On the amplitudes for the rare CP-conserving decays $K^\pm \to \pi^\pm \ell^+ \ell^-$ and $K_S \to \pi^0 \ell^+ \ell^-$

Marc Knecht

Centre de Physique Théorique UMR7332,
CNRS Luminy Case 907, 13288 Marseille cedex 09 - France
knecht@cpt.univ-mrs.fr

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based on: G. D'Ambrosio, D. Greynat, M.K., to appear in JHEP [arXiv:1812.00735]

OUTLINE

I. Introduction

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III. Predicting the amplitude for $K^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-}$

IV. Summary - Conclusions

I. Introduction

Rare kaon decays [$K \to \pi\gamma^{(*)}$, etc.] proceed through FCNC, are suppressed in the SM $\longrightarrow$ might provide an interesting window into new physics

For a review, see V. Cirigliano et al, Rev Mod Phys 84, 399 (2012)

Particularly interesting examples

$$K^+ \to \pi^+ \nu\bar{\nu} \ [\mathrm{NA62}] \qquad\qquad K_L \to \pi^0 \nu\bar{\nu} \ [\mathrm{KOTO}]$$

- dominated by short-distances
- clean SM prediction, hadronic matrix elements from $K_{\ell 3}$

F. Mescia, C. Smith, Phys. Rev. D 76, 034017 (2007)

$$Br(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} = 8.39(30) \cdot 10^{-11} \left[\frac{|V_{cb}|}{40.7 \cdot 10^{-3}}\right]^{2.8} \left[\frac{\gamma}{73.2^{\mathrm{o}}}\right]^{0.74}$$

$$Br(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{SM}} = 3.36(5) \cdot 10^{-11} \left[\frac{|V_{ub}|}{3.88 \cdot 10^{-3}}\right]^{2} \left[\frac{|V_{cb}|}{40.7 \cdot 10^{-3}}\right]^{2} \left[\frac{\sin\gamma}{\sin 73.2^{\mathrm{o}}}\right]^{2}$$

J. Brod et al., Phys Rev D 83, 034030 (2011)

A. J. Buras et al, JHEP 1511, 33 (2011)

For a review, see A. Buras et al, Rev Mod Phys 80, 965 (2008)

In general, long distances dominate the amplitudes

$\longrightarrow$ difficult to make predictions due to unknown hadronic matrix elements

The CP conserving decays considered here

$$K^\pm \to \pi^\pm \gamma^* \to \pi^\pm \ell^+ \ell^- \qquad\qquad K_S \to \pi^0 \gamma^* \to \pi^0 \ell^+ \ell^-$$

belong to this second category

- similar short-distance parts as in $K \to \pi \nu \bar{\nu}$
- analogues, in the kaon sector, of $b \to s \ell^+ \ell^-$ transitions
- any LFUV effect invoked in order to explain the anomalies seen at LHCb might also manifest itself here [A. Crivellin et al., Phys. Rev. D 93, 074038 (2016)]
- $K_S \to \pi^0 \ell^+ \ell^-$ gives the contribution of indirect CPV to $K_L \to \pi^0 \ell^+ \ell^-$
- $$R_{K^\pm} \equiv \frac{\mathrm{Br}[K^\pm \to \pi^\pm \mu^+ \mu^-]}{\mathrm{Br}[K^\pm \to \pi^\pm e^+ e^-]} = \begin{cases} 0.313(71) & \text{[PDG average]} \\ 0.309(43) & \text{[NA48/2 alone]} \end{cases}$$

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What does the SM predict?

II. Brief theory overview

Amplitudes $(K, \pi) = (K^{\pm}, \pi^{\pm}),\ (K_S, \pi^0)$

- Long-distance part (first order in G_F and in α)

$$\mathcal{A}_{\rm LD} = e^2 \times \bar{\rm u}(p_{\ell^-})\gamma_\sigma {\rm v}(p_{\ell^+}) \times \frac{(-1)}{s}\left[\eta^{\sigma\rho} - (1-\xi)\,\frac{(k-p)^\sigma (k-p)^\rho}{s}\right] \times \langle \pi(p)|j_\rho(0)|K(k)\rangle_{\rm SM}$$

$$\begin{aligned}\langle \pi(p)|j_\rho(0)|K(k)\rangle_{\rm SM} &= i\int d^4x\,\langle \pi(p)|T\{j_\rho(0)\mathcal{L}^{\Delta S=1}_{\rm non-lept}(x)\}|K(k)\rangle_{\rm QCD} \\ &= \left[s(k+p)_\rho - (M_K^2 - M_\pi^2)(k-p)_\rho\right] \times \frac{W_{\rm LD}(s;\nu)}{16\pi^2 M_K^2}\end{aligned}$$

$\longrightarrow$ pure three-flavour (four-flavour on the lattice) QCD problem

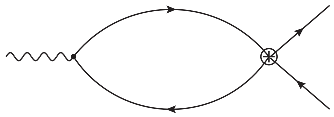
$$\mathcal{L}^{\Delta S=1}_{\rm non-lept}(x) = -\frac{G_{\rm F}}{\sqrt{2}} V_{us}^* V_{ud} \sum_{I=1}^{6} C_I(\nu) Q_I(x;\nu)$$

$$\nu\frac{d}{d\nu} j_\rho = 0 \qquad \nu\frac{d}{d\nu}\mathcal{L}^{\Delta S=1}_{\rm non-lept} = 0 \qquad \text{but } \nu\frac{d}{d\nu} T\{j_\rho(0)\mathcal{L}^{\Delta S=1}_{\rm non-lept}(x)\} \neq 0$$

- Short-distance part

$$\mathcal{L}^{\Delta S=1}_{\rm lept}(x) = -\frac{G_{\rm F}}{\sqrt{2}} V_{us}^* V_{ud}\left[C_{7V}(\nu) Q_{7V}(x) + C_{7A} Q_{7A}(x)\right]$$

$$Q_{7V} = (\bar{s}^i d_i)_{V-A}(\bar{\ell}\ell)_V,\ Q_{7A} = (\bar{s}^i d_i)_{V-A}(\bar{\ell}\ell)_A$$



E. Witten, Nucl. Phys. B 122, 109 (1977)

F. J. Gilman, M. B. Wise, Phys Rev D 21, 3150 (1980)

C. Dib et al., Phys Lett B 218, 487 (1989); Phys Rev D 39, 2639 (1989)

J. Flynn, L. Randall, Nucl Phys B 326, 31 (1989) [Nucl Phys B 334, 580 (1990)]

A. J. Buras et al., Nucl Phys B 423, 349 (1994)

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$$\mathcal{L}_{\rm lept}^{\Delta S=1}(x) = -\frac{G_{\rm F}}{\sqrt{2}}V_{us}^* V_{ud}\left[C_{7V}(\nu)Q_{7V}(x) + C_{7A}Q_{7A}(x)\right]$$

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$$\mathcal{A} = (-e^2) \times \bar{\rm u}(p_{\ell^-})\gamma_\rho {\rm v}(p_{\ell^+}) \times (k+p)^\rho \times \frac{W(s)}{16\pi^2 M_K^2}, \quad W(s) = W_{\rm LD}(s;\nu) + W_{\rm SD}(s;\nu)$$

Differential decay rate

$$\frac{d\Gamma}{dz} = \frac{\alpha^2 M_K}{3(4\pi)^5}\lambda^{3/2}(1,z,M_\pi^2/M_K^2)\sqrt{1-4\frac{m_\ell^2}{zM_K^2}}\left(1+2\frac{m_\ell^2}{zM_K^2}\right)|W(z)|^2 \quad z \equiv s/M_K^2$$

Several tools in order to evaluate the corresponding matrix elements:

- Chiral perturbation theory $[4m_\ell^2 \leq s \leq (M_K - M_\pi)^2]$

  - one loop — G. Ecker et al., Nucl Phys B 291, 692 (1987)

  - beyond one loop — G. D'Ambrosio et al., JHEP 9808, 004 (1998)

$\longrightarrow$ limitation: unknown low-energy constants

- Chiral perturbation theory and large-N_c

S. Friot et al., Phys Lett B 595, 301 (2004)
E. Coluccio Leskov et al., Phys Rev D 93, 094031 (2016)

- Lattice QCD

G. Isidori et al., Phys Lett B 633, 75 (2006)
N. H. Christ et al, Phys Rev D 92, 094512 (2015); D 94, 114516 (2016)

- $\cdots$

J. Portolés, J Phys Conf Series 800, 012030 (2017)

Concentrate on "beyond one loop" (b1L) model $[z \equiv s/M_K^2]$

$$W_{+,S;\mathrm{b1L}}(z) = G_F M_K^2(a_{+,S} + b_{+,S} z) + \frac{8\pi^2}{3}\frac{M_K^2}{M_\pi^2}\left[\alpha_{+,S} + \beta_{+,S}\frac{M_K^2}{M_\pi^2}(z-z_0)\right]\left(1+\frac{M_K^2}{M_V^2}z\right)\left[\frac{z-4\frac{M_\pi^2}{M_K^2}}{z}\bar{J}_{\pi\pi}(zM_K^2)+\frac{1}{24\pi^2}\right]$$

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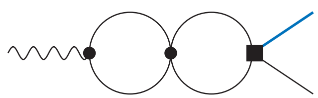

- α_+, β_+ $[\alpha_S, \beta_S]$ from slope and curvature of $K^+ \to \pi^+\pi^+\pi^-$ $[K_S \to \pi^0\pi^+\pi^-]$ amplitude

$$\alpha_+ = -20.84(74)\cdot 10^{-8}, \quad \beta_+ = -2.88(1.08)\cdot 10^{-8}, \quad \alpha_S = -6.81(74)\cdot 10^{-8}, \quad \beta_S = -1.5(1.1)\cdot 10^{-8}$$

J. Bijnens et al. Nucl. Phys. B 648, 317 (2003)

- Only loops with pions, loops with kaons included in the polynomial part

- Still not a complete two-loop representation, but neglected two-loop effects are small

G. D’Ambrosio, D. Greynat, M.K., arXiv:1812.00735

- $G_F M_K^2 a_{+,S} = W_{+,S;\mathrm{b1L}}(0), \quad G_F M_K^2 b_{+,S} = W'_{+,S;\mathrm{b1L}}(0) - \frac{1}{60}\left(\frac{M_K^2}{M_\pi^2}\right)^2\left(\alpha_{+,S} - \beta_{+,S}\frac{s_0}{M_K^2}\right)$

- Gives a good description of data

exp.	ref.	mode	number of events
BNL*	[1]	$K^+ \to \pi^+ e^+ e^-$	~ 500
BNL-E865*	[2]	$K^+ \to \pi^+ e^+ e^-$	$10\,300$
NA48/2*	[3]	$K^\pm \to \pi^\pm e^+ e^-$	$7\,263$
BNL-E787	[4]	$K^+ \to \pi^+ \mu^+ \mu^-$	~ 200
BNL-E865	[5]	$K^+ \to \pi^+ \mu^+ \mu^-$	~ 400
FNAL-E871	[6]	$K^\pm \to \pi^\pm \mu^+ \mu^-$	~ 100
NA48/2*	[7]	$K^\pm \to \pi^\pm \mu^+ \mu^-$	3120
NA48/1	[8]	$K_S \to \pi^0 e^+ e^-$	7
NA48/1	[9]	$K_S \to \pi^0 \mu^+ \mu^-$	6

[1] C. Alliegro *et al.*, Phys. Rev. Lett. **68**, 278 (1992)

[2] R. Appel *et al.* [E865 Collaboration], Phys. Rev. Lett. **83**, 4482 (1999) [hep-ex/9907045]

[3] J. R. Batley *et al.* [NA48/2 Collaboration], Phys. Lett. B **677**, 246 (2009) [arXiv:0903.3130 [hep-ex]]

[4] S. Adler *et al.* [E787 Collaboration], Phys. Rev. Lett. **79**, 4756 (1997) [hep-ex/9708012]

[5] H. Ma *et al.* [E865 Collaboration], Phys. Rev. Lett. **84**, 2580 (2000) [hep-ex/9910047]

[6] H. K. Park *et al.* [HyperCP Collaboration], Phys. Rev. Lett. **88**, 111801 (2002) [hep-ex/0110033]

[7] J. R. Batley *et al.* [NA48/2 Collaboration], Phys. Lett. B **697**, 107 (2011) [arXiv:1011.4817 [hep-ex]]

[8] J. R. Batley *et al.* [NA48/1 Collaboration], Phys. Lett. B **576**, 43 (2003) [hep-ex/0309075]

[9]J. R. Batley *et al.* [NA48/1 Collaboration], Phys. Lett. B **599**, 197 (2004) [hep-ex/0409011]

*: decay distribution

Concentrate on "beyond one loop" (b1L) model $[z \equiv s/M_K^2]$

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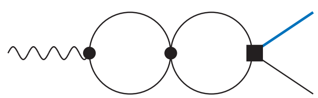

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- Fit to e^+e^- data (NA48/2 + E865)

$$a_+ = -0.593(9)_{\mathrm{stat}}(1)_{\alpha_+}(6)_{\beta_+}, \quad b_+ = -0.675(40)_{\mathrm{stat}}(16)_{\alpha_+}(2)_{\beta_+}, \quad \chi^2/\mathrm{d.o.f} = 45.7/39$$

- Fit to e^+e^- data (NA48/1)

$a_S = -1.29(3.15)$, $b_S = 17.8(10.6)$ and $a_S = 1.28(3.16)$, $b_S = -17.6(10.6)$

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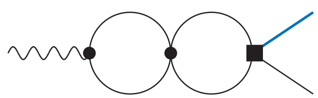

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III. Predicting the amplitude for

$$K^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-}$$

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Proposal:

$$W_+(z) = W_+^{\pi\pi}(z) + W_+^{\rm res}(z;\nu) + W_+^{\rm SD}(z;\nu)$$

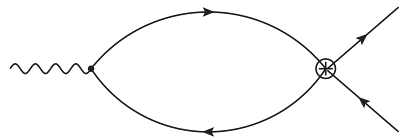
- $W_+^{\pi\pi}(z)$: contribution from (resonant) two-pion state

- $W_+^{\rm SD}(z)$: contributions from short distances

- $W_+^{\rm res}(z)$: contribution from intermediate energy range
  - described by a set of resonances
  - has to match the $\sim \ln(-s/\nu^2)$ behaviour at short distances

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Short-distance behaviour



$$\lim_{q\to\infty} i\int d^4x\, e^{iq\cdot x} T\{j^\mu(x)Q_I(0)\} = [q^\mu q^\rho - q^2\eta^{\mu\rho}]\times\bar{s}\gamma_\rho(1-\gamma_5)d\times\frac{1}{4\pi}\left[\xi^I_{00} - \xi^I_{01}\ln\frac{-q^2}{\nu^2}\right] + \mathcal{O}(q)$$

$$\xi^1_{01} = \frac{1}{4\pi}\,\frac{8}{9}\,N_c,\qquad \xi^2_{01} = \frac{1}{4\pi}\,\frac{8}{9},\qquad \xi^1_{00} = \frac{1}{4\pi}\times\begin{cases}\frac{16}{27}\,N_c\ \ \mathrm{NDR}\\ \\ \frac{40}{27}\,N_c\ \ \mathrm{HV}\end{cases},\qquad \xi^2_{00} = \frac{1}{4\pi}\times\begin{cases}\frac{16}{27}\ \ \mathrm{NDR}\\ \\ \frac{40}{27}\ \ \mathrm{HV}\end{cases},$$

$$\xi^3_{01} = -\frac{1}{4\pi}\,\frac{8}{9},\qquad \xi^4_{01} = -\frac{1}{4\pi}\,\frac{8}{9}\,N_c,\qquad \xi^3_{00} = \frac{1}{4\pi}\times\begin{cases}-\frac{16}{27}\ \ \mathrm{NDR}\\ \\ -\frac{40}{27}\ \ \mathrm{HV}\end{cases},\qquad \xi^4_{00} = \frac{1}{4\pi}\times\begin{cases}-\frac{16}{27}\,N_c\ \ \mathrm{NDR}\\ \\ -\frac{40}{27}\,N_c\ \ \mathrm{HV}\end{cases}$$

$$\xi^{5,6}_{00} = \xi^{5,6}_{01} = 0$$

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Short-distance behaviour

QCD corrections?

$$\lim_{q\to\infty} i\int d^4x\, e^{iq\cdot x} T\{j^\mu(x)Q_I(0;\nu)\} = [q^\mu q^\rho - q^2\eta^{\mu\rho}] \times [\bar{s}\gamma_\rho(1-\gamma_5)d](0) \times \frac{1}{4\pi}\xi_I(\alpha_s;\nu^2/q^2) + \mathcal{O}(q)$$

$$\xi_I(\alpha_s\,;\nu^2/q^2) = \sum_{p\geq 0}\sum_{r=0}^{p+1} \xi^I_{pr}\alpha_s^p(\nu)\ln^r(-\nu^2/q^2)$$

$$\lim_{q\to\infty} i\int d^4x\, e^{iq\cdot x} T\{j^\mu(x)\mathcal{L}_{\Delta S=1}(0)\} =$$

$$= \left(-\frac{G_\mathrm{F}}{\sqrt{2}}V^*_{us}V_{ud}\right)[q^\mu q^\rho - q^2\eta^{\mu\rho}] \times \bar{s}\gamma_\rho(1-\gamma_5)d \times \frac{1}{4\pi}\sum_{I=1}^{4} C_I(\nu)\,\xi_I(\alpha_s\,;\nu^2/q^2) + \mathcal{O}(q)$$

$$\nu\frac{dW_{+,S}(z;\nu)}{d\nu} = \pm 16\pi^2 M_K^2\left(\frac{G_\mathrm{F}}{\sqrt{2}}V^*_{us}V_{ud}\right) \times \frac{f_+^{K\pi}(zM_K^2)}{4\pi}\nu\frac{d}{d\nu}\sum_I C_I(\nu)\,\xi_I(\alpha_s\,;\nu^2/zM_K^2)$$

$$\nu\frac{d}{d\nu}\left[\frac{W_{+,S}(z;\nu)}{16\pi^2M_K^2} \pm \frac{G_\mathrm{F}}{\sqrt{2}}V^*_{us}V_{ud} \times \frac{C_{7V}(\nu)}{4\pi\alpha}f_+^{K\pi}(zM_K^2)\right] = \pm\left(\frac{G_\mathrm{F}}{\sqrt{2}}V^*_{us}V_{ud}\right)\frac{f_+^{K\pi}(s)}{4\pi}\sum_{I=1}^{6}\left(\frac{\gamma^{(0)}_{I,7}}{4\pi} + 2\xi^I_{01}\right)C_I(\nu) = 0 + \mathcal{O}(\alpha_s)$$

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$$\xi_I(\alpha_s\,;\nu^2/q^2) = \sum_{p\geq 0}\sum_{r=0}^{p+1}\xi^I_{pr}\alpha_s^p(\nu)\ln^r(-\nu^2/q^2)$$

$$\lim_{q\to\infty} i\int d^4x\, e^{iq\cdot x} T\{j^\mu(x)\mathcal{L}_{\Delta S=1}(0)\} =$$

$$= \left(-\frac{G_{\rm F}}{\sqrt{2}}V^*_{us}V_{ud}\right)[q^\mu q^\rho - q^2\eta^{\mu\rho}]\times\bar{s}\gamma_\rho(1-\gamma_5)d\times\frac{1}{4\pi}\sum_{I=1}^{4} C_I(\nu)\,\xi_I(\alpha_s\,;\nu^2/q^2) + \mathcal{O}(q)$$

$$\nu\frac{dW_{+,S}(z;\nu)}{d\nu} = \pm 16\pi^2 M_K^2\left(\frac{G_{\rm F}}{\sqrt{2}}V^*_{us}V_{ud}\right)\times\frac{f_+^{K\pi}(zM_K^2)}{4\pi}\nu\frac{d}{d\nu}\sum_I C_I(\nu)\,\xi_I(\alpha_s\,;\nu^2/zM_K^2)$$

$$\nu\frac{d}{d\nu}\left[\frac{W_{+,S}(z;\nu)}{16\pi^2 M_K^2}\pm\frac{G_{\rm F}}{\sqrt{2}}V^*_{us}V_{ud}\times\frac{C_{7V}(\nu)}{4\pi\alpha}f_+^{K\pi}(zM_K^2)\right] = \pm\left(\frac{G_{\rm F}}{\sqrt{2}}V^*_{us}V_{ud}\right)\frac{f_+^{K\pi}(s)}{4\pi}\sum_{I=1}^{6}\left(\frac{\gamma^{(0)}_{I,7}}{4\pi}+2\xi^I_{01}\right)C_I(\nu) = 0+\mathcal{O}(\alpha_s)$$

turn the argument around!

$$\text{order }\ \mathcal{O}(\alpha_s):\quad \xi^I_{11} = -\frac{1}{2}\frac{\gamma^{(1)}_{I,7V}}{(4\pi)^2}-\frac{1}{2}\sum_{J=1}^{6}\frac{\gamma^{(0)}_{IJ}}{4\pi}\xi^J_{00}\qquad \xi^I_{12} = -\frac{1}{4}\sum_{J=1}^{6}\frac{\gamma^{(0)}_{IJ}}{4\pi}\xi^J_{01}$$

coefficients ξ_{p0} not determined by RG

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Two-pion states

requires an unsubtracted dispersion relation

$$W_+(z)|_{\pi\pi} = \frac{1}{\pi}\int_0^\infty dx \frac{\mathrm{Abs}\, W_+(x/M_K^2)|_{\pi\pi}}{x - zM_K^2 - i0}$$

with

$$\frac{\mathrm{Abs}\, W_+(s/M_K^2)|_{\pi\pi}}{16\pi^2 M_K^2} = \theta(s-4M_\pi^2) \times \frac{s-4M_\pi^2}{s}\lambda_{K\pi}^{-1/2}(s) \times F_V^{\pi *}(s) \times f_1^{\pi^+\pi^-\to K^+\pi^-}(s)$$

Then a_+ and b_+ are given by spectral sum rules

$$G_F M_K^2 a_+|_{\pi\pi} = W_+(0)|_{\pi\pi} = \frac{1}{\pi}\int_0^\infty \frac{dx}{x}\, \mathrm{Abs}\, W_+(x/M_K^2)|_{\pi\pi}$$

and

$$G_F M_K^2 b_+|_{\pi\pi} = W_+'(0)|_{\pi\pi} = \frac{M_K^2}{\pi}\int_0^\infty \frac{dx}{x^2}\, \mathrm{Abs}\, W_+(x/M_K^2)|_{\pi\pi}$$

requires $F_V^{\pi *}(s)$ and $f_1^{\pi^+\pi^-\to K^+\pi^-}(s)$ beyond low-energy expansion

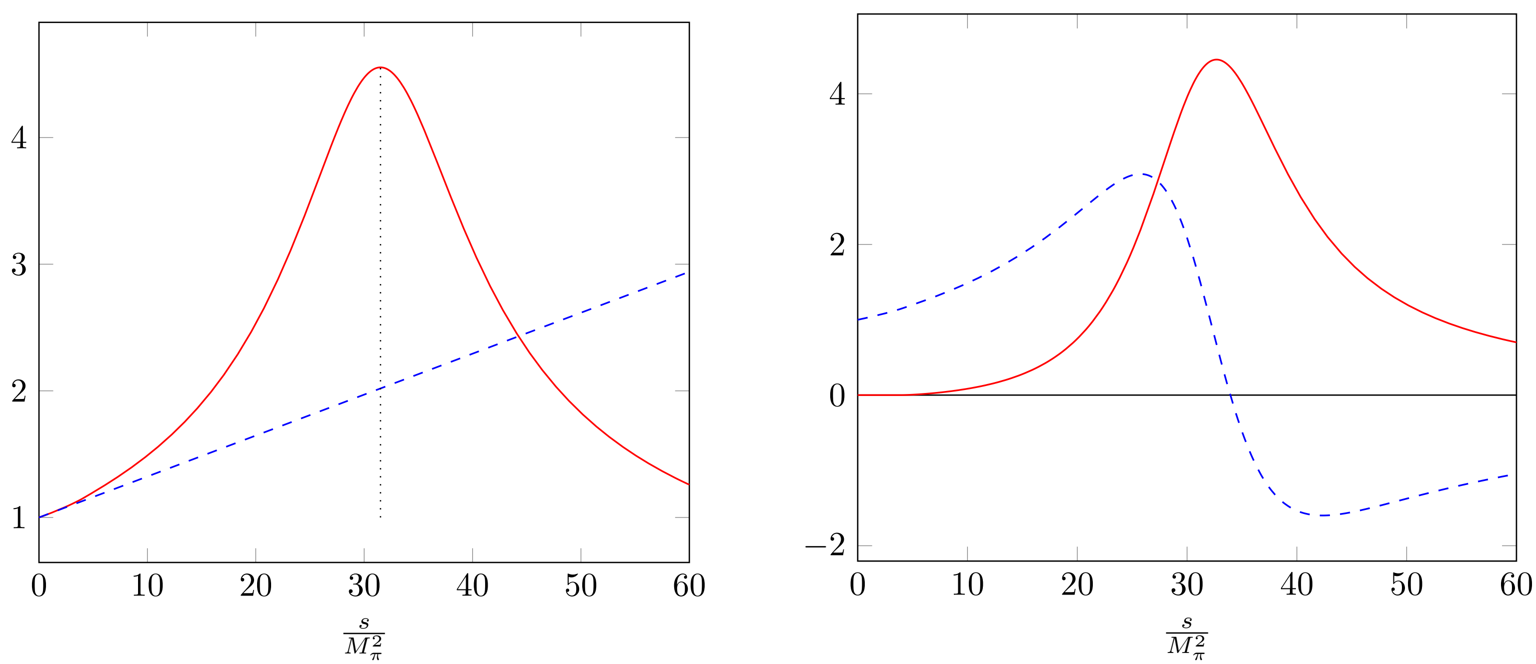
Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Two-pion states

Simple approach: unitarize both using the inverse amplitude method

$$F_V^\pi(s)|_{\rm IAM} = \frac{1}{1 - \frac{s}{M_V^2} - \frac{\beta}{6F_\pi^2}(s - 4M_\pi^2)\,\bar{J}_{\pi\pi}(s)}$$

T. N. Truong, Phys Rev Lett 61, 2526 (1988)
A. Dobado et al, Phys Lett B 235, 134 (1990)
T. Hannah, Phys Rev D 55, 5613 (1997)
A. Dobado, J. R. Pelaez, Phys Rev D 56, 3057 (1997)
J. Nieves et al., Phys Rev D 65, 036002 (2002)



left: $|F_V^\pi(s)|$; right: $\mathrm{Re}\,F_V^\pi(s)$ (dashed) and $\mathrm{Im}\,F_V^\pi(s)$ (solid)

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Two-pion states

Simple approach: unitarize both using the inverse amplitude method

$$f_1^{K^+\pi^-\to\pi^+\pi^-}(s) = \frac{\dfrac{\alpha_+}{96\pi M_\pi^2} \times \lambda_{K\pi}^{1/2}(s)\sqrt{1-\frac{4M_\pi^2}{s}}}{1 - \dfrac{\beta_+}{\alpha_+}\dfrac{s-s_0}{M_\pi^2} - \dfrac{\beta}{6}\dfrac{s-4M_\pi^2}{F_\pi^2}\left[\bar{J}_{\pi\pi}(s) - \mathrm{Re}\,\bar{J}_{\pi\pi}(s_0)\right] + \dfrac{\beta}{6}\dfrac{s_0-4M_\pi^2}{F_\pi^2}(s-s_0)\mathrm{Re}\,\bar{J}'_{\pi\pi}(s_0)}$$

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$$a_+|_{\pi\pi} = -1.58 \qquad b_+|_{\pi\pi} = -0.76 \qquad \text{for}\ \beta_+ = -0.85\cdot 10^{-8}$$

note: position of the ρ resonance in $f_1^{\pi^+\pi^-\to K^+\pi^-}(s)$ much too low for $\beta_+ = -2.88(1.08)\cdot 10^{-8}$... (phase goes through $\pi/2$ at $s\sim M_\rho^2/2$!)

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Two-pion states

Simple approach: unitarize both using the inverse amplitude method

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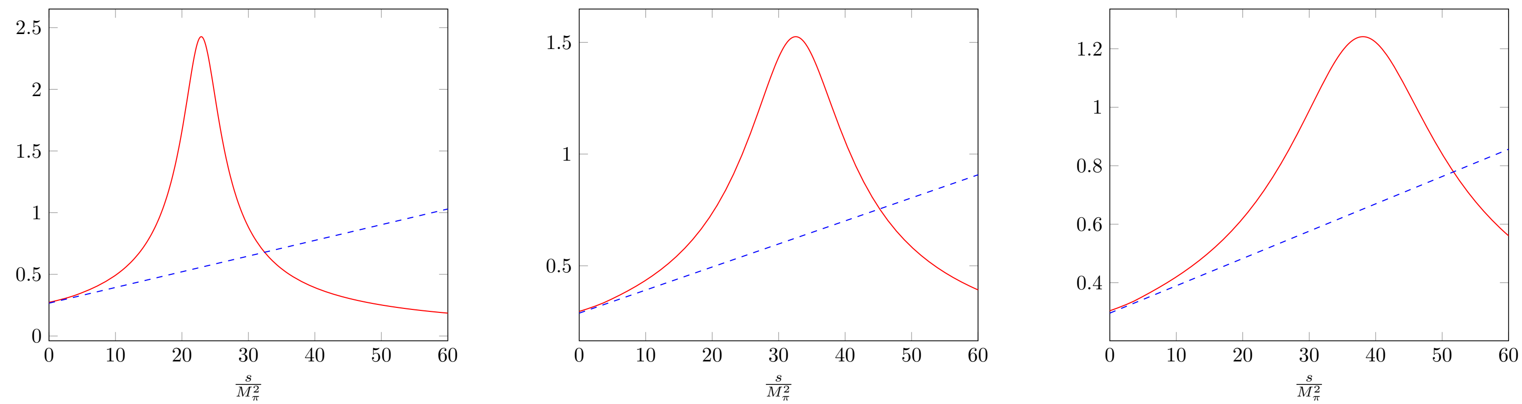

$|f_1^{\pi^+\pi^-\to K^+\pi^-}(s)/\lambda_{K\pi}^{1/2}(s)\sqrt{1-\frac{4M_\pi^2}{s}}|$ for $\alpha_+ = -20.84\cdot 10^{-8}$

left: $\beta_+ = -1.26\cdot 10^{-8}$; centre: $\beta_+ = -0.85\cdot 10^{-8}$; right: $\beta_+ = -0.72\cdot 10^{-8}$

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Two-pion states

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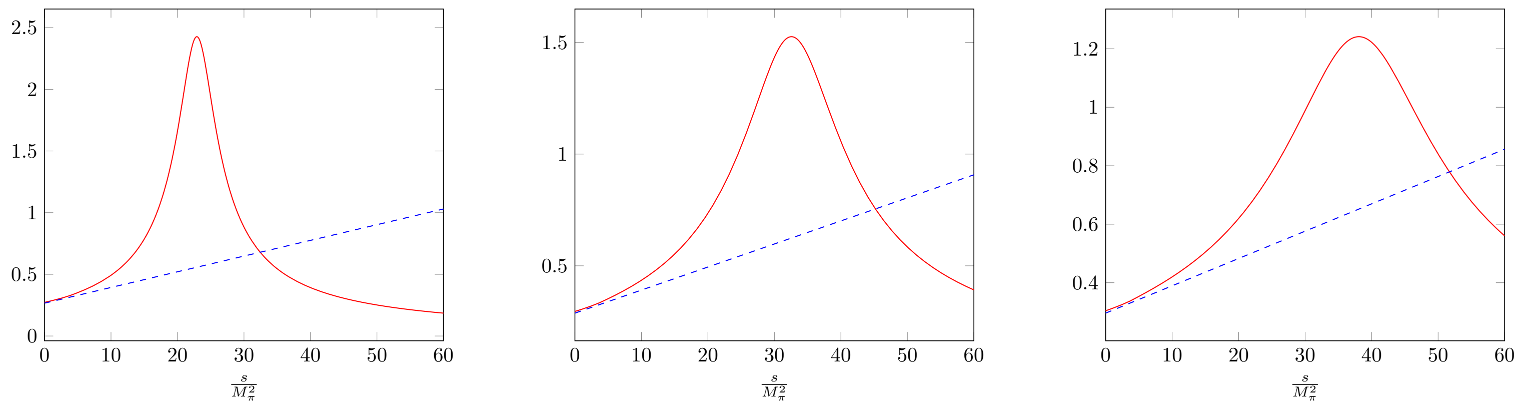


$|f_1^{\pi^+\pi^-\to K^+\pi^-}(s)/\lambda_{K\pi}^{1/2}(s)\sqrt{1-\frac{4M_\pi^2}{s}}|$ for $\alpha_+ = -20.84\cdot 10^{-8}$

left: $\beta_+ = -1.26\cdot 10^{-8}$; centre: $\beta_+ = -0.85\cdot 10^{-8}$; right: $\beta_+ = -0.72\cdot 10^{-8}$

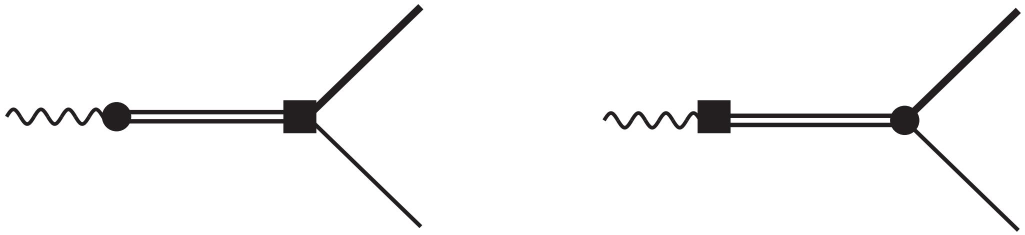
three-parameter fit to data also prefers smaller values of $|\beta_+|$...

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Other intermediate states?

Simplified proposal: modelled by an infinite tower of zero-width resonances, with couplings adjusted such as to match the short-distance behaviour

$$W_+(z) = W_+^{\pi\pi}(z) + W_+^{\rm res}(z;\nu) + W_+^{\rm SD}(z;\nu)$$



Possible resonances:
$J^{PC} = 1^{--}$, $I = 1$, $S = 0$, like $\phi(1020)$ [not $\rho(770)$!]
$J^P = 1^-$, $I = 1/2$, $S = \pm 1$, like $K^*(892)$

$$W_+^{\rm res}(z;\nu) = \sum_{V=\phi\cdots} \frac{f_V \widetilde{g}_V}{s - M_V^2 + i0} + \sum_{V=K^*\cdots} \frac{g_V \widetilde{f}_V}{s - M_V^2 + i0}$$

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Other intermediate states?

Look for

$$W_+^{\rm res}(z;D) = \frac{f_+^{K^\pm\pi^\mp}(zM_K^2)}{4\pi}\int dx\,\frac{\rho_{\rm res}(x;D)}{x - zM_K^2 - i0}$$

$$\rho_{\rm res}(s;D) = A(D)(4\pi)^{2-\frac{D}{2}}\left(\frac{M^2}{\nu_{\rm MS}^2}\right)^{\frac{D}{2}-2}\Gamma\left(2-\frac{D}{2}\right)\sum_{n\geq 1} M^2\mu_n(D)\delta(s-nM^2)\quad A(D) = A + A'(D-4)+\cdots$$

$$\int dx\,\frac{\rho_{\rm res}(x;D)}{x+wM^2} = A(D)(4\pi)^{2-\frac{d}{2}}\left(\frac{M^2}{\nu_{\rm MS}^2}\right)^{\frac{D}{2}-2}\Gamma\left(2-\frac{D}{2}\right)\sum_{n\geq 1}\frac{\mu_n(D)}{(n+w)}\quad w\equiv -s/M^2\quad M\sim 1{\rm GeV}$$

with the weights $\mu_n(D)$ satisfying

- $\sum_{n\geq 1}\frac{\mu_n(D)}{n} = 1$
- $\mu_n(D) = (D-4)\bar\mu_n + \mathcal{O}\left((D-4)^2\right)$ as $D\to 4$
- $\xi(w)\equiv\sum_{n\geq 1}\frac{\bar\mu_n}{n(n+w)}$ converges
- $\xi(w)\sim\ln w$ as $w\to +\infty$

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Other intermediate states?

Solution (not unique!) can be found in the form

$$\mu_n(D) = \mathsf{a}(D)\, n^{\frac{D-4}{2}} + \mathsf{b}(D)\, n^2 \left(\frac{D}{2} - 1\right)^n$$

$$\mathsf{a}(D) = \frac{\frac{\sqrt{\pi}}{2}}{\Gamma\left(\frac{4-D}{2}\right)\Gamma\left(\frac{D-1}{2}\right)} \quad \text{and} \quad \mathsf{b}(D) = \frac{1}{2}\,\frac{(D-4)^2}{D-2}\left[1 - \frac{\frac{\sqrt{\pi}}{2}\zeta\left(\frac{6-D}{2}\right)}{\Gamma\left(\frac{4-D}{2}\right)\Gamma\left(\frac{D-1}{2}\right)}\right]$$

$$\int dx\, \frac{\rho_{\text{res}}(x;D)}{x - zM_K^2} = A(D)(4\pi)^{\frac{4-D}{2}}\left(\frac{M^2}{\nu^2_{\overline{\text{MS}}}}\right)^{\frac{D-4}{2}}\Gamma\left(\frac{4-D}{2}\right) - A\left[\gamma_E + \psi(1+w)\right] + \mathcal{O}(D-4)$$

$$A = 16\pi^2 M_K^2\left(\frac{G_{\text{F}}}{\sqrt{2}}V_{us}^* V_{ud}\right)\sum_I C_I(\nu)\xi_{01}^I$$

$$A' = -16\pi^2 M_K^2\left(\frac{G_{\text{F}}}{\sqrt{2}}V_{us}^* V_{ud}\right)\sum_I C_I(\nu)\left[\frac{1}{2}\xi_{00}^I - (1-\ln 2)\xi_{01}^I\right]$$

$$W_+^{\text{res}}(z;\nu) = \frac{f_+^{K^\pm\pi^\mp}(zM_K^2)}{4\pi}\times 16\pi^2 M_K^2\left(\frac{G_{\text{F}}}{\sqrt{2}}V_{us}^* V_{ud}\right)\sum_I C_I(\nu)\left\{\xi_{00}^I - \xi_{01}^I\left[\ln\frac{M^2}{\nu^2} + \psi\left(1 - z\,\frac{M_K^2}{M^2}\right)\right]\right\}$$

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Putting everything together

$$a_+ = \int_0^\infty \frac{dx}{x}\,\frac{\rho_+^{\pi\pi}(x)}{G_F M_K^2} + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \left\{ \frac{C_{7V}(\nu)}{\alpha} + \sum_I C_I(\nu) \left[\xi_{00}^I - \xi_{01}^I \left(\ln \frac{M^2}{\nu^2} - \gamma_E \right) \right] \right\}$$

$$\begin{aligned} b_+ = & \int_0^\infty \frac{dx}{x^2}\,\frac{\rho_+^{\pi\pi}(x)}{G_F} + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \frac{\pi^2}{6}\,\frac{M_K^2}{M^2} \sum_I C_I(\nu)\,\xi_{01}^I \\ & + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times \lambda_+ \frac{M_K^2}{M_\pi^2} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \left\{ \frac{C_{7V}(\nu)}{\alpha} + \sum_I C_I(\nu) \left[\xi_{00}^I - \xi_{01}^I \left(\ln \frac{M^2}{\nu^2} - \gamma_E \right) \right] \right\} \\ & - \frac{1}{60} \left(\frac{M_K^2}{M_\pi^2} \right)^2 \left(\alpha_+ - \beta_+ \frac{s_0}{M_\pi^2} \right) \end{aligned}$$

$$a_+ = -1.58 + \begin{cases} -0.10 \div +0.03 & \text{NDR} \\ -0.14 \div +0.07 & \text{HV} \end{cases}$$

$$b_+ = -0.76 + \begin{cases} -0.04 \div +0.03 & \text{NDR} \\ -0.07 \div +0.03 & \text{HV} \end{cases}$$

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Putting everything together

$$a_+ = \int_0^\infty \frac{dx}{x}\,\frac{\rho_+^{\pi\pi}(x)}{G_F M_K^2} + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \left\{ \frac{C_{7V}(\nu)}{\alpha} + \sum_I C_I(\nu) \left[\xi_{00}^I - \xi_{01}^I \left(\ln \frac{M^2}{\nu^2} - \gamma_E \right) \right] \right\}$$

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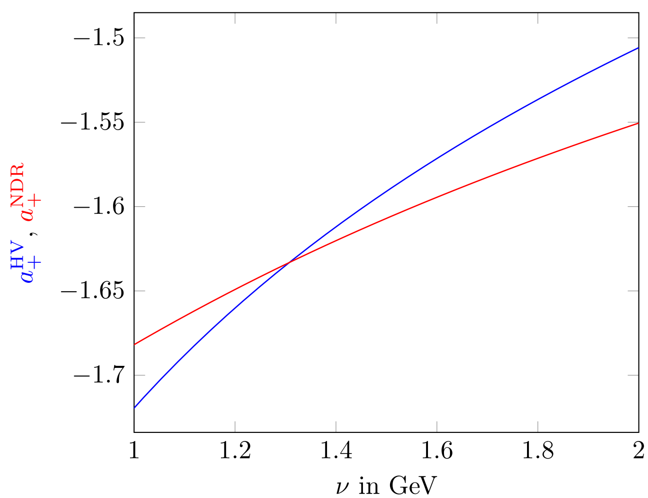

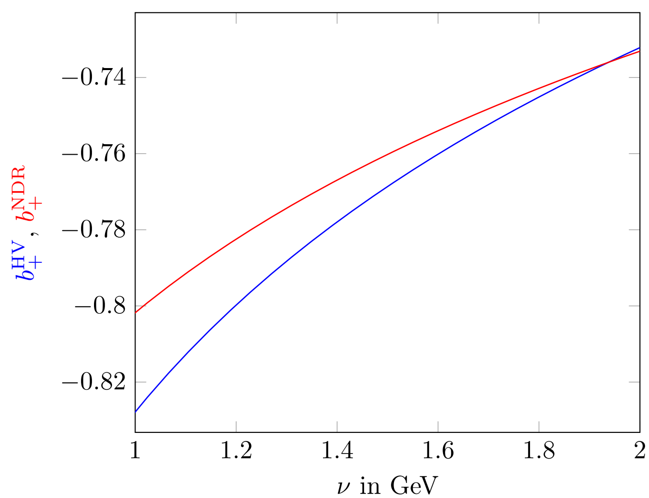

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Putting everything together

$$a_+ = \int_0^\infty \frac{dx}{x}\,\frac{\rho_+^{\pi\pi}(x)}{G_F M_K^2} + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \left\{ \frac{C_{7V}(\nu)}{\alpha} + \sum_I C_I(\nu) \left[\xi_{00}^I - \xi_{01}^I \left(\ln \frac{M^2}{\nu^2} - \gamma_E \right) \right] \right\}$$

$$\begin{aligned} b_+ = & \int_0^\infty \frac{dx}{x^2}\,\frac{\rho_+^{\pi\pi}(x)}{G_F} + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \frac{\pi^2}{6}\,\frac{M_K^2}{M^2} \sum_I C_I(\nu)\,\xi_{01}^I \\ & + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times \lambda_+ \frac{M_K^2}{M_\pi^2} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \left\{ \frac{C_{7V}(\nu)}{\alpha} + \sum_I C_I(\nu) \left[\xi_{00}^I - \xi_{01}^I \left(\ln \frac{M^2}{\nu^2} - \gamma_E \right) \right] \right\} \\ & - \frac{1}{60} \left(\frac{M_K^2}{M_\pi^2}\right)^2 \left(\alpha_+ - \beta_+ \frac{s_0}{M_\pi^2} \right) \end{aligned}$$

$$a_+ = -1.58 + \begin{cases} -0.21 \div -0.10 & \text{NDR} \\ -0.14 \div +0.11 & \text{HV} \end{cases}$$

$$b_+ = -0.76 + \begin{cases} -0.04 \div -0.02 & \text{NDR} \\ -0.05 \div +0.08 & \text{HV} \end{cases}$$

Difficult to assess theoretical errors at this stage

Values come in the right ballpark (encouraging)...

... for $\beta_+ \cdot 10^{-8} = -0.85$!

IV. Summary - Conclusions

- $K^\pm \to \pi^\pm \ell^+ \ell^-$ and $K_S \to \pi^0 \ell^+ \ell^-$ offer a window to BSM physics (as, in general, other rare kaon decays)

- Unfortunately, they are long-distance dominated

- Amplitudes can be described by two parameters, $a_{+,S}$ and $b_{+,S}$ (assuming $\alpha_{+,S}$ and $\beta_{+,S}$ known)

- Rather precise data on decay distribution available for $K^\pm \to \pi^\pm \ell^+ \ell^-$

- Can expect more in the future (NA62), also for $K_S \to \pi^0 \ell^+ \ell^-$ (LHCb)

E. Goudzovski, https://indico.cern.ch/event/648004/contributions/2987967

R. Aaij et al. (LHCb coll.), arXiv:1808.08865

A. A. Alves Junior et al., arXiv:1808.03477

Extracting a_+ and b_+ from recent data

- Combined fit to the BNL-E865 and NA48/2 data for $K^\pm \to \pi^\pm e^+ e^-$ clearly favours the solution where $a_+ < 0$, $b_+ < 0$, with $a_+ \sim b_+$

- Data prefer somewhat smaller (in absolute value) values of β_+ than those obtained by direct determinations from $K \to \pi\pi\pi$ data

Low-energy expansion of the form factors

- Two-loop expression of the form factors worked out ($\pi\pi$ intermediate states)

- Determinations of $a_{+,S}$ and $b_{+,S}$ not affected by corrections

- Existing one-loop calculations suggest that kaon loops could possibly have a sizable effect on a_+ and a_S
$\longrightarrow$ requires a two-loop ChPT calculation

Contribution from the two-pion state

- Phenomenological evaluation from unsubtracted dispersion relation

- Absorptive part provided by the e.m. form factor of the pion and by the P-wave projection of the $K^{\pm}\pi^{\mp} \to \pi^{+}\pi^{-}$ amplitude

- Simple approach (IAM unitarization)

- Issue of β_{+} resurfaces...

- More elaborate tools (e.g. Khuri-Treiman eqs.) available

- Room for improvement in the future ($\bar{K}K$?, $K\pi$?...)

Matching with the short-distance regime

- Form factors behave, in the asymptotic euclidian region, as $\sim \ln(-s/\nu)$, where ν is the renormalization scale

- Renormalization of SD singularity implemented through the operator Q_{7V} and its Wilson coefficient $C_{7V}(\nu)$

- SD behaviour results from the pile-up of more and more complex intermediate states

- This process has been described through a, necessary infinite, set of zero-width resonances intermediate states

- In the absence of QCD corrections, possible to adjust the couplings of these resonances such as to reproduce the correct high-energy behaviour

- Extend the construction to α_s corrections

Lepton flavour universality in the kaon sector?

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Prospects for phenomenological estimates of a_+ and b_+ look good, with some (reasonable) amount of additional theoretical work

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Prospects for phenomenological estimates of a_+ and b_+ look good, with some (reasonable) amount of additional theoretical work

Complementary to existing and future efforts to address this issue through lattice QCD

Thank you for your attention