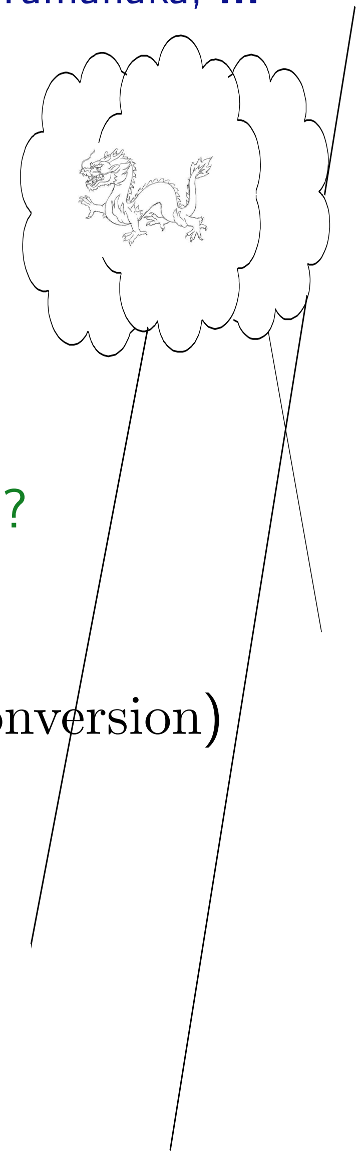
...., 1810.01884, 1710.06787, 1703.02057

(Bonne Année 2019!)

To Learn about Lepton Flavour Change ($\mu \to e$) ?

Vincenzo Cirigliano, Sacha Davidson , Yoshi Kuno, Albert Saporta, Masato Yamanaka, ...

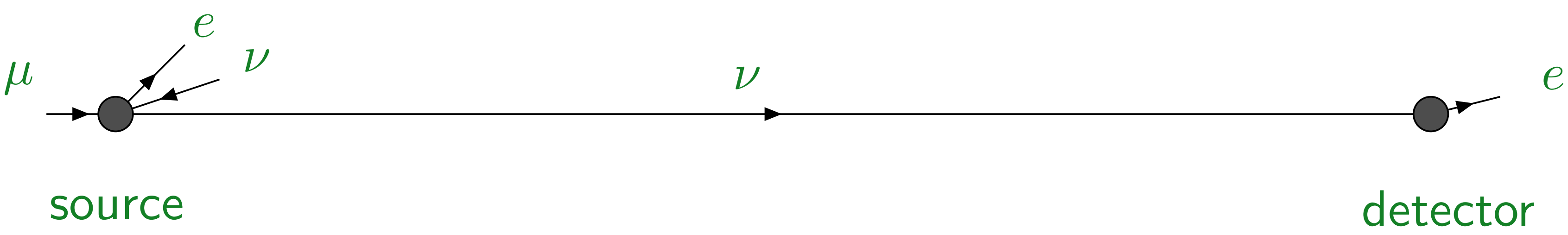


1. Lepton Flavour Violation
   - what is it, why interesting + what do we know ?
     (exptal reach in $\mu \leftrightarrow e$ to improve by $\sim 10^4$ in a few years)
2. From an EFT perspective, what can we learn?
   - at the experimental scale (distinguishing operator coefficients)?
   - at the New Physics scale? (... RGEs)
3. counting coefficients + constraints ($\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\boldsymbol{\mu \to e}$ conversion)
4. ?thinking about "fine-tuning"?

What is Lepton Flavour Violation?

- three lepton flavour in the Standard Model : $\boldsymbol{e}, \boldsymbol{\mu}, \tau$
(curious: 6 quark flavours)

- LFV $\equiv$ charged lepton flavour change, at a point.
  ν are shy, and quantum over thousands of km



$\Rightarrow$ ν oscillations don't count.

Some LFV processes and bounds

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (2018, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUM)	10^{-16} (Mu2e,COMET)
		10^{-18} (PRISM/PRIME)
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)

$\mu A \to eA \equiv \mu$ in $1s$ state of nucleus A converts to e

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		10^{-18} (PRISM/PRIME)
$\overline{K_L^0} \to \mu\bar{e}$	$< 4.7 \times 10^{-12}$ (BNL)	
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II, LHCb?)
$\tau \to e\phi$	$< 3.1 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$h \to \tau^{\pm}e^{\mp}$	$< 6.9 \times 10^{-3}$	
$Z \to e^{\pm}\mu^{\mp}$	$< 7.5 \times 10^{-7}$	

LFV in EFT

1. Lepton Flavour Change is interesting:
   - none in the Standard Model with $m_\nu = 0$
   - occurs with m_ν and mixing matrix U

   m_ν renormalisable Dirac: LFV amplitudes GIM-suppressed (like quarks)

$$\mathcal{A} \propto \frac{m_\nu^2}{m_W^2} \Rightarrow BR \lesssim 10^{-48}$$

$\Rightarrow$ if see LFV, lepton flavour sector different from quarks!

suppose m_ν NOT Dirac, New Physics heavy

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2. use EFT to learn about heavy NP for LFV
   - parametrize LFV as contact interactions with constant coefficients
   - extract coefficients from data

     $\Rightarrow$ how many operator coefficients can be constrained?

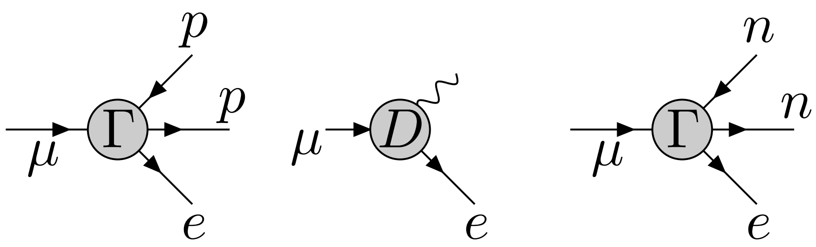
   - what do coefficients tell about New Physics?

     $\Rightarrow$ use SM RGEs to translate constraints from exptal scale to NP scale

lets try to do this with $\mu \to e$ conversion...

$\mu \to e$ conversion

- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM: muon capture $\mu + p \to \nu + n$
- bound μ interacts with nucleus, converts to e $(E_e \approx m_\mu)$



$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V\ ,\ A\ ,\ T\}$$

$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

$$BR_{SI} \sim \quad Z^2 |\sum \tilde{C}_{SI}|^2 \quad , \quad \tilde{C}_{SI} \in \{\tilde{C}^p_V, \tilde{C}^p_S, \tilde{C}^n_V, \tilde{C}^n_S, C_D\}$$

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over nucleons $\propto$ spin of only unpaired nucleon)

$$BR_{SD} \sim \quad |\sum \tilde{C}_{SD}|^2 \quad , \quad \tilde{C}_{SD} \in \{\tilde{C}^p_A, \tilde{C}^p_T\}, \{\tilde{C}^n_A, \tilde{C}^n_T\}$$

(The operator basis for $\mu \to e$ conversion)

At Λ_{expt}, μ interaction with nucleon $N \in \{n, p\}$ parametrised by 22 operators :

D	$\overline{e}\sigma^{\alpha\beta}P_{R,L}\mu F_{\alpha\beta}$	
$S,\ V$	$\overline{e}P_{R,L}\mu\overline{N}N$	$\overline{e}\gamma^{\alpha}P_{L,R}\mu\overline{N}\gamma_{\alpha}N$
$A,\ T$	$\overline{e}\gamma^{\alpha}P_{L,R}\mu\overline{N}\gamma_{\alpha}\gamma_5 N$	$\overline{e}\sigma^{\alpha\beta}P_{R,L}\mu\overline{N}\sigma_{\alpha\beta}N$
$P,\ Der$	$\overline{e}P_{L,R}\mu\overline{N}\gamma_5 N$	$\overline{e}\gamma^{\alpha}P_{L}\mu(\overline{N}i\overset{\leftrightarrow}{\partial}_{\alpha}\gamma_5 N)$

chiral basis for the lepton current (relativistic e), but not for the non-rel. nucleons.

Matching at LO in χPT gives Derivative. But can absorb into matching constants = quark matrix elements in nucleons.

add to $\mathcal{L}$ as :

$$2\sqrt{2}G_F \sum_{X\in\{L,R\}} \sum_{N\in\{p,n\}} \left(m_\mu \tilde{C}_{D,X}\mathcal{O}_{D,X} + \sum_O \tilde{C}_O^{NN}\mathcal{O}_{O,X}^{NN} \right)$$

To constrain/distinguish 22 coefficients

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$$\mathrm{BR}_{SD} \sim \left|\widetilde{C}^{NN}_{A,L} + 2\widetilde{C}^{NN}_{T,R}\right|^2 + \left|\widetilde{C}^{NN}_{A,R} + 2\widetilde{C}^{NN}_{T,L}\right|^2 \qquad (N \text{ odd})$$

$$\mathrm{BR}_{SI} \propto |\widetilde{C}^{pp}_{V,R}V^{(p)}_A + \widetilde{C}'^{pp}_{S,L}S^{(p)}_A + \widetilde{C}^{nn}_{V,R}V^{(n)}_A + \widetilde{C}'^{nn}_{S,L}S^{(n)}_A + C_{D,L}D|^2 + \{L \leftrightarrow R\}$$

$$\sim Z^2\left|\vec{C}_R \cdot \hat{v}_A\right|^2 + Z^2\left|\vec{C}_L \cdot \hat{v}_A\right|^2 \qquad \vec{v}_A \equiv \left(V^{(p)}_A, S^{(p)}_A, V^{(n)}_A, S^{(n)}_A, D_A\right)$$

(Can distinguish SD vs SI, L vs R,

KitanoKoikeOkada

$$S^{(p)}_A, V^{(p)}_A \sim \int d^3x \widetilde{\psi}^{1s}_\mu |f_p(x)|^2 \widetilde{\psi}^*_e (\bar{p}\{1, \gamma_0\}p)$$

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$$\sim Z^2\left|\vec{C}\cdot\hat{v}_A\right|^2 \qquad \vec{v}_A \equiv \left(V^{(p)}_A, S^{(p)}_A, V^{(n)}_A, S^{(n)}_A, D_A\right)$$

(Can distinguish SD vs SI, L vs R,also V vs S,p vs n ...)

KitanoKoikeOkada...
KKO calculate integrals

Focus on SI, where "target vector" $\vec{v}_A$ made of overlap integrals:

$$S^{(p)}, V^{(p)} \sim \int d^3x \widetilde{\psi}^{1s}_\mu |f_p(x)|^2 \widetilde{\psi}^*_e (\bar{p}\{1,\gamma_0\}p)$$

- neglect C_D (better sensitivity of upcoming $\mu \to e\gamma$ and $\mu \to e\bar{e}e$).
- 1st experimental search (eg Gold) probes projection of $\vec{C}$ of $\vec{v}_{Au}$
  ... next target needs to have component $\perp$ to Gold!
  $\Leftrightarrow$ find targets with sufficiently large misalignment angle
- by changing nucleus A (+improve theory caln), could probe three combinations of coefficients: Σ, $n-p$, $V-S$

(S vs V : wavefn ψ_e of outgoing e distorted in heavy nuclei (KKO))

What to learn at Λ_{exp}: setting constraints ($\mu A \to eA$,$\mu \to e\gamma$ + $\mu \to e\bar{e}e$)

parametrise with 22 nucleon ops (10 SI: D,S,V) + (12 SD: P,A,T)
2 dipole operators
6 four-lepton operators

1. constrain 2 dipoles +6 4ℓ coeffs with $\mu \to e\gamma$ + $\mu \to e\bar{e}e$

$$BR(\mu \to e\gamma) = 384\pi^2(|C_{D,R}|^2 + |C_{D,L}|^2) \Rightarrow |C_{D,X}| \lesssim 10^{-8}$$

$$BR(\mu \to e\bar{e}e) = \frac{1}{8}(|C_{(\overline{e}P_L\mu)(\overline{e}P_Le)}|^2 + |C_{(\overline{e}P_R\mu)(\overline{e}P_Re)}|^2) + 2|C_{(\overline{e}\gamma P_R\mu)(\overline{e}\gamma P_Re)}|^2 + 2|C_{(\overline{e}\gamma P_L\mu)(\overline{e}\gamma P_Le)}|^2$$
$$+|C_{(\overline{e}\gamma P_L\mu)(\overline{e}\gamma P_Re)}|^2 + |C_{(\overline{e}\gamma P_R\mu)(\overline{e}\gamma P_Le)}|^2 \qquad \Rightarrow \quad |C_{..}| \lesssim 10^{-6}$$

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$$+|C_{(\overline{e}\gamma P_L\mu)(\overline{e}\gamma P_Re)}|^2 + |C_{(\overline{e}\gamma P_R\mu)(\overline{e}\gamma P_Le)}|^2 \qquad \Rightarrow \quad |C_{..}| \lesssim 10^{-6}$$

2. constrain 6 combinations of 8 $\{S,V\}$ coefficients

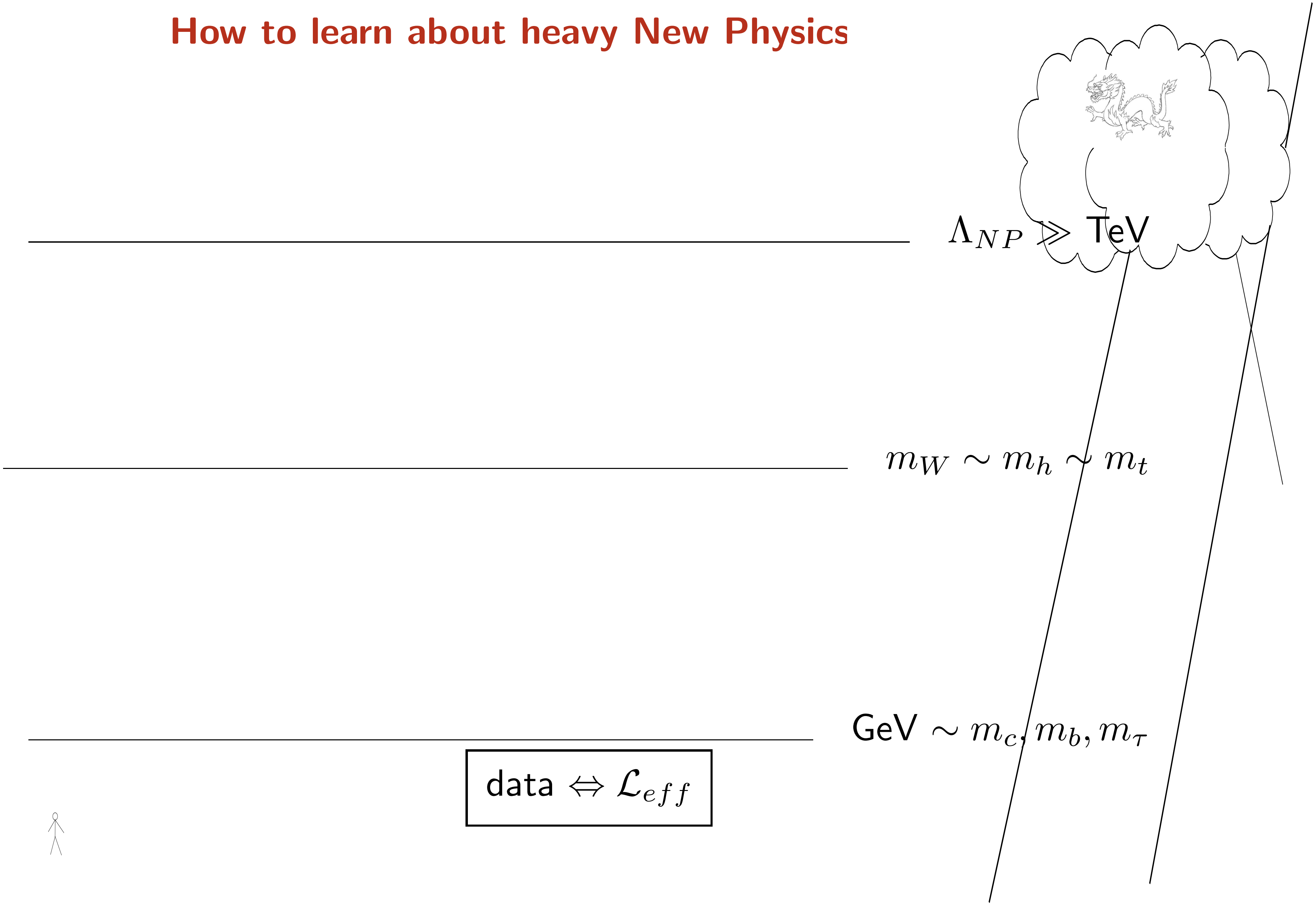
3. Spin-Dependent: $4 \to 8$ constraints ?

can do n vs p by comparing odd-p, A vs T vs P $\Leftrightarrow$ dedicated nucl.caln.)

$\Rightarrow$ 28 coefficients, 18 $\to$ 22 constraints

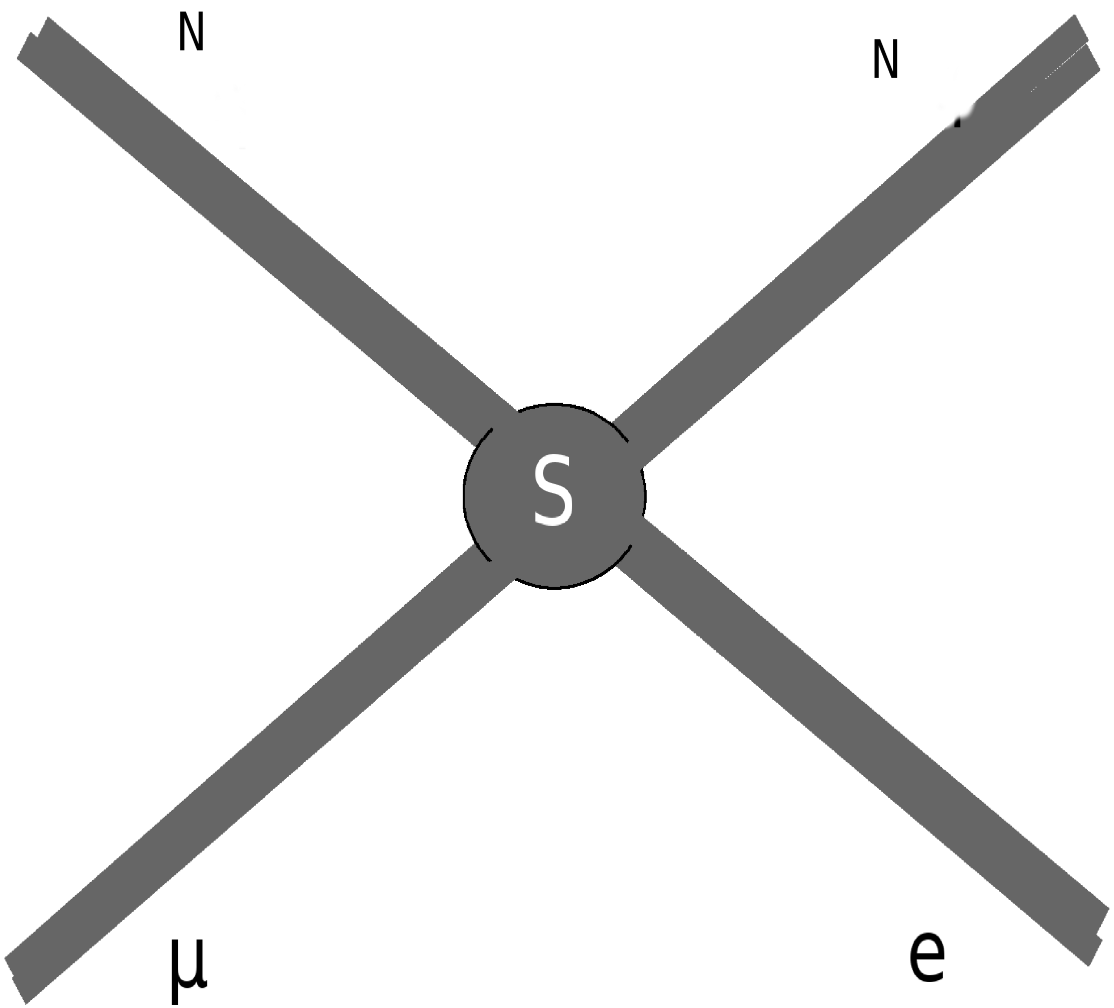
Georgi, EFT, ARNPP 43(93) 209

How to learn about heavy New Physics



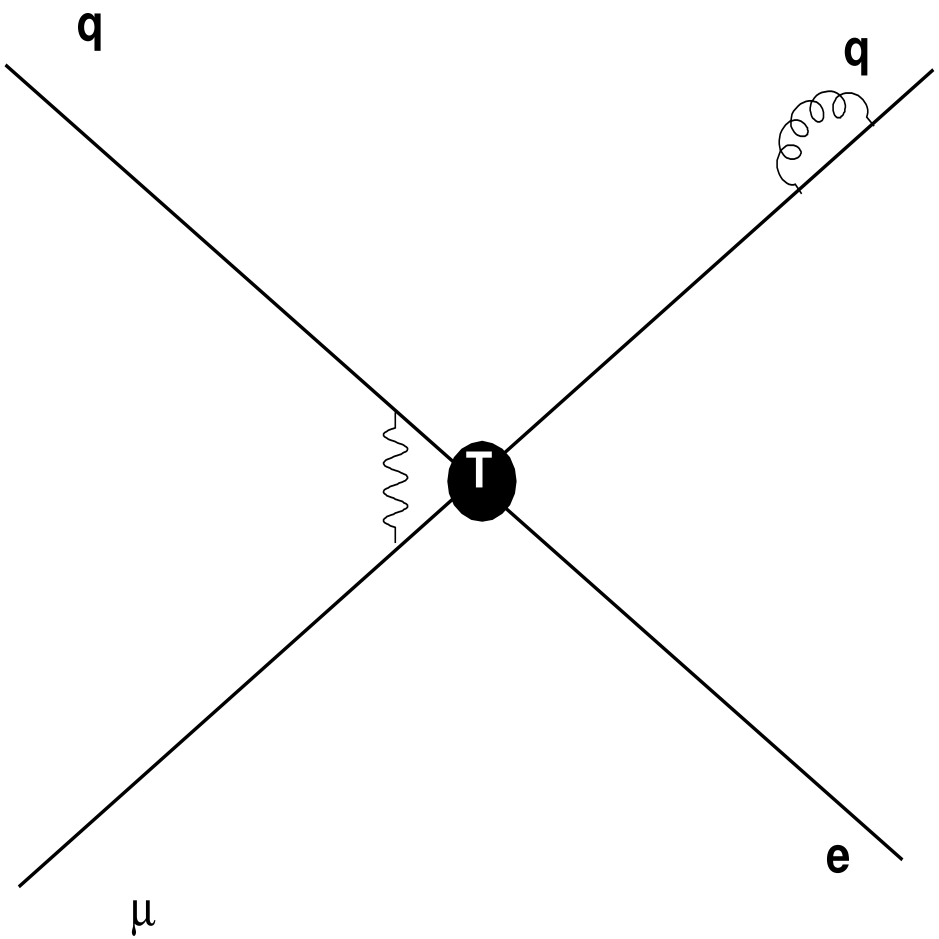
Peeling off the SM loop corrections

expt measures operator coefficient $\widetilde{C}(\mu_{exp})$, at exptal energy scale $\sim m_\mu \to m_\tau$, among external legs at same scale...



Peeling off SM loops

But if I look on shorter distance scale ($\sim 1/m_W$) I might see



The operator basis below m_W : 82 operators

Add QCD×QED-invar operators, representing all 3,4 point interactions of μ with e and *flavour-diagonal* combination of γ, g, u, d, s, c, b. $Y \in L, R$.

$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta}$ *dim* 5

$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e)$ $(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$

$(\overline{e}P_Y\mu)(\overline{e}P_Y e)$ *dim* 6

$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$ $(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$

$(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$

$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f)$ $(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$

$(\overline{e}P_Y\mu)(\overline{f}P_Y f)$ $(\overline{e}P_Y\mu)(\overline{f}P_X f)$ $f \in \{u, d, s, c, b, \tau\}$

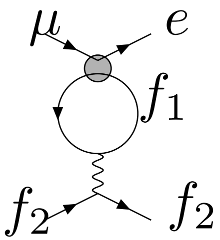
$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$

$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta}$ $\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta}$ *dim* 7

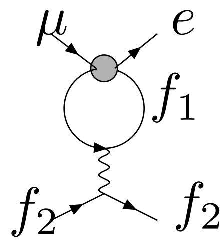
$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta}$ $\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$

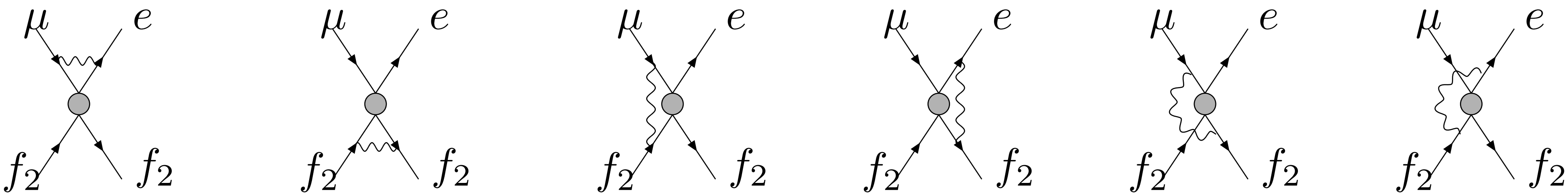
...zzz...but 82 coeffs!
(recall: 18-22 constraints)

$(P_X, P_Y = (1 \pm \gamma_5)/2)$, all operators with coeff $-2\sqrt{2}G_F C$.



Run with QED + QCD between m_μ and m_W





$$\mu\frac{\partial}{\partial\mu}\vec{C} = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}$$

QCD: not mix ops, should resum $\Rightarrow$ multiplicative renorm S,T ops(neglect)
QED: *does* mix ops, but $\alpha_{em} \ll$:

$$C_A(m_W)\left(\delta_{AB} - \frac{\alpha_{em}}{4\pi}[\Gamma]_{AB}\log\frac{m_W}{m_\tau} + \frac{\alpha_{em}^2}{32\pi^2}[\Gamma\Gamma]_{AB}\log^2\frac{m_W}{m_\tau} + ..\right) = C_B(m_\tau)$$

NB: at one loop: $\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Gamma}_V & 0 \\ 0 & \mathbf{\Gamma}_{STD} \end{bmatrix}$, so at one loop, almost all 82 operators contribute to $\boldsymbol{\mu \to e}$ conversion.

at tree, 14 (+2+2) coefficients contribute to $\mu \to e$ conversion (2 dipoles+2digluons)

$$\sqrt{\frac{BR_{Al}^{exp}}{33}} \gtrsim \left| 3C_{V,L}^{uu} + 3C_{V,L}^{dd} + 11C_{S,R}^{uu} + 11C_{S,R}^{dd} + 0.84C_{S,R}^{ss} + \frac{4m_N}{27m_c}C_{S,R}^{cc} + \frac{4m_N}{27m_b}C_{S,R}^{bb} \right|$$

also constraint on coeffs with $L \leftrightarrow R$ (the chirality of e)

quark coefficients at 2 GeV(lattice matching $\{G_S^{Nq}\}$)

at one loop, 44 +2+2 of 82 operators contribute to $\mu \to e$ conversion (2 dipoles+2digluons)

$$
\begin{aligned}
\sqrt{\frac{BR^{exp}_{Al}}{33}} \quad \gtrsim \quad & \Big| 3C^{uu}_{V,L} + 3C^{dd}_{V,L} + \frac{\alpha}{\pi}\Big[3C^{dd}_{A,L} - 6C^{uu}_{A,L}\Big] \log \\
& + \frac{\alpha}{3\pi}[C^{ee}_{V,L} + C^{\mu\mu}_{V,L}] \log - \frac{\alpha}{3\pi}[C^{ee}_{A,L} + C^{\mu\mu}_{A,L}] \log \\
& - \frac{2\alpha}{3\pi}\Big[2(C^{uu}_{V,L} + C^{cc}_{V,L}) - (C^{dd}_{V,L} + C^{ss}_{V,L} + C^{bb}_{V,L}) - (C^{ee}_{V,L} + C^{\mu\mu}_{V,L} + C^{\tau\tau}_{V,L})\Big] \log \\
& + \lambda^{-a_S}\Big(11C^{uu}_{S,R} + 11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R}\Big) \\
& + \lambda^{-a_S}\frac{\alpha}{\pi}\Big[\frac{13}{6}(11C^{uu}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R}) + \frac{5}{3}(11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R})\Big] \log \\
& - \lambda^{a_T} f_{TS}\frac{8\alpha}{\pi}\Big[22C^{uu}_{T,R} + \frac{8m_N}{27m_c}C^{cc}_{T,R} - 11C^{dd}_{T,R} - 0.84C^{ss}_{T,R} - \frac{4m_N}{27m_b}C^{bb}_{T,R})\Big] \log \Big|
\end{aligned}
$$

also constraint on coeffs with $L \leftrightarrow R$ (the chirality of e)

quark coefficients at m_W

$\log \equiv \log(m_W/2\text{GeV}) \simeq 3.7$,

$\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

How much cancellation to beleive? (“fine-tuning of coefficients”?)

suppose $\{C(\Lambda_{expt})\}$ parametrise renormalisable, natural high-scale model.

1. allow arbitrary cancellations among $\{C(\Lambda_{NP})\}$

   ($\{C(\Lambda_{NP})\}$ unknown functions of the model parameters, symmetry-based cancellations could appear fortuitous?)

2. assume model not know Λ_{expt} (despite that is determined by mass ratios which models knows)

   so coefficients not cancel against logs

$\Rightarrow$ allow: $|C_1 + nC_2| = 0$, not allow: $|C_1 + n\alpha C_2 \log| = 0$

• QCD-running of scalars and tensors $\Leftrightarrow$ not cancel S vs T vs V to more than one sig fig

$$\Rightarrow |\sum_j C_{V,j} + \lambda^{a_S} \sum_k C_{S,k} + \lambda^{a_T} \sum_i C_{T,i}| < \epsilon \longrightarrow \begin{cases} |\sum_j C_{V,j}| < 10\epsilon \\ |\lambda^{a_S} \sum_k C_{S,k}| < 10\epsilon \\ |\lambda^{a_T} \sum_i C_{T,i}| < 10\epsilon \end{cases}$$

Then within each subset, at each order in $\alpha \log$, allow cancellations up to next order $\sim \mathcal{O}(\frac{\alpha}{4\pi} \log)$:

$$\text{if} \quad |\Sigma_j n_j C_j| < \epsilon \Rightarrow C_j \lesssim \frac{4\pi}{\alpha} \frac{\epsilon}{n_j}$$

Which coefficients are missing? Why?

τ scalars and tensors that mix at one loop to $\mu \to e\gamma$:

$$(\overline{e}P_Y\mu)(\overline{\tau}P_Y\tau) \qquad (\overline{e}P_Y\mu)(\overline{\tau}P_X\tau) \qquad X \neq Y \in \{L, R\}$$

$$(\overline{e}\sigma P_Y\mu)(\overline{\tau}\sigma P_Y\tau)$$

e scalars $(\overline{e}P_Y\mu)(\overline{e}P_Y e)$ constrained at tree by $\mu \to e\bar{e}e$

μ scalars $(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$ contributes at 2 loop to $\mu \to e\gamma$

axial operators $(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha f)$ for $f \in \{\tau, c, s, b\}$

pseudoscalar operators $(\overline{e}P_Y\mu)(\overline{f}\gamma_5 f)$ for $f \in \{\tau, u, d, c, s, b\}$

diphotons + CPV digluons

Summary

Observable Lepton Flavour Violation would imply that the neutrino mass/lepton flavour sector is different from the quarks, and give information on the required New Physics.

sensitivive probes to $\mu \leftrightarrow e$ are $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and $\boldsymbol{\mu \to e}$ conversion (= conversion of a μ, in the $1s$ state of a nucleus, into an electron who escapes with $E_e \sim m_\nu$). Currently reach $\Lambda_{LFV} \sim 10^3$ TeV.
New expts will extend this reach to $\Lambda_{LFV} \sim 10^4$ TeV in the next few years.

In an EFT perspective, the experiments set ~ 20 independent constraints. But there are ~ 80 $\mu \leftrightarrow e$ (and otherwise flavour diagonal) operators in the QCD*QED invar operator basi below m_W...
$\Rightarrow$ find more constraints?
$\Rightarrow$ think about defining "fine-tuning" in EFT?

BackUp

To calculate the $\mu \to e$ conversion rate (like WIMP scattering on nuclei)

build the nucleus as a bound state of nucleons, ($|f_N(x)|^2$= distribution of N in nucleus A)
bind muon in $1s$ state. For 4-ferm operators :

SI overlap int: KitanoKoikeOkada

$$\mathcal{M} \sim \sum_{N,O} \widetilde{C}_O(\bar{u}_e \Gamma_O u_\mu) \int d^3x \widetilde{\psi}_\mu^{1s} |f_N(x)|^2 e^{-iqx} (\bar{N}\Gamma_O N)$$

SD overlap int: guess from SD DM targets

For light nuclei ($Z \lesssim 30$), $\widetilde{\psi}_\mu^{1s} \simeq$ constant in nucleus, $\Rightarrow$ use WIMP results.
eg for **Spin-Dependent**: ($S_N^A \equiv$ spin expect. value of nucleon N in nucleus A of spin J_A. $S_N^A \simeq 1/2$).

$$\sum_{N\in A} \int d^3x |f_N(\vec{x})|^2 (\overline{u}_N \gamma^k \gamma_5 u_N) = 4m_N S_N^A \frac{J_A^k}{|J_A|} \quad ,$$

Engel,...

also at $q^2 \to 0$: $\overline{N}\sigma N = 2\overline{N}\gamma\gamma_5 N$, $\overline{N}\gamma_5 N \to 0$ so with $\widetilde{C}'^{pp}_{A,L} \equiv \widetilde{C}^{pp}_{A,L} + 2\widetilde{C}'^{pp}_{T,R}$:

$$\frac{\Gamma_{SD}}{\Gamma_{capt}} \simeq \frac{8G_F^2 m_\mu^5}{\Gamma_{capt}\pi^2}(Z\alpha)^3 \frac{J_A+1}{J_A} \frac{S_A(m_\mu)}{S_A(0)} \left[\left| S_p^A \widetilde{C}'^{pp}_{A,L} + S_n^A \widetilde{C}'^{nn}_{A,L} \right|^2 + \{L \leftrightarrow R\} \right]$$

$$BR_{SD} \sim \left| \widetilde{C}^{NN}_{A,L} + 2\widetilde{C}^{NN}_{T,R} \right|^2 + \left| \widetilde{C}^{NN}_{A,R} + 2\widetilde{C}^{NN}_{T,L} \right|^2$$

CiriglianoDavidsonKuno

$S_A(q)$ finite momentum transfer correction (exists only for Axial) for Al $\simeq 0.29$ EngelRTO,KlosMGS
(also can make WIMP approx for low-Z SI $\mu \to e$ conversion)

The Spin-Independent $\mu \to e$ conversion rate

build the nucleus as a bound state of nucleons,
bind muon in $1s$ state. For 4-ferm operators :

SI overlap int: KitanoKoikeOkada

$$\mathcal{M} \sim \sum_{N,O} \widetilde{C}_O(\bar{u}_e \Gamma_O u_\mu) \int d^3x \widetilde{\psi}_\mu^{1s} |f_N(x)|^2 \widetilde{\psi}_e^* (\bar{N} \Gamma_O N)$$

SD overlap int: guess from SD DM targets

For heavy nuclei, $\widetilde{\psi}_\mu^{1s} \simeq$ varies in nucleus, $\Rightarrow$ evaluate overlap integrals.
For **Spin Independent** operators (D,S,V) KKO calculated "overlap integrals" of wavefns $\widetilde{\psi}_\mu^{1s}$, $\widetilde{\psi}_e \sim e^{iqx}$ and (for 4-f ops) operator $\times$nucleon density ($|f_N(x)|^2$):

$$S^{(p)}, V^{(p)} \quad \sim \quad \int d^3x \widetilde{\psi}_\mu^{1s} |f_p(x)|^2 \widetilde{\psi}_e^* (\bar{p}\{1, \gamma_0\} p)$$

Distortion of $\widetilde{\psi}_e$ at high Z causes $V^{(N)} > S^{(N)}$

$$\mathrm{BR}_{SI} = \frac{32 G_F^2 m_\mu^5}{\Gamma_{cap}} \Big[|\widetilde{C}_{V,R}^{pp} V^{(p)} + \widetilde{C}_{S,L}^{\prime pp} S^{(p)} + \widetilde{C}_{V,R}^{nn} V^{(n)} + \widetilde{C}_{S,L}^{\prime nn} S^{(n)} + C_{D,L} D|^2 + \{L \leftrightarrow R\} \Big]$$

$$\simeq \quad Z^2 |\sum \tilde{C}|^2 \quad \sim \quad Z^2 \frac{|\sum \tilde{C}_{SI}|^2}{|\sum \tilde{C}_{SD}|^2} BR_{SD}$$

caveats to (our) Spin Dep Estimates

make approximation
A,T overlap integrals $\leftrightarrow$ nuclear expectation value of spin current
SD $\mu \to e$ conversion DM WIMP scattering

1. to use SD WIMP results, must be able to factor ψ_μ out of overlap integral
   but for "heavier" nuclei, $R_{nucleus} > R_{\psi_\mu} \sim \alpha/m_\mu$

2. SD WIMP results for Axial currents of nucleons
   at $q^2 = 0$, pseudoscalar vanishes and tensor current $\propto$ axial:

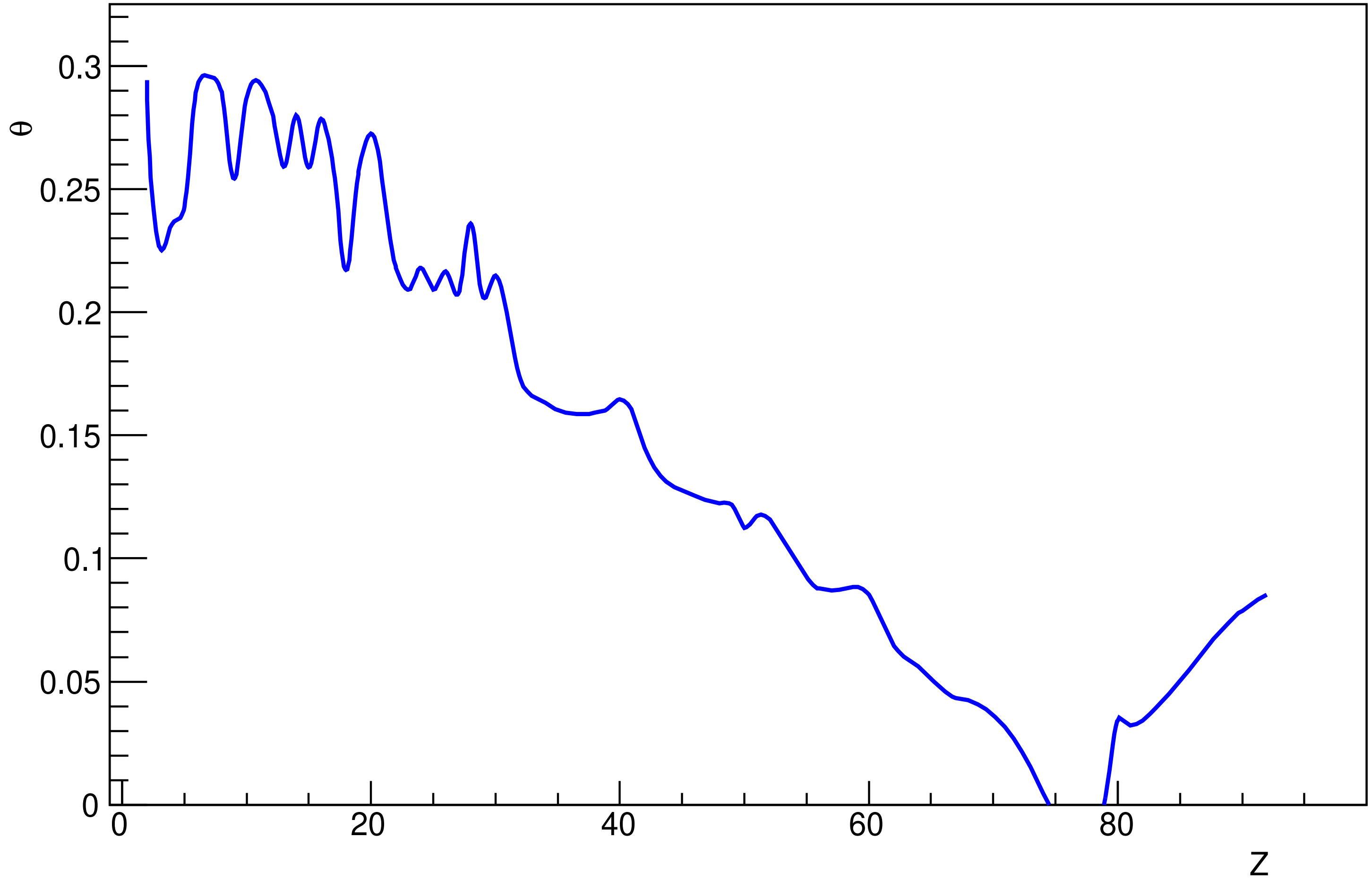
$$\begin{aligned} \overline{u}^o_N(P_f)\gamma_5 u^t_N(P_i) &\to 2\vec{q}\cdot\vec{S}_N \\ \overline{u}^o_N(P_f)\gamma^j\gamma_5 u^t_N(P_i) &\to 4m_N S^j_N \\ \overline{u}^o_N(P_f)\sigma_{ik} u^t_N(P_i) &\to 4m_N \epsilon_{ikj} S^j_N \end{aligned}$$

spin vector of the nucleon: $2\vec{S}_N = u^\dagger_N \vec{\Sigma} u_N / 2E_N$
rotation generator : $S^{ij} = \frac{i}{4}[\gamma^i, \gamma^j] = \frac{1}{2}\epsilon^{ijk}\Sigma^k$.

But $q^2 = m^2_\mu$...what about P, and $A \neq T$ because no pion exchange to T.

Misalignment angle between gold (Au,Z=79) and other targets

$$BR(\mu Au \to e Au) \leq 7 \times 10^{-13}$$



$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)})$, and $BR \propto |\vec{v}_A \cdot \vec{C}|^2$

$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta$ ...plot θ on vertical axis

Quantifying which targets give independent information

1. neglect Dipole (better sensitivity of $\mu \to e\gamma$ (MEGII) and $\mu \to e\bar{e}e$(Mu3e).
   remain to determine: $\vec{C} \equiv (\widetilde{C}^{pp}_{VR}, \widetilde{C}^{pp}_{SL}, \widetilde{C}^{nn}_{VR}, \widetilde{C}^{nn}_{SL})$

2. recall that
   $$BR_{SI}(A\mu \to Ae) \propto \left|\vec{C} \cdot \vec{v}_A\right|^2$$
   where target vector for nucleus A
   $$\vec{v}_A \equiv \left(V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)}\right)$$

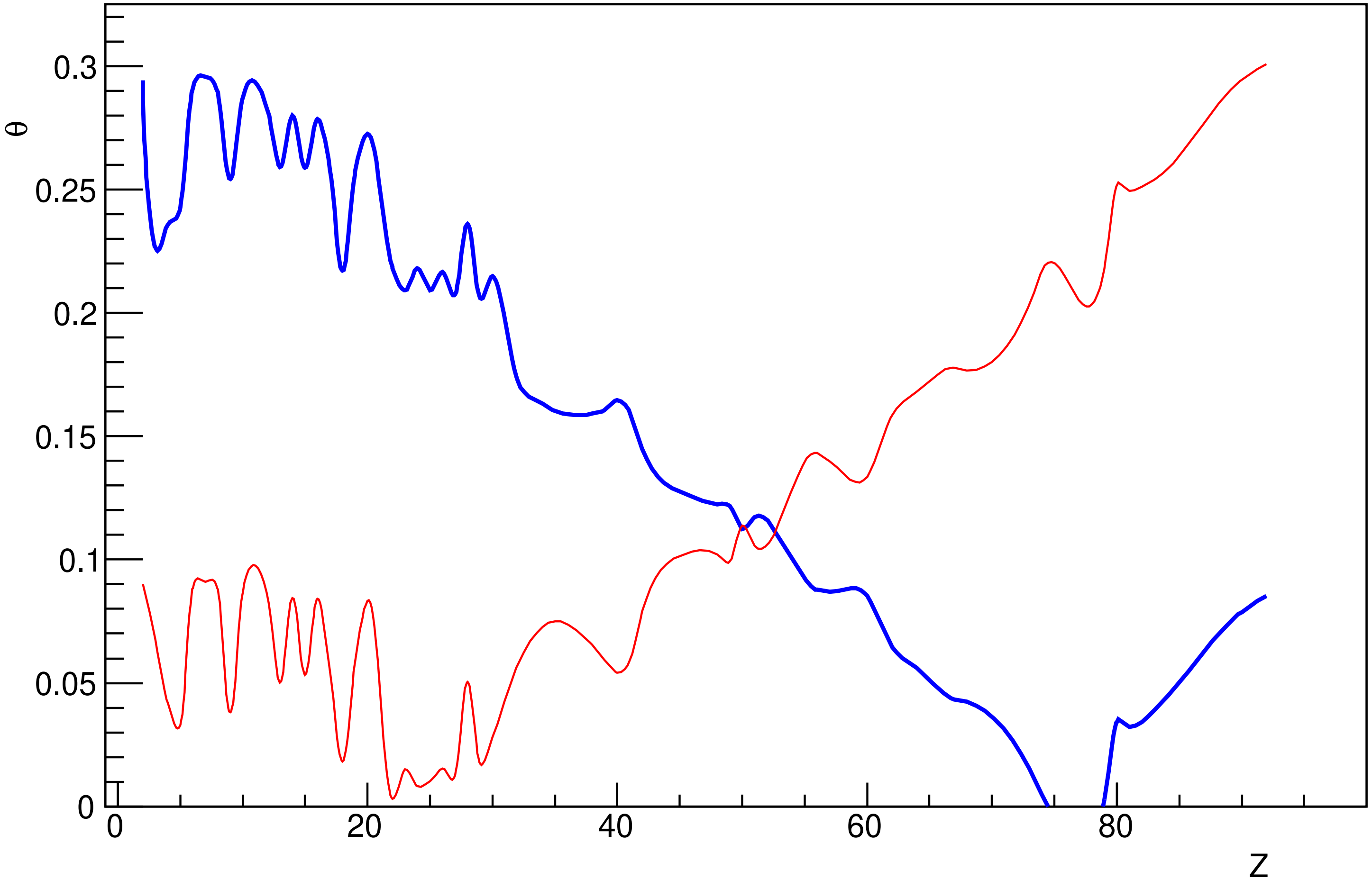
3. So first experimental search (eg on Aluminium) probes projection of $\vec{C}$ of $\vec{v}_{Al}$
   ... next target needs to have component $\perp$ to Aluminium!
   $\Leftrightarrow$ plot misalignment angle θ between target vectors

4. how big does θ need to be?
   overlap integrals have theory uncertainty: $\Delta\theta \left\{ \begin{array}{cc} \text{nuclear} & \sim 5\% (KKO) \\ NLO\ \chi\text{PT} & \sim 10\%(?) \end{array} \right.$
   Both vectors uncertain by $\Delta\theta$; need misaligned by $2\Delta\theta \approx 10 \to 20\%$

Current data+ theory uncertainty $\sim 10\%$: two targets give $\Delta\theta > 0.2$

$\boldsymbol{BR(\mu Au \to eAu) \leq 7 \times 10^{-13}}$ $(Au : Z = 79)$

$BR(\mu Ti \to eTi) \leq 4.3 \times 10^{-12}$ $(Ti : Z = 22)$



$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)})$, and $BR \propto |\vec{v}_A \cdot \vec{C}|^2$

$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta$ ...plot θ on vertical axis

In the future...with a 5% theory uncertainty:

First target of Mu2e, COMET: Aluminium (Z=13, A=27)

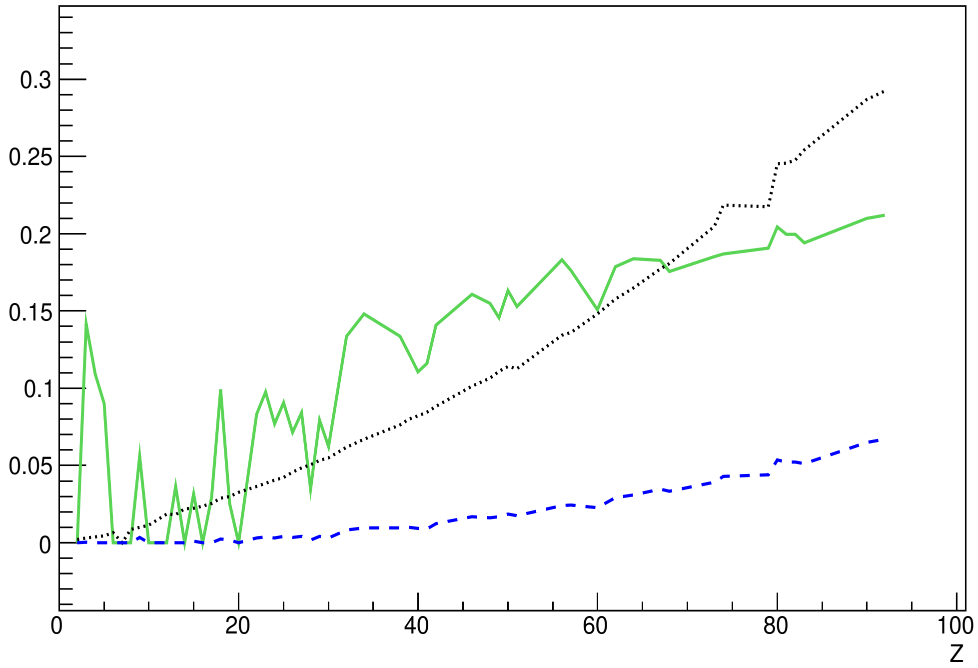
$$\hat{v}_{Al} \approx \tfrac{1}{2}(1,1,1,1)$$ (recall $\tilde{C}_V^{pp}, \tilde{C}_S^{pp}, \tilde{C}_V^{nn}, \tilde{C}_S^{nn}$)

basis of three other "directions":

$$\hat{v}_{np} \equiv \frac{1}{2}(-1,-1,1,1)$$

$$\hat{v}_{VS} \equiv \frac{1}{2}(1,-1,1,-1)$$

$$\hat{v}_{IsoSV} \equiv \frac{1}{2}(-1,1,1,-1)$$

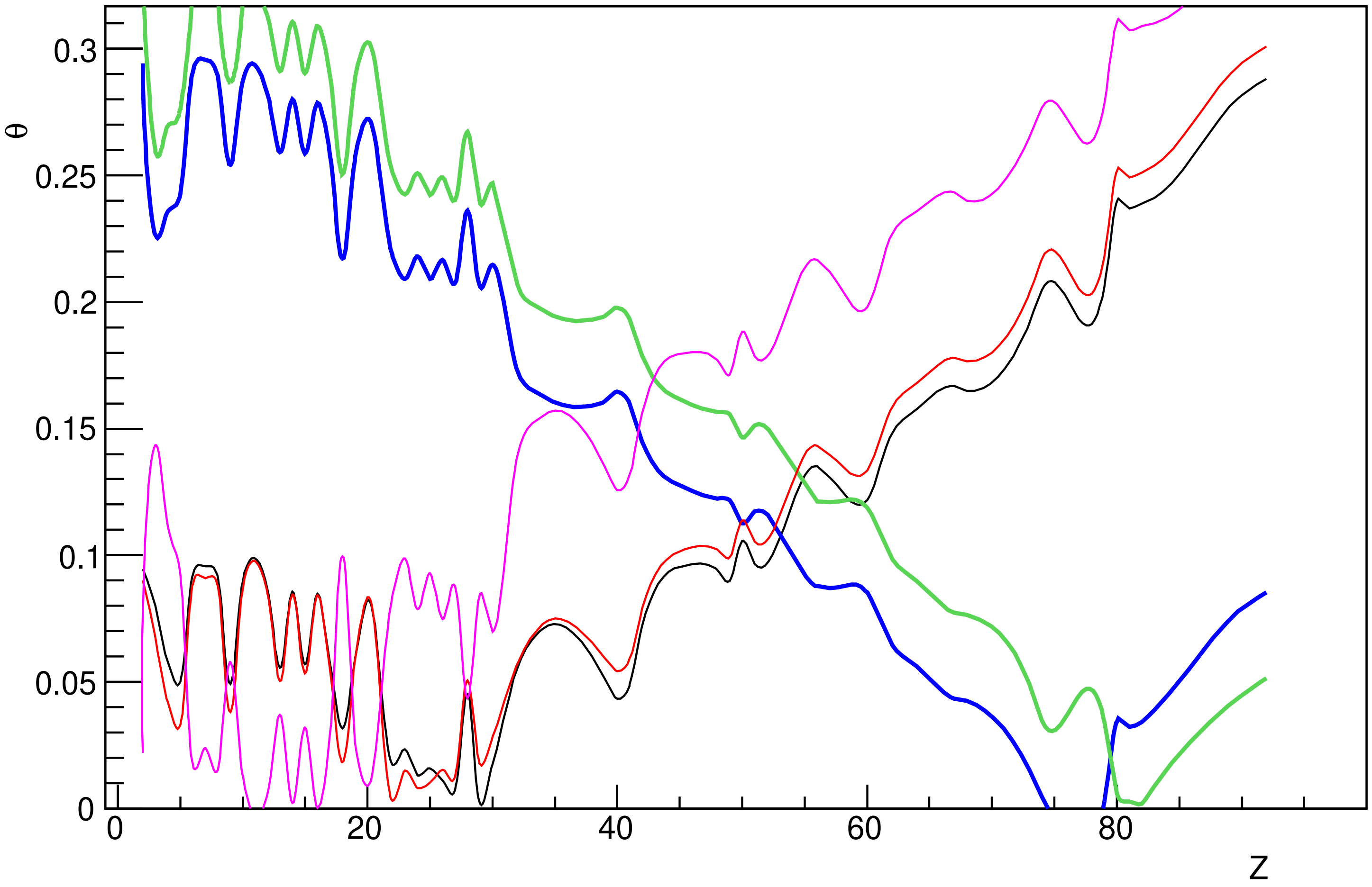


probe 3 combinations of SI coeffs

All current data...

$BR(\mu Au \to e Au) \leq 7 \times 10^{-13}$ $(Au : Z = 79)$

$BR(\mu Ti \to e Ti) \leq 4.3 \times 10^{-12}$ $(Ti : Z = 22)$



$BR(\mu Pb \to e Pb) \leq 4.6 \times 10^{-11}$

$BR(\mu S \to e S) \leq 7 \times 10^{-11}$ S = Sulpher, Z = 16

$BR(\mu Cu \to e Cu) \leq 1.6 \times 10^{-8}$ Cu = Copper, Z = 29

in practise: need to "match" and "run"

need a recipe to relate EFTs at different scales

1. when change EFTs ($eg\ N \leftrightarrow q$ at 2 GeV):
   $match$ ($=$ set equal) Greens functions in both EFTs at the matching scale
   match quark operators onto nucleon ($N \in \{n, p\}$) operators:

$$\bar{q}(x)\Gamma_O q(x) \to G_O^{N,q} \bar{N}(x)\Gamma_O N(x)$$

   eg, $\langle N|\bar{q}(x)q(x)|N\rangle = G_O^{N,q}\langle N|\bar{N}(x)N(x)|N\rangle = G_O^{N,q}\overline{u_N}(P_f)u_N(P_i)e^{-i(P_f-P_i)x}$
   So obtain, eg $\widetilde{C}^p_{S,L} = \sum_q G_S^{p,q} c_{S,L}^{qq}$

2. Within an EFT: Lagrangian parameters $(\alpha_s(\mu), \phi(\mu), C_I(\mu), ...)$ evolve with scale (due to loops).Described by Renormalisation Group Eqns. For $\{C_I\}$ below m_W:
   Davidson,CrivellinDPS

$$\mu\frac{\partial}{\partial\mu}(C_I, ...C_J, ...) = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}^e$$

   line up operator coefficients in $\vec{C}$, $\mathbf{\Gamma}$ $=$ anomalous dimension matrix :
   $\mathbf{\Gamma}^s \leftrightarrow$ rescales coefficients, $\mathbf{\Gamma}^e \leftrightarrow$ transform one coeff to another...
   Above m_W : $\mathbf{\Gamma}$ for $SU(3) \times SU(2) \times U(1)$ JenkinsManoharTrott

But QED loops are $\mathcal{O}(\alpha/4\pi)$... surely negligeable?

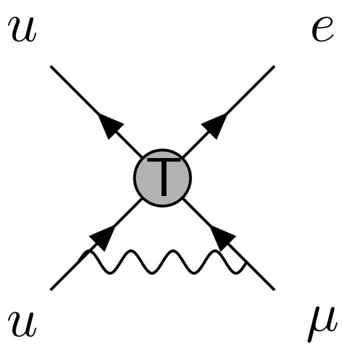
Work top-down = suppose a model that gives only tensor operator at m_W:

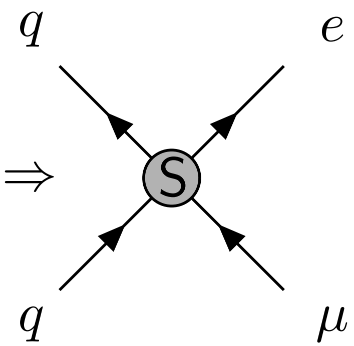
$$2\sqrt{2}G_F\ C_T(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$$

1: forget RGEs Match to nucleons $N \in \{n,p\}$ as $\widetilde{C}_T^{NN} = \langle N|\bar{u}\sigma u|N\rangle C_T^{uu} \lesssim \frac{3}{4}C_T^{uu}$

$$\Rightarrow \boldsymbol{BR \approx BR_{SD} \approx \tfrac{1}{2}|C_T|^2}$$

2: include RGEs

$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$ 

$+ \ldots \quad \Rightarrow$ 

$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\ C_T^{uu}\ (\overline{u}u)(\overline{e}P_Y\mu)$

$$\Delta C_S^{uu}(m_\tau) \sim \tfrac{1}{7}C_T^{uu}(m_W)$$

Then match to nucleons: $\widetilde{C}_S^{NN} = \langle N|\bar{u}u|N\rangle \Delta C_S^{uu} \sim C_T^{uu}$ so $\widetilde{C}_S^{pp} \gtrsim \widetilde{C}_T^{pp}$,

$$\boldsymbol{BR \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 8Z^2 BR_{SD}}$$

$\Rightarrow$ loop effects mix tensor to scalar.. change $BR(\mu A \to eA)$ by $\mathcal{O}(10^3)$

What does $BR < 10^{-12}$ mean? Is it restrictive?

LFV Branching Ratios normalised to *weak* muon decay, $\tau_\mu \sim 2 \times 10^{-6}$ sec

$$BR(\mu \to e\bar{e}e) \equiv \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \quad , \quad \Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_F^2 m_\mu^5}{192\pi^3} = \frac{m_\mu^5}{1536\pi^3 v^4} \qquad {\scriptstyle m_\mu = .105 \text{ GeV}, \; v = 174 \text{ GeV}}$$

...so if $\Gamma(\mu \to e\bar{e}e) \simeq \frac{m_\mu^5}{1536\pi^3\Lambda_{LFV}^4}$ then $BR \lesssim \begin{cases} 10^{-12} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 200 \text{ TeV} \\ 10^{-16} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 2000 \text{ TeV} \end{cases}$

NB: $\Lambda_{LFV} = (16\pi^2)^n M_{LFV}$/couplings; *not* the mass scale of new particles M_{LFV}

Compare to $\frac{(g-2)_\mu}{2} \equiv a \simeq \alpha_{em}/\pi$ (electromagnetic *amplitude*):

$$\begin{aligned} \Delta a &\equiv a^{SM} - a^{exp} \simeq 3 \times 10^{-9} \\ &\sim \frac{m_\mu^2}{16\pi^2\Lambda_{NP}^2} \end{aligned}$$

$\Rightarrow \Lambda_{NP} \sim m_t$.