



Institut für Theoretische Physik



Two-loop top mass effects in Higgs pair production at NLO QCD RPP 2019, LPC Clermont-Ferrand

[in collaboration with F. Campanario, S. Glaus, M. M. Mühlleitner, M. Spira, and J. Streicher (arXiv:1811.05692)]

23/01/2019, Julien Baglio



The SM ultimate test: probing the scalar potential

From the scalar potential before EWSB (ϕ as the Higgs field):

$$V(\phi) = -m^2 |\phi|^2 + \lambda |\phi|^4$$



To $V(\phi)$ after EWSB, with $M_H^2 = 2m^2$, $v^2 = m^2/\lambda$:

$$\phi = \left(\frac{0}{\frac{V+H(x)}{\sqrt{2}}}\right) \Rightarrow V(H) = \frac{1}{2}M_H^2H^2 + \frac{1}{2}\frac{M_H^2}{v}H^3 + \frac{1}{8}\frac{M_H^2}{v^2}H^4 + \text{constant}$$



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Overview HH production channels



[from J.B., Djouadi, Quevillon, Rept.Prog.Phys. 79 (2016) 116201]

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Heavy top-quark limit (HTL) calculation

$\mathsf{HTL}\equiv \textit{m}_t \rightarrow +\infty$

- \rightarrow Effective tree-level ggH and ggHH couplings
- \rightarrow Reduce the number of loop by one at each perturbative order



- HTL valid for $\hat{s} \ll 4m_t^2$, but *HH* production threshold $4M_H^2 \le \hat{s}$ ⇒ narrow energy range for which HTL is valid!
- Born-improved NLO QCD HTL: improve HTL result with

$$d\sigma_{
m NLO} \simeq d\sigma_{
m NLO}^{
m HTL} imes rac{d\sigma_{
m LO}^{
m full}}{d\sigma_{
m LO}^{
m HTL}}$$
 [Dawson, Dittmaier, Spira, PRD 58 (1998) 115012]



Gluon fusion: Where we stand in 2019

■ LO QCD (1-loop): Dominated by top-quark loops [Eboli, Marques, Novaes,

Natale, PLB 197 (1987) 269; Glover, van der Bij, NPB 309 (1988) 282; Dicus, Kao, Willenbrock, PLB 203 (1988) 457]

NLO QCD HTL (1-loop): +93% correction

[Dawson, Dittmaier, Spira, PRD 58 (1998) 115012]

Toward full NLO QCD (2-loop):

 \rightarrow NLOFT_{approx}, m_t -effects in real radiation: -10%

[Frederix et al, PLB 732 (2014) 079, Maltoni, Vryonidou, Zaro, JHEP 1411 (2014) 079]

$\rightarrow O(1/m_t^{12})$ terms in virtual amplitudes: $\pm 10\%$

[see e.g. Grigo, Hoff, Steinhauser, NPB 900 (2015) 412]

NNLO QCD HTL (2-loop): +20% on total cross section

[De Florian, Mazzitelli, PLB 724 (2013) 306; PRL 111 (2013) 201801]

■ Latest NNLO QCD: full NLO QCD + NNLO HTL + NNLO exact reals ⇒ +10% to +20% in distributions

[Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli, JHEP 1805 (2018) 059]



Full NLO QCD corrections to $gg \rightarrow HH$

[J.B., Campanario, Glaus, Mühlleitner, Spira, Streicher, arXiv:1811.05692]



NLO QCD in 2016 Distributions at 14 TeV



[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke, JHEP 1610 (2016) 107]

- Reduction to master integrals, sector decomposition, contour deformation
- Large mass effects in the tail up to $\sim -30\%$ w.r.t. HTL
- Born-improved HTL outside full NLO scale variation for m_{HH} > 410 GeV, p_{T,h} > 160 GeV
- No quantitative statement on the top-quark scheme uncertainty

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Overview of our calculation

 $\sigma_{\rm NLO}(pp \to HH + X) = \sigma_{\rm LO} + \Delta \sigma_{\rm virt}^{(1)} + \Delta \sigma_{\rm virt}^{(2)} + \Delta \sigma_{gg} + \Delta \sigma_{gq} + \Delta \sigma_{q\bar{q}}$





Technical setup for the virtual corrections

- Triangle from single Higgs, 1-particle reducible analytically calculated
 [see also Degrassi, Giardino, Gröber, EPJC 76 (2016) 411]
- Classification of the 47 2-loop tensor box diagrams into 6 topologies (+ corresponding fermion-flow reversed diagrams)
- With dimensional regularization $D = 4 2\epsilon$: calculate the matrix-element form factors F_1 and F_2 using FORM/Reduce/Mathematica for $g(k_1)g(k_2) \rightarrow H(k_3)H(k_4)$,

$$\mathcal{M} = arepsilon_{\mu}^{*}(\mathbf{k}_{1})arepsilon_{\nu}^{*}(\mathbf{k}_{2})\left(F_{1}\mathbf{T}_{1}^{\mu
u}+F_{2}\mathbf{T}_{2}^{\mu
u}
ight),$$

$$\begin{split} \mathbf{T}_{1}^{\mu\nu} &= \mathcal{G}^{\mu\nu} - \frac{k_{2}^{\mu}k_{1}^{\nu}}{k_{1}\cdot k_{2}}, \qquad p_{T}^{2} = 2\frac{(k_{2}\cdot k_{3})(k_{1}\cdot k_{3})}{k_{1}\cdot k_{2}} - k_{3}^{2}, \\ \mathbf{T}_{2}^{\mu\nu} &= \mathcal{G}^{\mu\nu} + \frac{k_{2}^{\mu}k_{1}^{\nu}}{(k_{1}\cdot k_{2})p_{T}^{2}}k_{3}^{2} - \frac{2}{p_{T}^{2}}\left[\frac{k_{2}\cdot k_{3}}{k_{1}\cdot k_{2}}k_{3}^{\mu}k_{1}^{\nu} + \frac{k_{1}\cdot k_{3}}{k_{1}\cdot k_{2}}k_{2}^{\mu}k_{3}^{\nu} - k_{3}^{\mu}k_{3}^{\nu}\right] \end{split}$$

Perform Feynman parametrization

 \rightarrow 6-dimensional integrals to be (numerically) evaluated

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2-loop virtual box corrections





2-loop virtual box corrections

Extraction of ultraviolet (UV) divergences: Endpoint subtraction of the Feynman integrals

$$\int_0^1 dx \, \frac{f(x)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \, \frac{f(x) - f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- Infrared (IR) divergences in the middle of the range
 - \Rightarrow Subtraction of the integrand and analytical integration
 - Generic denominator $N = ar^2 + br + c$, $N_0 = br + c$
 - Feynman parameters (x, s, t) and $\rho = \frac{\hat{s}}{m^2}$

•
$$a = O(\rho), \quad b = 1 + O(\rho), \quad c = -\rho x (1 - x)(1 - s)t$$

$$\int_0^1 dx dr \, \frac{rH(x,r)}{N^{3+2\epsilon}} = \int_0^1 dx dr \, \left[\left(\frac{rH(x,r)}{N^{3+2\epsilon}} - \frac{rH(x,0)}{N_0^{3+2\epsilon}} \right) + \frac{rH(x,0)}{N_0^{3+2\epsilon}} \right]$$



Handling threshold instabilities

- Threshold at $\hat{s} = m_{HH}^2 = 4m_t^2$:
 - \Rightarrow Analytical continuation in the complex plane with

$$m_t^2
ightarrow m_t^2 \left(1 - i ilde{\epsilon}
ight), \;\;\; ilde{\epsilon} \ll 1$$

• Enhance stability above threshold with integration by parts With N = a + bx,

$$\int_0^1 dx \, \frac{2b \, f(x)}{N^3} = \frac{f(0)}{a^2} - \frac{f(1)}{(a+b)^2} + \int_0^1 dx \, \frac{f'(x)}{N^2}$$

$$\int_0^1 dx \, \frac{4b \, f(x) \log N}{N^3} = \left[-\frac{f(x)(1+2\log N)}{N^2} \right]_0^1 + \int_0^1 dx \, \frac{f'(x)(1+2\log N)}{N^2}$$



UV renormalization and IR subtraction

- α_s and m_t input parameters to renormalize
 - \rightarrow $\overline{\text{MS}}$ renormalization for α_s with 5 active flavors, δ_{α_s}
 - \rightarrow Top-quark contribution to the external gluon self-energies, δ_{g}
 - \rightarrow On-shell renormalization for m_t , δ_{m_t}

IR subtraction, δ_{IR} :

Subtraction of Born-improved HTL virtual corrections to box diagrams \Rightarrow IR-safe virtual mass-effects



Richardson extrapolation

- Goal: From $m_t^2 (1 i\tilde{\epsilon})$, obtain the limit $\tilde{\epsilon} \to 0$
- Solution: Richardson extrapolation of the result! Assuming $f(\tilde{\epsilon}) - f(0)$ polynomial for small $\tilde{\epsilon}$, method to accelerate the convergence of $f(\tilde{\epsilon})$ to f(0)





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Partonic sub-processes $gg \rightarrow HHg, gq/\bar{q} \rightarrow HHq/\bar{q}, q\bar{q} \rightarrow HHg$



- Full matrix elements generated with FeynArts/FormCalc [see Hahn, PoS ACAT2010 (2010) 078], evaluated with 1-loop library COLLIER [Denner, Dittmaier, Hofer, CPC 212 (2017) 220]
- Then subtracted with Born-improved HTL matrix-element squared calculated analytically
 > IR safe mass effects in the reals



Putting everything together

- Numerical integration performed with VEGAS on a cluster, \hat{t} -integration
- Final hadronic result:

$$\Delta \hat{\sigma}_{\text{virt}} = \int d\Phi_{2\to 2} \left[(\delta_{\alpha_s} + \delta_g + \delta_{m_t} + \delta_{\text{IR}} + \mathcal{M}_{\text{virt}}^{\Box}) (\mathcal{M}_{\text{LO}})^* \right] + \Delta \hat{\sigma}_{\text{virt}}^{\Delta} + \Delta \hat{\sigma}_{\text{virt}}^{1\text{PR}}$$

$$m_{HH}^2 \frac{d\sigma_{\rm NLO}}{dm_{HH}^2} = m_{HH}^2 \frac{d\sigma_{\rm HPAIR}}{dm_{HH}^2} + m_{HH}^2 \frac{d\Delta\sigma_{\rm virt}}{dm_{HH}^2} + m_{HH}^2 \frac{d\Delta\sigma_{\rm reals}}{dm_{HH}^2}$$

HTL hadronic result calculated with HPAIR [Spira, 1996]

Input parameters: can be freely chosen! PDG values for M_W and M_Z , $M_H = 125 \text{ GeV}, m_t = 172.5 \text{ GeV}, G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}, \sqrt{s} = 14 \text{ TeV}$



Final results



 \blacksquare Mass effects in the real corrections $\sim -10\%$ as in <code>[Maltoni, Vryonidou,</code>

Zaro, JHEP 1411 (2014) 079]

- Mass effects in the virtual corrections $\sim -25\%$ at $m_{HH} = 1$ TeV
- First independent cross-check of the results in the literature:
 - $\sigma_{\rm PDF4LHC}^{\rm NLO} = 32.78(7)_{-12.5\%}^{+13.5\%}$ fb vs $\sigma_{\rm PDF4LHC}^{\rm literature} = 32.91(11)_{-12.5\%}^{+13.5\%}$ fb



Focus on the top-quark mass uncertainty Top-quark scheme uncertainty easily calculable!

Switch to $\overline{\text{MS}}$ scheme and calculate xs for $m_t = \overline{m}_t(\overline{m}_t)$, $m_t = \overline{m}_t(m_{HH}/4)$, $m_t = \overline{m}_t(m_{HH}/2)$, $m_t = \overline{m}_t(m_{HH})$

- Recalculate HPAIR xs and the real corrections \rightarrow fast!
- Recalculate 2-loop corrections $\rightarrow OK!$
- Switch the mass counterterm from OS to MS scheme

$$\frac{\delta m_t}{m_t} = \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{4\pi\mu_R^2}{m_t^2}\right)^{\epsilon} \left(\frac{1}{\epsilon} - \log\left(\frac{\mu_{R,t}^2}{m_t^2}\right)\right)$$

$$= \text{Results:} \frac{d\sigma}{dQ}\Big|_{Q=300\text{GeV}} = \frac{+10\%}{-22\%}, \frac{d\sigma}{dQ}\Big|_{Q=400\text{GeV}} = \frac{+7\%}{-7\%},$$

$$\frac{d\sigma}{dQ}\Big|_{Q=600\text{GeV}} = \frac{+0\%}{-26\%}, \frac{d\sigma}{dQ}\Big|_{Q=1200\text{GeV}} = \frac{+0\%}{-30\%} \quad (Q = M_{HH})$$

$$\Rightarrow \sigma^{\text{tot}} = 32.78(7)^{+2\%} \quad (\text{very preliminary interpolation})$$

 $\Rightarrow \sigma_{NLO}^{\text{tot}} = 32.78(7)^{+2\%}_{-25\%}$ (very preliminary interpolation!!)



$gg \rightarrow HH @$ NLO in 2019: Two independent calculations finally exist on the market!

 First independent cross-check since 2016 for the full 2-loop NLO QCD corrections in gluon fusion!

 \rightarrow Complete different method compared to the 2016 calculation [IBP, Richardson extrapolation, etc]

\rightarrow Code flexible: m_t , M_H not fixed a priori, can be changed at will

 \rightarrow results compatible with 2016 study

- Allows for the first evaluation of the top-quark scheme uncertainty, that is found to be large!
- Outlook: Extension to BSM physics and in particular EFT and 2HDM models



Backup slides



Details for the renormalization

UV renormalization: δ_{α_s} , δ_g , δ_{m_t}

 \rightarrow MS renormalization for α_s with 5 active flavors $N_F = 5$

$$\frac{\delta \alpha_s}{\alpha_s} = \frac{\alpha_s}{\pi} \, \Gamma(1+\epsilon) \, (4\pi)^{\epsilon} \left[-\frac{33 - 2(N_F + 1)}{12\epsilon} + \frac{1}{6} \log\left(\frac{\mu_R^2}{m_t^2}\right) \right], \quad \delta_{\alpha_s} = \frac{\delta \alpha_s}{\alpha_s} \, \mathcal{M}_{\mathrm{LO}}$$

 $\rightarrow\,$ Top-quark contribution to the external gluons self-energies

$$\delta_g = \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{4\pi\mu_R^2}{m_t^2}\right)^{\epsilon} \left(-\frac{1}{6\epsilon}\right) \mathcal{M}_{\rm LO}$$

 \rightarrow On-shell renormalization for m_t

$$\frac{\delta m_t}{m_t} = \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{4\pi\mu_R^2}{m_t^2}\right)^{\epsilon} \left(\frac{1}{\epsilon} + \frac{4}{3}\right), \quad \delta_{m_t} = -2\frac{\delta m_t}{m_t} m_t^2 \frac{\partial \mathcal{M}_{\rm LO}}{\partial m_t^2}$$

• **IR subtraction:**
$$\delta_{\rm IR} = \frac{\alpha_s}{\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu_R^2}{-m_{HH}^2}\right)^{\epsilon} \left[\frac{3}{2\epsilon^2} + \frac{33-2N_F}{12\epsilon} \left(\frac{\mu_R^2}{-m_{HH}^2}\right)^{-\epsilon} - \frac{11}{4} + \frac{\pi^2}{4}\right] \mathcal{M}_{\rm LO}$$



Details for the renormalization

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$$IR subtraction: \delta_{\rm IR} = \frac{\alpha_s}{\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu_R^2}{-m_{HH}^2}\right)^{\epsilon} \left[\frac{3}{2\epsilon^2} + \frac{33-2N_F}{12\epsilon} \left(\frac{\mu_R^2}{-m_{HH}^2}\right)^{-\epsilon} - \frac{11}{4} + \frac{\pi^2}{4}\right] \mathcal{M}_{\rm LO}$$



Building the local IR counterterm:

$$d\Delta \hat{\sigma}_{ij}^{\mathrm{mass}} = d\Delta \hat{\sigma}_{ij} - \frac{d\Delta \hat{\sigma}_{ij}^{\mathrm{HTL}}}{d\Delta \hat{\sigma}_{ij}}$$

Local IR counterterm with a projected on-shell LO 2 \rightarrow 2 kinematics to rescale the 2 \rightarrow 3 HTL

 $2 \rightarrow 2~OS~LO$ from $_{[Catani,~Seymour,~NPB~485~(1997)~291]}$ with initial-state emitter, initial-state spectator



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$$d\Delta \hat{\sigma}_{ij}^{\mathrm{mass}} = - d\hat{\sigma}_{\mathrm{LO}} - d\hat{\sigma}_{\mathrm{LO}}^{\mathrm{HTL}}$$

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 $2 \rightarrow 2 \mbox{ OS LO from}$ $_{[Catani, \mbox{ Seymour, NPB 485 (1997) 291}]}$ with initial-state emitter, initial-state spectator

\Rightarrow Mass effects IR safe in the real corrections



Distributions at higher center-of-mass energies

Still preliminary results!!



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