Loop Quantum Cosmology

How to confront Loop Quantum Gravity with cosmological observations

Killian Martineau

Overseen by Aurélien Barrau.

In collaboration with B.Bolliet, J. Grain, F. Moulin and S.Schander.

28th of November, 2018

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Summary

1) The cosmological sector of Loop Quantum Gravity (LQG):
Loop Quantum Cosmology (LQC)

2) Focus on the background dynamics of LQC:
The duration of inflation

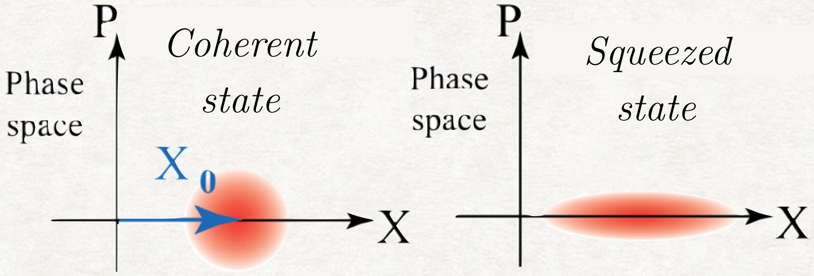
3) Cosmological perturbations in LQC

How to test Loop Quantum Gravity?

- **Existence of a minimal length-scale**

  → Modified dispersion relations $v_\gamma = c\left(1 \pm \frac{E}{E_{QG}}\right)$

  → Modification of quantum properties of light (squeezing,...)



"Integrated effects"

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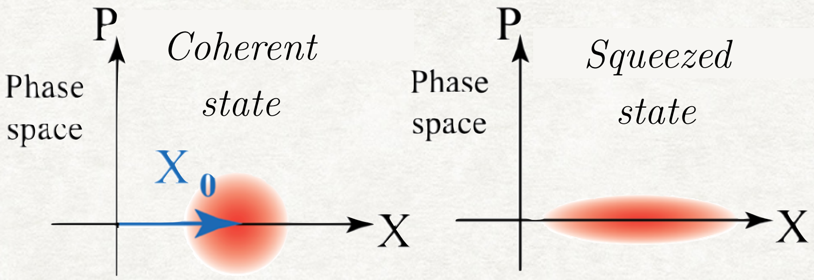

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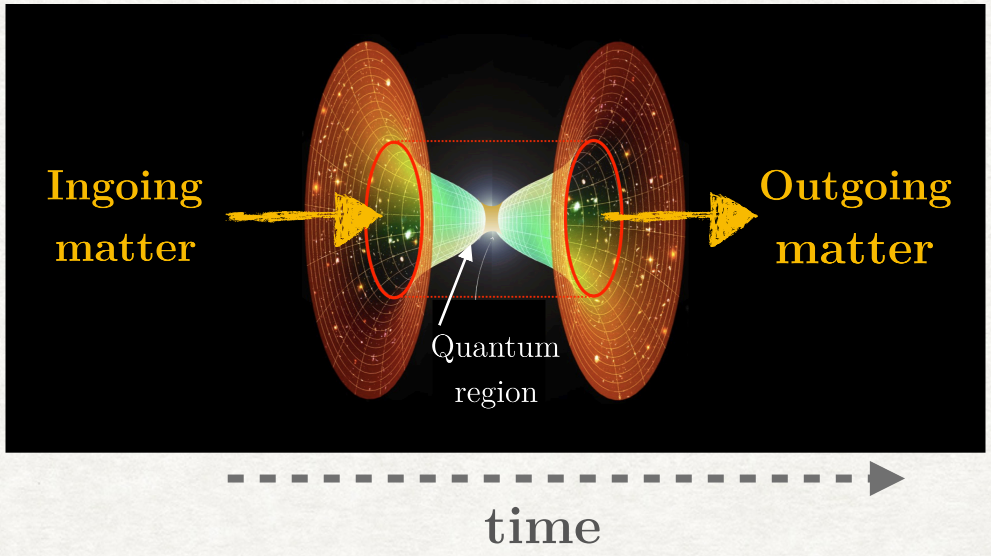
- **Black holes**



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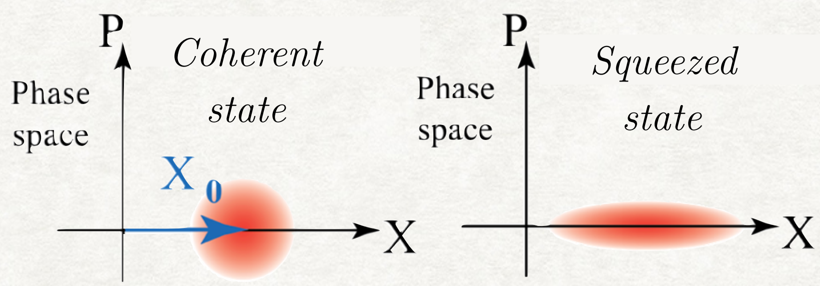

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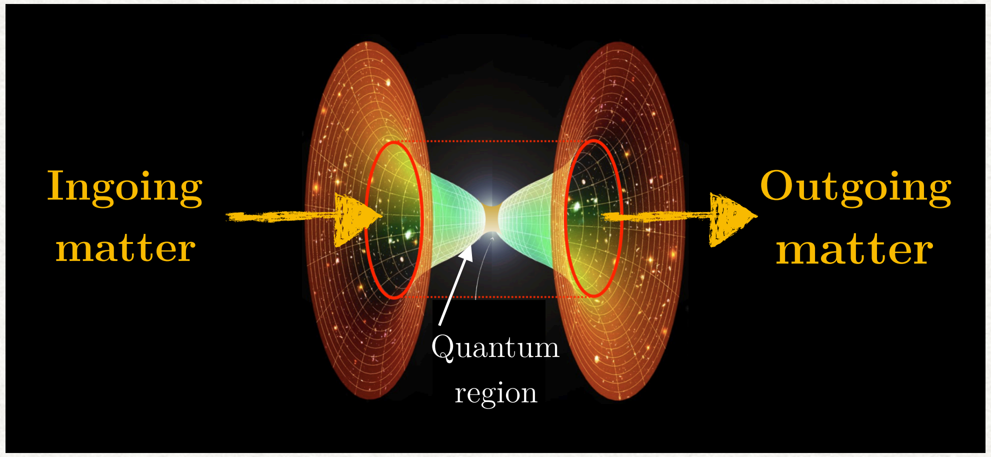

time

- **Primordial universe** → Loop Quantum Cosmology

Loop Quantum Cosmology (LQC)

Standard cosmology

Friedmann equation: $H^2 = \frac{8\pi G}{3}\rho$

Dynamics *Matter content*

$$H = -\sqrt{\frac{8\pi G}{3}\rho}$$

Contracting universe

$$H = +\sqrt{\frac{8\pi G}{3}\rho}$$

Expanding universe

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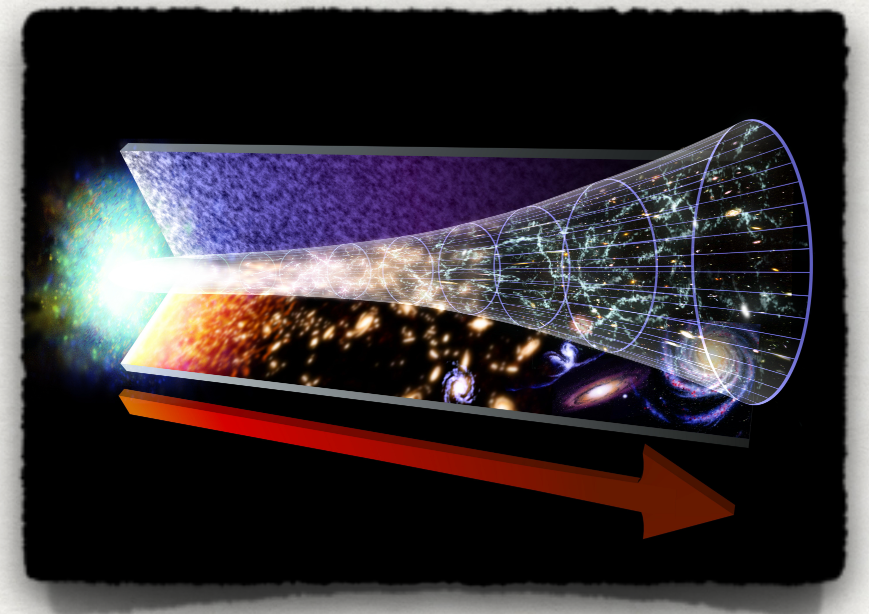

Big Bang

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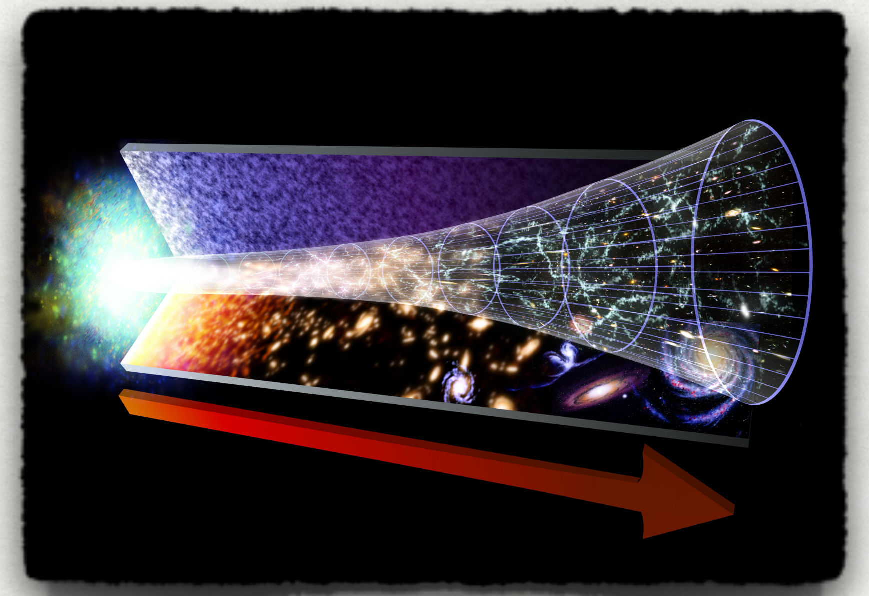

Big Bang

Loop Quantum Cosmology

Homogeneous and isotropic universe

↓ *LQG quantisation procedure*

Modified Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$

ρ_c : maximal energy density.

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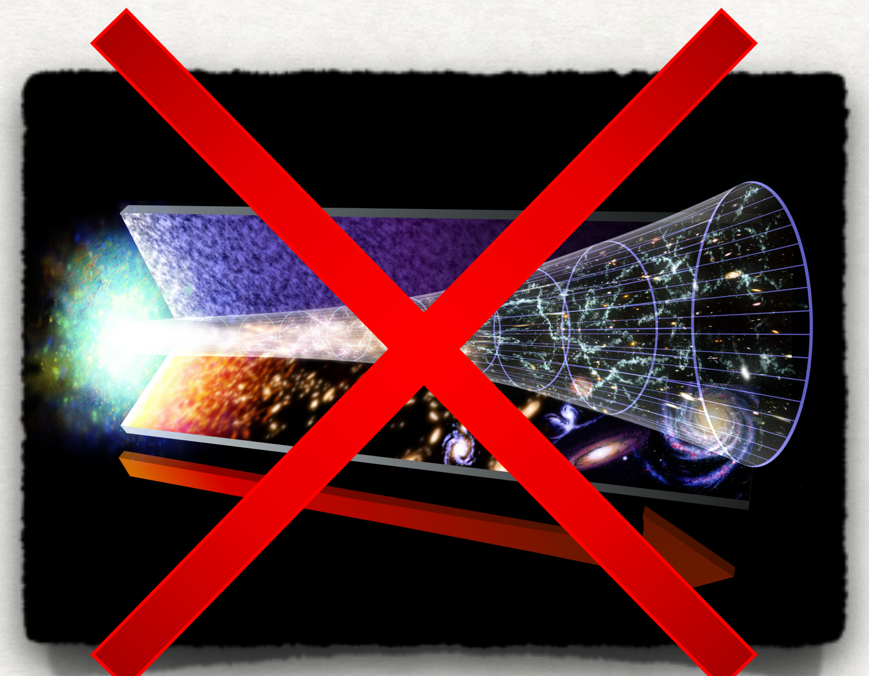

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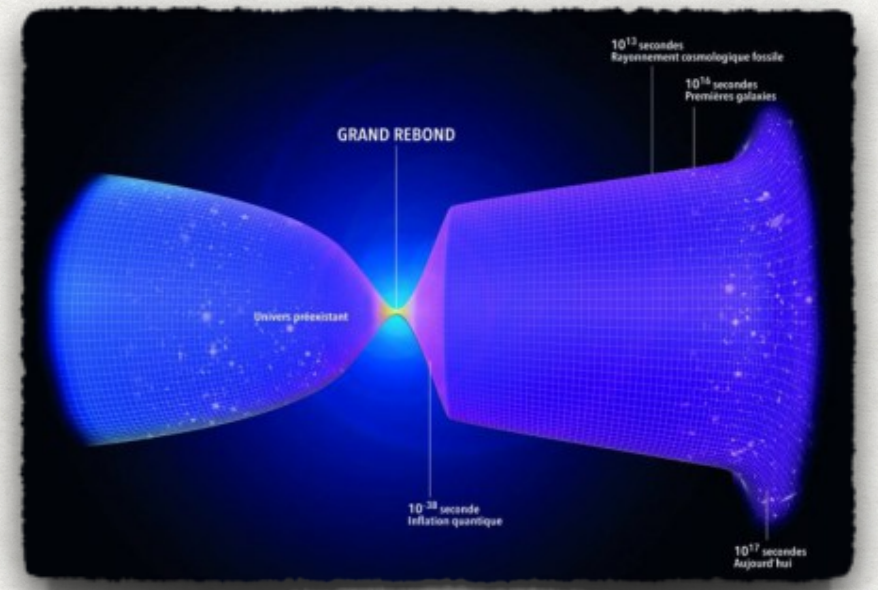
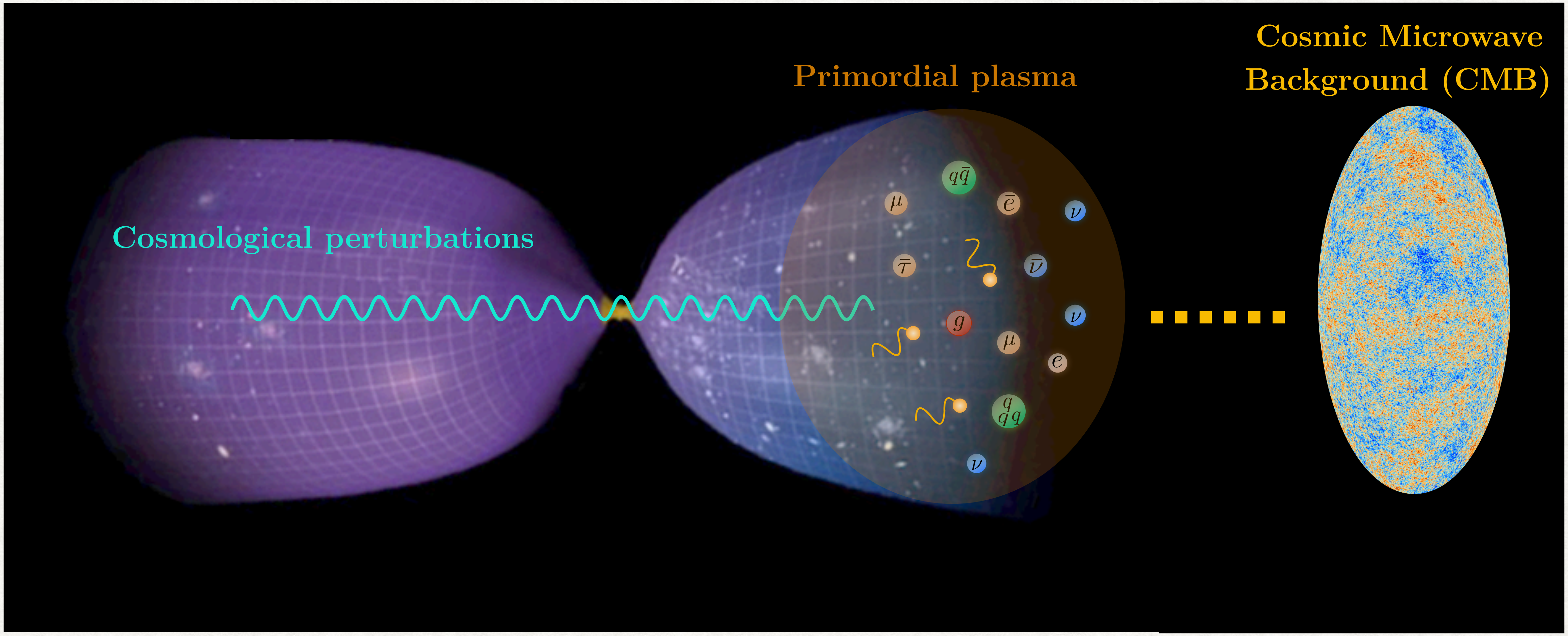


Big Bounce

Where to look for imprints of this bounce?



Solution: the Cosmic Microwave Background (CMB)

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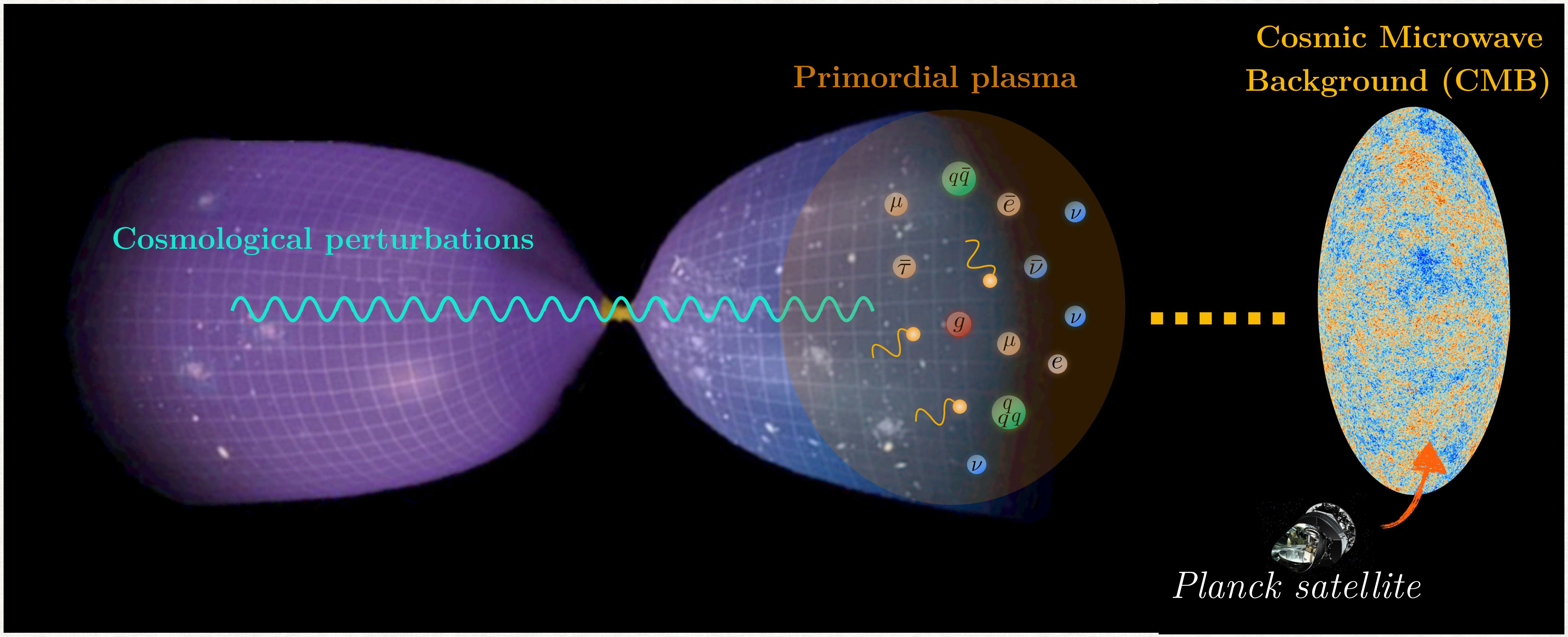

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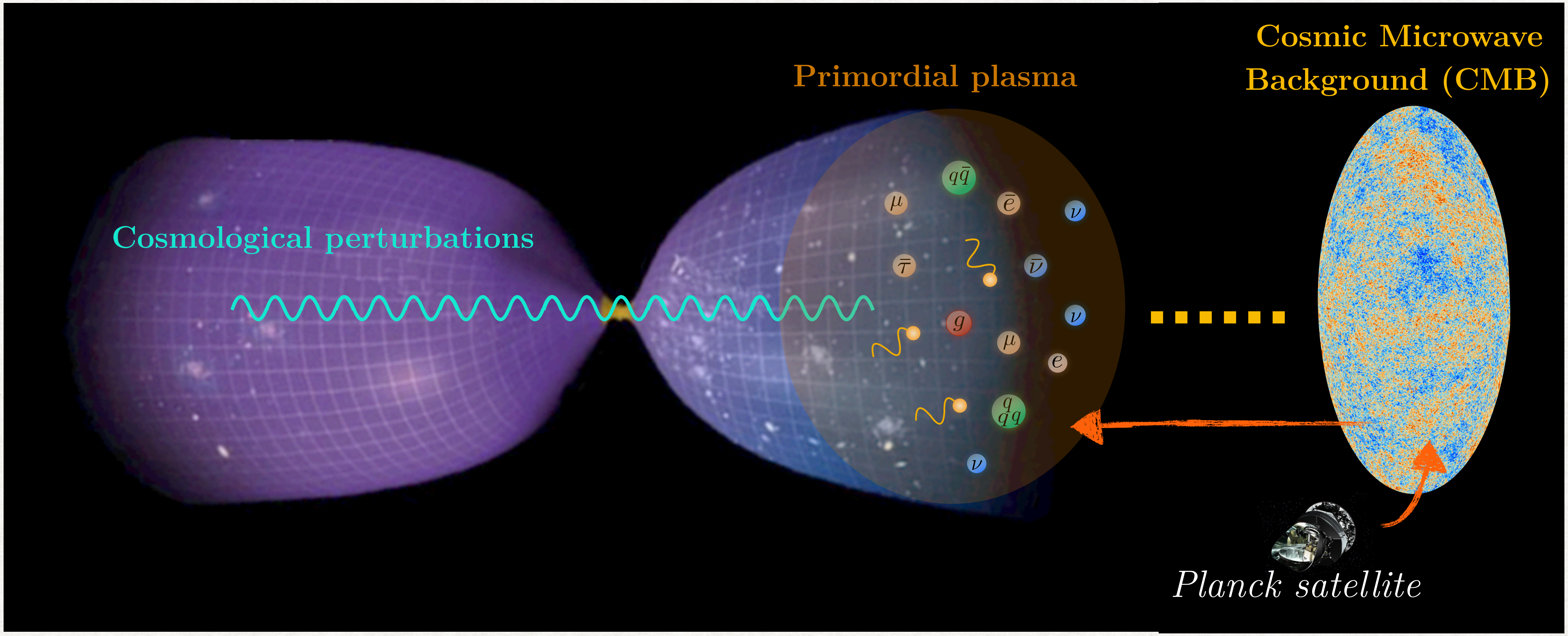

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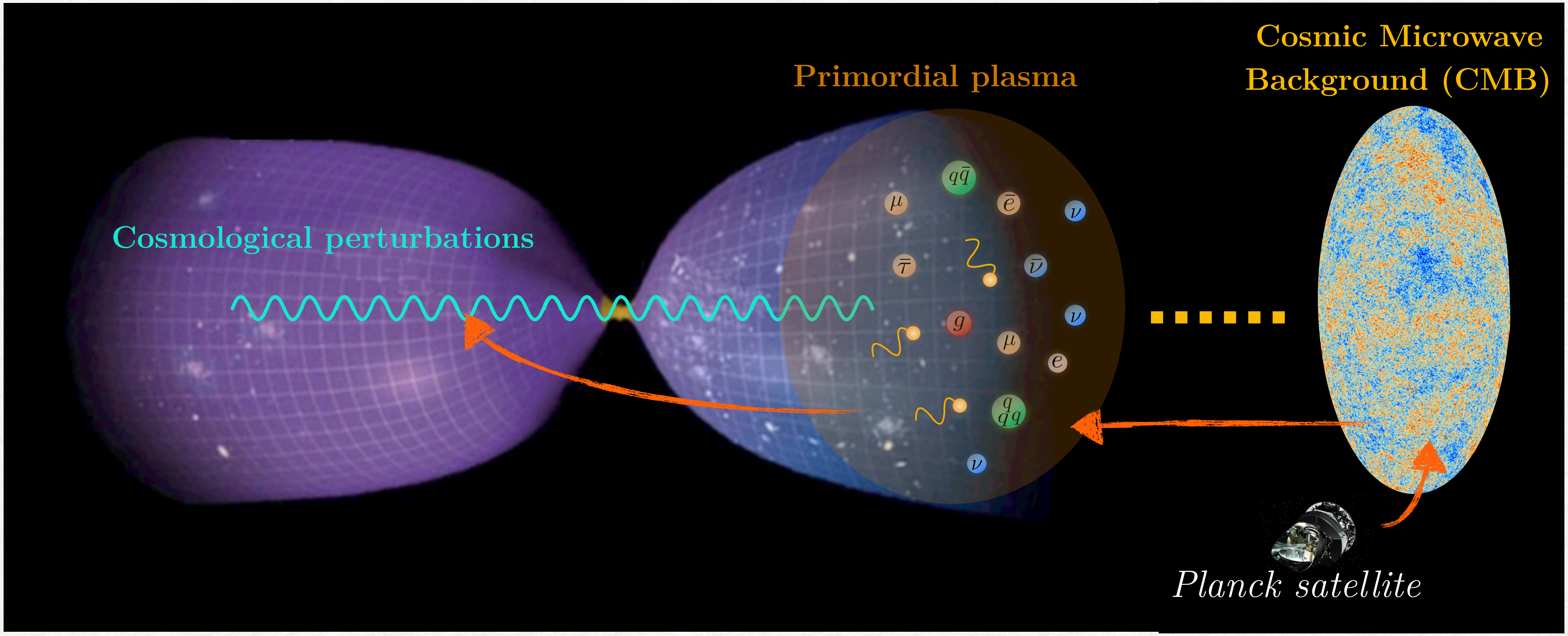

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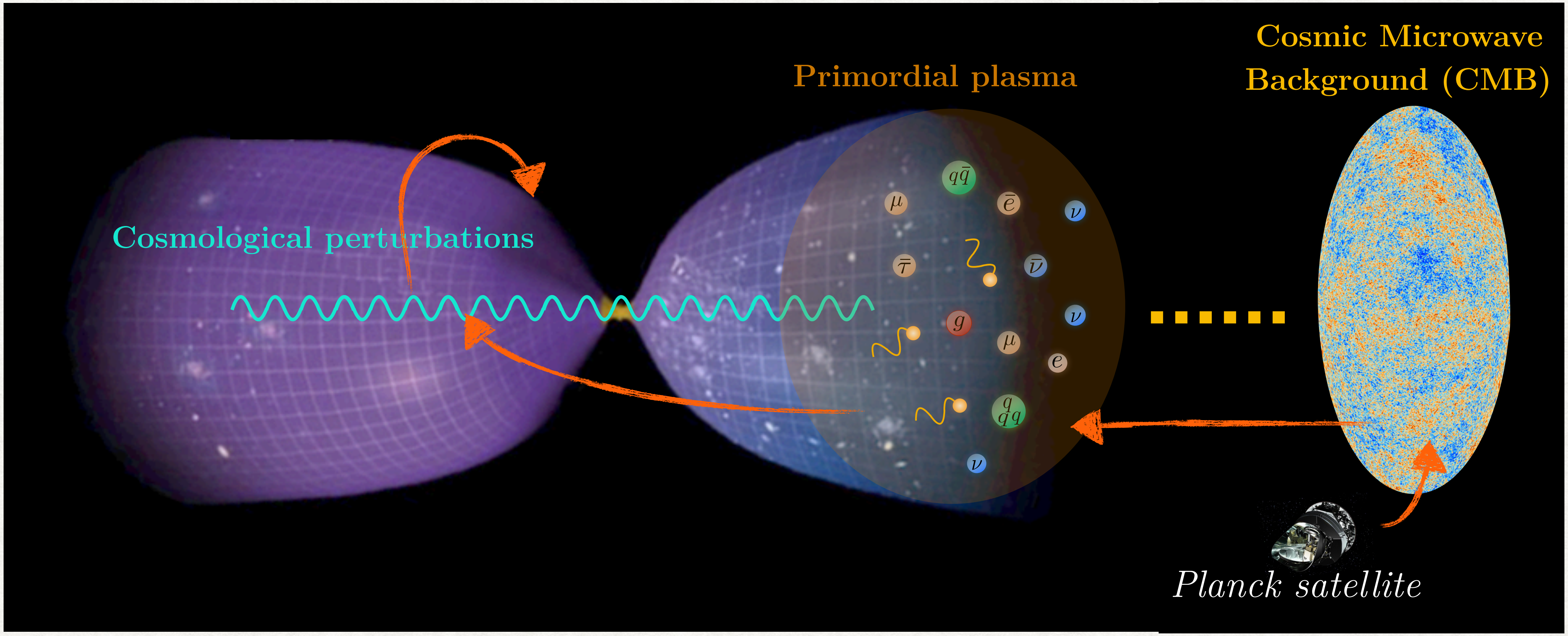

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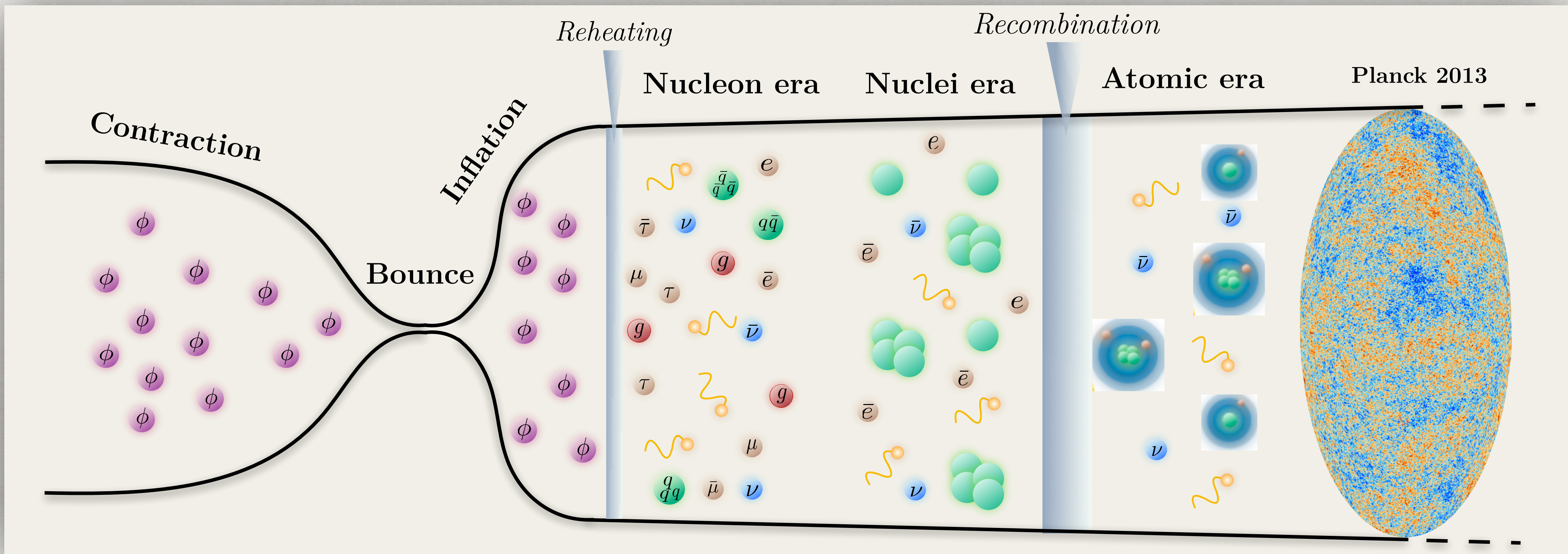
Evolution of the Universe in LQC



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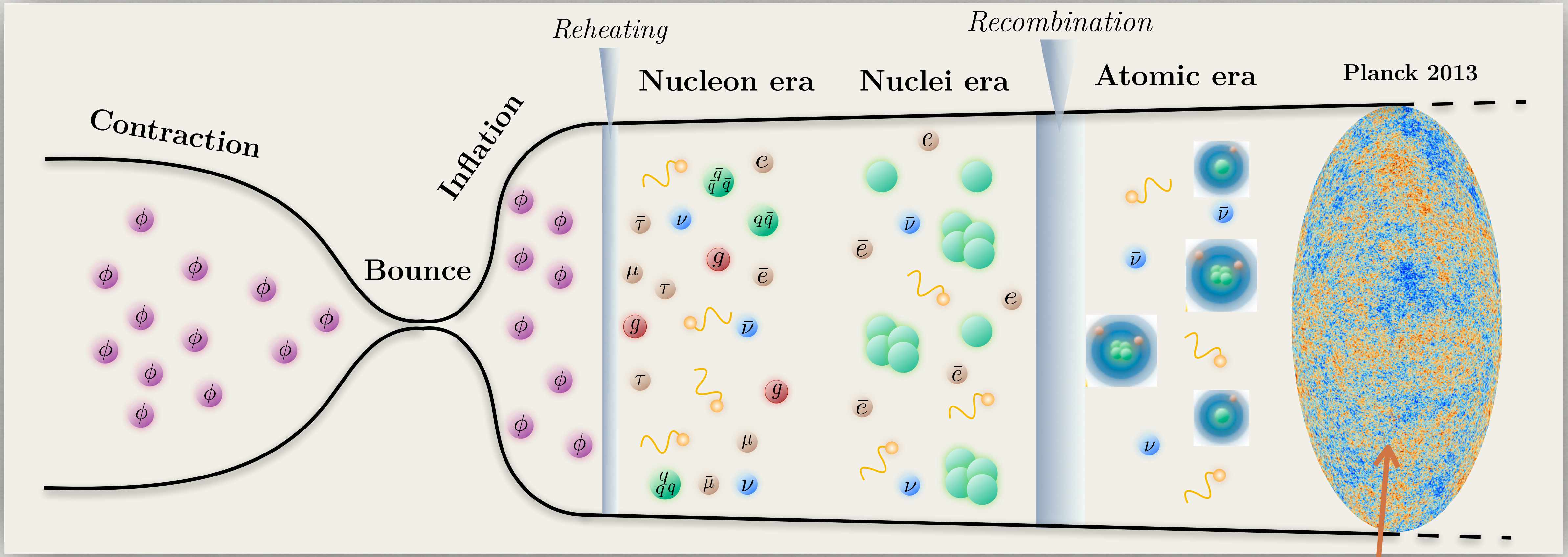

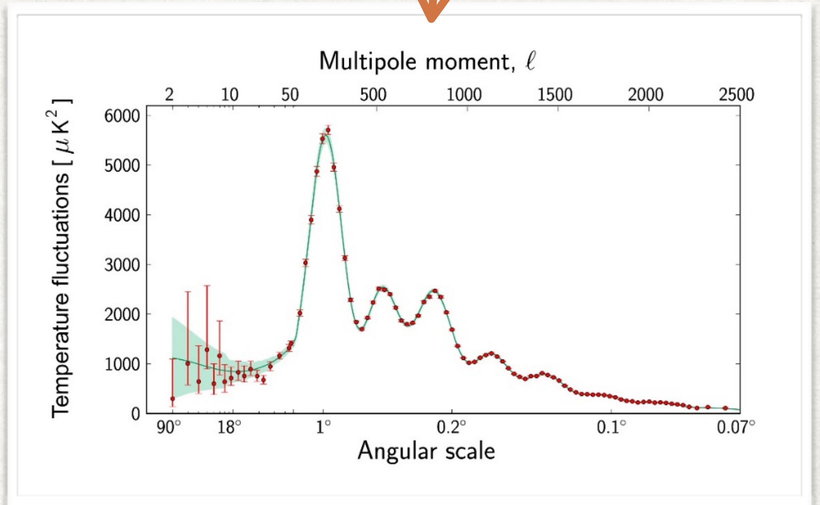

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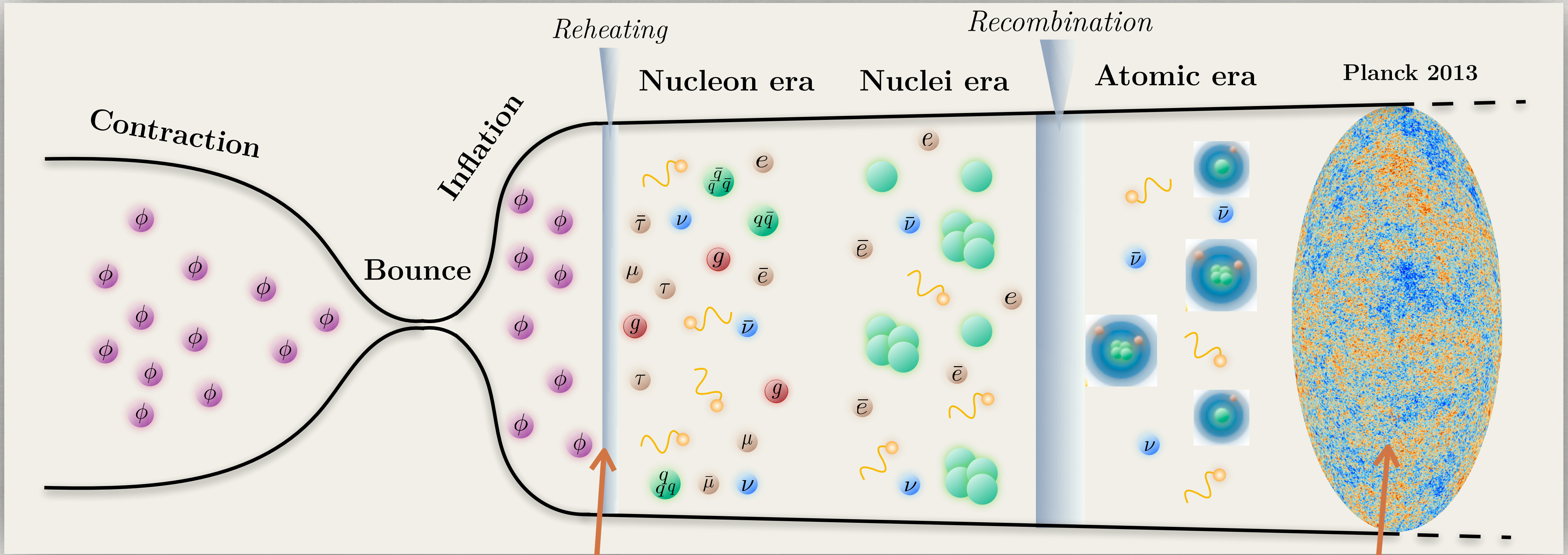

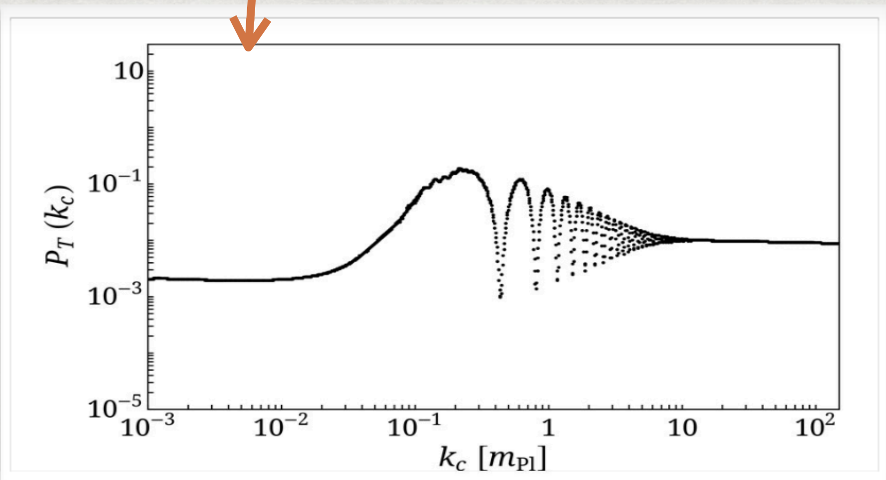

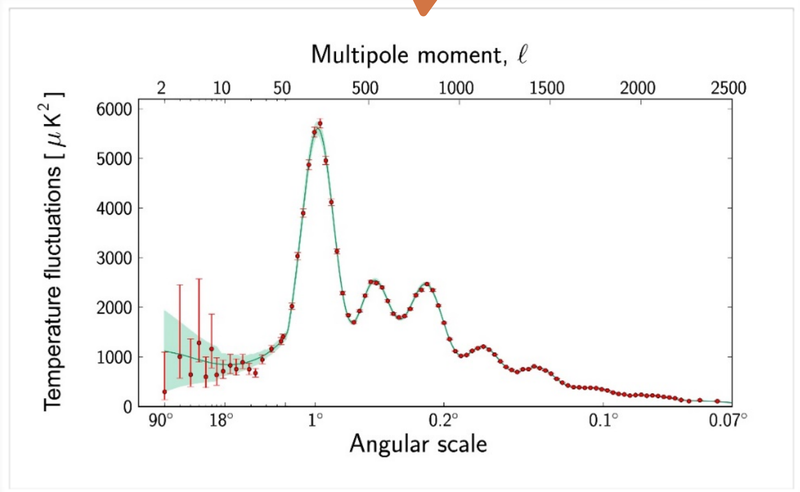

Primordial power spectra

Martineau, Barrau, Grain, arXiv:1709.03301

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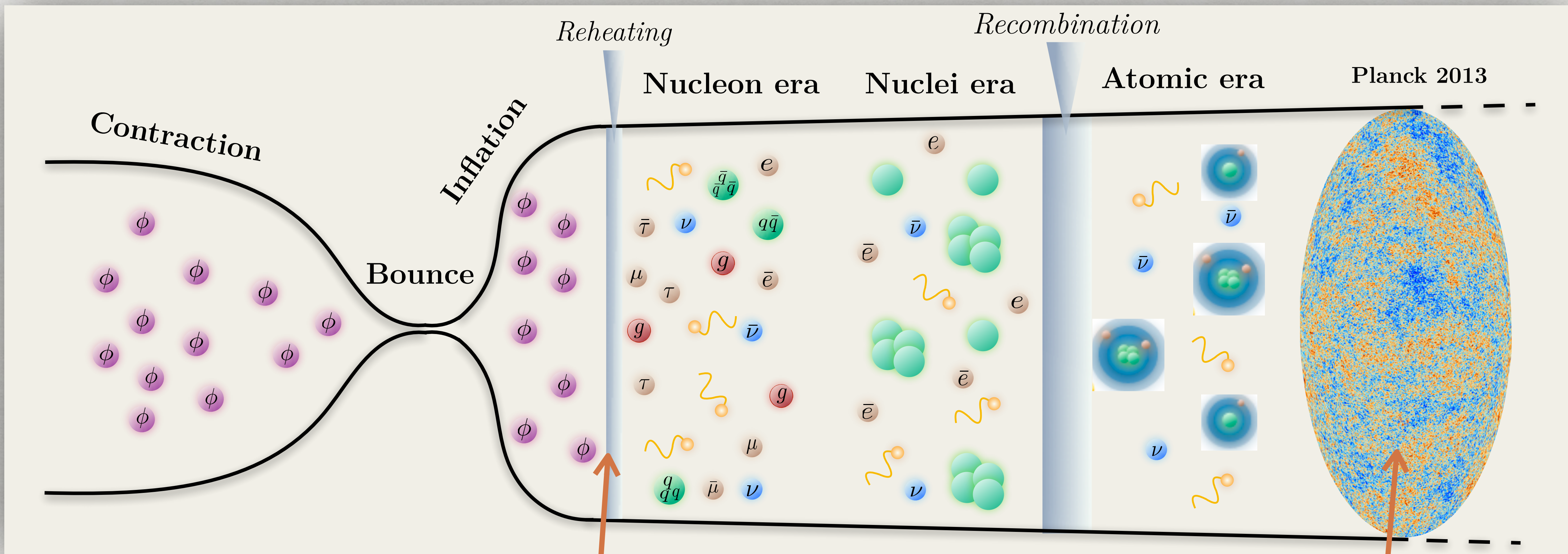

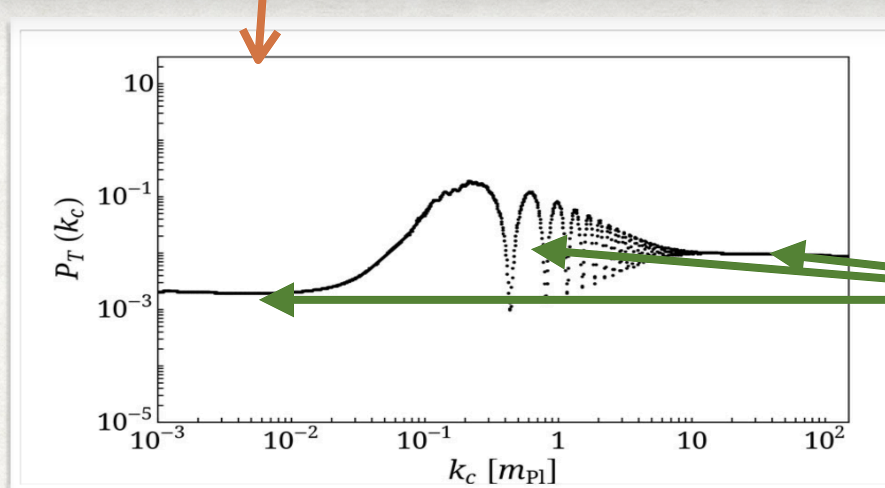

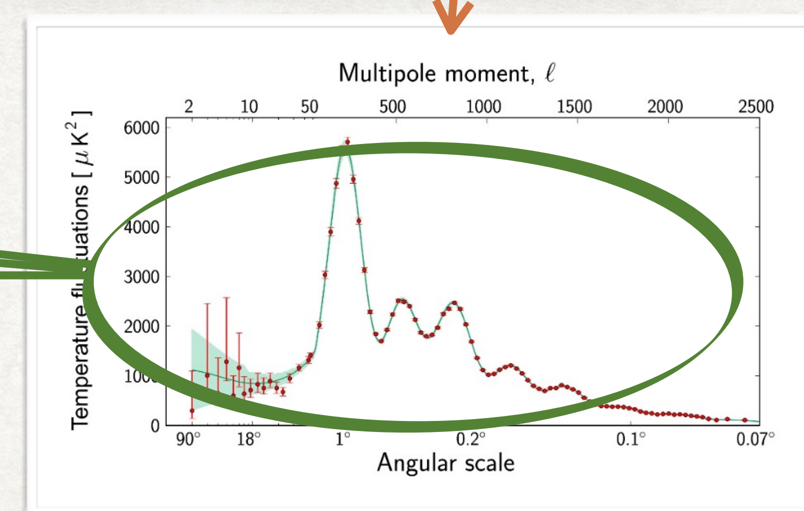

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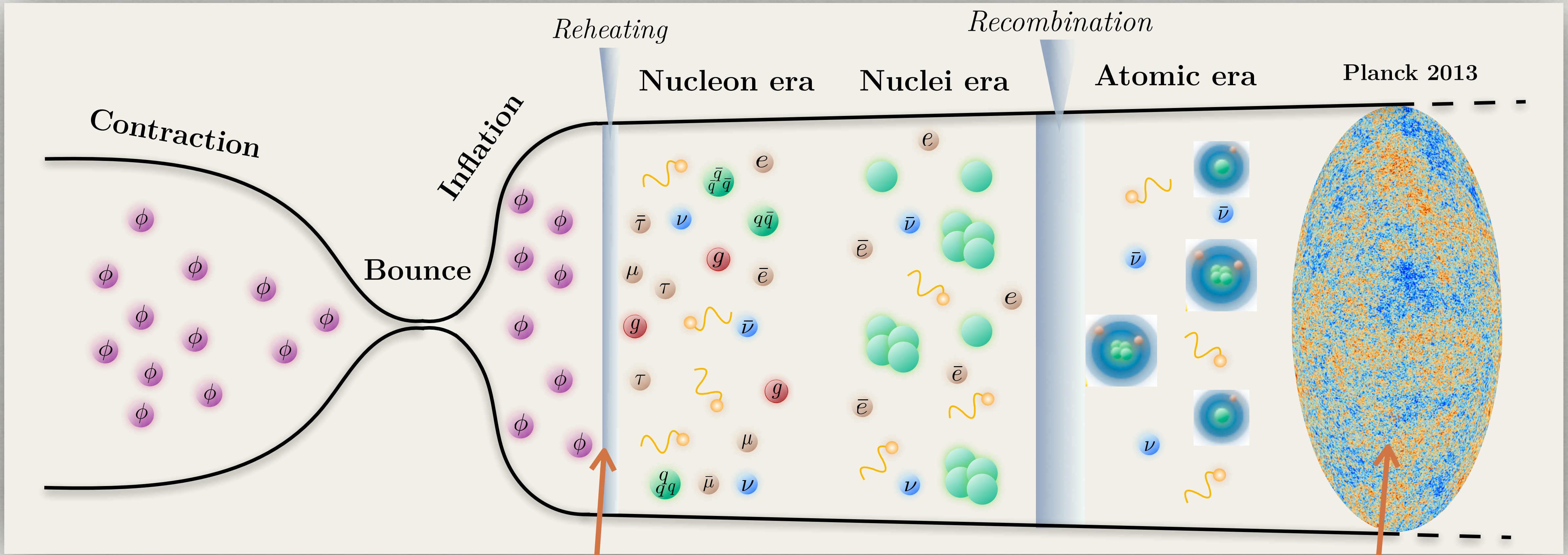

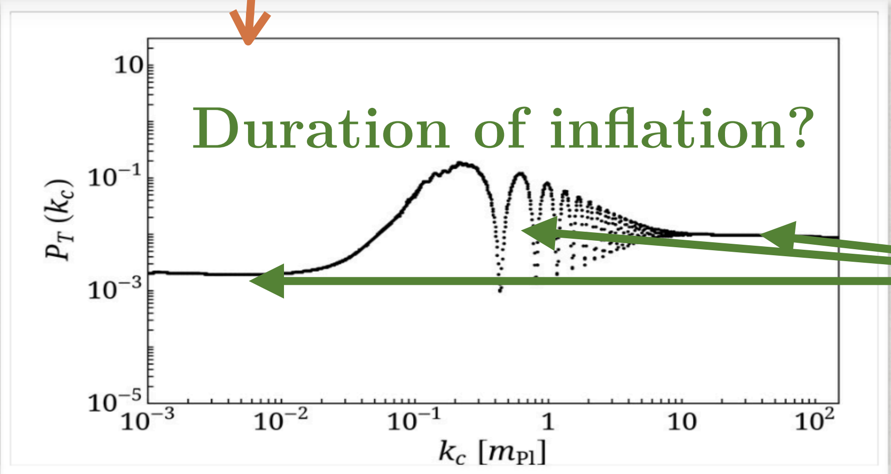

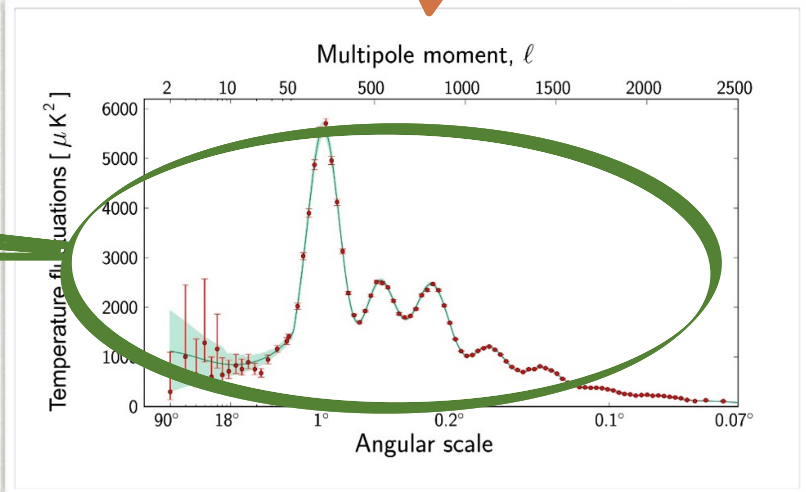

Important to know the duration of inflation

Primordial power spectra

Martineau, Barrau, Grain, arXiv:1709.03301

Duration of inflation in LQC

Number of inflationary e-folds:

$$N = \ln\left(\frac{a(t_f)}{a(t_i)}\right)$$

t_i *: beginning of inflation* t_f*: end of inflation*

Problem in standard cosmology:

$$N \in \underbrace{[60, 10^{14}]}$$

13 magnitude orders!

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13 magnitude orders!

Inflation duration investigation in LQC

Martineau, Barrau, Schander, arXiv:1701.02703

Varying the three main "unknowns" of the model

- **1st unknown: The amount of shear**

Anisotropies scale as $\mathbf{a^{-6}}$ *(Bianchi models)* ⟶ Important in any bouncing model

Bianchi I universe: $ds^2 = -dt^2 + a_1^2 dx^2 + a_2^2 dy^2 + a_3^2 dz^2$

Inflation duration investigation in LQC

Martineau, Barrau, Schander, arXiv:1701.02703

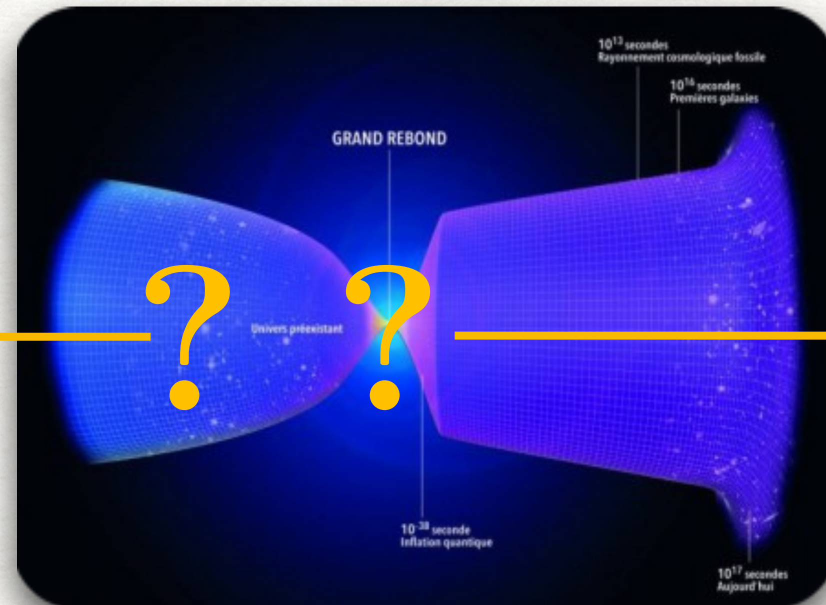
- **<u>2nd unknown: The way to set initial conditions</u>**

$$\ddot{\phi} + 3\cancel{H}\dot{\phi} + \frac{dV}{d\phi} = 0$$

Klein Gordon ⟶ Harmonic oscillator

δ : initial field phase

"Flat PDF on δ "



Flat PDF on $x_0 = \sqrt{\frac{V(0)}{\rho(0)}}$

No physical motivation

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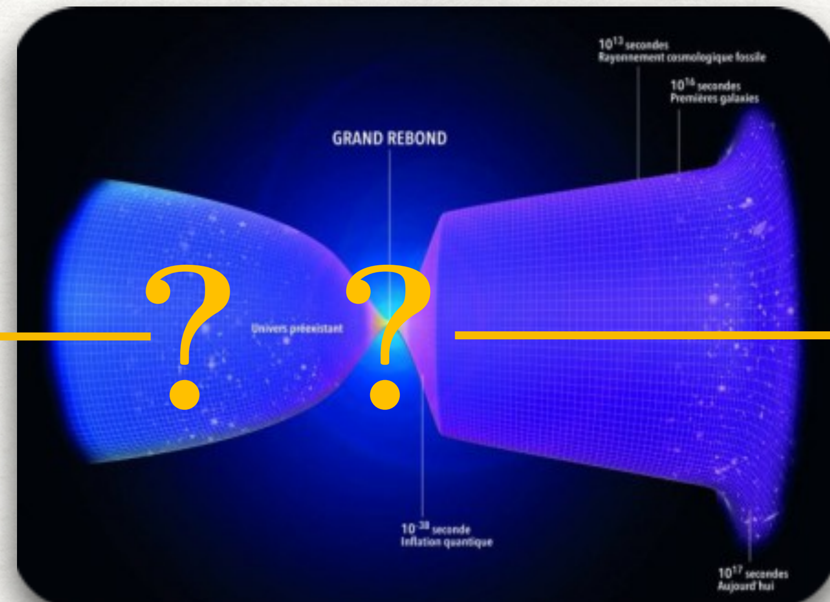

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- **<u>3^{rd} unknown: The universe matter content</u>**

$$\rho(t) = \frac{1}{2}\dot{\Phi}^2 + V(\Phi)$$

$V(\Phi)$ fixed according to 2015 Planck data

Planck 2015, arXiv:1502.02114

Duration of inflation in LQC

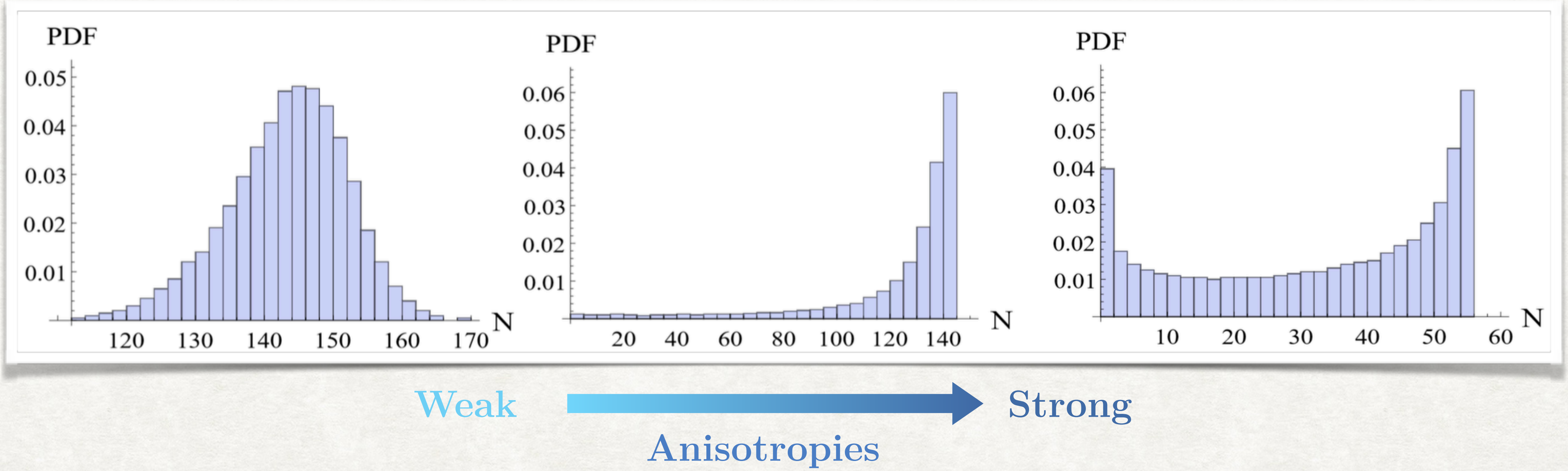
Martineau, Barrau, Schander, arXiv:1701.02703

- **Conclusion**

The number of e-folds is **well constrained** in LQC if:

- Initial conditions are set in the classical contracting branch
- The inflaton potential is confining.

- **Example** - Quadratic potential: $V(\Phi) = \frac{1}{2}m^2\Phi^2$
  - Initial conditions in the classical contracting branch



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How to compute the primordial power spectra?

Einstein equations of General Relativity:

(local) curvature of space-time

$$\mathcal{G}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

matter content

$$\delta\mathcal{G}_{\mu\nu} = 8\pi G \delta T_{\mu\nu} \quad \Leftrightarrow \quad v_k''(\eta) + \left(k_c^2 - \frac{z''_{T/S}(\eta)}{z_{T/S}(\eta)}\right) v_k(\eta) = 0$$

Mukhanov-Sasaki equation

η : conformal time, $d\eta = dt/a$

in which the perturbations type is contained in z:

$$z_T(\eta) = a(\eta)$$

Tensor

$$z_S(\eta) = a(\eta)\frac{\dot{\phi}(\eta)}{H(\eta)}$$

Scalar

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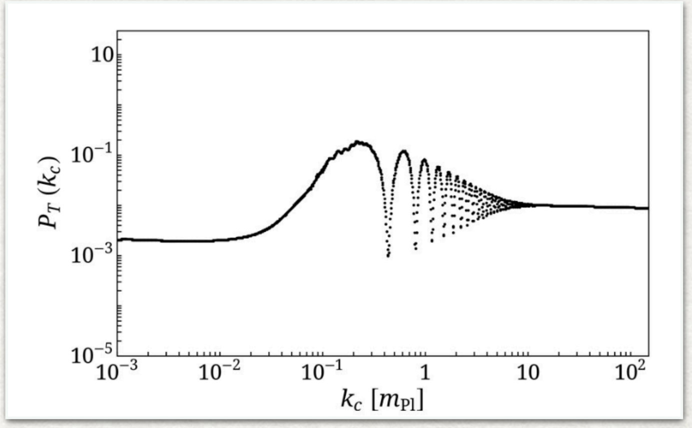
The primordial power spectra are obtained by computing the Mukhanov variables v and z **at the end of the inflationary phase η_e** .

$$\mathcal{P}_T(k_c) = \frac{4\kappa k^3}{\pi^2}\left|\frac{v_k(\eta_e)}{z_T(\eta_e)}\right|^2 \qquad \mathcal{P}_S(k_c) = \frac{k^3}{2\pi^2}\left|\frac{v_k(\eta_e)}{z_S(\eta_e)}\right|^2 \longrightarrow$$



Primordial power spectrum

Primordial power spectra in LQC

- Two different approaches for the perturbations in LQC

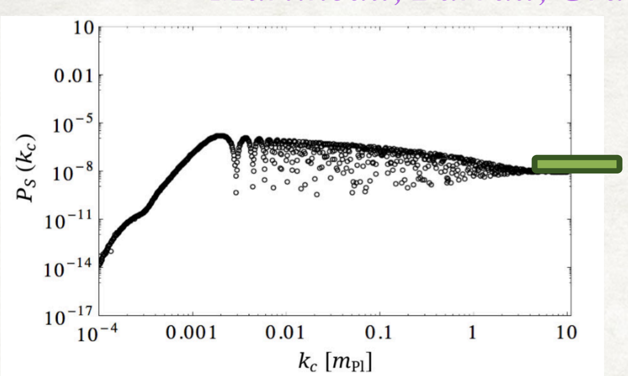
Dressed Metric (DM)	Deformed Algebra (DA)

The Mukhanov-Sasaki equation remains unchanged:

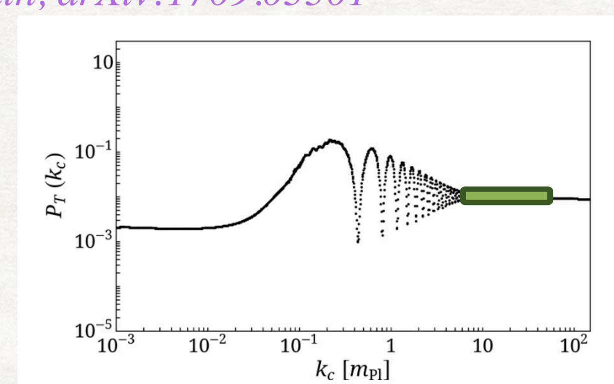
$$v_k''(\eta) + \left(k_c^2 - \frac{z_{T/S}''(\eta)}{z_{T/S}(\eta)}\right) v_k(\eta) = 0$$

: Observable window position for N ~ 75

Martineau, Barrau, Grain, arXiv:1709.03301



Scalar spectrum



Tensor spectrum

✓ **In agreement with Planck constrains**

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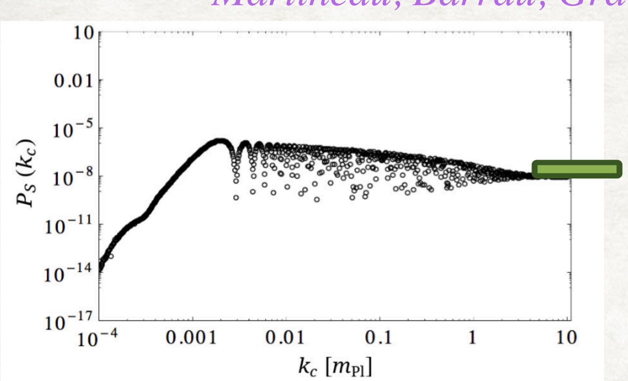

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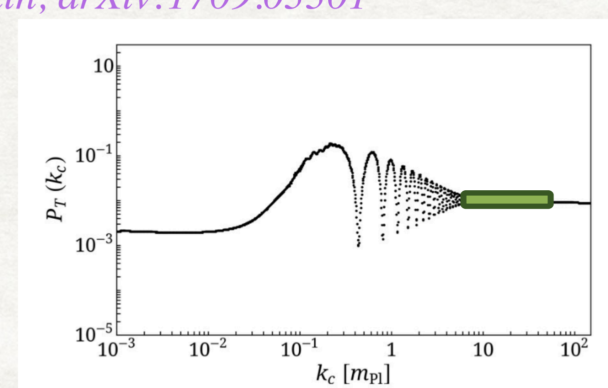

Tensor spectrum

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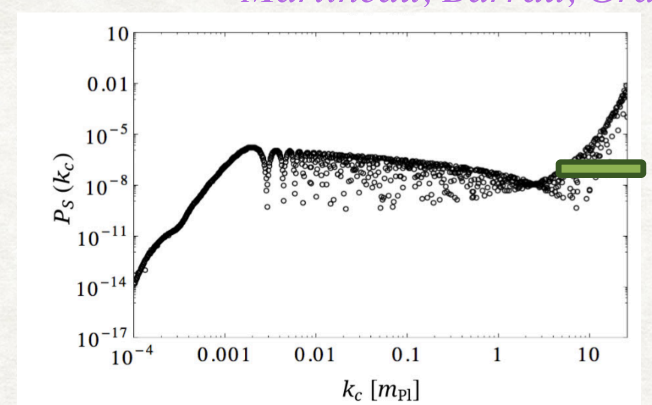
Deformed Algebra (DA)

The Mukhanov-Sasaki equation is modified:

$$v_k''(\eta) + \left(\Omega(\eta) k_c^2 - \frac{z_{T/S}''(\eta)}{z_{T/S}(\eta)}\right) v_k(\eta) = 0$$

$$\Omega(\eta) = 1 - 2\frac{\rho(\eta)}{\rho_c} \qquad -1 < \Omega < 1$$

Martineau, Barrau, Grain, arXiv:1709.03301



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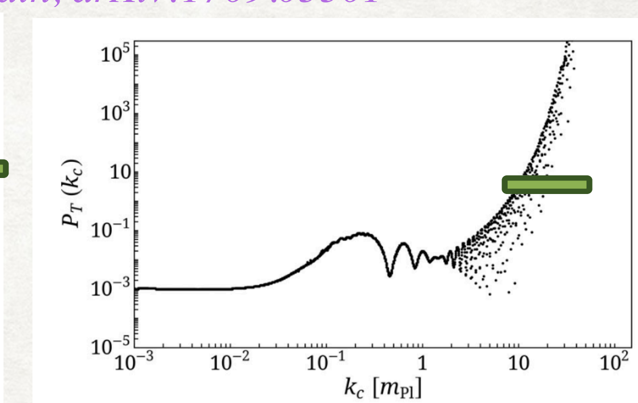


Tensor spectrum

✗ **Excluded by Planck**

Bolliet, Barrau, Grain, Schander arXiv:1510.08766

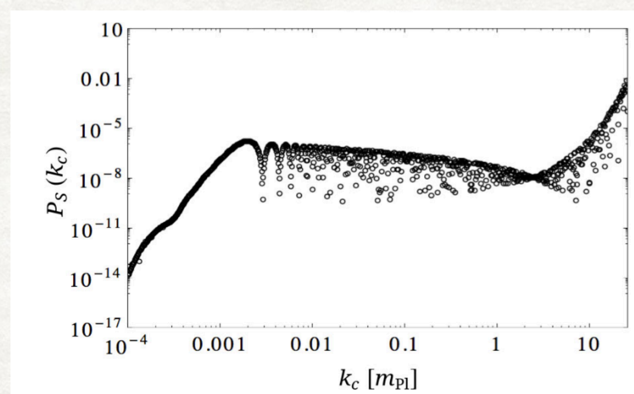
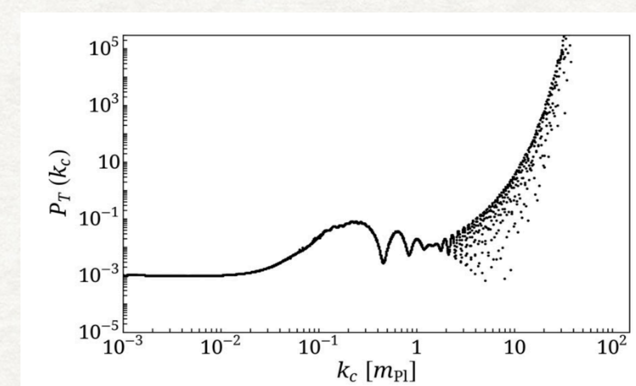
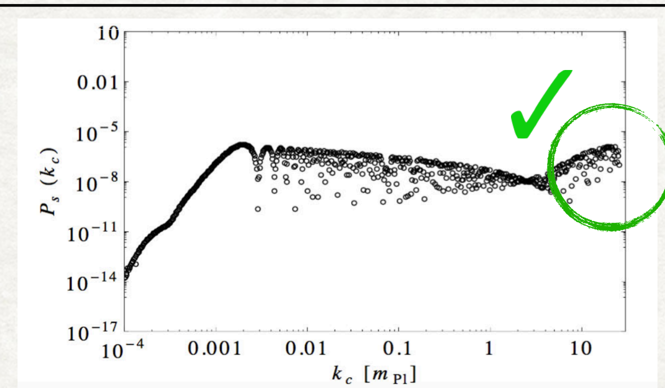
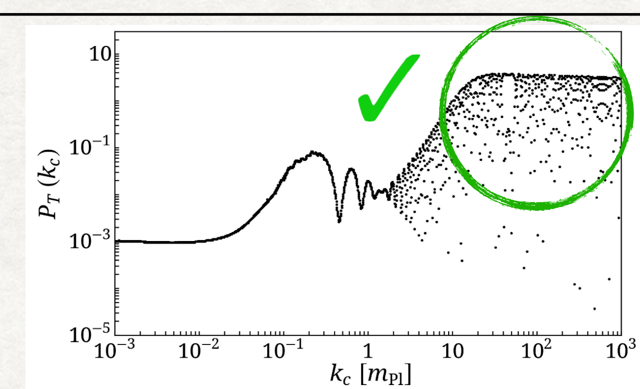
Trans-planckian problem in LQC

Martineau, Barrau, Grain, IJMP 2018, arXiv:1709.03301

But: **Trans-planckian problem:** If the number of inflationary e-folds N>60, the observed modes were much smaller than Planck length at the bounce.

↳ Introduction of **modified dipersion relations:**

$$v_k''(\eta) + \left(\Omega(\eta)a^2(\eta)\boxed{\mathcal{F}(k_\varphi)^2} - \frac{z_{T/S}''(\eta)}{z_{T/S}(\eta)}\right) v_k(\eta) = 0$$

Deformed Algebra approach	*Scalar spectra*	*Tensor spectra*
Without MDR $\mathcal{F}(k_\varphi) = k_\varphi$		
Unruh MDR $\mathcal{F}(k_\varphi) = k_0 \tanh\left[\left(\frac{k_\varphi}{k_0}\right)^p\right]^{\frac{1}{p}}$		

Conclusions

- **The bouncing background dynamics of LQC is well established** ✓

  Interesting prediction: *the number of inflationary e-folds N*

- **The fate of cosmological perturbations in the LQG framework remains unclear** ✗

  Different approaches, trans-planckian problem, …

  Still not possible to say if the full LQG framework is excluded or not by cosmological data

  BUT some scenarios have already been excluded