Amplitudes meet BSM pheno

Alex Pomarol, CERN & UAB (Barcelona)

of course, not the first encounter…

Two *little applications:*

I. Amplitude methods: useful for simplifying calculations

(**shortcut** from the Feynman way)

But not much used in BSM phenomenology!

Use of amplitudes for calculating **one-loop corrections from indirect BSM effects**

many surprises known!

☛ Crucial role plaid by helicity selection rules

II. Bottom-up approach to theories of Goldstones:

composite Higgs

Consistently from $\mathcal{A}(1234) \to \boldsymbol{q_i}$ (for $\boldsymbol{q_i \to 0}$)

I. EFT capturing the (indirect) impact of BSMs

Assuming **new-physics scale Λ is heavier than M_W**,
we can perform an expansion in derivatives and SM fields

(assuming lepton & baryon number)

$$\mathcal{L}_{\rm eff} = \frac{\Lambda^4}{g_*^2}\mathcal{L}\left(\frac{D_\mu}{\Lambda}\,,\ \frac{g_H H}{\Lambda}\,,\ \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}}\,,\ \frac{g F_{\mu\nu}}{\Lambda^2}\right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

SM

leading deviations from the SM

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SM

leading deviations from the SM

Λ —— $\mathcal{O}_i, \mathcal{O}_j, \ldots$

m_W —— $\mathcal{O}_i$

One-loop operator mixing important:

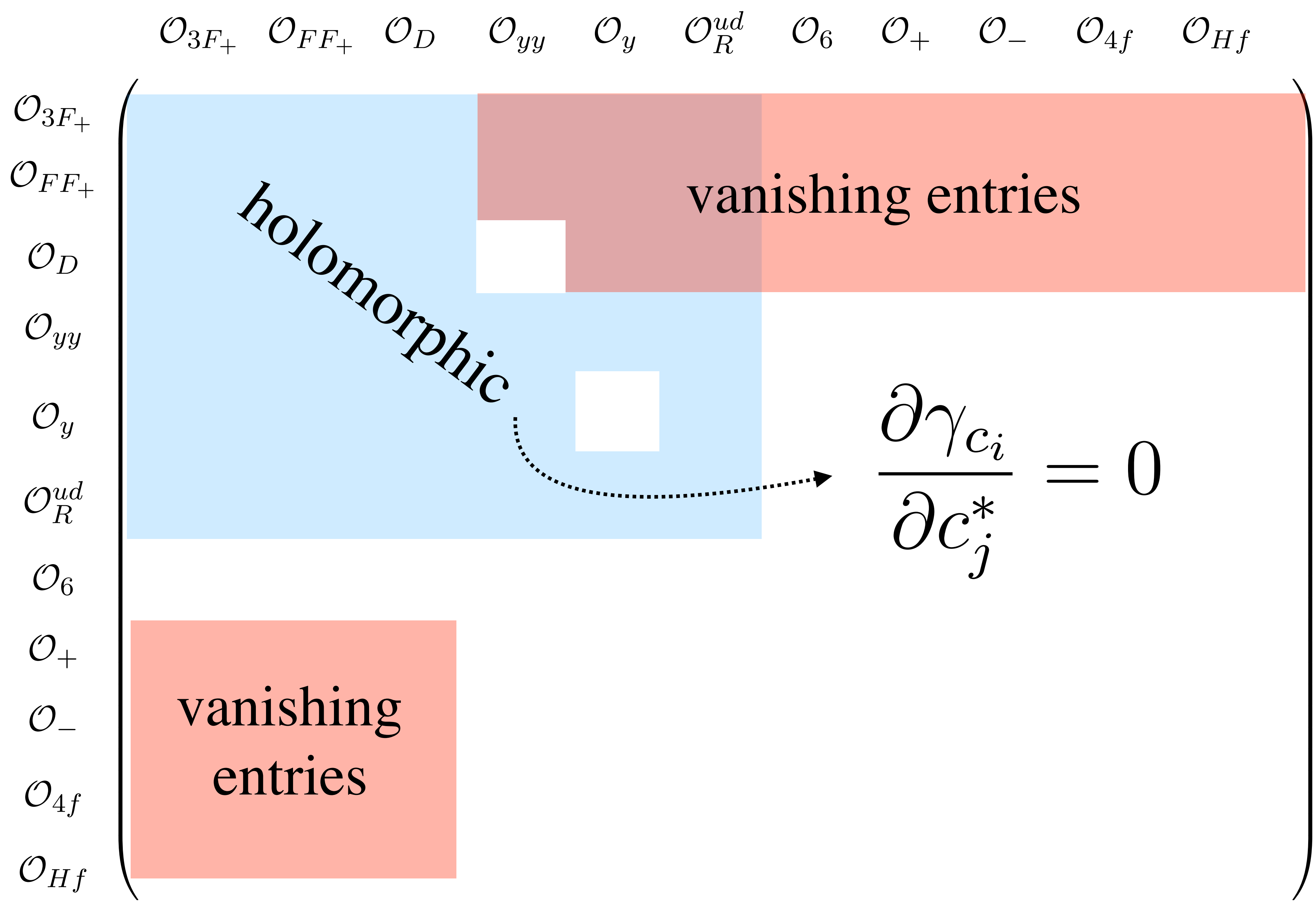
(tells us how BSM enter in observables)

$$\boxed{\gamma_{c_i} = \frac{dc_i}{d\log\mu} = \gamma_{c_i}(c_j)}$$

c_j = Wilson coefficient

One-loop anomalous dimension of dim-6 operators

$$\gamma_{c_i}(c_j) =$$



arXiv:1412.7151 (explained from susy)

Very practical example:

Renormalization of electron EDM

Recent strong bound by ACME experiment:

$$|d_e| < 1.1 \cdot 10^{-29}\,\mathrm{e}\cdot\mathrm{cm}$$

Can provide important constraints even if BSM enters at the 2-loop level!

$$\frac{d_e}{e} \sim \frac{1}{(16\pi^2)^2}\frac{m_e}{\Lambda^2} \qquad \to \Lambda > 3\,\mathrm{TeV}$$

Best weapon of BSM mass destruction!

or even on dimension-8 operators!

Panico, AP, Riembau arXiv:1810.09413

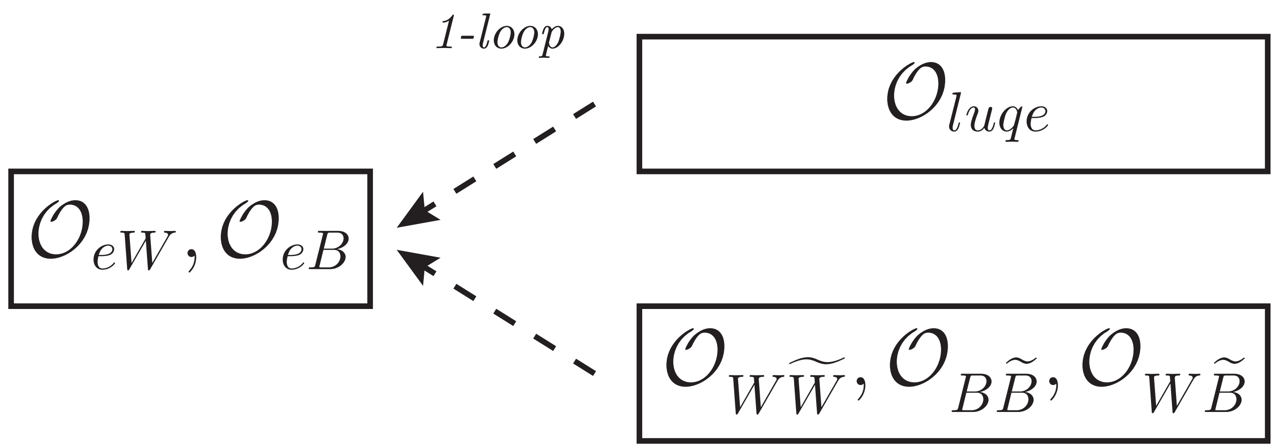
One-loop mixing:

$$\begin{tikzcd} & \boxed{\mathcal{O}_{luqe}} \arrow[dl, dashed, "\textit{1-loop}"'] \\ \boxed{\mathcal{O}_{eW}, \mathcal{O}_{eB}} & \\ & \boxed{\mathcal{O}_{W\widetilde{W}}, \mathcal{O}_{B\widetilde{B}}, \mathcal{O}_{W\widetilde{B}}} \arrow[ul, dashed] \end{tikzcd}$$

out of 59 operators

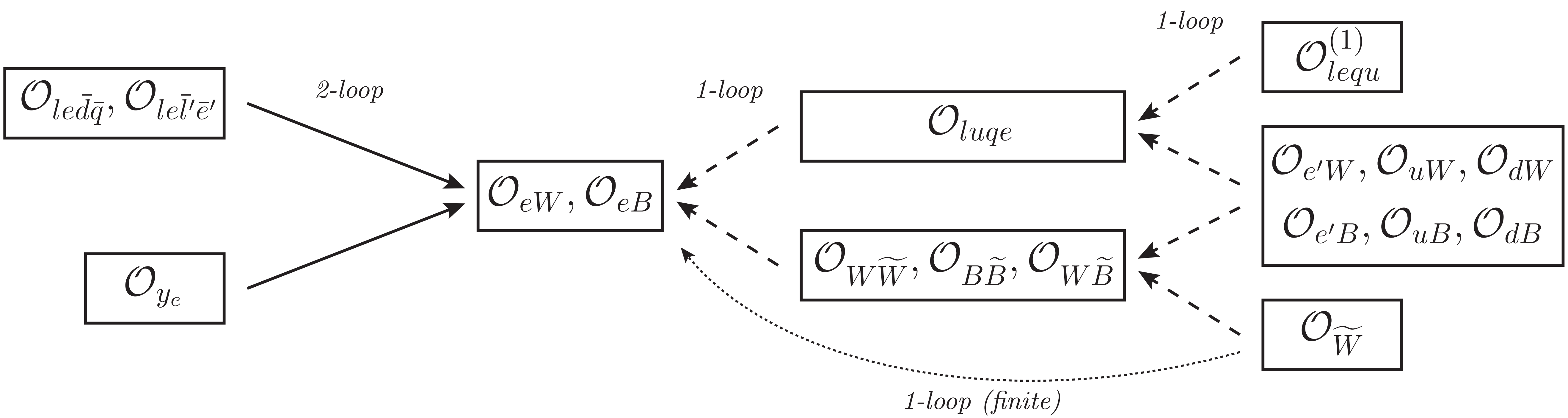
Panico, AP, Riembau arXiv:1810.09413

One-loop mixing:



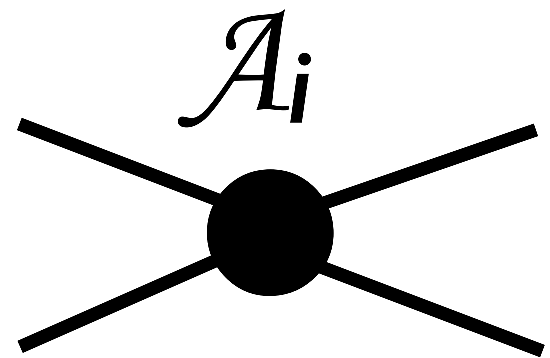
Two-loop mixing:

out of 59 operators



From operators to on-shell amplitudes

$\mathcal{O}_{\mathbf{i}} \longrightarrow$



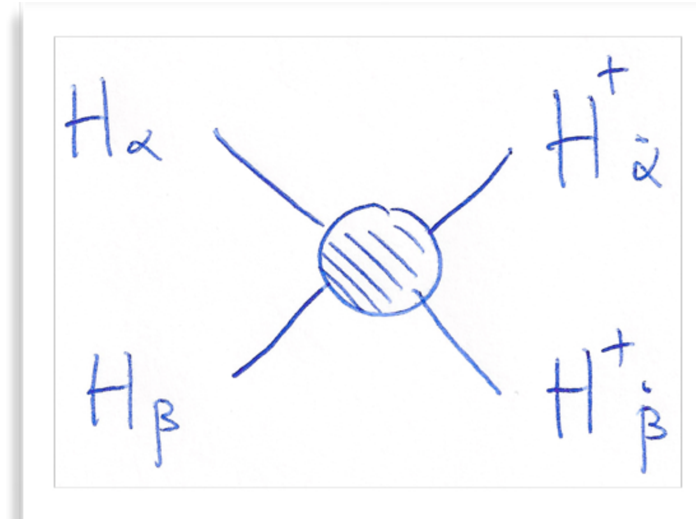
n = **number of external states**

h = **helicity of the amplitude**

Example $O(\partial^2 H^4)$:

$$\mathcal{O}_H = \frac{1}{2}(\partial^\mu |H|^2)^2$$

$$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$$

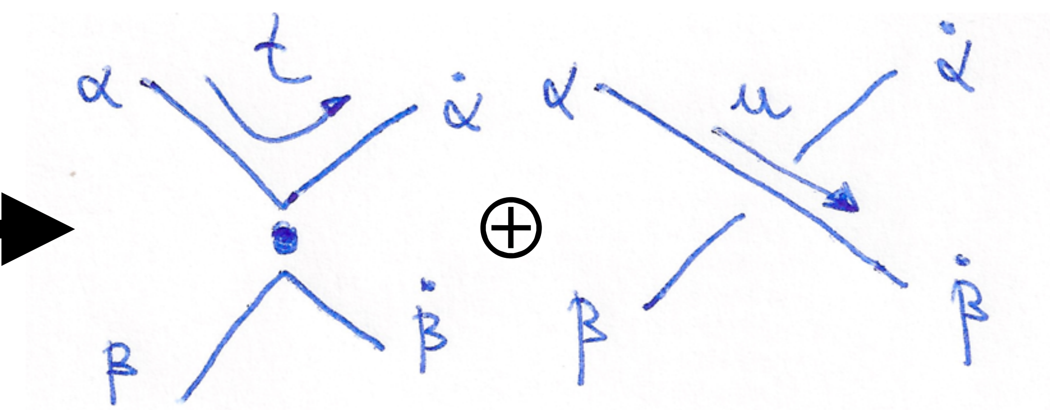


$\sim O(p^2)$

n=4; h=0

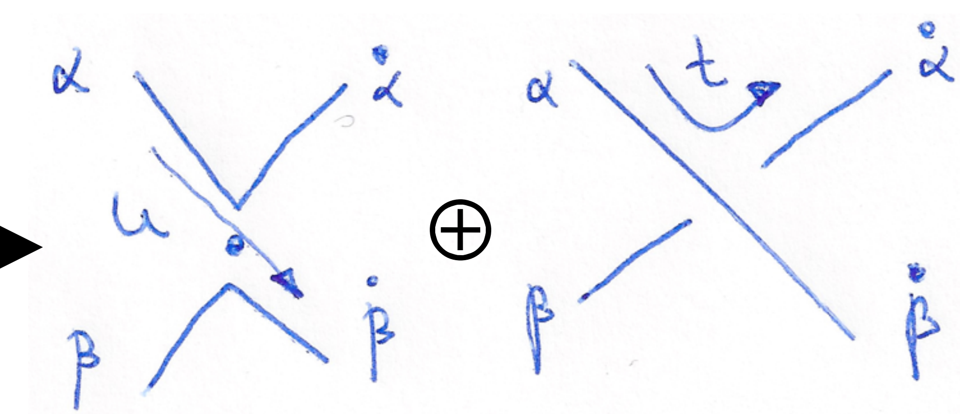
Two amplitudes:

$$A\left[t\,\delta_{\alpha\dot\alpha}\delta_{\beta\dot\beta} + u\,\delta_{\beta\dot\alpha}\delta_{\alpha\dot\beta}\right]$$



flavor-momentum "alignment"

$$B\left[u\,\delta_{\alpha\dot\alpha}\delta_{\beta\dot\beta} + t\,\delta_{\beta\dot\alpha}\delta_{\alpha\dot\beta}\right]$$



flavor-momentum "anti-alignment"

From operators to on-shell amplitudes

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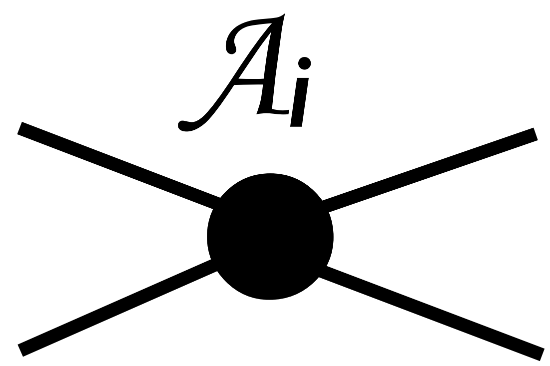
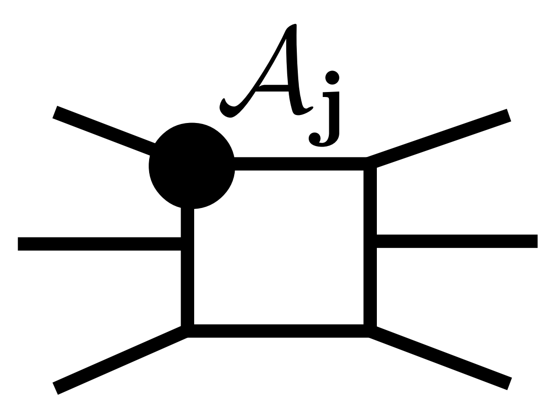


n **= number of external states**

h **= helicity of the amplitude**

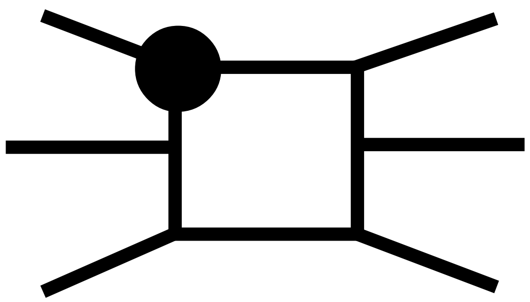
Interested here in one-loop corrections:

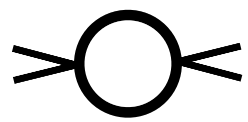
$\mathcal{A}_{\mathbf{i}} =$

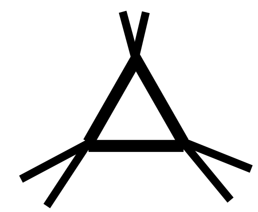


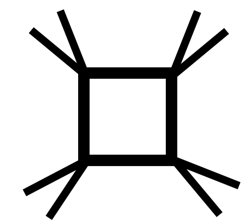
After one-loop reduction to Passarino-Veltman integrals

$$\mathcal{A}_{\mathbf{i}} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \textbf{rational}$$









divergent ☛ anomalous dimensions

After one-loop reduction to Passarino-Veltman integrals

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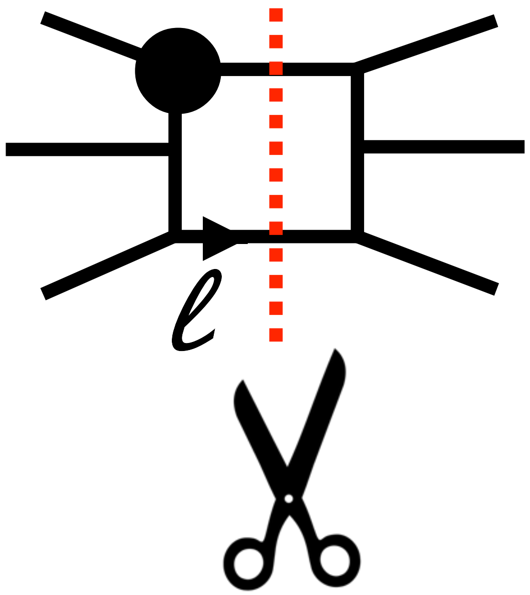
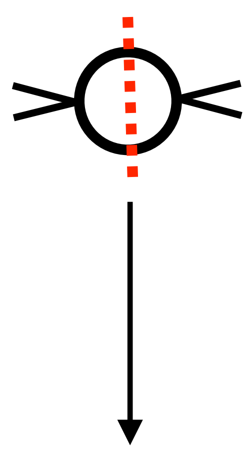
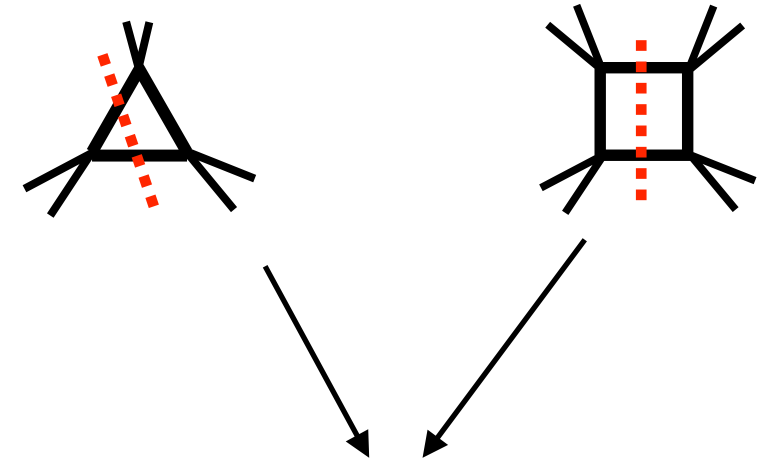


double cut



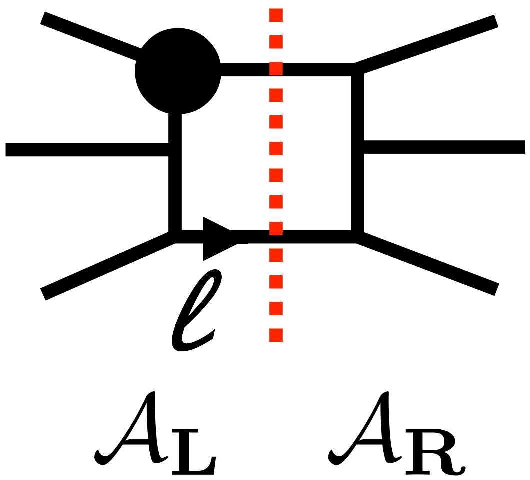
c_2

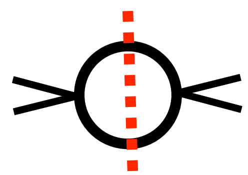


ℓ-dependent

After one-loop reduction to Passarino-Veltman integrals

$$\mathcal{A}_{\mathbf{i}} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \mathbf{rational}$$





c2

ℓ-dependent

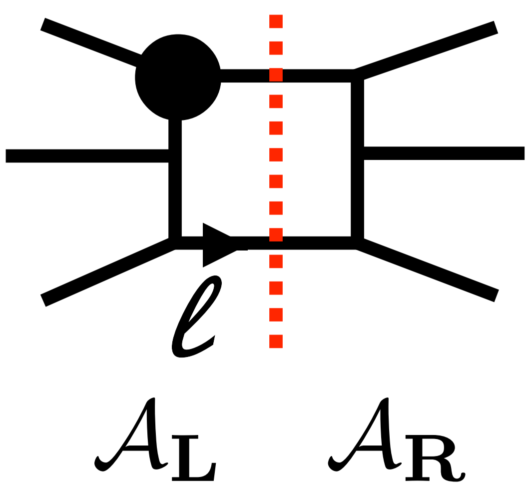
$$\sum\!\!\!\!\!\!\int \mathcal{A}_L \mathcal{A}_R$$

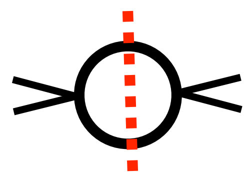
$$\boldsymbol{n_i = n_L + n_R}$$
$$\boldsymbol{h_i = h_L + h_R}$$

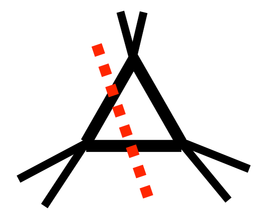
sum over internal states & phase-space integration

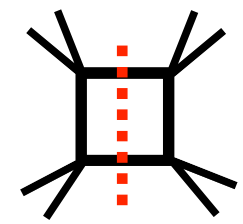
After one-loop reduction to Passarino-Veltman integrals

$$\mathcal{A}_\mathbf{i} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \textbf{rational}$$









c_2

ℓ-dependent

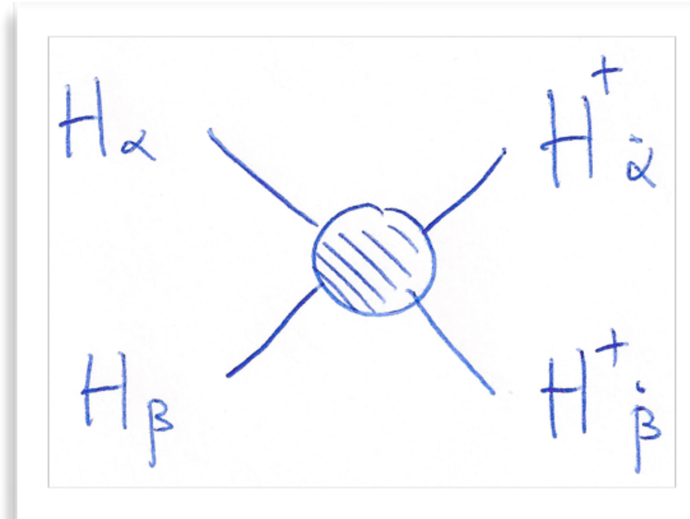
$$\sum\!\!\!\!\!\!\int \mathcal{A}_L \mathcal{A}_R$$

$n_i = n_L + n_R$
$h_i = h_L + h_R$

"tailoring" needed to get $\sum c_2 = \gamma_i$

Example $O(\partial^2 H^4)$:

$$\left.\begin{aligned} \mathcal{O}_H &= \frac{1}{2}(\partial^\mu |H|^2)^2 \\ \mathcal{O}_T &= \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2 \end{aligned}\right\} \longrightarrow$$

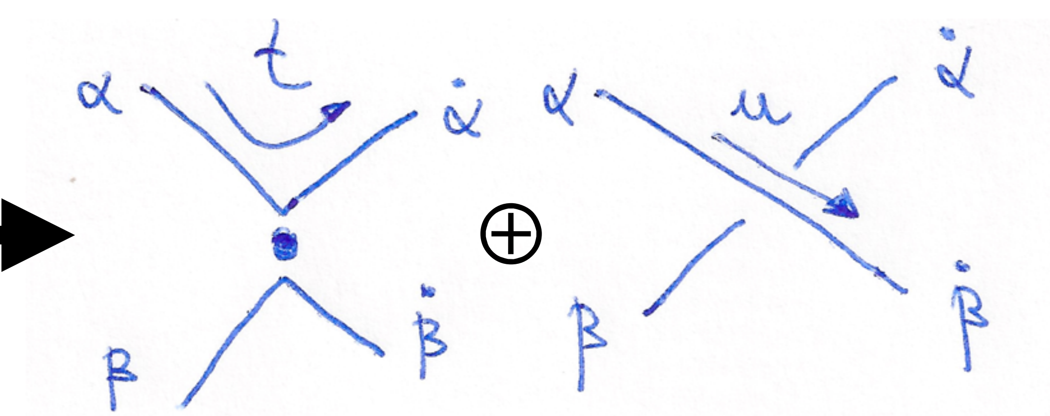


$\sim O(p^2)$

n=4; h=0

Two amplitudes:

$$A\left[t\,\delta_{\alpha\dot\alpha}\delta_{\beta\dot\beta} + u\,\delta_{\beta\dot\alpha}\delta_{\alpha\dot\beta}\right] \longrightarrow$$



flavor-momentum "alignment"

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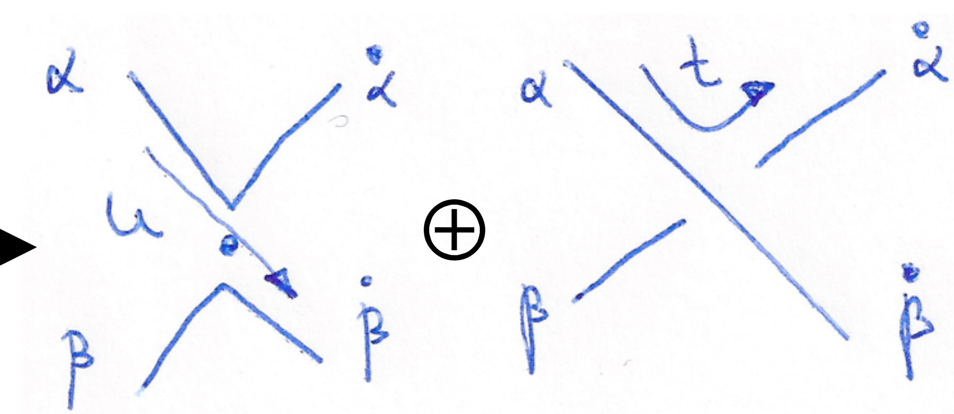
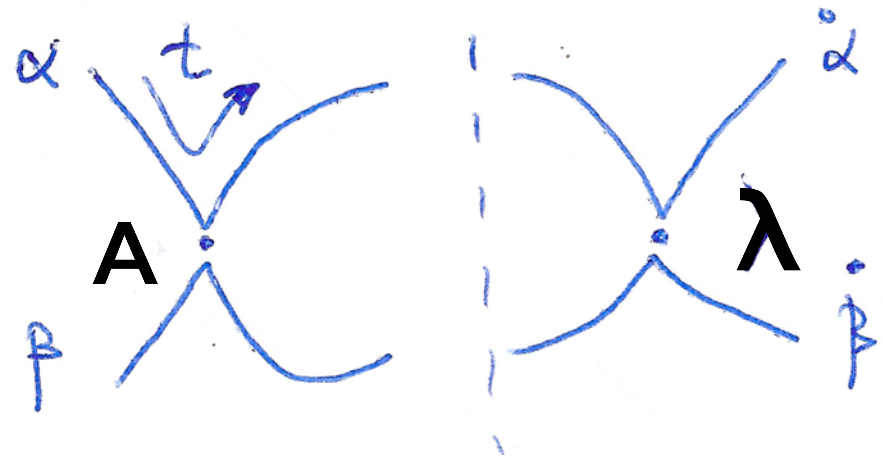


flavor-momentum "anti-alignment"

One-loop corrections

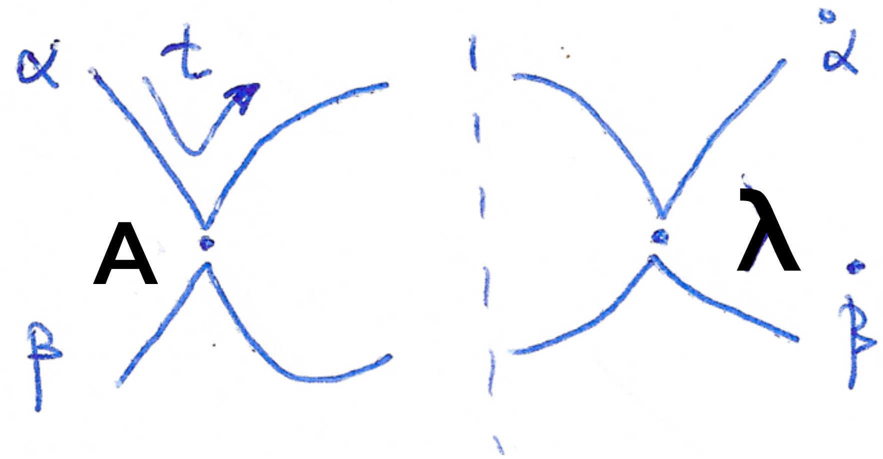
s-channel:



⊕ t-channel ⊕ u-channel

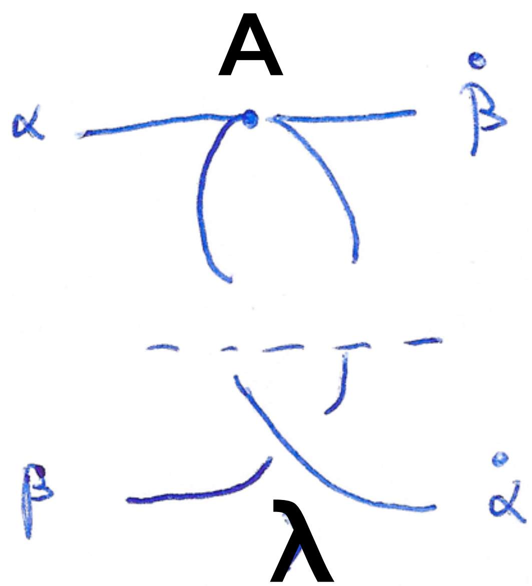
One-loop corrections

s-channel:



$s \leftrightarrow u$

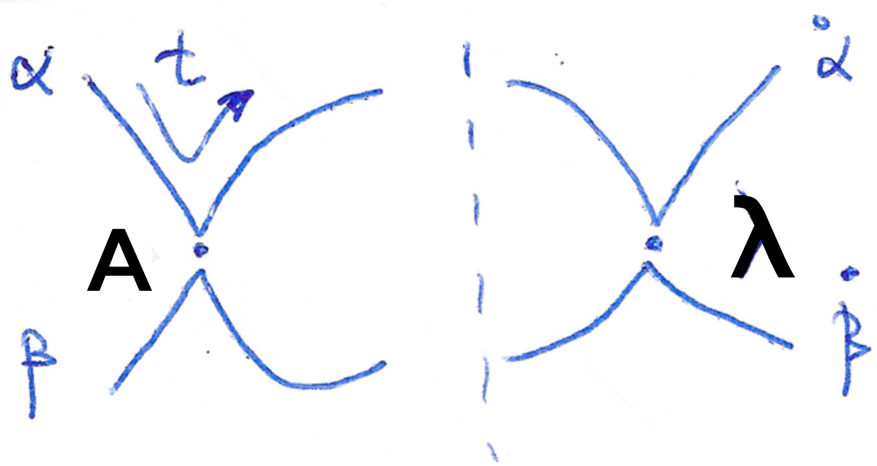
u-channel:



$\sim (s+u) = -t$

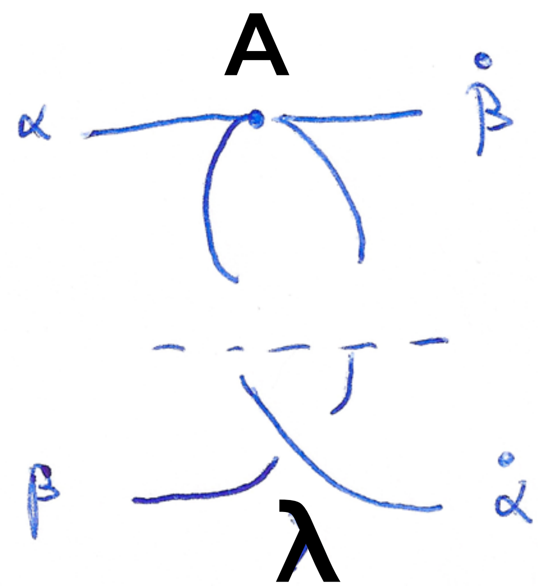
One-loop corrections

s-channel:

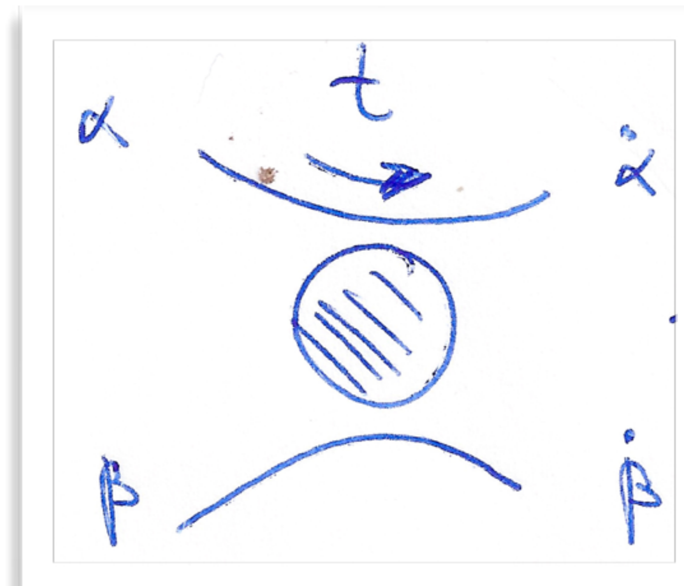


s↔u

u-channel:



⊕ t-channel

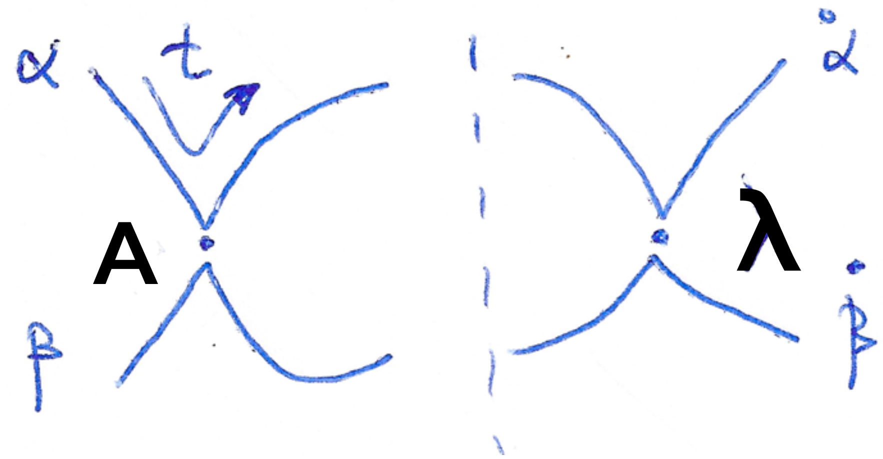


preservation of momentum-flavor "alignment" !

Custodial sym.!

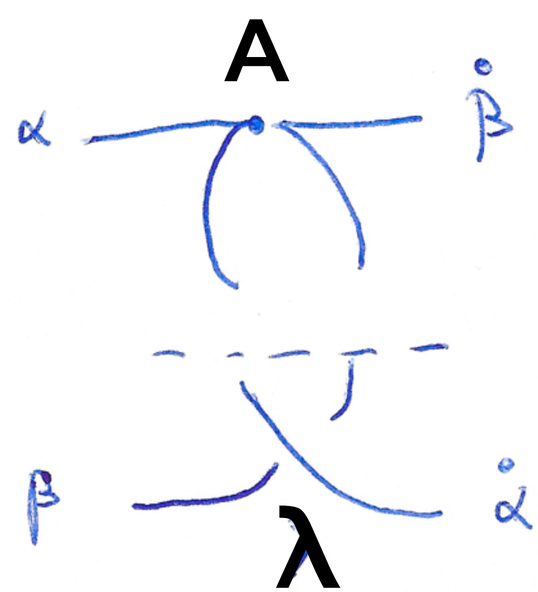
One-loop corrections

s-channel:

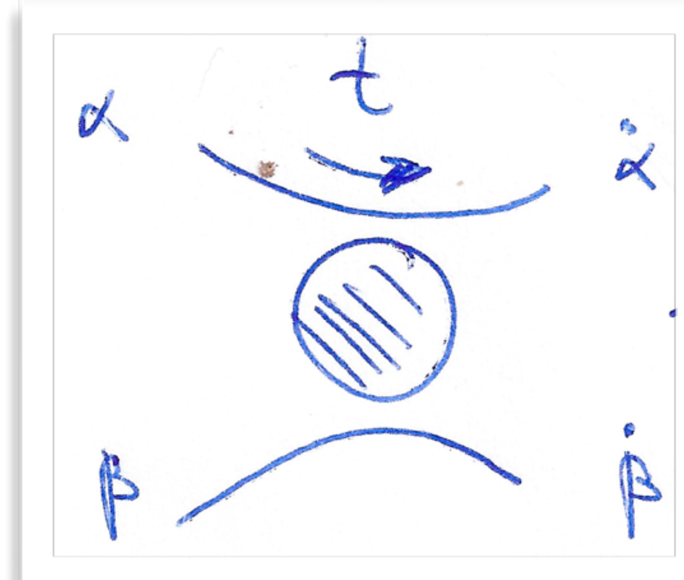


s↔u

u-channel:



⊕ t-channel



preservation of momentum-flavor "alignment" !

also preservation of momentum-flavor "anti-alignment" <u>for doublets</u>

Custodial sym.!

Helicity selection rules

arXiv:1505.01844

Helicity selection rules

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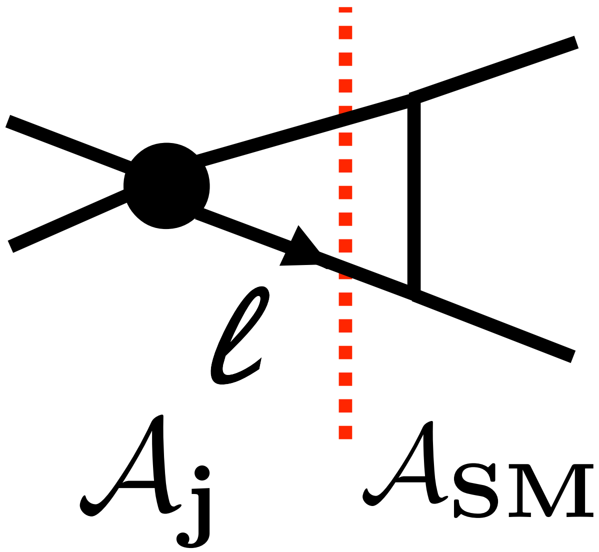


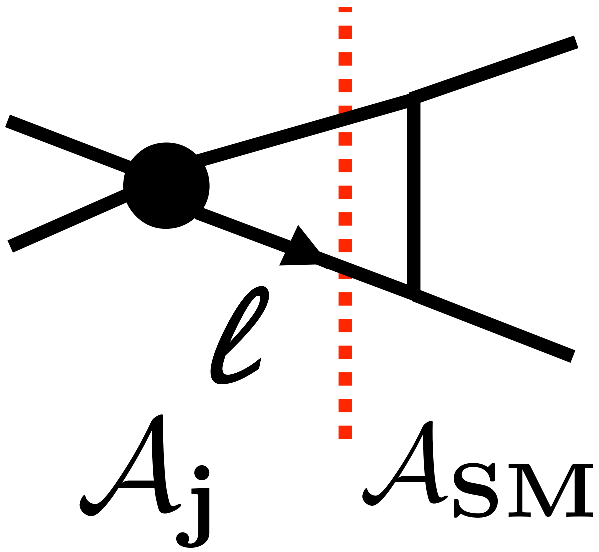
$$\longrightarrow \sum\!\!\!\!\!\!\int \mathcal{A}_{\mathbf{j}} \mathcal{A}_{\mathrm{SM}}$$

n_j , h_j $\quad$ $n = 4$, h

(no contribution from n=3)

Helicity selection rules

arXiv:1505.01844

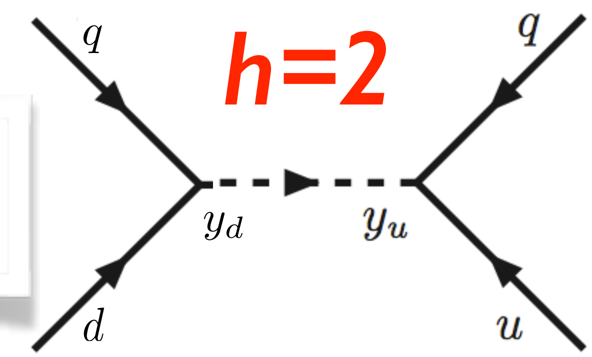


$$\longrightarrow \sum\!\!\!\!\!\!\int \mathcal{A}_{\mathrm{j}} \mathcal{A}_{\mathrm{SM}}$$

SM tree-level amplitude

n=4 SM amplitudes ☛ h=0

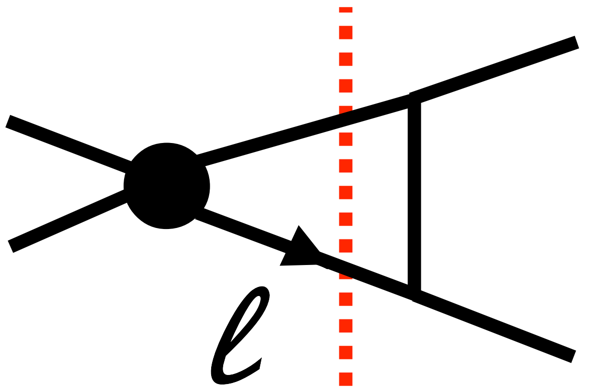
h=2



up to one exception!

Helicity selection rules

arXiv:1505.01844



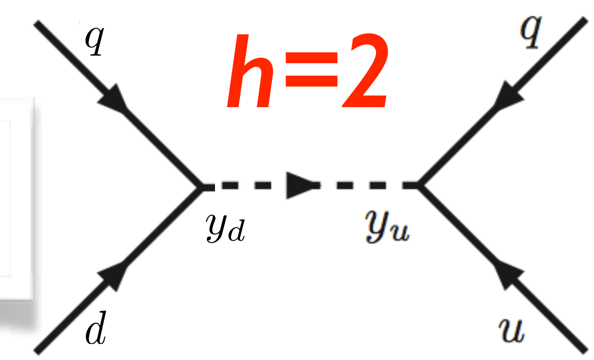
$$\mathcal{A}_{\rm j} \qquad \mathcal{A}_{\rm SM}$$

$$\longrightarrow \quad \sum\!\!\!\!\!\!\int \mathcal{A}_{\rm j}\mathcal{A}_{\rm SM}$$

SM tree-level amplitude

n=4 SM amplitudes ☛ h=0

***h*=2**



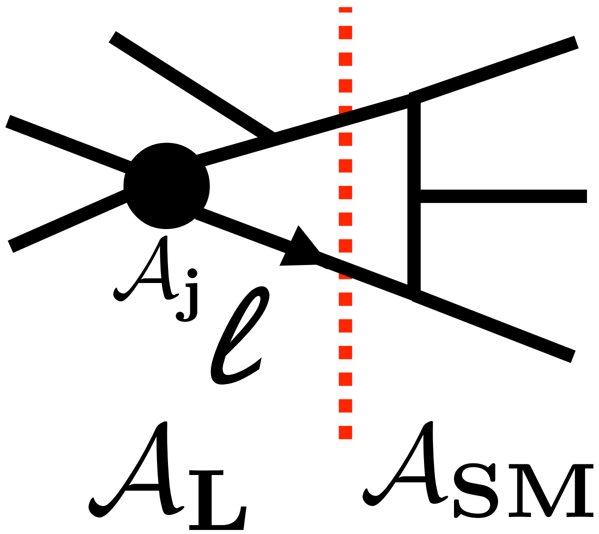
up to one exception!

$$\left.\begin{array}{l} n_i = n_j + 4 \\ h_i = h_j \end{array}\right\} n_i - n_j = h_i - h_j + 4 - 4$$

$$☛\ \Delta n = |\Delta h|$$

Helicity selection rules

arXiv:1505.01844



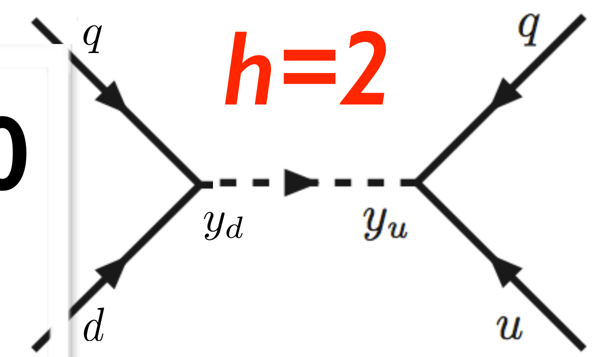
$$\sum\!\!\!\!\!\!\int \mathcal{A}_{\mathrm{j}}\mathcal{A}_{\mathrm{SM}}$$

SM tree-level amplitude

n ≥ 4 SM amplitudes ☛ h ≥ 0

☛ $n \geq |h|+4$

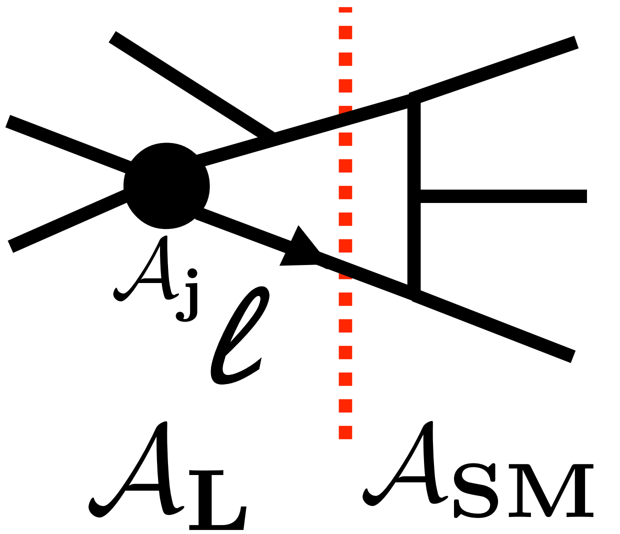
***h*=2**



up to one exception!

Helicity selection rules

arXiv:1505.01844

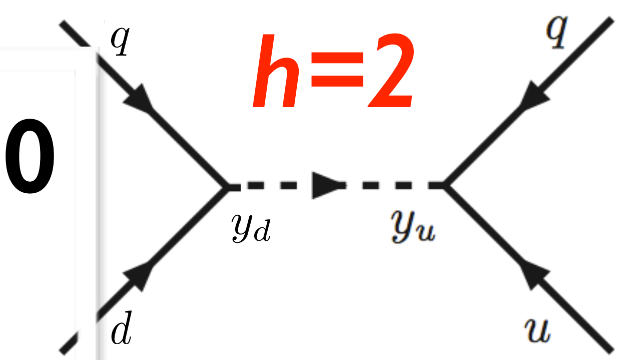


$$\longrightarrow \quad \sum\!\!\!\!\!\!\int \mathcal{A}_{\rm j}\mathcal{A}_{\rm SM}$$

SM tree-level amplitude

$n \geq 4$ SM amplitudes ☛ $h \geq 0$

☛ $n \geq |h|+4$



h=2

up to one exception!

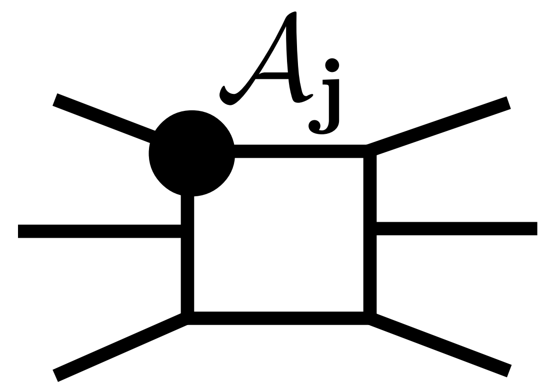
$$\left.\begin{array}{l} n_i = n_L + n_{SM} \\ h_i = h_L + h_{SM} \end{array}\right\} n_i - n_j \geq h_i - h_j + 4 - 4$$

$$☛ \ \Delta n \geq |\Delta h|$$

Helicity selection rules

arXiv:1505.01844

$$\mathcal{A}_{\mathbf{i}} =$$



$\Delta n = n_i - n_j$

$\Delta h = h_i - h_j$

$$\boxed{\Delta n \geq |\Delta h|}$$

up to the exception!

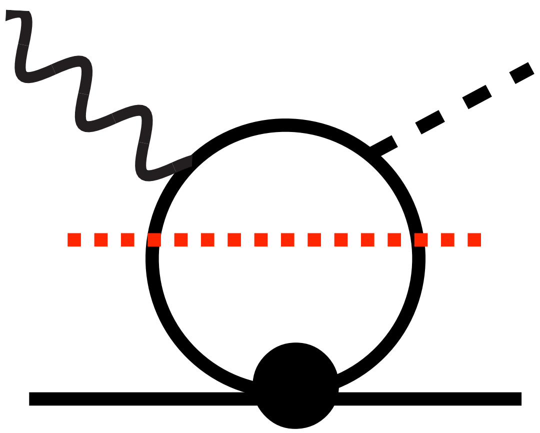
Examples:

also explained by susy techniques: arXiv:1412.7151

I. No 4-fermion $(\bar{\psi}\gamma^\mu\psi)^2$ corrections to dipoles

$$F_{\alpha\beta}\psi^\alpha\psi^\beta h$$

n=4; h=2



n=4; h=0

Examples:

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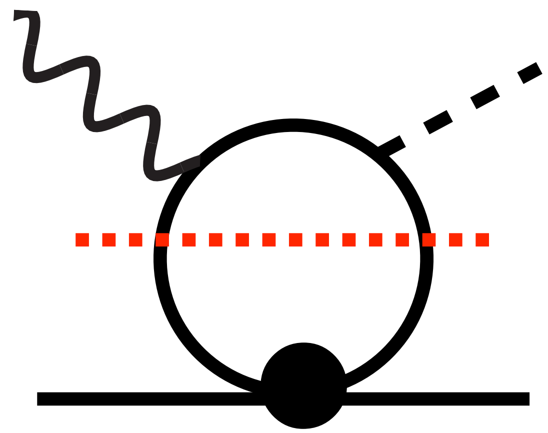

n=4; **h=2**

Examples:

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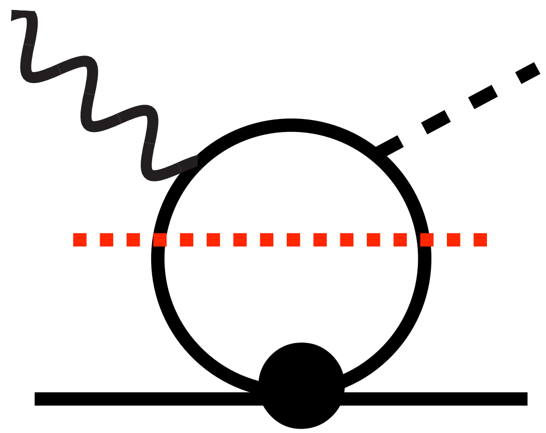

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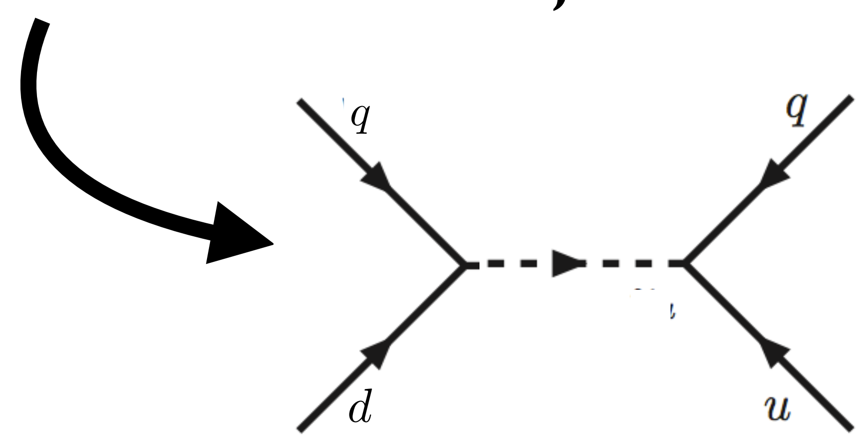


from scalar leptoquarks:
(3,2,7/6),(3,1,-1/3)
& extra Higgses

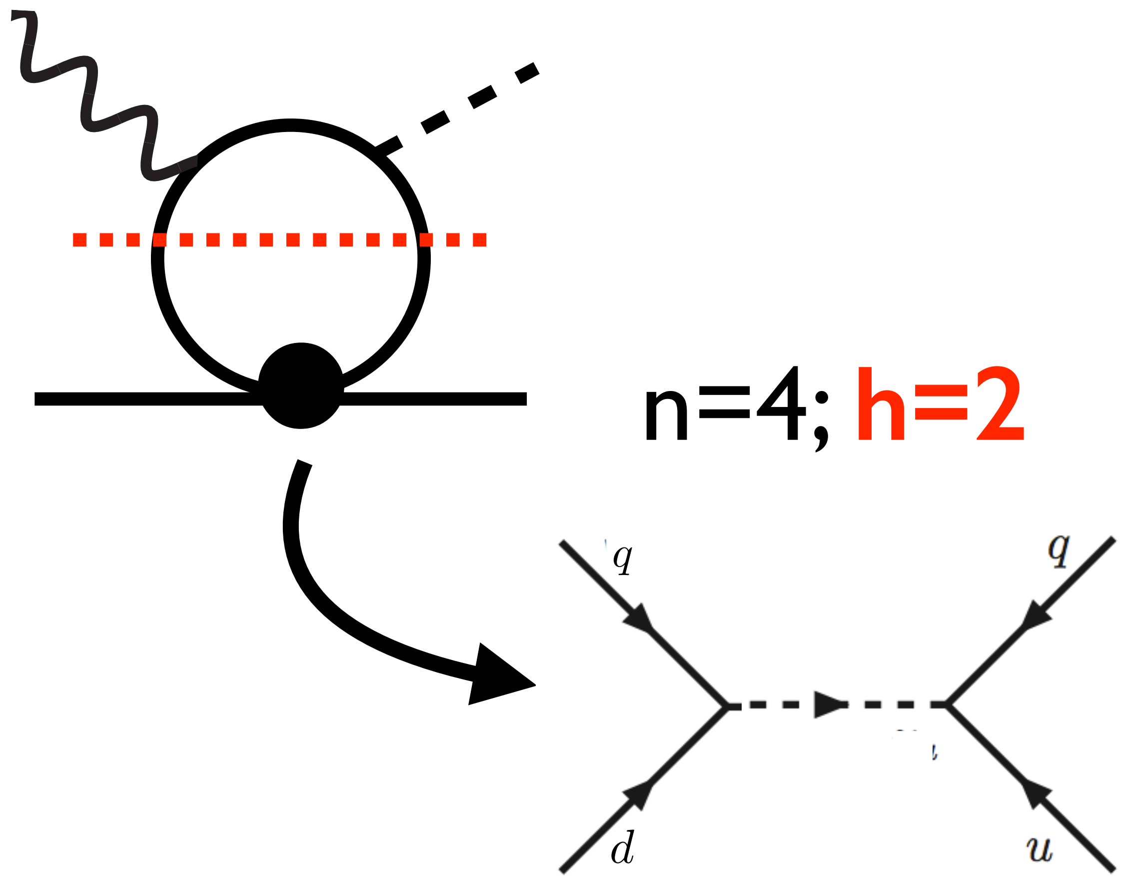
Examples:

also explained by susy techniques: arXiv:1412.7151

I. ~~No~~ 4-fermion $(\bar{\psi}\psi)^2$ corrections to dipoles

$$\boxed{F_{\alpha\beta}\psi^{\alpha}\psi^{\beta}h}$$

n=4; h=2



n=4; **h=2**

from scalar leptoquarks:
(3,2,7/6),(3,1,-1/3)
& extra Higgses

EDM ACME bound can reach: $\boxed{M_{LQ}>400\ \text{TeV}}$

Examples:

also explained by susy techniques: arXiv:1412.7151

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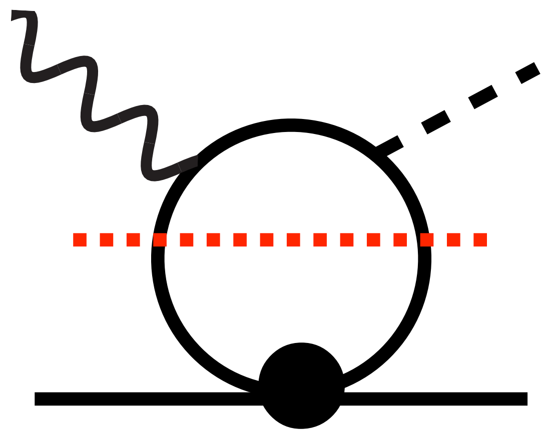
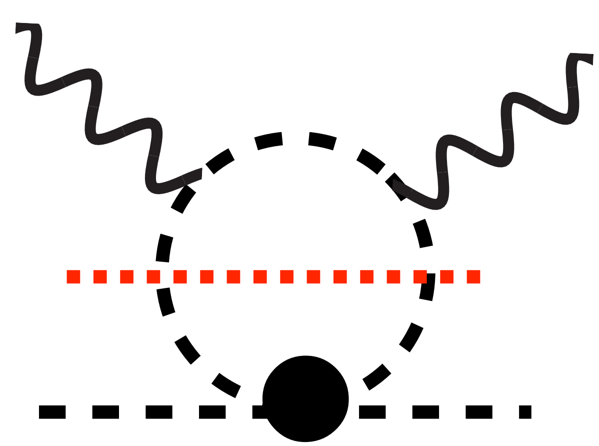


n=4; **h=2**

II. No p^2H^4 corrections to $H\gamma\gamma$

$$F_{\alpha\beta}F^{\alpha\beta}h^2$$

n=4; h=2



n=4; h=0

number of states

helicity	3	4	5	6
0		$\bar{\psi}^2\psi^2$ $p^2\|HH^\dagger\|^2$ $p\|H\|^2\bar{\psi}\psi$		$\|H\|^6$
1			$H\|H\|^2\psi\psi$	
2		$\|H\|^2F^2$ ψ^4 $HF\psi\psi$		
3	F^3			

number of states

helicity

	3	4	5	6
0		$\bar{\psi}^2\psi^2$ $p^2\lvert HH^\dagger\rvert^2$ $p\lvert H\rvert^2\bar{\psi}\psi$		$\lvert H\rvert^6$
1			$H\lvert H\rvert^2\psi\psi$	
2		$\lvert H\rvert^2F^2$ ψ^4 $HF\psi\psi$		
3	F^3			

$\Delta n \geq |\Delta h|$

renormalization cone

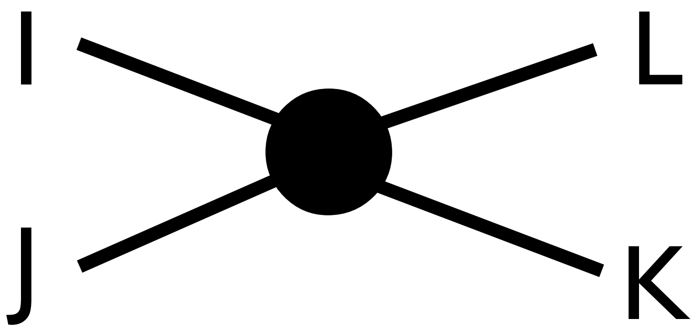
II. Bottom-up approach to Goldstone amplitudes:

Only assume:

a) $\pi_i \in$ reps of $\mathcal{H}$ (no coset input)

b) $\mathcal{A}(1234) \to q_i$ (for $q_i \to 0$) (Adler's zeros)

in collaboration with P. Baratella & B. Harling

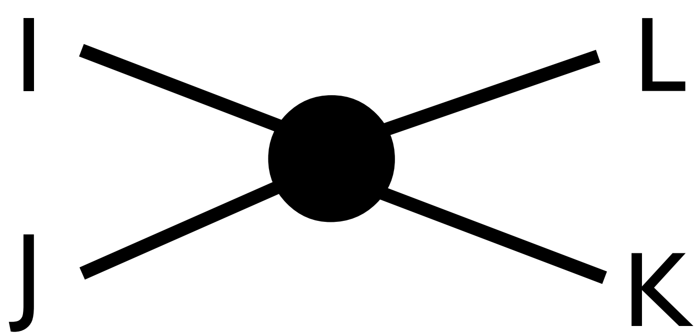


$$= \mathbf{f}(\mathbf{s},\mathbf{t},\mathbf{u})\,\mathbf{T}_{\mathbf{IJKL}} + \cdots$$

invariant under crossing

kin. functions

inv. tensors



$$= \mathbf{f}(\mathbf{s},\mathbf{t},\mathbf{u})\,\mathbf{T}_{\mathbf{IJKL}} + \cdots$$

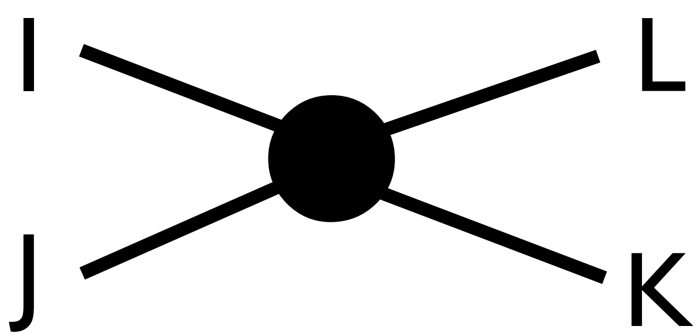
kin. functions

inv. tensors

invariant under crossing

Tensor invariants (for π ∈ Adj of SU(N)):

- single trace (6): $tr(t_I t_J t_K t_L)$
- double trace (3): $tr(t_I t_J)\, tr(t_K t_L)$



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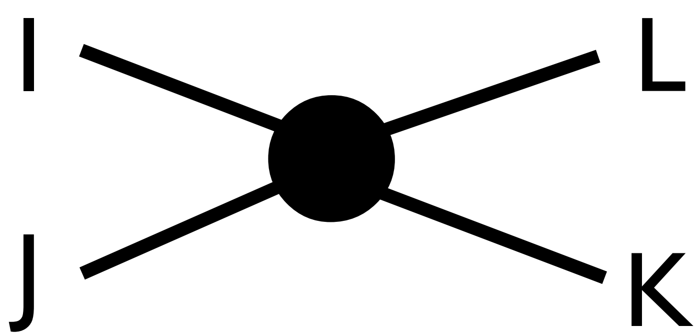
Kinematics:

$\mathbf{f}(\mathbf{s},\mathbf{t},\mathbf{u})$ $\qquad$ $s+t+u=0$



$$= \mathbf{f(s,t,u)}\,\mathbf{T_{IJKL}} + \cdots$$

kin. functions

inv. tensors

invariant under crossing

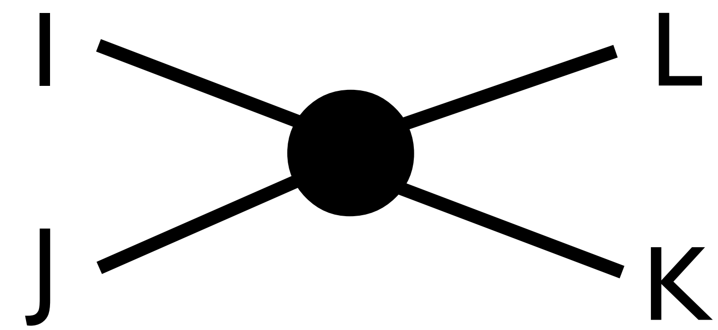
Tensor invariants (for π ∈ Adj of SU(N)):

Under permutations:

- single trace (6): $tr(t_I t_J t_K t_L)$ ∈ **1+2+3**
- double trace (3): $tr(t_I t_J)\,tr(t_K t_L)$ ∈ **1+2**

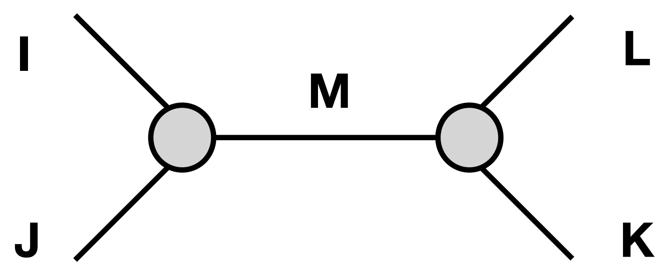
Kinematics:

$\mathbf{f(s,t,u)}$ $s+t+u=0$ ∈ **1+2**



$O(p^2)$: $\boxed{(t-u)\, f_s+(u-s)\, f_t+(s-t)\, f_u}$

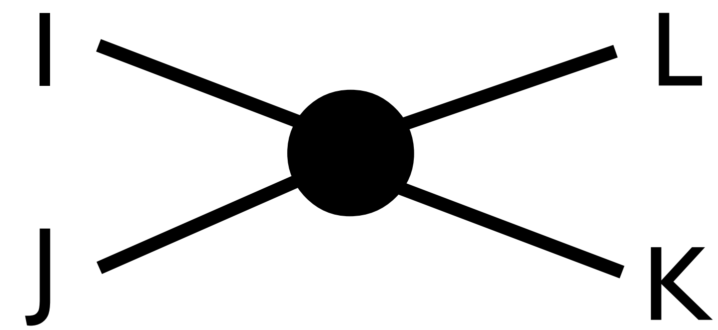
- single trace: $f_{IJM}\, f_{MKL}$



$f_s+f_t+f_u=0$ Jacobi

$s+t+u=0$

$O(p^2)$: $(t-u)\, f_s + (u-s)\, f_t + (s-t)\, f_u$



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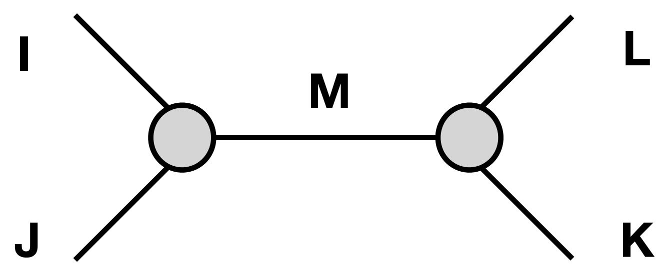


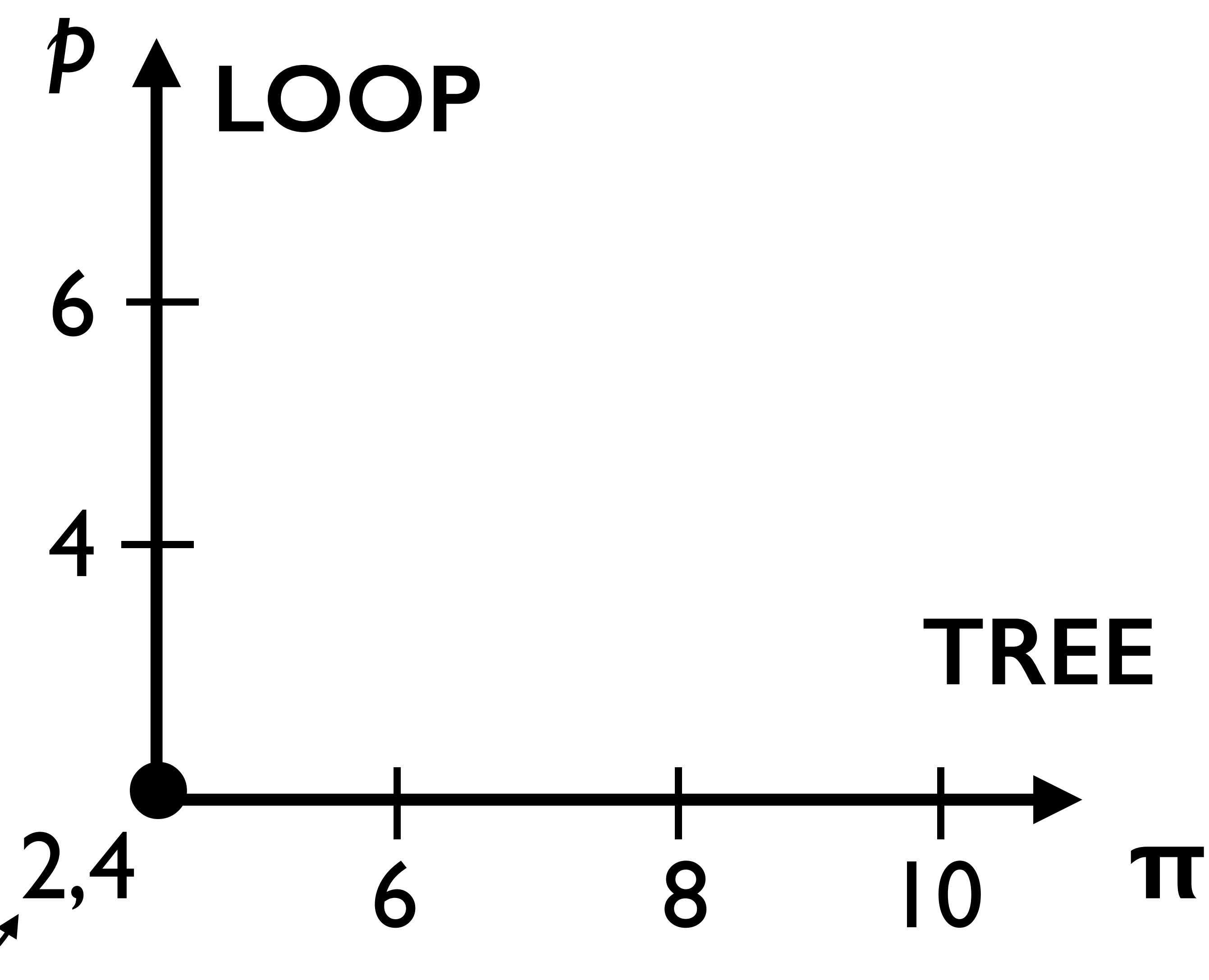
$f_s+f_t+f_u=0$ Jacobi

$s+t+u=0$

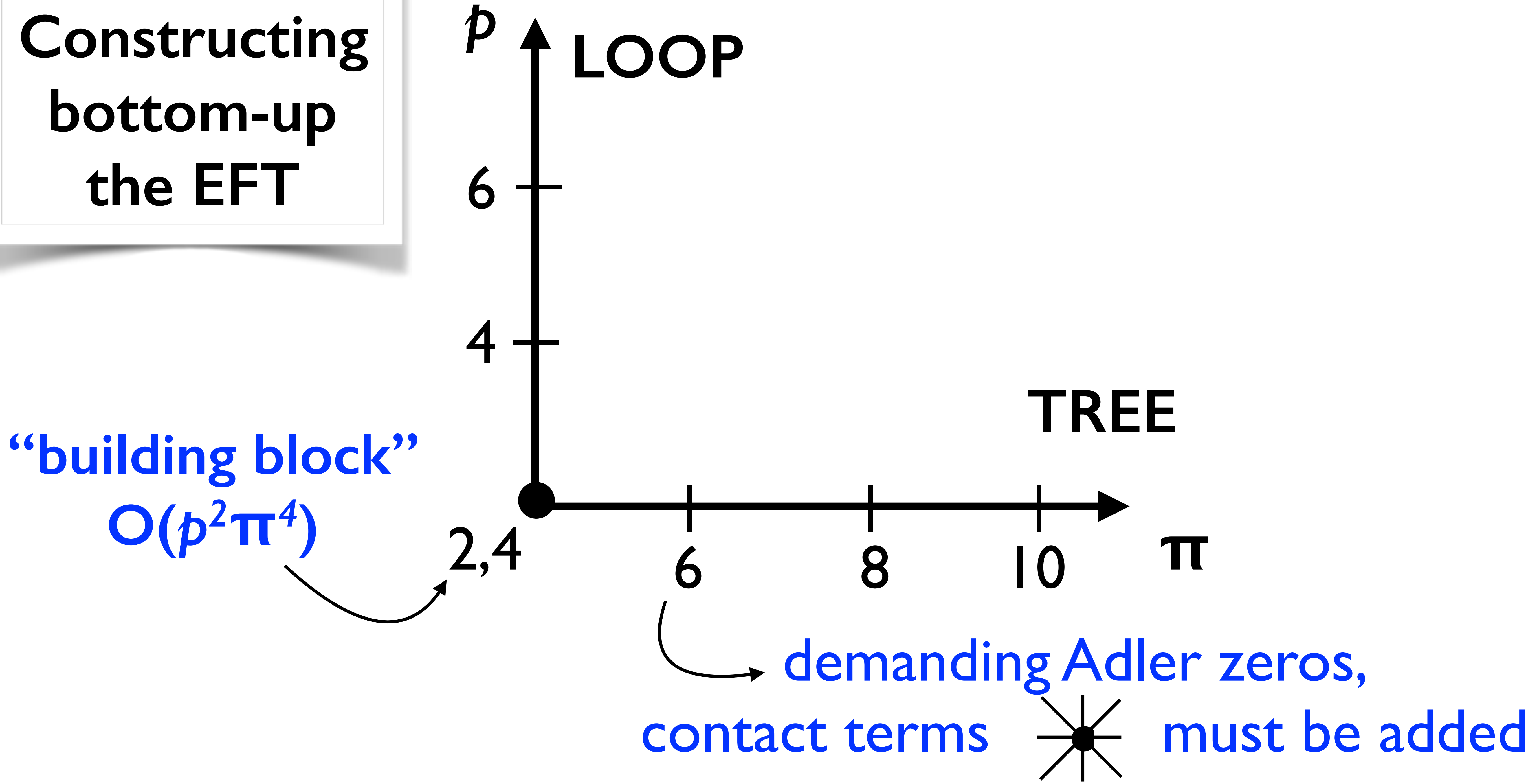
- double trace: $\delta_{IJ}\, \delta_{KL} - \delta_{IL}\, \delta_{JK}$

Constructing bottom-up the EFT

“building block”
$O(p^2\pi^4)$

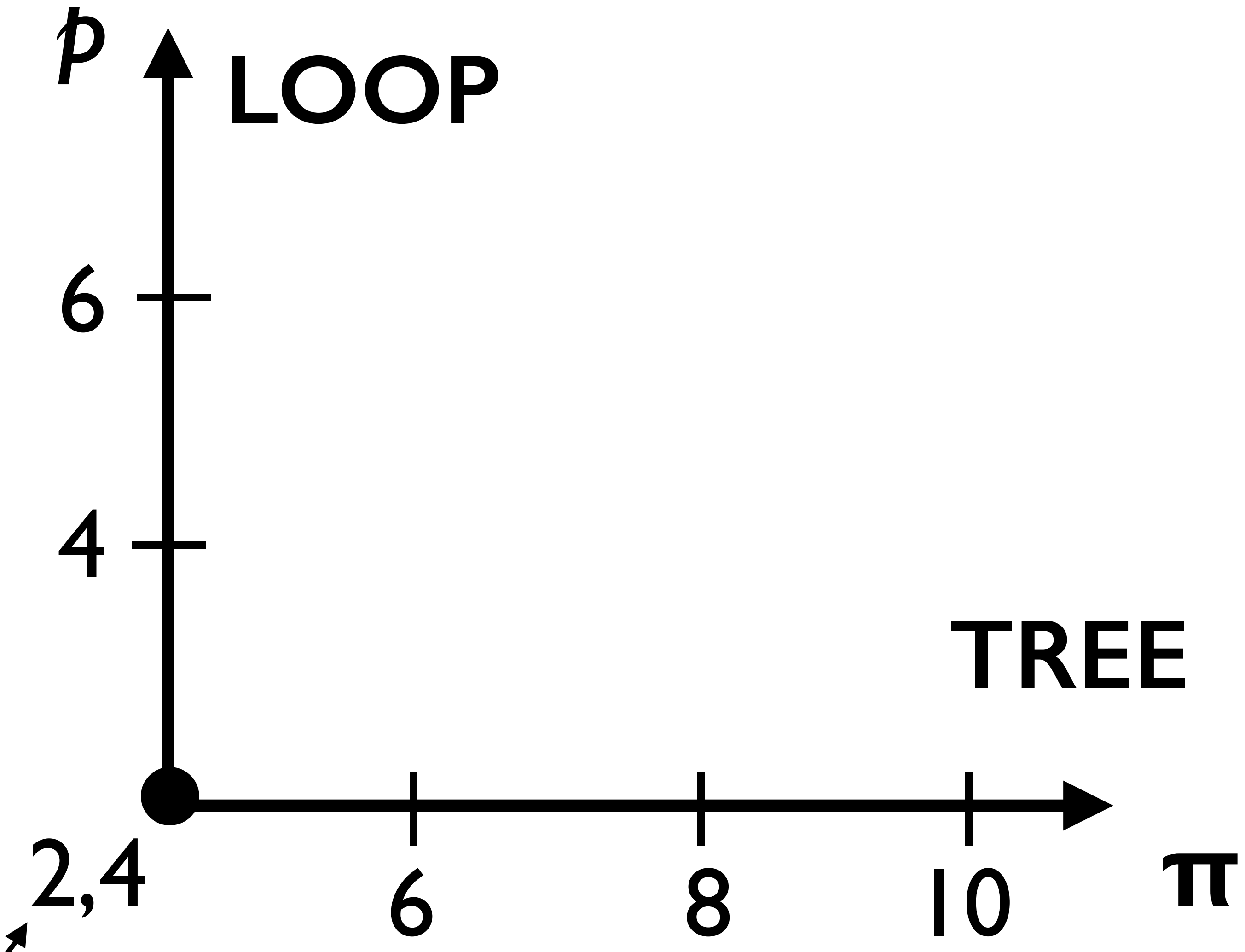


Constructing bottom-up the EFT



arXiv:1904.12859

Constructing bottom-up the EFT



“building block”
$O(p^2\pi^4)$

demanding Adler zeros,
contact terms must be added

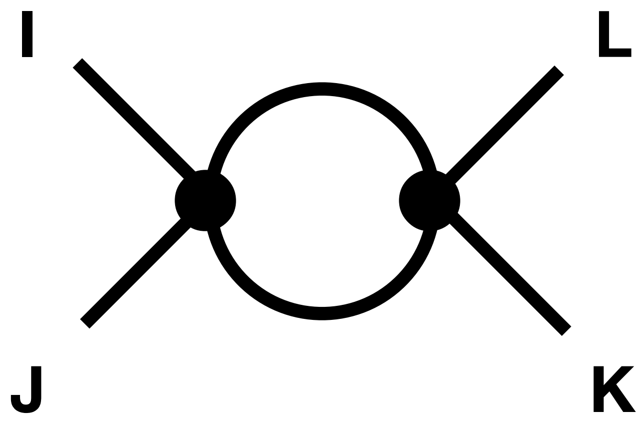
arXiv:1904.12859

<u>possible *only* for one choice:</u>

☛ single trace → reconstructing SU(N)xSU(N)/SU(N)

☞ double trace → reconstructing SO(N)/SO(N-1)

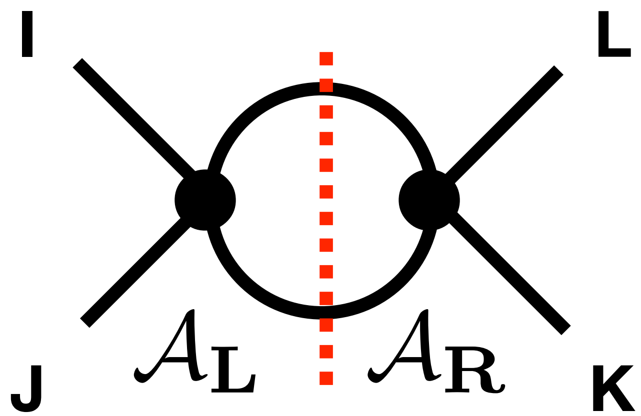
LOOPS



+ *crossing*

☛ O(p^4) amplitude

LOOPS

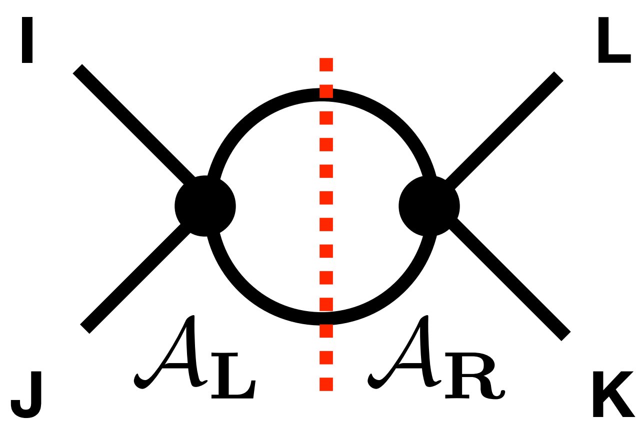


+ *crossing*

☛ O(p^4) amplitude

$$\sum\!\!\!\!\!\!\!\int \mathcal{A}_L \mathcal{A}_R$$

LOOPS



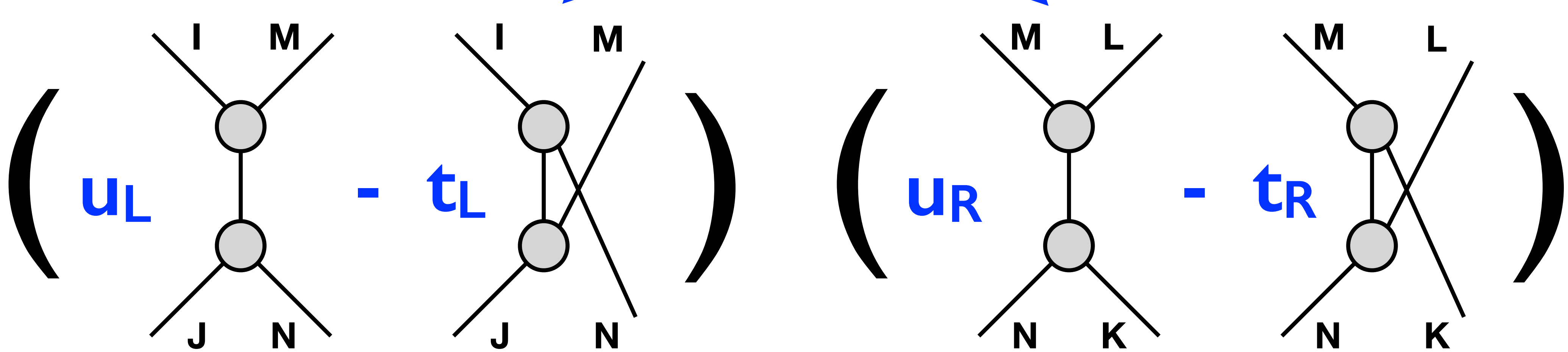
+ *crossing*

☛ $O(p^4)$ amplitude

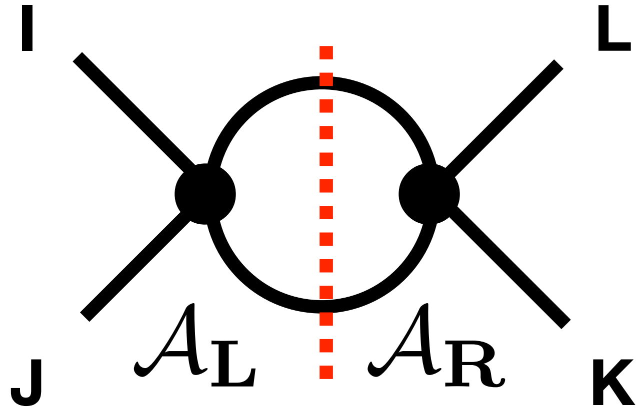
$$\sum\!\!\!\!\!\!\!\int \mathcal{A}_L \mathcal{A}_R$$

thanks to Jacobi identity & s+t+u=0

- single trace:



LOOPS



+ *crossing*

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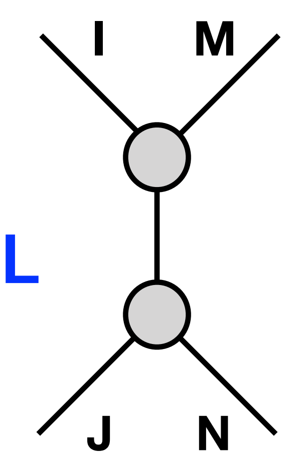
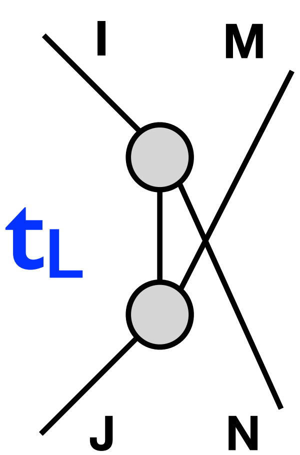
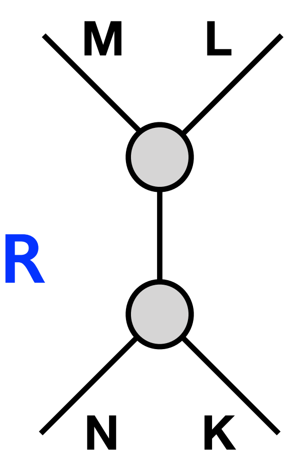
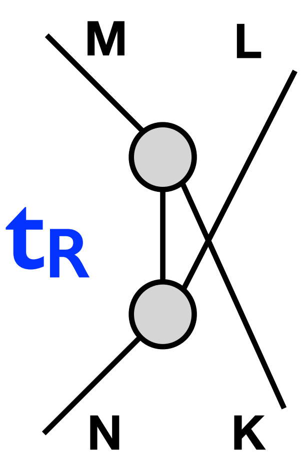
- single trace:

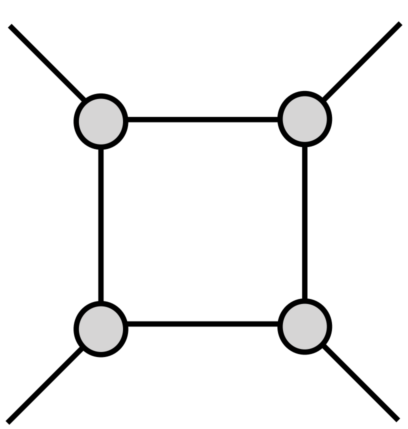
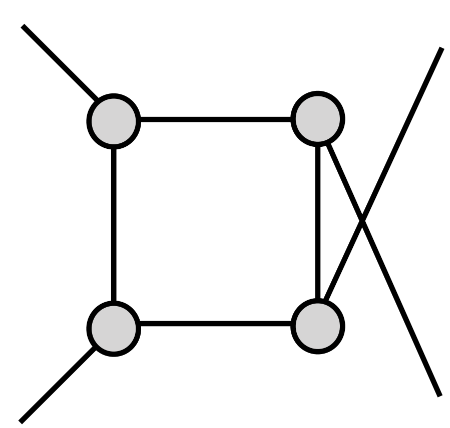
$\Big(u_L$ 

$- \; t_L$ 

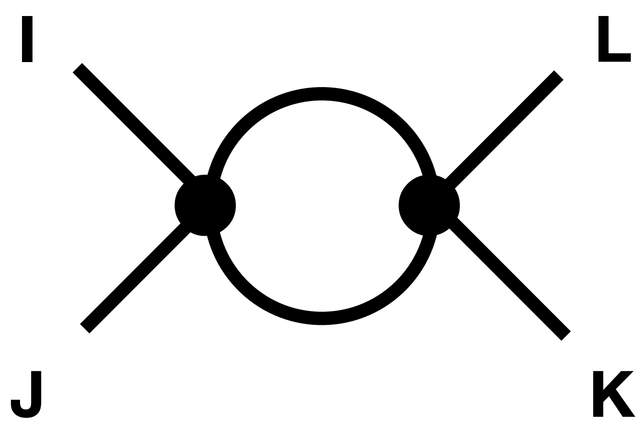
$\Big) \Big(u_R$ 

$- \; t_R$ 

$\Big)$

$=$  $\sum\!\!\!\!\!\!\int (u_L u_R + t_L t_R)$ $+$  $\sum\!\!\!\!\!\!\int (u_L t_R + t_L u_R)$

ONE LOOP



+ *crossing*

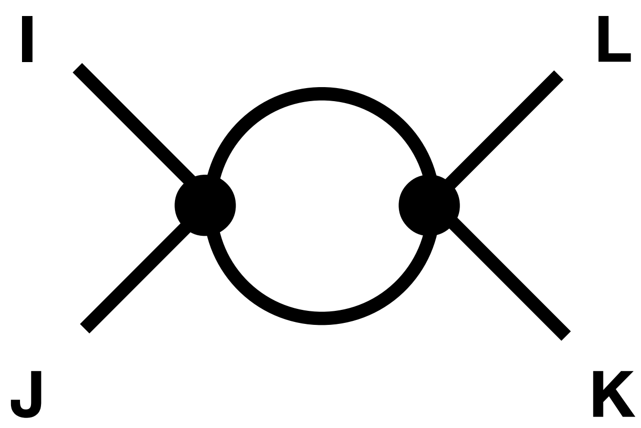
- single trace:

$$= (s^2 + t^2 + u^2)\,(\,\mathrm{Tr}[\,F^I\,F^J\,F^K\,F^L\,] + \textit{crossing}\,)$$

$$(F^I)_{JK} = f_{IJK}$$

Unclear why so simple!

ONE LOOP



+ *crossing*

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$$(F^I)_{JK} = f_{IJK}$$

Unclear why so simple!

- double trace:

$$= ((3N\text{-}7)/2 \; s^2 + t^2 + u^2) \; \delta_{IJ} \; \delta_{KL} + \textit{crossing}$$

Conclusions

- Amplitude methods seems quite suited for calculating indirect BSM effects ☛ e.g. anomalous dimensions of $\mathcal{O}_6$
- Helps to obtain selections rules
- Allows to construct models from bottom-up
- Further work: Automatize AD calculations, going beyond one-loop, unravel the $\pi^m p^n$ structure, …