

Parton-shower and matching uncertainties in top-pair production with Herwig 7



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BASICS

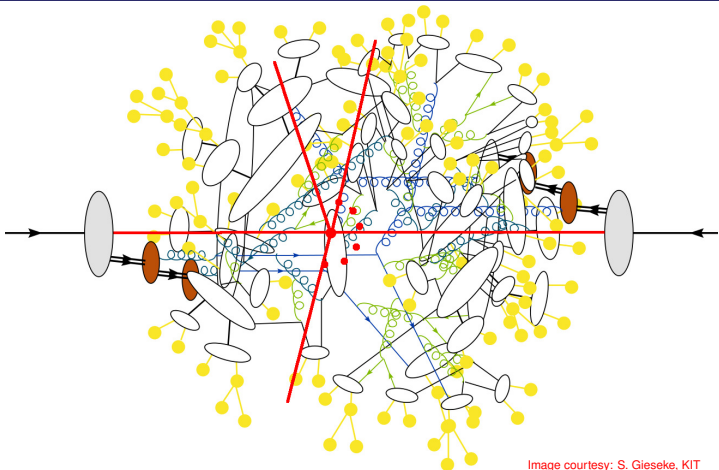


Image courtesy: S. Gieseke, KIT

- Parton picture of the proton (QCD asymptotic freedom); PDFs; resolve quarks and gluons at high energies.
- Single **hard interaction** between partons (hard matrix elements); determined from first principles.
- **QCD radiation** in initial and final state (PDFs and parton showers); evolution from first principles.
- **Hadronization and hadron decays** (QCD confinement); no first principles, needs modelling.
- **Multiple parton interactions** (MPI) / Underlying event; modelling of soft QCD.

- Event generators aim at exclusive final states with $O(100 - 1000)$ particles.
- Fixed order calculations can only do a limited number of legs.

LO and NLO automatization: Up to 5 - 10 particles in the final state.

NNLO: Selected processes with 2 or 3 particles in the final state.

- Logarithmic enhancements in the soft/collinear regions of phase space.

Need to be resummed to all orders in α_s .

- Fixed-order matrix elements (ME)

✓ For large-angle and/or hard radiation: Exact results at a certain fixed order.

✗ Soft/collinear regions: Missing effects of the all-order summation of large logarithms.

- Parton showers (PS)

✓ Generate a large amount of partons, due to multiple emissions in the soft/collinear approximation. Evolving an inclusive cross section at a hard scale into an exclusive final state at a lower cut-off scale.

✓ Approximate the matrix element to “all” orders in the soft/collinear regions, i.e. resummation at leading log (next-to-leading log) accuracy.

✗ The hard regions of phase space are naturally described badly.

- Matching/Merging

- Combine PS and ME by correcting (or replacing) the hardest emission(s) of the PS.
- The more MEs get involved, the better → Hard regions by the MEs, soft/collinear regions by the PS.
- There is double counting between PS and MEs → Matching and merging: Derive formulae for auxiliary cross sections, or algorithms to combine multiple inclusive calculations, upon which a parton shower can be applied without leading to double counting.
- Stick to matching in this talk.

- Condensed master formula to compute a physics observable:

omitting flux factors and initial-state averaging

$$\langle O \rangle \propto \sum_{a,b} \underbrace{\int dx_1 f_a(x_1) \int dx_2 f_b(x_2)}_{\substack{\text{parton distribution functions} \\ \text{(non-perturbative)}}} \sum_n \underbrace{\int d\phi_n}_{\substack{\text{final state} \\ \text{phase space integral} \\ \text{(numerically)}}} \underbrace{O(p_1, \dots, p_n)}_{\substack{\text{observable} \\ \text{(infrared safe)}}} \underbrace{|\mathcal{A}_{2+n}|^2}_{\substack{\text{amplitude squared} \\ \text{(perturbative)}}$$

- For an observable with LO prediction through an n -parton tree-level amplitude:

$$\alpha_s^{n-2} |\mathcal{A}_n|^2 = \alpha_s^{n-2} \left(|\mathcal{A}_n^{(0)}|^2 + \alpha_s 2\text{Re}(\mathcal{A}_n^{(0)*} \mathcal{A}_n^{(1)}) + \alpha_s^2 2\text{Re}(\mathcal{A}_n^{(0)*} \mathcal{A}_n^{(2)}) + \alpha_s^2 |\mathcal{A}_n^{(1)}|^2 + \dots \right)$$

$$\alpha_s^{n-2} |\mathcal{A}_{n+1}|^2 = \alpha_s^{n-2} \left(\alpha_s |\mathcal{A}_{n+1}^{(0)}|^2 + \alpha_s^2 2\text{Re}(\mathcal{A}_{n+1}^{(0)*} \mathcal{A}_{n+1}^{(1)}) + \dots \right)$$

$$\alpha_s^{n-2} |\mathcal{A}_{n+2}|^2 = \alpha_s^{n-2} \left(\alpha_s^2 |\mathcal{A}_{n+2}^{(0)}|^2 + \dots \right) \quad \text{etc.}$$

- LO: n -parton contribution; NLO: n - and $(n+1)$ -parton contribution
- O IR safe means $O_{n+1} \rightarrow O_n$ if +1 is soft or any two partons of $n+1$ are collinear
- NLO real $\left(R = |\mathcal{A}_{n+1}^{(0)}|^2 \right)$ plus virtual $\left(V = 2\text{Re}(\mathcal{A}_n^{(0)*} \mathcal{A}_n^{(1)}) \right)$ in the soft/collinear limit:

$$\int_{n+1} O_{n+1} R + \int_n O_n V \longrightarrow \int_n O_n \left(\int_{+1} R + V \right)$$

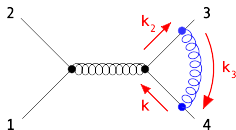
• LO and NLO contributions in a condensed notation:

the “inclusiveness” and thus IR safety breaks down for identified partons (here through initial state PDFs), thus if initial state partons are present, a counterterm $\int O_n C$ is needed to subtract additional collinear divergences;

$$\langle O \rangle^{LO} = \int_n O_n B \quad \langle O \rangle^{NLO} = \int_{n+1} O_{n+1} R + \int_n O_n V + \int_n O_n C$$

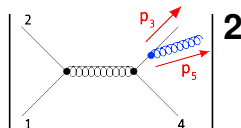
$$V \propto \int dV, \text{ w/ } dV \propto d^4 k f(p_i, m_i, k)$$

loop



Loop integration may lead to soft ($|k_i| \rightarrow 0$), collinear ($|k_i||k_j|$) and ultraviolet ($|k| \rightarrow \infty$) divergences.

$R \propto$ squared **real-emission** amplitude



Integration over **+1 real-emission phase space** may lead to soft ($|p_i| \rightarrow 0$) and collinear ($|p_i||p_j|$) divergences.

- UV divergences removed by renormalization: V denotes the renormalized virtual piece
- Squared amplitude level: IR divergences cancel between virtual and real emission
- Observable level: Only IR safe observables guarantee the cancellation of IR divergences

cf previous slide

- Dimensional regularization in $D = 4 - 2\varepsilon$ dimensions ($|\varepsilon| \ll 1$): Divergences yield ε -poles.
- Analytical many-body phase-space integration impossible \rightarrow Numerical integration.
keep in mind that numerical integration is in finite four dimensions
- Different phase-space dimensions of both NLO contributions \rightarrow Combined Monte Carlo integration in finite four dimensions impossible.
- Need to cancel IR divergencies for the n and $n+1$ parts separately.
keep in mind that UV poles are already removed
- A popular method is the subtraction method:

$$\langle O \rangle^{NLO} = \int_{n+1} \left[O_{n+1} R \Big|_{\varepsilon=0} - O_n A \Big|_{\varepsilon=0} \right] + \int_n \left[O_n V + O_n \int_{+1} A \right]_{\varepsilon=0}$$

Using QCD factorization properties, one may devise a subtraction term A , such that

- A has the same pointwise singular behaviour as R .
 $O_{n+1} \rightarrow O_n$ in the soft/collinear limit: $O_n A$ acts as local counterterm to $O_{n+1} R$.
- A is analytically integrable over the $+1$ -parton sub-space in D dimensions.
The resulting soft/collinear ε -poles cancel the explicit soft/collinear ε -poles of V .
together with the poles from the initial-state collinear counterterm, if present
- The first bracket is finite by definition. The second bracket is free of ε -poles.
Separate numerical integration of both brackets in four dimensions possible.

- ▶ Most NLO QCD calculations nowadays use the subtraction method to regularize the soft/collinear divergencies between the two different phase spaces of the virtual and real corrections.

- ▶ The bottleneck used to be the virtual contributions → NLO revolution:

Past 10 - 15 years. Automation of NLO QCD corrections. Breakthroughs in understanding underlying principles & implementation of efficient algorithms, particularly for one-loop calculations. NLO QCD corrections for virtually any SM process “at the push of a button”.

- ▶ Paradigm shift:

Dedicated ME providers (OLPs; one-loop providers) & MC event generators interface on the code level.

Let the MC event generator steer the computation (process setup, real subtraction, phase-space integration, ...), possibly also showering and hadronization, ... Use the OLPs for ME input, as e.g. suggested in the [BLHA\(2\)](#) accord.

- ▶ Automated QCD NLO OLPs:

OpenLoops, Recola, MadLoop w/ MG5_aMC@NLO, GoSam, NLOX, NJet, Helac-NLO, Black-Hat,

Automated QCD MC frameworks:

BBMC, MoCaNLO, Munich, Sherpa, MG5_aMC@NLO, Herwig 7, Powheg-Box, Powhel, VBFNLO, MCFM,

Some of the MC event generators are fixed-order event generators, to be interfaced to shower programs like Herwig 7, Sherpa, or Pythia (mostly via Les Houches event files).

Some of the MC event generators go one step further and collect all under one hood: e.g. Herwig 7 and Sherpa have their own showers and hadronization but rely largely on external OLPs (interface on the code level), on the other hand e.g. MG5_aMC@NLO has its own OLP but interfaces to external shower providers (internally also through LH event files).

PARTON SHOWERS

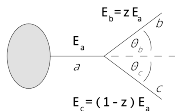
- ▶ Cutting perturbative calculations at a certain fixed order: always accompanied by spurious logarithms L of ratios of process dependent scales $\sqrt{s_i}$ by a non-physical cut-off scale μ :
 - ▶ Enhanced in regions where the ratios are small (or large) \rightarrow large logs.
 - ▶ Problematic if $L\alpha_s \approx 1$.
- ▶ Current fixed-order calculation are not sufficient to describe the momenta of outgoing jets well in an exclusive picture of the process (including jet structures, i.e. distributions of the associated particles).
- ▶ A way to deal with that is through parton showers, by dressing the hard process with additional soft/collinear QCD radiation:

- ▶ Parton showers approximate the matrix element in the soft and collinear regions, where contributions are enhanced.
- ▶ They are built on the factorization of an $n+1$ particle state into an n particle state times a one-particle phase space and a universal splitting function:

$$d\sigma_{n+1} = \sigma_n \frac{dt}{t} dz P(z)$$

- ▶ Iterate to generate multiple emissions, successively off the n particle state, the $n+1$ particle state, etc.

- ▶ Fast production of many-particle final states in enhanced regions.
- ▶ Summing up high orders in α_s ; in the soft/collinear approximation takes all the leading large logarithms into account.
- ▶ Evolving, in a cascade like fashion, from an inclusive hard process at some large scale, to an exclusive many-particle state at some low scale at which QCD confinement (hadronization) takes over.



For small angles:

$$p_a^2 = t = (p_b + p_c)^2 = z(1-z)E_a^2(\Theta_b + \Theta_c)^2$$

- ▶ Given a LO process with total cross section σ_n , the associated (differential) NLO cross section factorizes in the collinear limit ($\theta_{ji} \ll \pi/2$):

$$d\sigma_{n+1}^j \approx \sigma_n \sum_{\text{emitters } i} \frac{\alpha_s}{2\pi} \frac{d\theta_{ji}}{\theta_{ji}} dz_{ji} P_{ji}(z_{ji}, \phi_{ji}) \frac{d\phi_{ji}}{2\pi} = \sigma_n \sum_{\text{emitters } i} d\mathcal{P}_{ji}(\theta_{ji}, z_{ji}, \phi_{ji})$$

- ▶ Hard configuration of emitters i , accompanied by a collinear parton j with energy fraction z_{ji} , wrt i .
- ▶ θ_{ji} : Emission angle between i and j .
- ▶ ϕ_{ji} : Azimuth of j around the i -axis.
- ▶ $P_{ji}(z_{ji}, \phi_{ji})$: Spin-dependent splitting functions
Enhanced for $z_{ji} = 0$ or also $z_{ji} = 1$, depending on the type of splitting.
Independent of the precise def. of z in the collinear limit: energy fraction, light-cone momentum fraction - or similar.
Neglecting spin correlations \rightarrow spin-averaged splitting functions $P_{ji}(z_{ji})$.
- ▶ Instead of $\frac{dt}{t} = \frac{d\theta^2}{\theta^2}$ one may also choose e.g. $\frac{dq^2}{q^2}$, $\frac{dk_{\perp}^2}{k_{\perp}^2}$ or $\frac{d\tilde{q}^2}{\tilde{q}^2} \rightarrow$ diff. choices for the evol. variable:
 - ▶ $q^2 = z(1-z)E^2\theta^2$ the virtuality of the off-shell emitter propagator, with energy E .
 - ▶ $k_{\perp}^2 = z^2(1-z)^2E^2\theta^2$ the emitted parton's transverse momentum wrt the emitter.
 - ▶ $\tilde{q}^2 = E^2\theta^2$ (used e.g. by Herwig 7 in its angular-ordered shower, aka \tilde{q} shower).
 - ▶ All of these choices are identical in the collinear limit, but extrapolate differently away from it.

- ▶ So far: Inclusive emission distribution of all emissions j off i .
 - ▶ Consider e.g. only $j = \text{gluon}$. Consider further the virtuality q^2 of the internal emitter line.
 - ▶ The total probability for all gluon branchings off a parton i between q^2 and $q^2 + dq^2$ is

$$d\mathcal{P}_i(q^2) = \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz P_{ji}(z)$$

- ▶ How to single out the distributions of individual gluons?
How to know that a certain branching is the “first” or the “hardest”?
- ▶ Introduce order: the virtuality q^2 serves as “ordering” variable.
- ▶ Introduce $\Delta_i(Q^2, q^2)$: the probability that there is no branching between q^2 and a certain max. virtuality $Q^2 > q^2$.
- ▶ The probability to emit nothing at all, while evolving all the way down to a shower cut-off Q_0^2 , is $\Delta_i(Q^2, Q_0^2)$.
- ▶ The probability to have the first emission at a scale q^2 is

$$\frac{d\Delta_i(Q^2, q^2)}{dq^2} = \Delta_i(Q^2, q^2) \frac{d\mathcal{P}_i(q^2)}{dq^2}$$

- ▶ Thus the solution for the Sudakov form factor is

$$\Delta_i(Q^2, q^2) = \exp \left\{ - \int_{q^2}^{Q^2} d\mathcal{P}_i(k^2) \right\} = \exp \left\{ - \int_{Q_0^2/k^2}^{Q^2/k^2} \frac{\alpha_s}{2\pi} \frac{dk^2}{k^2} \int_{Q_0^2/k^2}^{1-Q_0^2/k^2} dz P_{ji}(z) \right\}$$

- ▶ We've sneaked in a resolution scale / a lower cut-off Q_0 . Why?
 - ▶ Soft/collinear configurations \rightarrow divergences arise universally
Physical measurements have a finite resolution \rightarrow Cannot differ an exact soft/collinear config. from just one parton
- ▶ Above Q_0 : (finite) resolvable emission
For the relative k_\perp between emitter and emission (cut-off $Q_0 = k_{\perp,0}$), we cut on soft and collinear simultaneously.
- ▶ Integrate the distribution below Q_0
Probability for non-resolvable emission: divergent \rightarrow add it to the loop correction of the hard process
- ▶ Unitarity: The total probability of either emitting s.th. or not emitting at all is one, or

$$\mathcal{P}(\text{no emission}) = 1 - \mathcal{P}(\text{all emissions})$$

- ▶ Parton showers built on this principle include loop corrections implicitly.
Exact for soft/collinear contributions. Hard non-collinear loops yield in general a finite correction.
- ▶ Formally, this defines a parton shower operation on an observable O , over an n -parton seed configuration: dropped sums over parton species shower cut-off $Q_0^2 = \mu_{\text{IR}}^2$

$$\text{PS}[Q^2, \mu_{\text{IR}}^2, \{\Phi_n, O\}] = \Delta(Q^2, \mu_{\text{IR}}^2) O_n + \int_{\mu_{\text{IR}}^2}^{Q^2} d\mathcal{P}(q^2) \Delta(Q^2, q^2) \text{PS}[q^2, \mu_{\text{IR}}^2, \{\Phi_{n+1}, O\}]$$

- ▶ Back substitution gives the Neumann series solution to the modified DGLAP evolution equations.
- ▶ Each splitting generates one order more in α_s . Summing the leading contributions of repeated parton branchings to all orders in α_s .

- ▶ By choosing the angle as ordering variable one can show that coherence effects of soft gluons are properly included → **angular-ordered showers**
 - ▶ One can show that color coherence leads to emissions of successively smaller opening angles.
 - ▶ In a p_T or virtuality ordered shower, one would have to manually veto any emission that is larger in angle than the previous one.
- ▶ However, for certain applications one would like to include color coherence and still be able to have the transverse momentum as ordering variable → **dipole showers**
 - ▶ Uses color dipoles and $2 \rightarrow 3$ splittings: emission off a color dipole; conserve on-shell conditions by using additional color-connected parton as momentum-balancing third party.
 - ▶ Since color coherence effects enter naturally, one does not have to enforce them through explicit angular ordering.
- ▶ Using a running α_s in the splittings, **with p_\perp as argument, i.e. $\alpha_s(q^2 = p_\perp^2)$** , a tower of certain higher order loop insertions can be resummed.
 - ▶ The value of α_s increases with decreasing $p_\perp \rightarrow$ increasing multiplicity; phase space fills with soft gluons.
 - ▶ To avoid regions of $\alpha_s \approx 1$ **need to place the cut-off $Q_0 = \mu_{\text{IR}}$ higher** → not a technical cut anymore.

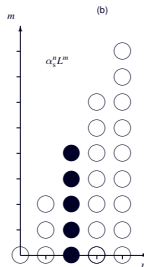
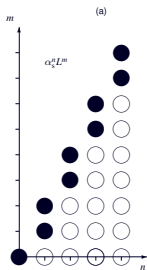
Above is in the form of a final-state evolution. An initial-state evolution can be formulated too, as backwards evolution, taking into account PDF evolution.

- ▶ HERWIG 7 *e.g.* implements an angular-ordered parton shower (using \tilde{q} as ordering variable) as well as a dipole shower, based on Catani-Seymour dipoles (which are also used in the NLO real subtraction in HERWIG 7).
- ▶ In terms of scales so far:
Hard process scale ($\mu_H = \mu_R = \mu_F$), **hard veto scale, aka shower start scale** ($Q = Q_\perp$), **shower scale** (μ_S in shower $\alpha_s(\mu_S)$), lower cut-off scale (μ_{IR}). For matching uncertainties we are interested in the first three.

MATCHING

Plots: [arXiv:1101.2599]

- ▶ Look at terms that enter in the α_s expansion of a jet cross section, e.g. in $e^+e^- \rightarrow \text{jets}$.
- ▶ For each order α_s^n there is a set of large logs L^m , with $m \leq 2n$.



- ▶ Terms in a jet cross section in e.g. $e^+e^- \rightarrow \text{jets}$ that are correctly included (filled blobs) e.g. by a NLL parton shower.
- ▶ Terms correctly included in a tree-level matrix element of order α_s^n . Here, a 4-jet observable in e.g. $e^+e^- \rightarrow \text{jets}$ with LO prediction at order α_s^2 .
- ▶ To get as many as possible blobs: combine fixed-order calculations and parton showers.
- ▶ However, notice that there is double counting if one does that naively.
- ▶ Avoid double counting in NLO+PS: NLO matching; essentially two methods, MC@NLO and Powheg.
- ▶ Variants thereof are built into various programs nowadays.

HERWIG 7 e.g. implements variants of both \rightarrow we'll see the idea behind the MC@NLO variant later on

- Continue with

$$\text{PS}[Q^2, \mu_{\text{IR}}^2, \{\Phi_n, O\}] = \Delta(Q^2, \mu_{\text{IR}}^2) O_n + \int_{\mu_{\text{IR}}^2}^{Q^2} d\mathcal{P}(q^2) \Delta(Q^2, q^2) \text{PS}[q^2, \mu_{\text{IR}}^2, \{\Phi_{n+1}, O\}]$$

$$\text{w/ } \Delta(Q^2, \mu_{\text{IR}}^2) = \exp \left[- \int_{\mu_{\text{IR}}^2}^{Q^2} d\mathcal{P}(q^2) \right] = 1 - \int_{\mu_{\text{IR}}^2}^{Q^2} d\mathcal{P}(q^2) + \mathcal{O}(\alpha_s^2)$$

and further

$$O_{\text{NLO}} = \int d\Phi_n (\mathcal{B} + \bar{\mathcal{V}}) O_n + \int d\Phi_{n+1} (\mathcal{R} O_{n+1} - \mathcal{A} O_n), \text{ with } \bar{\mathcal{V}} = \mathcal{V} + \int d\Phi_1 \mathcal{A}$$

- Up to NLO in α_s : $\text{PS}[Q^2, \mu_{\text{IR}}^2, \{\Phi_n, O_{\text{NLO}}\}]$

$$= \int d\Phi_n (\mathcal{B} + \bar{\mathcal{V}}) O_n + \int d\Phi_{n+1} (\textcolor{red}{\mathcal{R}} O_{n+1} - \textcolor{blue}{\mathcal{A}} O_n) + \int d\Phi_n \mathcal{B} \int_{\mu_{\text{IR}}^2}^{Q^2} d\mathcal{P}(q^2) (\textcolor{red}{O}_{n+1} - \textcolor{blue}{O}_n)$$

Double-counting between **PS & real emission** and **PS & real subtraction**

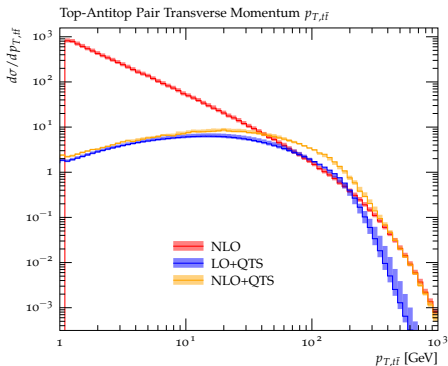
Total inclusive cross section unaffected

NLO Matching: Restore the correct NLO expression at the observable level

- Subtract PS contribution, such that $\text{PS}[O_{\text{NLO}}^{\text{matched}}] = O_{\text{NLO}} + \mathcal{O}(\text{NNLO } \alpha_s)$

$$O_{\text{NLO}}^{\text{matched}} = \int d\Phi_n (\mathcal{B} + \bar{\mathcal{V}}) O_n + \int d\Phi_{n+1} (\mathcal{R} O_{n+1} - \mathcal{A} O_n) - \int d\Phi_n \mathcal{B} \int_{\mu_{\text{IR}}^2}^{Q^2} d\mathcal{P}(q^2) (O_{n+1} - O_n)$$

Plot by D. Rauch (first steps with HERWIG++ and MATCHBOX beta for top-pair production; Master's thesis in 2014)



- Red: NLO calculation; the low energy region is dominated by large logarithms
- Blue: LO + parton shower
 - the low energy region is properly Sudakov suppressed
 - the high energy region is only described with LO accuracy
- Yellow: MC@NLO-like matching (pretty much the matching subtraction formula) with angular-ordered parton shower
 - the low energy region shows the correct sudakov suppression
 - the high energy tail is described with NLO accuracy

NLO MATCHING IN HERWIG 7

- ▶ Exclusive event generation in particle collisions at had-had, had-lep and lep-lep colliders, up to the particle level

- ▶ HERWIG 7 is written in C++ and based on THEPEG

[<https://herwig.hepforge.org/>]

[<https://thepeg.hepforge.org/>]

Supports LHAPDF and HepMC event output and/or Rivet

- ▶ Perturbative physics

- ▶ **Hard process** at NLO QCD

- ▶ LHE file input
- ▶ Built-in LO and NLO matrix elements (MEs)
- ▶ Automated assembly of NLO QCD calculations
- ▶ Interfacing various external ME providers

- ▶ **Parton shower** Monte Carlo

- ▶ Angular-ordered parton shower
- ▶ Dipole shower
- ▶ Decays of heavy resonances (incl. spin correlations)
- ▶ Dedicated Powheg matched / matrix element corrected
- ▶ Automated matching machinery, algorithms based on MC@NLO and Powheg

- ▶ Non-perturbative physics

- ▶ **Hadronization**

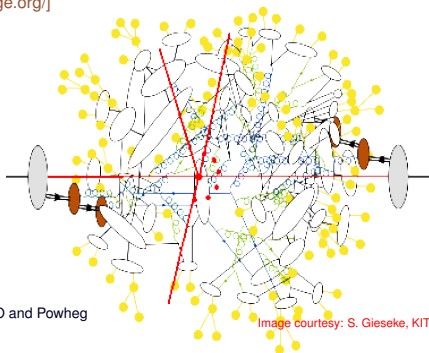
- ▶ Cluster hadronization model
- ▶ Color reconnection
- ▶ Decays

- ▶ Underlying event

- ▶ **MPI** (eikonal multiple interaction model)

- ▶ Diffractive processes

- ▶ BSM machinery: Built-in processes as well as UFO model file input



► HERWIG 7 \in MCnet

[<http://www.montecarlonet.org/>]

[<http://www.montecarlonet.org/index.php?p=Projects/herwig>]



► HERWIG++ has been developed over the course of ~ 10 years [[hep-ph/0311208](#), ..., [0803.0883](#), ..., [1310.6877](#)]

Intention: Fully replace and supersede the capabilities of FORTRAN HERWIG [[hep-ph/0011363](#), [hep-ph/0210213](#)]

► Paradigm shift towards NLO \rightarrow Release of HERWIG 7 [[arXiv:1512.01178](#)]

The current version is Herwig 7.1.5 (top-pair study in [[arXiv:1810.06493](#)] done with 7.1.4)

► As the successor of the HERWIG++ 2 and HERWIG 6 series

HERWIG 7 supersedes the physics capabilities of both its predecessors

Focusing greatly on precision and NLO automatization

Greatly improved installation, steering and documentation

► The MATCHBOX module forms the basis for the automated NLO capabilities of HERWIG 7

Fully integrated framework for automated NLO matching, with full control over the fixed order input

► Automated setup for a full NLO QCD calculation in the subtraction formalism

► Implementation of the CS dipole subtraction method (massless [[Catani, Seymour, '96](#)] and massive [[Catani, Dittmaier, Seymour, Trocsanyi, '02](#)])

► Fixed-order input: In-house calculations and Interfaces to various external matrix-element providers

► Automated diagram based multi-channel phase-space sampling and adaptive phase-space integration

► Fully automated matching algorithms: Subtractive (based on MC@NLO [[Frixione, Webber, '02, '06](#)]) and multiplicative (based on Powhag [[Nason '04](#); [Aliolo, Nason, Oleari, Re '08](#)])

► Plug-ins to the two shower variants in HERWIG 7

All in one framework: External matrix-element codes fully interfaced, no event files to move around anymore
MATCHBOX already introduced previously [[Plätzer, Gieseke '12](#)]. Beta tested in Herwig++.

- Remember

$$O_{\text{NLO}}^{\text{matched}} = \int d\Phi_n (\mathcal{B} + \bar{\mathcal{V}}) O_n + \int d\Phi_{n+1} (\mathcal{R} O_{n+1} - \mathcal{A} O_n) - \int d\Phi_n \mathcal{B} \int_{\mu_{\text{IR}}^2}^{\mathcal{Q}^2} d\mathcal{P}(q^2) (O_{n+1} - O_n)$$

- Rearranging wrt O_n and O_{n+1} (so-called S - and H -events)

$$\begin{aligned} O_{\text{NLO}}^{\text{matched}} = & \left[\int d\Phi_n (\mathcal{B} + \bar{\mathcal{V}}) - \int d\Phi_{n+1} \mathcal{A} + \int d\Phi_n \mathcal{B} \int_{\mu_{\text{IR}}^2}^{\mathcal{Q}^2} d\mathcal{P}(q^2) \right] O_n \\ & + \left[\int d\Phi_{n+1} \mathcal{R} - \int d\Phi_n \mathcal{B} \int_{\mu_{\text{IR}}^2}^{\mathcal{Q}^2} d\mathcal{P}(q^2) \right] O_{n+1} \end{aligned}$$

- O_n and O_{n+1} contributions are separately not finite: Add extra term $\mathcal{A}_{\text{bridge}}$ below shower cut-off μ_{IR}

$$\int d\Phi_{n+1} \mathcal{A}_{\text{bridge}}(\Phi_{n+1}) \Theta(\mu_{\text{IR}}^2 - q^2) (O_n - O_{n+1})$$

Subtract real-emission divergencies in the $n+1$ -parton bin and those of \mathcal{A} in the n -parton bin
 For IR safe observables only adds power corrections below μ_{IR} (conserves the log behaviour)
 Choose $\mathcal{A}_{\text{bridge}} = \mathcal{A}$

$$O_{\text{NLO}}^{\text{matched}} = \left[\int d\Phi_n \left(\mathcal{B} + \bar{\mathcal{V}} \right) + \int d\Phi_n \mathcal{B} \int_{\mu_{IR}^2}^{Q^2} d\mathcal{P}(q^2) - \int d\Phi_{n+1} \mathcal{A} \Theta(q^2 - \mu_{IR}^2) \right] O_n$$

$$+ \left[\int d\Phi_{n+1} \mathcal{R} - \int d\Phi_n \mathcal{B} \int_{\mu_{IR}^2}^{Q^2} d\mathcal{P}(q^2) - \int d\Phi_{n+1} \mathcal{A} \Theta(\mu_{IR}^2 - q^2) \right] O_{n+1}$$

► Interfaces at amplitude level

- Built-in (one-loop) helicity sub-amplitudes, spinor helicity library and caching facilities
- **MG5_AMC@NLO** [<https://launchpad.net/mg5amcnlo>]
(color-ordered sub-amplitudes)
Color bases: **COLORFULL** [M. Sjö Dahl, S. Plätzer],
CVOLVER [S. Plätzer]
- In-house calculations, e.g. parts of **HJETS++**
[F. Campanario, T. Figy, S. Plätzer, M. Sjö Dahl]

► Interfaces at squared amplitude level

- Dedicated interfaces [**HEJ** [<https://hej.hepforge.org/>];
NLOJET++ [www.desy.de/~znagy/Site/NLOJet++.html]]
- **BLHA(2)** [**GoSAM** [<https://gosam.hepforge.org/>];
NJET [<https://bitbucket.org/njet/njet/>];
OPENLOOPS [<https://openloops.hepforge.org/>];
VBFNLO [<https://www.itp.kit.edu/vbfnlo/>]]

► Shower plugins:

- Angular ordered $P(\vec{q})$ or Dipole shower $D(p_\perp)$
- MEC $R_{\text{MEC}}(p_\perp)$

$$\begin{aligned}
O_{\text{NLO}}^{\text{matched}} = & \left[\int d\Phi_n \left(\mathcal{B} + \bar{\nu} \right) + \int d\Phi_n \mathcal{B} \int_{\mu_{IR}^2}^{\mathcal{Q}^2} d\mathcal{P}(q^2) - \int d\Phi_{n+1} \mathcal{A} \Theta(q^2 - \mu_{IR}^2) \right] O_n \\
& + \left[\int d\Phi_{n+1} \mathcal{R} - \int d\Phi_n \mathcal{B} \int_{\mu_{IR}^2}^{\mathcal{Q}^2} d\mathcal{P}(q^2) - \int d\Phi_{n+1} \mathcal{A} \Theta(\mu_{IR}^2 - q^2) \right] O_{n+1}
\end{aligned}$$

- ▶ MC@NLO-type (subtractive matching; $\text{NLO} \oplus$): The matching subtraction formula from before.
 For the dipole shower things get particularly easy, as $\mathcal{BP} \approx \mathcal{A}$.
 For the \tilde{q} shower the emission of a single emission also reduces to that of Catani-Seymour dipoles.
- ▶ Powheg-type (multiplicative matching; $\text{NLO} \otimes$): Replace $\mathcal{BP} \approx \mathcal{R}$.
 The amount of negative events, i.e. the ones in the O_{n+1} bracket, reduces. But not completely!
 Powheg matching goes along with having to have the first emission also the hardest.
 For the \tilde{q} shower this is not the case \rightarrow truncated, vetoed shower.
 For practical reasons only the first emission is replaced by $\mathcal{BP} \approx \mathcal{R}$.

- ▶ The parton shower hard scale is limited from above: $Q = Q_\perp$ an upper limit on the transverse momentum available to the shower.
- ▶ Instead of a fixed scale, smear the hard veto scale Q_\perp by a functional profile $\kappa(Q_\perp, p_\perp)$, with p_\perp the transverse momentum of the splitting.

Known from Powhag-Box, where they use $\kappa(Q_\perp, p_\perp) = 1/(1+x^2)$, with $x = p_\perp/Q_\perp$, as hfact.

- ▶ Profile scales can be applied generically, though. We will look at it in MC@NLO matching, comparing with another profile scale choice, i.e. the resummation profile

$$\kappa(Q_\perp, p_\perp) = \begin{cases} 1 & , x \leq 1 - 2\rho , \\ 1 - \frac{(1-2\rho-x)^2}{2\rho^2} & , x \in (1 - 2\rho, 1 - \rho] , \\ \frac{(1-x)^2}{2\rho^2} & , x \in (1 - \rho, 1] , \\ 0 & , x > 1 , \end{cases} \quad (1)$$

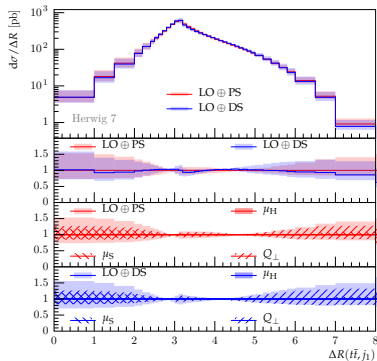
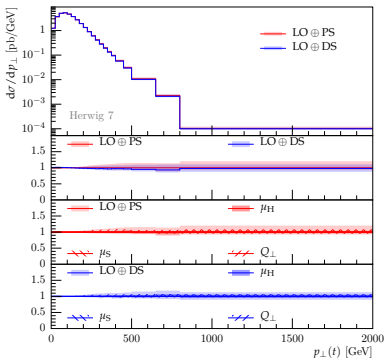
In Herwig 7.1.4 $\rho = 0.3$.

- ▶ The **resummation profile** $\rightarrow 1$ for $p_\perp < (1 - 2\rho)Q_\perp$, $\rightarrow 0$ for $p_\perp > Q_\perp$, and quadratically interpolates between these regions. It reproduces the correct towers of logarithms, and switches off the resummation smoothly towards the hard region.
- ▶ **hfact** $\rightarrow 1$ in the resummation region, for $p_\perp < Q_\perp$, \rightarrow zero in the fixed-order region, for $p_\perp > Q_\perp$. hfact does not produce the desired towers of logarithms. Not close enough to one in the Sudakov region, $p_\perp \ll Q_\perp$, and does not enforce a sufficient cutoff on the shower emissions in the hard region, for $p_\perp \gg Q_\perp$.

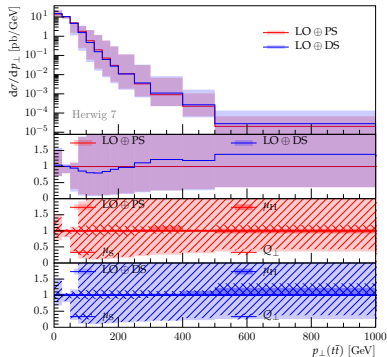
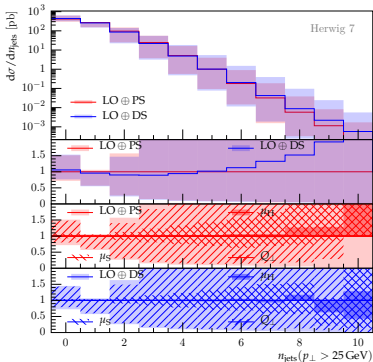
- ▶ Improved radiation in production and decay of heavy quark flavours, especially in the dipole shower.
 - ▶ MC@NLO- ($\text{NLO} \oplus$) / Powheg-type ($\text{NLO} \otimes$) matching to angular-ordered (PS) / dipole shower (DS).
 - ▶ MG5 for tree amplitudes, OpenLoops for one-loop amplitudes.
 - ▶ Variations of hard process scales (μ_H), hard veto scale (Q_\perp), shower scales (μ_S in shower $\alpha_s(\mu_S)$).
 - ▶ Choices of central scales and hard cut-off profiles (e.g. Powheg-type hfact or resummation inspired).
-
- ▶ For the shower improvements, please have a look at [arXiv:1810.06493], containing a comprehensive overview on the currently implemented details.
 - ▶ In the following focus on the uncertainty studies.
 - ▶ For the non-data-comparison scale-variation “benchmarks” we consider $t\bar{t}$ pair production at 13 TeV using parton-level predictions for stable top quarks.
 - ▶ The factorization and renormalization scales are set to the same value $\mu_R = \mu_F \equiv \mu_H$, where our default for the central hard process scale choice is

$$\mu_H = \frac{m_{\perp,t} + m_{\perp,\bar{t}}}{4},$$

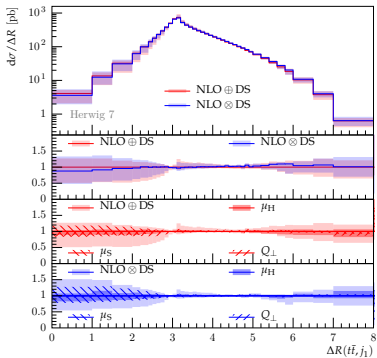
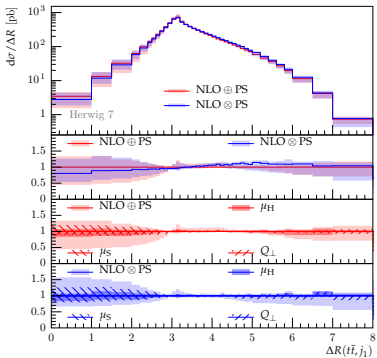
- ▶ Scale variations are by factors of 2 up and down: either combined (27-fold scale variation), or broken down. Default profile is the resummation profile.
- ▶ Further details, please see [arXiv:1810.06493] as well. Also for more data comparisons against ATLAS data. At the end I'll only show some data comparisons against CMS data (Sorry!)



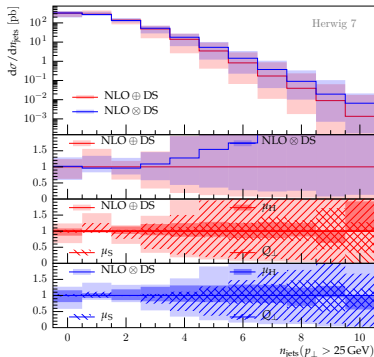
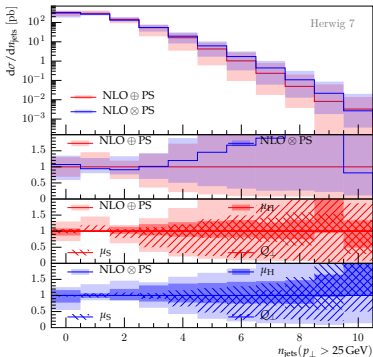
- Plot envelope of all scale variations (overall scale variations), but also breakdown into variations of μ_H , μ_S , Q_{\perp} separately
- $p_{\perp}(t)$ well described by LO ME, showers have limited impact (overall scale variation equally dominated by μ_H , μ_S , Q_{\perp})
- With only LO ME, $\Delta R(t\bar{t}, j_1)$ sensitive to showers (overall scale variation equally dominated by μ_S , Q_{\perp} below π and by Q_{\perp} only above π)
- Both showers describe similar distributions for $p_{\perp}(t)$ and $\Delta R(t\bar{t}, j_1)$



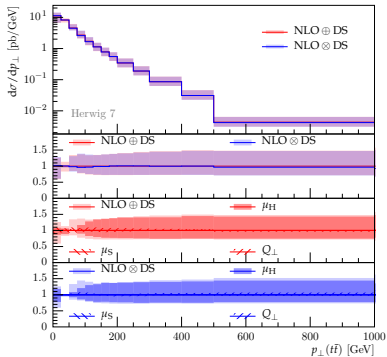
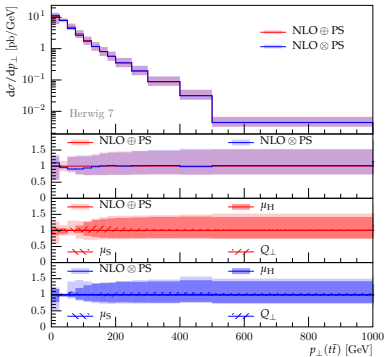
- ▶ With only LO ME, $n_{\text{jets}} (p_{\perp} > 25 \text{ GeV})$ sensitive to showers (Q_{\perp} , μ_S have increasing effect with increasing n_{jets} ; Q_{\perp} dominates), DS predicts more many-jet events
- ▶ With only LO ME, $p_{\perp}(t\bar{t})$ very sensitive to showers (Q_{\perp} dominates), DS predicts more high- $p_{\perp}(t\bar{t})$ events



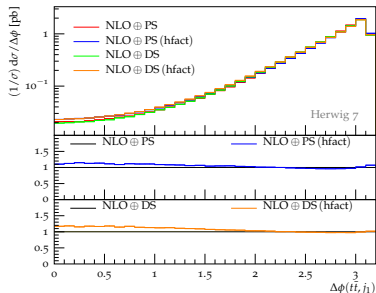
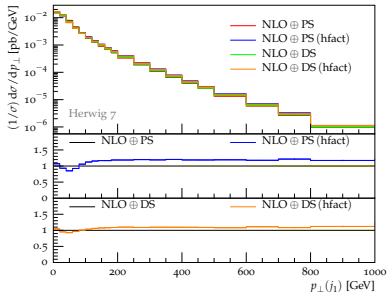
- NLO matched, $\Delta R(\bar{t}l, j_1)$ probes hard process and parton shower (above π already described by only NLO ME; largest uncertainty below π from μ_S), for both showers both matchings describe similar distributions



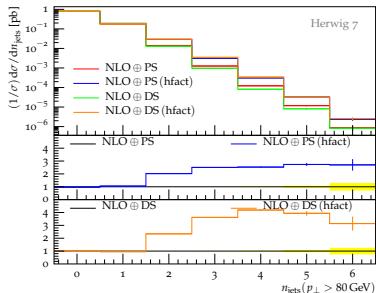
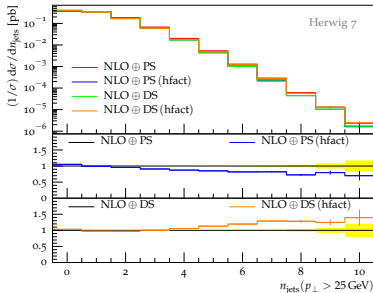
- NLO matched, $n_{\text{jets}}(p_{\perp} > 25 \text{ GeV}) = 0, 1$ formally accurate to NLO; for both showers both matchings agree up to 3 jets, above 3 jets Powheg-type matching predicts more many-jet events; Overall Q_{\perp} significant, but μ_S and μ_H contribute visibly (in NLO \otimes PS μ_S seems more pronounced)



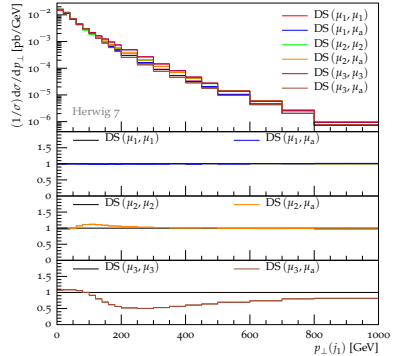
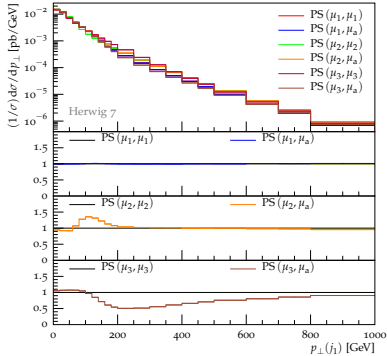
- NLO matched, much improved overall scale uncertainties in $p_{\perp}(t\bar{t})$, as the showers have a smaller impact (exclusively dominated by μ_H); for both showers both matchings agree well



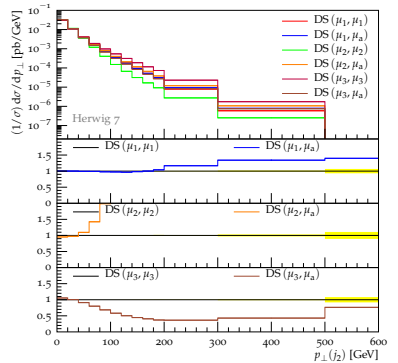
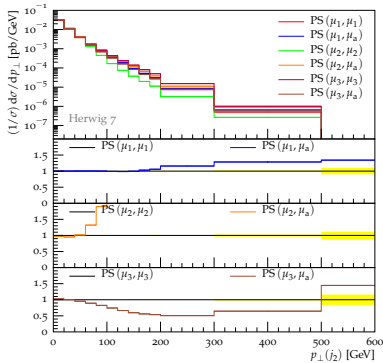
- ▶ Looking at different cut-off profiles of the hard veto scale Q_{\perp} in MC@NLO-type matching, one reminiscent of hfact in PowhegBox, one inspired by resummation
- ▶ In $p_{\perp}(j_1)$ hfact overshoots resummation for high p_{\perp}
- ▶ In $\Delta\phi(t\bar{t}, j_1)$ hfact overshoots resummation for low $\Delta\phi$
- ▶ So, comparing hfact and resummation inspired for $p_{\perp}(j_1)$ we have $\sim 20\%$ effects at high $p_{\perp}(j_1)$



- ▶ Looking at different cut-off profiles of the hard veto scale Q_{\perp} in MC@NLO-type matching, one reminiscent of hfact in PowhegBox, one inspired by resummation
- ▶ In $n_{\text{jets}}(p_{\perp} > 25\text{GeV})$ for large n_{jets} (shower described) hfact undershoots resummation with PS, and overshoots it with DS
- ▶ In $n_{\text{jets}}(p_{\perp} > 80\text{GeV})$ hfact always overshoots for large n_{jets}
- ▶ So, comparing hfact and resummation inspired for $n_{\text{jets}}(p_{\perp} > 25\text{GeV})$ we have $\sim 20\%$ effects, while for $n_{\text{jets}}(p_{\perp} > 80\text{GeV})$ we observe $\sim 300\%$ effects in shower described regions

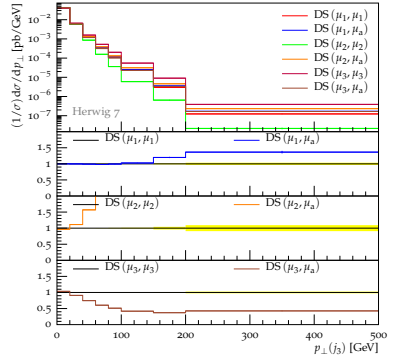
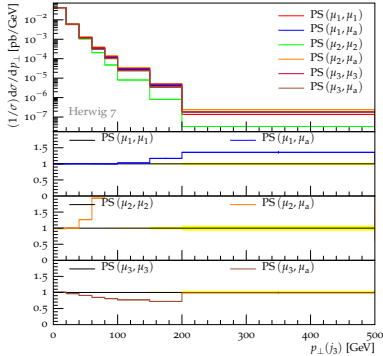


- ▶ Hard veto scale effects: vary choices of μ_H , while looking at different choices for Q_\perp , either $(\mu_H, Q_\perp = \mu_H)$ or $(\mu_H, Q_\perp = \mu_a)$
- ▶ $\mu_H = \mu_1 = (m_{\perp,t} + m_{\perp,\bar{t}})/2$, or $\mu_H = \mu_2 = (m_{\perp,t} + m_{\perp,\bar{t}})/4$, or $\mu_H = \mu_3 = m_{t\bar{t}}$
- ▶ $\mu_a^2 = (\sum_{i \in n_{\text{out}}} m_{\perp,i}^2)/n_{\text{out}}$, taking into account the extra emission in the hard event, giving a more sensible choice for the shower start scale for a larger range of real-emission p_\perp 's
- ▶ Here, $p_\perp(j_1)$. j_1 is described correctly at NLO.

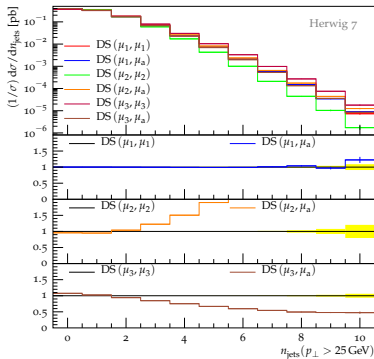
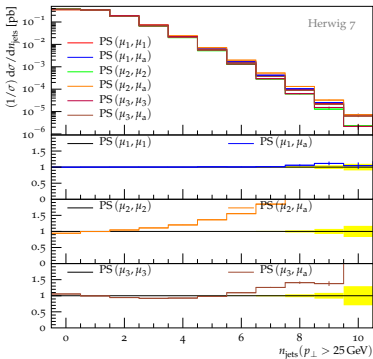


- Hard scale and hard veto scale effects, $p_{\perp}(j_2)$
- Major effects. j_2 is produced from the shower.

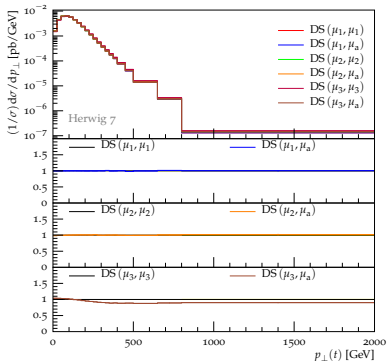
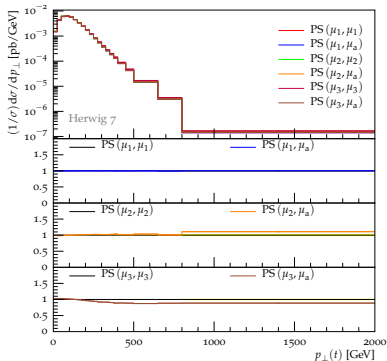
[arXiv:1810.06493]



- ▶ Hard scale and hard veto scale effects, $p_{\perp}(j_3)$
- ▶ Major effects. Also j_3 is produced from the shower.

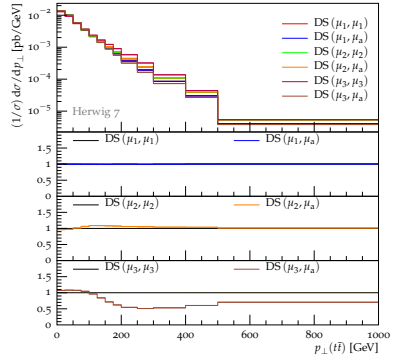
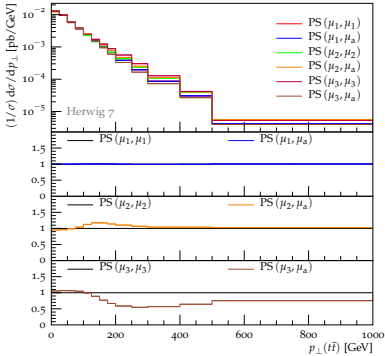


- Hard scale and hard veto scale effects, major effects on $n_{\text{jets}}(p_{\perp} > 25 \text{ GeV})$, especially with DS

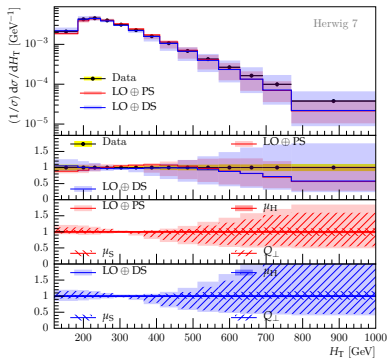


► Hard scale and hard veto scale effects, $p_{\perp}(t)$

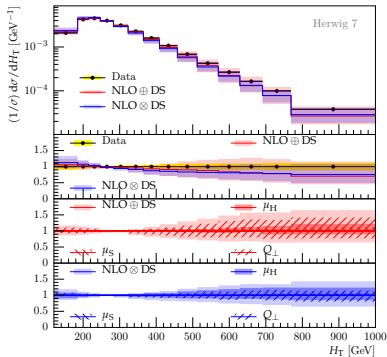
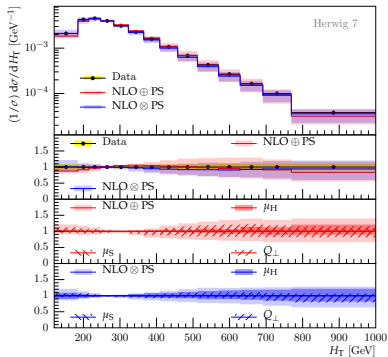
[arXiv:1810.06493]



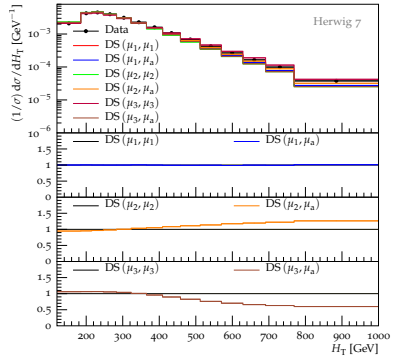
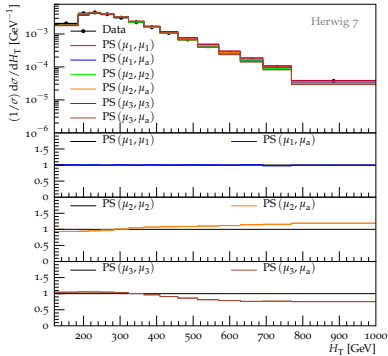
► Hard scale and hard veto scale effects, $p_{\perp}(t\bar{t})$



- ▶ H_{\perp} distribution measured in semileptonic 8 TeV $pp \rightarrow t\bar{t}$ events by CMS [arXiv:1607.00837]
- ▶ H_{\perp} = sum of all jet p_{\perp} 's
- ▶ Plot envelope of all scale variations (overall scale uncertainties), but also breakdown into variations of μ_H , μ_S , Q_{\perp} separately
- ▶ LO + shower description: above 300 GeV sensitive to showers (Q_{\perp} variation dominates, μ_S variation visible), PS and DS predict similar rates (except in the lower bins), DS has larger overall uncertainties towards high- H_{\perp} events (driven by Q_{\perp} variation; reflecting the difference in phase space of the two showers)



- ▶ H_{\perp} distribution measured in semileptonic 8 TeV $pp \rightarrow t\bar{t}$ events by CMS [arXiv:1607.00837]
- ▶ NLO + shower description: better uncertainties and description of data in both showers with both matching variants, DS undershoots a bit towards higher H_{\perp} , but has slightly larger overall uncertainties
- ▶ In both showers the overall uncertainties with MC@NLO-type are larger than with Powheg-type matching, but in neither there is a clear single dominant source of uncertainty (in NLO⊕DS the Q_{\perp} variation seems more prominent, though)



- H_\perp distribution measured in semileptonic 8 TeV $pp \rightarrow t\bar{t}$ events by CMS [arXiv:1607.00837]
- Hard veto scale effects: vary choices of μ_H , while looking at different choices for Q_\perp , either $(\mu_H, Q_\perp = \mu_H)$ or $(\mu_H, Q_\perp = \mu_a)$
- $\mu_H = \mu_1 = (m_{\perp,t} + m_{\perp,\bar{t}})/2$, or $\mu_H = \mu_2 = (m_{\perp,t} + m_{\perp,\bar{t}})/4$, or $\mu_H = \mu_3 = m_{\bar{t}}$
- $\mu_a^2 = (\sum_{i \in n_{\text{out}}} m_{\perp,i}^2)/n_{\text{out}}$, taking into account the extra emission in the hard event, giving a more sensible choice for the shower start scale for a larger range of real-emission p_\perp 's

Thank you!