Shift Symmetries on (A)dS

Kurt Hinterbichler (Case Western)

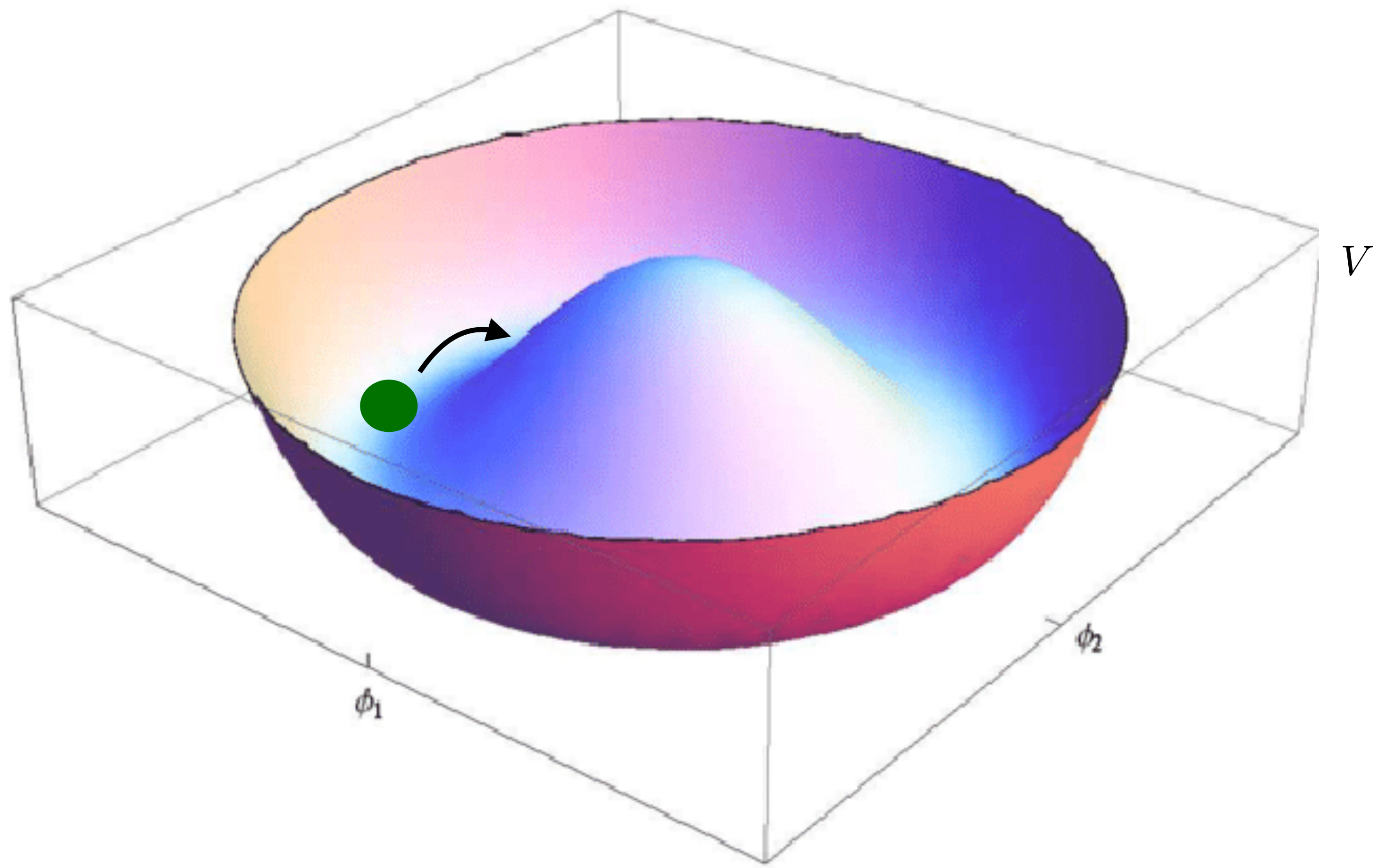
24th Rencontres Itzykson, Saclay, June 7, 2019

arxiv:1812.08167, arxiv:1906.xxxxx

w/ James Bonifacio, Laura Johnson, Austin Joyce, Rachel Rosen

Broken symmetries

Spontaneously broken symmetries → Goldstone Bosons



Goldstone Bosons have shift symmetry

$$\delta\phi = c + \cdots$$

Shift symmetry

A broken symmetry transformation starts with a field-independent term:

$$\delta\phi = c + \mathcal{O}(\phi) + \mathcal{O}(\phi^2) + \cdots$$

Broken symmetry (does not preserve vacuum $\phi = 0$)

$$\delta\phi = \mathcal{O}(\phi) + \mathcal{O}(\phi^2) + \cdots$$

Unbroken symmetry (preserves vacuum $\phi = 0$)

Shift invariant Lagrangian: $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \cdots$

Interactions for an exact shift symmetry: $\delta\phi = c$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + F(\partial\phi, \partial\partial\phi, \cdots) + \lambda\,\phi$$

Function of invariant building block $\partial_\mu\phi$

Wess-Zumino term

Galileon symmetry

Scalar kinetic term also has *galileon* symmetry:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 \quad , \qquad \delta\phi = b_\mu x^\mu$$

constant vector

Boring interactions: $F(\partial\partial\phi, \partial\partial\partial\phi, \cdots)$

Function of invariant building block $\Pi_{\mu\nu} = \partial_\mu\partial_\nu\phi$

Wess-Zumino terms (*galileons*):

Luty, Porrati, Rattazzi (2003)
Nicolis, Rattazzi, Trincherini (2008)

Garrett Goon, KH, Austin Joyce, Mark Trodden (2012)

$$\mathcal{L}_1 = \phi \ ,$$

$$\mathcal{L}_2 = -\frac{1}{2}(\partial\phi)^2 \ ,$$

$$\mathcal{L}_3 = -\frac{1}{2}(\partial\phi)^2[\Pi] \ ,$$

$$\mathcal{L}_4 = -\frac{1}{2}(\partial\phi)^2\left([\Pi]^2 - [\Pi^2]\right) \ ,$$

$$\mathcal{L}_5 = -\frac{1}{2}(\partial\phi)^2\left([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\right)$$

Deformation of the symmetry: $\delta\phi = b_\mu x^\mu + \frac{1}{\Lambda^4} b^\mu \phi\,\partial_\mu\phi$ ➡ DBI theory

$$\mathcal{L} = -\Lambda^4\sqrt{1 + \frac{1}{\Lambda^4}(\partial\phi)^2}$$

Extensions of Galileon symmetry

Scalar kinetic term also has *extended galileon* symmetry:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 \quad , \qquad \delta\phi = s_{\mu\nu}x^\mu x^\nu$$

symmetric, traceless constant tensor

special galileon:

Clifford Cheung, Karol Kampf, Jiri Novotny, Jaroslav Trnka (2014)

KH, Austin Joyce (2015)

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{12\Lambda^6}(\partial\phi)^2\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right]$$

$$\delta\phi = s_{\mu\nu}x^\mu x^\nu + \frac{1}{\Lambda^6}s^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

nonlinear deformation of the symmetry

Extensions of Galileon symmetry

Scalar kinetic term has extended galileon symmetry of all orders:

,

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2$$

$$\delta\phi = c + c_\mu x^\mu + c_{\mu_1\mu_2} x^{\mu_1} x^{\mu_2} + c_{\mu_1\mu_2\mu_3} x^{\mu_1} x^{\mu_2} x^{\mu_3} + \cdots$$

symmetric, traceless constant tensors

There do not seem to be interesting interactions at higher orders.

KH, Austin Joyce (2014)

Clifford Cheung, Karol Kampf, Jiri Novotny, Chia-Hsien Shen, Jaroslav Trnka (2016)

Mark Bogers, Tomas Brauner (2018)

Diederik Roest, David Stefanyszyn, Pelle Werkman (2019)

How does all this extend to (A)dS and to higher spins?

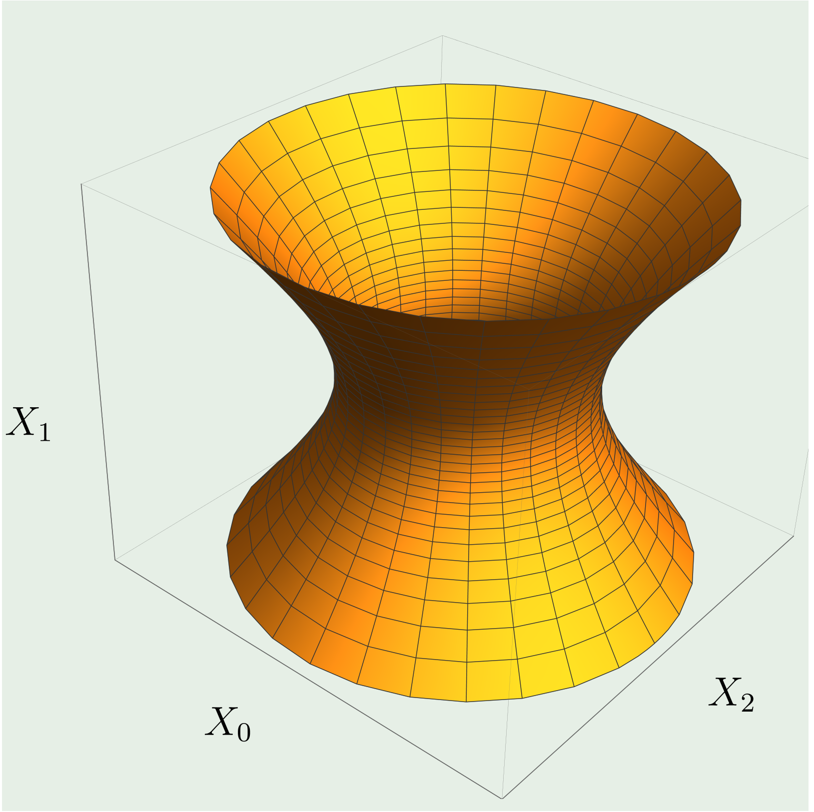
(A)dS embedding space

Dirac (1936)

Embed D dimensional (A)dS into D+1 dimensional Minkowski:

$$X^{\mu}(x) \quad , \qquad \eta_{AB}X^A X^B = \pm\mathcal{R}^2$$

embedding space coordinates   intrinsic (A)dS coordinates



(A)dS tensors correspond to embedding space tensors:

$$T_{\mu_1\cdots\mu_s}(x) \longleftrightarrow T_{A_1\cdots A_s}(X)$$

Homogeneity, transverse-ness conditions:

$$\left(X^A\partial_A - \mu\right)T_{A_1\cdots A_s} = 0 \qquad X^{A_1}T_{A_1\cdots A_s} = 0$$

Rules for projecting derivatives:

$$\partial_{(A_1}\ldots\partial_{A_n}\Phi_{A_{n+1}\ldots A_{n+s})} \to \nabla_{(\mu_1}\ldots\nabla_{\mu_n}\phi_{\mu_{n+1}\ldots\mu_{n+s})} + \cdots$$

Scalars in (A)dS

- Massless scalar preserves shift symmetry:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}(\partial\phi)^2 \,, \qquad \delta\phi = c$$

Higher symmetries all broken: $\delta\phi = c + c_\mu x^\mu + c_{\mu_1\mu_2}x^{\mu_1}x^{\mu_2} + c_{\mu_1\mu_2\mu_3}x^{\mu_1}x^{\mu_2}x^{\mu_3} + \cdots$

- There is a special mass which preserves a galileon symmetry:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}\left[(\partial\phi)^2 - DH^2\phi^2\right] \,, \qquad \delta\phi = S_A X^A\Big|_{(A)dS}$$

constant embedding space vector

Flat limit:

galileon

shift

$$\begin{array}{ccccc} S_A & & S_\mu & & S_D \\ \square & \rightarrow & \square & \oplus & \bullet \end{array}$$

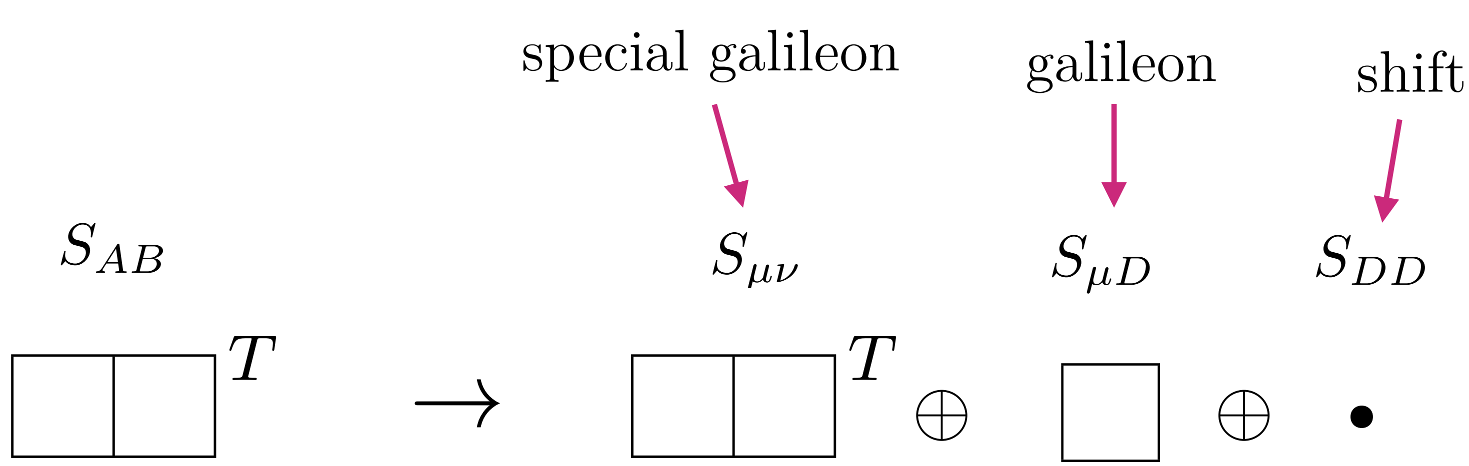
Scalars in (A)dS

- There is a different mass which preserves second-order galileon symmetry:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}\left[(\partial\phi)^2 - 2(D+1)H^2\phi^2\right] , \qquad \delta\phi = S_{AB}X^A X^B\big|_{(A)dS}$$

symmetric, traceless, embedding space vector

Flat limit:



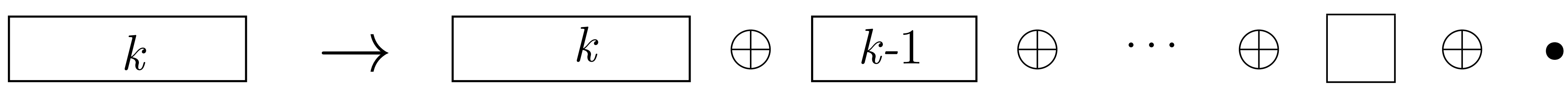
Scalars in (A)dS

- Sequence of special mass values: $k = 0, 1, 2, \ldots$

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - \frac{m_k^2}{2}\phi^2\right) , \qquad \delta\phi = S_{A_1 \ldots A_k} X^{A_1} \cdots X^{A_k}\Big|_{(A)dS}$$

$$m_k^2 = -k(k+D-1)H^2$$

Flat limit:



Masses come from higher dimensional laplacian:

$$\partial^2\Phi \to \nabla^2\phi - m_k^2\phi$$

Massive higher spins in (A)dS

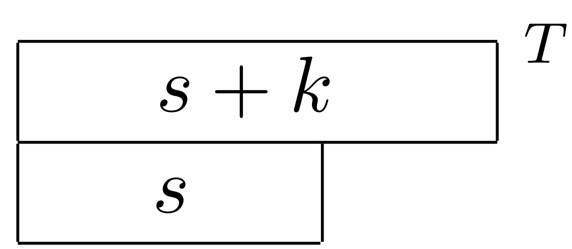
Massive spin s field on (A)dS:

$$\left(\Box - H^2\left[D + (s-2) - (s-1)(s+D-4)\right] - m^2\right)\ \phi_{\mu_1\cdots\mu_s} + \cdots = 0$$

$$m^2_{s,k} = -(k+2)(k+D-3+2s)H^2\,, \qquad\qquad k = 0, 1, 2, \ldots$$

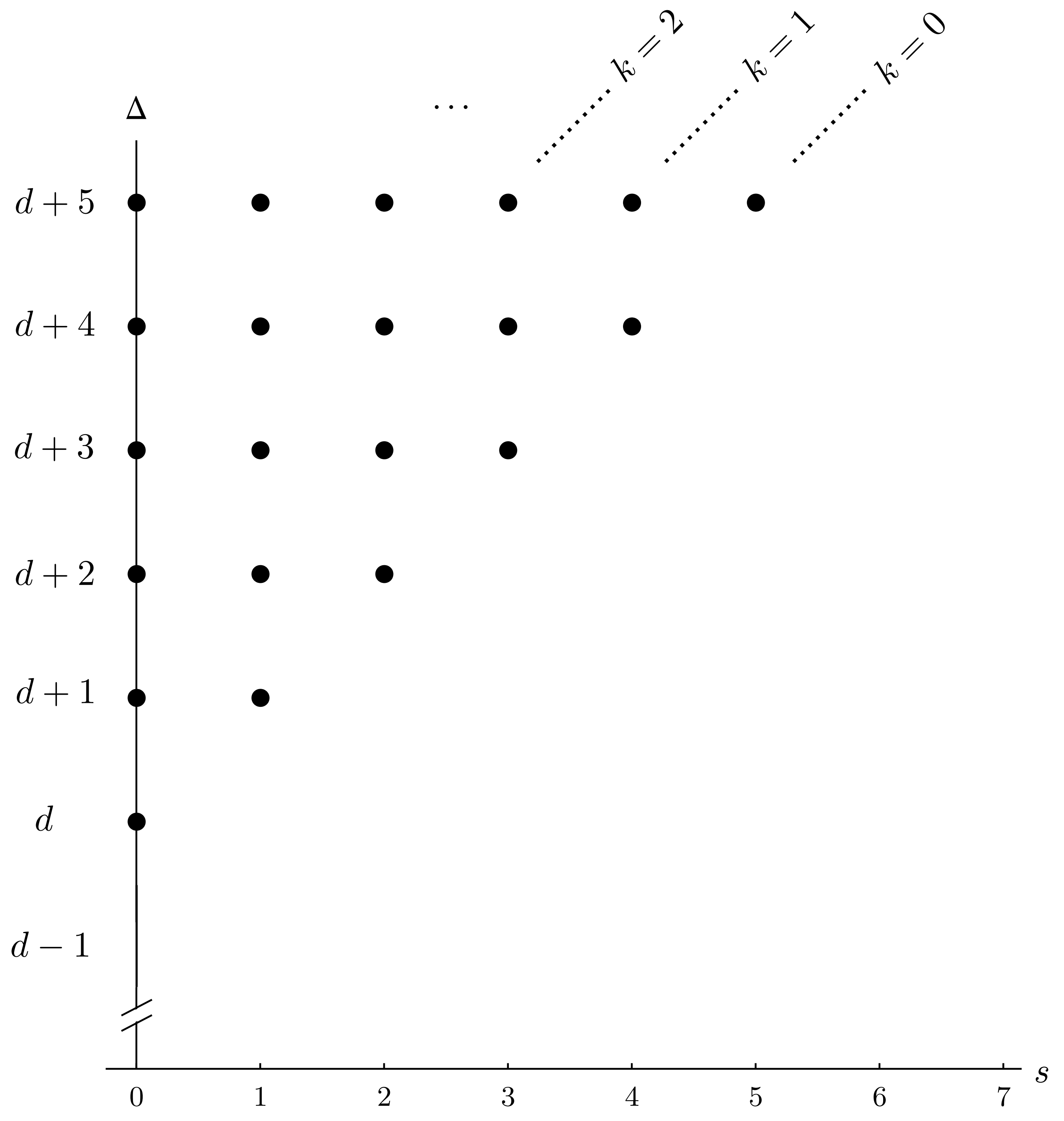
Symmetry under shifts parametrized by a mixed symmetry ambient space tensor:

$$\delta\phi_{\mu_1\ldots\mu_s} = S_{A_1\ldots A_{s+k},B_1\ldots B_s} X^{A_1}\ldots X^{A_{s+k}} \frac{\partial X^{B_1}}{\partial x^{\mu_1}} \ldots \frac{\partial X^{B_s}}{\partial x^{\mu_s}}\Bigg|_{\text{(A)dS}}$$

$S_{A_1\cdots A_{s+k},B_1\cdots B_s} \in$ 

Higher spins in (A)dS

Dual CFT$_d$ operators: $\Delta = k + s + D - 1$



Shift-symmetric fields are "longitudinal modes" of partially massless fields.

Partially massless fields

Massive spin s field on (A)dS:

$$\left(\Box - H^2\left[D+(s-2)-(s-1)(s+D-4)\right]-m^2\right)\phi_{\mu_1\cdots\mu_s}+\cdots=0$$

At special values of the mass there are enhanced gauge symmetries:

$$\bar{m}^2_{s,t}=(s-t-1)(s+t+D-4)H^2\ ,\qquad t=0,1,2,\cdots,s-1$$

depth

$$\delta\phi_{\mu_1\cdots\mu_s}=\nabla_{(\mu_{t+1}}\nabla_{\mu_{t+2}}\cdots\nabla_{\mu_s}\xi_{\mu_1\cdots\mu_t)}+\ldots$$

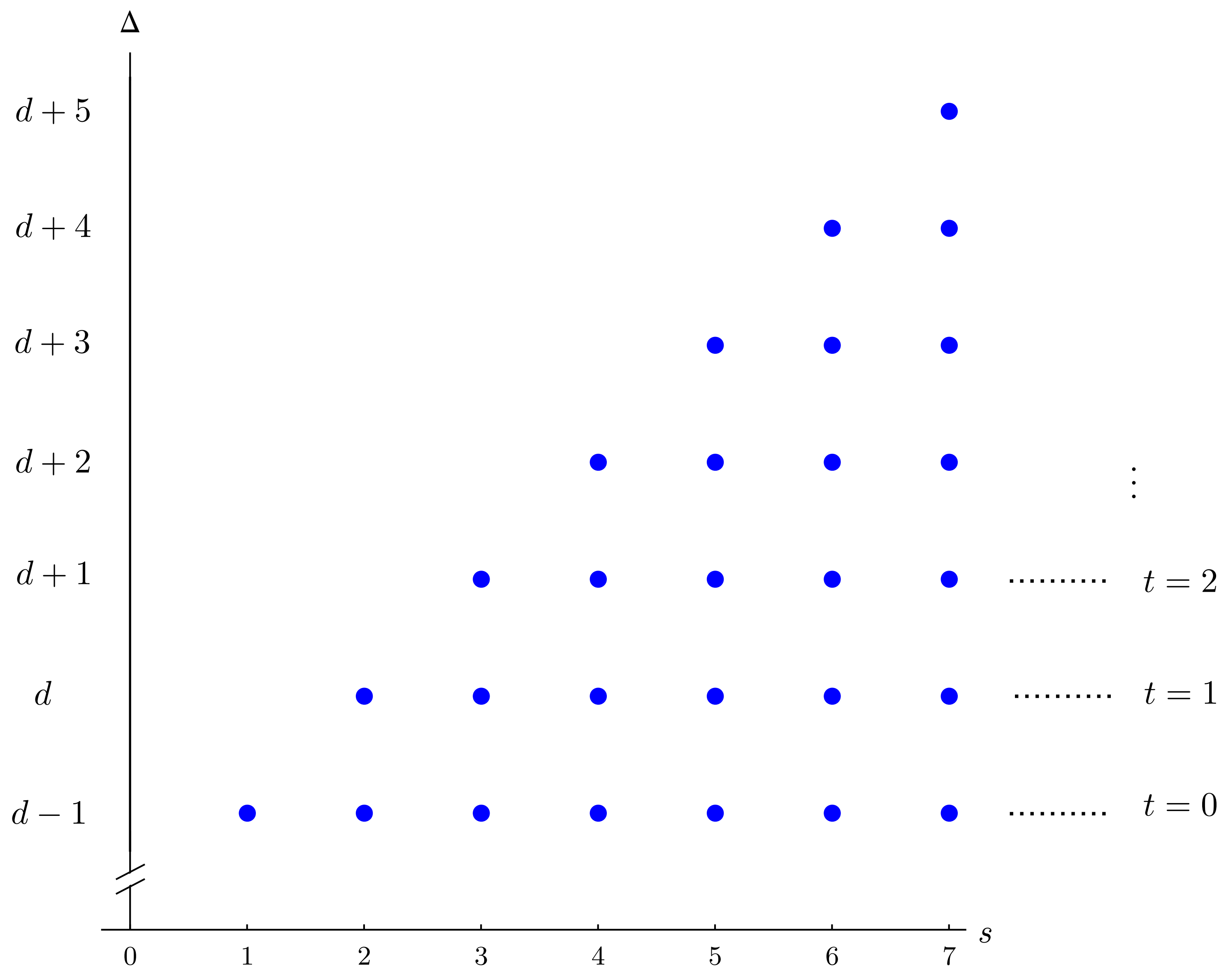
Gauge symmetry eliminates helicities 0,1,...,t

$$\underbrace{0,1,\cdots,t},t+1,\cdots,s$$

Partially massless fields

Dual CFT$_d$ operators: $\Delta_{s,t} = t + d - 1$

Short multiplets with a level s-t null state: $P_{i_1} \dots P_{i_{s-t}} |\Delta\rangle^{i_1 \cdots i_s} = 0$



Shift symmetries from partially massless fields

- Example: Massless limit of a massive vector:

$$\frac{1}{\sqrt{-g}}\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu$$

$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

Introduce Stückelberg field:

$$A_\mu \to A_\mu + \frac{1}{m}\partial_\mu \phi$$

$$\delta A_\mu = \partial_\mu \Lambda, \quad \delta\phi = -m\Lambda$$

Massless limit $m \to 0$

$$\frac{1}{\sqrt{-g}}\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\delta A_\mu = \partial_\mu \Lambda, \quad \delta\phi = 0$$

Reducibility parameter: if Λ is such that $\partial_\mu \Lambda = 0$, then symmetry survives the massless limit:

$$\delta A_\mu = 0, \quad \delta\phi = \hat{\Lambda}$$

$\hat{\Lambda} = m\Lambda$

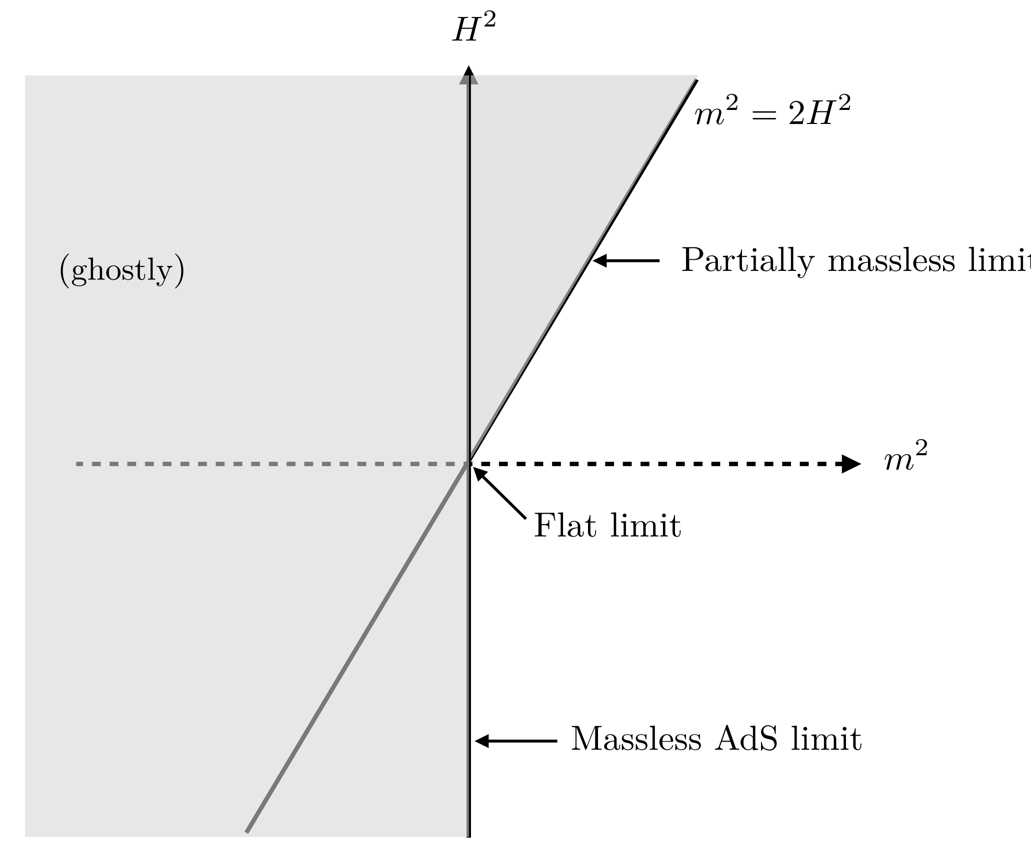
reducibility parameter $\to$ shift symmetry of longitudinal mode

Shift symmetries from partially massless fields

- Example: massless limit of a massive spin-2:

Claudia de Rham, KH, Laura A. Johnson (2018)

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$



Vector Stückelberg field:

$$h_{\mu\nu} \to h_{\mu\nu} + \frac{1}{m}\left(\nabla_\mu A_\nu + \nabla_\nu A_\mu\right)$$

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \quad \delta A_\mu = -m\xi_\mu$$

Massless limit $m \to 0$

$$\mathcal{L} = \mathcal{L}_{\text{massless graviton}} + \sqrt{-g}\left[-\frac{1}{2}F_{\mu\nu}^2 - \frac{6}{L^2}A^2\right]$$

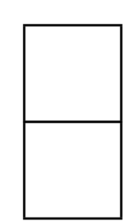
$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \quad \delta A_\mu = 0$$

Reducibility parameter: if ξ_μ is such that $\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$

$$\delta h_{\mu\nu} = 0, \quad \delta A_\mu = \hat{\xi}$$

$\hat{\xi}_\mu = m\xi_\mu$

Reducibility parameters are (A)dS Killing vectors:

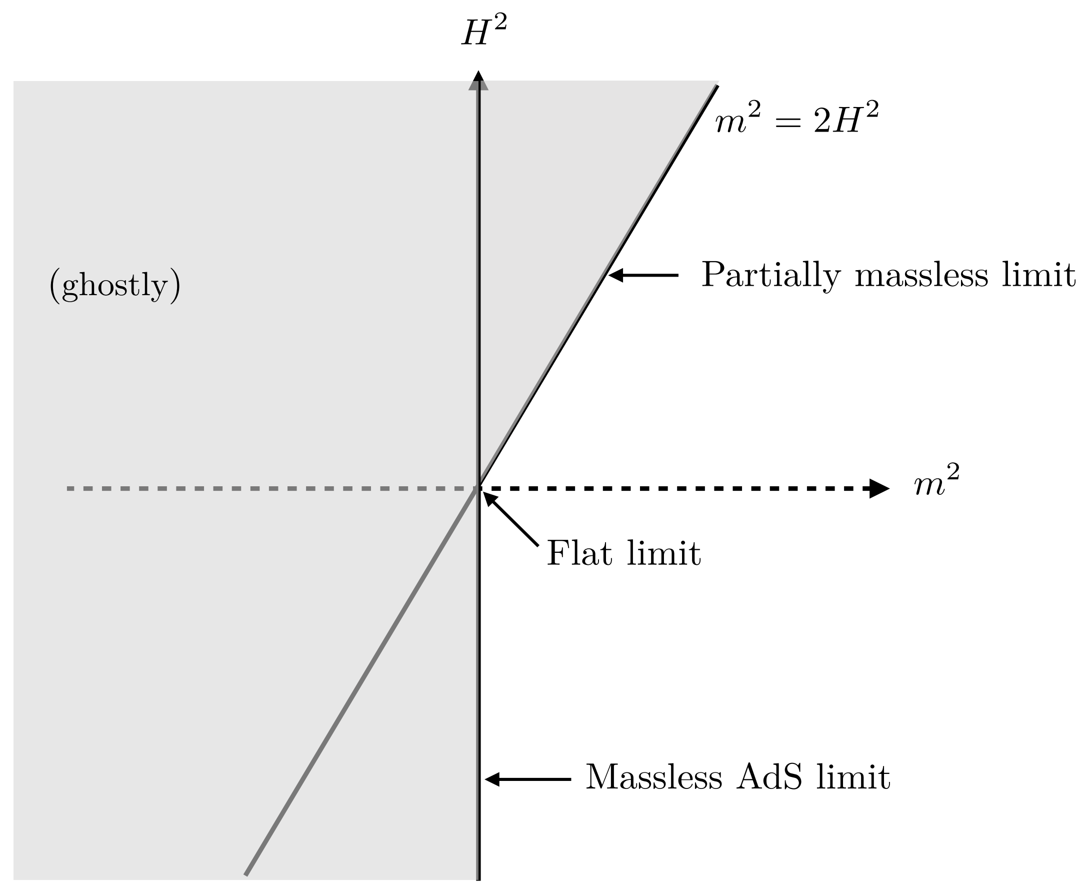
$$\delta A_A = M_{AB} X^B, \quad M_{AB} \in$$ 

Shift symmetries from partially massless fields

- Example: PM limit of a massive spin-2 $m^2 \to 2H^2$

Claudia de Rham, KH, Laura A. Johnson (2018)

$$\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \xi + H^2 g_{\mu\nu}\xi$$



Scalar Stückelberg field:

$$h_{\mu\nu} \to h_{\mu\nu} + \frac{1}{H\epsilon}\left(\nabla_\mu \nabla_\nu \phi + H^2 g_{\mu\nu}\phi\right)$$

$$\epsilon^2 \equiv m^2 - 2H^2$$

$$\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \chi + H^2 \chi g_{\mu\nu}, \quad \delta\phi = -H\epsilon\chi$$

Partially massless limit:

$$\mathcal{L} = \mathcal{L}_{\rm PM} - \sqrt{-g}\frac{1}{2}\left[(\partial\phi)^2 - DH^2\phi^2\right]$$

Reducibility parameter: $\nabla_\mu \nabla_\nu \chi + H^2 \chi g_{\mu\nu} = 0$

Partially massless reducibility parameters:

$$\delta\phi = S_A X^A\big|_{\rm (A)dS}\,, \qquad S_A \in \square$$

Shift symmetries from partially massless fields

General rule:

$$(m^2, s) \xrightarrow[m^2 \to \bar{m}^2_{s,t}]{} (\bar{m}^2_{s,t}, s) \oplus (m^2_{t,k}, t)$$

PM field

shift symmetric field

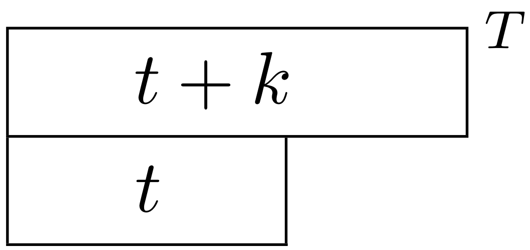
Reducibility parameters: $\delta\phi_{\mu_1\cdots\mu_s} = \nabla_{(\mu_{t+1}} \cdots \nabla_{\mu_s} \xi_{\mu_1\cdots\mu_t)} + \cdots$

$$\nabla_{(\mu_{t+1}} \nabla_{\mu_{t+2}} \cdots \nabla_{\mu_{t+k+1}} K^{(k)}_{\mu_1\cdots\mu_t)} + \cdots = 0 \ , \qquad k = s - t - 1$$

Generalized Killing tensors. Finite space of solutions:

$$K^{(k)}_{\mu_1\cdots\mu_t} = K_{A_1\cdots A_{t+k}, B_1\cdots B_t} X^{A_1} \cdots X^{A_{t+k}} \frac{\partial X^{B_1}}{\partial x^{\mu_1}} \cdots \frac{\partial X^{B_t}}{\partial x^{\mu_t}},$$

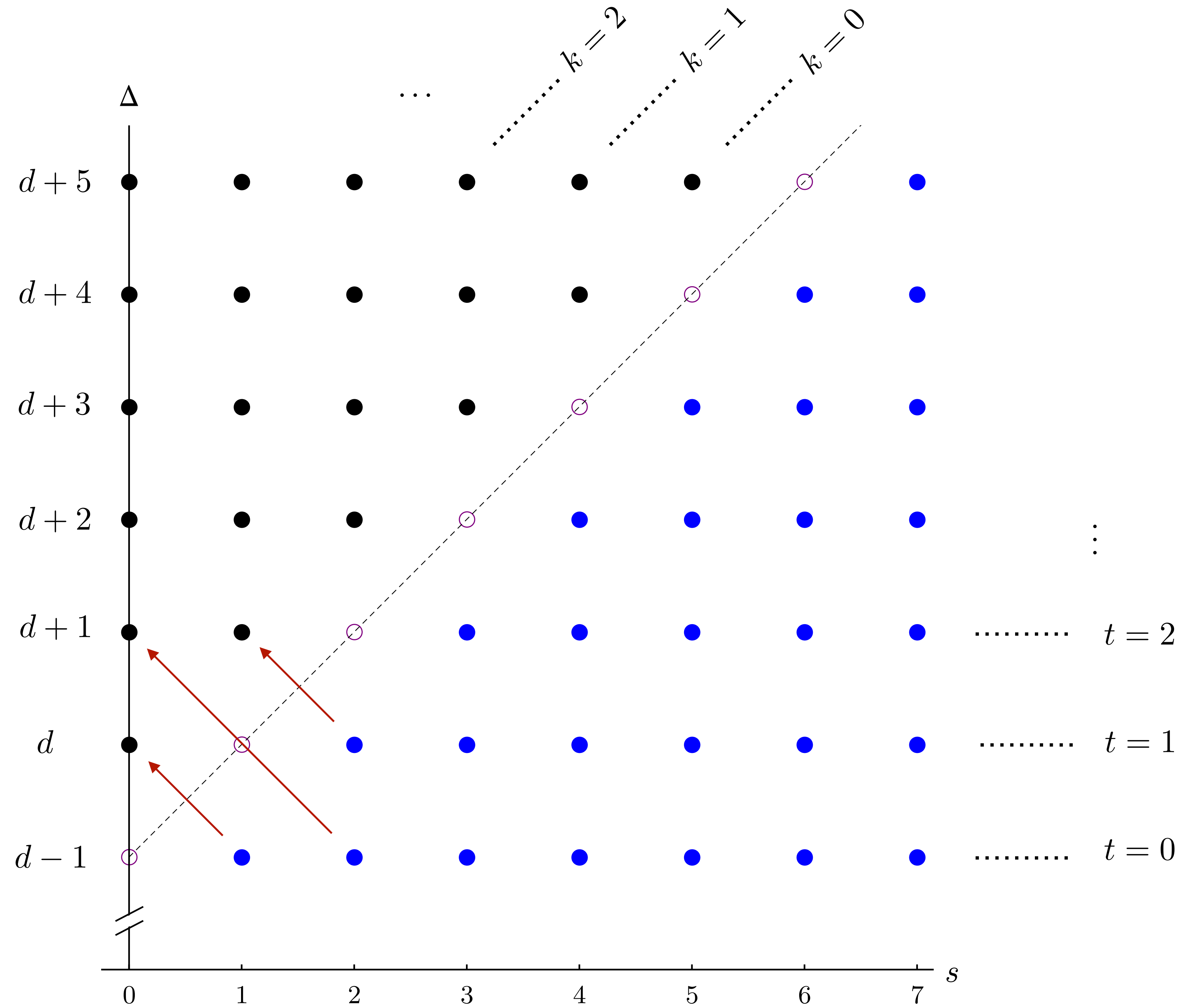
$K_{A_1\cdots A_{t+k}, B_1\cdots B_t} \in$



Shift symmetries from partially massless fields

CFT Branching rule: $$(\Delta, s) \xrightarrow[\Delta \to t+d-1]{} (t+d-1, s) \oplus (s+d-1, t)$$

Null module



Are there interactions preserving these shift symmetries?

Algebra of symmetries

(A)dS isometries (unbroken):

$$J_{AB}\Phi = X_A \partial_B \Phi - X_B \partial_A \Phi$$

Commutators give (a real form of) so(D+1) algebra:

$$[J_{AB}, J_{CD}] = \eta_{AC} J_{BD} - \eta_{BC} J_{AD} + \eta_{BD} J_{AC} - \eta_{AD} J_{BC}$$

Shift symmetries (broken):

$$S_{A_1 \cdots A_k}\Phi = X_{(A_1} \cdots X_{A_k)_T} + \mathcal{O}\left(\Phi\right)$$

possible non-linear deformation

Shifts transform as tensors under (A)dS isometries

$$[J_{BC}, S_{A_1 \cdots A_k}] = \sum_{i=1}^{k} \left(\eta_{BA_i} S_{A_1 \ldots A_{i-1} C A_{i+1} \ldots A_k} - \eta_{CA_i} S_{A_1 \ldots A_{i-1} B A_{i+1} \ldots A_k}\right)$$

Algebra of symmetries

Remaining commutator has one possible structure (k>0):

$$[S_{A_1\ldots A_k}, S^{B_1\ldots B_k}] = \alpha k!^2 \delta_{(A_1}^{\ (B_1} \ldots \delta_{A_{k-1}}^{\ B_{k-1}} J_{A_k)}^{\ B_k)} + \cdots$$

arbitrary constant

Jacobi identities:

$$[S_{A(k)}, [S_{B(k)}, S_{C(k)}]] + [S_{B(k)}, [S_{C(k)}, S_{A(k)}]] + [S_{C(k)}, [S_{A(k)}, S_{B(k)}]] = 0$$

⬇

$\alpha = 0$ for k>2

α arbitrary for k=1,2

“Abelian” theories

$\alpha = 0$ is the algebra of the free theory

$$S_{A_1 \cdots A_k} \Phi = X_{(A_1} \cdots X_{A_k)_T}$$

Interactions can be constructed from the building blocks:

$$\partial_{(A_1} \ldots \partial_{A_{k+1})} \Phi \rightsquigarrow \Phi^{(k)}_{\mu_1 \cdots \mu_{k+1}} = \nabla_{(\mu_1} \ldots \nabla_{\mu_{k+1})} \phi + \mathcal{O}(H^2)$$

$$\mathcal{L} = \sqrt{-g}\, F\left(\Phi^{(k)}_{\mu_1 \cdots \mu_{k+1}}, \nabla_\mu \Phi^{(k)}_{\mu_1 \cdots \mu_{k+1}}, \cdots \right)$$

arbitrary function of the building blocks and its derivatives

These will generally have ghosts.

“Abelian” theories

For k=1 there is a set of ghost-free terms:

$$\partial_A \partial_B \Phi \rightsquigarrow \Phi^{(1)}_{\mu\nu} = \left(\nabla_\mu \nabla_\nu + H^2 g_{\mu\nu}\right)\phi \quad , \quad \delta\phi = S_A X^A\big|_{(A)dS}$$

$$\mathcal{L}_n = \sqrt{-g}\,\Phi^{(1)[\mu_1}_{\mu_1} \cdots \Phi^{(1)\ \mu_n]}_{\mu_n}, \quad n = 1, \ldots, D$$

These are the (A)dS galileons: Garrett Goon, KH, Mark Trodden (2011)

PRL **106,** 231102 (2011) PHYSICAL REVIEW LETTERS week ending 10 JUNE 2011

New Class of Effective Field Theories from Embedded Branes

Garrett L. Goon, Kurt Hinterbichler, and Mark Trodden
Center for Particle Cosmology, Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
(Received 6 April 2011; published 10 June 2011)

We present a new general class of four-dimensional effective field theories with interesting global symmetry groups. These theories arise from purely gravitational actions for (3 + 1)-dimensional branes embedded in higher dimensional spaces with induced gravity terms. The simplest example is the well known Galileon theory, with its associated Galilean symmetry, arising as the limit of a DGP brane world

$$\begin{aligned}
\delta_+ \hat\pi &= \frac{1}{u}\left(u^2 - y^2\right) \\
\delta_- \hat\pi &= -\frac{1}{u} \ , \\
\delta_i \hat\pi &= \frac{y_i}{u} \ .
\end{aligned}$$

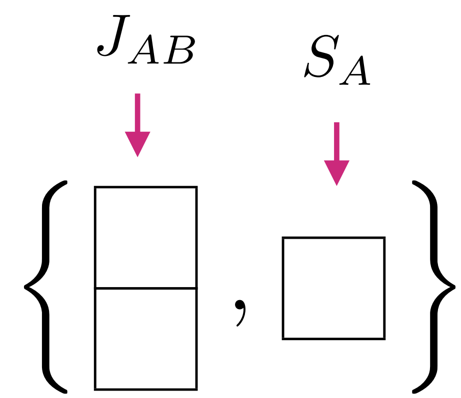
$$\begin{aligned}
\hat{\mathcal{L}}_1 &= \sqrt{-g}\hat\pi \ , \\
\hat{\mathcal{L}}_2 &= -\frac{1}{2}\sqrt{-g}\left((\partial\hat\pi)^2 - \frac{4}{L^2}\hat\pi^2\right) \ , \\
\hat{\mathcal{L}}_3 &= \sqrt{-g}\left(-\frac{1}{2}(\partial\hat\pi)^2[\hat\Pi] - \frac{3}{L^2}(\partial\hat\pi)^2\hat\pi + \frac{4}{L^4}\hat\pi^3\right) \ , \\
\hat{\mathcal{L}}_4 &= \sqrt{-g}\left[-\frac{1}{2}(\partial\hat\pi)^2\left([\hat\Pi]^2 - [\hat\Pi^2] + \frac{1}{2L^2}(\partial\hat\pi)^2 + \frac{6}{L^2}\hat\pi[\hat\Pi] + \frac{18}{L^4}\hat\pi^2\right) + \frac{6}{L^6}\hat\pi^4\right] \ , \\
\hat{\mathcal{L}}_5 &= \sqrt{-g}\left[-\frac{1}{2}\left((\partial\hat\pi)^2 + \frac{1}{5L^2}\hat\pi^2\right)\left([\hat\Pi]^3 - 3[\hat\Pi][\hat\Pi^2] + 2[\hat\Pi^3]\right)\right. \\
&\quad \left. - \frac{12}{5L^2}\hat\pi(\partial\hat\pi)^2\left([\hat\Pi]^2 - [\hat\Pi^2] + \frac{27}{12L^2}[\hat\Pi]\hat\pi + \frac{5}{L^4}\hat\pi^2\right) + \frac{24}{5L^8}\hat\pi^5\right] \ ,
\end{aligned}$$

“Non-abelian” theories

For k=1 there is a possible deformation of the algebra:

$$[S_A, S_B] = \alpha J_{AB}$$

$$S_A \Phi = X_A + \alpha\, \Phi \partial_A \Phi$$



This forms an so(D+2) algebra: $\mathcal{J}_{\mathcal{AB}} = \left(\begin{array}{c|c} 0 & S_A \\ \hline -S_A & J_{AB} \end{array}\right)$

$$[\mathcal{J}_{\mathcal{AB}}, \mathcal{J}_{\mathcal{CD}}] = \eta_{\mathcal{AC}} \mathcal{J}_{\mathcal{BD}} + \cdots$$

Symmetry breaking pattern:

$$\text{so}(D{+}2) \to \text{so}(D{+}1)$$

D+1 dimensional (A)dS D dimensional (A)dS

This gives (A)dS DBI galileons.

Clark, Love, Nitta, Veldhuis (2005)

Garrett Goon, KH, Mark Trodden (2011)

KH, Austin Joyce, Justin Khoury (2011)

“Non-abelian” theories

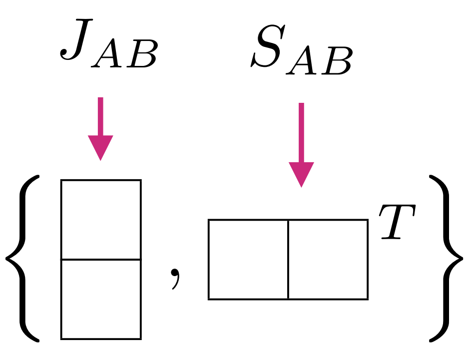
For k=2 there is a possible deformation of the algebra:

$$[S_{A_1A_2}, S_{B_1B_2}] = \alpha\left(\eta_{A_1B_1}J_{A_2B_2} + \eta_{A_2B_1}J_{A_1B_2} + \eta_{A_1B_2}J_{A_2B_1} + \eta_{A_2B_2}J_{A_1B_1}\right)$$

$$S_{AB}\Phi = X_{(A}X_{B)_T} + \alpha\partial_{(A}\Phi\partial_{B)_T}\Phi$$

This forms an sl(D+1) algebra

$$M_{AB} \equiv -\frac{1}{2}J_{AB} \pm \frac{i}{2\sqrt{\alpha}}S_{AB}$$



$$[M_{AB}, M_{CD}] = \eta_{BC}M_{AD} - \eta_{AD}M_{CB}$$

Symmetry breaking pattern:

$$\mathrm{sl}(D{+}1) \rightarrow \ \mathrm{so}(D{+}1)$$

k=2 theory

Lagrangian for D=4: ghost-free, completely fixed by the symmetry

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\rm SG} = -\frac{\Lambda^6}{H^2}\frac{(y^2-8y+8)\left(8X^2-3y^{3/2}\sqrt{X+y}+12Xy-3X\sqrt{y}\sqrt{X+y}+3y^2\right)}{15y^3(X+y)^{3/2}}$$
$$-\frac{\Lambda^6}{H^2}\left(\frac{5(y-4)y+16}{10y^{5/2}}-\frac{1}{10}\right)+\frac{2(y-4)\phi}{15Xy^{5/2}}\left(\frac{\sqrt{y}(2X+3y)}{(X+y)^{3/2}}-3\right)\frac{H^2}{\Lambda^6}\partial^\mu\phi\partial^\nu\phi X^{(1)}_{\mu\nu}(\Pi)$$
$$+\frac{y-2}{30X^2y^2}\left(2\sqrt{y}-\frac{2X^2+3Xy+2y^2}{(X+y)^{3/2}}\right)\frac{1}{\Lambda^6}\partial^\mu\phi\partial^\nu\phi X^{(2)}_{\mu\nu}(\Pi)$$
$$+\frac{\phi}{45X^2y^{3/2}}\left(\frac{\sqrt{y}(3X+2y)}{(X+y)^{3/2}}-2\right)\frac{H^2}{\Lambda^{12}}\partial^\mu\phi\partial^\nu\phi X^{(3)}_{\mu\nu}(\Pi),$$

$$y\equiv 1+4\frac{H^4}{\Lambda^6}\phi^2,\quad X\equiv\frac{H^2}{\Lambda^6}(\partial\phi)^2\ ,\qquad \Pi_{\mu\nu}\equiv\nabla_\mu\nabla_\nu\phi\ ,$$

$$X^{(n)\mu}{}_\nu(M) = (n+1)!\,\delta^{[\mu}_\nu M^{\mu_2}{}_{\mu_2}\dots M^{\mu_{n+1}]}{}_{\mu_{n+1}}$$

k=2 theory

Expansion in powers of the field:

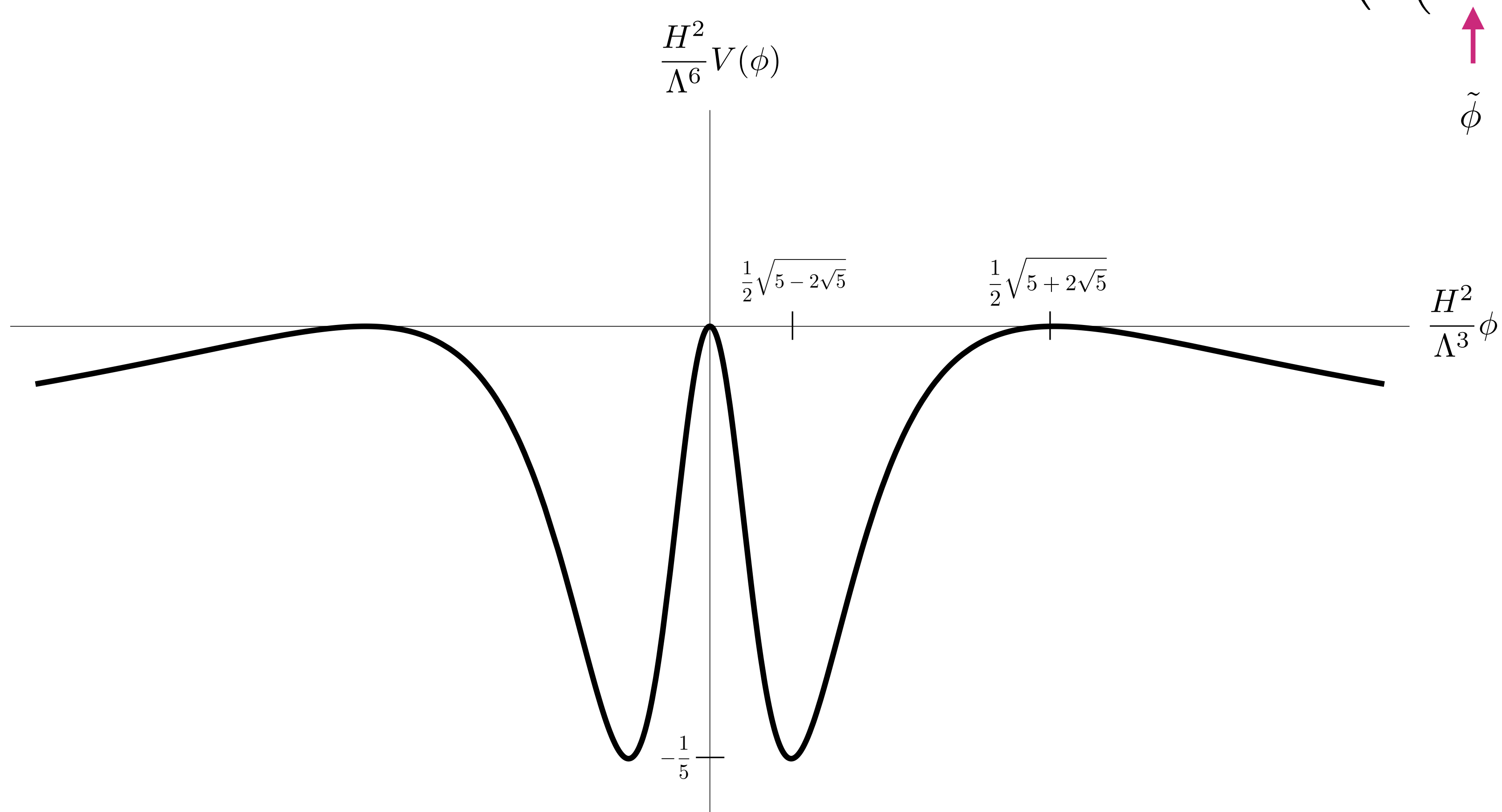
$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\rm SG} = -\frac{1}{2}\left[(\partial\phi)^2 - 10H^2\phi^2\right] + \frac{1}{24\Lambda^6}\left[\partial^\mu\phi\partial^\nu\phi X^{(2)}_{\mu\nu}(\Pi) + \mathcal{O}\left(H^2\right)\right] + \mathcal{O}\left(\phi^6\right)$$

Flat space limit $H \to 0$ is the special galileon:

$$\mathcal{L}_{\rm SG} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{24\Lambda^6}\partial^\mu\phi\partial^\nu\phi X^{(2)}_{\mu\nu}(\Pi)$$

Non-trivial potential fixed by the symmetry: $V(\tilde{\phi}) = -\frac{1}{\sqrt{-g}}\mathcal{L}_{\rm SG}\Big|_{\partial\phi=0} = \frac{\Lambda^6}{10H^2}\left(\frac{80\tilde{\phi}^4 - 40\tilde{\phi}^2 + 1}{\left(4\tilde{\phi}^2+1\right)^{5/2}} - 1\right)$

$\tilde{\phi} \equiv H^2\phi/\Lambda^3$



k=2 theory

Lagrangian in general D:

$$\frac{\mathcal{L}_{\rm SG}}{\sqrt{-g}} = \sum_{j=0}^{D-1} \frac{\psi^{D-j} + (-1)^j \psi^{*\,D-j}}{i^j \Lambda^{j(D+2)/2} \left|\psi\right|^{D+3} 2\,\Gamma(j+3)} \left[(j+2) f_j\left(\frac{X}{|\psi|^2}\right) - (j+1) f_{j+1}\left(\frac{X}{|\psi|^2}\right)\right] \partial^\mu\phi\partial^\nu\phi X^{(j)}_{\mu\nu}(\Pi)$$
$$+ \frac{\Lambda^{D+2}}{2(D+1)H^2}\left(1 - \frac{\psi^{*\,D+1} + \psi^{D+1}}{2\left|\psi\right|^{D+1}}\right),$$

$$f_j(x) \equiv {}_2F_1\left(\frac{D+3}{2}, \frac{j+1}{2}; \frac{j+3}{2}; -x\right), \qquad \psi \equiv 1 - 2i\frac{H^2}{\Lambda^{\frac{D}{2}+1}}\phi\,, \qquad X \equiv \frac{H^2}{\Lambda^{D+2}}(\partial\phi)^2$$

Expansion in powers of the field:

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\rm SG} = -\frac{1}{2}(\partial\phi)^2 + (D+1)H^2\phi^2 + \frac{1}{24\Lambda^{D+2}}\left[\partial^\mu\phi\partial^\nu\phi X^{(2)}_{\mu\nu}(\Pi) + \mathcal{O}\left(H^2\right)\right] + \mathcal{O}\left(\phi^6\right)$$

Flat space limit $H \to 0$

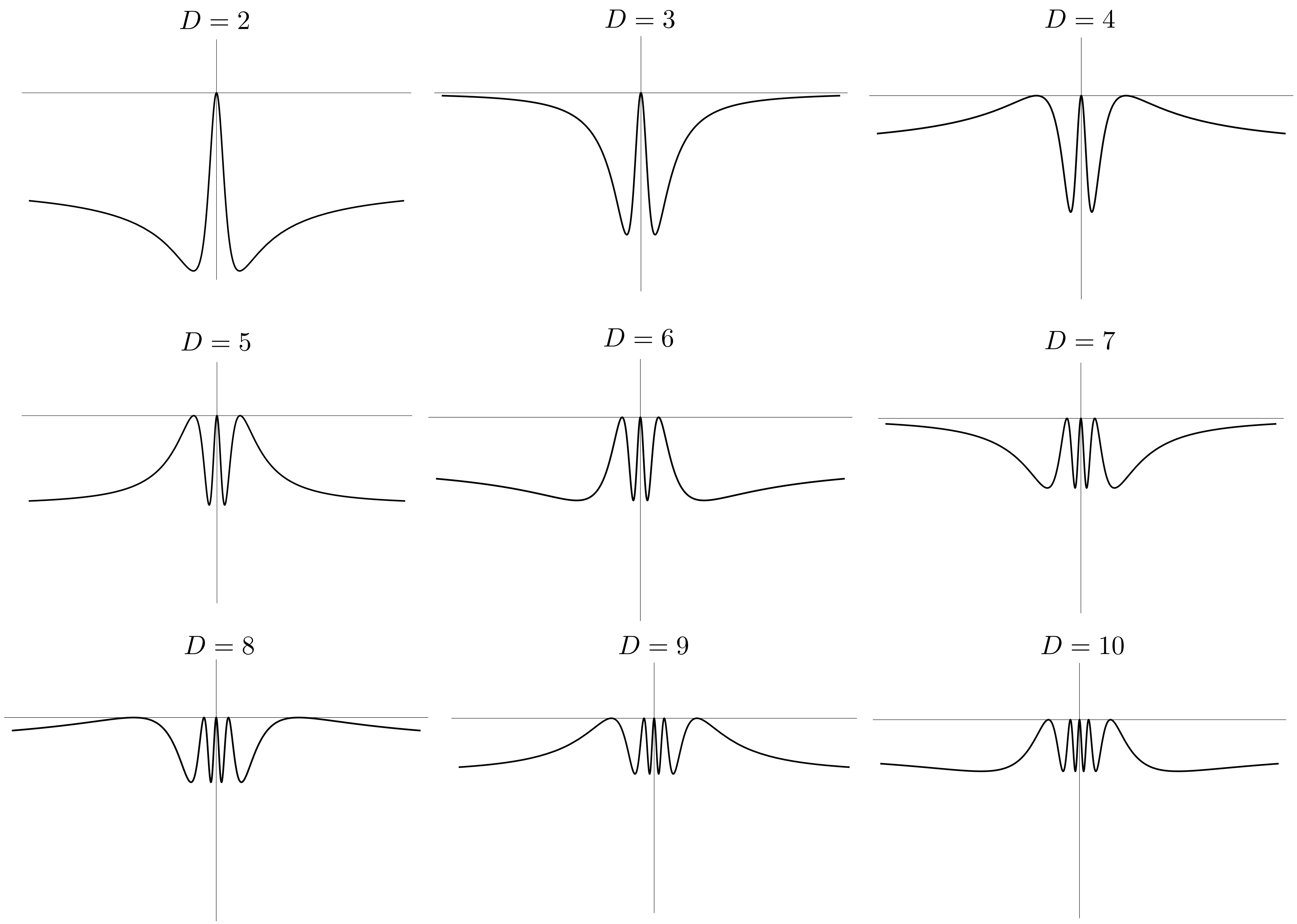
$$\mathcal{L}_{\rm SG}\Big|_{H=0} = -\sum_{\substack{j=0,\\ j\,\text{even}}}^{D-1} \frac{1}{\Lambda^{j(D+2)/2}} \frac{(-1)^{j/2}}{(j+2)!} \partial^\mu\phi\partial^\nu\phi X^{(j)}_{\mu\nu}(\Pi)$$

k=2 theory

The potential in general D :

$$\psi \equiv 1 - 2iH^2\phi/\Lambda^{(D+2)/2}$$

$$V(\phi) = -\frac{1}{\sqrt{-g}}\mathcal{L}_{\rm SG}\Big|_{\partial\phi=0} = \frac{\Lambda^{D+2}}{2(D+1)H^2}\left(\frac{\psi^{D+1}+\psi^{*\,D+1}}{2\,|\psi|^{D+1}} - 1\right)$$



Vector Interactions

Massless decoupling limit of fully non-linear massive gravity on AdS

Claudia de Rham, KH, Laura A. Johnson (2018)
James Bonifacio, KH, Laura A. Johnson, Austin Joyce (to appear)

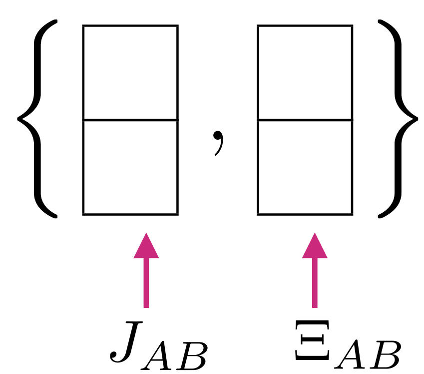
Non-linear proca theory: $\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{6}{L^2}A_\mu A^\mu - \frac{6}{L^2}A_\mu A^\mu \nabla^\nu A_\nu + \cdots\right)$

Non-abelian extension of k=0 spin-1 shift symmetry:

$$\delta A_\mu = \xi_\mu + \xi^\nu \nabla_\nu A_\mu - \xi_\mu \sqrt{1 - A^2/L^2}$$

Killing vector $\xi_\mu = \Xi_{AB} X^A \frac{\partial X^B}{\partial x^\mu}$

This forms an so(D+1)⊕so(D+1) algebra:

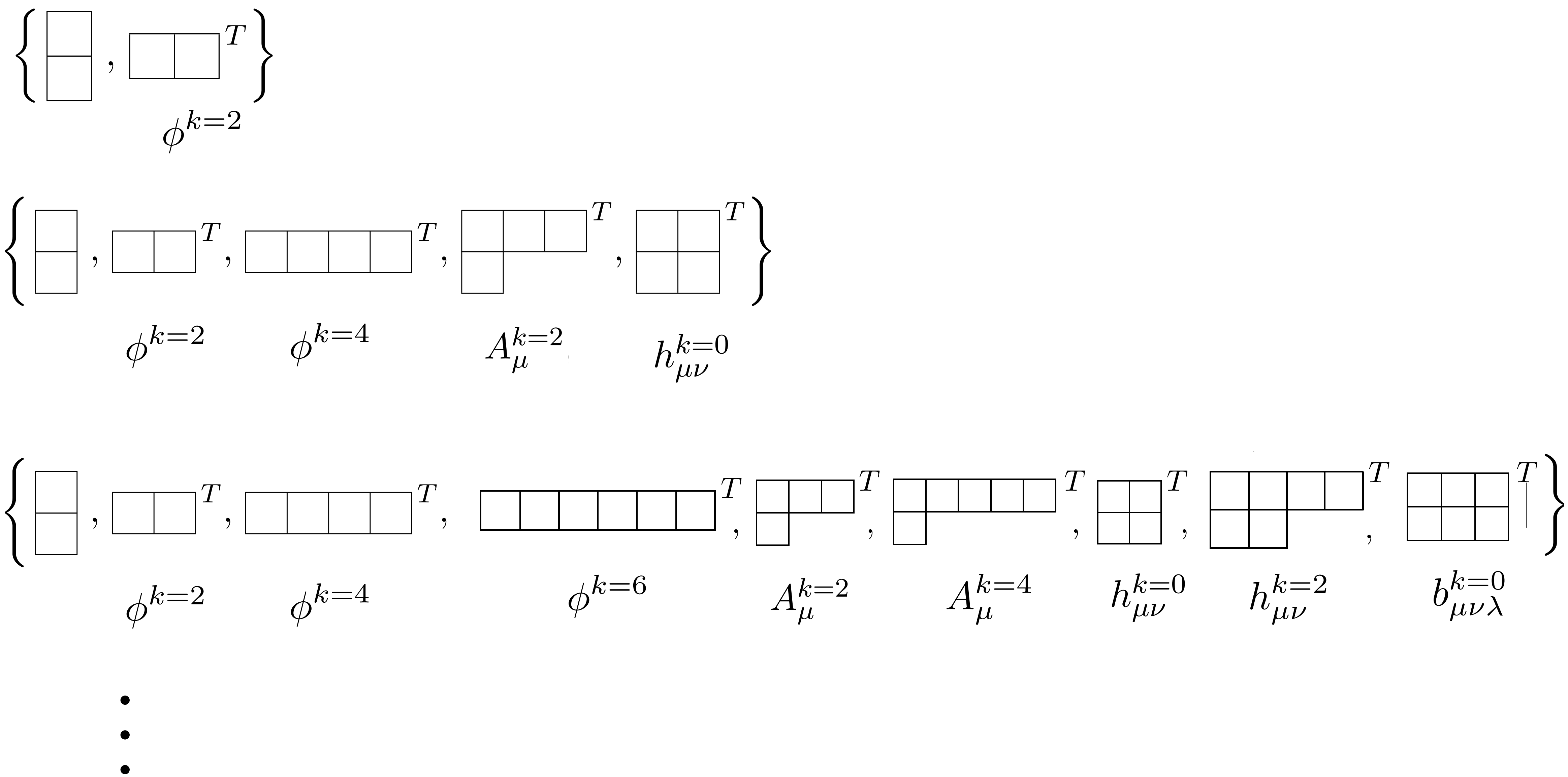


Symmetry breaking pattern:

$$\mathrm{so}(D+1)\oplus\mathrm{so}(D+1) \rightarrow \mathrm{so}(D+1)_{\text{diagonal}}$$

Other higher spin interactions?

There is a series of algebras which result from finite truncations of various higher spin algebras: Joung, Mkrtchyan (2015)



Is there a shift-symmetric theory with an infinite tower of fields coming from the longitudinal modes of Vasiliev theory?

Summary

- Massive fields of all spins on (A)dS develop shift symmetries at particular values of the masses, labelled by an integer k=0,1,2...

- These fields correspond to the longitudinal modes of partially massless gauge fields.

- We found interactions that preserve these symmetries in the scalar case when $k \leq 2$ (giving the AdS galileons and special galileon) and in the vector case when k=0.

- We believe there are more complicated multi-field interacting examples, including those with infinite numbers of fields (longitudinal modes of Vasiliev).