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Deviations from scale invariance, from scale invariance

(w/ Austin Joyce and Guanhao Sun, in preparation)

“Scale invariance”

Cosmology: Correlators do not depend on distance (Up to logs)

$$\langle \zeta\zeta \rangle \propto \log|x - y| \sim \frac{1}{k^3}$$

QFT: Correlators = distance$^{\text{power}}$

$$\langle \mathcal{O}_\Delta \mathcal{O}_\Delta \rangle = \frac{1}{|x-y|^{2\Delta}}$$

However

Inflation, first order in slow roll:

$$\langle \zeta\zeta \rangle \propto \frac{1}{k^{3+2\epsilon+\eta}}$$

$$\langle \gamma\gamma \rangle \propto \frac{1}{k^{3+2\epsilon}}$$

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Still scale invariant in QFT sense...

Why?

Warmup: massless scalar in dS

$$S = \int d^4x \sqrt{g} (\partial\phi)^2 \to \int d\tau d^3k \frac{1}{H^2\tau^2} \left[|\phi'|^2 - k^2 |\phi|^2 \right]$$

Invariant under:

$$\tau \to \lambda\tau$$
$$k \to \lambda^{-1} k$$
$$\phi \to \lambda^3 \phi \qquad \text{(in } k, \tau \text{ space)}$$

Two-point function

$$\Longrightarrow \quad \langle \phi\phi \rangle \sim \lambda^6 \to \delta^3(\ldots) \times \frac{H^2}{k^3} F(k\tau)$$

For $k\tau \to 0$, EOM:

$$\partial_\tau \left(\frac{1}{\tau^2} \phi' \right) \simeq 0 \quad \Longrightarrow \quad \phi \to \text{const}$$

$$\Longrightarrow \quad \langle \phi\phi \rangle \sim \delta^3(\ldots) \times \frac{H^2}{k^3}$$

Slow-roll inflation

Tensors:

$$S = \int d\tau d^3k\, a^2 \left[|\gamma'|^2 - k^2 |\gamma|^2 \right] \qquad a^2(\tau) \propto \frac{1}{\tau^{2+2\epsilon}}$$

Invariant under:

$$\tau \to \lambda \tau$$
$$k \to \lambda^{-1} k$$
$$\gamma \to \lambda^{3+\epsilon} \gamma$$

$$\Rightarrow \quad \langle \gamma\gamma \rangle \sim \delta^3(\ldots) \times \frac{1}{k^{3+2\epsilon}} F(k\tau)$$

Slow-roll inflation

Scalars:

$$S = \int d\tau d^3k\, \epsilon a^2 \left[|\zeta'|^2 - k^2 |\zeta|^2 \right] \qquad \epsilon(\tau)a^2(\tau) \propto \frac{1}{\tau^{2+2\epsilon+\eta}}$$

Invariant under:

$$\tau \to \lambda\tau$$
$$k \to \lambda^{-1}k$$
$$\zeta \to \lambda^{3+\epsilon+\eta/2}\zeta$$

$$\Rightarrow \quad \langle\zeta\zeta\rangle = \delta^3(\ldots) \times \frac{1}{k^{3+2\epsilon+\eta}} F(k\tau)$$

What is this scale invariance?

- Approximate isometry? No (only dS)
- Conformal Killing vector? No (all FRW's)
- Accident for quadratic order? Maybe
- Accident for slow-roll inflation? Apparently not...
- Notice: totally not manifest before constraints (all $\mathcal{O}(\epsilon^0)$ terms cancel)

More data

Solid inflation:

$$S_\gamma = \int d\tau d^3k \, a^2 \left[|\gamma'|^2 - k^2|\gamma|^2 - \epsilon a^2 H^2 |\gamma|^2 \right]$$

$$S_T = \int d\tau d^3k \, a^2 \left[\frac{k^2}{1 + k^2/\epsilon a^2 H^2} |\pi_T'|^2 - \epsilon a^2 H^2 c_T^2 k^2 |\pi_T|^2 \right]$$

$$S_L = \int d\tau d^3k \, a^2 \left[\frac{k^2}{1 + k^2/\epsilon a^2 H^2} |\pi_L' + \epsilon a H \pi_L|^2 - \epsilon a^2 H^2 c_L^2 k^2 |\pi_L|^2 \right]$$

with $a(\tau), H(\tau), \epsilon(\tau), c_T(\tau), c_L(\tau) \sim \tau^\#$

Three regimes: k^2/a^2 vs. $H^2/c_{L,T}^2$ and ϵH^2

No obvious scale invariance. Still…

$$\langle \gamma\gamma \rangle = \delta^3(\ldots) \times \frac{1}{k^{3+2\epsilon}} F(k\tau) \to \delta^3(\ldots) \times \frac{\tau^{8\epsilon c_T^2/3}}{k^{3-2\epsilon c_L^2}}$$

$$\langle \mathscr{R}\mathscr{R} \rangle = \delta^3(\ldots) \times \frac{1}{k^{3+2\epsilon+\eta}} F(k\tau^{1-s_L}) \to \delta^3(\ldots) \times \frac{\tau^{8\epsilon c_T^2/3-4s}}{k^{3-2\epsilon c_L^2+\eta+5s}}$$

which exhibit some scale invariance.

Flow

Scaling can change between UV and IR.
E.g., in flat space:

$$S = \int d^4k \left[k^2|\phi|^2 - m^2|\phi|^2\right] \qquad (k \equiv k^\mu)$$

UV: $k \to \lambda k$, $\phi \to \lambda^{-3}\phi$ $\Rightarrow$ $\langle\phi\phi\rangle \sim \delta^4(\ldots) \times \frac{1}{k^2}$

IR: $k \to \lambda k$, $\phi \to \lambda^{-2}\phi$ $\Rightarrow$ $\langle\phi\phi\rangle \sim \delta^4(\ldots) \times \frac{1}{m^2}$

So, for cosmology find the correct IR scale-invariance...

No!

It does not work. Nor should it:

- All modes start inside the horizon.
- Their wave function is normalized to the UV, flat-space one (for Bunch-Davies state).
- Time-evolution takes them outside.

More formally:

$$\Psi[\bar{\phi}(\overrightarrow{x}); \bar{\tau}] = \int^{\phi(\overrightarrow{x},\bar{\tau})=\bar{\phi}(\overrightarrow{x})} D\phi(\overrightarrow{x}, \tau)\, e^{i(S+i\epsilon\,\text{terms})}$$

Looks like we need scale-invariance for all $k\tau$'s.

In fact, for solid inflation:

$$\langle \gamma\gamma \rangle = \delta^3(\ldots) \times \frac{1}{k^{3+2\epsilon}} F(k\tau)$$

$$\langle \mathscr{R}\mathscr{R} \rangle = \delta^3(\ldots) \times \frac{1}{k^{3+2\epsilon+\eta}} F(k\tau^{1-s_L})$$

Not manifest at all in the action, or in computation.

What does work

Actual recipe is extremely simple (but makes no sense):

- Find scaling in the UV: easy (neglect mixing).
- Find time-evolution in the IR: easy (neglect k)
- Combine them.

Example

Slow-roll inflation:

UV: $$S \simeq \int d\tau d^3k\, a^2 \left[|\delta\phi'|^2 - k^2 |\delta\phi|^2 \right]$$

Neglect mass and mixing, keep $a^2(\tau) \propto \dfrac{1}{\tau^{2+2\epsilon}}$ (WKB)

Invariant under:

$$\tau \to \lambda\tau$$
$$k \to \lambda^{-1}k$$
$$\delta\phi \to \lambda^{3+\epsilon}\delta\phi$$

$\Rightarrow$ $$\langle \delta\phi\, \delta\phi \rangle \propto \delta^3(\ldots) \times \frac{1}{k^{3+2\epsilon}} F(k\tau)$$

IR: Time-evolution, easy for

$$\zeta = H\frac{\delta\phi}{\dot{\phi}_0} \propto \frac{\delta\phi}{\sqrt{\epsilon}} \propto \delta\phi\,\tau^{\eta/2}$$

$$\langle\delta\zeta\,\delta\zeta\rangle \propto \delta^3(\ldots) \times \frac{\tau^\eta}{k^{3+2\epsilon}}F(k\tau)$$

$\zeta \to \mathrm{const}$ for $k\tau \to 0$

$$\Longrightarrow \quad \langle\delta\zeta\,\delta\zeta\rangle \propto \delta^3(\ldots) \times \frac{1}{k^{3+2\epsilon+\eta}}$$

Ultimate check

Gaugid inflation:

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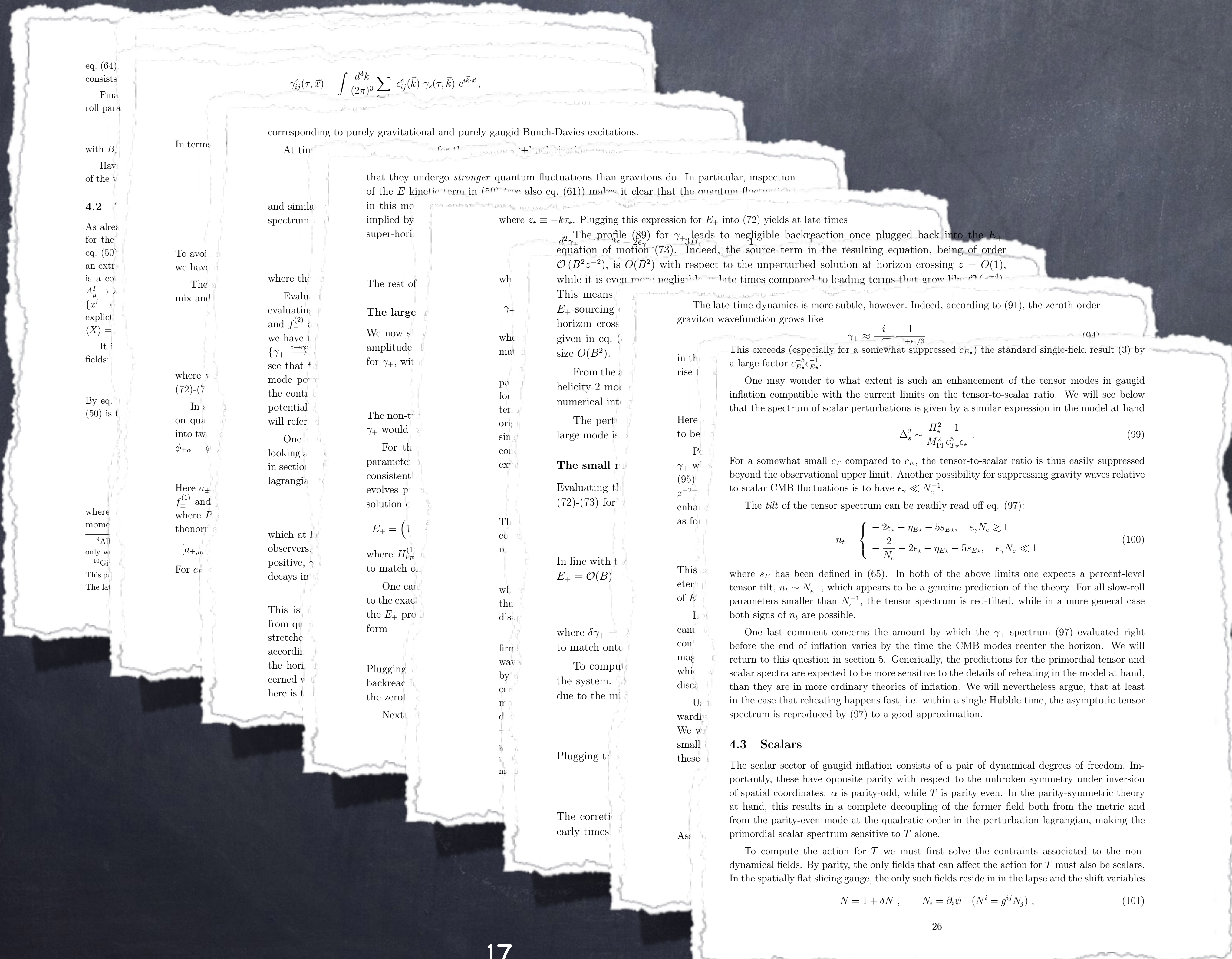
This exceeds (especially for a somewhat suppressed $c_{E\star}$) the standard single-field result (3) by a large factor $c_{E\star}^{-5}\epsilon_{E\star}^{-1}$.

One may wonder to what extent is such an enhancement of the tensor modes in gaugid inflation compatible with the current limits on the tensor-to-scalar ratio. We will see below that the spectrum of scalar perturbations is given by a similar expression in the model at hand

$$\Delta_s^2 \sim \frac{H_\star^2}{M_{\rm Pl}^2}\frac{1}{c_{T\star}^5\epsilon_\star} \, . \tag{99}$$

For a somewhat small c_T compared to c_E, the tensor-to-scalar ratio is thus easily suppressed beyond the observational upper limit. Another possibility for suppressing gravity waves relative to scalar CMB fluctuations is to have $\epsilon_\gamma \ll N_e^{-1}$.

The *tilt* of the tensor spectrum can be readily read off eq. (97):

$$n_t = \begin{cases} -2\epsilon_\star - \eta_{E\star} - 5s_{E\star}, & \epsilon_\gamma N_e \gtrsim 1 \\ -\dfrac{2}{N_e} - 2\epsilon_\star - \eta_{E\star} - 5s_{E\star}, & \epsilon_\gamma N_e \ll 1 \end{cases} \tag{100}$$

where s_E has been defined in (65). In both of the above limits one expects a percent-level tensor tilt, $n_t \sim N_e^{-1}$, which appears to be a genuine prediction of the theory. For all slow-roll parameters smaller than N_e^{-1}, the tensor spectrum is red-tilted, while in a more general case both signs of n_t are possible.

One last comment concerns the amount by which the γ_+ spectrum (97) evaluated right before the end of inflation varies by the time the CMB modes reenter the horizon. We will return to this question in section 5. Generically, the predictions for the primordial tensor and scalar spectra are expected to be more sensitive to the details of reheating in the model at hand, than they are in more ordinary theories of inflation. We will nevertheless argue, that at least in the case that reheating happens fast, i.e. within a single Hubble time, the asymptotic tensor spectrum is reproduced by (97) to a good approximation.

4.3 Scalars

The scalar sector of gaugid inflation consists of a pair of dynamical degrees of freedom. Importantly, these have opposite parity with respect to the unbroken symmetry under inversion of spatial coordinates: α is parity-odd, while T is parity even. In the parity-symmetric theory at hand, this results in a complete decoupling of the former field both from the metric and from the parity-even mode at the quadratic order in the perturbation lagrangian, making the primordial scalar spectrum sensitive to T alone.

To compute the action for T we must first solve the contraints associated to the non-dynamical fields. By parity, the only fields that can affect the action for T must also be scalars. In the spatially flat slicing gauge, the only such fields reside in in the lapse and the shift variables

$$N = 1 + \delta N \, , \qquad N_i = \partial_i \psi \quad (N^i = g^{ij} N_j) \, , \tag{101}$$

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Tensors: γ_{ij}, E_{ij}

$$S = \int d\tau d^3k \left\{ a^2\left[|\gamma'|^2 - k^2|\gamma|^2 - \epsilon_\gamma a^2 H^2 |\gamma|^2\right] \right.$$

$$+\epsilon_E a^4 H^2 \left[|E'|^2 - c_E^2 k^2 |E|^2\right]$$

$$\left. +2\epsilon_\gamma a^4 H^2 \left[kE\gamma\right] \right\}$$

UV: Neglect mass and mixing, keep τ-dependence of coefficients

$\Rightarrow$ $\gamma_{ij}:$

$$\tau \to \lambda\tau$$
$$k \to \lambda^{-1} k$$
$$\gamma \to \lambda^{3+\epsilon} \gamma$$

$E_{ij}:$

$$\tau \to \lambda\tau$$
$$k \to \lambda^{-1+s_E} k$$
$$E \to \lambda^{4+\epsilon+\eta_E/2-3s_E/2} E$$

IR: Time-evolution. Leading order terms:

$$S = \int d\tau d^3k \left\{ a^2 \left[|\gamma'|^2 - \epsilon_\gamma a^2 H^2 |\gamma - kE|^2 \right] + \epsilon_E a^4 H^2 |E'|^2 \right\}$$

⇨ $E \to \text{const}$ $\gamma \to kE$

⇨ $$\langle\gamma\gamma\rangle \sim k^2 \langle EE\rangle \propto \delta^3(\ldots) \times \frac{1}{k^{3+2\epsilon+\eta_E+5s_E}}$$

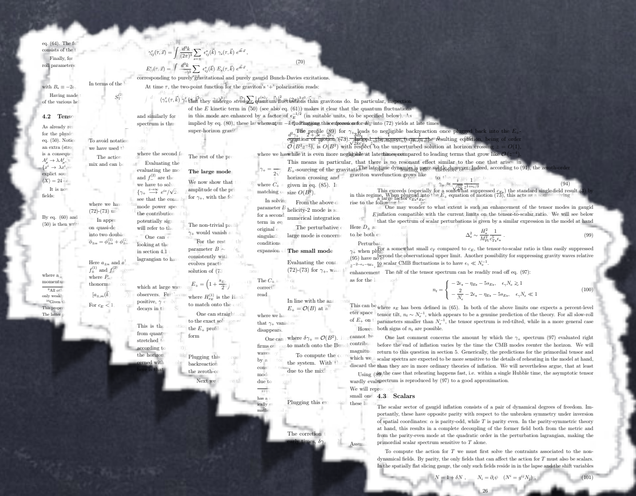
IR: Time-evolution. Leading order terms:

$$S = \int d\tau d^3k \left\{ a^2 \left[|\gamma'|^2 - \epsilon_\gamma a^2 H^2 |\gamma - kE|^2 \right] + \epsilon_E a^4 H^2 |E'|^2 \right\}$$

$\Longrightarrow$ $E \to \text{const}$ $\qquad \gamma \to k\,E$

$\Longrightarrow$ $\langle \gamma\gamma \rangle \sim k^2 \langle EE \rangle \propto \delta^3(\ldots) \times \dfrac{1}{k^{3+2\epsilon+\eta_E+5s_E}}$

Matches the



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- Deeper? We'll learn something about inflation
- Relationship to Baumann's talk or to holographic methods (Skenderis et al.)?