Testing general relativity with gravitational waves

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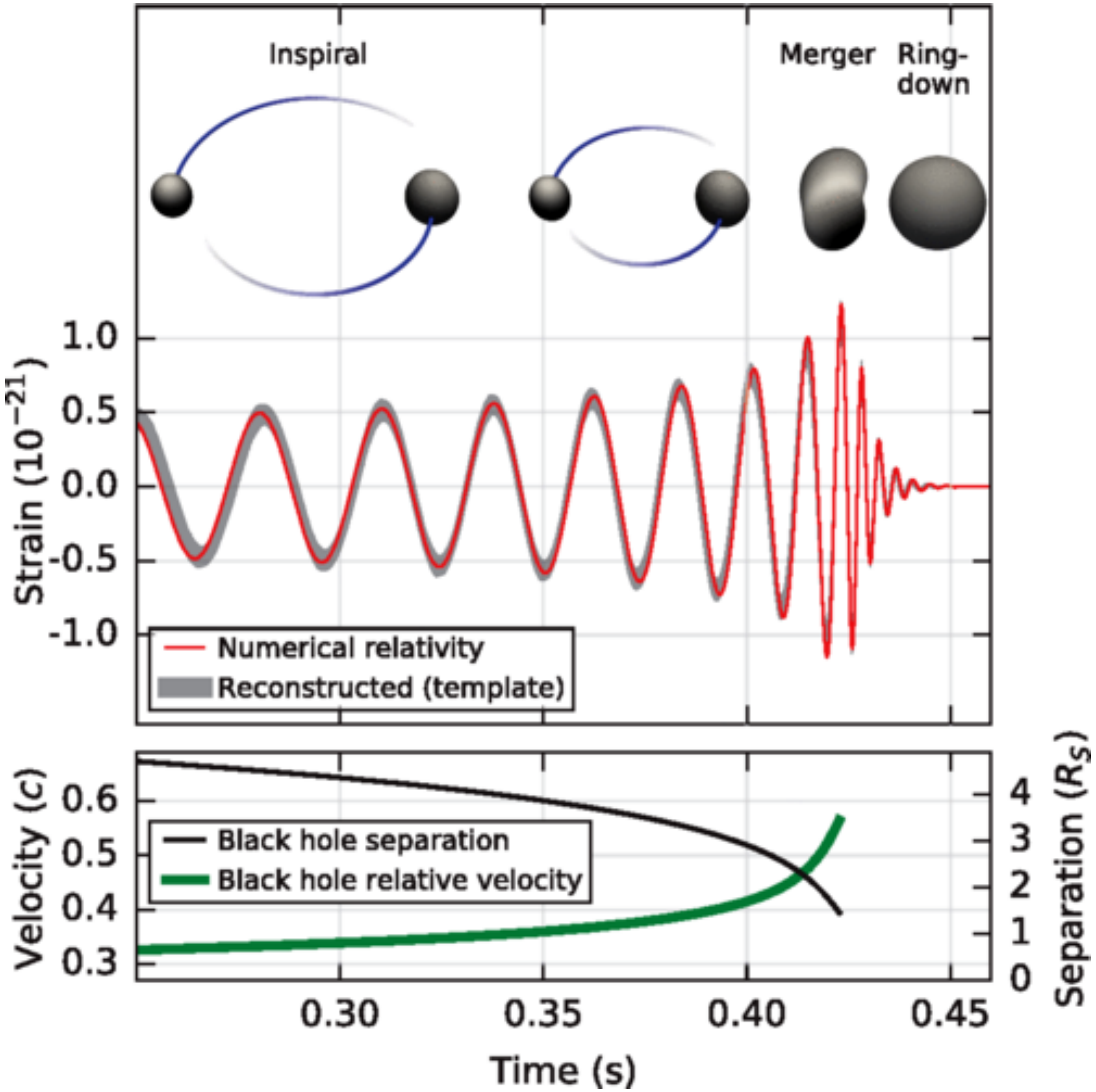
Outline

- Fundamental aspects of GW physics
- Strategies
- Data analysis primer
- Null tests
- On the nature of black holes
- Conclusions

Fundamental aspects of GW physics

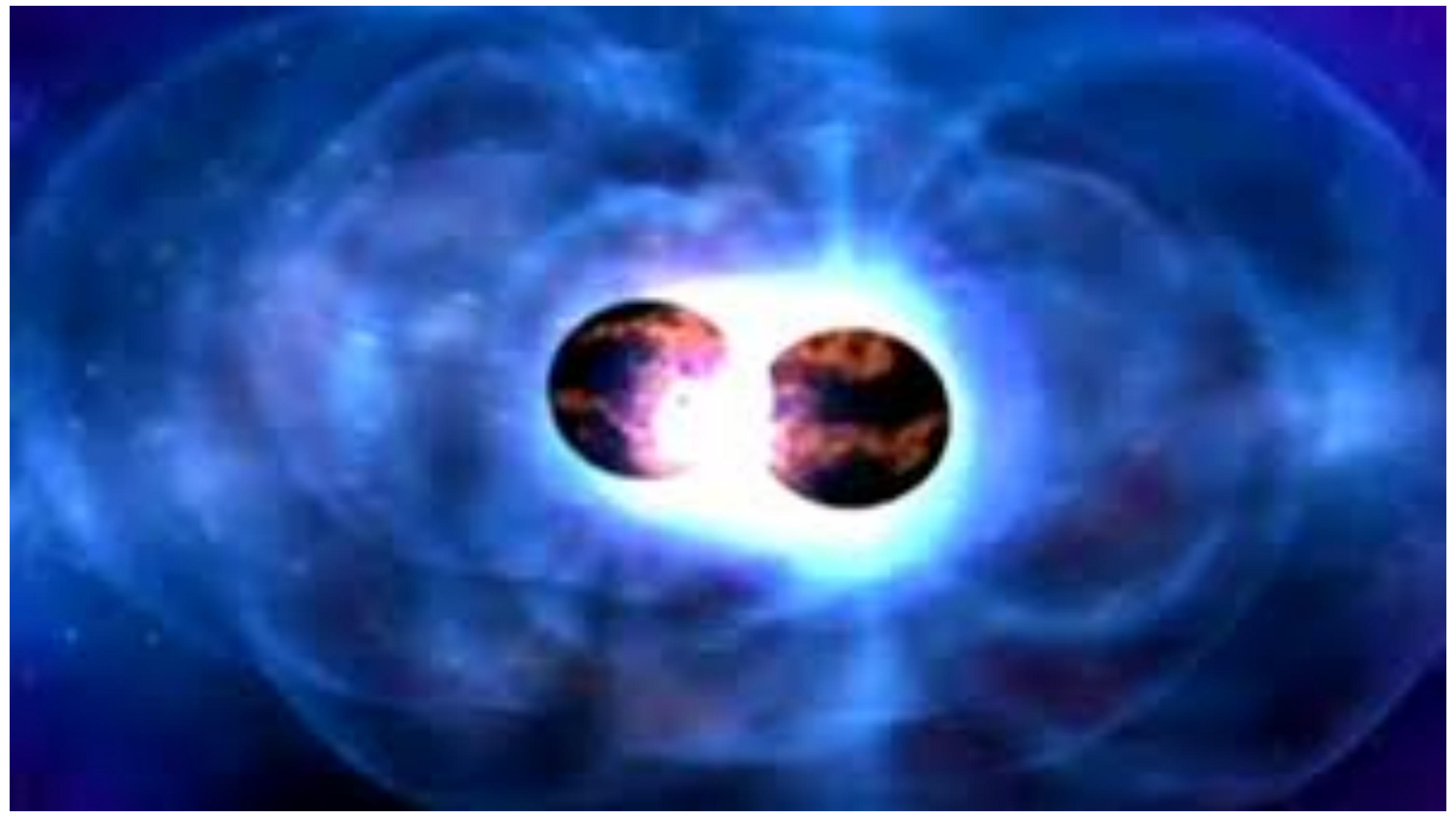
- In GR, gravitational waves (GW) are wave solutions to Einstein's equations generated from time varying mass quadrupoles and propagating at the speed of light
- Shape of GW signal carries information about
  - binary dynamics and component nature
  - non-linear dynamics of space-time
  - final object nature



LVC, arXiv:1602.03837

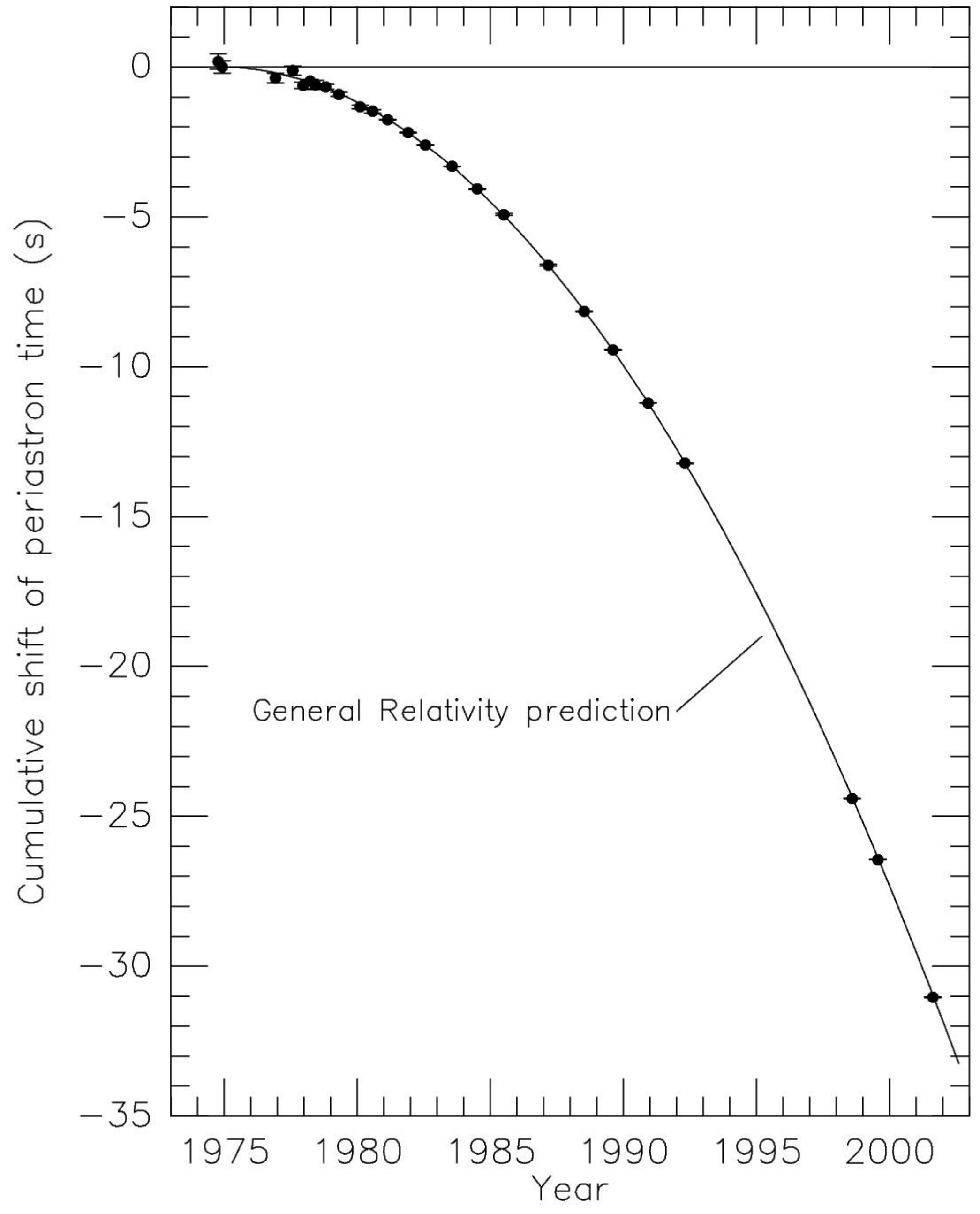
What physics can be probed

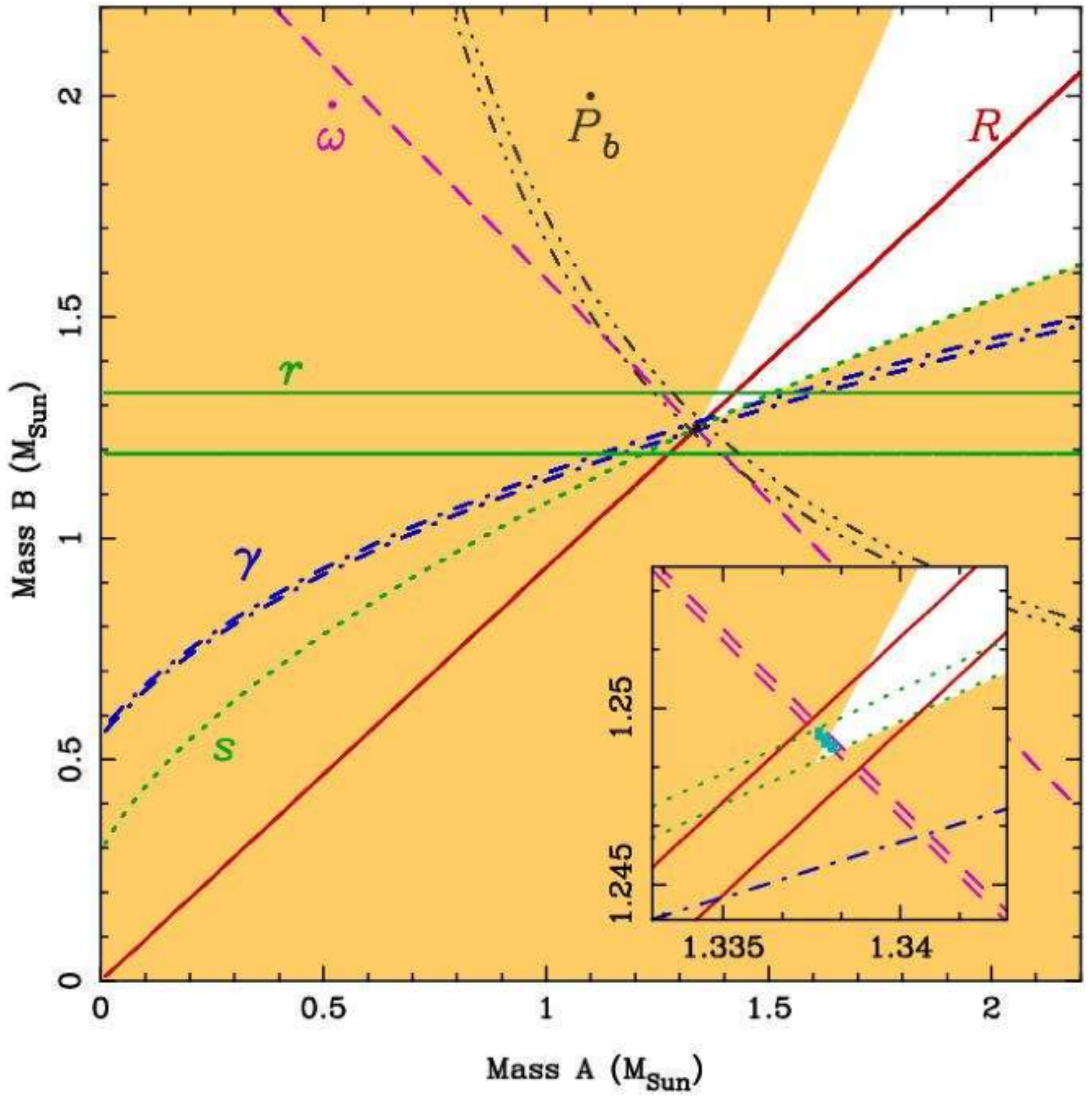
- Matching observed data with a solution to Einstein's equations allows to probe
  - Laws of space-time dynamics
  - Nature of black holes
  - Equation of state of neutron stars
  - Cosmology



Dynamics of space-time

- GR is non renormalisable
  - higher order terms in the action
- Dark matter & dark energy
  - signature of modified gravity?
- GR is extremely well tested in between these regimes (Will, arXiv:1403.7377, Psaltis, arXiv: 0806.1531)



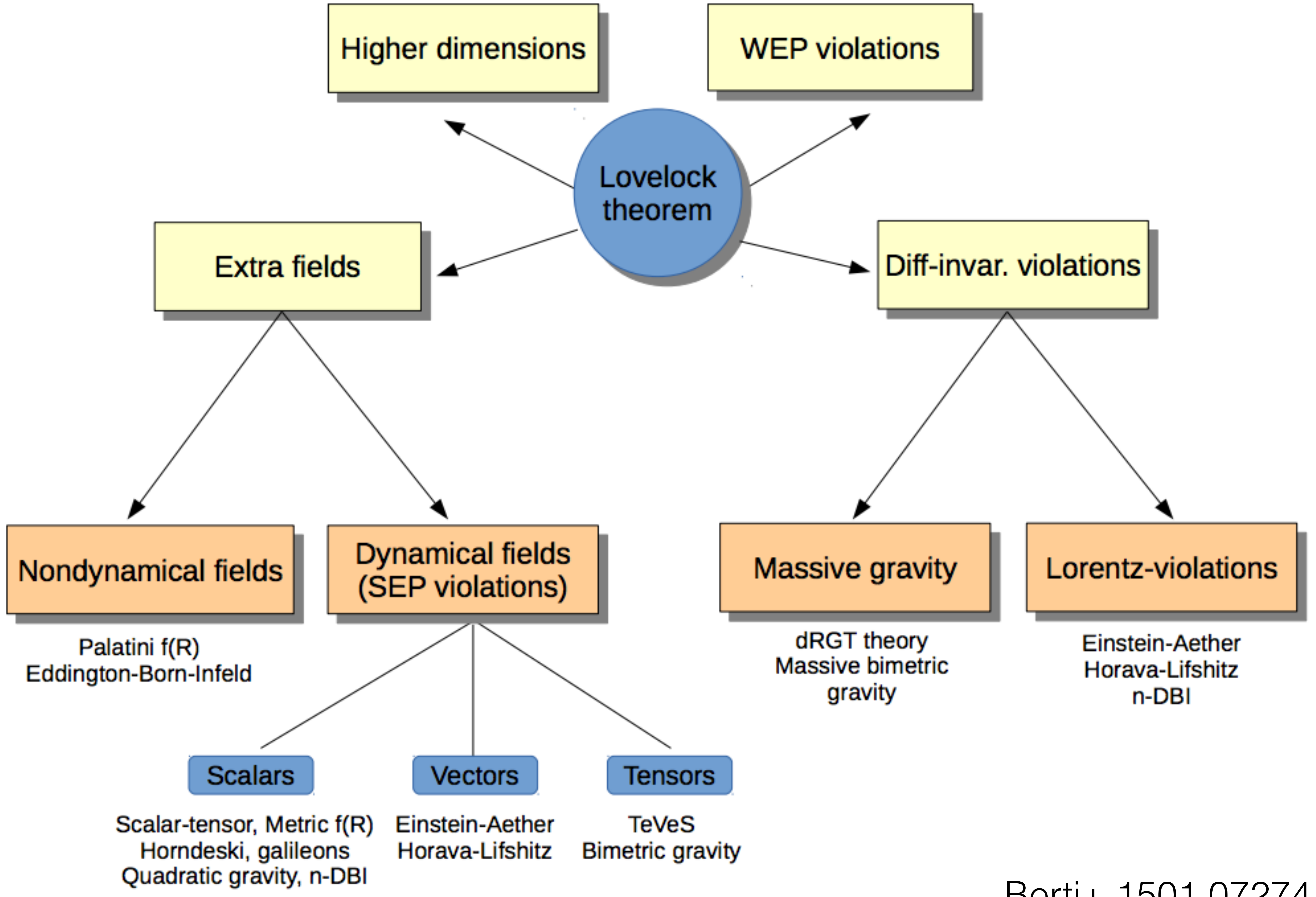


Weisberg & Taylor, arXiv:0407149
Kramer+,arXiv:0609417

Extensions of GR

- Alternative theories
  - Introduce extra degrees of freedom:
    - additional fields
    - higher-curvature terms
  - Challenge GR assumptions:
    - Lorentz invariance
    - Equivalence principle
- Need tests in the strong-field

Lovelock theorem: In 4D, the only divergence free symmetric rank-2 tensor constructed only by the metric and its derivatives up to 2nd order and preserving diffeomorphism invariance is the Einstein tensor plus a constant.



Berti+,1501.07274

Gravitational strong-field

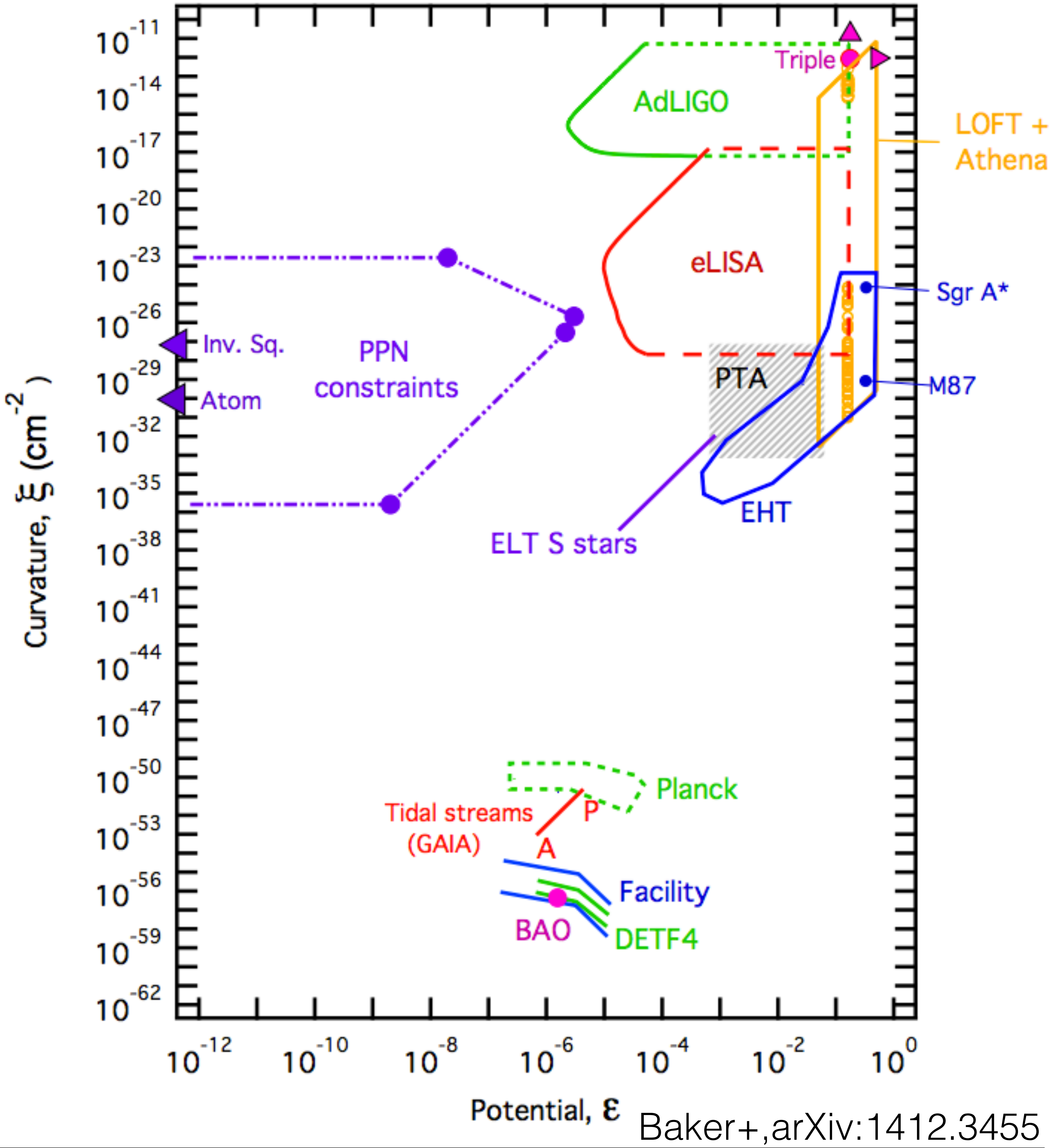
- Field strength

$$\epsilon = \frac{GM}{c^2 R}$$

- Curvature (Kretschmann scalar)

$$\xi = (R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})^{1/2}$$

- Gravitational waves from binary black holes are the optimal probes



Baker+,arXiv:1412.3455

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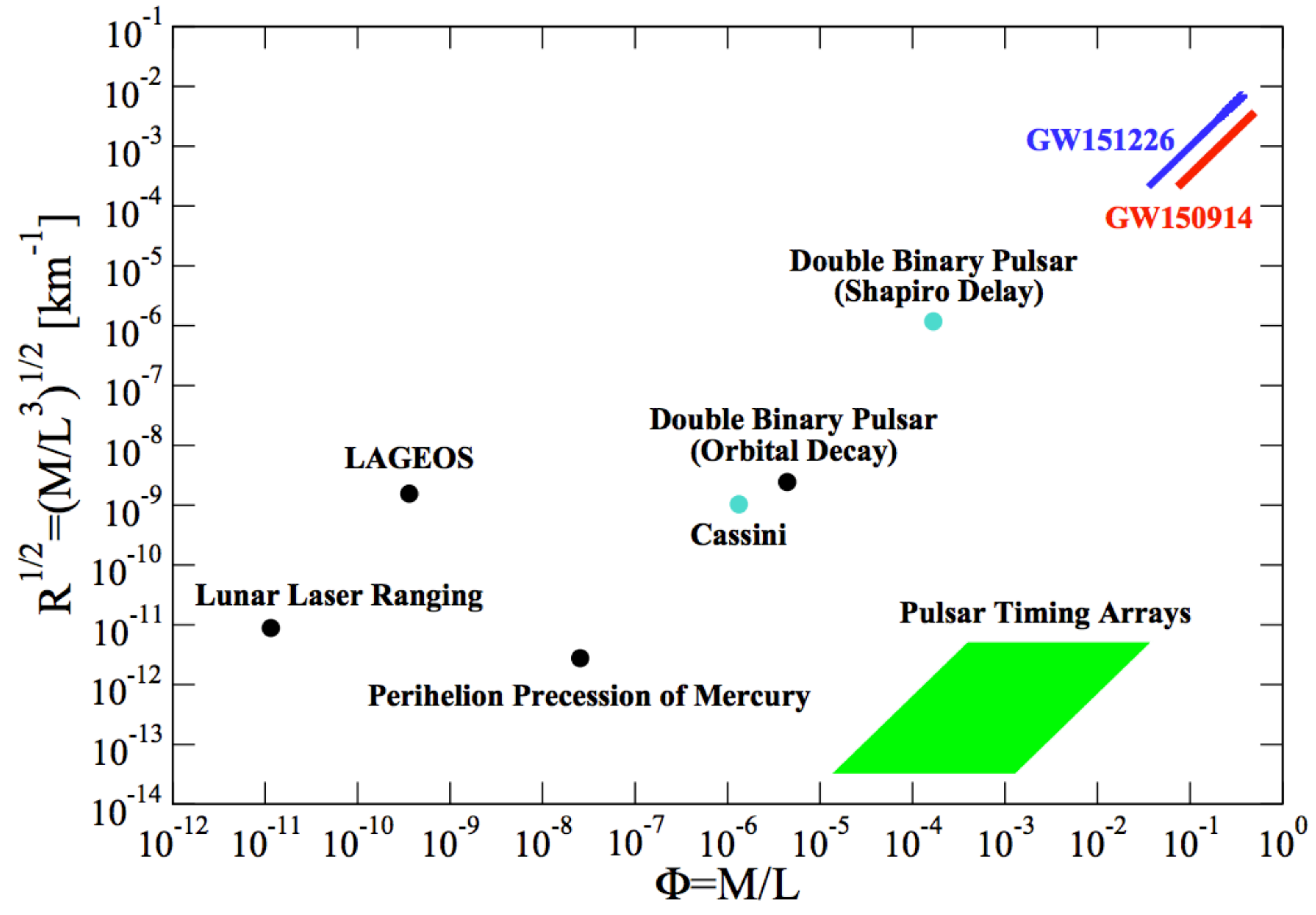
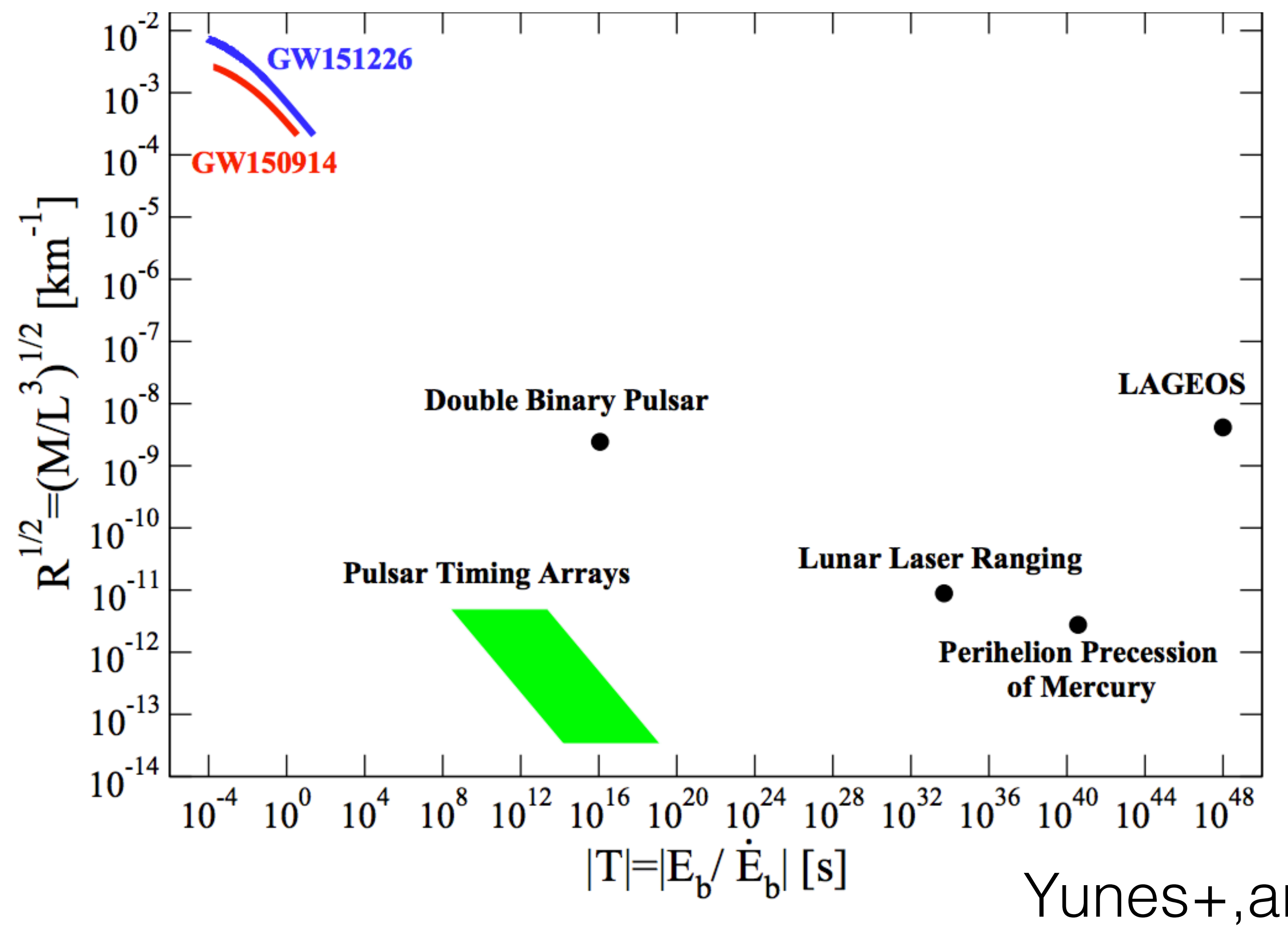
- Space-time is *dynamic*





Yunes+,arXiv:1603.08955

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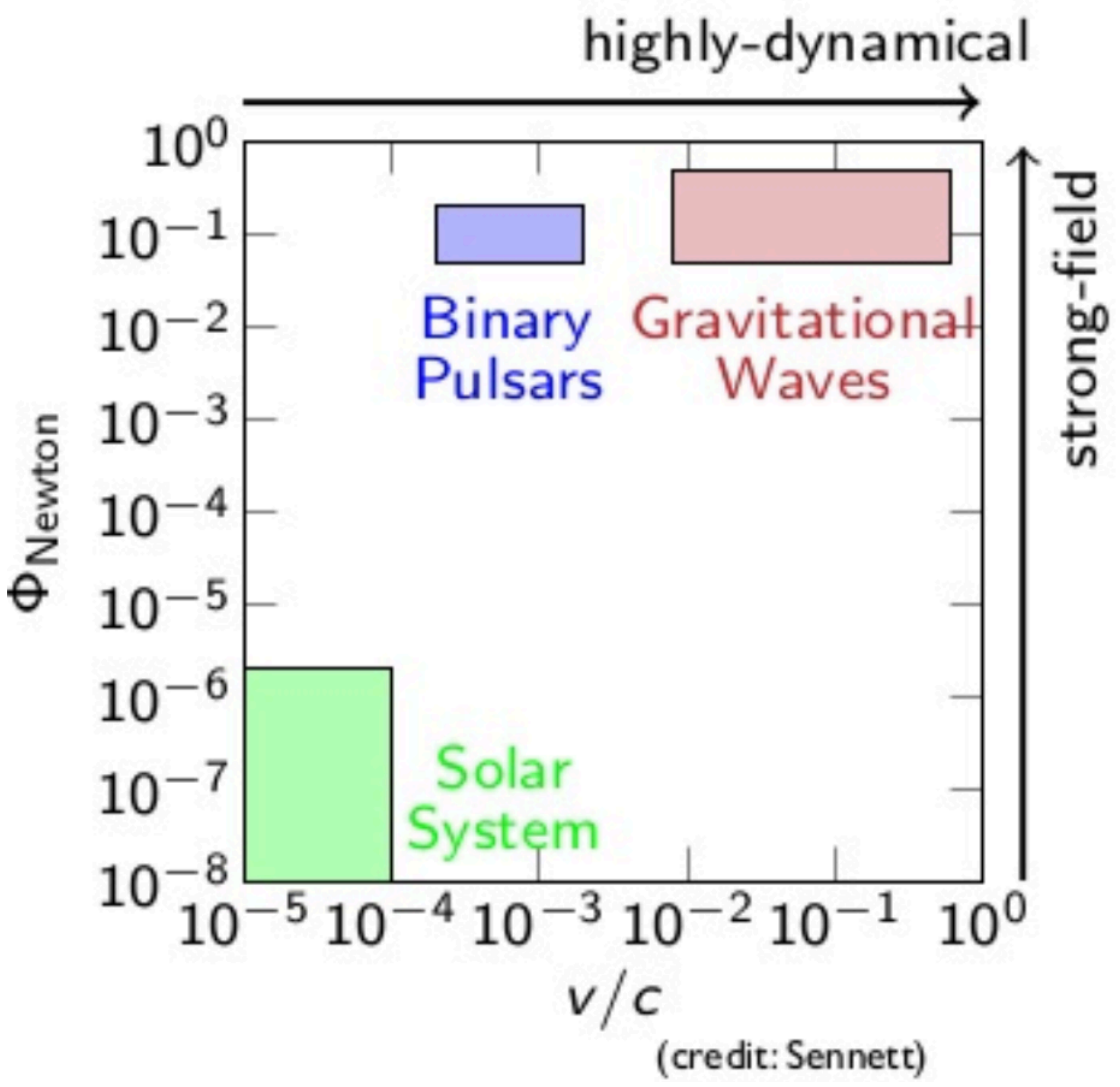
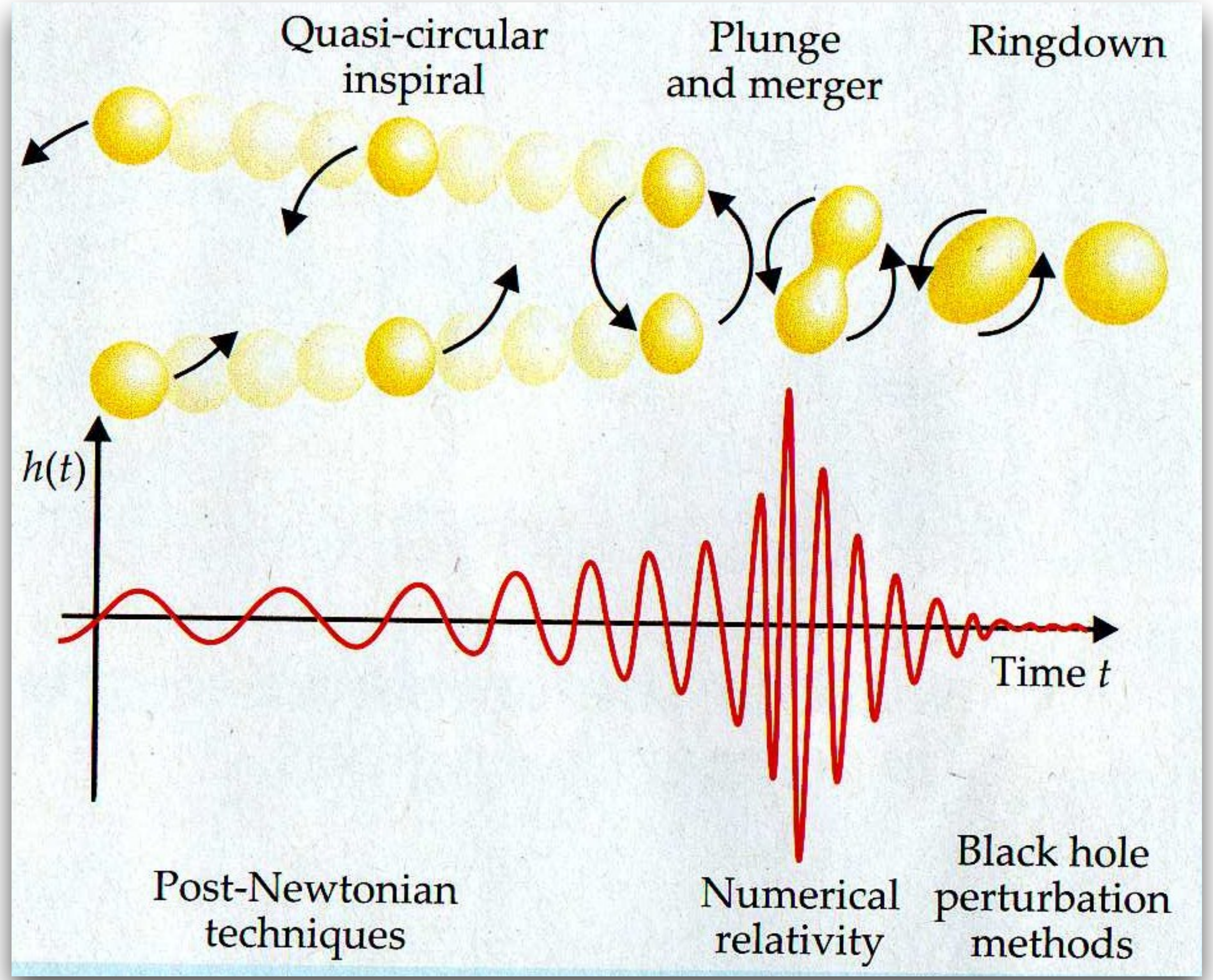


Gravitational wave solutions in GR

- Binary black holes solutions are constructed combining:
  - post-Newtonian theory in the weakly non-linear inspiral regime
  - direct numerical solution in the highly non-linear merger regime
  - perturbation theory in the ringdown regime

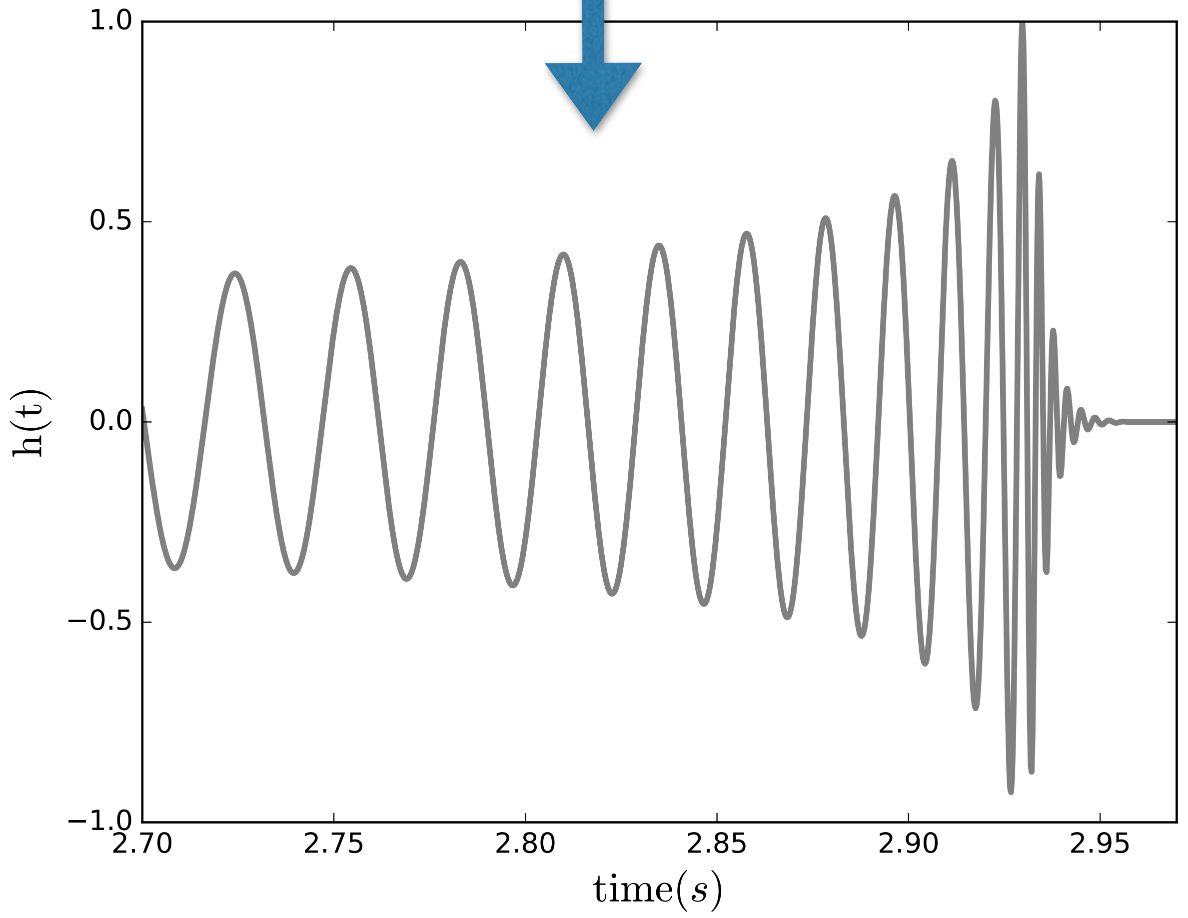


Strong-field GR solutions

- Accurate solutions obtained by direct integration
- Formulation and implementation highly non-trivial
- Computationally challenging
- Numerical solution used to inform and complement analytical formulations:
    - Effective one body (Buonanno & Damour, arXiv:9811091, Bohe+,arXiv:1611.03703, Nagar+, arXiv:1806.01772)
    - Phenomenological (e.g. Khan+,arXiv:1508.07253)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

few weeks later



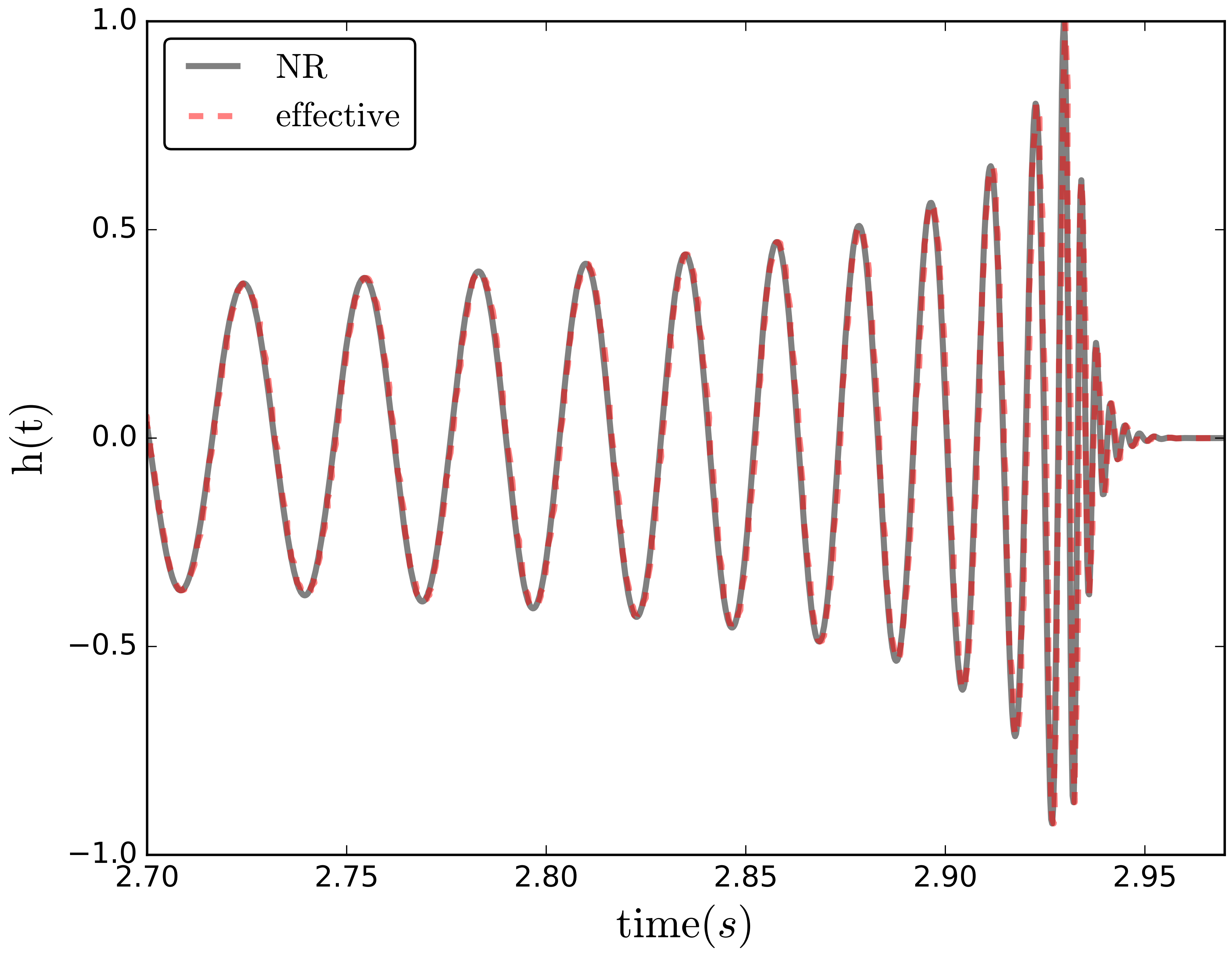
GW templates in GR

- Analytical, parametric description of GW solution in GR

$$h(f;\theta) = A(f;\theta)e^{i\Phi(f;\theta)}$$

$$\Phi(f;\theta) \equiv \Phi(f; m_1, m_2, \vec{s}_1, \vec{s}_2)$$

- Suitable for detection, parameter estimation and parametric tests of general relativity



GW in alternative gravity

- Alternative to GR can introduce extra-fields, curvature terms, challenge GR pillars, …
- Almost no full solution in non-GR known (but see Okounkova et al, arXiv:1705.07924)
- GW phase is modified:
  - non-GR action (extra fields, higher curvature, …): no full non-linear description, only post-Newtonian
  - Propagation (Lorentz violations, graviton mass, …): GR-like BBH dynamics, but modified GW propagation
  - non-GR BHs (extra-fields, exotic objects):
    - tidal deformability
    - ringdown spectrum
    - Echoes

Strategies

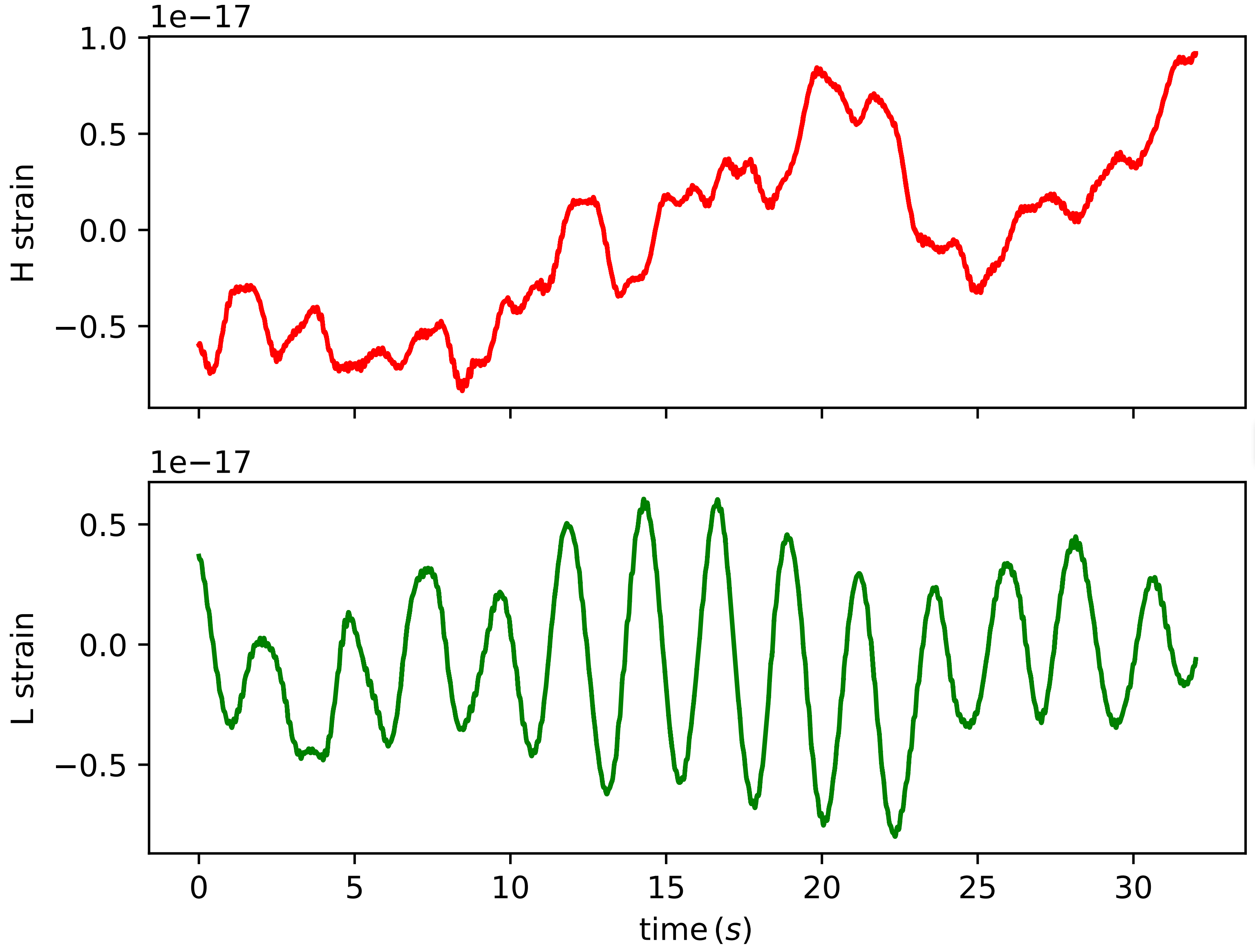
- Two strategies to test GR

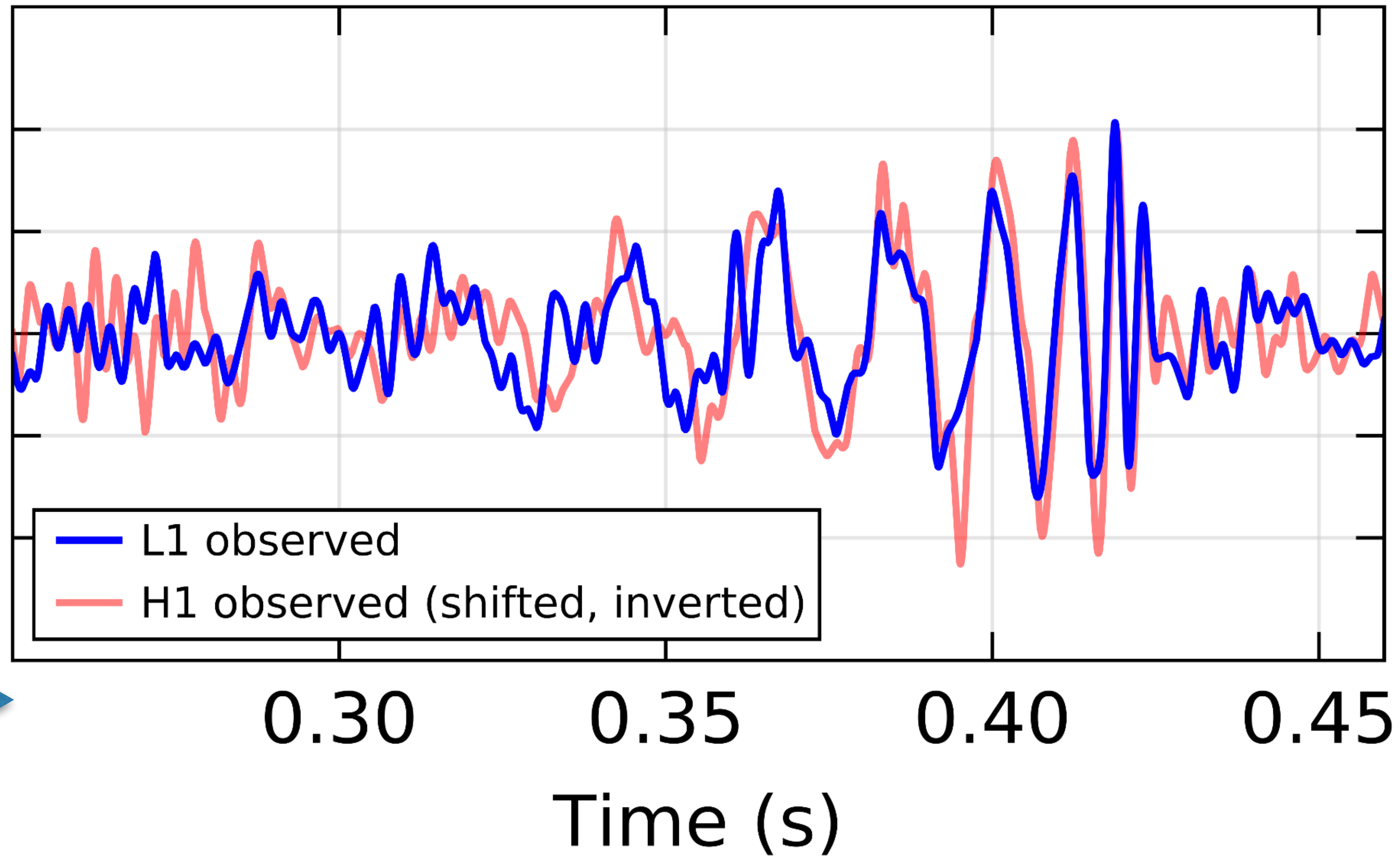
  1. Self-consistency tests: perturb around GR and check for evidence of inconsistencies (e.g. Li+, arXiv:1110.0530)

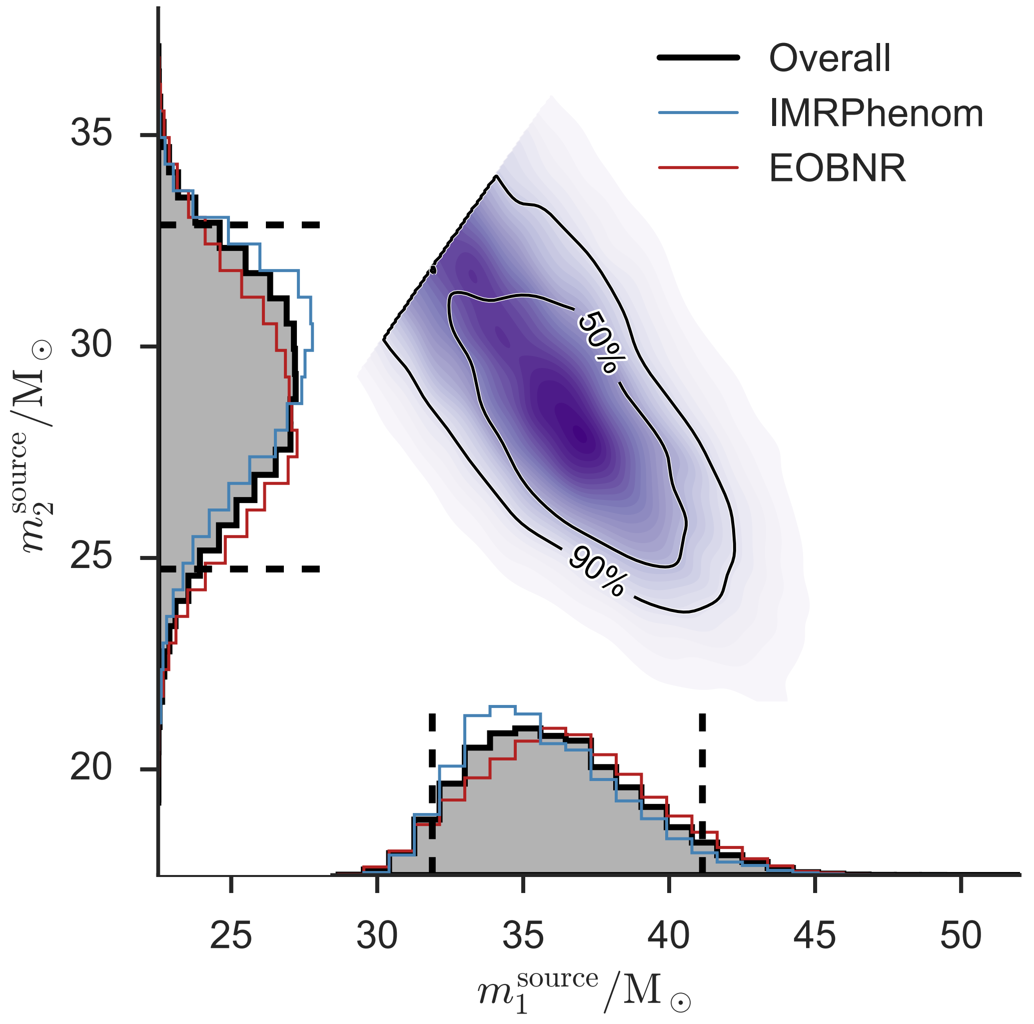
  2. Targeted tests: assume an alternative theory of gravity and try to constrain its parameters (e.g. Del Pozzo+, arXiv:1101.1391)

- How to decide which model better describes the data?

What is “data analysis”?







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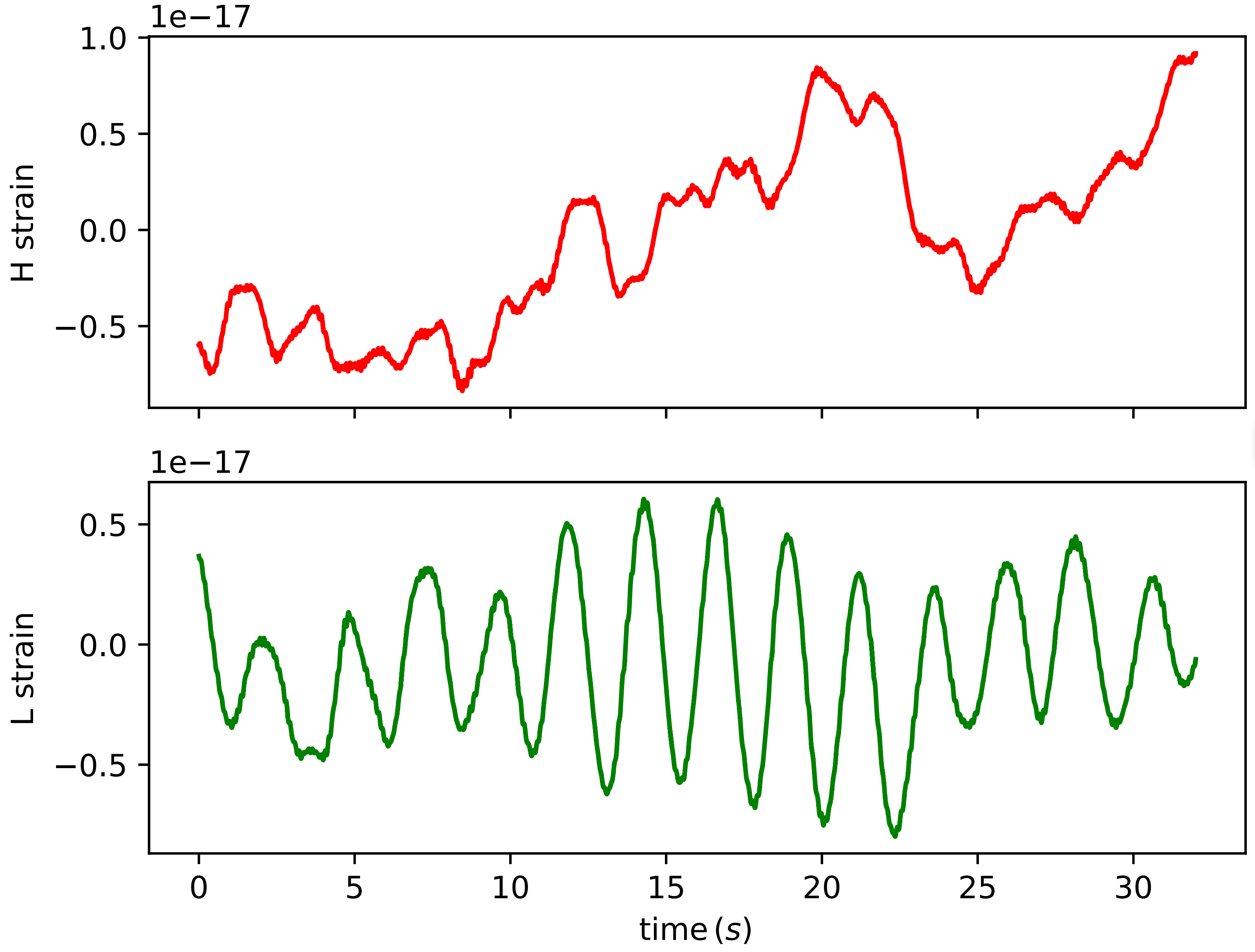
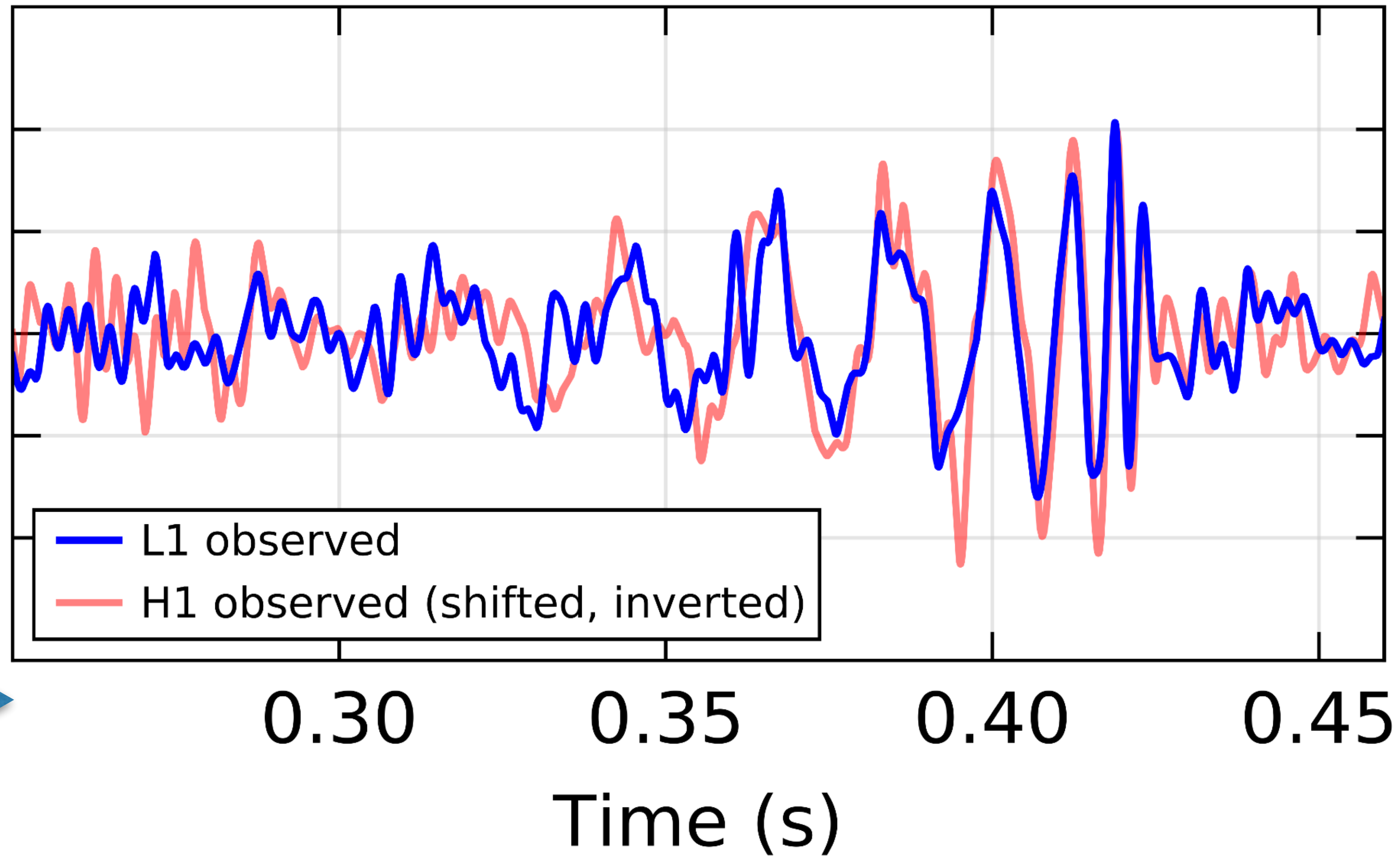
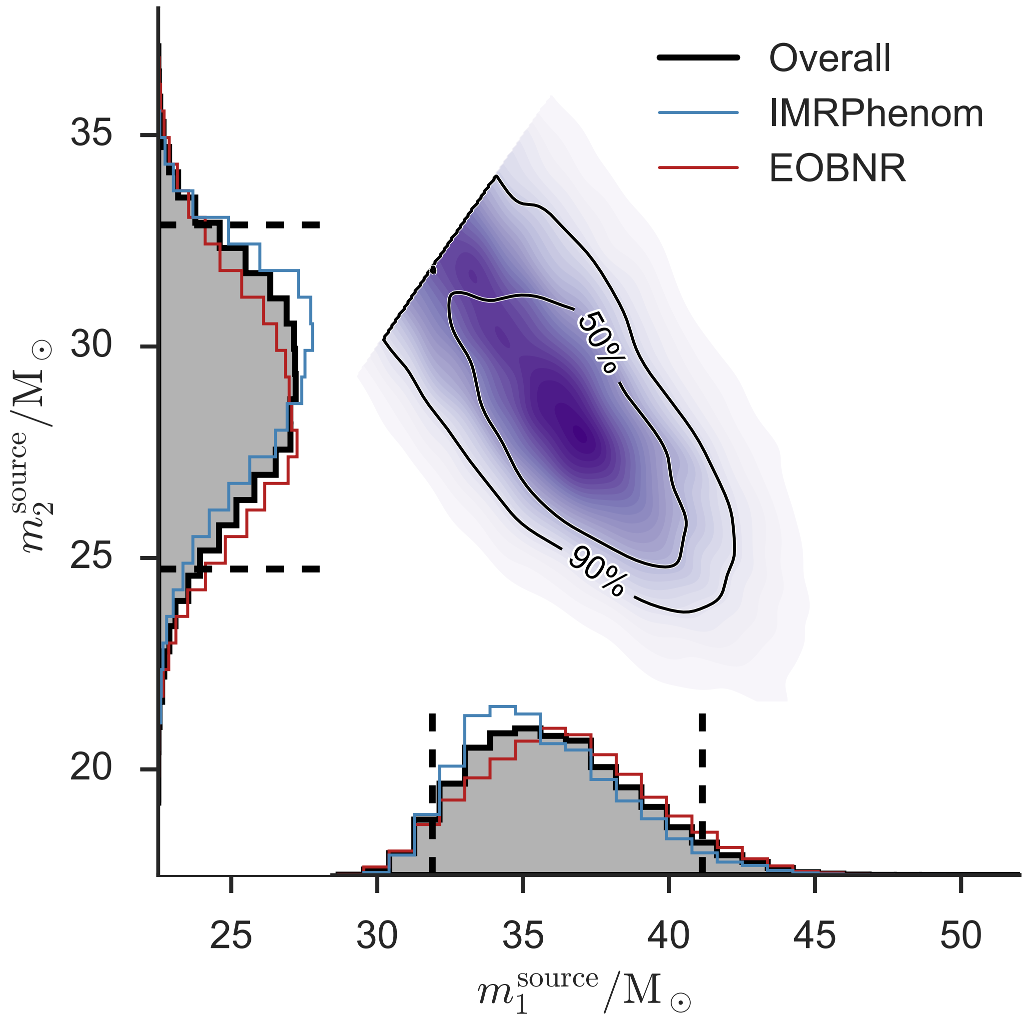


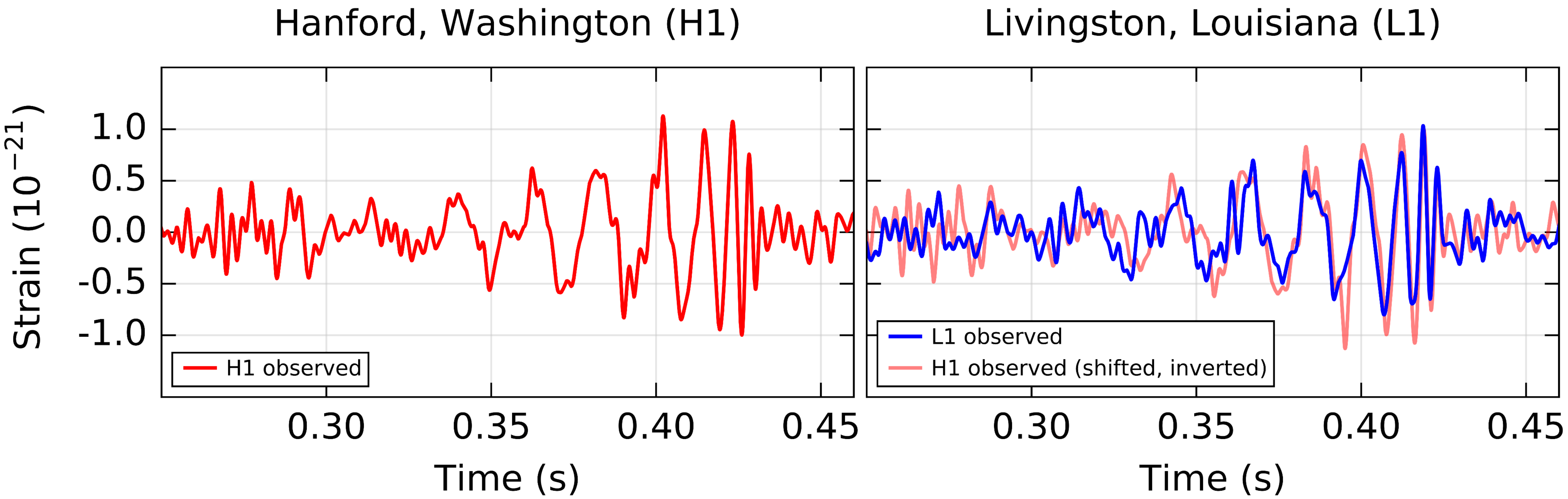


$$p(H|DI) = p(H|I)\frac{p(D|HI)}{p(D|I)}$$



Why Bayes' theorem?

- Gravitational wave events are rare
- Noise dominated detectors
- Need to know what we are looking for VERY well to detect it/measure its properties
  - Matched filtering



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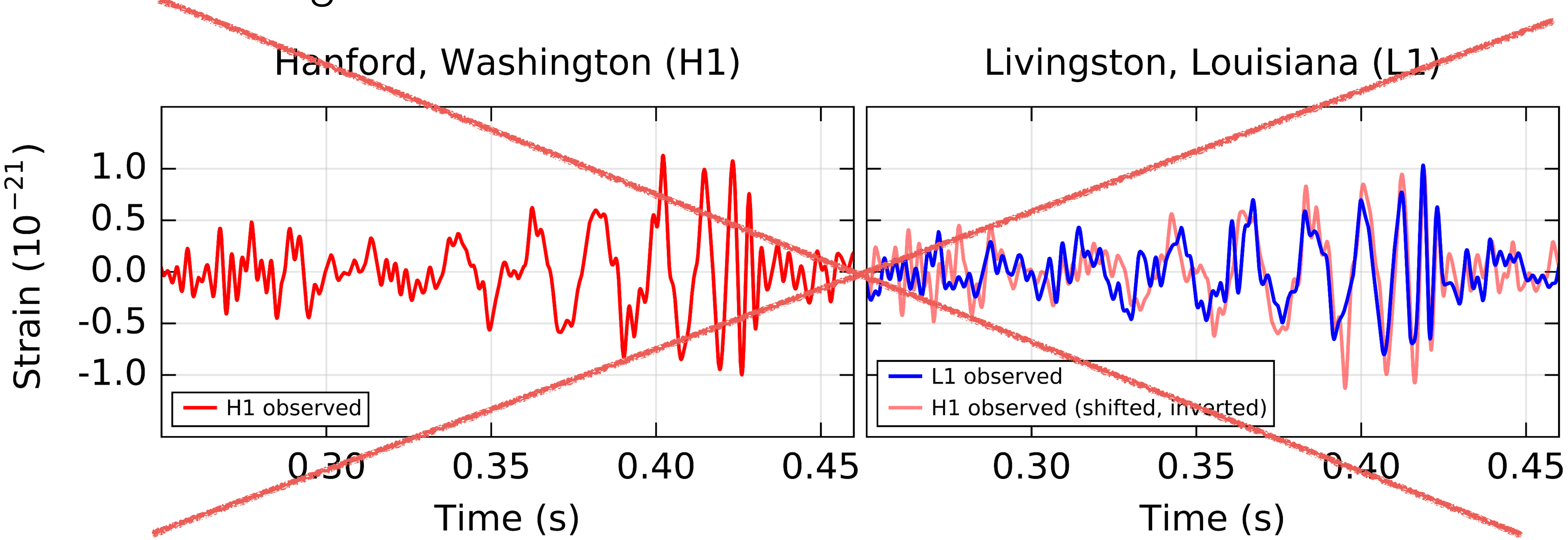
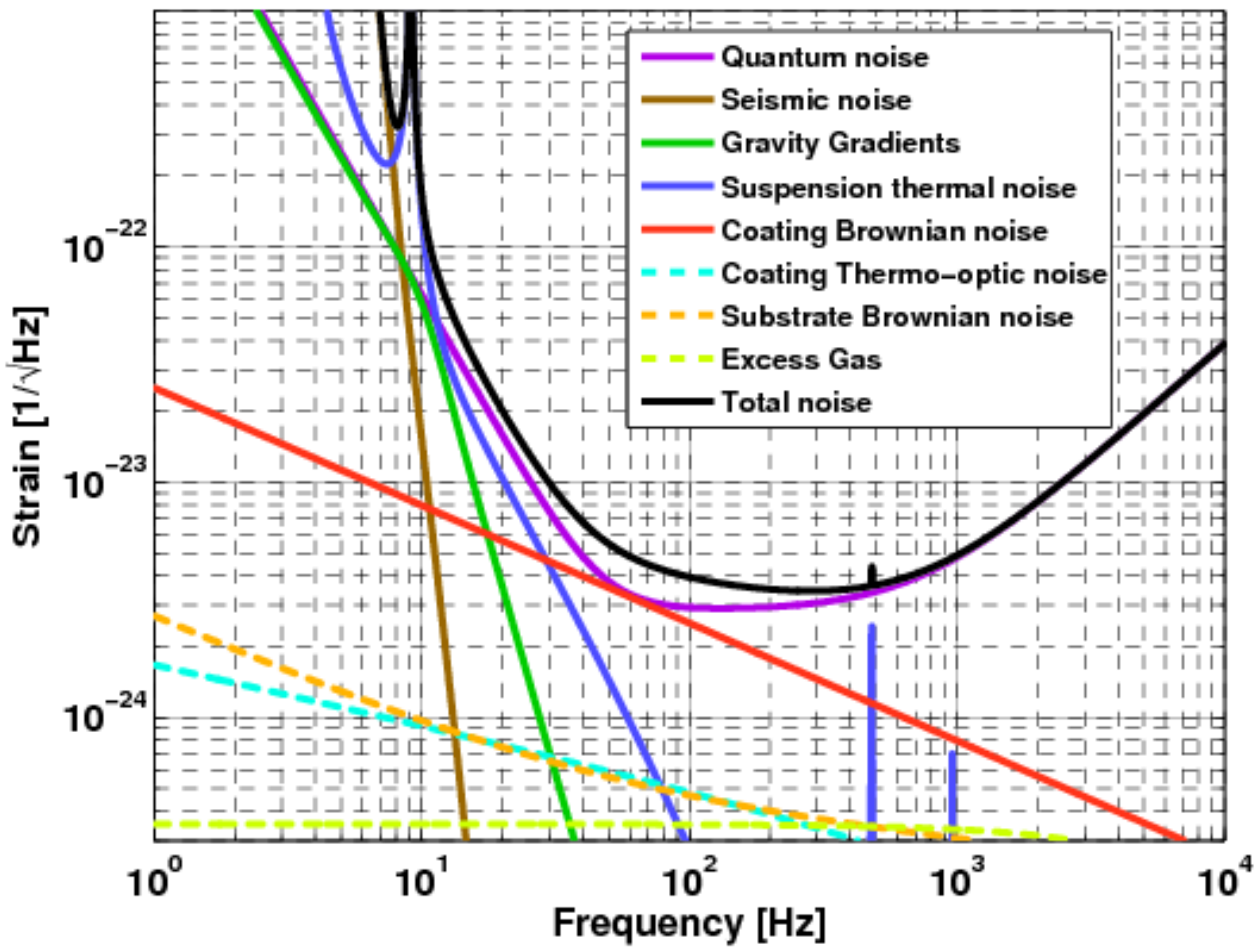


Noise

- IFO noise is a superposition of several (independent) processes
- Invoke the central limit theorem (or the maximum entropy principle)
    - Noise modelled as a wide-sense stationary stochastic process with known autocovariance

$$< n(t)n(t+\tau) > = C(\tau)$$

$$< \tilde{n}(f)\tilde{n}(f') > = \frac{1}{2}S_n(f)\delta(f-f')$$



Matched filtering

- Detector output $$d = h + n$$
- Filter $$A(t) = \int d\tau F(t+\tau)a(\tau) = \int df \tilde{F}^*(f)\tilde{a}(f)$$
- Filtered output

$$\int df \tilde{F}^*(f)\tilde{d}(f) = \int df \tilde{F}^*(f)\tilde{h}(f) + \int df \tilde{F}^*(f)\tilde{n}(f)$$

$$D = H + N$$

- signal-to-noise ratio

$$SNR \equiv \rho = \frac{H^2}{< N^2 >}$$

Matched filtering

$$< N^2 > = \int df |\tilde{F}(f)|^2 S_n(f)$$

$$|H|^2 = \left| \int df \tilde{F}^*(f)\tilde{h}(f) \right|^2$$

- Multiply and divide numerator by $\sqrt{S_n(f)}$ and use the Schwarz inequality

- Optimal filter

$$\tilde{F}^*(f) = C \frac{\tilde{h}(f)}{S_n(f)}$$

Signal-to-noise ratio

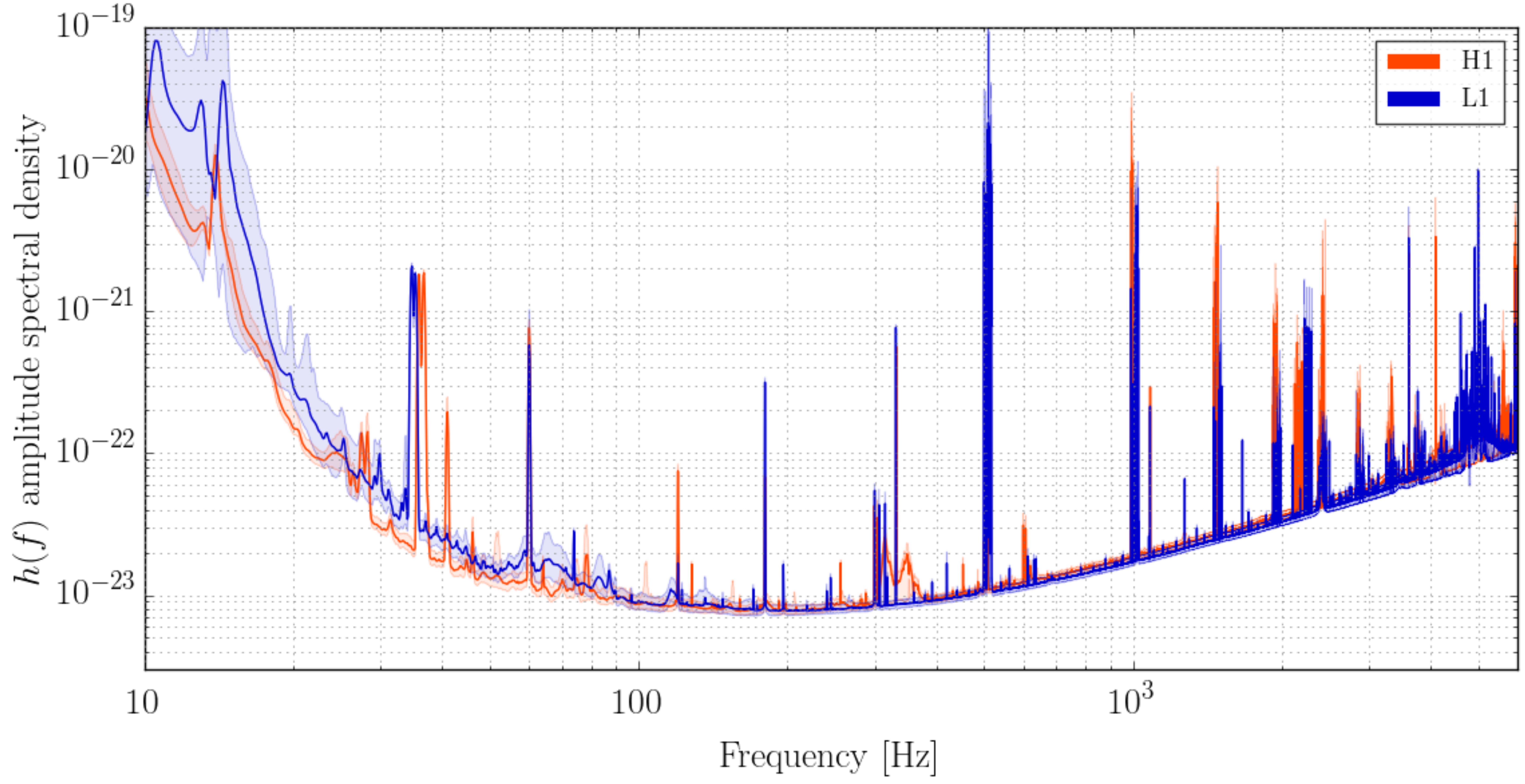
- Use optimal filter to define a scalar product

$$(a|b) = 4\mathrm{Re}\int df\,\frac{\tilde{a}^*\tilde{b}+\tilde{a}\tilde{b}^*}{S_n(f)}$$

- matched filter SNR $\rho^2 = (d|h)$
- optimal SNR $\rho^2_{opt} = (h|h)$
- Frequency domain Likelihood

$$p(D|\theta HI) = e^{-\frac{(d-h|d-h)}{2}}$$

Power spectral density



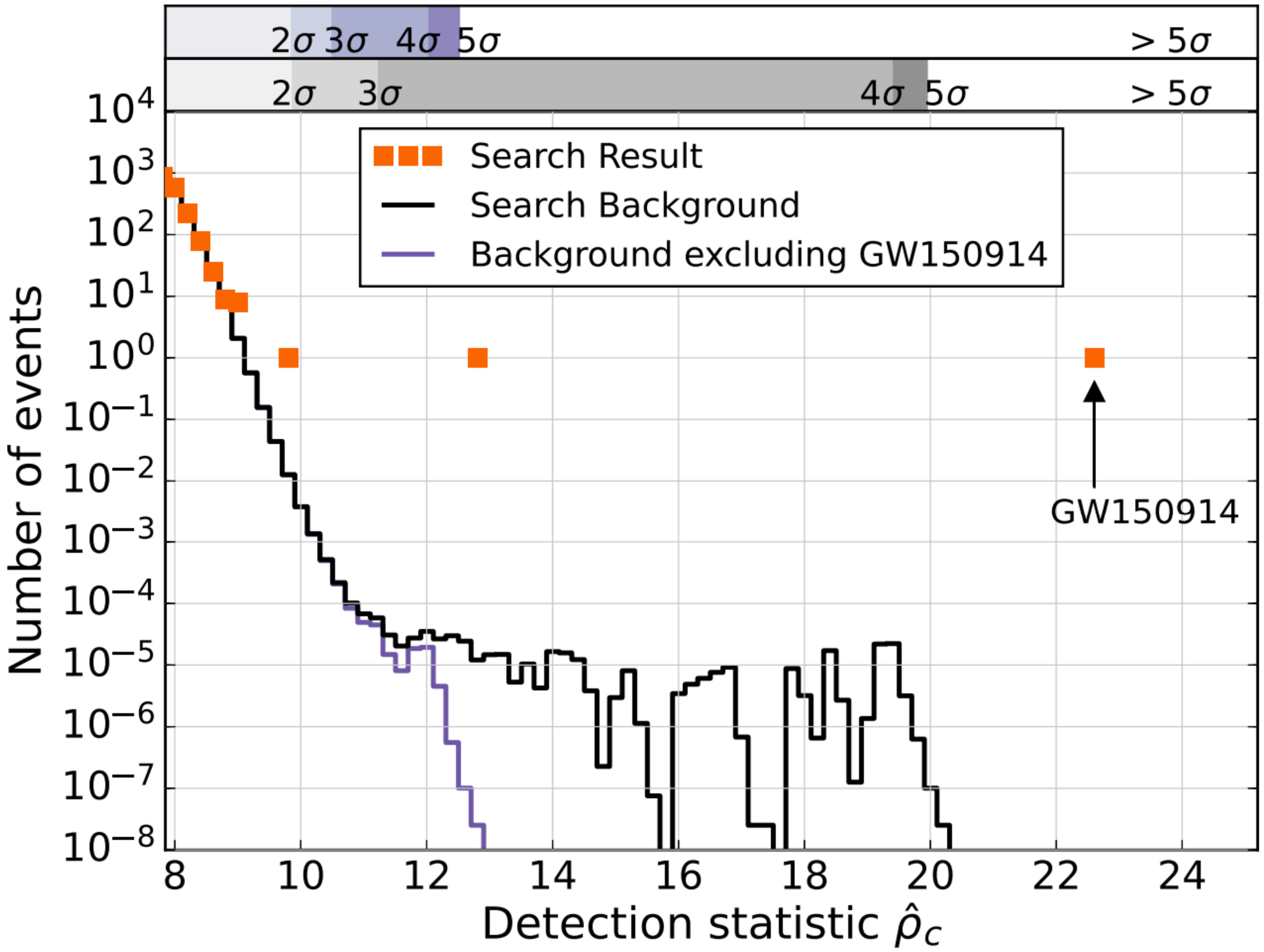
Detection of GW

- Compute odds ratio for the hypotheses
  - S: (Unknown) signal plus noise
  - N: pure noise

$$O_{SN} = \frac{p(S|I)}{p(N|I)} \frac{\int_{\Theta} d\theta \, p(\theta|SI)p(D|\theta SI)}{p(D|NI)}$$

- Computationally unfeasible, yet
- Instead of marginalising, maximise the likelihood
- Estimate significance by generating noise realisations

Significance



LVC, arXiv:1602.03837

Parameter estimation

- $h(t;\theta)$ depends on a set of parameters θ
  - D=9 for non-spinning binaries: masses, orientation, sky location, reference time and phase, luminosity distance
  - D=15 in general: spin vectors
  - More parameters for extra physics (e.g. BH charges, tests of GR, tidal effects, etc…)

Parameter estimation

- Parameters are estimated computing the posterior distribution for all of them
  - joint posterior distribution

$$p(\theta|DSI) = \frac{p(\theta|SI)p(D|\theta SI)}{\int_\Theta p(\theta|SI)p(D|\theta SI)}$$

Large dimensional integral
numerical methods

- Account for all correlations among parameters and all known physical information

Assigning confidence

- In addition to posteriors, one can compute the odds between the signal and signal + noise hypotheses

$$O_{SN} = \frac{p(S|I)}{p(N|I)} \frac{\int_\Theta d\theta\, p(\theta|SI)p(D|\theta SI)}{p(D|NI)}$$

Bayes' factor: ratio of marginal likelihoods

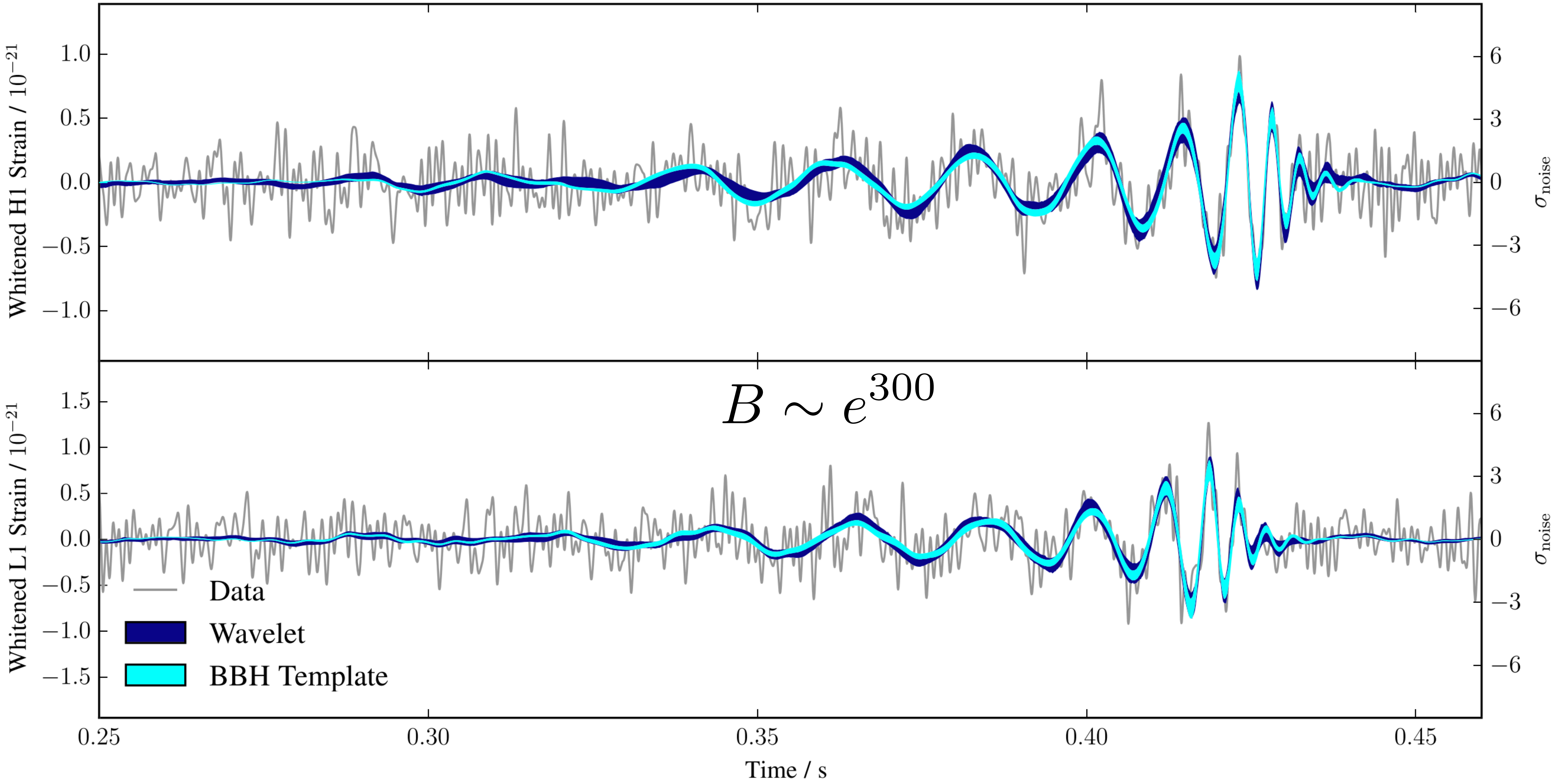
Model selection

- Compute odds ratio (Bayes factor) between competing models

$$O_{12} = \frac{p(M_1|I)}{p(M_2|I)} \frac{p(D|M_1\,I)}{p(D|M_2\,I)}$$

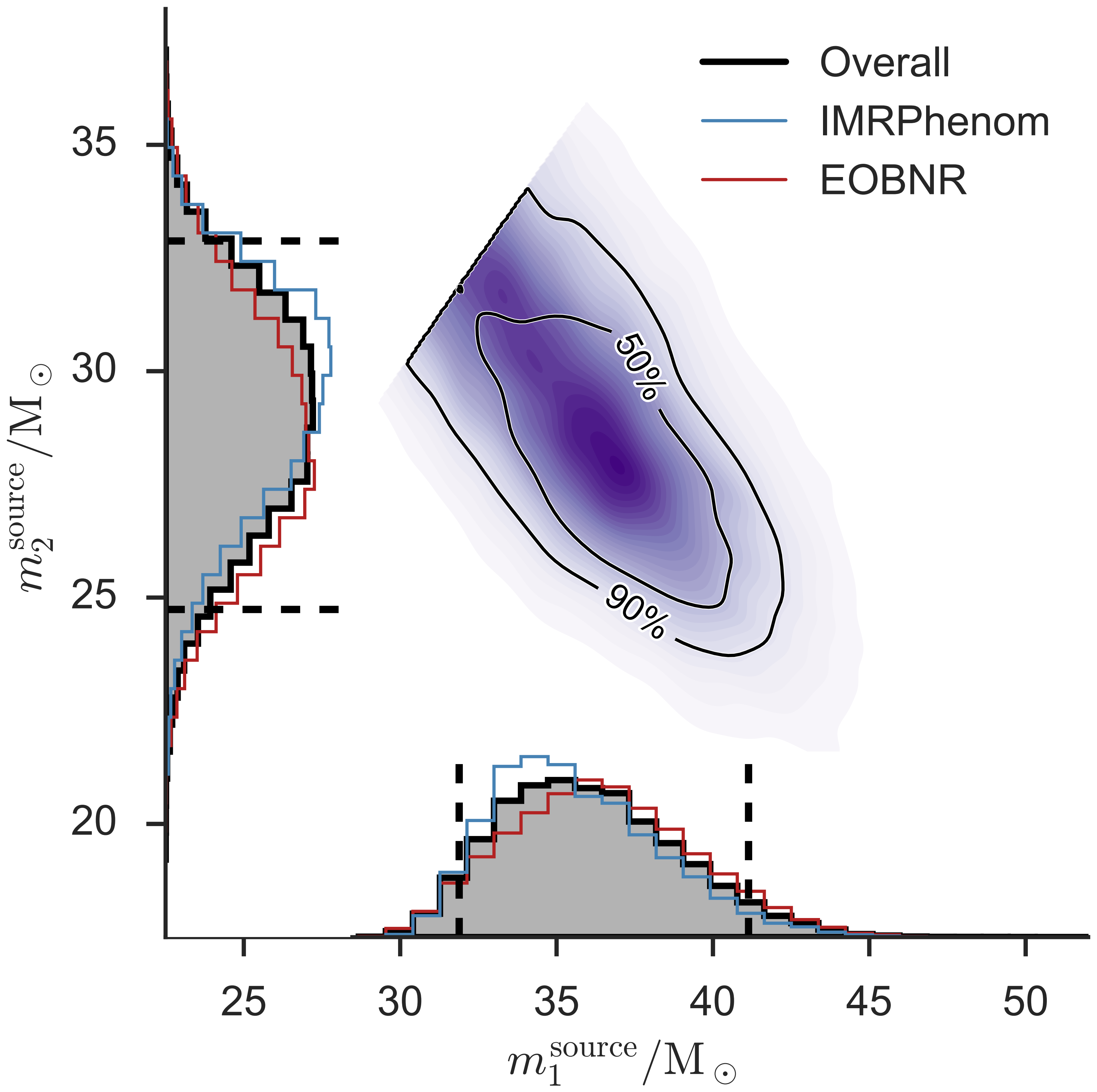
- Evidence accumulates over multiple events

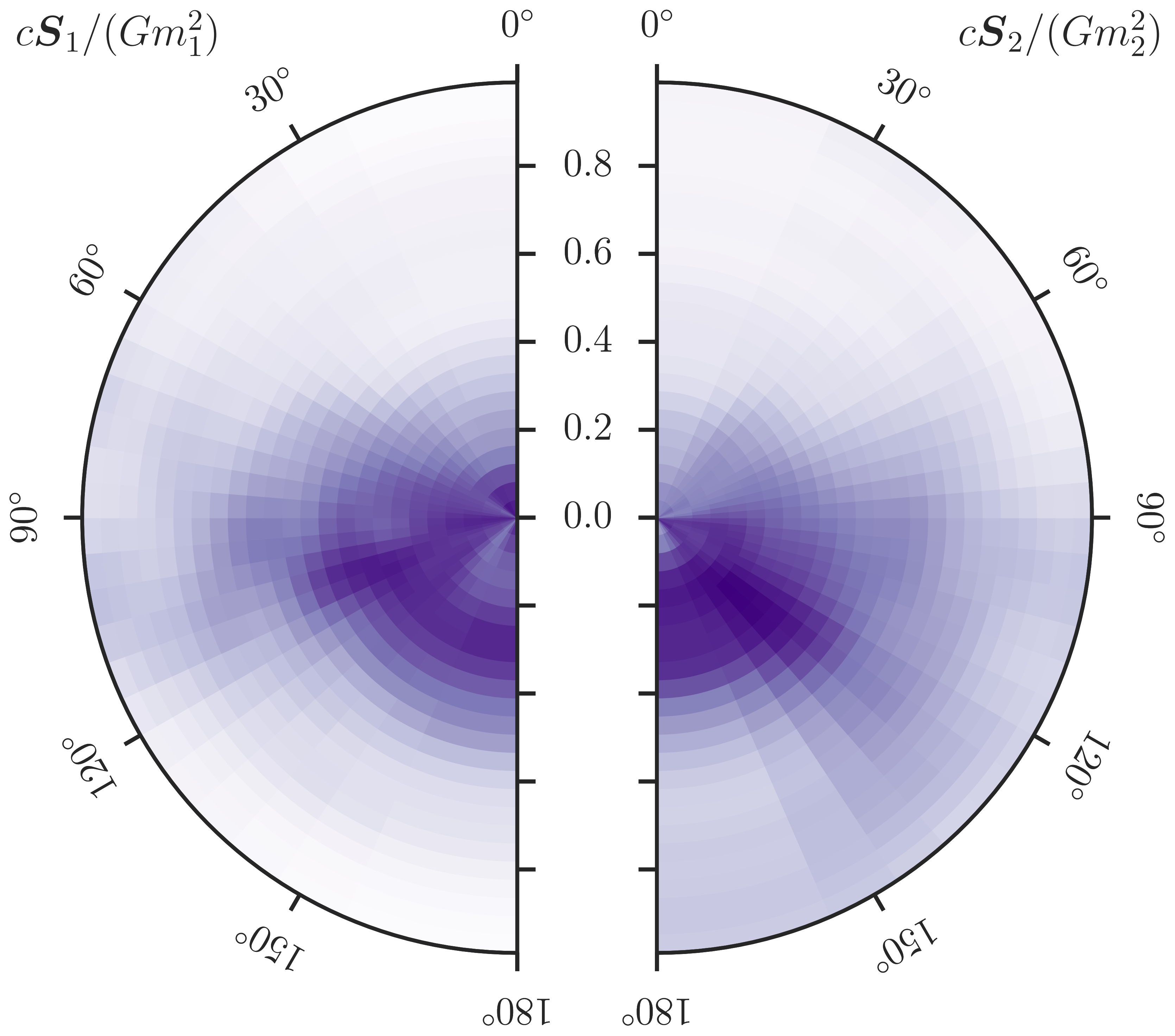
GW150914



Abbott et al, arXiv:1602.03840

GW150914



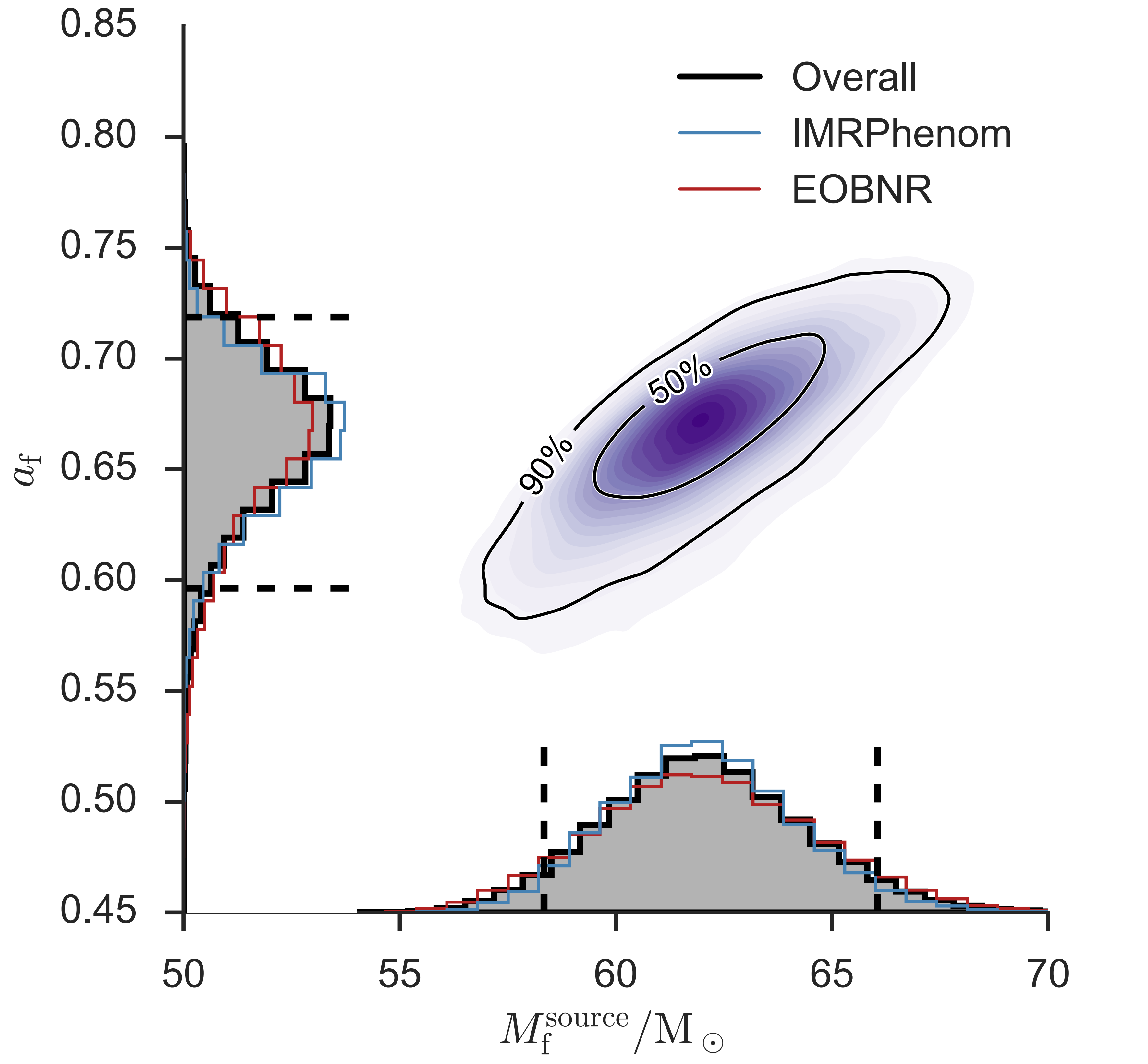


Abbott et al, arXiv:1602.03840

GW150914

- Accurate measurements require accurate and precise predictions

"Inference is as good as the information you put into it"



Abbott et al, arXiv:1602.03840

Residuals

- After subtraction of the best fit GR waveform, the residuals must be consistent with Gaussian noise

$$p(residuals) \sim p(n)$$

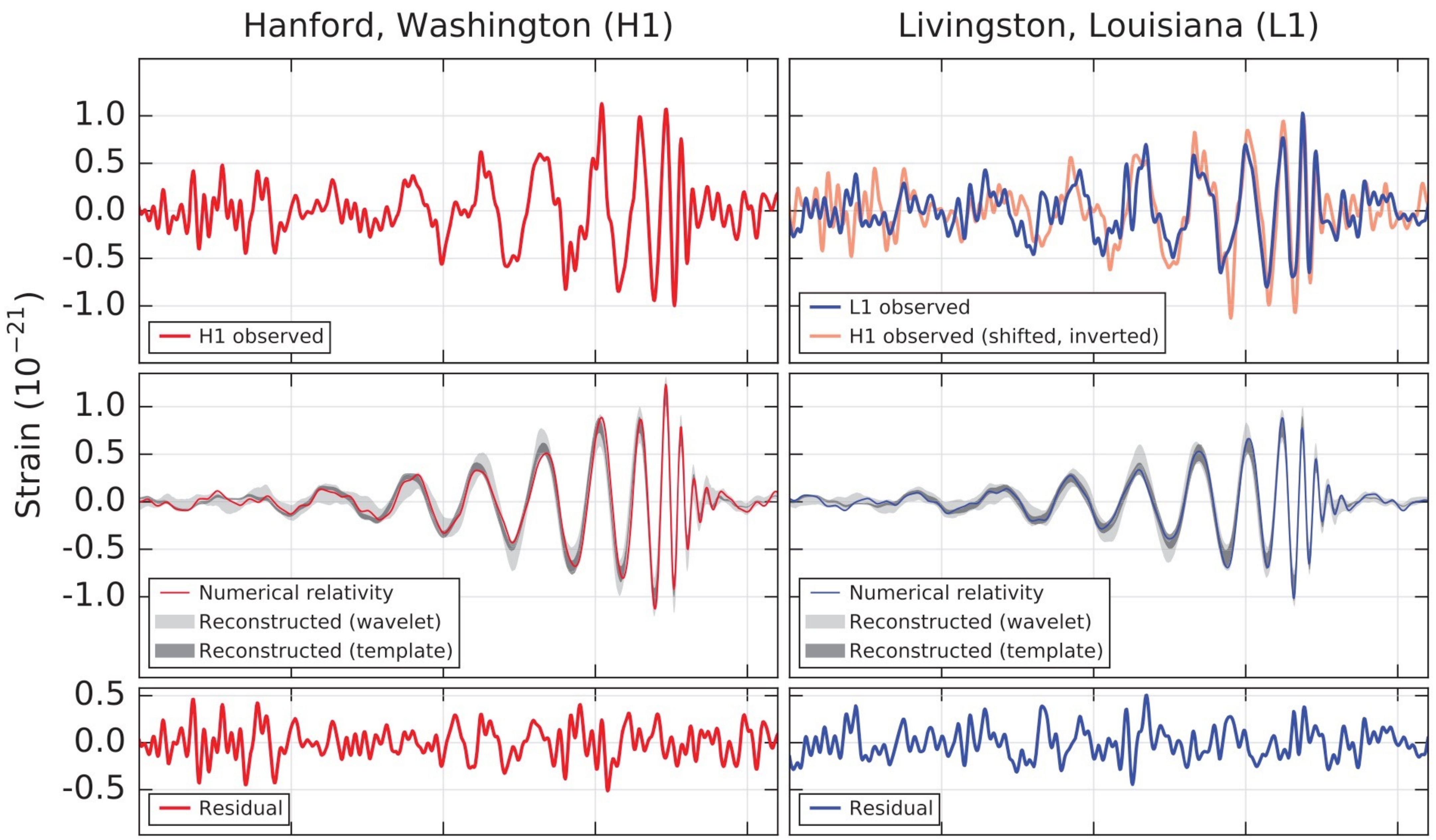
- Use un-modelled methods (Cornish & Littenberg, 1410.3835) to search for coherent power in the residuals

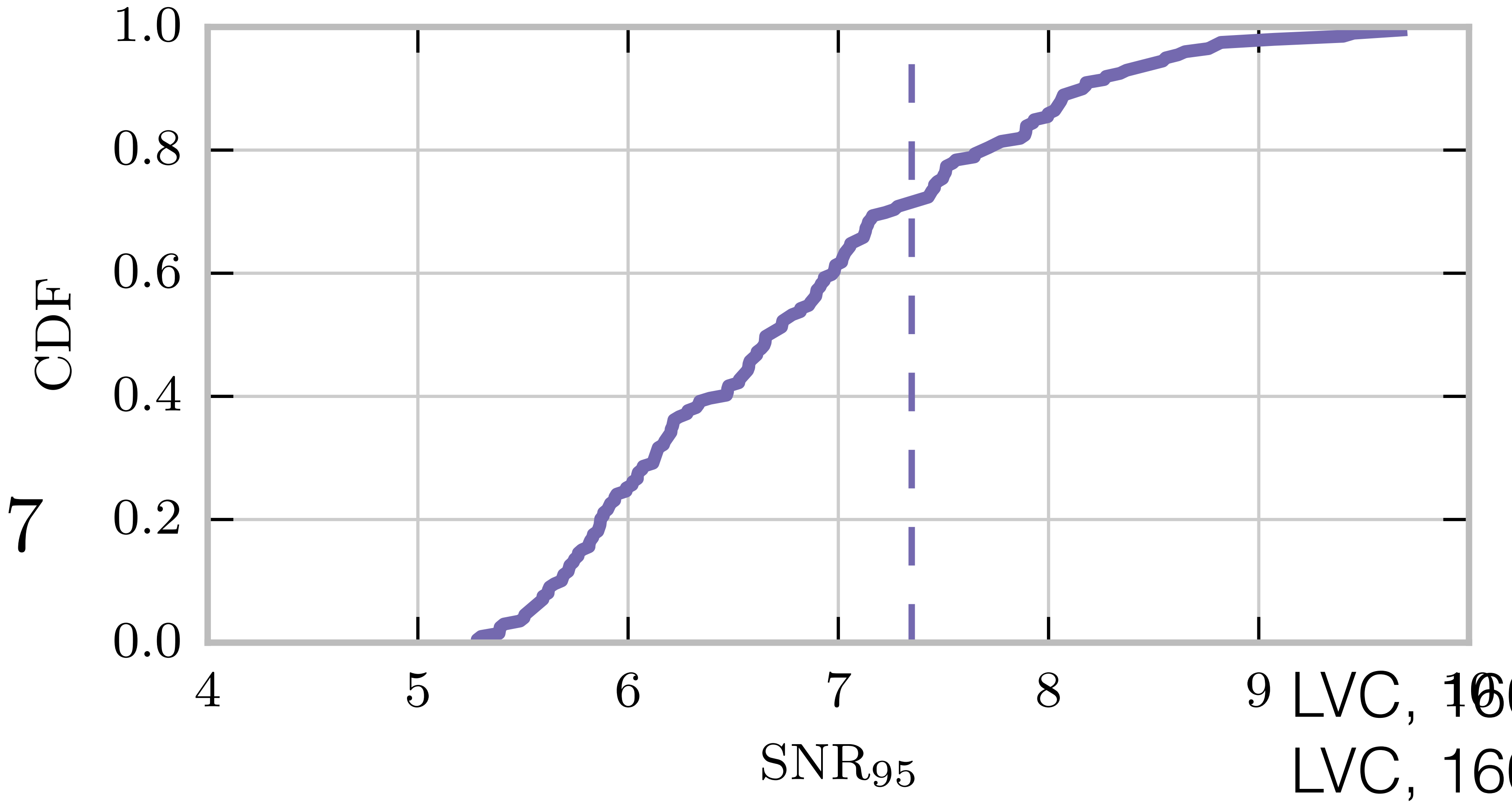
- GW150914 residual SNR < 7 at 95% confidence

- Match between GW150914 and the best GR template > 96%

$$FF = \sqrt{\frac{SNR^2_{det}}{SNR^2_{det} + SNR^2_{res}}}; \quad SNR_{det} = 25; \quad SNR_{res} \leq 7$$

$$FF \geq 0.96$$





LVC, 1602.03837
LVC, 1602.03841

Parametrised tests of GR

- GW waveforms are expressed in terms of effective series, for the Phenom family:

$$h(f;\theta) = A(f;\theta)e^{i\Phi(f;\theta)}$$

$$\Phi(f;\theta) = \sum_{k=0}^{7}(\varphi_k + \varphi_k^{(l)})f^{(k-5)/3} + \sum_{i\neq k}\varphi_i g(f)$$

post-Newtonian series — effective series

$$\varphi_j \equiv \varphi_j(m_1, m_2, \vec{s}_1, \vec{s}_2)$$

- Modified theories of gravity change the series (e.g. PPE: Yunes & Pretorius, arXiv:0909.3328, Cornish+,arXiv:1105.2088)

- Perturb the GW phase around GR (Li+,arXiv:1110.0530, Agathos+,arXiv:1311.0420)

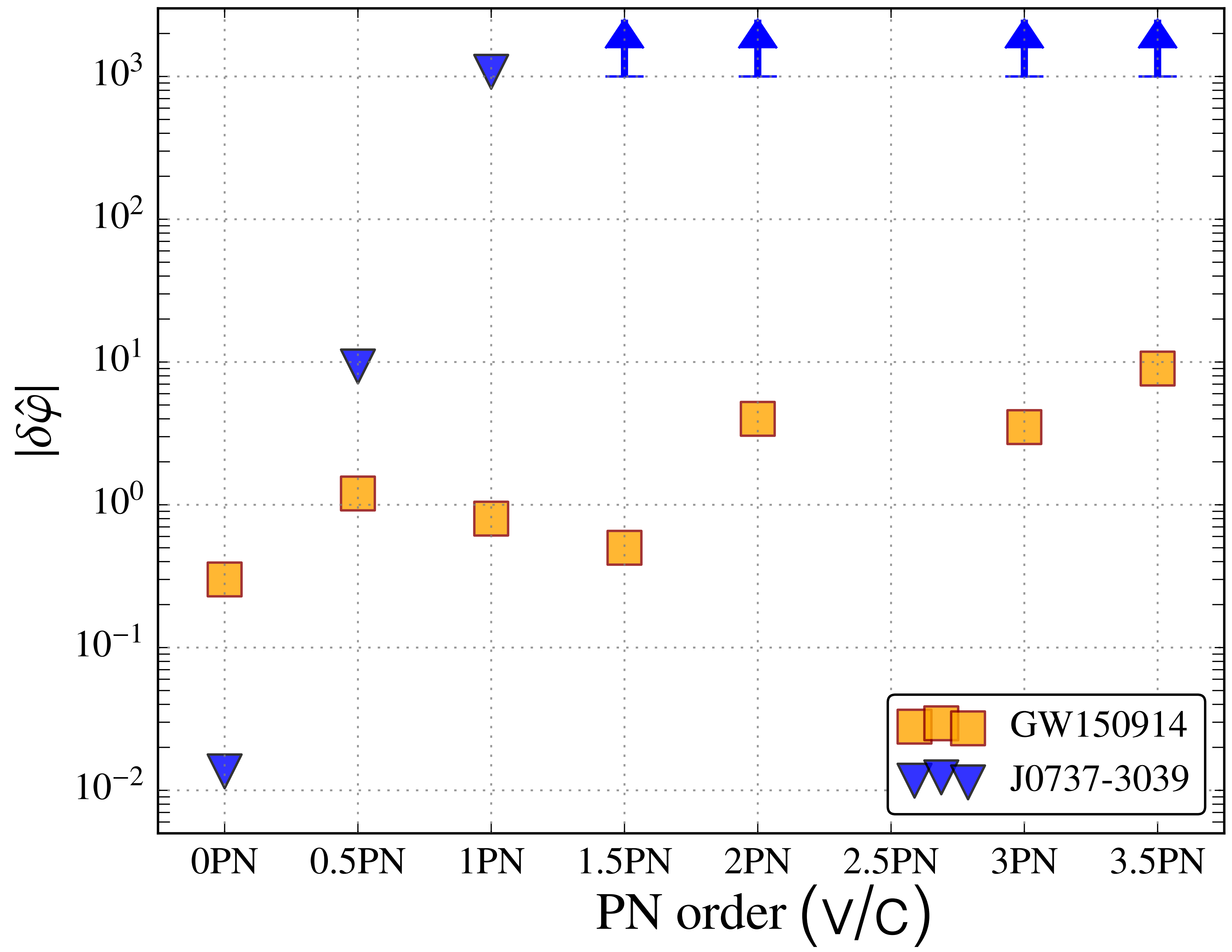
$$\hat{\varphi}_j \equiv \varphi_j^{GR}(1+\delta\hat{\varphi}_j) \qquad \delta\hat{\varphi}_j = 0 \iff \text{GR}$$

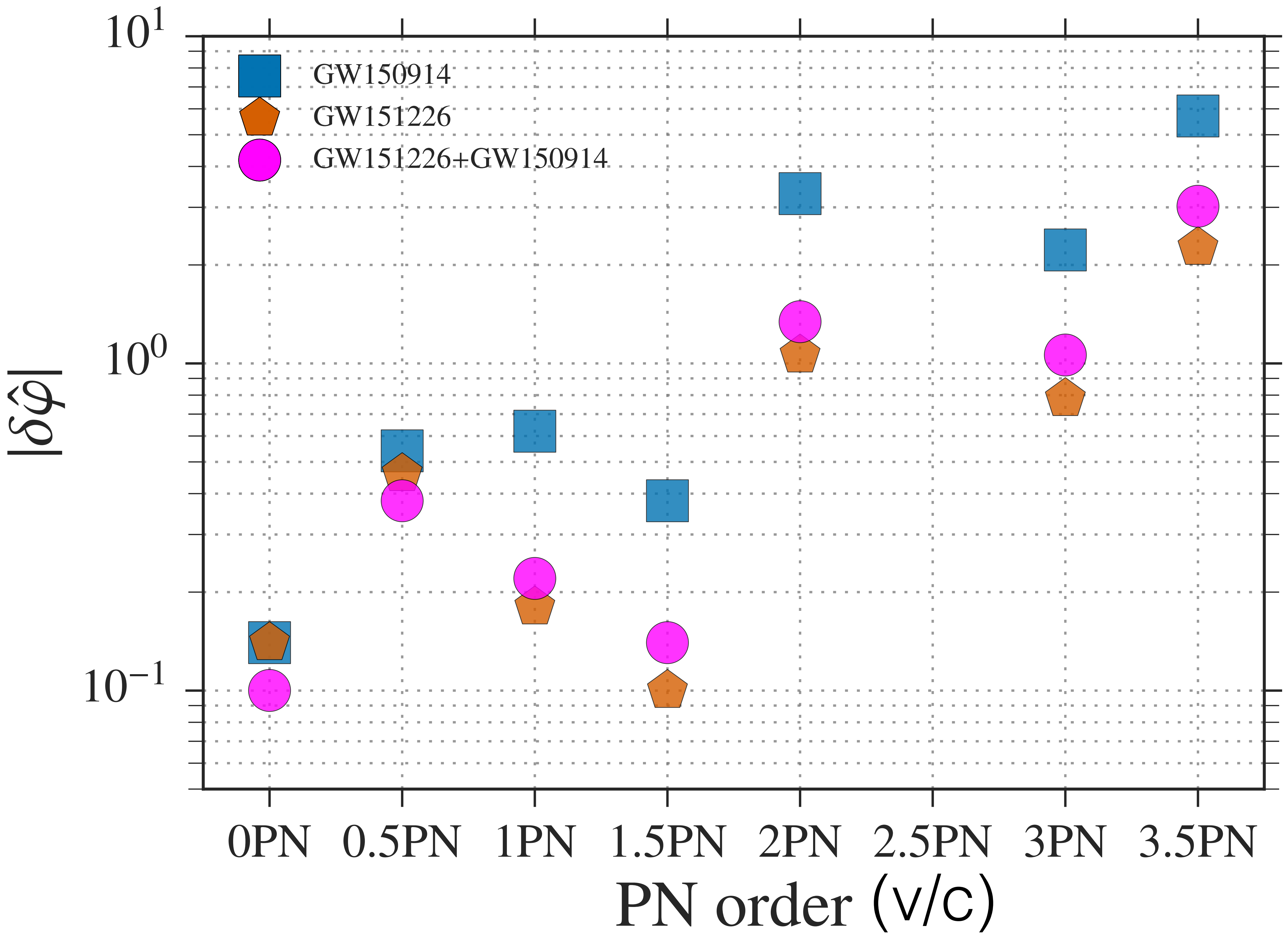
- Bound violations by computing posterior distributions for the $\delta\hat{\varphi}_j$ in concert with the physical parameters of the system

waveform regime	parameter	f–dependence	
early-inspiral regime	$\delta\hat{\varphi}_0$	$f^{-5/3}$	post-Newtonian
	$\delta\hat{\varphi}_1$	$f^{-4/3}$	
	$\delta\hat{\varphi}_2$	f^{-1}	
	$\delta\hat{\varphi}_3$	$f^{-2/3}$	
	$\delta\hat{\varphi}_4$	$f^{-1/3}$	
	$\delta\hat{\varphi}_{5l}$	$\log(f)$	
	$\delta\hat{\varphi}_6$	$f^{1/3}$	
	$\delta\hat{\varphi}_{6l}$	$f^{1/3}\log(f)$	
	$\delta\hat{\varphi}_7$	$f^{2/3}$	
intermediate regime	$\delta\hat{\beta}_2$	$\log f$	effective
	$\delta\hat{\beta}_3$	f^{-3}	
merger–ringdown regime	$\delta\hat{\alpha}_2$	f^{-1}	
	$\delta\hat{\alpha}_3$	$f^{3/4}$	
	$\delta\hat{\alpha}_4$	$\tan^{-1}(af+b)$	

LVC, arXiv:1602.03841

Post-Newtonian constraints





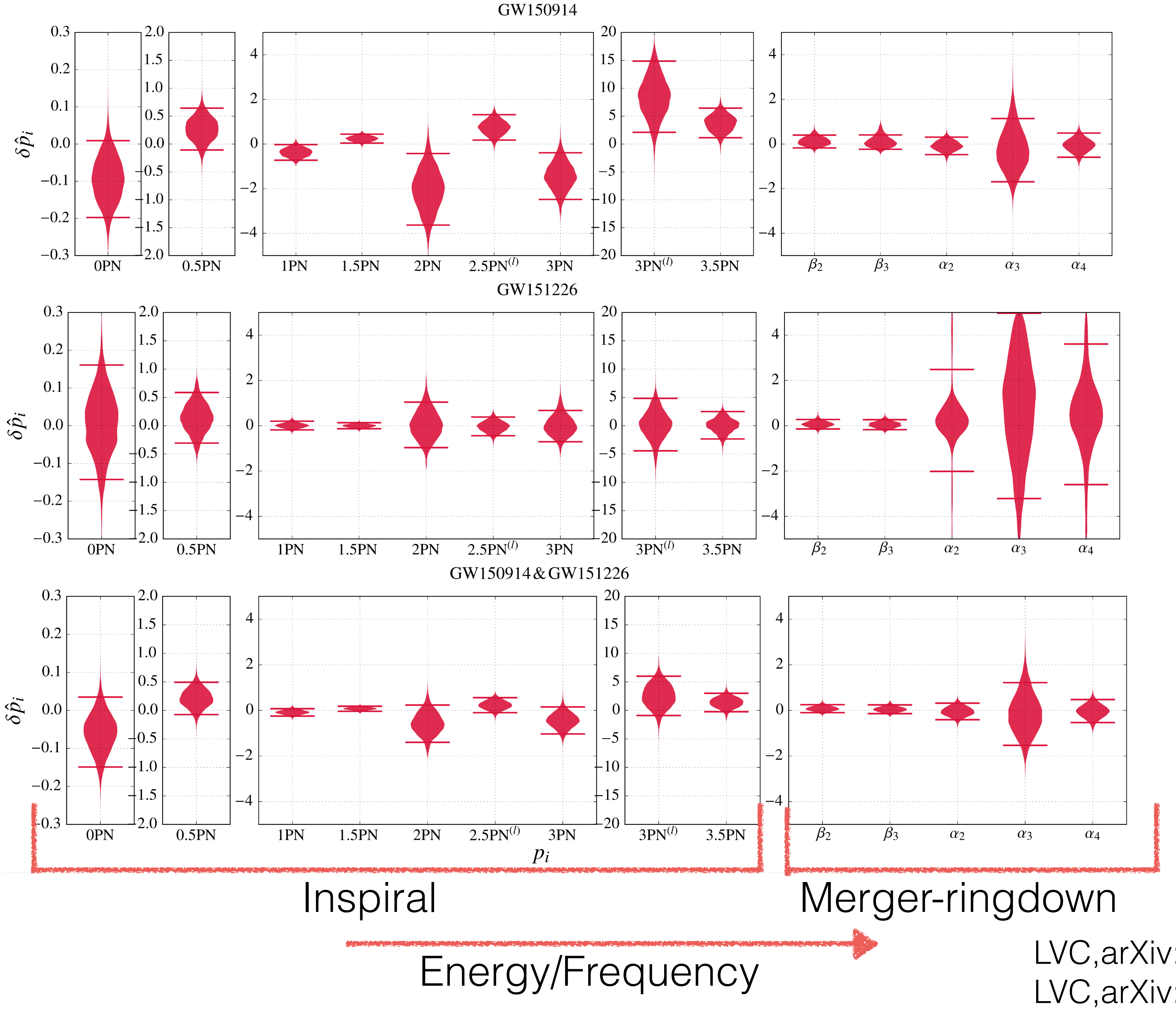
- Constraints not achievable by any other means
- Can be mapped to the space of specific theories (e.g. Yunes+,arXiv: 1603.08955)

LVC,arXiv:1602.03841
LVC,arXiv:1606.04855

Constraints on space-time dynamics

- Only constraints on space-time dynamics

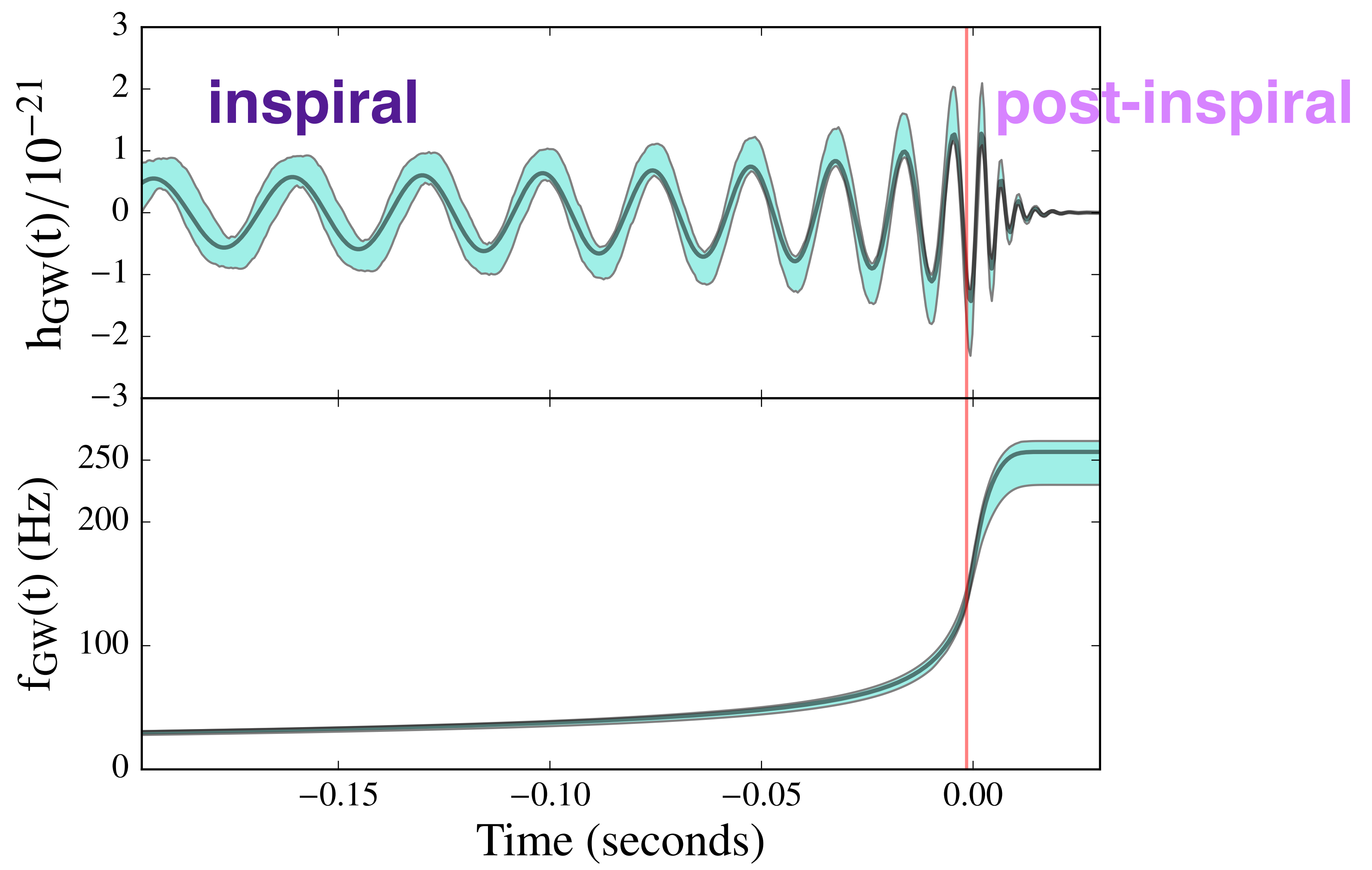
- Posterior distributions for $\delta\hat{\varphi}_j$ show no evidence for violations of GR

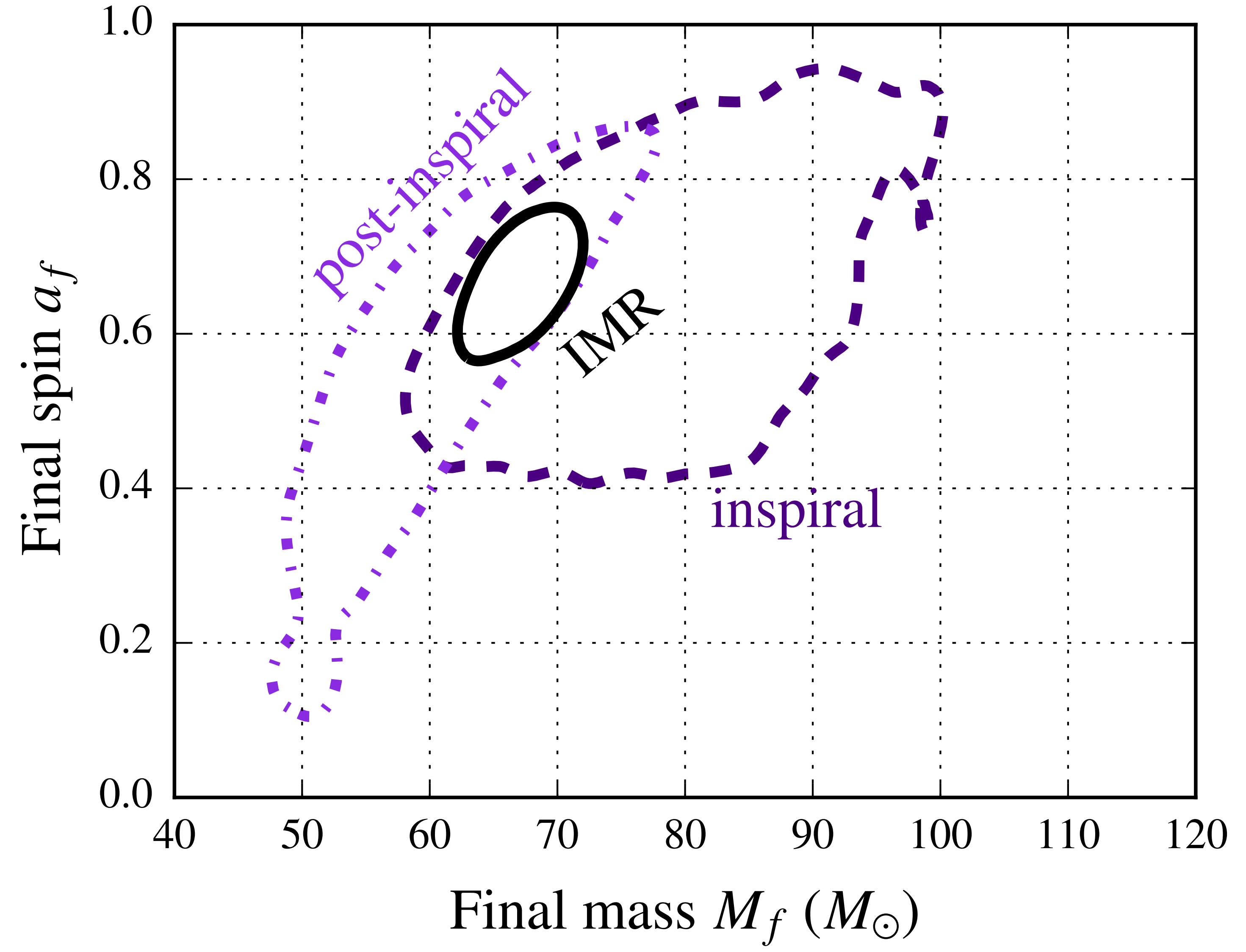


LVC,arXiv:1602.03841
LVC,arXiv:1606.04856

Waveform self-consistency

- The comparison of the final mass and spin predicted from the "inspiral" with the ones measured directly from the "merger-ringdown" is a consistency test on the waveform (Ghosh et al, 2016) and thus, on the corresponding GR solution





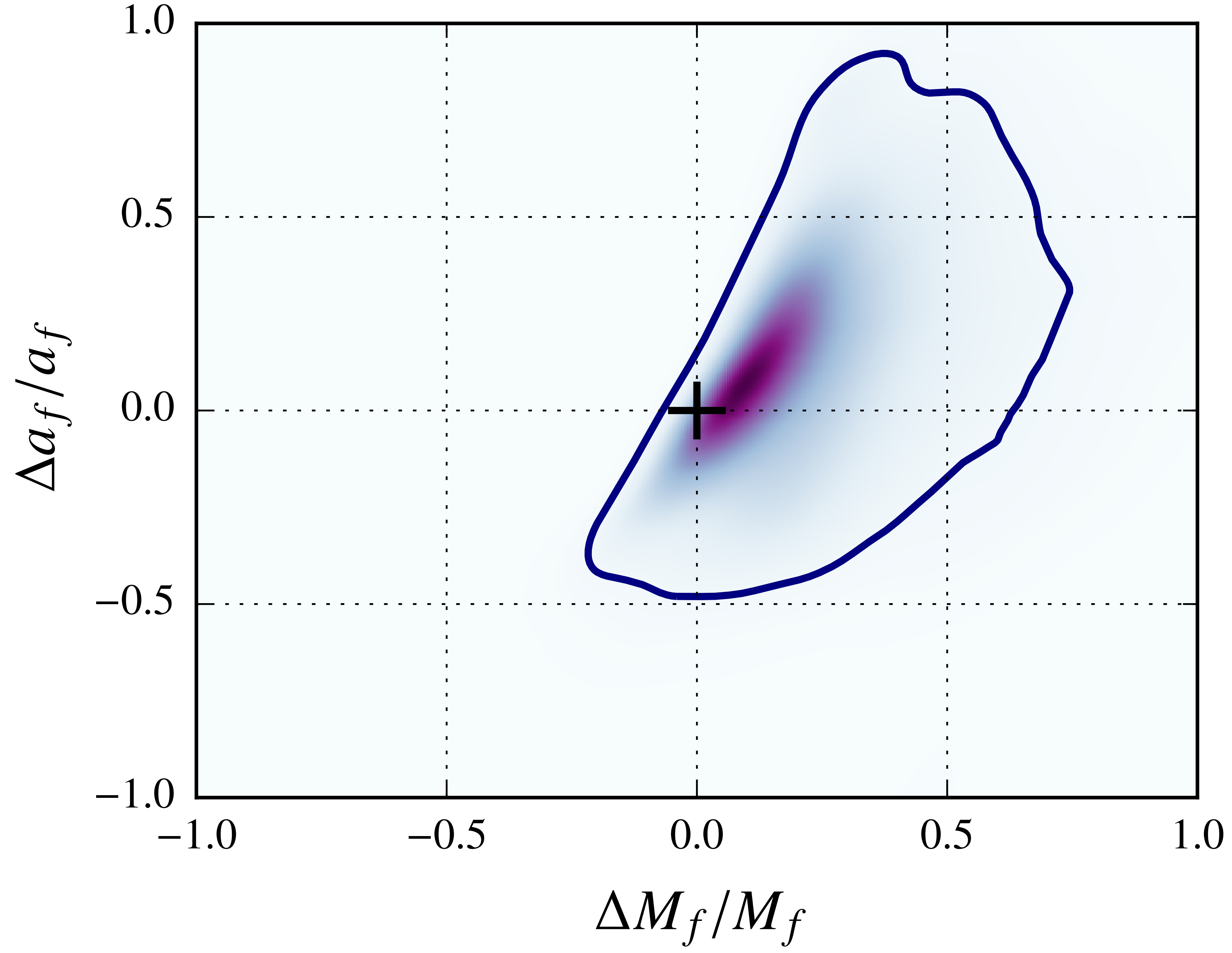
LVC,arXiv:1602.03841

Waveform self-consistency

- Re-parametrising to the relative difference between the inspiral and post-inspiral estimates, one can quantify the confidence level where GR (0,0) lies

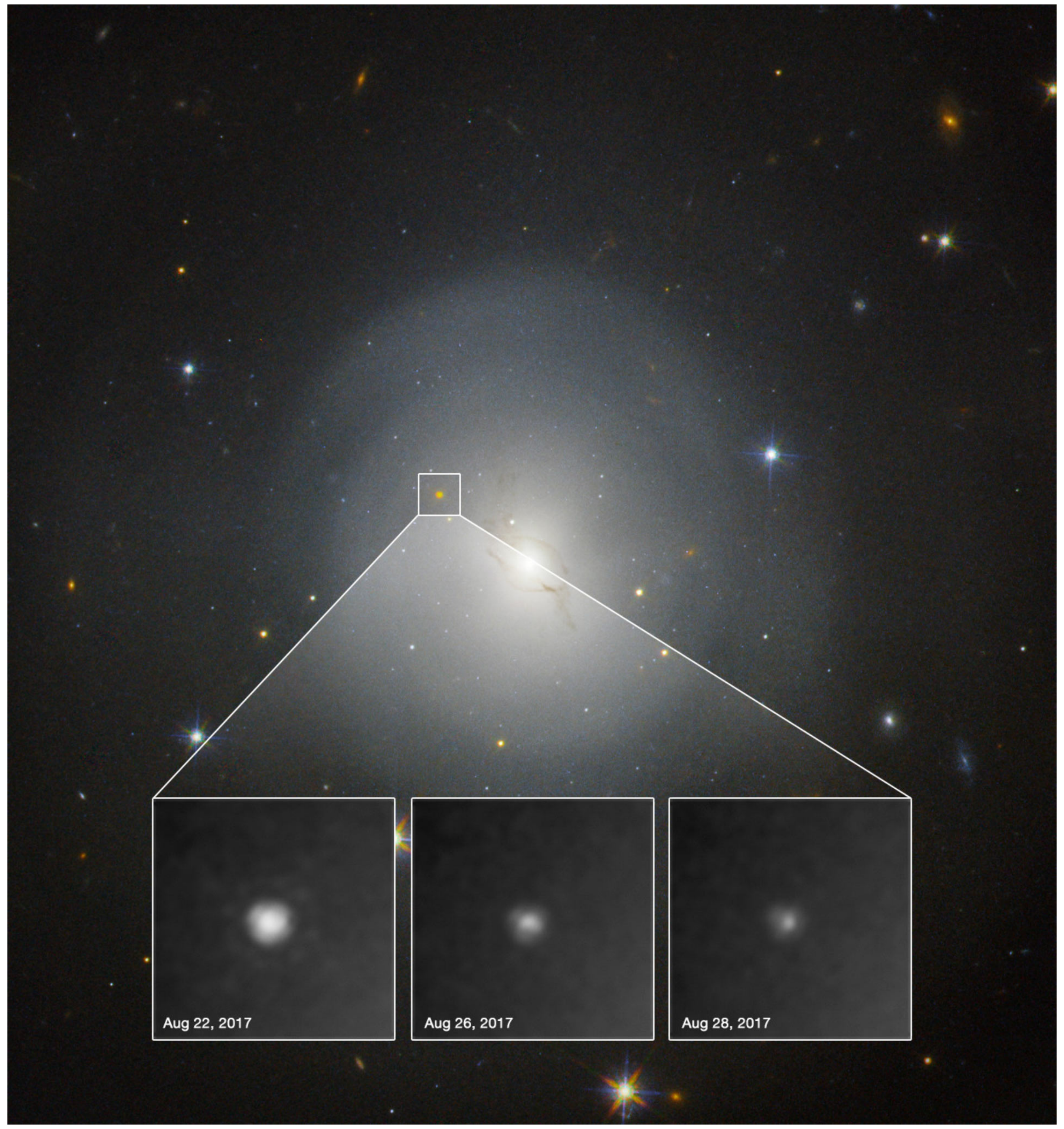
- For GW150914, GR lies on the isoprobability contour that encloses 28% of the posterior

- No evidence for deviations from GR

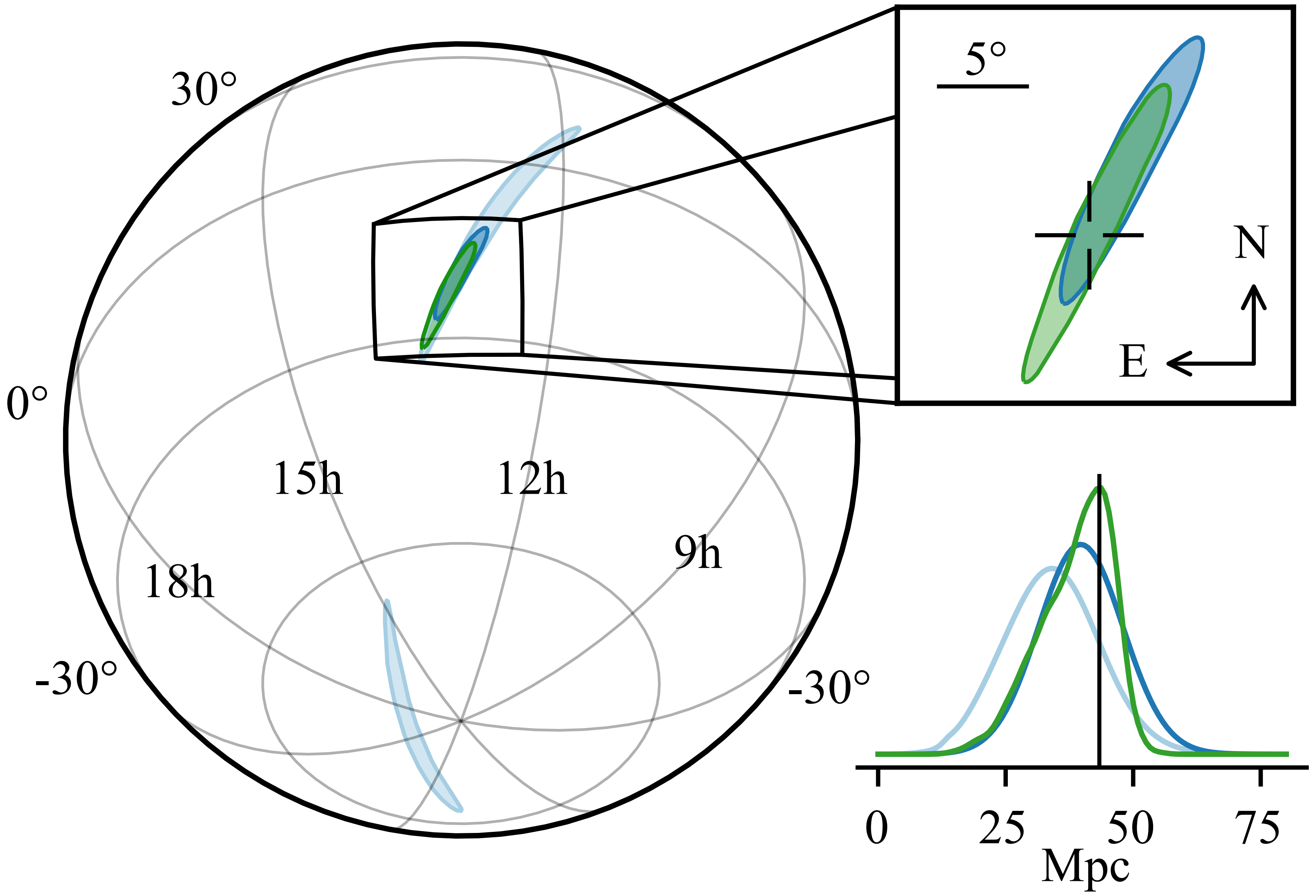


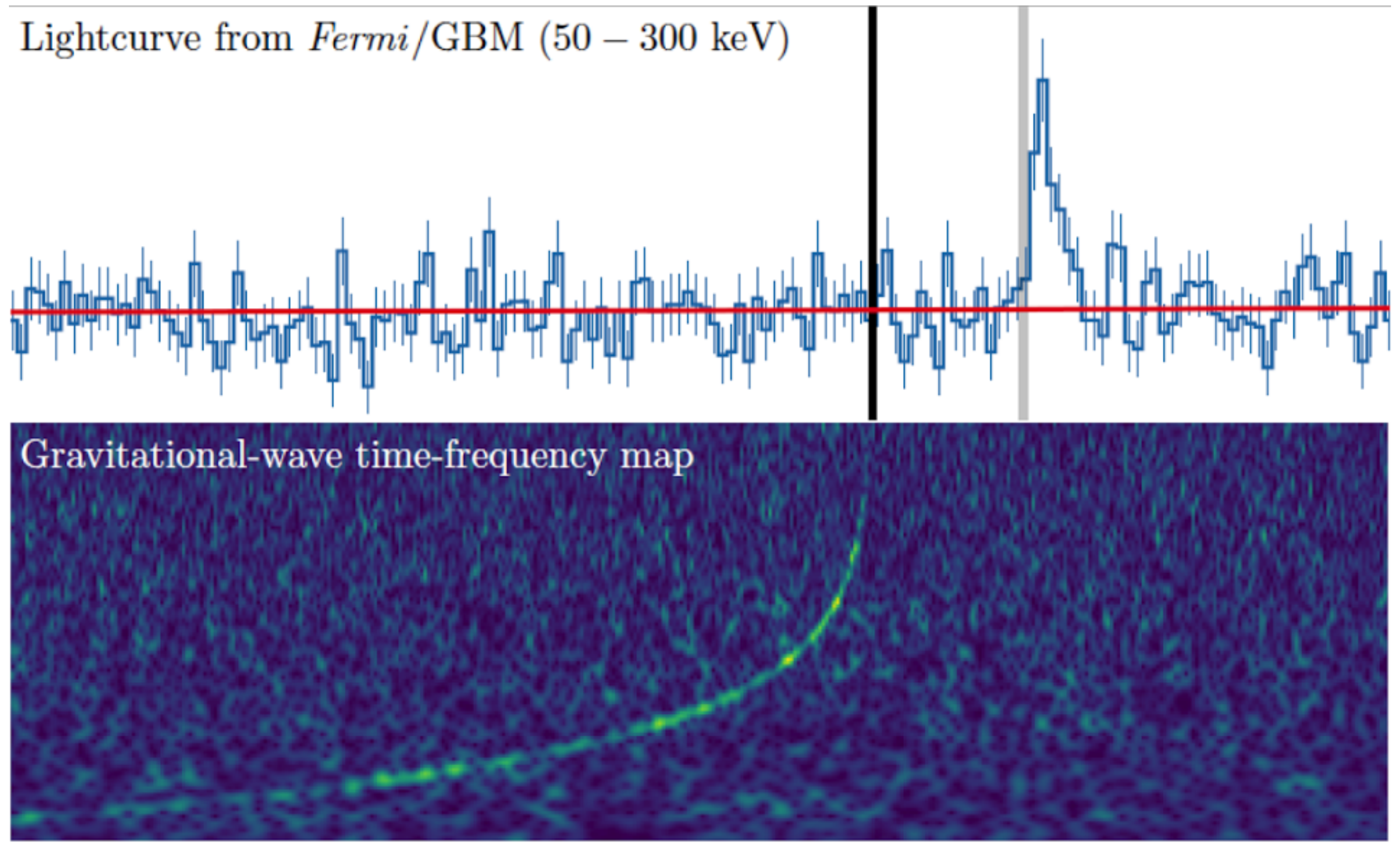
LVC,arXiv:1602.03841

GW170817



- Electromagnetic counterpart!





GW170817

$$-3 \times 10^{-15} \leq \frac{\Delta v}{v_{EM}}{}^{\star} \leq 7 \times 10^{-16}$$

$$\Downarrow$$

$$S = \int d^4x \sqrt{-g} \Big\{ K(\phi, X) - G_3(\phi, X) \Box\phi$$

$$\cancel{+ G_4(\phi, X) R + G_{4,X}(\phi, X) \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) \right]}$$

$$\cancel{+ G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{G_{5,X}(\phi, X)}{6} \left[(\Box\phi)^3 - 3\Box\phi (\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) \right.}$$

$$\cancel{\left. + 2(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla_\sigma \phi)(\nabla^\nu \nabla^\sigma \phi) \right]} \Big\}$$

Many models that could potentially explain the accelerated expansion yet evade solar system constraints via screening have been ruled out

* note that if gravity did not propagate at c, timing of binary pulsars would be impossible (Damour & Deruelle 81)

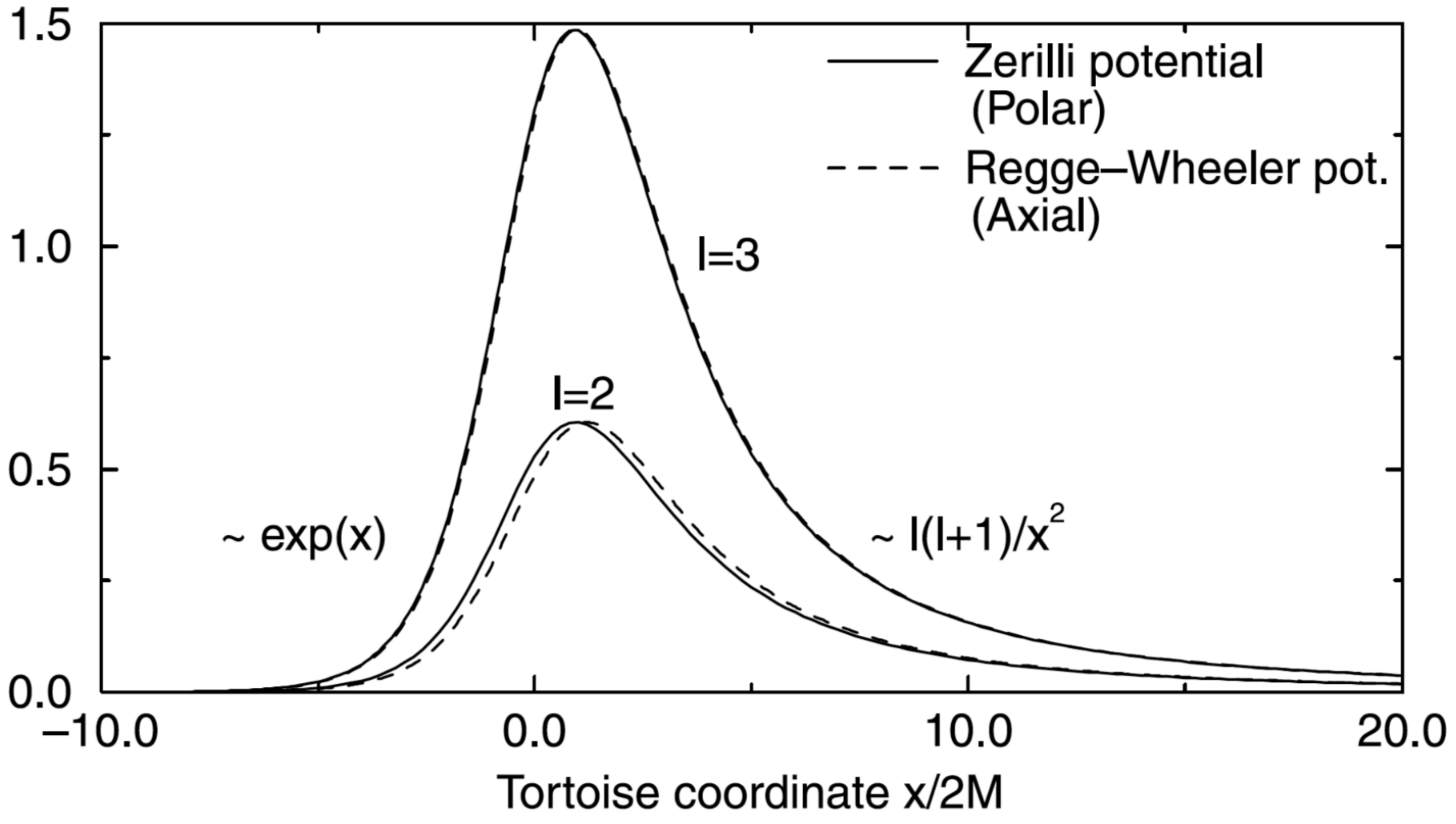
Creminelli, Vernizzi, arXiv:1710.05877
Sakstein, Jain, arXiv:1710.05893
Baker et al, arXiv:1710.06394

On the nature of black holes

BH perturbation theory

- Study linear perturbation around Schwarzschild or Kerr metrics
- Perturbations obey Schrodinger-like equation (Regge & Wheeler 56, Zerilli 70 Teukolski 72)
- Potential barrier around a BH

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + V_{lm}(x)\right)\psi_{lm}(x,t) = 0$$

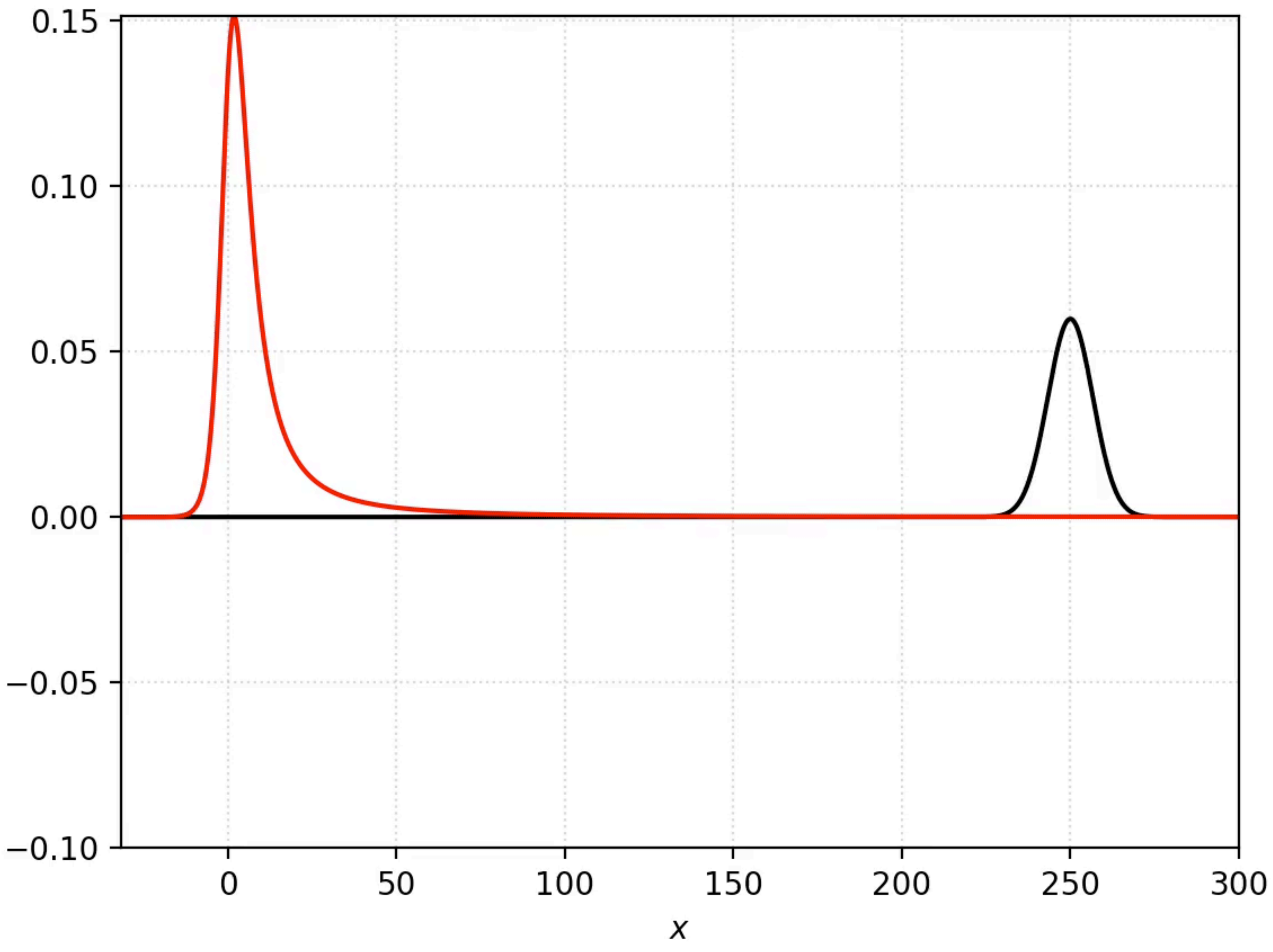


Nollert 99

Quasi-normal modes

- BH responds to perturbations by "ringing" (Vishveshawara 70, Press 71, Ruffini et al, 72, Chandrasekhar 75)
  - Quasi-normal modes excited by light-ring crossing (Goebel 72)
- Waveform

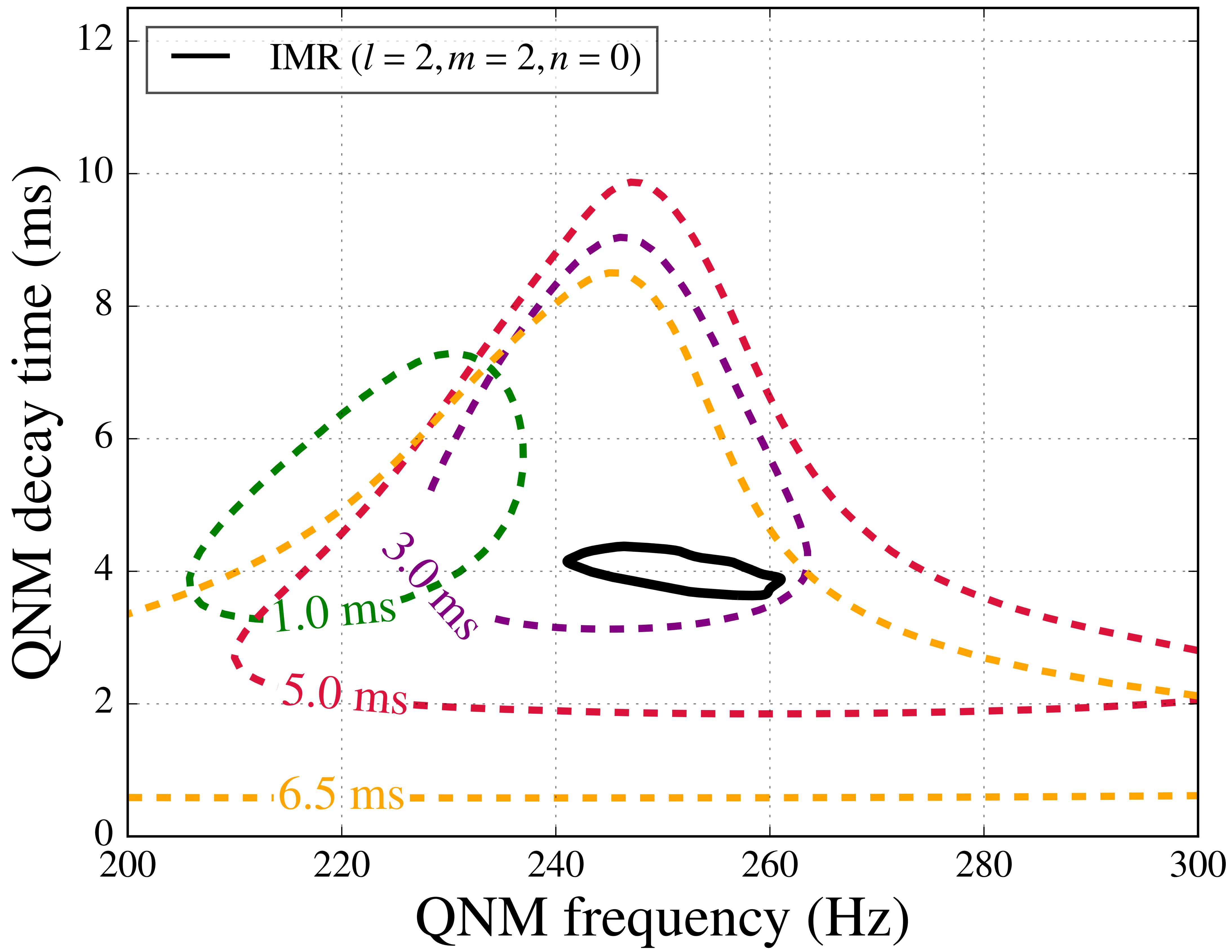
$$h(t) = \sum_{nlm} A_{nlm} e^{-\frac{t-t_0}{\tau_{nlm}}} \cos(\omega_{nlm}(t-t_0) + \varphi_{nlm})$$



The nature of the final object

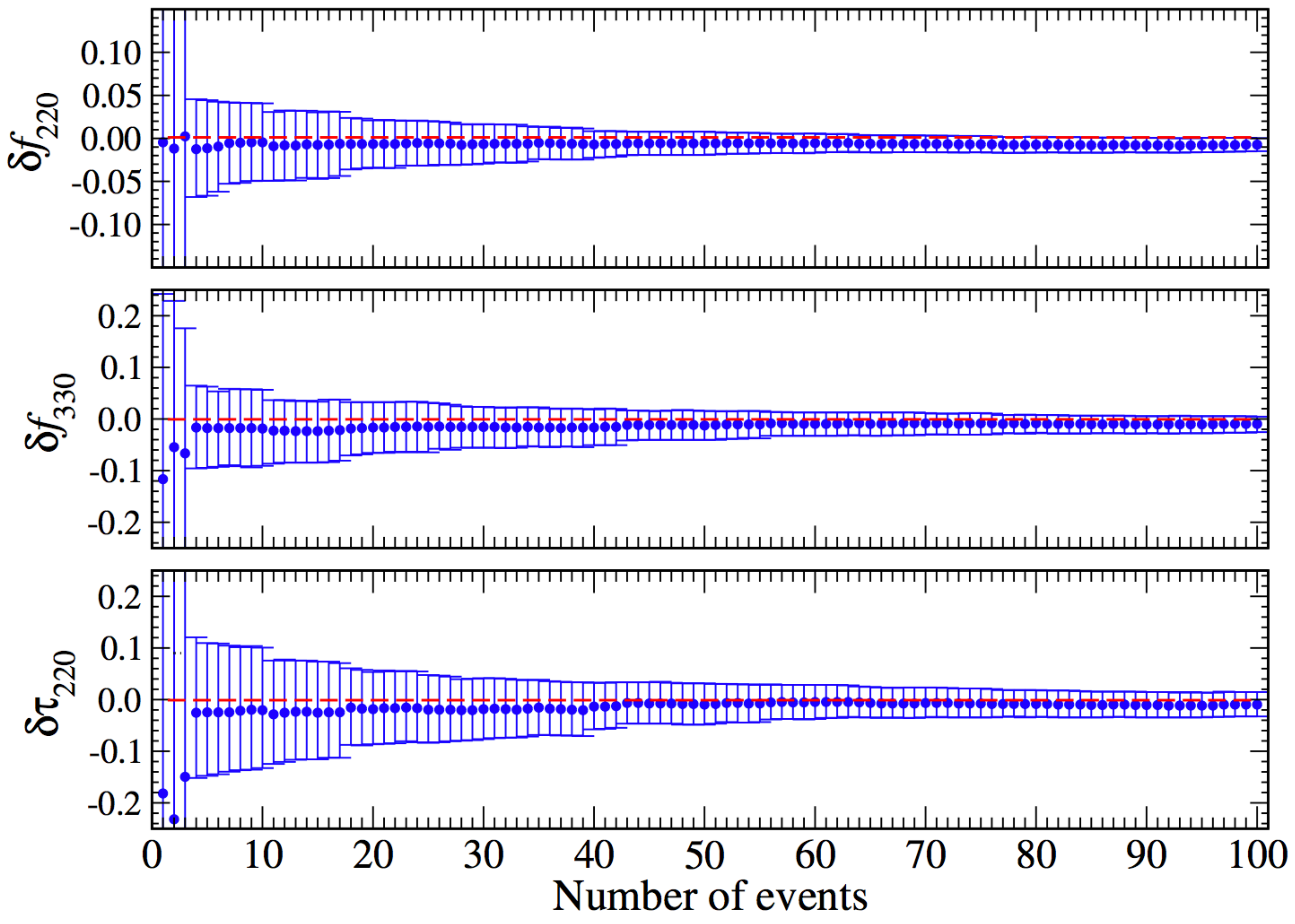
- Central frequencies ω_{nlm} and decay times τ_{nlm} are functions of BH mass and spin only (manifestation of the BH uniqueness hypothesis, Berti et al, arXiv:0512160)

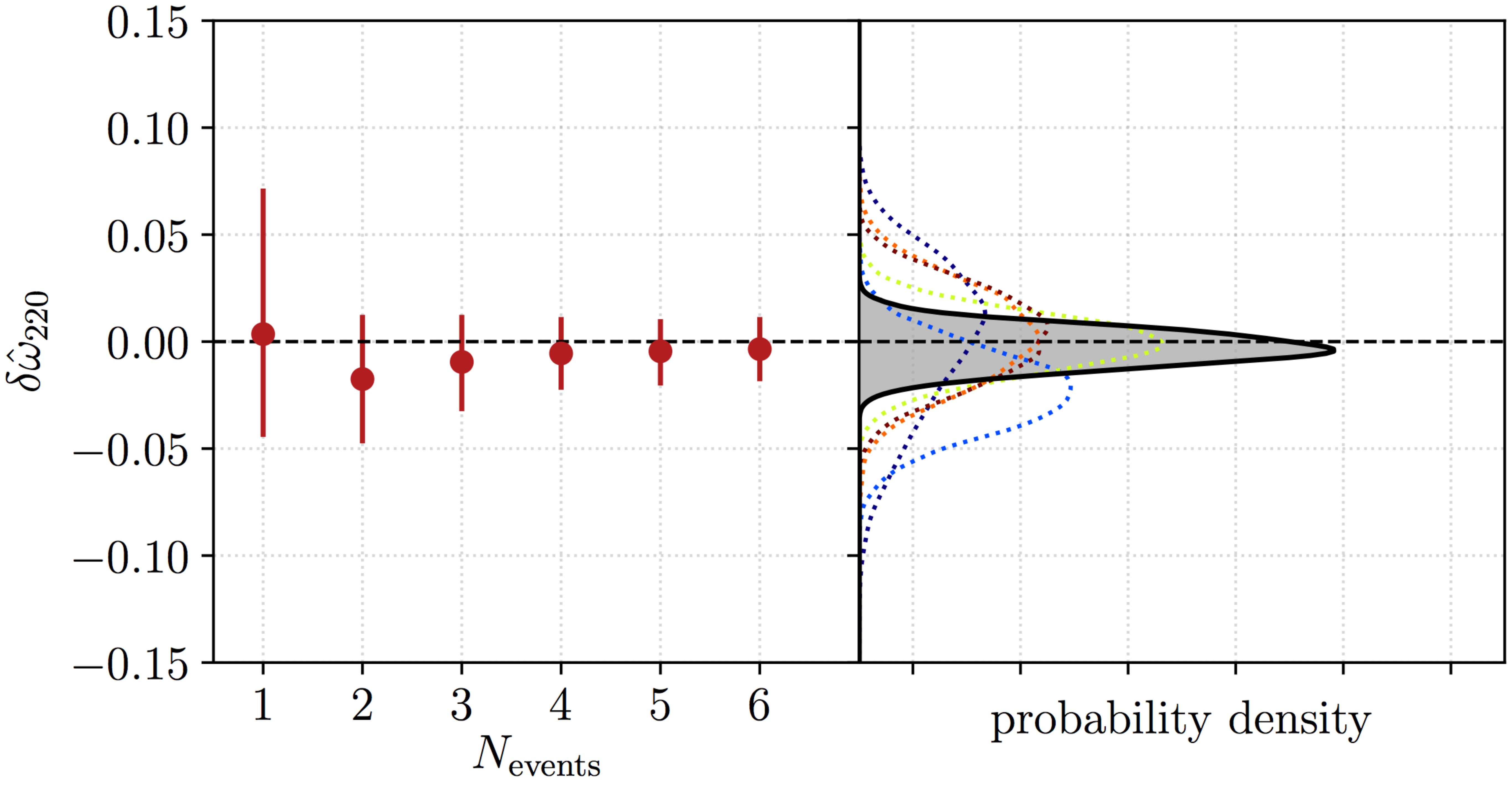
- Need at least two modes detected to test BH nature and “no-hair” theorem (e.g. Gossan et al, arXiv:1111.5819, Meidam et al,arXiv:1406.3201)



LVC,arXiv:1602.03841

Prospects for ringdown constraints

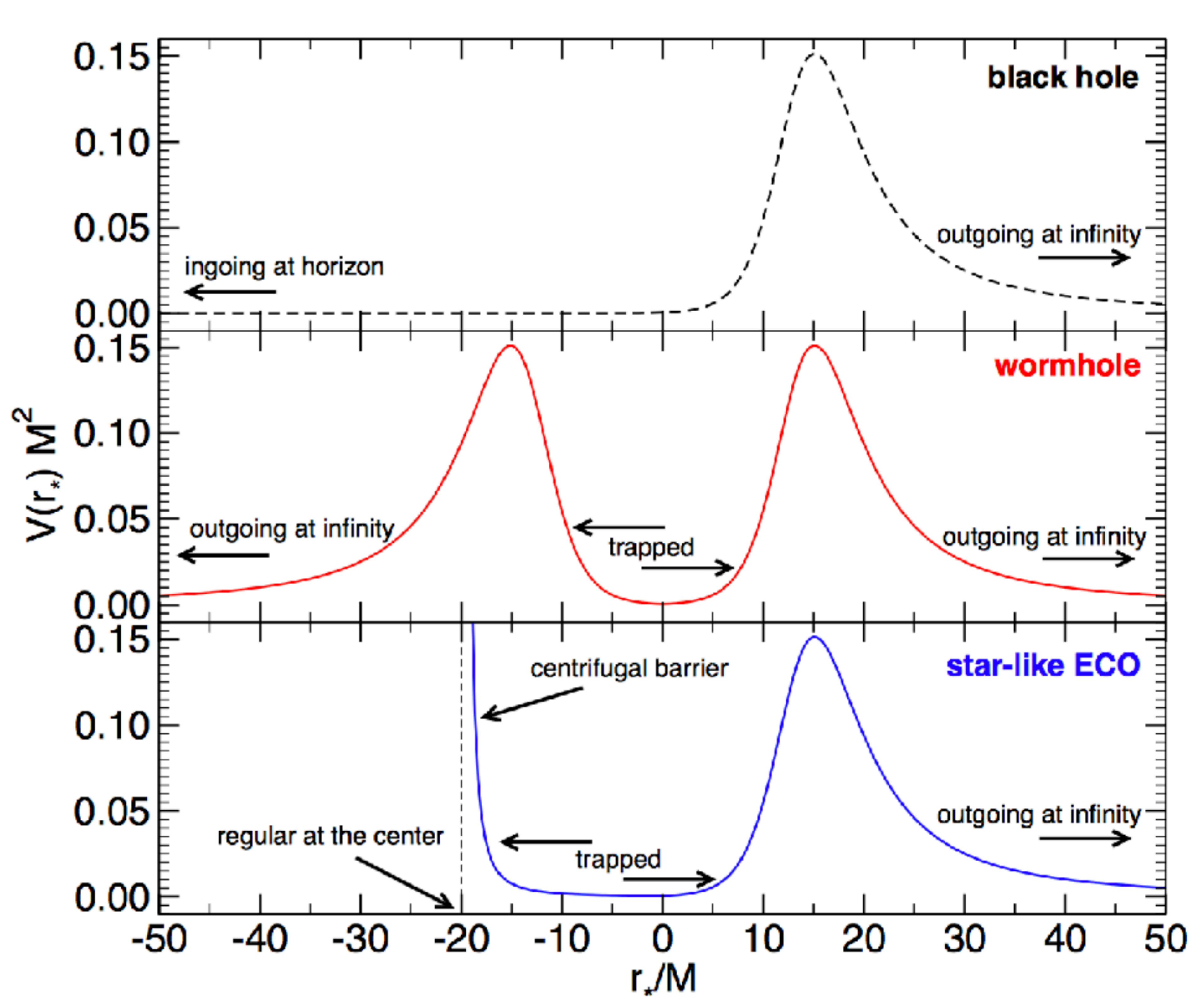


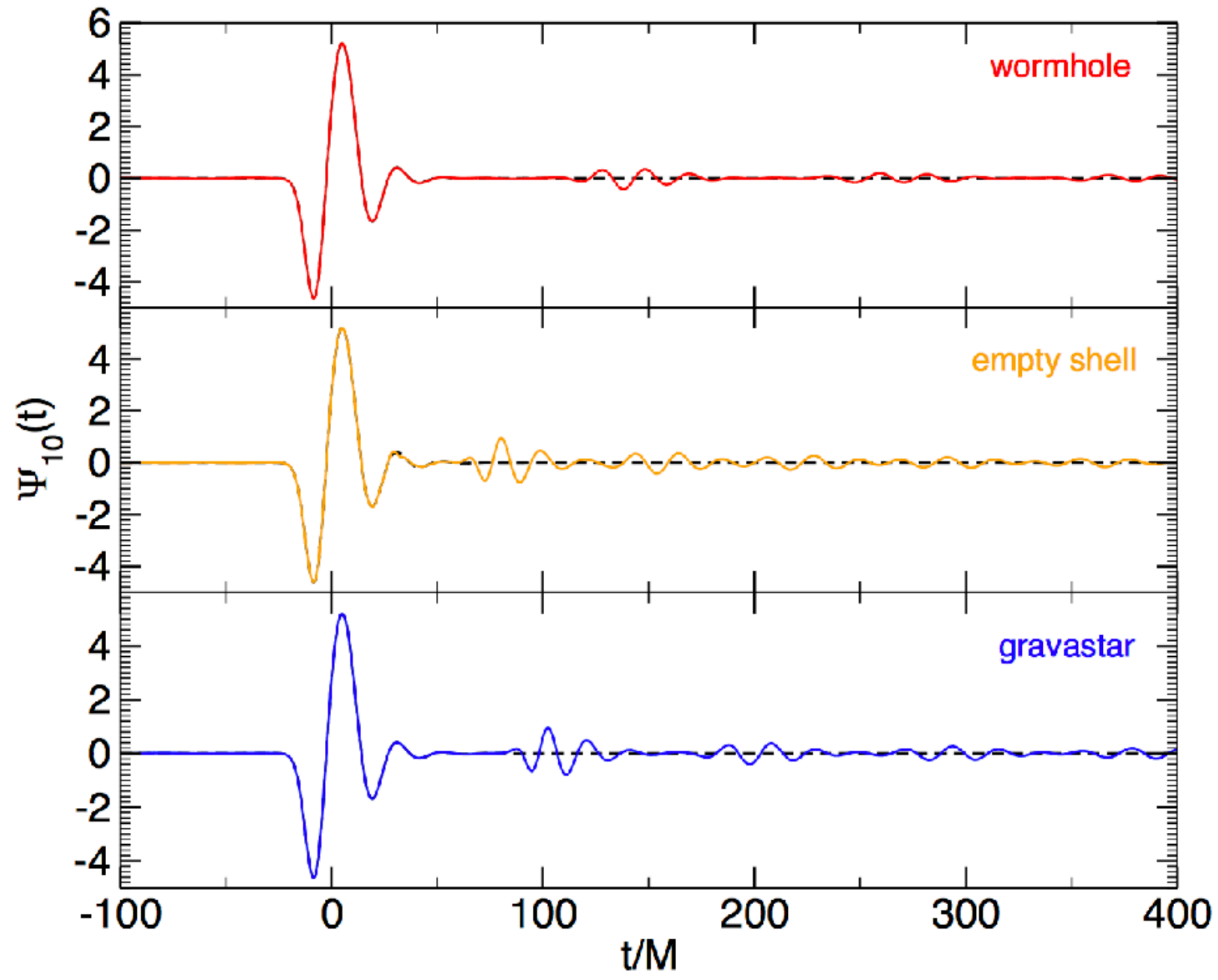


Brito et al, arXiv:1805.00293

Carullo et al, arXiv:1805.04760

Echoes



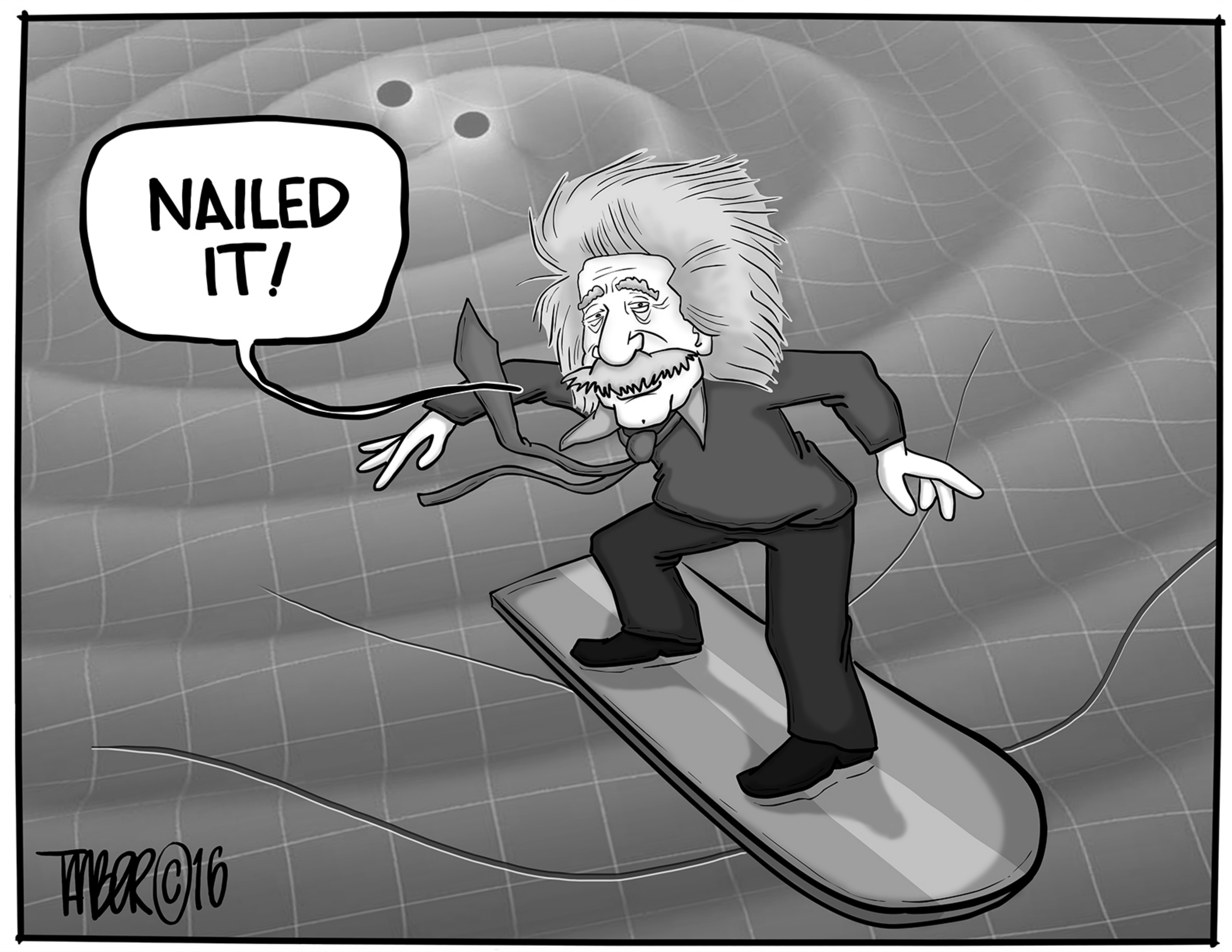


- Claims of echoes (Abedi et al arXiv:1612.00266) unconfirmed (Westerweck et al, arXiv:1712.09966)

Cardoso et al, arXiv:1608.08637

Conclusions

- The era of GW astrophysics is started
- First glimpse at space-time extreme regimes:
  - **BBHs and GW behave just like GR predicts**
- Just the beginning:
  - many more detections in the future
  - improved sensitivities
  - multi-wavelength studies
- Look forward to a prolific season in gravitational physics
  - NS equation of state
  - Cosmography



Conclusions II

- Nature of black holes
  - Technical challenges related to ringdown detection
  - BH uniqueness?
  - BH thermodynamics?
  - Quantum nature of space-time?