Gravitational Waves and Fundamental Physics

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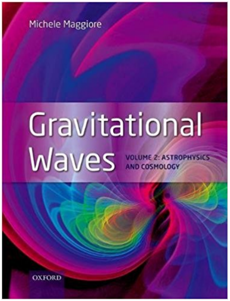
APC 2018

The first detections of binary coalescence opens up the era where we begin to explore the Universe using GWs.

We will focus on three selected topics:

- Dark energy and GWs
- Stochastic backgrounds of GWs of cosmological origin
- (BH quasi-normal modes)

More details in
MM, *Gravitational Waves*, Vol. 2, 848 p., Oxford Univ. Press 2018.



I. Dark energy and GWs

GWs from coalescing binaries provide an absolute measurement of the distance to the source. To lowest order

$$
\begin{aligned}
h_+(t) &= \frac{4}{r}\,\frac{1+\cos^2\iota}{2}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f}{c}\right)^{2/3}\cos(2\pi f t)\\
h_\times(t) &= \frac{4}{r}\cos\iota\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f}{c}\right)^{2/3}\sin(2\pi f t)
\end{aligned}
$$

$$\dot f = \frac{96}{5}\,\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f^{11/3} \qquad\qquad M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

$\rightarrow$ measure r without the need of calibration (“standard sirens")

For sources at cosmological distances $\qquad \frac{1}{r} \longrightarrow \frac{1}{d_L}$

$$d_L(z) = \frac{1+z}{H_0} \int_0^z \frac{d\tilde{z}}{\sqrt{\Omega_M (1+\tilde{z})^3 + \rho_{\rm DE}(\tilde{z})/\rho_0}}$$

$[\Omega_M = \rho_M(0)/\rho_0, \quad \rho_0 = 3H_0^2/(8\pi G)]$

- low z: Hubble law, $d_L \simeq H_0^{-1} z$ (GW170817)
- moderate z: access $\Omega_M, \rho_{\rm DE}(z)$

Need an independent determination of z (electromagnetic counterpart, statistical methods)

$\rightarrow$ multi-messenger astronomy

– low z:

Planck 2018+BAO+SNe: $H_0 = 68.34 \pm 0.83$
local measurements (Riess et al.): $H_0 = 73.48 \pm 1.66$

3.7σ discrepancy: indication for deviation from ΛCDM?

GW170817: $H_0 = 70.0^{+12.0}_{-8.0}$
$O(50-100)$ standard sirens at advanced LIGO/Virgo needed to arbitrate the discrepancy

– moderate z: non-trivial DE? $p_{\rm DE} = w_{\rm DE}\rho_{\rm DE}$

typical parametrization $w_{\rm DE}(z) = w_0 + \frac{z}{1+z} w_a$

Planck 2018+BAO+SNe:

w_0 only: $w_0 = -1.0281 \pm 0.031$

(w_0, w_a): $w_0 = -0.961 \pm 0.077, \quad w_a = -0.28^{+0.31}_{-0.27}$

Modified GW propagation

Belgacem, Dirian, Foffa, MM (2017,2018)

in GR, in a FRW metric $ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$

$$\tilde{h}_A'' + 2\mathcal{H}\tilde{h}_A' + k^2\tilde{h}_A = 0 \qquad (\mathcal{H} \equiv a'/a)$$

$$\tilde{h}_A(\eta, \mathbf{k}) = \frac{1}{a(\eta)}\tilde{\chi}_A(\eta, \mathbf{k}) \qquad \rightarrow \qquad \tilde{\chi}_A'' + \left(k^2 - \frac{a''}{a}\right)\tilde{\chi}_A = 0$$

inside the horizon $\quad a''/a \ll k^2 \quad \rightarrow \quad \tilde{\chi}_A'' + k^2\tilde{\chi}_A \simeq 0$

1. GWs propagate as the speed of light
2. $h_A \propto 1/a$. For coalescing binaries this gives $h_A \propto 1/d_L(z)$

In several modified gravity models

$$\tilde{h}_A'' + 2\mathcal{H}[1 - \delta(\eta)]\tilde{h}_A' + k^2\tilde{h}_A = 0$$

$$\tilde{h}_A(\eta, \mathbf{k}) = \frac{1}{\tilde{a}(\eta)}\tilde{\chi}_A(\eta, \mathbf{k}) \qquad \text{where} \qquad \frac{\tilde{a}'}{\tilde{a}} = \mathcal{H}[1 - \delta(\eta)]$$

then

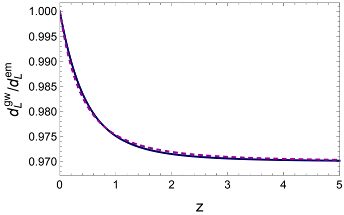
$$\tilde{\chi}_A'' + \left(k^2 - \frac{a''}{a}\right)\tilde{\chi}_A = 0 \quad \rightarrow \quad \tilde{\chi}_A'' + k^2\tilde{\chi}_A \simeq 0$$

1. GWs still propagate as the speed of light (ok with GW170817)
2. $h_A \propto 1/\tilde{a}$

the “GW luminosity distance" is different from the standard (electromagnetic) luminosity distance !

$$d_L^{\rm gw}(z) = d_L^{\rm em}(z) \exp\left\{ - \int_0^z \frac{dz'}{1+z'}\, \delta(z') \right\}$$

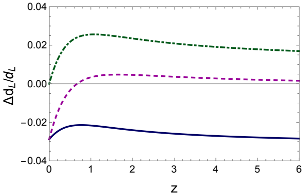
Deffayet and Menou 2007
Saltas et al 2014
Lombriser and Taylor 2016
Nishizawa 2017
Belgacem et al 2017, 2018



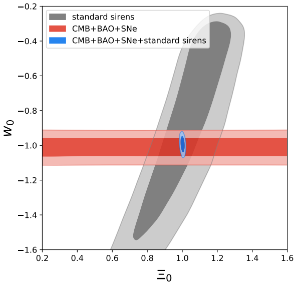
well fitted by

$$\frac{d_L^{\rm gw}(z)}{d_L^{\rm em}(z)} = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n}$$

(Ξ_0, n) parametrize deviations in the tensor sector



For a generic modified gravity model Ξ_0 is more easily observable than w_0 because of the degeneracies with H_0, Ω_M !



Forecasts for the Einstein Telescope

ET could detect $\sim 10^5 - 10^6$ BNS/yr up to large z; assume 10^3 em counterparts

(more detailed modelization of the GRB detection in progress, with T.Regimbau and E. Howell)

$$\Delta w_0 = 3.2\%, \quad \Delta \Xi_0 = 0.8\%$$

Forecast for LISA in progress (with the LISA CosmoWG)

II. Stochastic backgrounds of GWs

GWs allow us to probe early Universe cosmology down to a primordial epoch unaccessible to electromagnetic observations

Particles decouple from the primordial plasma when the reaction rate $\Gamma < H(t)$

- Photons decouple shortly after recombination, $T \simeq 0.26$ eV, $z \simeq 1090$
- For neutrinos

$$\frac{\Gamma}{H} \sim \frac{G_F^2 T^5}{T^2/M_{\rm Pl}} \simeq \left(\frac{T}{1{\rm MeV}}\right)^3$$

- For gravitons

$$\frac{\Gamma}{H} \sim \left(\frac{T}{M_{\rm Pl}}\right)^3$$

Some definitions

$$h_{ij}(t,\mathbf{x}) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d^2\hat{\mathbf{n}}\, \tilde{h}_A(f,\hat{\mathbf{n}})\, e_{ij}^A(\hat{\mathbf{n}})\, e^{-2\pi i f(t-\hat{\mathbf{n}}\cdot\mathbf{x}/c)}$$

$$\langle \tilde{h}_A^*(f,\hat{\mathbf{n}})\tilde{h}_{A'}(f',\hat{\mathbf{n}}')\rangle = \delta(f-f')\,\frac{\delta^2(\hat{\mathbf{n}},\hat{\mathbf{n}}')}{4\pi}\,\delta_{AA'}\,\frac{1}{2}S_h(f)$$

$$\begin{aligned}\rho_{\rm gw} &= \frac{1}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle \\ &\equiv \int_{f=0}^{f=\infty} d(\log f)\;\frac{d\rho_{\rm gw}}{d\log f}\end{aligned}$$

The critical density of the Universe is $\rho_c = 3H_0^2/(8\pi G)$.

$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_c}\,\frac{d\rho_{\rm gw}}{d\log f} = \frac{4\pi^2}{3H_0^2} f^3 S_h(f)$$

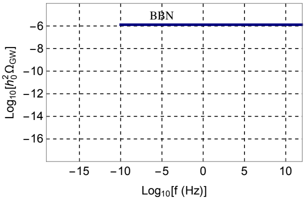
$h_0^2\Omega_{\rm gw}(f)$ is independent of the uncertainty on H_0

What do we know on $h_0^2\Omega_{\rm gw}(f)$?

BBN is a balance between nuclear reaction rate and the expansion rate.

Any extra energy at time of BBN alters it. $\Rightarrow$ limit on $\rho_{\rm GW}$

$$\int_{f=f_{\rm BBN}}^{f=\infty} d(\log f)\ h_0^2\Omega_{\rm gw}(f) < 1.3\times 10^{-6}$$



Only applies to modes that were inside the horizon at time of BBN

Note the huge range of frequencies on the horizontal axis!

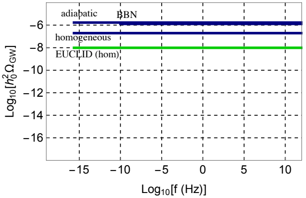
Limits on extra radiation at CMB.

Extra radiation at the epoch of RD-MD equilibrium shifts the epoch of RD-MD transition. This affects the acoustic peaks and delays structure formation

If the perturbations induced by GWs are adiabatic, they act as extra neutrinos:

$$\int_{f=f_{\rm CMB}}^{f=\infty} d(\log f)\ h_0^2\Omega_{\rm gw}(f) < 1.7\times 10^{-6}$$

More likely, for GWs generated by cosmic string, phase transitions, etc. use homogeneous initial conditions. The degeneracy with neutrinos is broken, stronger limits



How to detect stochastic GW backgrounds at interferometers?
For a single detector, it is just an extra noise!

For a single GW signal $s(t) = n(t) + h(t)$. If we know $h(t)$:

$$\int_0^T dt\, s(t)h(t) = \int_0^T dt\, n(t)h(t) \;+ \int_0^T dt\, h^2(t) = O(T^{1/2}) + O(T)$$

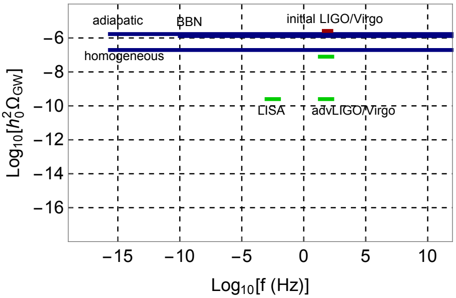
Optimal filtering: $S = \int_0^T dt\, s(t)K(t), \qquad \tilde{K}(f) = \tilde{h}(f)/S_n(f)$

For a stochastic GW signal, we do not know $h(t)$, but we can perform correlations between two detectors:

$$S = \int_0^T dt_1 \int_0^T dt_2\, s_1(t)s_2(t)K(t_1 - t_2)$$

again, $S/N \propto T^{1/2}$

Existing limits from adv LIGO/Virgo:
$h_0^2 \Omega_{\rm gw} < 7.9 \times 10^{-8}$ in the band $20 - 86$ Hz

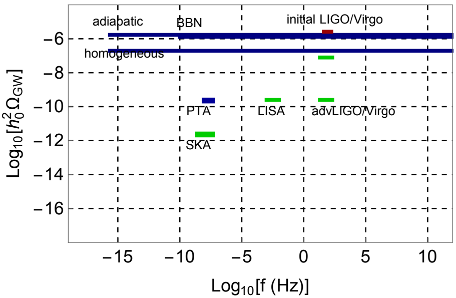


LISA competitive as a single intf because $\Omega_{\rm gw}(f) \propto f^3 S_h(f)$

Pulsar timing arrays (EPTA, NANOGrav,PPTA, forming the IPTA)

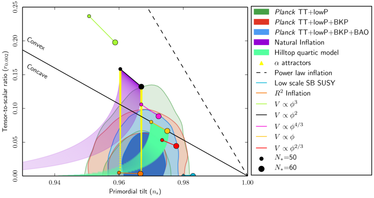
monitor networks of stable ms pulsars. Sensitive to $f \simeq 1/T$ ($T \sim 10$ yr $\to f \sim$ nHz)

$h_0^2\Omega_{\rm gw}(f) < 2.3 \times 10^{-10}$ @ $f = 6.3$ nHz (PPTA)
$h_0^2\Omega_{\rm gw}(f) < 4.2 \times 10^{-10}$ @ $f = 2.8$ nHz (NANOGrav)
$h_0^2\Omega_{\rm gw}(f) < 4.1 \times 10^{-9}$ @ $f = 2.8$ nHz (EPTA)

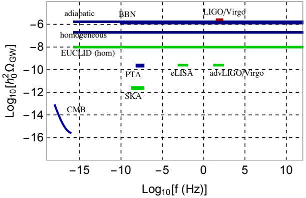


Limits on GWs from CMB

Temperature anisotropies:



Planck, paper XX (2015)



Polarization anisotropies:

scalar perturbations only generate E-modes
tensor perturbations generate both E- and B-modes

Joint analysis of data from BICEP2/Keck Array and Planck.

- strong evidence for dust and no statistically significant evidence for tensor modes
- $r_{0.05} < 0.12$ at 95% c.l. ($k_* = 0.05\,\mathrm{Mpc}^{-1}$)

Theoretical expectations

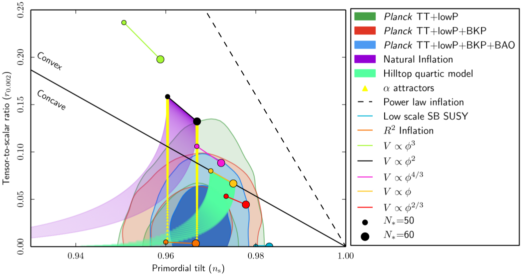
PTA limits possibly in tension with some current models of SMBH mergers, but theoretical uncertainties are still large

Ingredients:

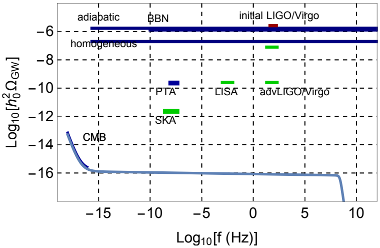
- measured galactic merger rate
- all large galaxies host SMBHs
- radiation-reaction from GWs alone is not sufficient to induce coalescence within the age of the Universe. Three-body scattering on stars or friction against circumbinary gas is needed (and is expected)

  On the other hand, if the coupling to the environment is too efficient, the time for GW emission is reduced

Planck data already rule out some inflationary models

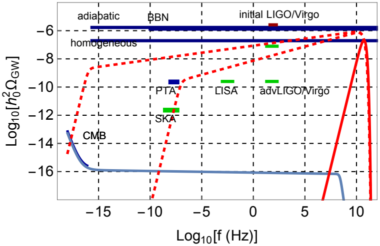


Inflationary prediction from amplification of vacuum fluctuations



Several (more hypothetical) generation mechanism have been studied

Alternatives to inflation
e.g., pre-big-bang



1st-order phase transition
at the EW scale
(requires extensions of the SM)

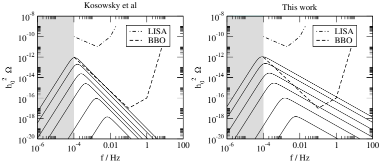
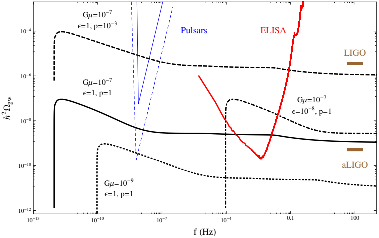


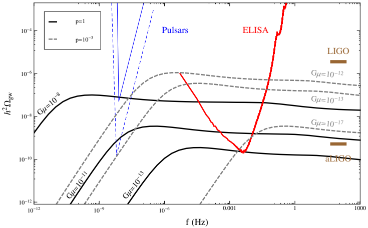
figure from Huber and Konstandin (2008)

Cosmic strings

small initial loop size



large initial loop size



from Binetruy, Bohe, Caprini and Dufaux (2012)

the background due to bursts from cusps and kinks on cosmic strings is also potentially detectable at advanced LIGO/Virgo or at eLISA

Damour and Vilenkin (2001)

several other interesting mechanisms proposed (e.g. from pre-heating after inflation, field condensates, etc.)

Conclusions

- Stochastic GW background are being actively searched over a huge range of frequencies, $10^{-17} - 10^{3}$ Hz
- temperature anisotropies and B-mode polarization of CMB already exclude plausible inflationary models
- PTA results already challenge some models of GWs from SMBH
- Several production mechanisms predict signals detectable at advanced IFOs or PTA.

A new territory will be explored in the next few years

III. BH quasi-normal modes. The vibrations of pure space-time configurations

By now a classic chapter of GR. A long history, going back to works of Regge and Wheeler (1957), Zerilli (1970), Vishveshwara (1970), Press (1971), Teukolsky (1973), Chandrasekhar (1975,1983), Chandrasekhar and Detweiler (1975), ...

A simpler example: scalar field on a Schwarzschild background

$$\Box\phi \equiv (-\bar{g})^{-1/2}\partial_\mu \left[(-\bar{g})^{1/2}\bar{g}^{\mu\nu}\partial_\nu\right]\phi = 0$$

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$A(r) = 1 - \frac{R_S}{r}\,, \qquad B(r) = \frac{1}{A(r)}\,, \qquad R_S = 2GM$$

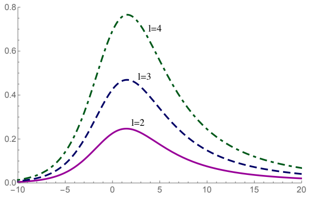
$$\begin{aligned}\phi(t,\mathbf{x}) &= \frac{1}{r}\sum_{l,m} u_{lm}(t,r) Y_{lm}(\theta,\phi) \\ r_* &\equiv r + R_S \log\frac{r-R_S}{R_S} \\ u_{lm}(t,r) &= \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\tilde{u}_{lm}(\omega,r)e^{-i\omega t}\end{aligned}$$

and we get

$$\left[-\frac{d^2}{dr_*^2} + V_l(r)\right]\tilde{u}_{lm} = \omega^2\,\tilde{u}_{lm}$$

where

$$V_l(r) = A(r)\left[\frac{l(l+1)}{r^2} + \frac{R_S}{r^3}\right]$$



Boundary conditions

at $r_* \to \pm\infty$ becomes a free wave equations, with solutions $e^{-i\omega(t \pm r_*)}$

To study how an initial localized perturbation evolve, we select pure outgoing waves at infinity,

$$u_{lm}(t,r) \to \int_{-\infty}^{\infty} d\omega\, A_{lm}^{\text{out}}(\omega) e^{-i\omega(t-r_*)}\,, \qquad (r_* \to +\infty)$$

and pure ingoing at the horizon

$$u_{lm}(t,r) \to \int_{-\infty}^{\infty} d\omega\, A_{lm}^{\text{in}}(\omega) e^{-i\omega(t+r_*)}\,, \qquad (r_* \to -\infty)$$

Why this selects a discrete set of frequencies?

Equivalent scattering problem: prepare an initial right-moving wavepacket,

$$u^0_{lm}(t, r_*) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A^0_{lm}(\omega) \exp\{-i\omega(t - r_*)\} , \qquad (r_* \to -\infty)$$

At $r_* \to -\infty$ there will also be a reflected, left-moving, wavepacket

$$u^{\rm refl}_{lm}(t, r_*) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A^{\rm refl}_{lm}(\omega) \exp\{-i\omega(t + r_*)\} , \qquad (r_* \to -\infty)$$

while at $r = +\infty$

$$u^{\rm trans}_{lm}(t, r_*) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A^{\rm trans}_{lm}(\omega) \exp\{-i\omega(t - r_*)\} , \qquad (r_* \to +\infty)$$

Amplitude for reflection: $S_{lm}(\omega) = A^{\rm refl}_{lm}(\omega)/A^0_{lm}(\omega)$.

Our b.c. correspond to $A^0_{lm}(\omega) = 0$ with $A^{\rm refl}_{lm}(\omega) \neq 0$, and therefore to poles of $S_{lm}(\omega)$

For metric perturbation, conceptually similar but technically more complicated:

- $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$. Linearize the Einstein eqs
- expand $h_{\mu\nu}$ in tensor spherical harmonics. Separate axial and polar perturbations
- choose a clever gauge (Regge-Wheeler gauge)

$$
\begin{aligned}
g_{\mu\nu} dx^{\mu} dx^{\nu} \; = \; & -A(r) \left[1 - \sum_{l=0}^{\infty} \sum_{m=-l}^{l} H_{lm}^{(0)} Y_{lm} \right] dt^2 + 2dtdr \left[\sum_{l=1}^{\infty} \sum_{m=-l}^{l} H_{lm}^{(1)} Y_{lm} \right] \\
& + B(r) dr^2 \left[1 + \sum_{l=0}^{\infty} \sum_{m=-l}^{l} H_{lm}^{(2)} Y_{lm} \right] + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \left[1 + \sum_{l=2}^{\infty} \sum_{m=-l}^{l} K_{lm} Y_{lm} \right] \\
& - 2dtd\theta \frac{1}{\sin \theta} \left[\sum_{l=2}^{\infty} \sum_{m=-l}^{l} h_{lm}^{(0)} \, \partial_{\phi} Y_{lm} \right] + 2dtd\phi \sin \theta \left[\sum_{l=2}^{\infty} \sum_{m=-l}^{l} h_{lm}^{(0)} \, \partial_{\theta} Y_{lm} \right] \\
& - 2drd\theta \frac{1}{\sin \theta} \left[\sum_{l=1}^{\infty} \sum_{m=-l}^{l} h_{lm}^{(1)} \, \partial_{\phi} Y_{lm} \right] + 2drd\phi \sin \theta \left[\sum_{l=1}^{\infty} \sum_{m=-l}^{l} h_{lm}^{(1)} \, \partial_{\theta} Y_{lm} \right]
\end{aligned}
$$

Find two master functions Q_{lm}, Z_{lm} in the axial and polar sectors. Then

$$\begin{aligned}
\frac{\partial^2}{\partial r_*^2}\tilde{Q}_{lm} + \left[\omega^2 - V_l^{\rm RW}(r)\right]\tilde{Q}_{lm} &= 0 \qquad \text{Regge} - \text{Wheeler eq.} \\
\frac{\partial^2}{\partial r_*^2}\tilde{Z}_{lm} + \left[\omega^2 - V_l^{\rm Z}(r)\right]\tilde{Z}_{lm} &= 0 \qquad \text{Zerilli eq.}
\end{aligned}$$

$V_l^{\rm RW,Z}(r)$ qualitatively similar to the scalar case $\Rightarrow$ define QNMs in the same way

$$(\omega_{\rm QNM})_{nl} = (\omega_R)_{nl} - i(\omega_I)_{nl}$$

Furthermore, axial and polar perturbations are isospectral

Several techniques developed for computing numerically $(\omega_R)_{nl}$, $(\omega_I)_{nl}$

The least damped QNM emits GWs at a frequency

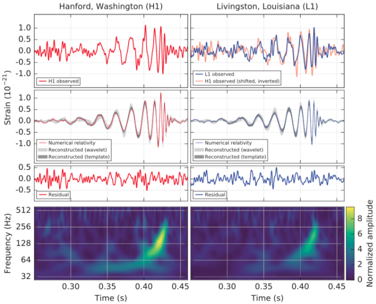
$$f \simeq 12\,\mathrm{kHz}\left(\frac{M_\odot}{M}\right)$$

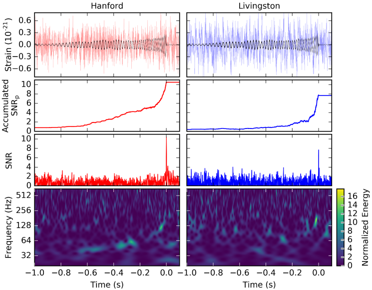
$$\tau = \frac{1}{\omega_I} \simeq 5.5 \times 10^{-5}\,\mathrm{s}\left(\frac{M}{M_\odot}\right)$$

- for $M = 10 M_\odot$: $f \sim 1$ kHz, $\tau \sim 0.5$ ms
- for $M = 10^6 M_\odot$: $f \sim 10$ mHz, $\tau \sim 1$ min

For realistic astrophysical applications, we need to perturb over Kerr BHs $\Rightarrow$ Teukolsky eq.

We can now compare with the observations!





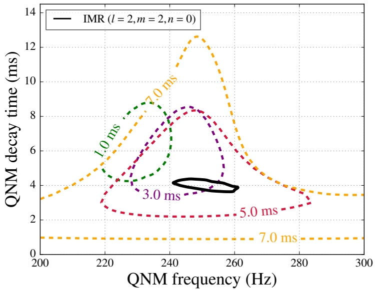
GW150914 visible even without matched filtering. Ringdown near the best sensitivity in frequency

GW151226 only visible after matched filtering.
Ringdown phase too small to be separately tested

Can we detect the BH quasinormal modes?

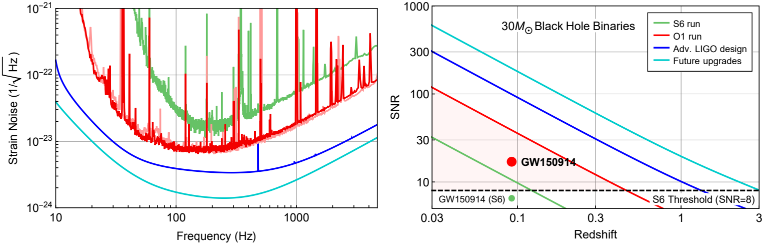
From the reconstructed mass and spin of the final BH we can compute the frequency and damping time of the dominant QNM

We can then compare with parameter estimation on the ringdown part of the signal, starting from $t_{max} + 1, 3, 5, 7$ ms.



LIGO and Virgo collaborations, Tests of GR paper, 2016

What about the near future?



LIGO and Virgo collaborations, 2016

Each improvement by factor of 10 in h means that we explore a volume 10^3 times bigger

I am looking forward to the detection of a large number of BH-BH coalescences, at cosmological distance, and SNR=O(100) !