Rates and differential distributions in heavy neutral lepton decays

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1 Introduction

The following text is extracted from a longer article in the making. Its aim is to show how one can calculate the production rate, decay rate and angular distribution of the decay products of a hypothetical "heavy neutrino" (or heavy neutral lepton, often abbreviated HNL in the following) with the couplings of an ordinary neutrino to standard model particles, up to a (very small) mixing matrix element. It is aimed at experimental groups wishing to evaluate the sensitivity of their apparatus to such heavy neutrino production and decay through simulation. Masses have to be taken into account at every stage of the production/decay process since, for example, the well known helicity suppression of ν_e production in two-body 0^- mesons decays no longer works when the neutrino is hypothetized to have a mass of a few MeV. Also, polarization of the neutrino must be taken into account because it bears on the angular distribution of its decay products and therefore on the acceptance of the experimental set-up to a given combination of mass and mode. Most results given here can probably be found in the litterature, see e.g. [1, 2, 3] but they are scattered among many experimental or theoretical papers, which is why we think this one might have some

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usefulness. Moreover, some of these papers contain errors, see e.g. [4], which we came across while trying to help a young experimentalist colleague preparing his thesis [5].
We will therefore give fairly complete derivations so as to allow anyone with a minimal litteracy in Dirac algebra to check our results. We apologize in advance to the many people who published such or such formula for not quoting them. It is a task beyond our capacity and anyway, quite useless in a work of this kind.
The present extract is devoted to "heavy neutrino" production by decay of charged 0^- mesons and is restricted to 2-body (0^- charged meson plus charged lepton) and the simplest 3-body not involving neutral currents (non self-conjugate charged lepton pair plus light neutrino).

2 Generalities - effective lagrangian

Neutrino states related to charged leptons through weak charged currents are thought to be linear combinations of mass eigenstates. The mechanism giving rise to these combinations is not known, but given the successes of "standard" physics, we assume that the interaction lagrangian is that of the Standard Model, namely:

$$\mathcal{L}_{int} = eA^\alpha J^{em}_\alpha + \frac{g}{cos\theta_w} Z^\alpha J^{neut}_\alpha + \frac{g}{\sqrt{2}}(W^{\alpha\dagger} J^{ch}_\alpha + W^\alpha J^{ch\dagger}_\alpha)$$

[1] where:

$$J^{ch}_\alpha = \sum_{\beta=e,\mu,\tau} \overline{\nu}_\beta \gamma_\alpha \mathcal{P}_L l_\beta + quark\ currents$$

$$J^{neut}_\alpha = \sum_f \overline{f}\gamma_\alpha(\mathcal{P}_L T^3_w - sin^2\theta_w Q)f$$

$$J^{em}_\alpha = \sum_f \overline{f}\gamma_\alpha Q f$$

- f is any elementary fermion field, ν_β and l_β stand for the neutrino and charged lepton fields of "flavour" β $(= e, \mu, \tau)$.

[1] Einstein's summation convention is used thourough for space-time indices

- T_w^3 and Q are the third weak isospin component and electric charge operators.
- $\mathcal{P}_L = \frac{1}{2}(1-\gamma^5)$ is the left-handed projector.
- $g = \frac{e}{sin\theta_w}$

ν_β's are assumed to be linear superpositions of fields corresponding to definite mass quanta which can be either Dirac or Majorana. The notation will be as follows:

$$\nu_\beta = \sum_h U_{\beta h} N_h$$

where N_h represents the field of a neutrino of mass μ_h. Greek indices will be used for leptonic "flavours" and latin indices for definite mass fields. It is known that there must exist three different light masses, but in the following, we will assume that there is at least an extra "heavy neutrino" (Heavy neutral lepton or HNL henceforth) . U is therefore a rectangular extension of the PMNS mixing matrix.

The processes of interest are at low energies and will always involve virtual W and Z's. Therefore, they will be at least second order in $\mathcal{L}_{int}$. Neglecting q^2 w.r.t. m^2 in the bosons propagators written in momentum space, one finds the effective lagrangian:

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}}(J^{neut,\alpha}J_\alpha^{neut} + J^{ch,\alpha}J_\alpha^{ch\dagger}) \tag{1}$$

with $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$ in tree approximation.

In the following, we shall note j for a leptonic current and J for a hadronic current.

3 Production through 2-body 0^- charged mesons decay

The relevant part of the effective lagrangian is here:

$$\frac{4G_F}{\sqrt{2}}(j^{ch,\alpha}J_\alpha^{ch\dagger} + h.c.)$$

From now on, we assume a M^+ (momentum P, mass M) decaying to HNL N_h (momentum p_N, mass μ), and antilepton l^+ of "flavour" β (momentum p_l mass m). M^+ being spinless, the only vector available to parametrize the hadronic current matrix element is its 4-momentum P^α.[2] Introducing M^+ 'decay constant' f_M and using Lorentz invariance one makes the usual *ansatz* for the current matrix element:

$$< O|A^{ch\ \alpha\ \dagger}(x)|M^+ >= i f_M V_{..} e^{-iP\cdot x} P^\alpha$$

where $V_{..}$ is the relevant CKM matrix element for $M^+ \to W^{+*}$ [3].

The leptonic current matrix element for producing a state N_h of definite mass μ and 4-momentum p_N together with a charged antilepton β^+ of flavour β mass m and 4-momentum p_l is:

$$< \beta^+ \ N_h| \sum_{k,\delta} U^*_{\delta k} \overline{N_k} \gamma_\alpha \mathcal{P}_L l_\delta(x)|O >= U^*_{\beta h} \overline{u}(N_h) \gamma_\alpha \mathcal{P}_L v(\beta) e^{i(p_l+p_N)\cdot x}$$

so that the transition matrix element for $M^+ \to N_h \beta^+$ will be:

$$-i\sqrt{2} G_F f_M U^*_{\beta h} V_{..} \overline{u}(N_h) \not{P} (1-\gamma^5) v(\beta) \ ^4$$

This result is obviously independant of the Dirac or Majorana nature of the N_h field.

As said in the introduction, we will give here a complete derivation. We only assume that the reader knows how to calculate traces of products of Dirac algebra matrices. Our way of calculating the HNL polarization vector and using it in the second decay is inspired by [6]

1. using $P = p_N + p_l$ and the Dirac equations:

   $\bar{u}\not{p}_N = \mu \bar{u}$ and $\not{p}_l v = -mv$

   simplify the matrix element to $-i\kappa\bar{u}(\alpha - \gamma^5\beta)v$
   where $\kappa = \sqrt{2} G_F f_M U^*_{\beta h} V_{..}$, $\alpha = \mu - m$, $\beta = \mu + m$

[2] Further notice that only the axial part of the hadronic current can have a non zero matrix element between a pseudoscalar state and the hadronic vacuum.
[3] W^{+*} is an off-shell W^+
[4] $\not{P}$ stands for $P^\alpha \gamma_\alpha$ (Feynman 's notation.)

2. multiply the m.e. by its complex conjugate:

   $\kappa^2 \bar{u}(\alpha - \gamma^5\beta)v\bar{v}(\alpha + \gamma^5\beta)u$

   $= \kappa^2\,\mathrm{Tr}(u\bar{u}(\alpha - \gamma^5\beta)v\bar{v}(\alpha + \gamma^5\beta)$

3. sum over antilepton polarizations, which amounts to the replacement:

   $v\bar{v} \rightarrow (\not{p}_l - m).$

4. In order to calculate the HNL polarization, keep its full density matrix for both momentum and spin:

   $u\bar{u} \rightarrow (\not{p}_N + \mu)\frac{1}{2}(1 + \gamma^5\not{s})$

   where s is the HNL polarization 4-vector which reduces, in the rest frame, to $(0, \mathbf{P})$ with $\mathbf{P}$ the usual polarization 3-vector for spin 1/2, i.e. twice the spin expectation value.

5. the squared m.e. thus becomes:

   $\kappa^2\,\mathrm{Tr}\,(\not{p}_N + \mu)\frac{1}{2}(1 + \gamma^5\not{s})(\alpha - \gamma^5\beta)(\not{p}_l - m)(\alpha + \gamma^5\beta)$

6. Calculate the trace. Using again 4-momentum conservation, this yields:

$$1/4\,\mathrm{Tr} = M^2(m^2 + \mu^2) - (m^2 - \mu^2)^2 + 2\mu(\mu^2 - m^2)s \cdot p_l \tag{2}$$

- To calculate the rate, sum over HNL spin states by replacing $s \rightarrow 0$ and multiplying by 2.

  Adding normalization and phase-space factors, one gets the width:

$$\Gamma(M^+ \rightarrow \beta^+ N_h) = \frac{G_F^2 f_M^2 |V_{..}|^2 |U_{\beta h}|^2}{8\pi M}\left(m^2 + \mu^2 - \frac{(m^2 - \mu^2)^2}{M^2}\right)\lambda^{1/2}(M^2, m^2, \mu^2)$$

where M, m, μ are the masses of M^+, β and N_h respectively and λ is the usual kinematical function

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$$

By letting $\mu \to 0$ (and forgetting about U) one retrieves the ordinary well known formula for decay into an antilepton and a standard model massless neutrino.

$$\Gamma = \frac{G_F^2 f_M^2 |V_{..}|^2}{8\pi} M m^2 (1 - \frac{m^2}{M^2})^2 \tag{3}$$

which, being proportionnal to m^2, explains the tiny ratio $\Gamma(e^+\nu)/\Gamma(\mu^+\nu)$, due to helicity conservation by V and A vertices in the ultra-relativistic limit. For the domain envisioned here ($\mu \geq$ a few MeV) the suppression no longuer works and both modes acquire the same order of magnitude modulo the coefficients $|U_{\beta h}|$

- To find the HNL polarization:

The squared m.e.(cf. 2) is proportionnal to the probability of finding 4-polarization s and must therefore be equal to $\mathrm{Tr}(\rho\rho_f)$ (with ρ_f the true HNL polarization matrix) up to a factor.
In the rest frame, ρ reduces to $\mathbf{1} + \sigma \cdot \mathbf{P}$ with σ the Pauli matrices and $\mathbf{P}$ the polarization 3-vector, so that the expression obtained is proportionnal to $\mathrm{Tr}\,(\mathbf{1} + \sigma \cdot \mathbf{P})(\mathbf{1} + \sigma \cdot \mathbf{P^f})$ or to $\mathbf{1} + \mathbf{P} \cdot \mathbf{P}^f$

By expliciting the proportionality of this last expression with (2) written in the HNL rest frame, we find the following for the HNL polarization vector to be used when simulating its decay:

$$\mathbf{P} = \frac{(m^2 - \mu^2)\lambda^{1/2}(M^2, m^2, \mu^2)}{M^2(m^2 + \mu^2) - (m^2 - \mu^2)^2}\hat{\mathbf{n}} = \mathbb{P}\hat{\mathbf{n}} \tag{4}$$

where $\hat{\mathbf{n}}$ is a unit vector in the direction of the parent meson or of the decay lepton in the N_h rest frame and the second equality defines $\mathbb{P}$. Although the formula obtained by the authors of [4] is not given in their paper, it is readily seen graphically (compare with fig. 1)that it must coincide with our above result for the case where the initial particle is a charged kaon decaying into muon and HNL. In particular, it is seen from the graph (fig. 1) or formula (4), that if the HNL mass μ coincides with the muon mass, its polarization vector is zero. The graph or formula (4) also show that when $\mu \to 0$, the coefficient of $\hat{\mathbf{n}}$ $\to 1$ that is, the massless neutrino will be pure -1 helicity.

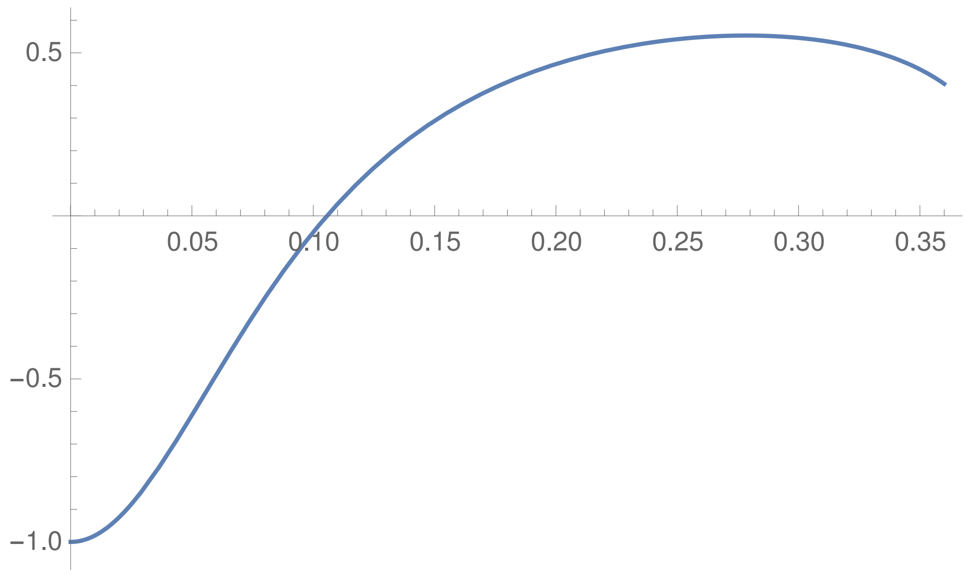

Figure 1: Polarization of HNL produced in $K^+ \to \mathrm{HNL} + \mu^+$ as a function of HNL mass. It is seen that when the latter coincides with the muon mass, the polarization vanishes

4 Two-body HNL decay into 0^- meson and lepton

These are crossed channels of those envisionned above for production. Amplitudes are trivial to write; in the 'charged' case, one finds e.g., for a decay into π^+, l^-:
(Here k, q and p are the 4-momenta of N_h(mass μ_h), l^- (mass m_l flavour γ) and π^+ (mass m_π) so that $k = p + q$)

$$A_{N_h \to \pi^+ l^-} = -i\frac{G_F}{\sqrt{2}} f_\pi V_{u,d} U_{h,\gamma} \bar{u}(l) \not{p} (1 - \gamma^5) u(N_h)$$

Using Dirac equation, one gets:

$$A_{N_h \to \pi^+ l^-} = -i\frac{G_F}{\sqrt{2}} f_\pi V_{u,d} U_{h,\gamma} \bar{u}(l)(\alpha + \beta\gamma^5) u(N_h)$$

with $\alpha = \mu_h - m_l$ and $\beta = \mu_h + m_l$

Squaring, summing over l polarizations and introducing the HNL polarization matrix with polarization 4-vector s one gets:

$$\overline{|A|^2} = \frac{G_F^2}{2} f_\pi^2 |U_{h,\gamma}|^2 |V_{u,d}|^2 \,\mathrm{Tr}\,(\not{q} + m_l)(\alpha + \beta\gamma^5)(\not{k} + \mu_h)\frac{1}{2}(1 + \gamma^5 \not{s})(\alpha - \beta\gamma^5)$$

Calculating the trace, one gets

$$\begin{aligned} 1/4\,\mathrm{Tr} &= (\alpha^2 + \beta^2) q \cdot k + 2\mu_h \alpha\beta q \cdot s + m_l \mu_h (\alpha^2 - \beta^2) \\ &= 2(\mu_h^2 + m_l^2) q \cdot k + 2\mu_h (\mu_h^2 - m_l^2) q \cdot s - 4 m_l^2 \mu_h^2 \\ &= (\mu_h^2 - m_l^2)^2 - m_\pi^2 (\mu_h^2 + m_l^2) + 2\mu_h (\mu_h^2 - m_l^2) q \cdot s \text{ [5]} \\ &= (\mu_h^2 - m_l^2)^2 - m_\pi^2 (\mu_h^2 + m_l^2) - 2\mu_h (\mu_h^2 - m_l^2) \mathbf{q} \cdot \mathbf{P} \end{aligned}$$

in the N_h rest frame, therefore:

$$\overline{|A|^2} = G_F^2 f_\pi^2 |U_{h,\gamma}|^2 |V_{u,d}|^2 [(\mu_h^2 - m_l^2)^2 - m_\pi^2 (\mu_h^2 + m_l^2) - 2\mu_h (\mu_h^2 - m_l^2) \mathbf{q} \cdot \mathbf{P}] \quad (5)$$

With phase space (integrated over the angles) equal to $\frac{|\mathbf{q}|}{4\pi\mu_h}$ or $\frac{\lambda^{1/2}(\mu_h^2, m_l^2, m_\pi^2)}{8\pi\mu_h^2}$ we find for the rate:

$$\Gamma(N_h \to l^- \pi^+) = \frac{G_F^2 f_\pi^2}{16\pi\mu_h^3} \{(\mu_h^2 - m_l^2)^2 - m_\pi^2 (\mu_h^2 + m_l^2)\} \lambda^{1/2}(\mu_h^2, m_l^2, m_\pi^2) |U_{h,\gamma}|^2 |V_{u,d}|^2$$

If $\gamma = e$, m_l can be neglected and this becomes:

$$\frac{G_F^2 f_\pi^2}{16\pi} \mu_h^3 \left(1 - \frac{m_\pi^2}{\mu_h^2}\right)^2 |U_{h,e}|^2 |V_{u,d}|^2$$

The angular distribution is non isotropic due to polarization (cf. (4)) as shown by (5)

Normalizing formula (5) in such a way that the constant term be equal to 1/2 (so that the integral over $\cos(\hat{\mathbf{n}}, \hat{\mathbf{q}})$ equals 1) we get:

$$\frac{dN}{d\cos\theta} = 1/2 - 1/2 \frac{\mu_h^2 - m_l^2}{(\mu_h^2 - m_l^2)^2 - m_\pi^2 (\mu_h^2 + m_l^2)} \lambda^{1/2}(\mu_h^2, m_l^2, m_\pi^2) \mathbb{P} \cos\theta$$

[5] We have used $2q \cdot k = \mu_h^2 + m_l^2 - m_\pi^2$

$\theta = (\hat{\mathbf{n}}, \hat{\mathbf{q}})$ is the angle between the recoil lepton direction ($\hat{\mathbf{n}}$) in the parent's decay $M^+ \to \mathrm{HNL} + \beta^+$ and the secondary lepton direction ($\hat{\mathbf{q}}$) from HNL decay seen in the HNL rest frame. $\mathbb{P}$ has been defined in (4).
It is clear that, contrary to formula (16) of ref.[4] the HNL decay is isotropic when its mass equals that of the lepton recoiling against it in the parent's decay. This formula is incoherent on different other grounds, making, for example, no distinction between the c.o.m. momenta in the HNL-generating meson two-body decay and the HNL two-body decay itself.

4.1 A pedagogical remark

It is interesting to note that the heavy neutrino which is in general only partially polarized is NOT a quantum mechanical linear superposition of helicity 1 and helicity -1 states contrary to what is stated in many places (see e.g. [7]). Since its polarization vector modulus is not one, there is no direction in which a spin measurement will yield 1/2 with certainty and this system, which is in a mixed state, cannot be represented by a wave function. Although the spin 0 initial meson can be thought of as being in a pure state, the HNL, being but a subsystem of the -evolved- initial state can only be represented by a density matrix (see e.g.[8])

5 Decay into a light neutrino and a non charge-conjugate lepton pair ($\beta^- \ \beta'^+ \ \nu$)

5.1 Decay matrix element

For a Dirac HNL and in tree approximation, the decay takes place through:

$$N_h \to \beta^- W^{+*} \to \beta^- \beta'^+ \nu_{\beta'}$$

It is only the β component of N_h that contributes to the first vertex so that a factor of $U^*_{\beta,h}$ enters the matrix element at this level. For the final state with a charged lepton of flavour β', the final neutrino has also flavour β'. Obviously, for kinematical calculations we will consider it to be massless and no mixing matrix elements need to be introduced here.

On the other hand, if neutrinos are Majorana particules, the following is also possible:

$$N_h \to \beta'^+ W^{-*} \to \beta'^+ \beta^- \overline{\nu}_\beta$$

which, although the final light neutrino is of a different flavour, cannot be practically distinguished from the above described process in a heavy neutrino search experiment. [6] As above, this final neutrino mass is neglected. Moreover, since the final neutrino flavours are different in the two processes, no interference between the amplitudes has to be considered. Clearly, neutral currents play no role here.

The relevant part of the effective lagrangian is now :

$$\mathcal{L}' = \frac{4G_F}{\sqrt{2}} \sum_{kk'\alpha\alpha'} U_{\alpha k} U^*_{\alpha' k'} \bar{l}_\alpha \gamma^\mu \mathcal{P}_L N_k \overline{N}_{k'} \gamma_\mu \mathcal{P}_L l_{\alpha'}$$

If the N_h are Dirac fields, the transition amplitude is simply:

$$4\frac{G_F}{\sqrt{2}} U_{\beta h} \overline{u}_\beta \gamma^\mu \mathcal{P}_L u_h \overline{u}^\nu_{\beta'} \gamma_\mu \mathcal{P}_L v_{\beta'}$$

here u's and v's are Dirac spinors and $\mathcal{P}_L$ is the projector on their left-handed part.
In conformity with the remark made about the final state neutral lepton, the $U^*_{\beta' k'} \overline{N}_{k'}$ sum has been replaced by the sole $\overline{u}^\nu_{\beta'}$ standing for a standard model neutrino of flavour β' produced together with the β'^+ charged anti-lepton.

For the Majorana case, there will be the extra piece:

$$4\frac{G_F}{\sqrt{2}} U^*_{\beta' h} \overline{u}_\beta \gamma^\mu \mathcal{P}_L v^\nu_\beta \overline{v}_h \gamma_\mu \mathcal{P}_L v_{\beta'}$$

with an analogous remark for the absence of final neutrino mixing matrix element and for the spinor v^ν_β which stands now for the light neutrino of flavour β produced together with the β-flavoured charged lepton.

[6] Neutrino fields are assumed to be Majorana's here, therefore the notation $\overline{\nu}_\beta$ can be taken as indicating the right-handed part of the ν_β field quantum.

We now Fierz-transform these amplitudes so as to render both first factors equal, getting:

$$-4\frac{G_F}{\sqrt{2}}U_{\beta h}\overline{u}_\beta\gamma^\mu\mathcal{P}_L v_{\beta'}\overline{u}^\nu_{\beta'}\gamma_\mu\mathcal{P}_L u_h$$

and

$$-4\frac{G_F}{\sqrt{2}}U^*_{\beta' h}\overline{u}_\beta\gamma^\mu\mathcal{P}_L v_{\beta'}\overline{v}_h\gamma_\mu\mathcal{P}_L v^\nu_\beta$$

and we use the relation

$$\overline{v}_h\gamma^\mu(1-\gamma^5)v_l = \overline{u}_l\gamma^\mu(1+\gamma^5)u_h$$

in order to have (almost) the same spinors sandwiching the second factors: indeed, l stands for β or β' which correspond to orthogonal states, but mathematically the spinors are the same.

5.2 Differential distribution

This being done, the two amplitudes can be added, yielding:

$$-2\frac{G_F}{\sqrt{2}}\overline{u}_\beta\gamma^\mu\mathcal{P}_L v_{\beta'}\overline{u}_l\gamma_\mu((U_{\beta h}+U^*_{\beta' h})-(U_{\beta h}-U^*_{\beta' h})\gamma^5)u_h$$

where it is understood here that since $\beta \neq \beta'$ interference terms (containing products like $U_{\beta h}U^*_{\beta' h}$) are to be cancelled in the end.

To simplify let $U_{\beta h}+U^*_{\beta' h} = \alpha$, $U_{\beta h}-U^*_{\beta' h} = \beta$. Squaring the above expression, we get:

$$2G_F^2\overline{u}_\beta\gamma^\mu\mathcal{P}_L v_{\beta'}\overline{v}_{\beta'}\gamma^\nu\mathcal{P}_L u_\beta\overline{u}_l\gamma_\mu(\alpha-\beta\gamma^5)u_h\overline{u_h}\gamma_\nu(\alpha^*-\beta^*\gamma^5)u_l$$

which we rewrite, summing over final polarizations and introducing the HNL density matrix:

$$2G_F^2\,\mathrm{Tr}\,((\not{p}_- + m_\beta)\gamma^\mu\mathcal{P}_L(\not{p}_+ - m_{\beta'})\gamma^\nu\mathcal{P}_L)\,\mathrm{Tr}\,(\not{q}\gamma_\mu(\alpha-\beta\gamma^5)(\not{k}+\mu_h)\frac{1}{2}(1+\gamma^5\not{s})\gamma_\nu(\alpha^*-\beta^*\gamma^5))$$

where k, p_-, p_+, q, s are the 4-momenta of $N_h, \beta^-, \beta'^+, \nu_l$ and the N_h 4-polarization. μ_h, m_β and $m_{\beta'}$ are the masses of N_h and of the two charged leptons. Taking the traces and contracting the Lorentz indices then yields:

$$64G_F^2(|U_{\beta h}|^2 q\cdot p_-(k-\mu_h s)\cdot p_+ + |U_{\beta' h}|^2 q\cdot p_+(k+\mu_h s)\cdot p_-) \tag{6}$$

It is seen that no spurious interference terms need to be cancelled explicitly.

The last expression can easily be transformed to:

$$64G_F^2\mu_h^2(|U_{\beta h}|^2(E_+^* - E_+)(E_+ + \mathbf{P}\cdot\mathbf{p}_+) + |U_{\beta' h}|^2(E_-^* - E_-)(E_- - \mathbf{P}\cdot\mathbf{p}_-)) \quad (7)$$

here: $E_{\mp}^* = (\mu_h^2 \pm m_\beta^2 \mp m_{\beta'}^2)/(2\mu_h)$ and $E_\mp$, $\mathbf{p}_\mp$ are β^- and β'^+ energies and 3-momenta in the decaying HNL rest-frame and $\mathbf{P}$ is its 3-polarization vector as calculated in Part I.

The three-body final state phase space depends on five variables only which can be taken, in the HNL center of mass frame, as E_+, E_- and three angles defining the final state orientation. By energy-momentum conservation, the three final momenta are coplanar in this frame and the angle θ_{+-} between $\mathbf{p}_+$ and $\mathbf{p}_-$ is fixed once E_+ and E_- are given [7]. One can then choose the polar angles of $\mathbf{p}_+$ with respect to the HNL parent direction $\hat{\mathbf{n}}$, which is itself parallel to $\mathbf{P}$ (see (4)), call them θ_+ and ϕ_+ and the angle of the decay plane around $\mathbf{p}_+$, say Φ, to completely define the final state. In order to use formula (7), one only needs the cosine of the angle of $\mathbf{p}_-$ and $\hat{\mathbf{n}}$ which is found to be

$$\cos\theta_- = \cos\theta_+ \cos\theta_{+-} + \sin\theta_+ \sin\theta_{+-} \cos\Phi \quad (8)$$

by a standard spherical trigonometry formula (see e.g. [9]) in the spherical triangle defined by $(\hat{\mathbf{n}}, \mathbf{p}_+, \mathbf{p}_-)$

Note that (7) is valid for a Majorana neutrino. For a Dirac neutrino going to β^-, β'^+, the second term must be dropped and conversely, the first term must be dropped for a Dirac anti-neutrino decaying into the same charged channel.

This result is again very different from those of [4], which nowhere gives the full differential decay distribution necessary for a proper simulation. Observe however, that in order to use formula (7) to estimate the acceptance of the apparatus to the channel studied, some estimate of the ratio $|U_{\beta h}|^2/|U_{\beta' h}|^2$ will have to be used.

[7] One finds: $2p_+p_-\cos\theta_{+-} = \mu_h^2 + m_+^2 + m_-^2 - 2\mu_h(E_+ + E_-) + 2E_+E_-$

5.3 Decay width

Practically, since $\beta \neq \beta'$, one has e.g. $m_\beta = m_e \ll m_\mu = m_{\beta'}$ so that m_β will be neglected.

With this approximation, the width can be analytically integrated with the results:

$$\Gamma = \frac{G_F^2 \mu_h^5}{192\pi^3} \left\{|U_{\beta h}|^2 + |U_{\beta' h}|^2\right\} f(r) \tag{9}$$

Here: $r = (m_{\beta'}/\mu_h)^2$ and $f(r) = (1 - 8r + r^2)(1 - r^2) - 12r^2 Log(r)$

(9) is valid for Majorana's neutrinos. The remarks already made above concerning the Dirac case apply.

For neutrinos produced by pions or kaons decays, the only kinematically allowed case is $\mu^\mp e^\pm \nu_l$. Therefore, the channel $N_h \to \mu^- e^+ \nu_l$ yields a measure of $|U_{\mu h}|^2$ for Dirac neutrinos and $N_h \to \mu^+ e^- \nu_l$ measures $|U_{eh}|^2$. For Dirac anti-neutrinos, the channels are permuted. Lastly, for Majorana neutrinos, the sum $|U_{\mu h}|^2 + |U_{eh}|^2$ is measured by either channel.

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