Scale Invariant Optimized Perturbation at ~~zero and~~ finite temperature

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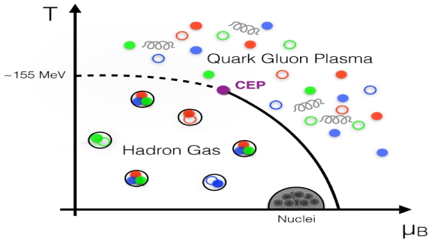
Context: QCD phase diagram/ Quark Gluon Plasma

Complete QCD phase diagram far from being confirmed:



$T \neq 0, \mu = 0$ well-established from lattice: no sharp phase transition, rather continuous crossover at $T_c \simeq 154 \pm 9$ MeV (Aoki et al '06).

Goal: more analytical approximations, ultimately in regions not much accessible on the lattice: large density (chemical potential) due to the famous "sign problem"

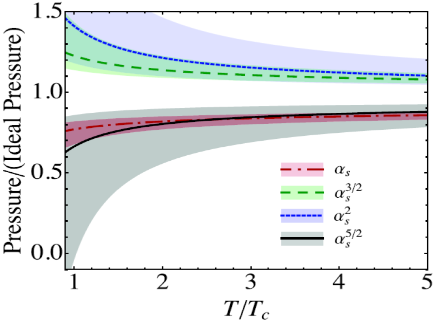
Tool: unconventional 'RG optimized' (RGOPT) resummation of perturbative expansions

Outline

- Introduction/Motivation
- Optimized (or 'screened') perturbation (OPT $\sim$ SPT)
- RG-compatible OPT $\equiv$ RGOPT
- Application: thermal nonlinear sigma model
  (many similarities with QCD but simpler)
- Application: thermal (pure gauge) QCD: hard thermal loops
- Conclusions

1. Motivation: problems of thermal perturbative expansion (QCD, ϕ^4, ...)

poorly convergent and very scale-dependent (ordinary) perturbative expansions:



QCD (pure gauge) pressure at successive (standard) perturbation orders
shaded regions: scale-dependence for $\pi T < \mu < 4\pi T$
(illustration from Andersen, Strickland, Su '10)

Problems of thermal perturbation (QCD but generic)

Usual suspect: mix up of *hard* $p \sim T$ and *soft* $p \sim \alpha_S T$ modes.

Thermal 'Debye' screening mass $m_D^2 \sim \alpha_S T^2$ gives IR cutoff,
BUT $\Rightarrow$ perturbative expansion in $\sqrt{\alpha_S}$ in QCD
$\rightarrow$ often advocated reason for slower convergence

Yet many interesting QGP physics features happen at not that large coupling $\alpha_S(\sim 2\pi T_c) \sim .5$, ($\alpha_S(\sim 2\pi T_c) \sim 0.3$ for pure gauge)

Many efforts to improve this (review e.g. Blaizot, Iancu, Rebhan '03):

Screened PT (SPT) (Karsch et al '97) $\sim$ Hard Thermal Loop (HTLpt) resummation (Andersen, Braaten, Strickland '99); Functional RG, 2PI formalism (Blaizot, Iancu, Rebhan '01; Berges, Borsanyi, Reinosa, Serreau '05)

Our RGOPT($T \neq 0$) essentially treats thermal mass 'RG consistently':
$\rightarrow$ UV divergences induce mass anomalous dimension.

2. Optimized (or Screened) Perturbation (OPT/SPT)

Trick ($T = 0$): add and subtract a mass, consider $m\,\delta$ as interaction:

$$\mathcal{L}(g, m) \to \mathcal{L}(\delta\, g, m(1-\delta)) \quad \text{(e.g. in QCD } g \equiv 4\pi\alpha_S\text{)}$$

$0 < \delta < 1$ interpolates between $\mathcal{L}_{free}$ and *massless* $\mathcal{L}_{int}$;
$\to m$: arbitrary trial parameter

• Take any (renormalized) pert. series, (re)expand in δ *after:*
$m \to m\,(1-\delta)$; $\quad g \to \delta\, g$
then $\delta \to 1$ (to recover original massless theory):

BUT m-dependence remains at finite δ^k-order:
fixed by stationarity prescription: optimization (OPT):
$\frac{\partial}{\partial m}$(physical quantity) $= 0$ for $m = \bar{m}_{opt}(g) \neq 0$:

• $T = 0$: exhibits *dimensional transmutation*: $\bar{m}_{opt}(g) \sim \mu\, e^{-const./g}$

• $T \neq 0$ similar idea: "screened perturbation" (SPT), or *resummed* "hard thermal loop (HTLpt)" (QCD) = expand around a quasi-particle mass.

Does this 'cheap trick' always work? and why?

•Exponentially fast convergence of this procedure for $D = 1$ ϕ^4 oscillator (cancels large pert. order factorial divergences!) Guida et al '95

•In Quantum Field Theories (QFT):

May be viewed as enforcing low order, best approximation of (all order) scale-invariance (massless limit!)

(NB *genuine* masses (e.g. $m_{quarks} \neq 0$) neglected/treated as small perturbations)

•OPT resums standard perturbation: in $O(N), SU(N), \cdots$ models, one-loop $\simeq$ large-N approximation.

•But QFT multi-loop calculations (specially $T \neq 0$) (very) difficult:
$\to$ empirical convergence? not clear

•Main problem (largely overlooked): OPT (=mass optimization) not much consistent with RG properties

•Other pb at higher order: OPT: $\partial_m(...) = 0$ has multi-solutions (some complex!), how to choose right one, if no nonperturbative "insight"??

3. RG-compatible OPT ($\equiv$ RGOPT) (JLK, A. Neveu '2010)

Consider a *physical* quantity (perturbatively RG invariant)
e.g. in thermal context the pressure $P(m, g, T)$:

in addition to 'OPT' Eq.: $\frac{\partial}{\partial m} P^{(k)}(m, g, \delta = 1)|_{m \equiv \tilde{m}} \equiv 0$,

Require (δ-modified!) $P(m, g)$ at order δ^k to satisfy (perturbative) Renormalization Group (RG) equation:

$$\mathrm{RG}\left(P^{(k)}(m, g, \delta = 1)\right) = 0$$

$$\mathrm{RG} \equiv \mu \frac{d}{d\,\mu} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_m(g)\, m \frac{\partial}{\partial m}$$

$\beta(g) \equiv -b_0 g^2 - b_1 g^3 + \cdots, \;\; \gamma_m(g) \equiv \gamma_0 g + \gamma_1 g^2 + \cdots$

$\rightarrow$ combined with OPT, RG Eq. reduces to massless form:

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right] P^{(k)}(m, g, \delta = 1) = 0$$

•Using both OPT AND RG completely fix $m \equiv \bar{m}$ and $g \equiv \bar{g}$.

$\rightarrow$ Parameter-free determination!: intermediate optimal $\bar{m}(g, T)$ but final (physical) result from $P(\bar{m}, \bar{g}, T) \rightarrow P(\Lambda^{QCD}_{\overline{MS}}, T)$

OPT + RG = RGOPT main features

•Basic interpolation: why not $m \to m\,(1-\delta)^a$?
Most previous works ($T = 0$ OPT, $T \neq 0$ Screened PT, HTLpt) do $a = 1$
but generally (we have shown) $a = 1$ spoils RG invariance!

•Standard OPT gives multiple $\bar{m}(g, T)$ solutions at increasing δ^k-orders

$\to$ Our approach restores RG, +requiring matching to perturbation (i.e. Asymptotic Freedom (AF) for QCD):
$\ln \frac{\mu}{\bar{m}} \sim \frac{1}{b_0\, g} + \cdots$ for $g \to 0$, $\bar{m} \sim \Lambda_{QCD}$

$\to$ At successive orders, AF-compatible optimal $\bar{m}$ (often unique) *only* appears for a universal critical a:
$m \to m\,(1-\delta)^{\frac{\gamma_0}{b_0}}$ (in general $\frac{\gamma_0}{b_0} \neq 1$)

$\to$ RG consistency goes beyond simple "add and subtract" trick

and removes any spurious solutions (incompatible with AF)

•But does not always avoid complex $\bar{m}$ solutions
(artifacts of perturbative coefficients, possibly cured by renormalization scheme change)

NB: some previous results with RGOPT ($T = 0$)

Chiral symmetry breaking order parameter $F_\pi(m_{u,d,s} = 0)/\Lambda_{\overline{MS}}^{\rm QCD}$:

F_π exp input $\to \Lambda_{\overline{MS}}^{n_f=3} \to \alpha_S^{\overline{MS}}(\mu = m_Z)$.

N^3LO: $F_\pi^{m_q=0}/\Lambda_{\overline{MS}}^{n_f=3} \simeq 0.25 \pm .01 \to \alpha_S(m_Z) \simeq 0.1174 \pm .001 \pm .001$
(JLK, A.Neveu, PRD88 (2013))
(compares well with α_S lattice and world average values [PDG2016-17])

Also applied to $\langle \bar{q} q \rangle$ at N^3LO (using spectral density of Dirac operator):
$\langle \bar{q} q \rangle^{1/3}_{m_q=0}(2\,{\rm GeV}) \simeq -(0.84 \pm 0.01)\Lambda_{\overline{MS}}$ (JLK, A.Neveu, PRD 92 (2015))
compares well with latest lattice result

Digression: basic Thermal QFT calculations in a nutshell

partition function: $Z = Tr\, e^{-\beta H}$, $\beta \equiv 1/T$,
derive from it free energy $F \sim -$Pressure, entropy,... $\rightarrow$ equation of state

At equilibrium (no time dependence), connected with field theory (imaginary time formalism) by

•going to Euclidean time $t \rightarrow -i\tau$;

•restrict τ in a box of size $\beta = 1/T$
$\Rightarrow$ p_0 has discrete (Matsubara) modes (bosons):
$\omega_n = 2\pi n/\beta = 2\pi n T, n \in Z$
$\rightarrow$ e.g. scalar propagator:

$$\frac{1}{p^2 - m^2} \rightarrow -\frac{1}{\omega_n^2 + \mathbf{p}^2 + m^2}$$

Loop integration (dimensional regularization, $\overline{MS}$-scheme):

$$\int \frac{dp_0\, d^3\mathbf{p}}{(2\pi)^4} f(p_0, \mathbf{p}) \rightarrow (\mu e^{\gamma_E})^\epsilon\ T \sum_{n=-\infty}^{+\infty} \int \frac{d^{3-2\epsilon}\mathbf{p}}{(2\pi)^{3-2\epsilon}} f(i\omega_n, \mathbf{p}) \equiv \sum\!\!\!\!\!\!\int_P f(P)$$

Remark: beyond one-loop $\sum\!\!\!\!\!\!\int_P$ generally difficult:
often done as systematic m/T expansions (when justified).

RGOPT($T \neq 0$) generic features (JLK, M.B Pinto, PRL116 '16)

1. Start from pert. 1-, 2-loop free energy/pressure $\mathcal{F}(m \neq 0, T \neq 0)$

2. One crucial point [*before* doing δ-expansion +RG-optimization]:

$\mathcal{F}(m \neq 0, T = 0)$ involves leading order scale dependence: $\sim m^4 \ln \mu$

compensated by $\mathcal{E}_0$ *finite* T-independent 'vacuum energy' subtraction:
$\mathcal{E}_0(g, m) = -m^4 \left(\frac{s_0}{g} + s_1 + s_2 g + \cdots \right)$
determined order by order such that $\mu \frac{d}{d\,\mu} \mathcal{E}_0$ cancels the $\ln \mu$ dependence:

$s_0 = \frac{1}{2(b_0 - 4\gamma_0)} = 8\pi^2$; $\quad s_1 = \frac{(b_1 - 4\gamma_1)}{8\gamma_0\,(b_0 - 4\gamma_0)} = -1$, ...:

Well-known at $T = 0$, BUT MISSED by $T \neq 0$ PT, SPT, HTLpt(QCD):
largely explains the (huge) scale dependence seen at higher order in these approaches! (more on this below)

3. Next expand in δ after $m^2 \to m^2(1-\delta)^a$; $g \to \delta g$; $\delta \to 1$:
RG only consistent for $a = 2\gamma_0/b_0$ ($= 1/3$ e.g.for ϕ^4 while $a = 1$ in SPT)

•All together, RGOPT gives much better residual scale dependence

4. Not quite QCD: $O(N)$ $D=2$ nonlinear σ model (NLSM)

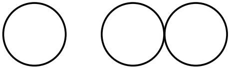
- Shares nice properties with QCD (asymptotic freedom, mass gap).
- $T \neq 0$: pressure, trace anomaly, etc have QCD-like shapes

- Nonperturbative $T \neq 0$ results available for comparison:

(lattice ($N=3$)[Giacosa et al '12], $1/N$-expansion [Andersen et al '04])

$$\mathcal{L}_0 = \frac{1}{2}(\partial \pi_i)^2 + \frac{g(\pi_i \partial \pi_i)^2}{2(1-g\pi_i^2)} - \frac{m^2}{g}\left(1-g\pi_i^2\right)^{1/2}$$

Two-loop pressure from:



- Advantage w.r.t. QCD: exact T-dependence at 2-loops:

$$P_{\rm pert.2loop} = -\frac{(N-1)}{2}\left[I_0(m,T) + \frac{(N-3)}{4} m^2 g I_1(m,T)^2\right] + \mathcal{E}_0,$$

$$I_0(m,T) = \frac{1}{2\pi}\left(m^2(1-\ln\frac{m}{\mu}) + 4T^2 K_0(\frac{m}{T})\right)$$

$$K_0(x) = \int_0^\infty dz \ln\left(1-e^{-\sqrt{z^2+x^2}}\right), \quad I_1(m,T) = \partial I_0(m,T)/\partial m^2$$

One-loop RGOPT pressure (NLSM, but generic features)

"Exact" (T-dependence) mass gap $\bar{m}(g,T)$ from $\partial_m P(m)=0$:

$$\ln\frac{\bar{m}}{\mu}=-\frac{1}{b_0\,g(\mu)}-2\frac{\partial}{\partial m^2}K_0(\frac{\bar{m}}{T}),\qquad (b_0^{\rm nlsm}=\frac{N-2}{2\pi})$$

•Exhibits *"exact"* (one-loop) scale invariance (generic RGOPT feature):
since one-loop running $g^{-1}(\mu)=g^{-1}(M_0)+b_0\ln\frac{\mu}{M_0})$

$T=0$: $\bar{m}(T=0)=\mu e^{-\frac{1}{b_0\,g(\mu)}}=\Lambda_{\overline{\rm MS}}^{1-{\rm loop}}$

$T\gg m$: $$\frac{\bar{m}(T)}{T}=\frac{\pi\,b_0\,g}{1-b_0\,g\,L_T},\quad (L_T\equiv\ln\frac{\mu\,e^{\gamma_E}}{4\pi T})$$

•Other remarkable property: running $g(\mu\sim T)$ emerges automatically:

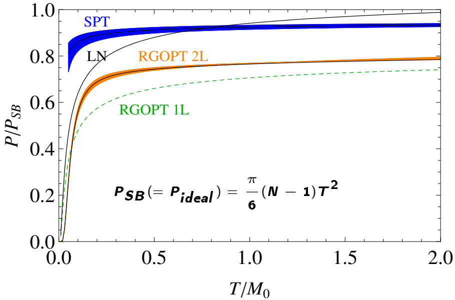
$$\frac{P_{1L}^{\rm RGOPT}}{P_{ideal}}(T\gg m)\simeq 1-\frac{3}{2}b_0 g(\frac{4\pi\,T}{e^{\gamma_E}})$$

while in standard perturbation/SPT/HTLpt/... $\mu\sim 2\pi T$ is *imposed*

e.g. to get correct ideal gas limit for large T

NLSM pressure [G. Ferrari, JLK, M.B. Pinto, R.0 Ramos, 1709.03457,PRD]

$P/P_{ideal}(N=4, g(M_0)=1)$: RGOPT 1- and 2-loop versus large N (LN) and SPT ($\equiv$ ignoring RG-induced subtraction + $m^2 \to m^2(1-\delta)$):

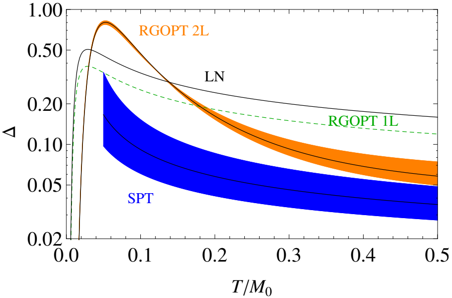


(shaded range: scale-dependence $\pi T < \mu \equiv M < 4\pi T$)

$\to$ At two-loops a moderate scale-dependence reappears, although less pronounced than 2-loop standard PT, SPT.

NLSM interaction measure (trace anomaly)

(normalized) $\Delta_{\rm NLSM} \equiv (\mathcal{E} - P)/T^2 \equiv T\partial_T(\frac{P}{T^2})$



$N = 4, g(M_0) = 1$ (shaded regions: scale-dependence $\pi T < \mu = M < 4\pi T$)

•Δ_{SPT}: perturbative monotonic behaviour + sizeable scale dependence.

•RGOPT qualitatively more similar to 4D lattice QCD, showing a peak: originates from nonperturbative RGOPT mass gap from $T \neq 0$ to $T = 0$.

(But no phase transition in 2D NLSM (Mermin-Wagner-Coleman theorem): just reflects broken conformal invariance (mass gap)).

5. Thermal (pure gauge) QCD: hard thermal loop (HTL)

QCD(glue) adaption of OPT → HTLpt [Andersen, Braaten, Strickland '99]:
same trick now operates on a gluon mass term [Braaten-Pisarski '90]:

$$\mathcal{L}_{QCD}(\text{gauge}) - \frac{m^2}{2} Tr \left[G_{\mu\alpha} \langle \frac{y^\alpha y^\beta}{(y.D)^2} \rangle_y G^\mu_\beta \right], \quad D^\mu = \partial^\mu - ig\, A^\mu, \quad y^\mu = (1, \hat{\mathbf{y}})$$

(effective, explicitly gauge-invariant but nonlocal Lagrangian):

originally describes screening mass $m^2 \sim \alpha_S T^2$ + other HTL contributions [dressing gluon vertices and propagators]

But here m arbitrary: to be determined by RGOPT optimization.

$$P^{HTL}_{\text{1-loop}} = -\frac{(N_c^2-1)}{2} \sum\!\!\!\!\!\!\!\int_P \{(d-1)\ln[P^2 + \Pi_T(P)] + \ln[p^2 + \Pi_L(P)]\}$$

with HTL-dressed T,L propagators:

$$\Pi_L(P) = m^2(1 - T_P); \quad \Pi_T(P) = \frac{m^2}{(d-1)n_P^2}(T_P - 1 + n_P^2); \quad T_P \equiv \int_0^1 dc \frac{P_0^2}{P_0^2 + c^2 p^2}$$

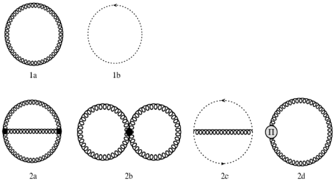
+HTL dispersion relation e.g.: $k^2 + m^2 \left[1 - \frac{\omega_L}{2k} \ln(\frac{\omega_L + k}{\omega_L - k})\right] = 0; \cdots$

• Exact 2-loop? daunting task...

NB possibly simpler effective gluon mass models/prescriptions exist...
[e.g. Reinosa, Serreau, Tissier, Weschbor '15, see J. Maelger's previous talk]

...But HTLpt advantage: calculated up to 3-loops α_S^2 (NNLO)

e.g. at two-loops:



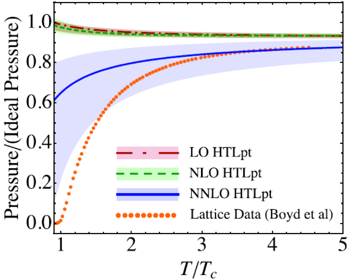
BUT only as m/T expansions [Andersen et al '99-'15]
Drawback: HTLpt $\equiv$ high-T approximation by definition.

$$P^{HTLpt}_{\text{1-loop},\overline{\text{MS}}} = P_{\text{ideal}} \times$$
$$\left[1 - \frac{15}{2}\hat{m}^2 + 30\hat{m}^3 + \frac{45}{4}\hat{m}^4(\ln\frac{\mu}{4\pi T} + \gamma_E - \frac{7}{2} + \frac{\pi^2}{3})\right]$$

$\hat{m} \equiv \frac{m}{2\pi T}$ $\qquad$ $P_{\text{ideal}} = (N_c^2 - 1)\pi^2 \frac{T^4}{45}$

Standard HTLpt results

Pure gauge at NNLO (3-loops) [Andersen, Strickland, Su '10]:



Reaching closer to lattice results (down to $T \sim 2 - 3\,T_c$) only emerges at NNLO (3-loop) for lower end of scale $\mu \sim \pi T - 2\pi T$.

Main HTLpt issue: drastically increasing scale dependence at NNLO order
Moreover HTLpt perturbative mass prescription: $\bar{m} \to m_D^{pert}(\alpha_S)$
[rather than $\partial_m P(m) = 0$, to avoid complex solutions]

[NB when including quarks, 3-loop HTLpt in agreement with lattice only for central scale, but even larger scale dependence (Andersen et al '13-16)]

RGOPT adaptation of HTLpt =RGOHTL

Main points/changes:

• Crucial RG invariance-restoring subtractions in Free energy (pressure):
$P_{HTLpt} \to P_{HTLpt} - m^4(\frac{s_0}{\alpha_S} + s_1 + \cdots)$: reflects its anomalous dimension.

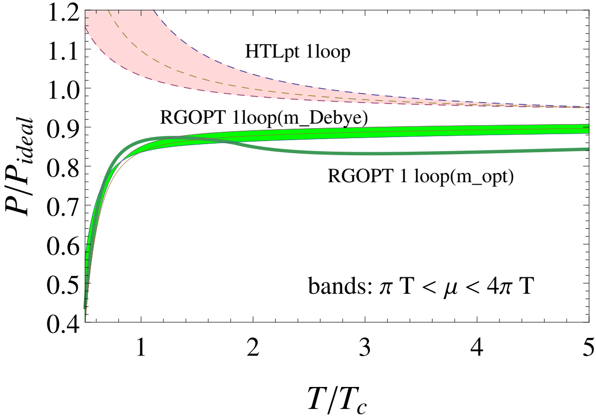
• Interpolate with RG-preserving $m^2(1-\delta)^{\frac{\gamma_0}{b_0}}$, where gluon mass anomalous dimension defined from (available) counterterm.

→ Scale dependence improves at higher orders since RG invariance maintained at all stages

•SPT,HTLpt,... do not fulfill (had missed) this:
RG-inconsistent, yet moderate scale dependence up to 2-loops: screened because the (leading order) RG-unmatched term $\mathcal{O}(m^4 \ln \mu)$ is perturbatively '3-loop' $\mathcal{O}(\alpha_S^2)$ from $m^2 \sim \alpha_S T^2$.

→ But explains why HTLpt scale dependence dramatically resurfaces at 3-loops!

RGOPT vs HTLpt: one-loop pressure



NB: the bending of P_{RGOPT} for small T/T_c is essentially due to $-P_{RGOPT}(T=0) \neq 0$, absent from HTLpt.

2-loop RGOHTL: needs new complicated calculations...

Crucial RG-consistent 2-loop subtractions determined by $\alpha_S m^4 \ln \mu$ term (non-logarithmic terms also very relevant).

These are $\mathcal{O}(\alpha_S T^4 \frac{m^4}{T^4})$: not calculated in HTLpt as perturbatively $\mathcal{O}(\alpha_S^3)$

Involves $\sim$ 30 independent integrals, $\sim$ 20 being (very) complicated, e.g.

$$\sum\!\!\!\!\!\!\!\int_{P,Q} \frac{T_P T_Q (p+q)^2}{p^2 P^2 q^2 Q^2 (P+Q)^2}; \quad T_P \equiv \int_0^1 dc \frac{P_0^2}{P_0^2 + c^2 p^2}$$

$(P^2 = P_0^2 + p^2)$, $c \equiv$HTL angle (averaging).

$\rightarrow$ formally "2-loop" but effectively 4-loop (HTL-dressed propagators)

(Techniques: contour integration + D-dim int. by parts+ other tricks)

Present status: our calculations in good progress but need more checks, specially the difficult non-logarithmic parts (i.e. finite parts in dim.reg.)

NB high-T $\leftrightarrow$ $T = 0$ correspondance:

$C_{20} \ln^2 \frac{\mu}{T} + C_{21} \ln \frac{\mu}{T} + C_{22}^{(T \gg m)} \leftrightarrow C_{20} \ln^2 \frac{\mu}{m} + C_{21} \ln \frac{\mu}{m} + C_{22}^{(T=0)}$

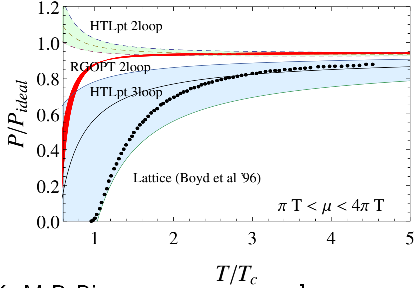
•Leading Log (LL) C_{20} determined simply from RG from one-loop.

•Similarly: 2-loop (perturbative) scale invariance guaranteed by RG giving s_1 simply in terms of C_{21} independently of precise C_{21} value!

(Preliminary) RGO(HTL) results (2-loop, pure gauge)

illustrated here for simplest LL approx.: $C_{21} = C_{22} = 0$
but RG-consistent subtraction $s_1(C_{21} = 0) \neq 0$

Moderate scale-dependence reappears at 2-loops
...but sensible improvement wrt HTLpt



[JLK, M.B Pinto, to appear soon]

NB scale dependence should further improve at 3-loops, generically:

RGOPT at $\mathcal{O}(\alpha_S^k) \to \bar{m}(\mu)$ appears at $\mathcal{O}(\alpha_S^{k+1})$ for *any* $\bar{m}$, but
$\bar{m}^2 \sim \alpha_S T^2 \to P \simeq \bar{m}_G^4/\alpha_S + \cdots$: leading μ-dependence at $\mathcal{O}(\alpha_S^{k+2})$.

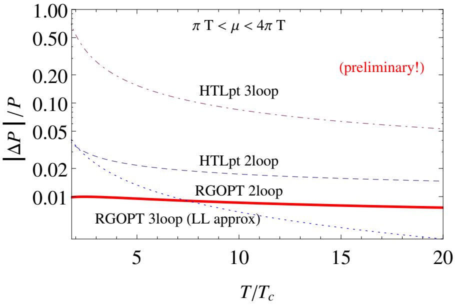
- Warning: low $T \sim T_c$ genuine $P(T)$ shape sensitive to true C_{21}, C_{22}
(crucially needed before possibly comparing RGOPT with lattice...)

Preliminary RGO(HTL) approximate 3-loop results

3-loops: exact $m^4\alpha_S^2$ terms need extra complicated calculations, but
$P^{3l}_{RGOHTL} \sim P^{2l}_{RGOHTL} + m^4\alpha_S^2(C_{30}\ln^3\frac{\mu}{2\pi T} + C_{31}\ln^2\frac{\mu}{2\pi T} + C_{32}\ln\frac{\mu}{2\pi T} + C_{33})$:

3-lopp LL and NLL coefficients C_{30}, C_{31} fully determined from lower orders from RG invariance

Within NLL approximation and in $T/T_c \gtrsim 2$ range where it is more trustable:



We assume that the true coefficients will not spoil this improved scale dependence.

Summary and Outlook

•RGOPT includes 2 major differences w.r.t. previous OPT/SPT/HTLpt... approaches:

1) OPT +/or RG optimizations fix optimal $\bar{m}$ *and possibly* $\bar{g} = 4\pi\bar{\alpha}_S$:
$\rightarrow$ parameter free determination in terms of only Λ_{QCD}

2) Maintaining RG invariance uniquely fixes the basic interpolation $m \rightarrow m(1-\delta)^{\gamma_0/b_0}$: discards spurious solutions and accelerates convergence.

• At $T \neq 0$, exhibits improved stability + drastically improved scale dependence (with respect to standard PT, but also w.r.t. HTLpt)

•Paves the way to extend such RG-compatible methods to full QCD thermodynamics specially for exploring also finite density