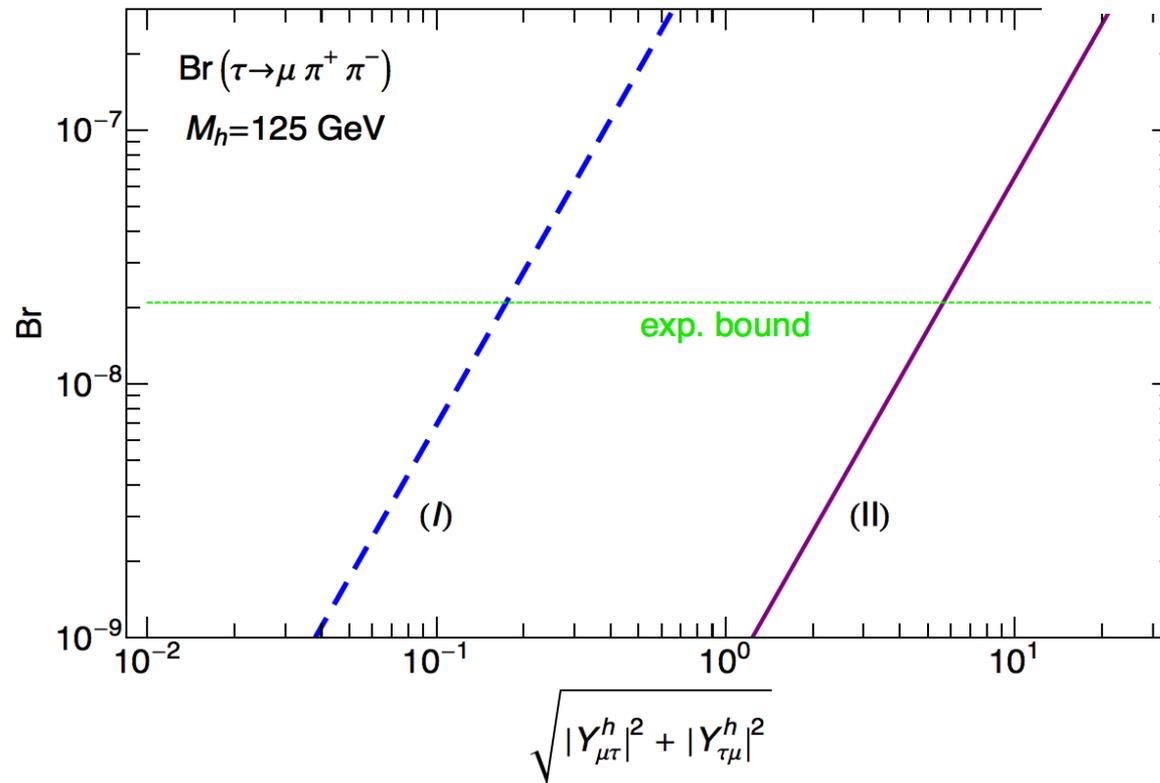
Less stringent but more robust handle on LFV Higgs couplings

? →

BaBar'10, Belle'10'11'13 except last from CLEO'97

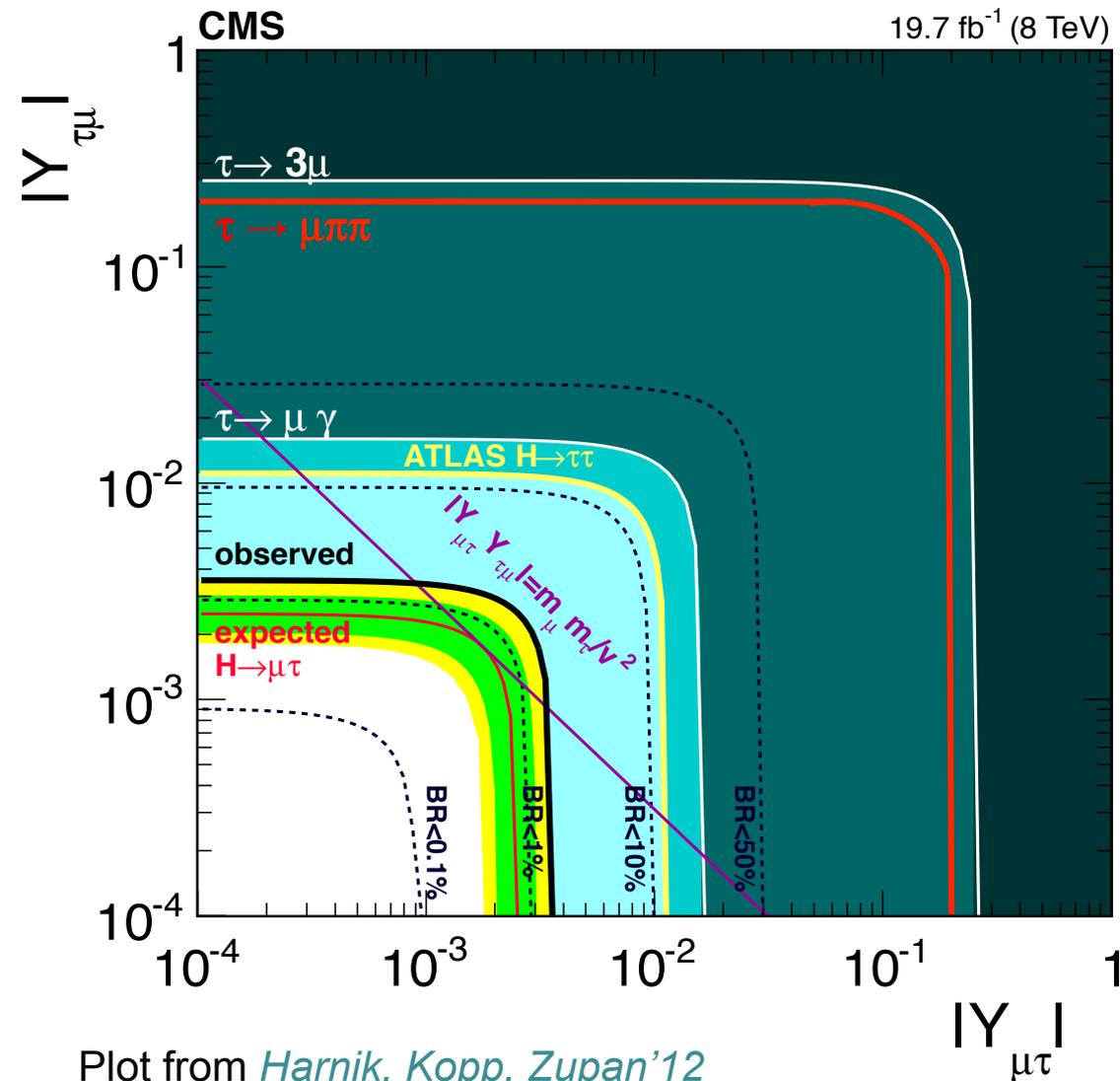
4.3 Impact of our results

Celis, Cirigliano, E.P.'14



- Dispersive treatment of hadronic part \Rightarrow bound reduced by one order of magnitude!
- ChPT, EFT only valid at low energy for $\mathbf{p \ll \Lambda = 4\pi f_\pi \sim 1 \text{ GeV}}$
 \Rightarrow *not valid up to $E = (m_\tau - m_\mu)$!*

4.4 Constraints in the $\tau\mu$ sector



Plot from *Harnik, Kopp, Zupan'12*
updated by *CMS'15*

- Constraints from LE:
 - $\tau \rightarrow \mu\gamma$: best constraints but loop level
 - ➔ sensitive to UV completion of the theory
 - $\tau \rightarrow \mu\pi\pi$: tree level diagrams
 - ➔ robust handle on LFV
- Constraints from HE:
 - LHC** wins for $\tau\mu$!
- Opposite situation for μe !
- For LFV Higgs and nothing else: LHC bound

➔ $BR(\tau \rightarrow \mu\gamma) < 2.2 \times 10^{-9}$

➔ $BR(\tau \rightarrow \mu\pi\pi) < 1.5 \times 10^{-11}$

4.5 Hint of New Physics in $h \rightarrow \tau\mu$?

$$BR(h \rightarrow \tau\mu) = (0.84^{+0.39}_{-0.37})\%$$

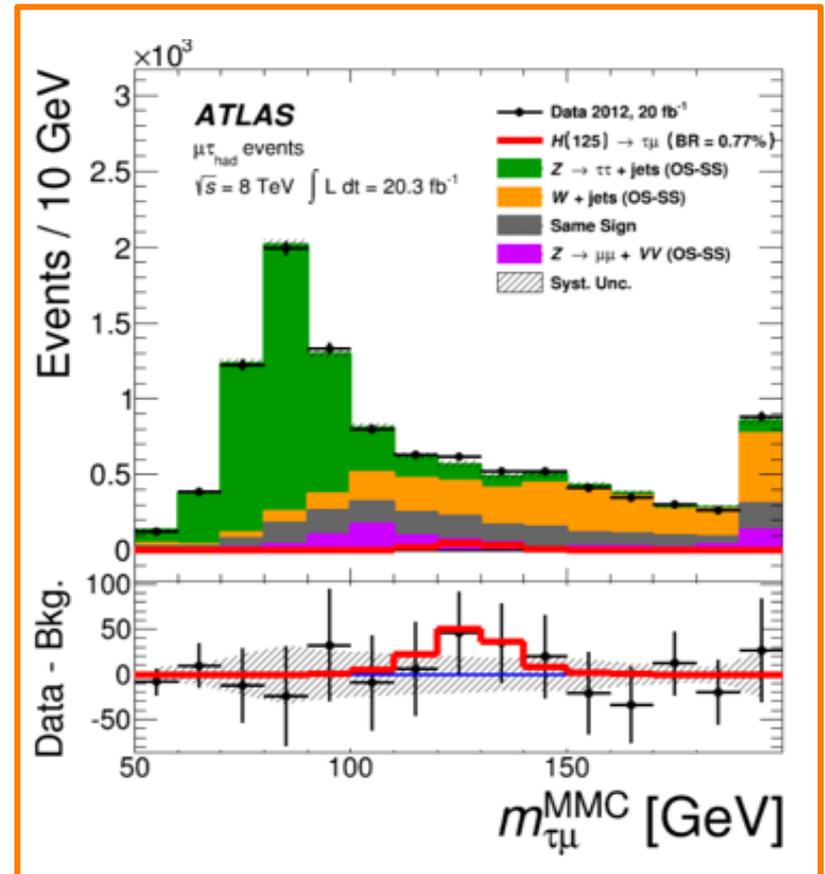
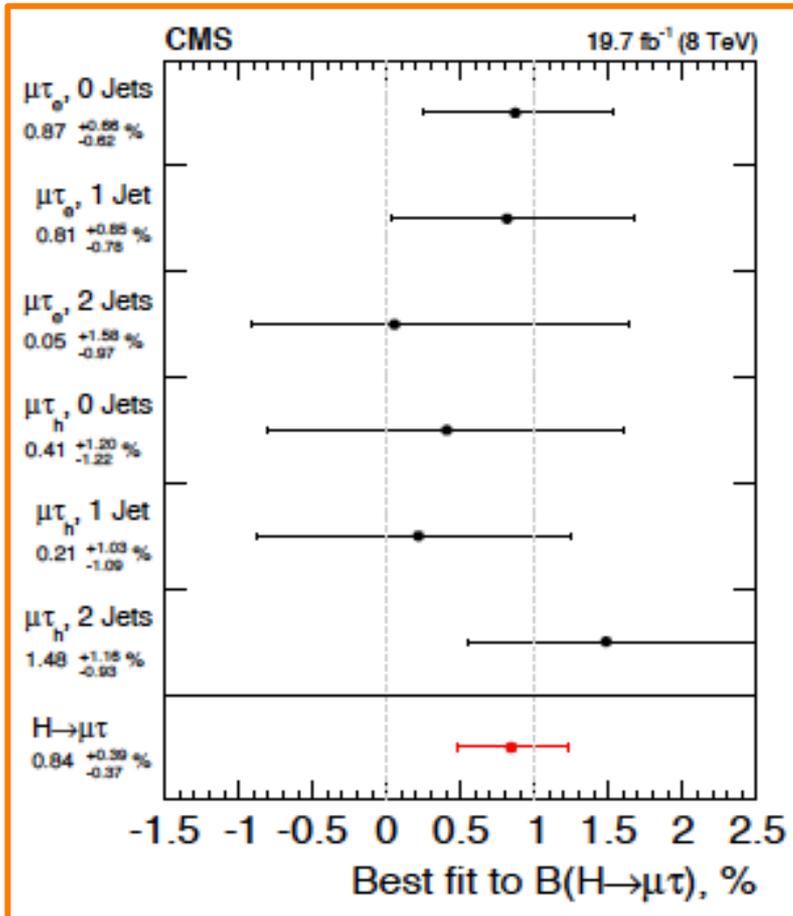
@2.4 σ

CMS'15

$$BR(h \rightarrow \tau\mu) = (0.53 \pm 0.51)\%$$

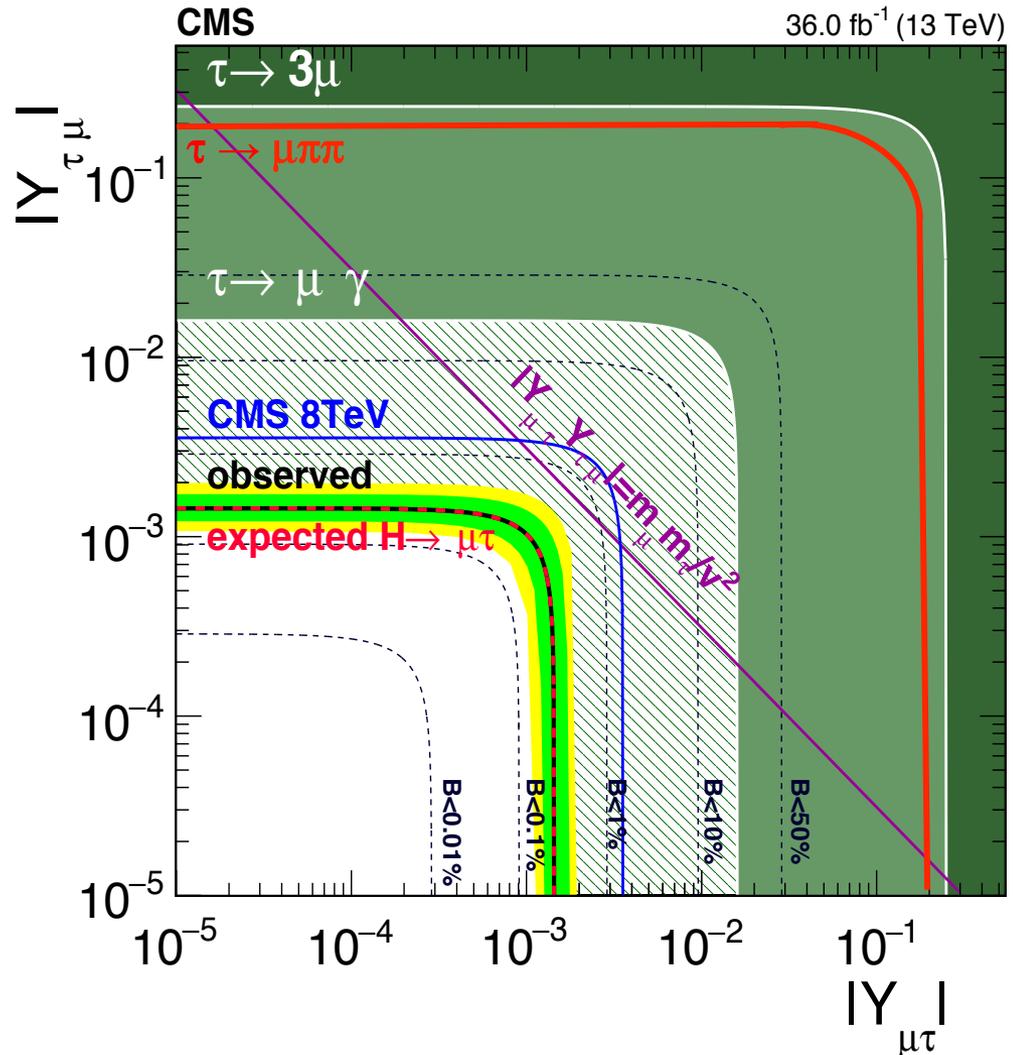
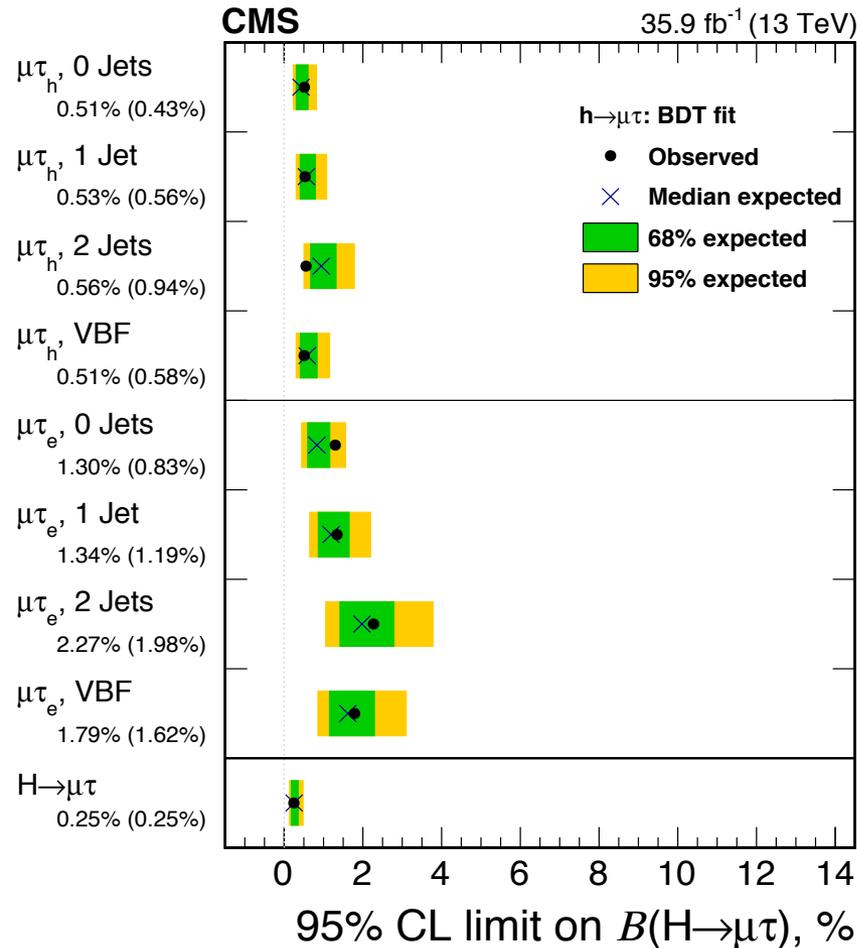
@1 σ

ATLAS'15



4.5 Hint of New Physics in $h \rightarrow \tau\mu$?

CMS'17



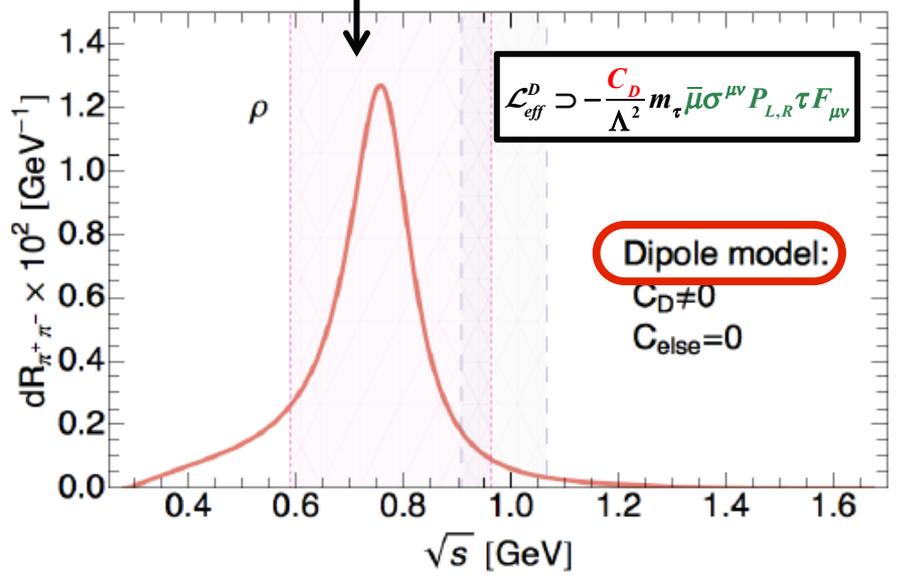
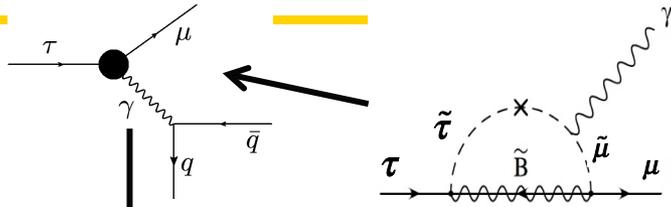
$$BR(h \rightarrow \tau\mu) = (0.25 \pm 0.25)\%$$

13 TeV@CMS

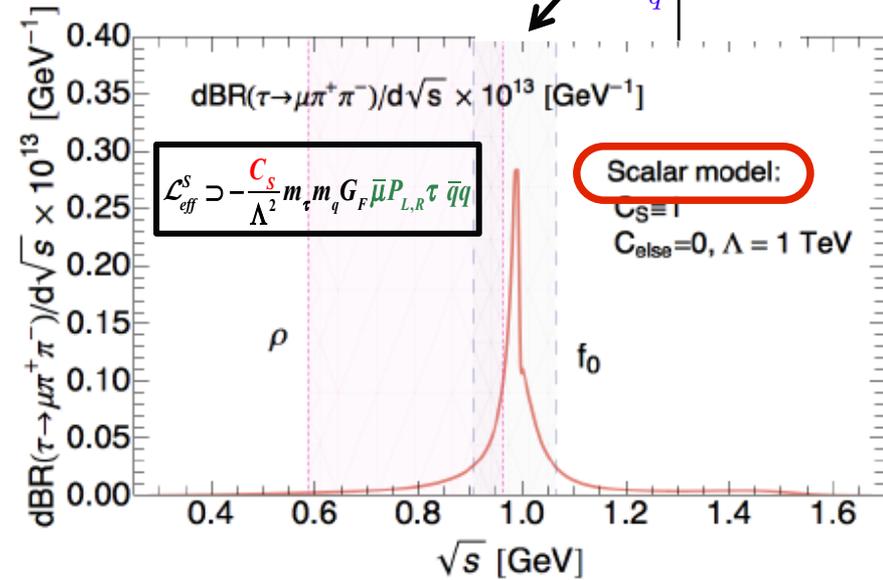
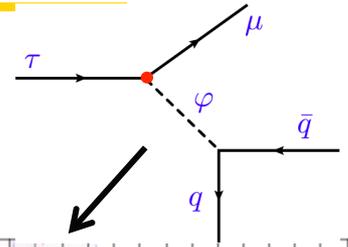
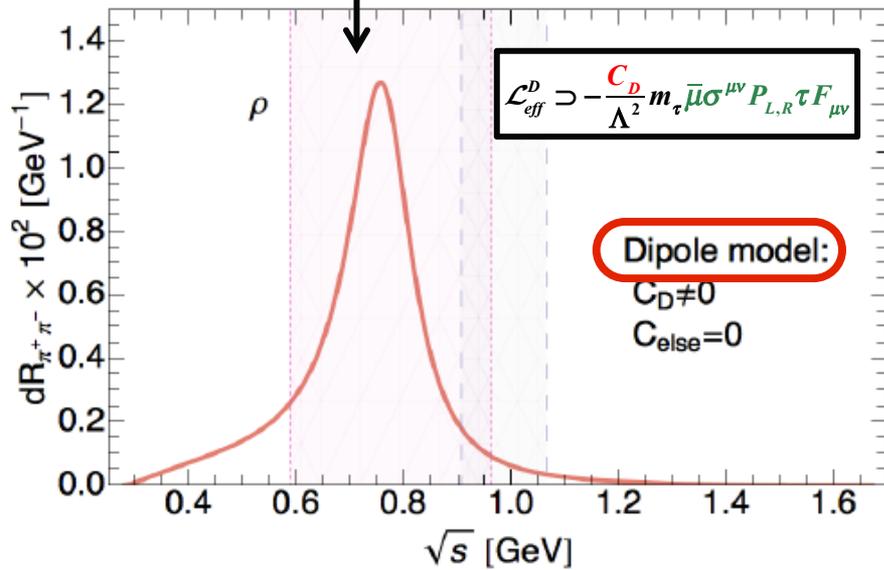
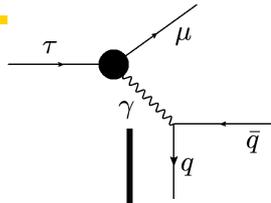
CMS'17

4.6 Discriminating power of $\tau \rightarrow \mu(e)\pi\pi$ decays

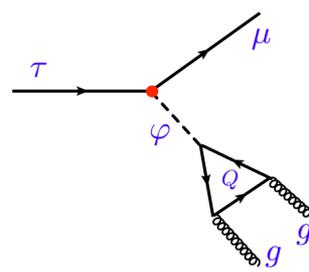
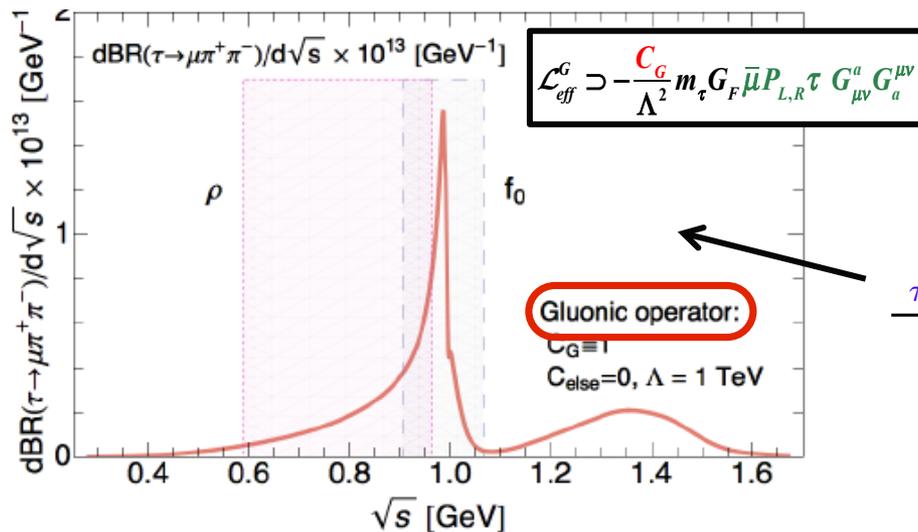
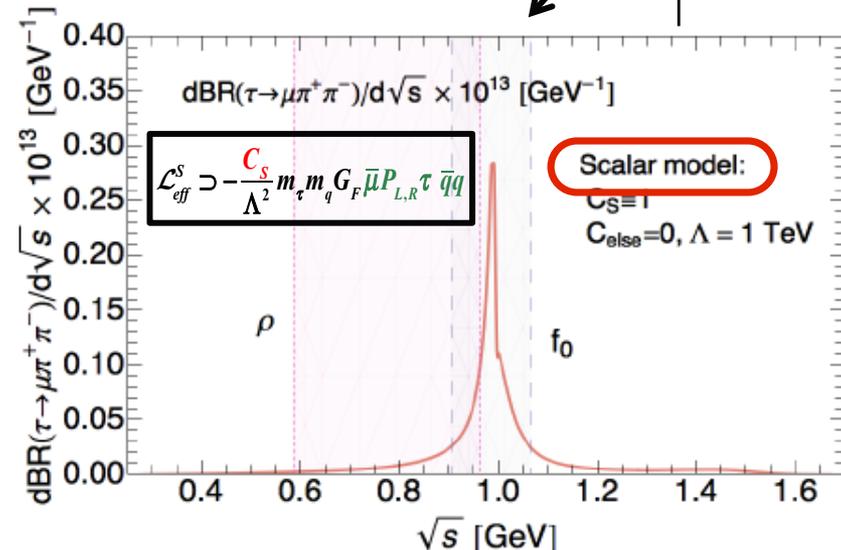
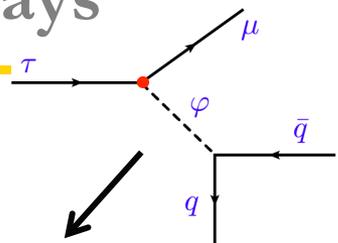
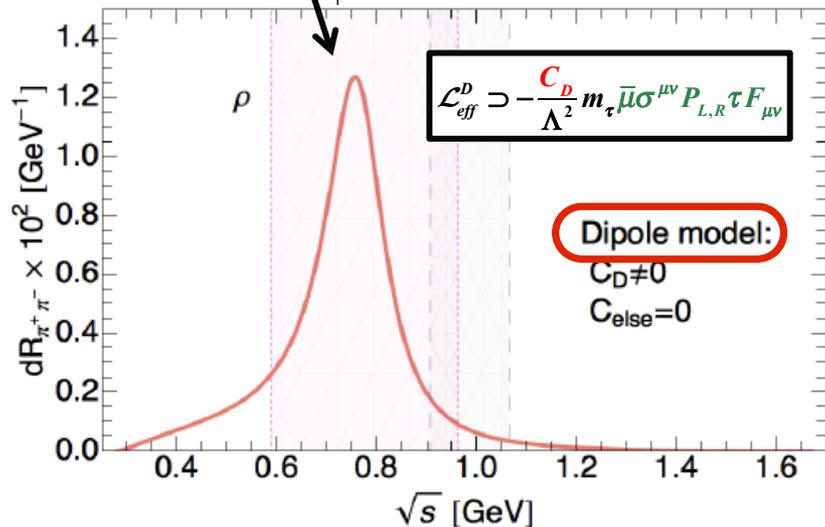
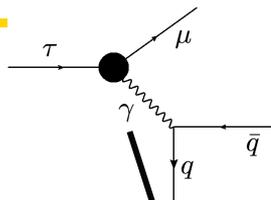
Celis, Cirigliano, E.P.'14



4.6 Discriminating power of $\tau \rightarrow \mu(e)\pi\pi$ decays



4.6 Discriminating power of $\tau \rightarrow \mu(e)\pi\pi$ decays



Different distributions according to the *operator!*

5. Conclusion and Outlook

Summary

- LFV can probe new physics scales much higher than those directly observable at the LHC
- It is possible to observe signals of new physics via *LFV transitions* in the near future, despite the lack of new physics observed so far at the LHC
- Different operators expected at low scales, we need to measure as many processes as possible
 - ➔ *Hadronic decays* such as $\tau \rightarrow \mu(e)\pi\pi$ important!
- Use a combination of dispersive methods and ChPT/Lattice QCD in order to determine the relevant hadronic matrix elements in a robust way
- Possible extension $\tau \rightarrow \mu(e)K^+K^-$

Summary

- LFV can probe new physics scales much higher than those directly observable at the LHC
- It is possible to observe signals of new physics via *LFV transitions* in the near future, despite the lack of new physics observed so far at the LHC
- Different operators expected at low scales, we need to measure as many processes as possible
 - ➡ *Hadronic decays* such as $\tau \rightarrow \mu(e)\pi\pi$ important!
- Possible extension $\tau \rightarrow \mu(e)K^+K^-$
- Interplay low energy and collider physics: LFV of the Higgs boson
- Complementarity with LFC sector: EDMs, g-2 and colliders:
 - ➡ New physics models usually strongly correlate these sectors

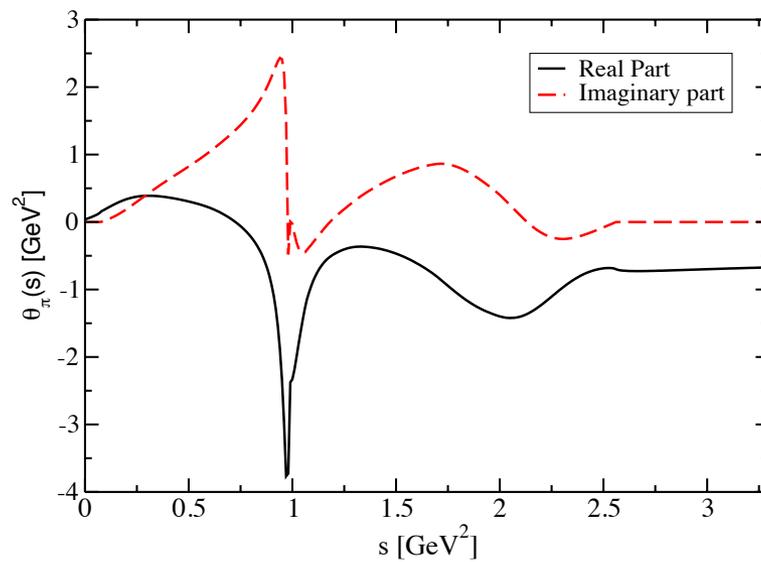
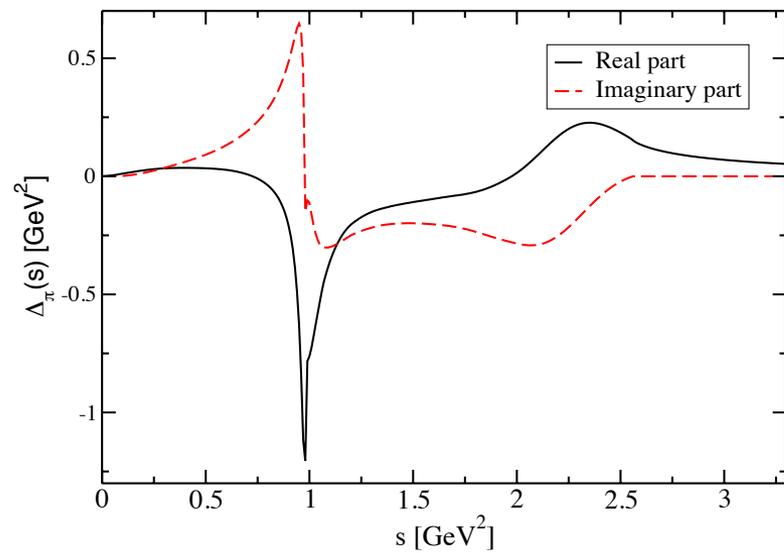
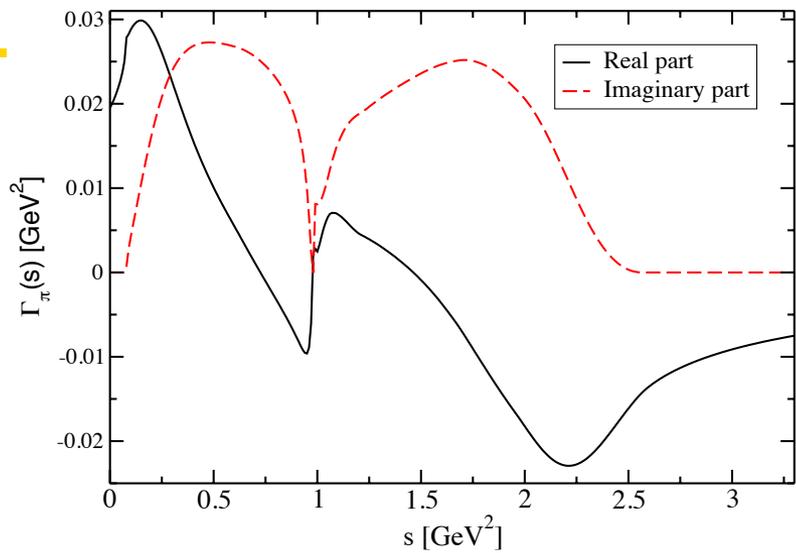
6. Back-up

T matrix parametrization

$$S_{mn} = \delta_{mn} + 2i \sqrt{\sigma_m \sigma_n} T_{mn}$$

$$S = \begin{pmatrix} \cos \gamma e^{2i\delta_\pi} & i \sin \gamma e^{i(\delta_\pi + \delta_K)} \\ i \sin \gamma e^{i(\delta_\pi + \delta_K)} & \cos \gamma e^{2i\delta_K} \end{pmatrix}$$

- Inelasticity: $\eta_0^0 \equiv \cos \gamma$.
- $\delta_\pi(s)$: $\pi\pi$ S wave phase shift
- $\delta_K(s)$: KK S wave phase shift

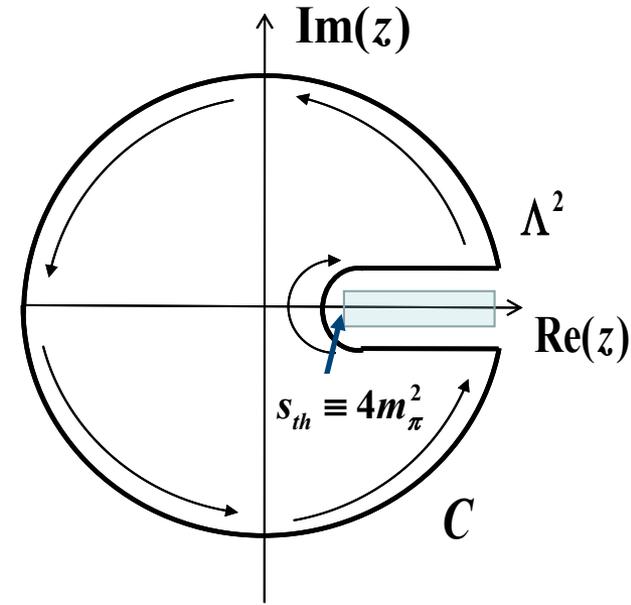


Canonical solution $X(s) = C(s), D(s)$:

- Knowing the discontinuity of $X(s)$ \Rightarrow write a dispersion relation for it

- Analyticity of the FFs: $X(z)$ is
 - real for $z < s_{th}$
 - has a branch cut for $z > s_{th}$
 - analytic for complex z

- Cauchy Theorem and Schwarz reflection principle:



$$\begin{aligned}
 X(s) &= \frac{1}{\pi} \oint_C dz \frac{X(z)}{z-s} \\
 &= \frac{1}{2i\pi} \int_{s_{th}=4M_\pi^2}^{\Lambda^2} dz \frac{\text{disc}[F(z)]}{z-s-i\epsilon} + \frac{1}{2i\pi} \int_{|z|=\Lambda^2} dz \frac{F(z)}{z-s}
 \end{aligned}$$

$\Lambda \rightarrow \infty$



$$X(s) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} dz \frac{\text{Im}[X(z)]}{z-s-i\epsilon}$$

$X(s)$ can be reconstructed everywhere from the knowledge of $\text{Im}X(s)$

3.4 Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}}F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution falling as $1/s$ for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$X(s) = C(s), D(s)$$

$$\text{Im}X_n^{(N+1)}(s) = \sum_{m=1}^2 T_{mn}^* \sigma_m(s) X_m^{(N)}(s)$$

$$X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}X_n^{(N+1)}(s')}{s' - s}$$

Determination of the polynomial

- Fix the polynomial with requiring $F_P(s) \rightarrow 1/s$ + ChPT:

Brodsky & Lepage '80

- Feynman-Hellmann theorem:

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d) B_0 + O(m^2)$$

$$M_{K^+}^2 = (m_u + m_s) B_0 + O(m^2)$$

$$M_{K^0}^2 = (m_d + m_s) B_0 + O(m^2)$$

Determination of the polynomial

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$$M_{K^+}^2 = (m_u + m_s) B_0 + O(m^2)$$

$$M_{K^0}^2 = (m_d + m_s) B_0 + O(m^2)$$



$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \dots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \dots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \dots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \dots \end{aligned}$$

Determination of the polynomial

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d) B_0 + O(m^2)$$

$$M_{K^+}^2 = (m_u + m_s) B_0 + O(m^2)$$

$$M_{K^0}^2 = (m_d + m_s) B_0 + O(m^2)$$

- For the scalar FFs:

$$P_\Gamma(s) = \Gamma_\pi(0) = M_\pi^2 + \dots$$

$$Q_\Gamma(s) = \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \dots$$

$$P_\Delta(s) = \Delta_\pi(0) = 0 + \dots$$

$$Q_\Delta(s) = \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \dots$$

- Problem: large corrections in the case of the kaons!

➡ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1) M_\pi^2$$

$$\Delta_K(0) = 1_{-0.05}^{+0.15} (M_K^2 - 1/2 M_\pi^2)$$

Daub, Dreiner, Hanart, Kubis, Meissner'13

Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

- For θ_p enforcing the asymptotic constraint is not consistent with ChPT
The unsubtracted DR is not saturated by the 2 states

➡ Relax the constraints and match to ChPT

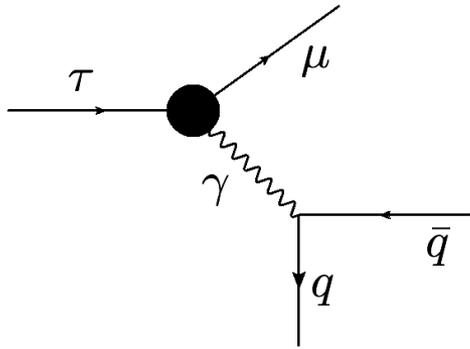
$$P_\theta(s) = 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1 \right) s$$
$$Q_\theta(s) = \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2 \right) s$$

with $\dot{f} = \left(\frac{df}{ds} \right)_{s=0}$

- At LO ChPT: $\dot{\theta}_{\pi,K} = 1$
- Higher orders ➡ $\dot{\theta}_K = 1.15 \pm 0.1$

3.1 Constraints from $\tau \rightarrow \mu\pi\pi$

- Contribution from dipole diagrams



$$L_{eff} = c_L Q_{L\gamma} + c_R Q_{R\gamma} + h.c.$$

with the dim-5 EM penguin operators :

$$Q_{L\gamma,R\gamma} = \frac{e}{8\pi^2} m_\tau (\mu \sigma^{\alpha\beta} P_{L,R} \tau) F_{\alpha\beta}$$

$$\frac{d\Gamma(\tau \rightarrow \ell \pi^+ \pi^-)}{d\sqrt{s}} = \frac{\alpha^2 |F_V(s)|^2 (|c_L|^2 + |c_R|^2)}{768\pi^5 m_\tau} \frac{(s - 4m_\pi^2)^{3/2} (m_\tau^2 - s)^2 (s + 2m_\tau^2)}{s^2}$$

with the vector form factor :

$$C_{L,R} = f(Y_{\tau\mu})$$

$$\langle \pi^+(p_{\pi^+}) \pi^-(p_{\pi^-}) | \frac{1}{2} (\bar{u} \gamma^\alpha u - \bar{d} \gamma^\alpha d) | 0 \rangle \equiv F_V(s) (p_{\pi^+} - p_{\pi^-})^\alpha$$

- Diagram only there in the case of $\tau^- \rightarrow \mu^- \pi^+ \pi^-$ absent for $\tau^- \rightarrow \mu^- \pi^0 \pi^0$
➡ neutral mode more model independent

Determination of $F_V(s)$

- Vector form factor
 - Precisely known from experimental measurements
 $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau^- \rightarrow \pi^0\pi^-\nu_\tau$ (isospin rotation)
 - Theoretically: Dispersive parametrization for $F_V(s)$

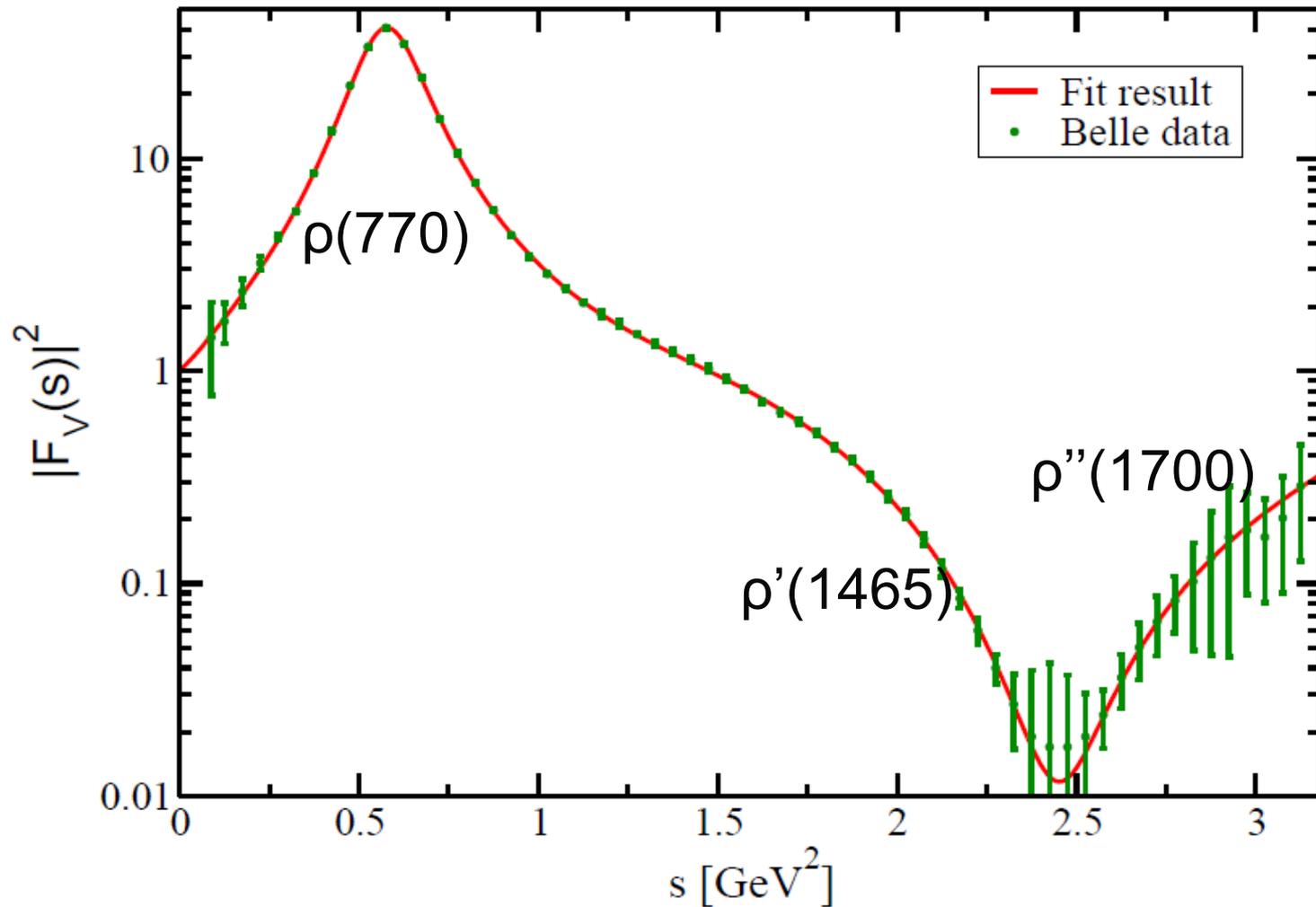
*Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13*

$$F_V(s) = \exp \left[\lambda_V' \frac{s}{m_\pi^2} + \frac{1}{2} (\lambda_V'' - \lambda_V'^2) \left(\frac{s}{m_\pi^2} \right)^2 + \frac{s^3}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s' - s - i\epsilon)} \right]$$

Extracted from a model including
3 resonances $\rho(770)$, $\rho'(1465)$
and $\rho''(1700)$ fitted to the data

- Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \rightarrow \pi^0\pi^-\nu_\tau$

Determination of $F_V(s)$

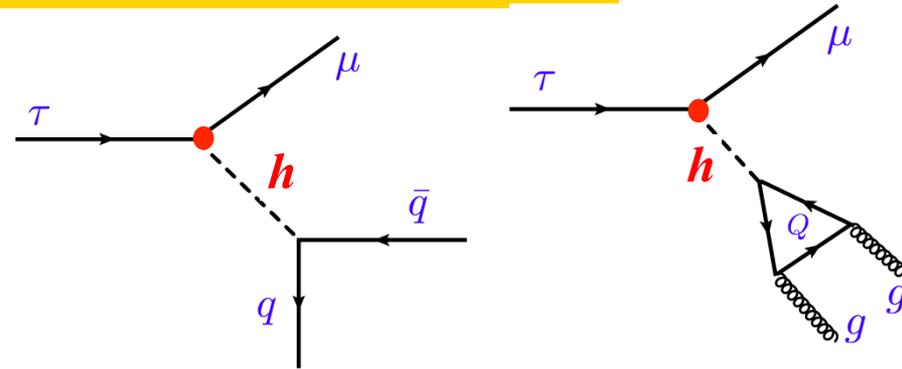


Determination of $F_V(s)$ thanks to precise measurements from Belle!

4.6 What if $\tau \rightarrow \mu(e)\pi\pi$ is observed?

Talk by J. Zupan
@ KEK-FF2014FALL

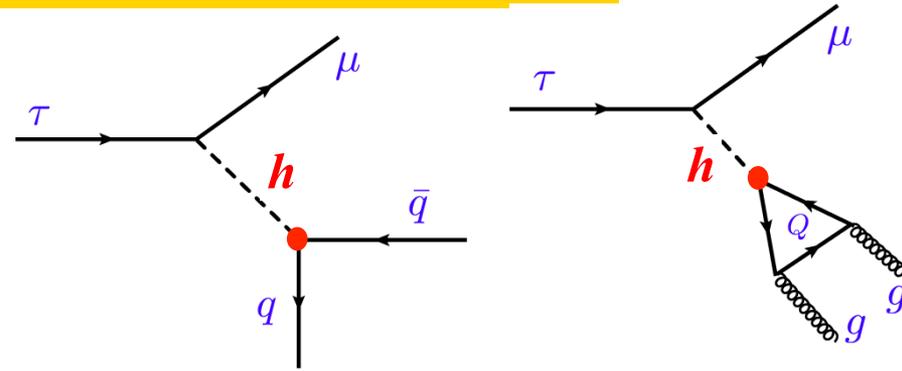
- $\tau \rightarrow \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$



4.6 What if $\tau \rightarrow \mu(e)\pi\pi$ is observed?

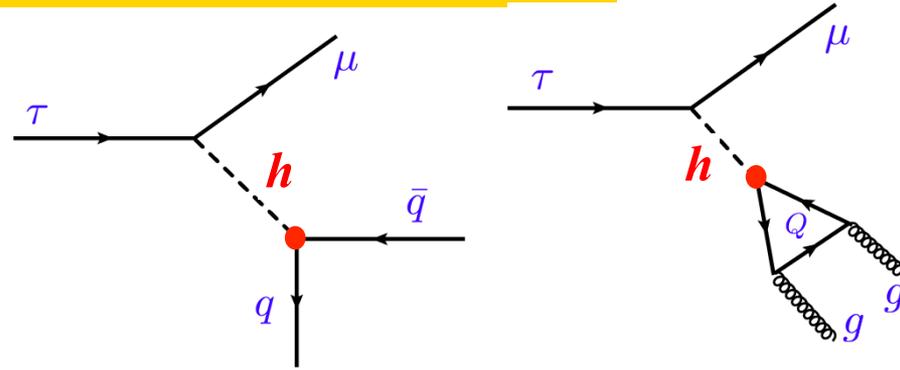
Talk by J. Zupan
@ KEK-FF2014FALL

- $\tau \rightarrow \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$
but also to $Y_{u,d,s}$!



4.6 What if $\tau \rightarrow \mu(e)\pi\pi$ is observed?

- $\tau \rightarrow \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!



- $Y_{u,d,s}$ poorly bounded

- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \rightarrow \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \rightarrow \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$

$$Br(\tau \rightarrow e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \rightarrow e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

$$Br(\tau \rightarrow \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \rightarrow \mu\pi^0\pi^0) < 1.5 \times 10^{-8}$$

$$Br(\tau \rightarrow e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \rightarrow e\pi^0\pi^0) < 2.1 \times 10^{-7}$$

below present experimental limits!

- If discovered \Rightarrow **upper limit** on $Y_{u,d,s}$!
 \Rightarrow Interplay between high-energy and low-energy constraints!

3.3 Handles

- Two handles:

➤ Branching ratios: $R_{F,M} \equiv \frac{\Gamma(\tau \rightarrow F)}{\Gamma(\tau \rightarrow F_M)}$ with F_M dominant LFV mode for model M

➤ Spectra for > 2 bodies in the final state:

$$\frac{dBR(\tau \rightarrow \mu\pi^+\pi^-)}{d\sqrt{s}} \quad \text{and} \quad dR_{\pi^+\pi^-} \equiv \frac{1}{\Gamma(\tau \rightarrow \mu\gamma)} \frac{d\Gamma(\tau \rightarrow \mu\pi^+\pi^-)}{d\sqrt{s}}$$

- Benchmarks:

➤ Dipole model: $C_D \neq 0$, $C_{\text{else}} = 0$

➤ Scalar model: $C_S \neq 0$, $C_{\text{else}} = 0$

➤ Vector (gamma, Z) model: $C_V \neq 0$, $C_{\text{else}} = 0$

➤ Gluonic model: $C_{GG} \neq 0$, $C_{\text{else}} = 0$

3.3 Branching ratios

- Two handles:

➤ Branching ratios:

$$R_{F,M} \equiv \frac{\Gamma(\tau \rightarrow F)}{\Gamma(\tau \rightarrow F_M)}$$

with F_M dominant LFV mode for model M

		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$

Benchmark

- $\rho(770)$ resonance ($J^{PC}=1^-$): cut in the $\pi^+\pi^-$ invariant mass:
 $587 \text{ MeV} \leq \sqrt{s} \leq 962 \text{ MeV}$
- $f_0(980)$ resonance ($J^{PC}=0^{++}$): cut in the $\pi^+\pi^-$ invariant mass:
 $906 \text{ MeV} \leq \sqrt{s} \leq 1065 \text{ MeV}$

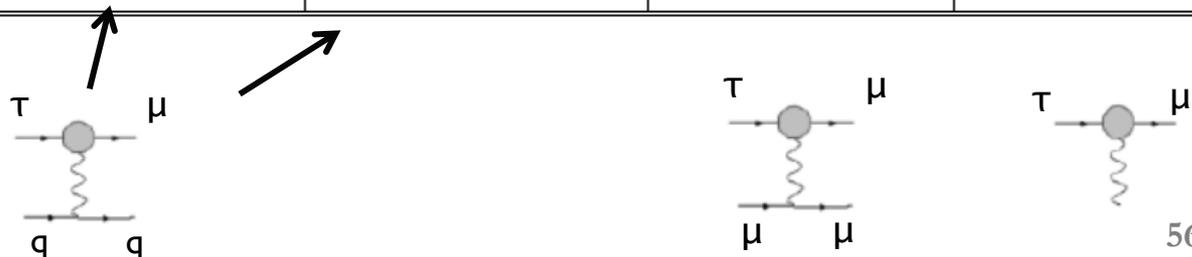
3.3 Branching ratios

- Two handles:

➤ Branching ratios: $R_{F,M} \equiv \frac{\Gamma(\tau \rightarrow F)}{\Gamma(\tau \rightarrow F_M)}$ with F_M dominant LFV mode for model M

		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$ BR	0.26×10^{-2} $< 1.1 \times 10^{-10}$	0.22×10^{-2} $< 9.7 \times 10^{-11}$	0.13×10^{-3} $< 5.7 \times 10^{-12}$	0.22×10^{-2} $< 9.7 \times 10^{-11}$	1 $< 4.4 \times 10^{-8}$
S	$R_{F,S}$ BR	1 $< 2.1 \times 10^{-8}$	0.28 $< 5.9 \times 10^{-9}$	0.7 $< 1.47 \times 10^{-8}$	- -	- -
$V(\gamma)$	$R_{F,V(\gamma)}$ BR	1 $< 1.4 \times 10^{-8}$	0.86 $< 1.2 \times 10^{-8}$	0.1 $< 1.4 \times 10^{-9}$	- -	- -
Z	$R_{F,Z}$ BR	1 $< 1.4 \times 10^{-8}$	0.86 $< 1.2 \times 10^{-8}$	0.1 $< 1.4 \times 10^{-9}$	- -	- -
G	$R_{F,G}$ BR	1 $< 2.1 \times 10^{-8}$	0.41 $< 8.6 \times 10^{-9}$	0.41 $< 8.6 \times 10^{-9}$	- -	- -

↑
Benchmark



4.1 Constraints from $\tau \rightarrow l\mathbb{P}$

- Tree level Higgs exchange
 - η, η'

$$\Gamma(\tau \rightarrow l\eta^{(l)}) = \frac{\bar{\beta}(m_\tau^2 - m_\eta^2)(|Y_{\mu\tau}^A|^2 + |Y_{\tau\mu}^A|^2)}{256\pi M_A^4 v^2 m_\tau} \left[(y_u^A + y_d^A)h_{\eta'}^q + \sqrt{2}y_s^A h_{\eta'}^s - \sqrt{2}a_{\eta'} \sum_{q=c,b,t} y_q^A \right]^2$$

with the decay constants :

$$\langle \eta^{(l)}(p) | \bar{q} \gamma_5 q | 0 \rangle = -\frac{i}{2\sqrt{2}m_q} h_{\eta^{(l)}}^q \quad \langle \eta^{(l)}(p) | \bar{s} \gamma_5 s | 0 \rangle = -\frac{i}{2m_s} h_{\eta^{(l)}}^s$$

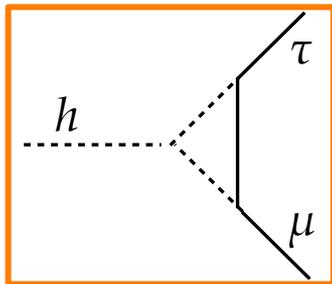
$$\langle \eta^{(l)}(p) | \frac{\alpha_s}{4\pi} G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a | 0 \rangle = a_{\eta^{(l)}}$$

$$\text{➤ } \boldsymbol{\pi} : \Gamma(\tau \rightarrow l\pi^0) = \frac{f_\pi^2 m_\pi^4 m_\tau}{256\pi M_A^4 v^2} (|Y_{\tau\mu}^A|^2 + |Y_{\mu\tau}^A|^2) (y_u^A - y_d^A)^2$$

4.5 Interplay between LHC & Low Energy

Dorsner et al.'15

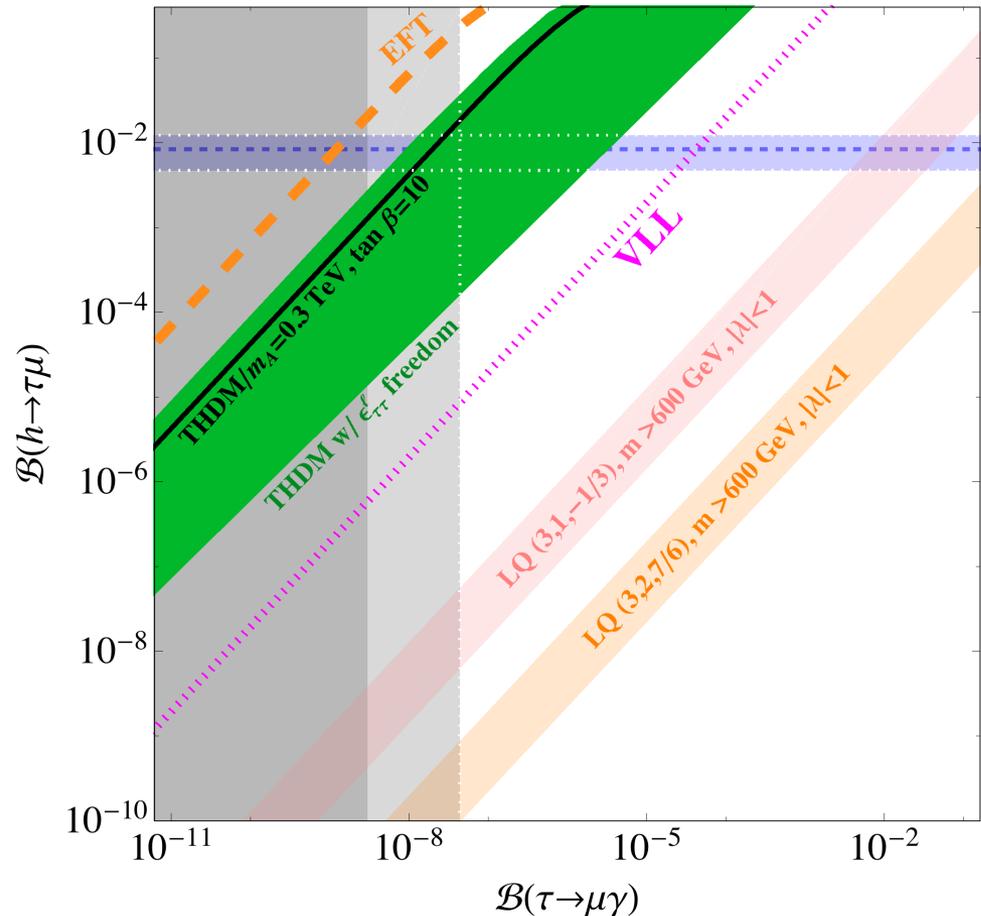
- If real what type of NP?
- If $h \rightarrow \tau \mu$ due to loop corrections:
 - extra charged particles necessary
 - $\tau \rightarrow \mu \gamma$ too large



- $h \rightarrow \tau \mu$ possible to explain if extra scalar doublet:

➡ *2HDM of type III*

- Constraints from $\tau \rightarrow \mu \gamma$ important! ➡ *Belle II*



4.5 Interplay between LHC & Low Energy

- **2HDMs** with gauged $L_\mu - L_\tau$
 - ➔ Z' , explain anomalies for
 - $h \rightarrow \tau\mu$
 - $B \rightarrow K^*\mu\mu$
 - $R_K=B \rightarrow K\mu\mu / B \rightarrow Kee$
- Constraints from $\tau \rightarrow 3\mu$
 - crucial** ➔ *Belle II, LHCb*
- See also:
 - Aristizabal-Sierra & Vicente'14,*
 - Lima et al'15,*
 - Omhura, Senaha, Tobe '15*

Altmannshofer & Straub'14, Crivellin et al'15
Crivellin, D'Ambrosio, Heeck.'15

$\cos(\alpha_{23}-\beta_{23}) = 0.25, a = 1/3$

