Qubits in Space

Late-time evolution with Horizons

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CPB, Kaplanek, Rummel
1806.11415 & 1906.xxx

Contents

- Motivation
- Secular terms in de Sitter space
    - Secular effects near dS space
- Qubits in dS
    - late-time resummation evaluated explicitly; critical slowing down; non-Markovian evolution
- Towards black holes: Rindler space
    - Secular thermal effects in flat space; accelerated qubits in flat space

The Punchline

- Gravity as an EFT

  low-energy quantum effects in gravity can be understood as a Wilsonian EFT

The Punchline

e.g. Hawking radiation calculation relies on $Q \ll M_p$ with $Q = 1/r_s$

$$\mathscr{L} \sim \sqrt{-g}\left[M_p^2 R + c_1 R^2 + \frac{c_2}{m^2} R^3 + \cdots\right]$$

$$\mathscr{A}_E(Q) \sim \left(\frac{Q^2}{M_p^{E-2}}\right)\left(\frac{Q}{4\pi M_p}\right)^{2L} \prod_{n,d>2}\left[\frac{Q^2}{M_p^2}\left(\frac{Q}{m}\right)^{d-4}\right]^{V_{nd}}$$

Leading order: tree-level GR

Next order: 1-loop GR

tree GR with one R^2 insertion

see e.g. gr-qc/0311082

The Punchline

cosmology similarly involves a series in H/M_p

$$\mathscr{L} \sim \sqrt{-g}\,\left[M_p^2[R + (\partial\phi)^2] + \boxed{v^4 U(\phi)} + c_1 R^2 + c_2(\partial\phi)^4 + \frac{c_3}{m^2}R^3 + \cdots\right]$$

$$\mathscr{B}_E(Q) \sim \frac{M_p^2}{H^2}\left(\frac{H^2}{M_p}\right)^E \left(\frac{H}{4\pi M_p}\right)^{2L} \boxed{\prod_{n,d=0}\left(\frac{v^4}{H^2 M_p^2}\right)^{V_{n0}}} \prod_{n,d>2}\left(\frac{H}{M_p}\right)^{2V_{nd}} \left(\frac{H}{m}\right)^{(d-4)V_{nd}}$$

New feature: dangerous low-energy contributions from zero-derivative interactions

see e.g. 1708.07443

The Punchline

- BUT: there are hints EFT methods fail at late times even at small curvatures
  - secular growth and IR sensitivity in cosmological perturbations
  - information loss at late times for BHs

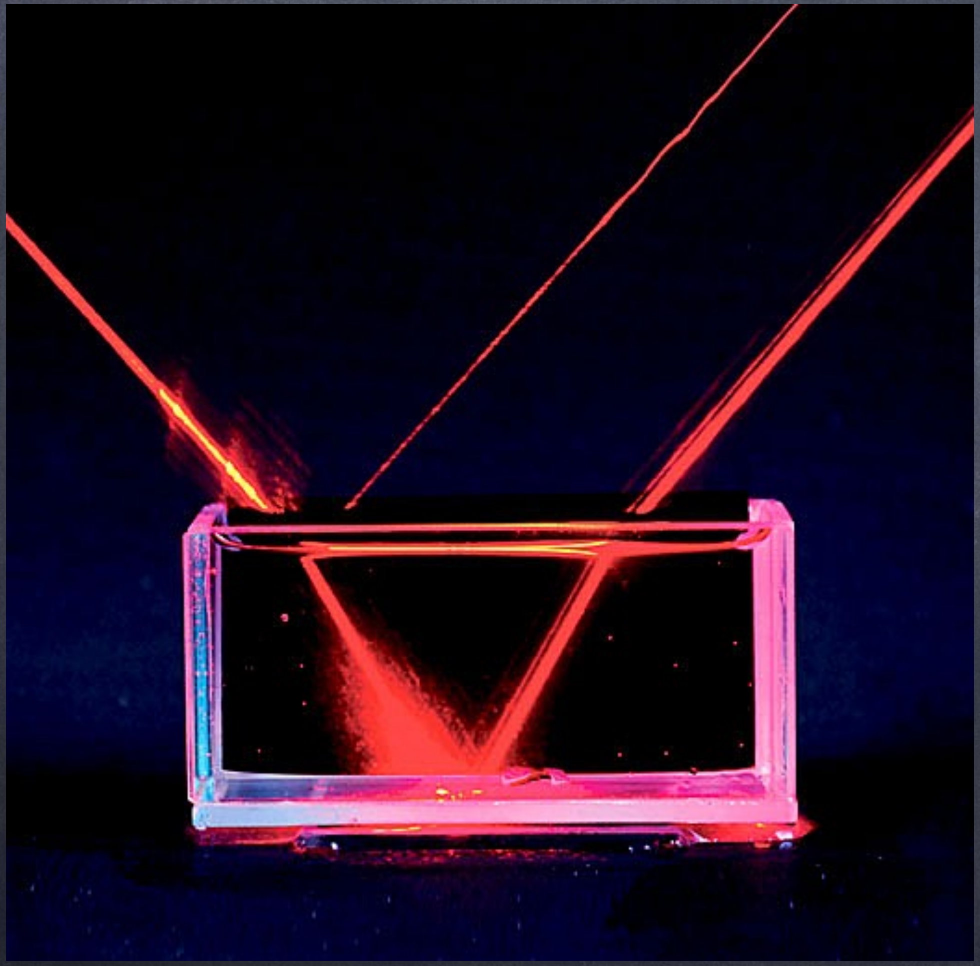
Can domain of validity of EFT sometimes require more than small curvatures?

The Punchline

- Will argue yes: EFT with gravity can differ from ordinary Wilsonian EFT
  - similar to effective description of particles in a medium
  - can involve issues of open systems (particularly in presence of horizons)
  - generic problems with late-time perturbative predictions (tho resummation methods exist)

The Punchline

- Late-time breakdown of perturbation theory illustrated by geometrical optics regime



Naive perturbation theory fails at late times

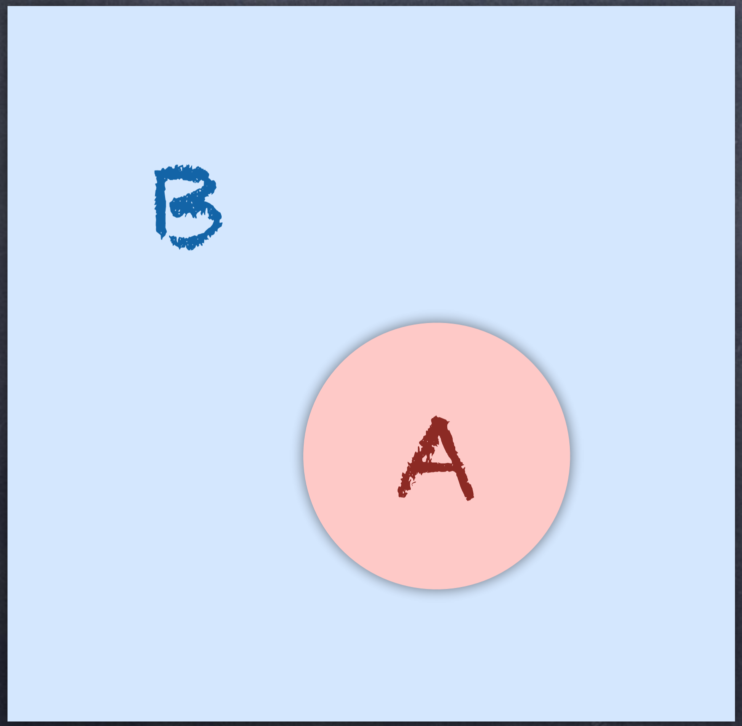
$$U(t) = e^{-i(H_0+V)t}$$

Physics is not powerless in geometrical optics regime: tools exist for understanding late time evolution

Open EFTs

- Open EFTs: consider the evolution of a subset A of a larger system B

  eg: light in glass or neutrinos in Sun or super-Hubble modes during inflation



$$\rho_A = \mathrm{Tr}_B\, \rho$$

$$\frac{\partial \rho}{\partial t} = -i\Big[\rho, H_{\rm int}\Big]$$

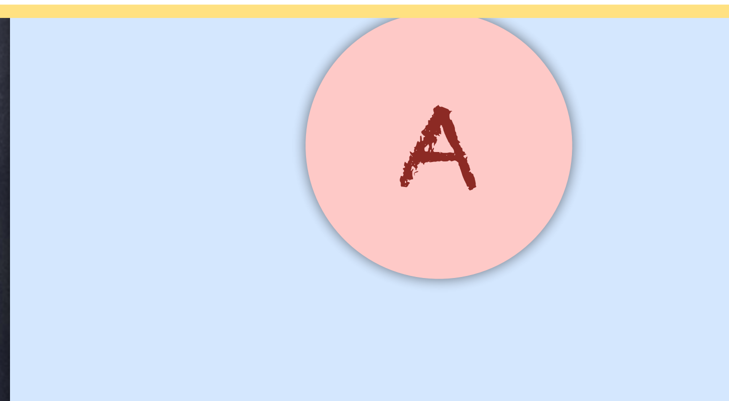
EFT part: evolution often simplifies for t much longer than typical correlation time

$$\frac{\partial \rho_A}{\partial t} \simeq F[\rho_A, \langle H_{\rm int}(t)\rangle, \langle H_{\rm int}(t) H_{\rm int}(t')\rangle, \cdots]$$

Open EFTs

An Introduction to Effective Field Theory

Thinking Effectively About Hierarchies of Scale



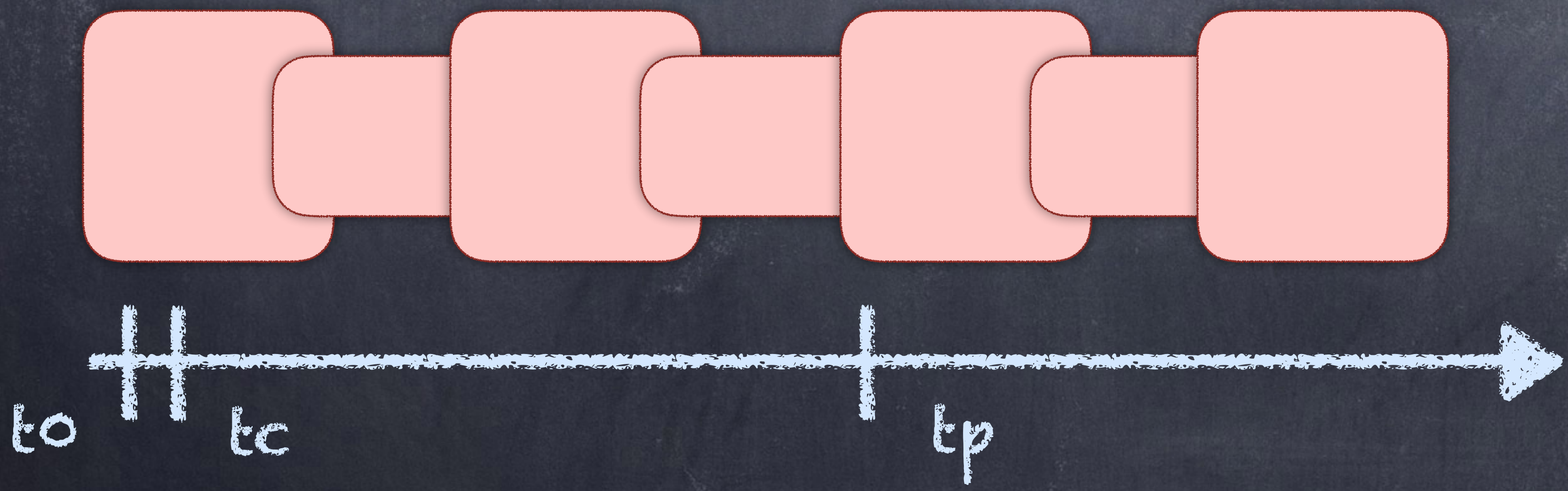
than typical correlation time

$$\frac{\partial \rho_A}{\partial t} \simeq F[\rho_A, \langle H_{\rm int}(t) H_{\rm int}(t') \rangle, \cdots]$$

Open EFTs

- Simplest example of late-time resummation: exponential decay

$$n(t) = n_0\, e^{-\Gamma t} \quad vs \quad n(t) \simeq n_0\big(1 - \Gamma t\big) \qquad \Gamma = O(g^2)$$



$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\,\Gamma n$$

all orders in $g^2 t$

see Leonardo's talk

Open EFTs

* Starobinsky (86)
Salopek & Bond (91)
Starobinsky Yokohama (94)
Vennin, Starobinsky (15)
Collins, Holman, Vardanyan

- As applied to inflationary cosmology captures stochastic methods* and decoherence

System consists of super-Hubble modes; environment is Hubble and sub-Hubble modes

$$\boxed{\frac{\partial \rho_A}{\partial t} \simeq i[\overline{H}_{\rm int}, \rho_A]} + \sum_{ij} c_{ij} \left[2L_i^* \rho_A L_j - L_i^* L_j \rho_A - \rho_A L_i^* L_j \right]$$

Diagonal terms give stochastic inflation + corrections

$$P[\varphi] = \langle \varphi \,|\, \varrho \,|\, \varphi \rangle \qquad \frac{\partial P}{\partial t} = \frac{\partial^2}{\partial \varphi^2}(NP) + \frac{\partial}{\partial \varphi}(FP)$$

Schrodinger evolution becomes stochastic in WKB limit

see e.g. 1512.00169

Lesgourges, Polarski & Starobinsky
CB, Holman, Tasinato, Williams

Open EFTs

- As applied to inflationary cosmology captures stochastic methods* and decoherence

System consists of super-Hubble modes; environment is Hubble and sub-Hubble modes

$$\frac{\partial \rho_A}{\partial t} \simeq i[\overline{H}_{\rm int}, \rho_A] + \boxed{\sum_{ij} c_{ij} \left[2L_i^* \rho_A L_j - L_i^* L_j \rho_A - \rho_A L_i^* L_j \right]}$$

Off-diagonal terms give decoherence and more

$$\langle \varphi \,|\, \varrho \,|\, \varphi' \rangle$$

Quantum fluctuations rapidly decohere (in field basis) in few Hubble times

see e.g. 1408.5002

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Qubits in Space

- Evolution of two-level qubit in various spacetimes:

  example of late-time resummation in qubit coupling; shows how field interaction changes naive evolution

$$H = I \otimes H_{\text{field}} + \mathfrak{h} \otimes I + \boxed{\mathfrak{m} \otimes \int_{y(\tau)} \mathrm{d}\tau\, \phi[y(\tau)]}$$

Unruh

$$\mathfrak{h} = \frac{\omega}{2}\sigma_3 \qquad \mathfrak{m} = g\,\sigma_1$$

H_{int}

$$H_{\text{field}} = -\int \mathrm{d}^3x \sqrt{-g}\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{m^2}{2}\phi^2 + \boxed{\frac{\lambda}{4!}\phi^4}\right]$$

H_λ

In dS choose qubit on co-moving trajectory

Qubits in Space

- Compute perturbative evolution in interaction picture

$$\rho_I(t) \simeq \rho(0) - i\int_0^t \mathrm{d}s_1\,\big[V(s_1),\rho(0)\big] + (-i)^2\int_0^t \mathrm{d}s_1\int_0^{s_1}\mathrm{d}s_2\,\Big[V(s_2),\big[V(s_1),\rho(0)\big]\Big] + \mathcal{O}(V^3)$$

$$V(s) := e^{iH_0 s} H_{\mathrm{int}} e^{-iH_0 s} \qquad\qquad \rho(0) = \big[|\Omega\rangle\langle\Omega|\big]\otimes\varrho_0$$

- Trace out field degrees of freedom to track evolution of reduced qubit density matrix

$$\begin{aligned}\varrho_I(\tau) \simeq \varrho_0 - g^2\int_0^{\tau}\mathrm{d}s_1\int_0^{s_1}\mathrm{d}s_2\Big\{ & W_\Omega(s_1,s_2)\big[\varrho_0\,\mathfrak{m}_I(s_1),\mathfrak{m}_I(s_2)\big] \\ & + W^*_\Omega(s_1,s_2)\,\big[\varrho_0\,\mathfrak{m}_I(s_1),\mathfrak{m}_I(s_2)\big]^\dagger\Big\} + \mathcal{O}(g^4)\end{aligned}$$

$$\varrho(t) = \mathrm{Tr}_\phi\,\rho(t) \qquad\qquad W_\Omega(s_1,s_2) = \langle\Omega\,|\,\phi[y(s_1)]\,\phi[y(s_2)]\,|\,\Omega\rangle$$

Late Times V1.0

- Choosing qubit initially in ground state gives

$$\varrho(\tau) \simeq |\downarrow\rangle\langle\downarrow| + g^2\sigma_3 \int_0^\tau \mathrm{d}s_1 \int_0^\tau \mathrm{d}s_2\, W_\Omega(s_1 - s_2)\, e^{-i\omega(s_1 - s_2)}$$

- In late-time limit ($\tau \to \infty$) the integral over s_1+s_2 is ill-defined, so compute the rate for a transition

$$\lim_{\tau\to\infty} \frac{\partial\varrho(\tau)}{\partial\tau} = g^2\sigma_3\, \mathscr{R}_\Omega(\omega)$$

Sciama & Candelas

$$\mathscr{R}_\Omega(\omega) := \int_{-\infty}^{\infty} \mathrm{d}\tau\, W_\Omega(\tau)\, e^{-i\omega\tau}$$

Late Times V2.0

- Liouville equation and projection onto reduced density matrix are both linear processes, so can do a better job of time evolution

$$\partial_t \rho = \mathscr{L}_t(\rho) \quad \text{where} \quad \mathscr{L}_t(\rho) := -i\Big[V(t), \rho\Big]$$

and $\mathscr{P}(\mathscr{O}) := |\Omega\rangle\langle\Omega| \otimes \mathrm{Tr}_\phi(\mathscr{O})$ so that $\mathscr{P}[\rho(t)] = |\Omega\rangle\langle\Omega| \otimes \varrho(t)$

can check $\mathscr{P}^2 = \mathscr{P}$ so that $\mathscr{Q}^2 = \mathscr{Q}$ where $\mathscr{Q} := 1 - \mathscr{P}$

- Then Liouville equation for the full density matrix can be expressed as a integro-differential equation for the reduced density matrix

$$\mathscr{P}(\partial_t \rho) = \mathscr{P}\mathscr{L}_t(\rho) = \mathscr{P}\mathscr{L}_t\mathscr{P}(\rho) + \mathscr{P}\mathscr{L}_t\mathscr{Q}(\rho)$$

$$\mathscr{Q}(\partial_t \rho) = \mathscr{Q}\mathscr{L}_t(\rho) = \mathscr{Q}\mathscr{L}_t\mathscr{P}(\rho) + \mathscr{Q}\mathscr{L}_t\mathscr{Q}(\rho)$$

Nakajima
Zwanzig

Late Times V2.0

- **Weak Coupling:** Evaluated to second order in perturbation theory the Nakajima-Zwanzig equation becomes

$$\partial_t \varrho(t) = -i\big[\mathfrak{m}(t), \varrho(t)\big]\,\langle\Omega|\,\mathscr{A}(t)\,|\Omega\rangle$$
$$+(-i)^2\int_{t_0}^{t} \mathrm{d}s\Big\{\Big[\mathfrak{m}(t), \mathfrak{m}(s)\varrho(s)\Big]\,\langle\Omega|\,\delta\mathscr{A}(t)\delta\mathscr{A}(s)\,|\Omega\rangle$$
$$-\Big[\mathfrak{m}(t), \varrho(s)\mathfrak{m}(s)\Big]\,\langle\Omega|\,\delta\mathscr{A}(s)\delta\mathscr{A}(t)\,|\Omega\rangle\Big\} + \mathscr{O}(V^3)$$

where $V(t) = \mathfrak{m}(t) \otimes \mathscr{A}(t)$

and $\delta\mathscr{A}(t) = \mathscr{A}(t) - \langle\Omega|\,\mathscr{A}(t)\,|\Omega\rangle$

Although nonlocal in time, on both sides this refers directly only to the reduced density matrix.

Qubits in Space

- For the qubit system of interest the Nakajima-Zwanzig equation becomes (in Schrodinger picture)

$$\frac{\partial \varrho_{11}}{\partial \tau} = g^2 \int_{-\tau}^{\tau} \mathrm{d}s\, W_\Omega(s)\, e^{-i\omega s} - 4g^2 \int_0^{\tau} \mathrm{d}s\, \mathrm{Re}[W_\Omega(s)]\, \cos(\omega s)\, \varrho_{11}(\tau - s)$$

$$\frac{\partial \varrho_{12}}{\partial \tau} = -\, i\omega \varrho_{12}(\tau) - 4ig^2 \int_0^{\tau} \mathrm{d}s\, \mathrm{Re}[W_\Omega(s)]\, \mathrm{Im}[\varrho_{12}(\tau - s)]$$

so off-diagonal and diagonal terms evolve independent of each other at this order

In general evolution is non-Markovian due to the integration over the qubit's past history ('memory effect')

Qubits in Space

- For the qubit system of interest the Nakajima-Zwanzig equation becomes (in Schrodinger picture)

$$\frac{\partial \varrho_{11}}{\partial \tau} = g^2 \int_{-\tau}^{\tau} \mathrm{d}s \; W_\Omega(s)\, e^{-i\omega s} - 4g^2 \int_0^{\tau} \mathrm{d}s \; \mathrm{Re}[W_\Omega(s)] \; \cos(\omega s)\, \varrho_{11}(\tau - s)$$

$$\frac{\partial \varrho_{12}}{\partial \tau} = -\, i\omega \varrho_{12}(\tau) - 4ig^2 \int_0^{\tau} \mathrm{d}s \; \mathrm{Re}[W_\Omega(s)]\, \mathrm{Im}[\varrho_{12}(\tau - s)]$$

so off-diagonal and diagonal terms evolve independent of each other at this order

Late-time simplicity follows if W falls off and evolution sought for times longer than falloff time

(if ω is large must also coarse grain W)

Qubits in Space

- Approximately markovian form at very late times if W(s) is sufficiently sharply peaked in time since $\varrho_{ij}(\tau - s) \simeq \varrho_{ij}(\tau)$ within the integral, leading to a Lindblad equation:

Candelas-Sciama result

$$\boxed{\frac{\partial \varrho_{11}}{\partial \tau} \simeq g^2 \mathscr{R}_\Omega(\omega)} \boxed{- 2g^2 \mathscr{C}_\Omega(\omega)\, \varrho_{11}(\tau)}$$

'thermalization' time:

$\xi_T = [2g^2 \mathscr{C}_\Omega(\omega)]^{-1}$

$$\frac{\partial \varrho_{12}}{\partial \tau} \simeq \boxed{- i \left[\omega + g^2 \Delta_\Omega\right] \varrho_{12}} \boxed{- g^2 \mathscr{C}_\Omega(\omega)\, \mathrm{Im}[\varrho_{12}(\tau)]}$$

frequency renormalization

'decoherence' time:

$\xi_D = 2\xi_T$

with $\mathscr{C}_\Omega(\omega) := 2 \int_0^\infty \mathrm{d}\tau\ \mathrm{Re}[W_\Omega(\tau)]\ \cos(\omega\tau)$

$$\Delta_\Omega(\omega) := 2 \int_0^\infty \mathrm{d}\tau\ \mathrm{Re}[W_\Omega(\tau)]\ \sin(\omega\tau)$$

Describes relaxation to asymptotic static solution:

if $W_\Omega(\tau - i\beta) = W_\Omega(-\tau)$ then $\varrho_{\text{static}} = \begin{bmatrix} e^{-\beta\omega} & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{e^{-\beta\omega} + 1}$

Qubits in de Sitter

- For de Sitter space evaluate Wightman function using BD vacuum and qubit along a co-moving curve to compute relaxation rates as system approaches static solution at very late times

$$W_{\rm BD}(\tau) \;=\; \frac{H^2(\frac{1}{4}-\nu^2)}{16\pi\cos(\pi\nu)}\;{}_2F_1\left[\tfrac{3}{2}+\nu,\tfrac{3}{2}-\nu;2;1+\left[\sinh\left(\tfrac{H\tau}{2}\right)-i\epsilon\right]^2\right]$$

$$\nu \;=\; \sqrt{\frac{9}{4}-\frac{M^2}{H^2}} \;=\; \sqrt{\frac{9}{4}-\frac{m^2}{H^2}+12\xi}$$

conformal scalar: $W_{\rm BD}(\tau) \;=-\,\dfrac{1}{16\pi^2}\,\dfrac{H^2}{\big[\sinh\left(H\tau/2\right)-i\epsilon\big]^2}$

- Also satisfies the thermal 'KMS' condition

if $W_\Omega(\tau-i\beta) \;=\; W_\Omega(-\tau)$ for $T=\dfrac{1}{\beta}=\dfrac{H}{2\pi}$

Qubits in de Sitter

- For de Sitter space evaluate Wightman function using BD vacuum and qubit along a co-moving curve to compute relaxation rates as system approaches static solution at very late times

e.g. for conformal scalars: $\xi_D = 2\xi_T \simeq \frac{2\pi}{g^2\omega} \tanh\left(\frac{\pi\omega}{H}\right)$

Trust the above Markovian limit on timescales $\xi \gg H^{-1}$

For m << H the Markovian limit instead requires $\xi \gg H/m^2$ (critical slowing down)

Qubits in de Sitter

- These expressions also allow resummation of scalar self-interaction because the leading resummed graphs correspond to there being a coupling-dependent mass shift:

$$M_{\rm eff}^2 = M^2 + \frac{3\lambda H^4}{16\pi^2 M^2}$$

CB, LeBlond, Holman & Shandera

Resums all orders in $(3\lambda H^2/16\pi^2 M^2)\ \ln(k\tau)$

As M becomes smaller $M_{\rm eff}$ is bounded from below, with

$$M_{\rm min}^2 = \frac{\sqrt{3\lambda}}{2\pi}\, H^2 \qquad\qquad \nu = \sqrt{\frac{9}{4} - \frac{M^2}{H^2}}$$

$$W_{\rm BD}(\tau) \sim -\frac{1}{4\pi^{5/2}} H^2 \sin(\pi\nu)\Gamma\left(\frac{3}{2}-\nu\right)\Gamma(\nu)\exp\left[-\left(\frac{3}{2}-\nu\right)H\tau\right]$$

Conclusions

- IR and secular issues likely generic for light bosons in gravitational fields, and cause perturbative failure at late times
  - Small curvatures/couplings need not be sufficient for calculation control
  - Resummation techniques available: Open EFTs
- Practical implications for black hole information loss and/or late-time cosmology?