Tests of general relativity: From waveform models to propagation tests to future prospects

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Outline

- Waveform modelling: Need accurate models for waveforms in GR to test for deviations and models in alternate theories to bound specific deviations
- Combining together constraints from multiple events
- Propagation tests: Dephasing of waveforms and comparison with EM
- Polarization tests: Use 3+ detectors to constrain deviations from purely tensor polarizations
- Black hole mimickers: Use tidal and/or rotational deformabilities to constrain possible massive, compact objects that could produce signals similar to black holes.
- Future prospects (3G and LISA)

Overview of waveform modelling in GR and beyond

- Current status of waveform models for BBH and BNS.
  NSBH not considered here explicitly, as they have not yet been observed, and do not have quite as well-developed waveform models.
- Ongoing work on improving these waveform models.

Motivation for waveform modelling

- NR simulations are expensive.
  - Can only run them for selected points in parameter space
  - Can't run them for long enough to cover the entire (design sensitivity) Advanced LIGO band (down to 10 Hz) except for high-mass systems.

    [Current longest NR BBH simulation (Szilágyi et al. PRL 2015) covers AdvLIGO band for total masses > 44 Msun; most only cover AdvLIGO band for masses > 65 Msun or higher]

- Have to design fast-to-evaluate models to interpolate between NR simulations and include as much physics from perturbation theory calculations (post-Newtonian and black hole/neutron star perturbation theory and self-force calculations) as possible.

General frameworks for waveform modelling

- Effective-one-body formalism: Physics-inspired resummations of perturbation theory results, with free coefficients tuned to NR simulations. Solve ODEs for dynamics and obtain waveform in the time domain.

  Relatively slow, though there are ways of interpolating these in the frequency domain (reduced order models) and recent work (Nagar and Rettegno, arXiv 2018) that gives much faster evolutions, at least in the aligned-spin quasicircular case.

- Phenom models: Natively frequency-domain model. Add phenomenological parameters to PN and BHPT results and calibrate by fits to FFTs of hybrids of PN/uncalibrated EOB and NR waveforms. Model precession by "twisting up" aligned-spin waveform using PN precession expressions. Relatively fast to evaluate.

- Surrogate models or Gaussian process regression: Directly interpolate NR waveforms. Can achieve accuracies comparable to those of the waveforms used for training, but requires a large number of waveforms, and thus covers a narrow parameter space than the other models.

Waveforms used for current tests of GR

- All current waveform-based tests of GR are carried out with the precessing Phenom model (in its extension to include tidal effects for binary neutron stars).
- The aligned-spin EOB model is used as a consistency check where possible.

Current and planned improvements to waveform models in GR

- Aligned-spin waveform models including higher modes now exist; these are now being extended to model precession. (Surrogate models already include higher modes in a restricted region of parameter space.)
- Work on modelling small-to-moderate eccentricity binary black holes continues, both on the PN side as well as several initial NR-tuned waveform models.
- There are some very preliminary models for BNS post-merger, but nothing ready for use in data analysis.
- There is ongoing work to push BBH simulations to higher spins and more extreme mass ratios. However, significantly extending beyond the current mass ratio record of 18:1 (for accurate simulations) likely requires significant new ideas.

Combining together constraints from multiple events

How to combine together multiple events?

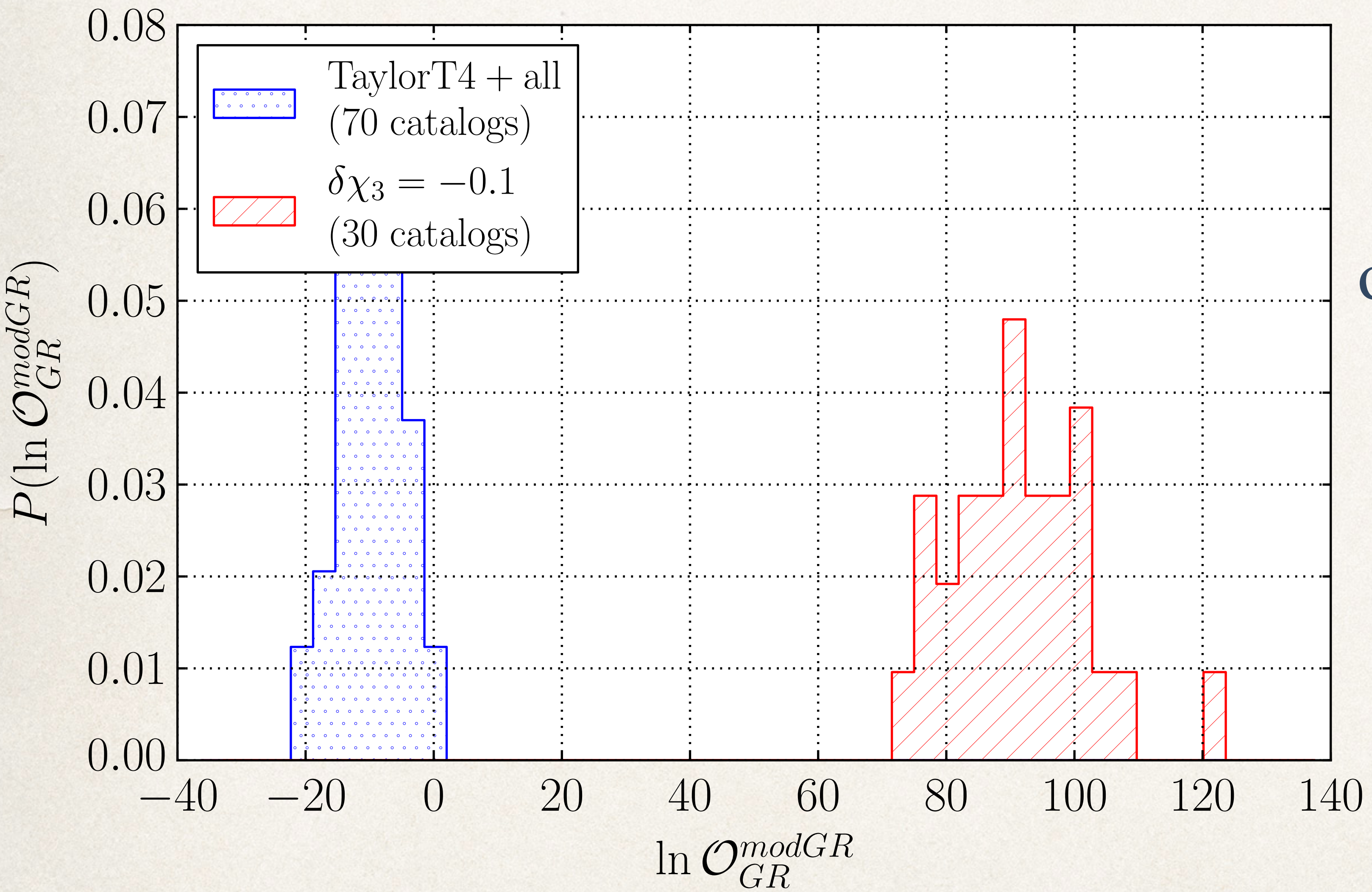
- When testing a given theory, where one knows the deviations one expects for a given binary, one can simply combine posteriors on the parameters of the theory.

- Alternatively, when making consistency tests, one can combine together Bayes factors for GR versus non-GR.

- One can even simply combine together the posteriors for the deviations in consistency tests—this is what is done in current LVC analyses.

  Studies show that it is possible for tests to pick up on GR violations this way, though they are not guaranteed to for an arbitrary violation.

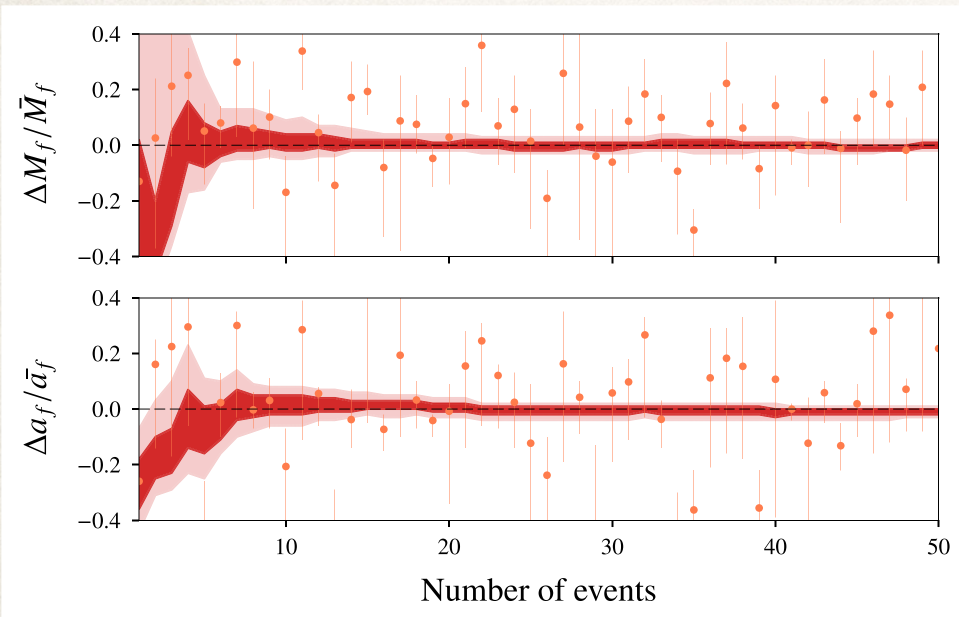
Efficacy of combining together events: Bayes factors

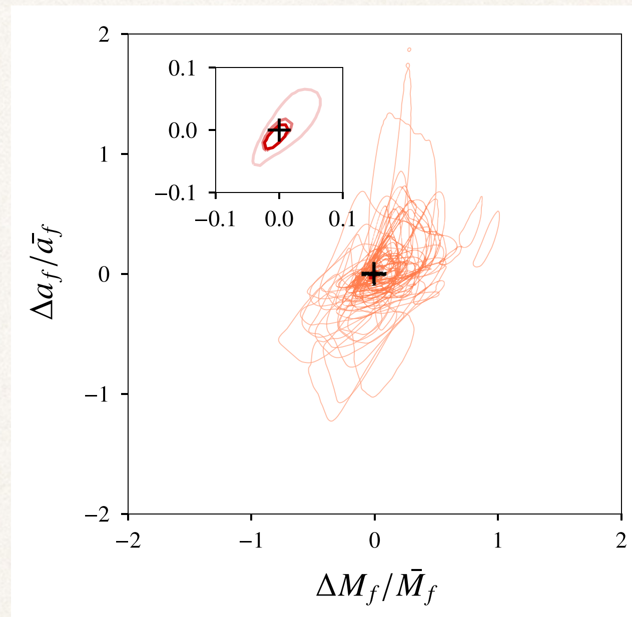


Post-Newtonian constraints from Agathos et al. PRD 2014, showing how one can obtain strong evidence against GR when combining together Bayes factors if the data are described by a deviation from GR.

Here a catalog consists of 15 events

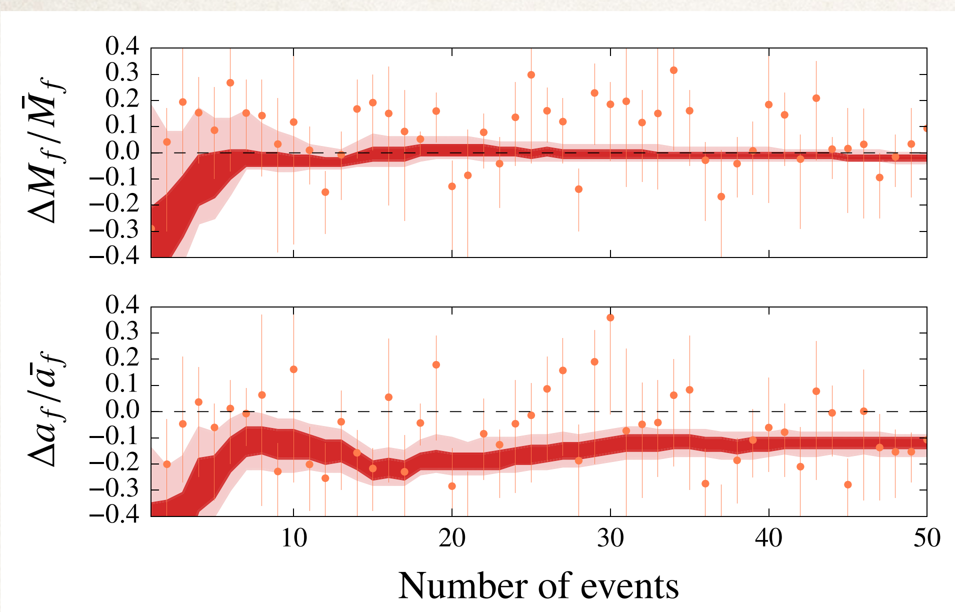
Efficacy of combining together events: Posteriors

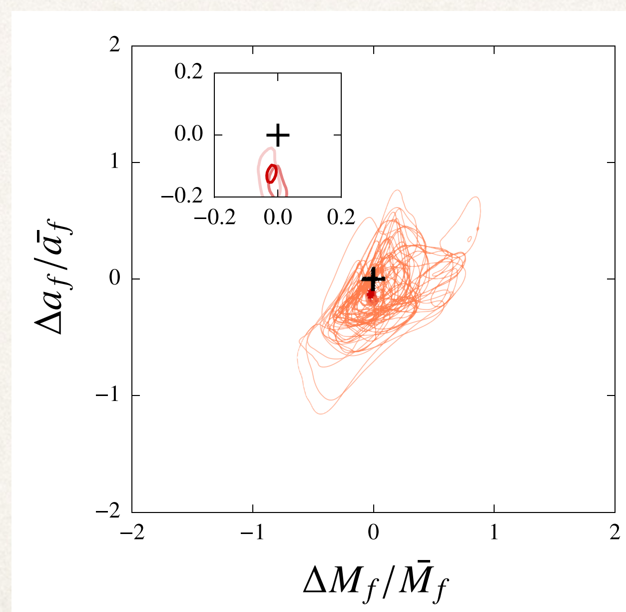




GR

Results from Ghosh et al. CQG 2018





modified GR [EOB waveforms with some higher modes in the energy flux scaled by a constant]

Have to worry about waveform systematics (with current waveforms) when combining together ~100 events

Propagation tests

Motivation

- The large distances to which we can detect GWs from compact binaries, compared to the size of the source, means that it is possible to put stringent constraints on modifications to the propagation of gravitational waves.
- In particular, it is possible to test modified dispersion and birefringence with a single GW detection; constraining a modified speed of propagation with no dispersion requires an EM counterpart.

  An EM counterpart or population of sources is also required for constraints on modifications to the $1/d_L$ falloff with distance.
- However, possible to perform a direct Rømer delay-style measurement of GW speed using a continuous GW source [Finn and Romano, PRD 2013].

Constraints on a modified dispersion relation

- The LVC has placed constraints on a modified dispersion relation of the form

$$E^2 = p^2c^2 + Ap^\alpha c^\alpha$$

  [Mirshekari, Yunes, and Will, PRD 2012]

- This reduces to the massive graviton dispersion relation for $\alpha = 0$, $A > 0$, and gives the leading contributions from multi-fractal spacetime ($\alpha = 2.5$), doubly special relativity ($\alpha = 3$), and Hořava-Lifshitz and extra dimensional theories ($\alpha = 4$). $\alpha = 4$ also corresponds to the leading non-birefringent contribution in the Standard Model Extension.

  $\alpha = 2$ gives a nondispersive modification of the speed of gravitational waves.

Effects on GW phasing of a modified dispersion relation

- Mirshekari, Yunes, and Will (MYW) obtain the frequency-domain dephasing associated with this dispersion relation for PN waveforms using the stationary phase approximation (SPA), obtaining

$$\delta\Psi = \begin{cases} \dfrac{\pi}{\alpha - 1} \dfrac{A D_\alpha}{(hc)^{2-\alpha}} \left[\dfrac{(1+z)f}{c}\right]^{\alpha-1} & \alpha \neq 1 \\ \dfrac{\pi A D_\alpha}{hc} \ln\left(\dfrac{\pi G \mathcal{M}^{\mathrm{det}} f}{c^3}\right) & \alpha = 1 \end{cases} .$$

$$D_\alpha = \frac{(1+z)^{1-\alpha}}{H_0} \int_0^z \frac{(1+z')^{\alpha-2}}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}} dz',$$

- They also use the "particle velocity"

$$v_p/c = pc/E = 1 - A E^{\alpha-2}/2 + O(A^2)$$

in the derivation.

Effects on GW phasing of a modified dispersion relation

- The LVC implementation applies this dephasing to IMR waveforms, where the SPA is not necessarily applicable.
- However, it is possible to derive this dephasing without using the SPA by starting from the PDE associated with the dispersion relation (a nonlocal PDE involving the fractional Laplacian when α is not an even integer).

  This gives the result one would obtain using the group velocity

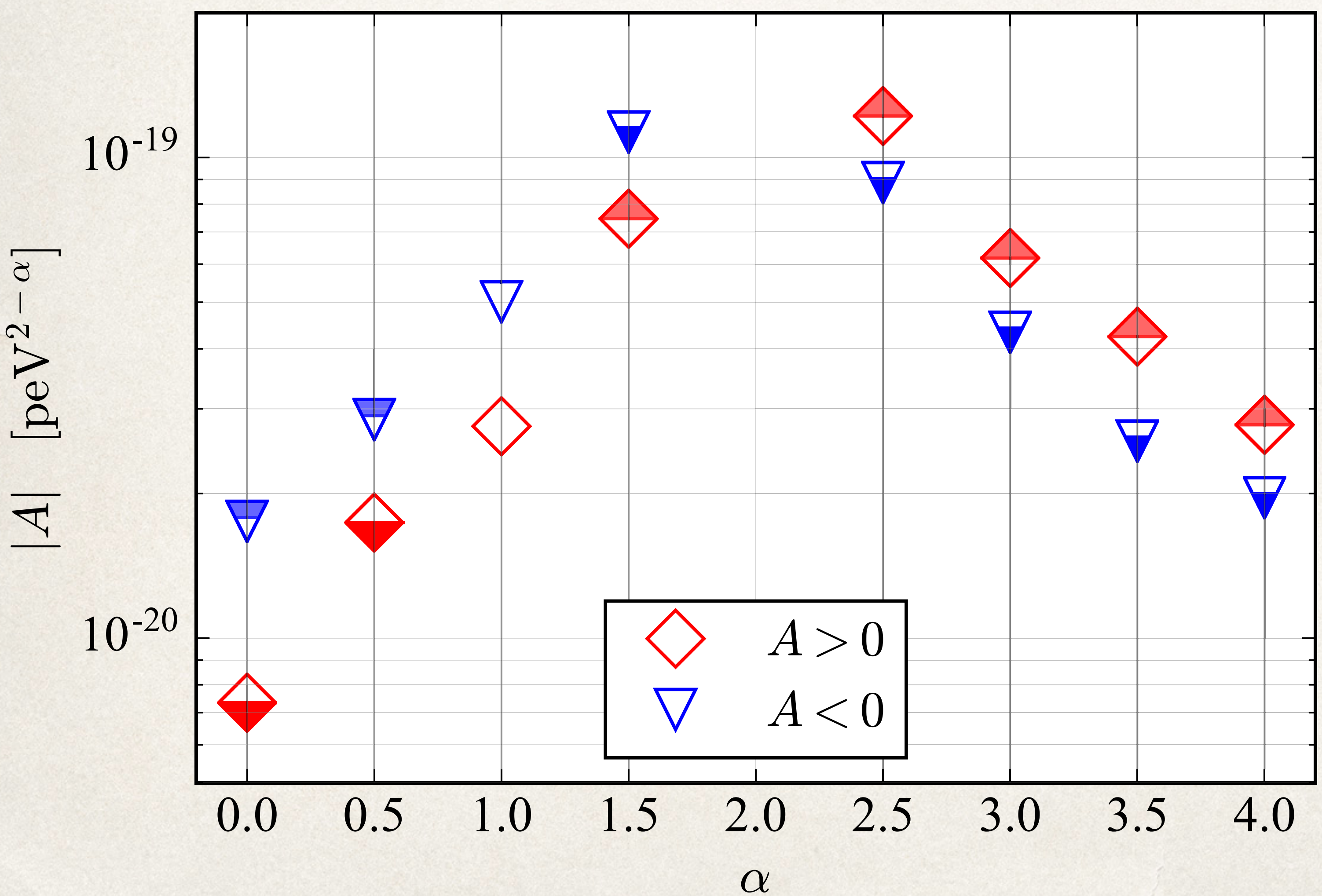
  $$v_g/c = (dE/dp)/c = 1 + (\alpha - 1)AE^{\alpha-2}/2 + O(A^2)$$

  in the MYW derivation instead of the particle velocity, which corresponds to rescaling the bounds on A by a factor of $1/(1 - \alpha)$ for $\alpha \neq 1$ and an unobservable constant dephasing for $\alpha = 1$.
- The constraints on the length scale associated with the dispersion relation are much larger than the size of the binary for $\alpha < 2$. However, they are much *smaller* than the size of the binary for $\alpha > 2$. Thus, one has to posit a screening mechanism for the GR + propagation dephasing waveform model to make sense in these cases.

  As discussed in Perkins and Yunes [arXiv 2018] in the massive graviton context, if the screening length is a significant fraction of the distance to the source, then this could affect the constraints (assuming that the waveform is not dephased significantly in regions where the potential is screened).

Results from GW detections

- This test was first applied to GW data in the GW170104 paper [LVC PRL **118**, 221101 (2017)].



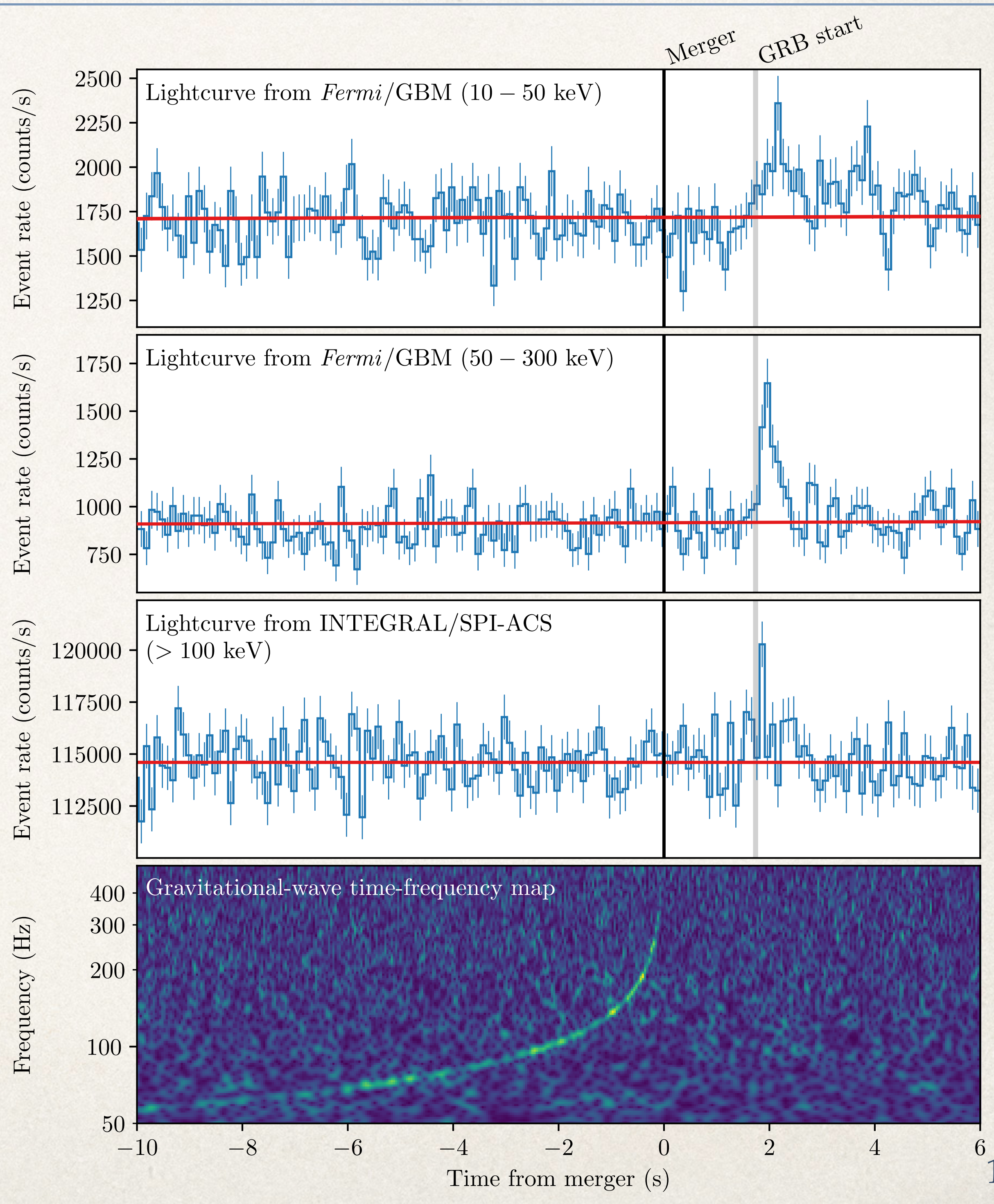
Combined constraints from GW150914, GW151226, and GW170104.

In almost all cases the strongest individual constraints come from GW170104, due to its larger distance.

GW170817 doesn't give improved constraints, since it is relatively close

GW + EM propagation constraints

- The time delay of 1.74 ± 0.05 s between GW170817 and GRB170817A gave several new constraints on propagation [LVC + Fermi + INTEGRAL, ApJL **848**, L13 (2017)]:
    - Speed of propagation of GWs
    - Shapiro delay of GWs
    - Constraints on SME coefficients
- Additionally, the EM distance estimate to the host galaxy allowed for constraints on the number of dimensions from the amplitude of the GWs [Pardo et al. JCAP 2018; LVC arXiv:1811.00364].



GW propagation speed constraint

- Since the timescales associated with launching the GRB after a BNS merger are much smaller than the propagation time of the signals to Earth, one can place stringent constraints on the propagation speed of GWs even with relatively weak assumptions about the astrophysics involved in launching the GRB.

- To obtain the constraints of

v_{GW} - v_{EM}

$$-3 \times 10^{-15} \leqslant \frac{\Delta v}{v_{\mathrm{EM}}} \leqslant +7 \times 10^{-16}$$

quoted in LVC + Fermi + INTEGRAL, ApJL **848**, L13 (2017), the GRB was assumed to be launched from 0 to 10 s after the merger.

These bounds are weakened by two orders of magnitude if one allows for more extreme scenarios, where the could be emitted up to ~100 s before the merger (if it came from crust cracking), or ~1000 s after it, in the case of a long-lived hypermassive neutron star.

GW Shapiro delay constraint

- As was first appreciated for EM and neutrino signals for SN1987A, there is an appreciable Shapiro delay (> 50 days) due to propagating through the Milky Way's gravitational potential.

- One can phenomenologically assume that there are different parameterized post-Newtonian parameters for GW and EM propagation in the Shapiro delay expression

$$\delta t_{\mathrm{S}} = -\frac{1+\gamma}{c^3}\int_{\boldsymbol{r}_{\mathrm{e}}}^{\boldsymbol{r}_{\mathrm{o}}} U(\boldsymbol{r}(l))\,dl,$$

and constrain the difference.

Newtonian potential

- One obtains

$$-2.6\times 10^{-7} \leqslant \gamma_{\mathrm{GW}} - \gamma_{\mathrm{EM}} \leqslant 1.2\times 10^{-6}.$$

using the same 0 to 10 s intrinsic time delay as well as conservative bounds on the mass of the Milky Way.

GW Shapiro delay constraint

- Note, however, that in discussions with Olivier Minazzoli on how to extend these constraints to include the contributions of all nearby galaxies, we realized that one really needs to use cosmological perturbation theory to perform this calculation, even for a relatively close source like GW170817/GRB170817A.

- The constraints we have deduced using the Milky Way's Newtonian potential are likely still conservative, but should be revisited.

Polarization tests

Work to date

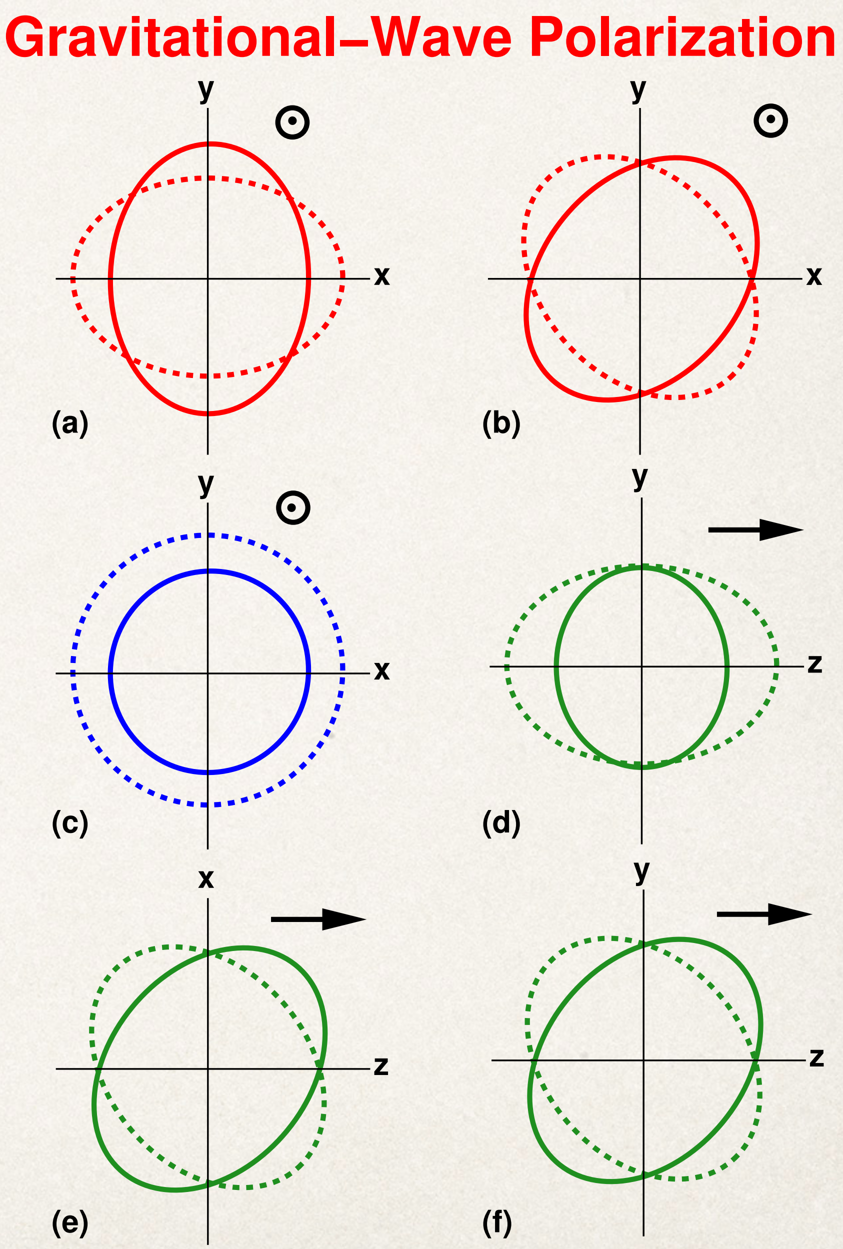
- The two LIGO detectors have very similar orientations => see almost the same polarization.

  Thus, not possible to make strong constraints on alternative polarizations using compact binary signals until Virgo came online.

  Illustrated in the GW150914 testing GR paper using Bayes factors between the plus and cross polarizations and the scalar breathing mode using unmodeled BayesWave reconstructions. These Bayes factors do not favour either model.

- With Virgo, one can distinguish between purely tensor and purely scalar or purely vector polarizations.

  However, one needs more detectors to distinguish between mixed polarizations using compact binary signals. See, e.g., Isi and Weinstein arXiv 2017.



GW polarizations in a general theory from Will LRR 2014

Polarization test results

- GW170814 [LVC PRL **119**, 141101 (2017)] provided the first constraints on purely vector or purely scalar GWs: Bayes factors in favour of purely tensor GWs of > 200 (vs vector) and 1000 (vs scalar), respectively.
- The precise EM sky location of GW170817 gives much stronger results [LVC arXiv:1811.00364]: Bayes factors of $\sim 10^{21}$ (vs vector) and 10^{23} (vs scalar), respectively.
- Both of these constraints are obtained using standard GR waveform models and non-tensorial detector responses.

Polarization tests: Continuous waves and stochastic backgrounds

- Due to the Earth's rotation, it is possible to disentangle additional polarization states with only two detectors for long-lived signals.

- This has been studied for continuous wave signals in Isi et al. PRD 2015 and Isi, Pitkin, and Weinstein PRD 2017. Additionally, LVC PRL **120**, 031104 (2018) places upper bounds on GWs from known pulsars allowing for generic polarizations using O1 data.

- There is a similar study for stochastic backgrounds in Callister et al. PRX 2017; LVC PRL **120**, 201102 (2018) presents upper bounds from O1.

Constraints on black hole mimickers

Motivation

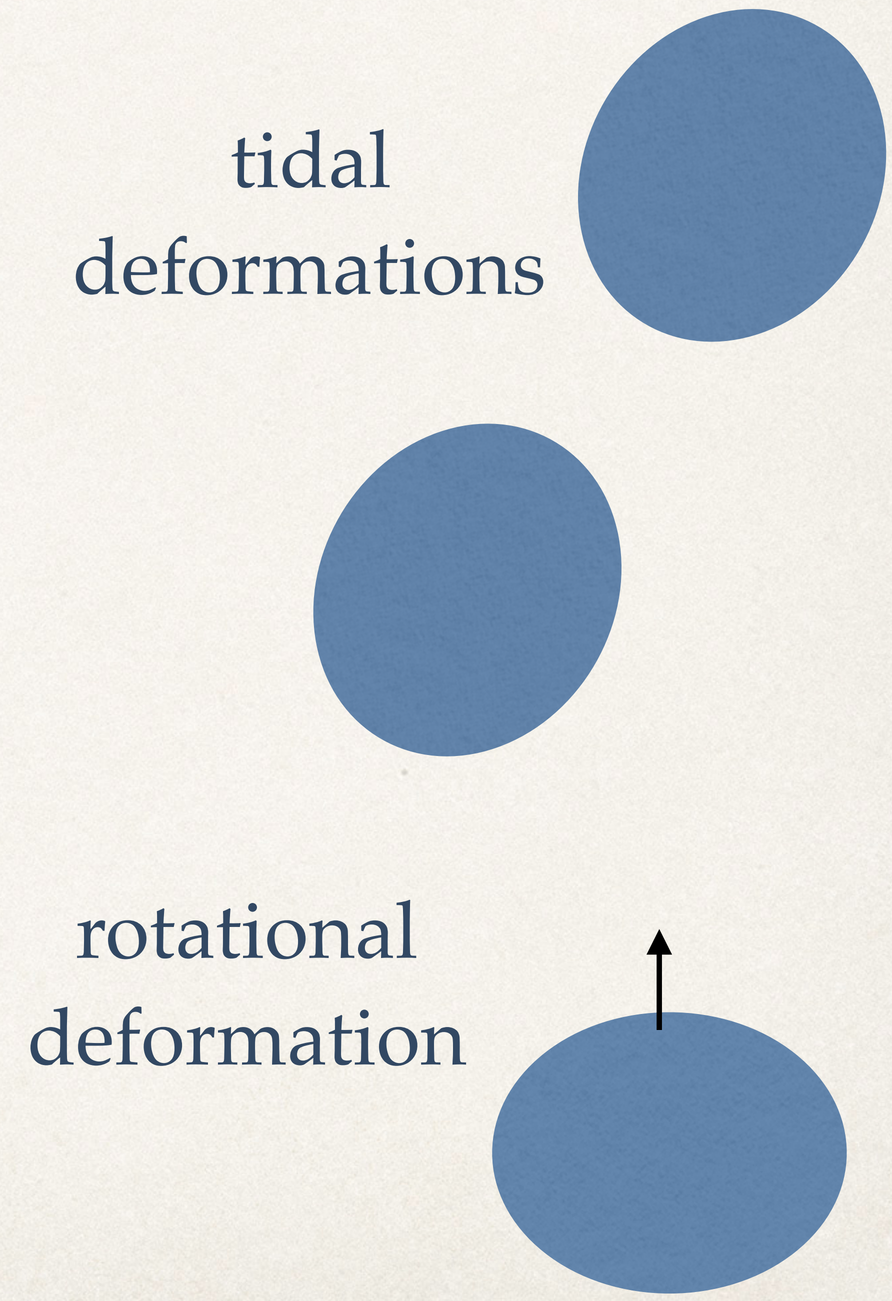
- There could be dark, massive, compact objects that are similar enough to black holes that binaries of such objects could produce the GW signals identified as coming from binary black holes.
- The best-motivated such objects are boson stars, which are described by the Einstein equations coupled to a complex scalar field.
- Other possibilities are dark matter stars or (to be quite speculative) gravastars, which have a de Sitter interior surrounded by a shell of matter.
- Any of these non-black hole objects' gravitational fields will have different responses to spin and tidal perturbations than black holes do.

Tidal and rotational deformabilities

- In the Newtonian picture, the applied tidal field induces a change to the body’s gravitational field, which is described by an infinite set of coefficients, one for each multipole moment, which depend upon the body’s internal structure. These first enter the waveform at (formal) 5PN order; actually Newtonian order. They are zero for black holes.

- In the relativistic case, this split is not so clean (see Gralla CQG 2018), and there is an ambiguity in going from the standard calculation of the Love number to the contributions. However, this should only be significant for objects very close to black holes.

- For rotating objects, there is no such ambiguity, as one can read off the multipoles at infinity, and one again has an infinite set of coefficients encoding how the body’s gravitational field responds to rotation. These first enter the waveform at 2PN and are all unity (by definition) for black holes.



Projected constraints from tidal deformations

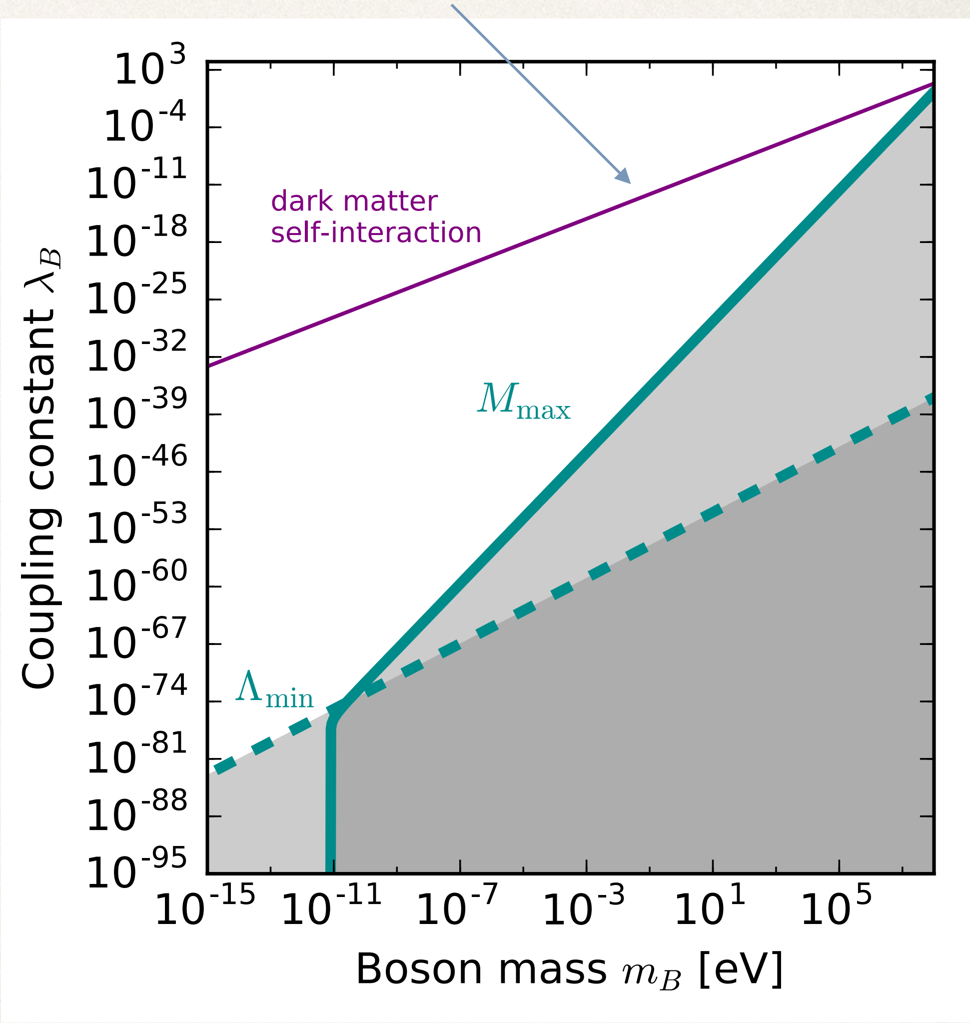
- For the case of tidal deformations, NKJ-M et al. arXiv 2018 used a waveform model given by adding the post-Newtonian tidal phasing to a frequency-domain binary black hole waveform model, to improve the point particle description of the waveform.

- Since this model will not describe the merger phase accurately, they consider various cutoff frequencies in computing the likelihood integral, and choose the highest one that is still below the contact frequency they estimate from the, assuming polytropic stars, as a simple model.

- They find that observations of binary black holes like the signals LIGO has seen so far in O3 will allow one to rule out noninteracting boson stars as and constrain the parameter space for boson stars with $\lambda_B|\phi|^4$ a interaction.

[These are obtained using the polytropic star results, but are likely representative of what one would obtain with an analysis using a binary boson star contact frequency calculation.]

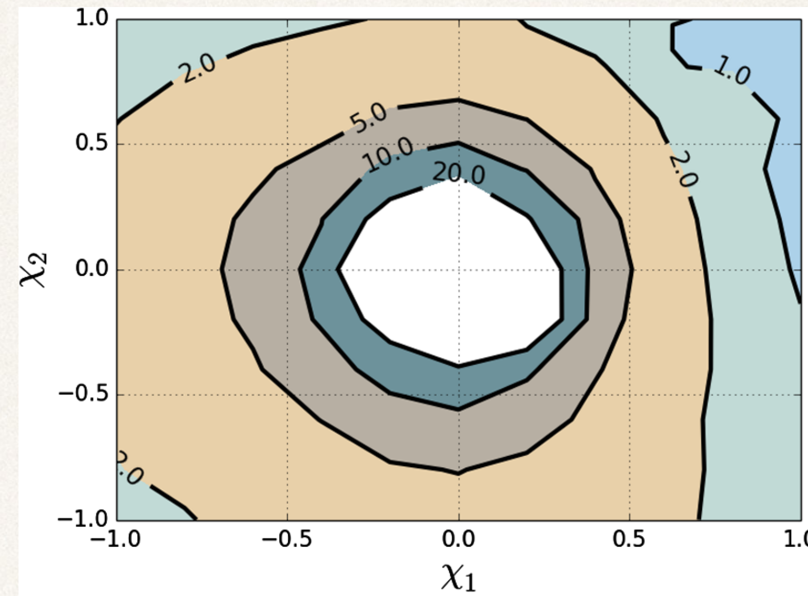
Dark matter self-interaction cross-section of 0.1–1 cm^2/g, needed to account for various observations



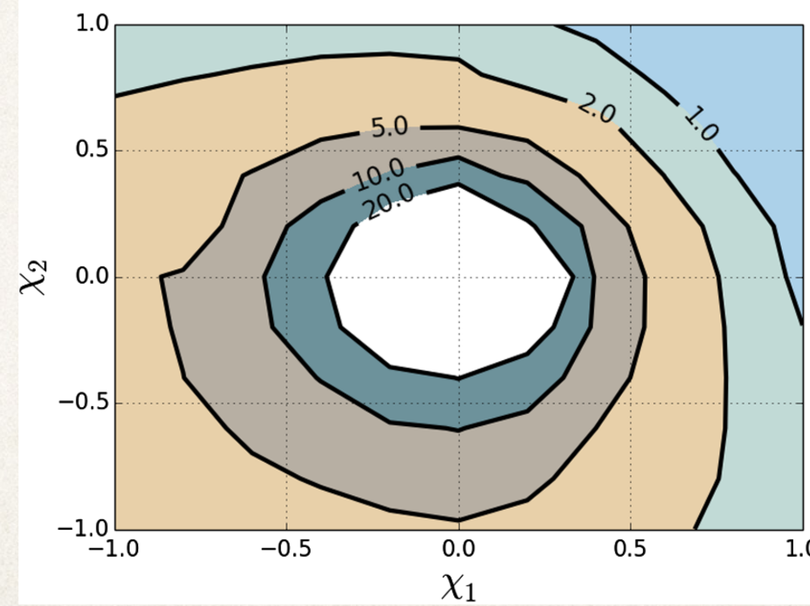
Projected constraints from rotational deformations

- For the case of rotational deformations, Krishnendu, Arun, and Mishra PRL 2017 use a PN waveform (including higher harmonics) and the Fisher matrix to obtain estimates of the accuracy with which they can measure the rotational deformations in the high-SNR limit.

- They only consider the quadrupole deformabilities $\kappa_{1,2}$ and find that the individual deformabilities are not measured very precisely, but the symmetric combination $\kappa_s = (\kappa_1 + \kappa_2)/2$ is measured fairly accurately. $\kappa_s = 1$ for a binary black hole, so one can use deviations from.



$5+4\ M_\odot$



$10+9\ M_\odot$

both for an SNR of 10 in Advanced LIGO

Future work

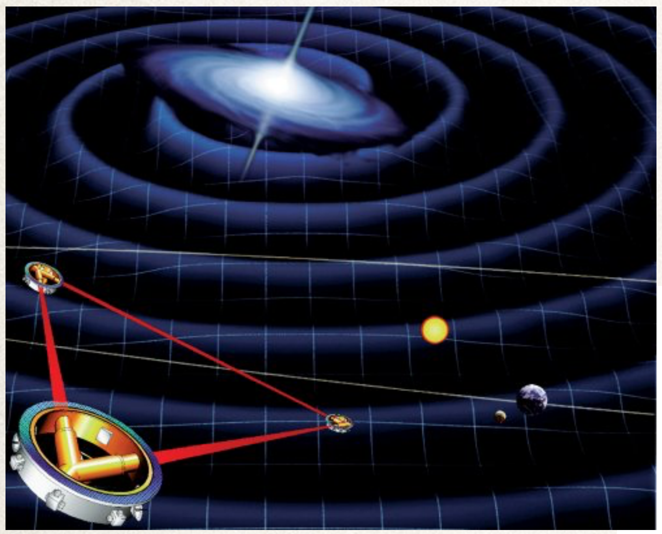
- Since one expects both rotational and tidal deformations to be important, the next step is to combine these analyses together (and include rotational-tidal couplings, computed in, e.g., Abdelsalhin, Pani, and Gualtieri, PRD 2018 and Landry, arXiv 2018).

- After this has been completed, it will be time to consider the constraints that can be placed using real data.

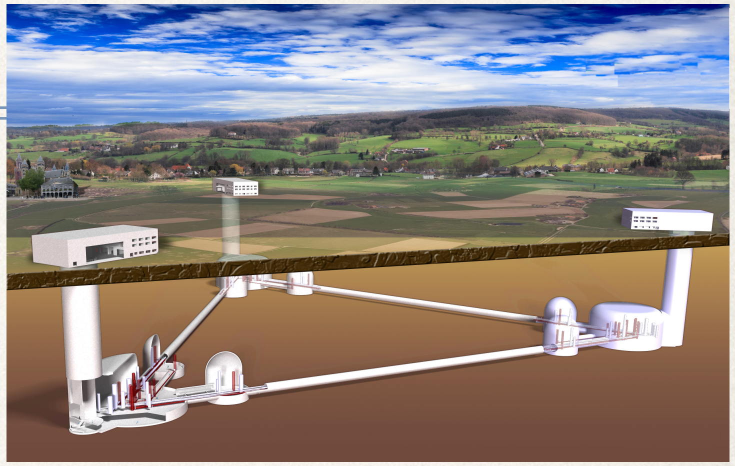
Future detectors

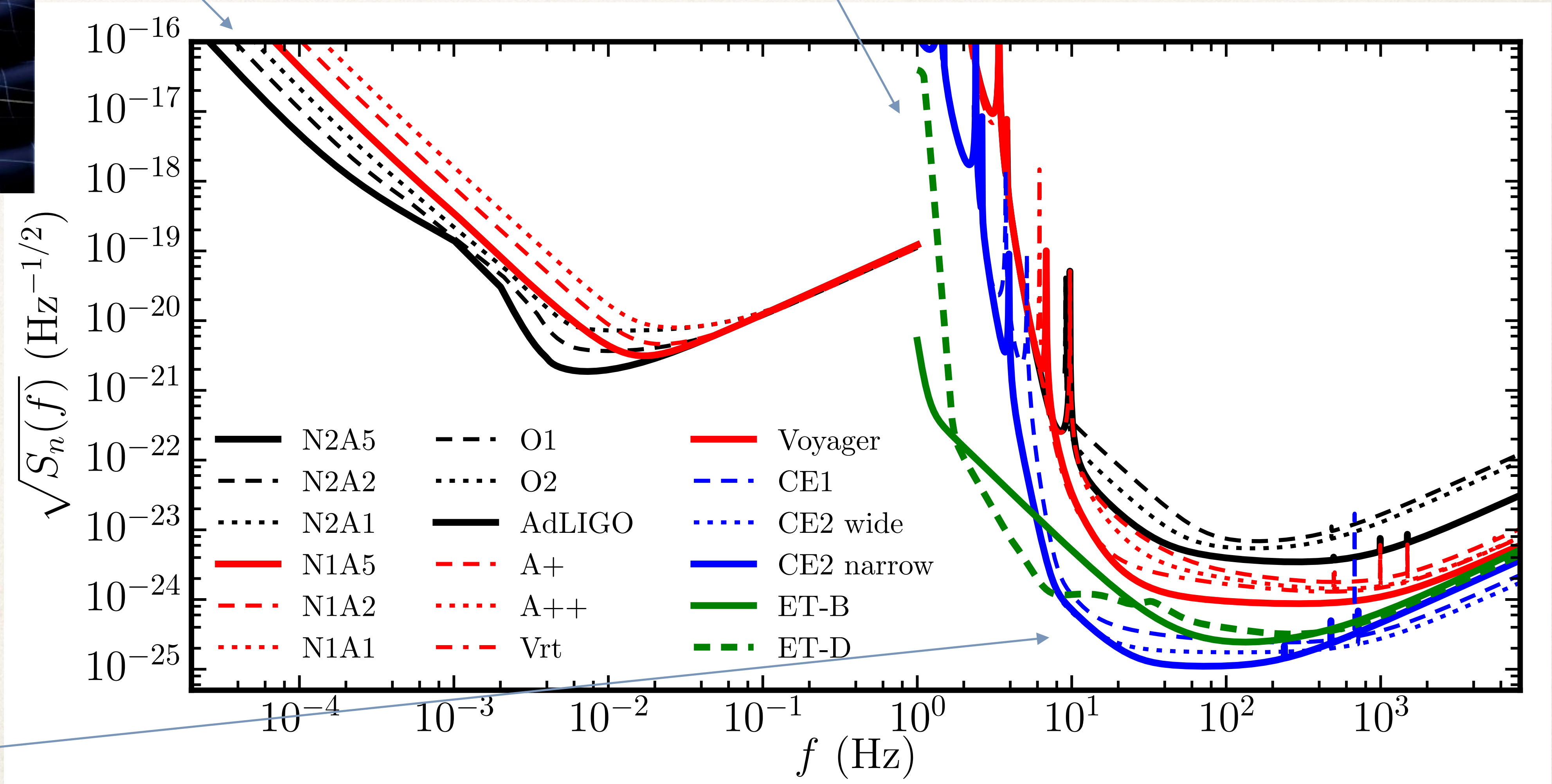
Planned and proposed future detectors

LISA



Einstein Telescope





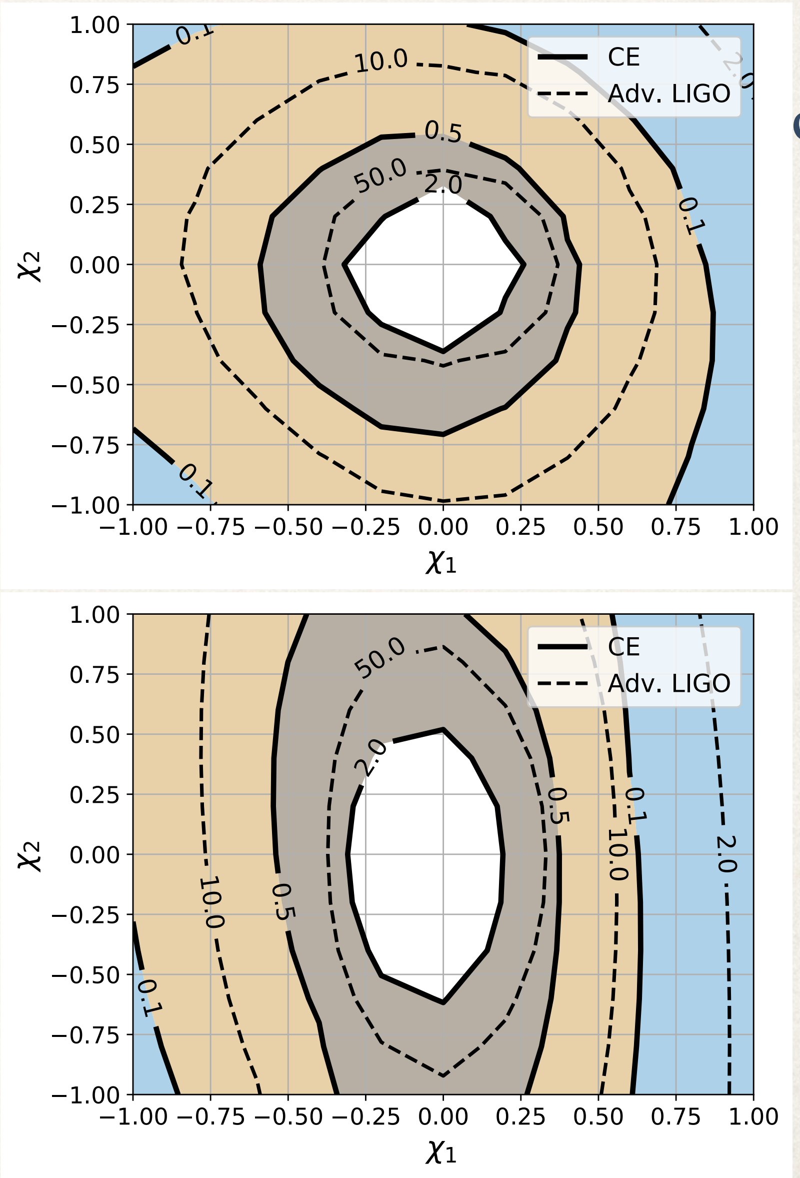
Cosmic Explorer

Figure from Berti et al. PRL **117**, 101102 (2016)

Testing GR with 3G ground-based detectors

- There are likely not any radically new tests of GR that are only possible with 3G detectors (except possibly the intermediate mass-ratio analogues of extreme mass-ratio inspirals). However, it is possible that, e.g., CW sources will only be detected once the detectors reach 3G sensitivity.
  [Astrophysical stochastic backgrounds are expected to be detected with a few years at design sensitivity; see, e.g., LVC PRL **120**, 091101 (2018).]

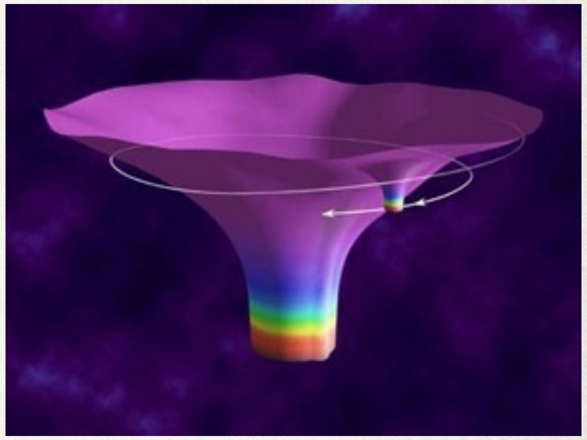
- However, applications of current and future tests of GR using 3G detectors will be much more sensitive, due to higher SNRs and more sources.



$q = 1.2$
$M_{tot} =$

Figure from Krishnendu, Mishra, and Arun arXiv 2018

Tests of gravity with LISA



- LISA should allow for completely new tests, most notably with extreme mass-ratio inspirals. Here one can read off the multipole moments of the large Kerr black hole from the waveform, as first noted by Ryan [PRD 1992]. This give a direct test of the black hole nature and the no-hair conjecture.

  However, there is much work required on self-force calculations in order to perform such measurements.

- It also has guaranteed loud (SNR ~100) CW sources (the verification white dwarf binaries) that can be used for various tests, e.g., tests of polarizations or a Rømer delay-style measurement of the speed of GWs (not as sensitive as the higher-frequency ground-based detector CW version, only ~10^{-3}, much less the GW170817 result, but very direct).

Conclusions

- There are currently fast-to-evaluate waveform models that are suitably accurate for tests, with ongoing work to improve accuracy and parameter space coverage/physics included.
- One can use these waveforms to make various tests of GR, including propagation effects, polarizations, and black hole mimickers.
- 3G detectors will make much more sensitive tests. LISA will allow for completely new tests, such as those from EMRIs, though much work on waveform modeling is required for those cases.