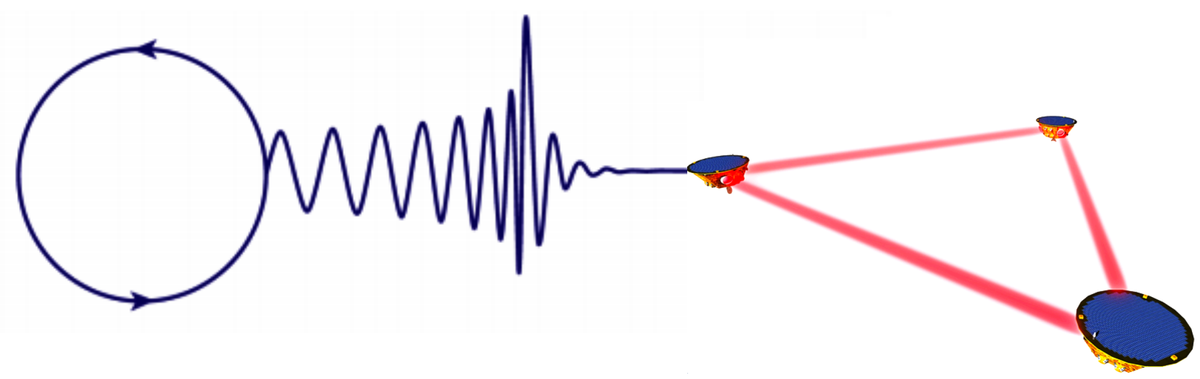
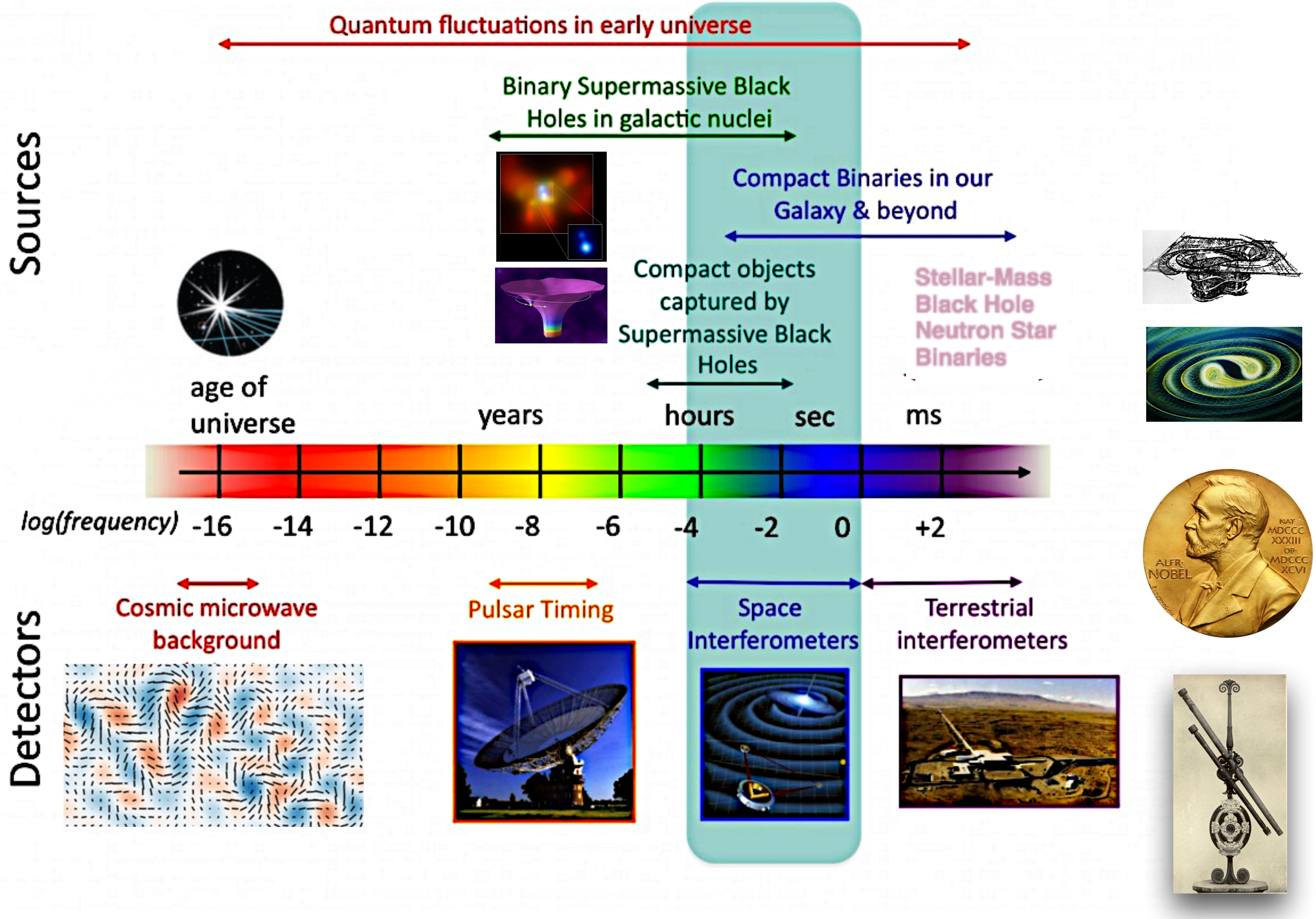
Precision Gravity: From the LHC to LISA



Rafael A. Porto

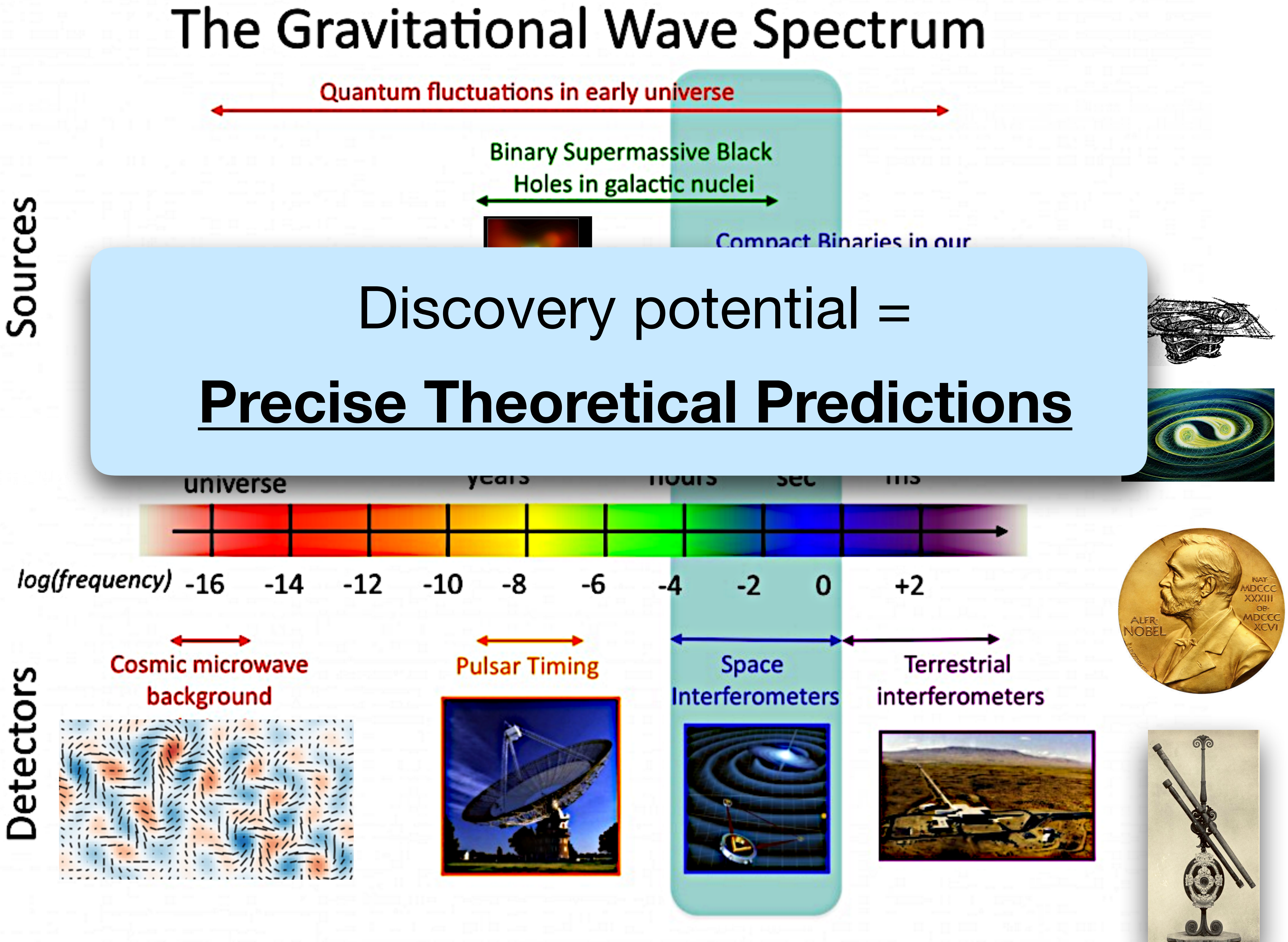
with Stefano Foffa, Chad Galley, Adam Leibovich, Andreas Ross, Ira Rothstein, Riccardo Sturani

The Gravitational Wave Spectrum

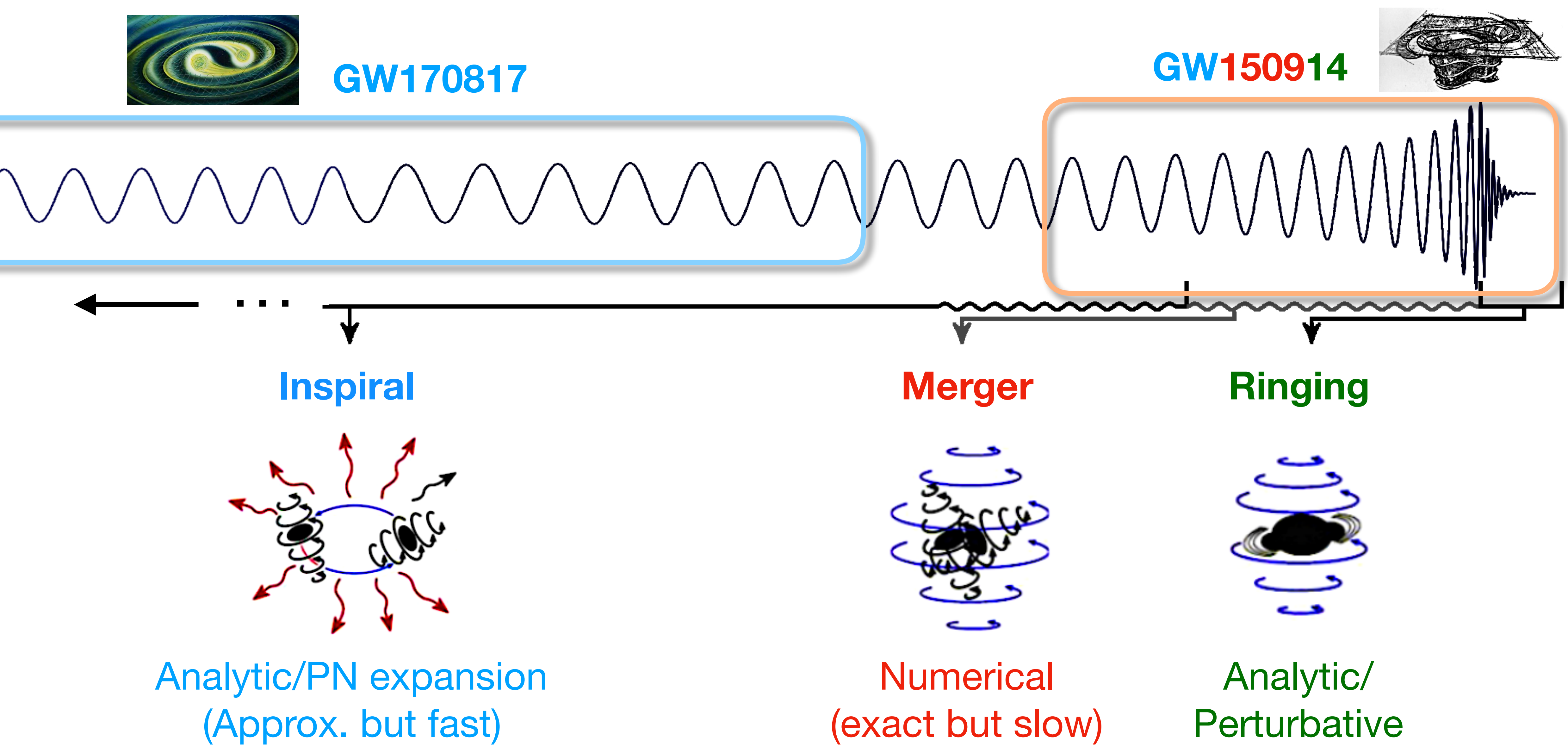


Discovery potential =

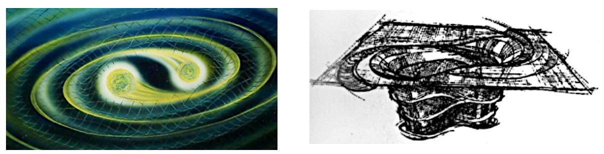
Precise Theoretical Predictions

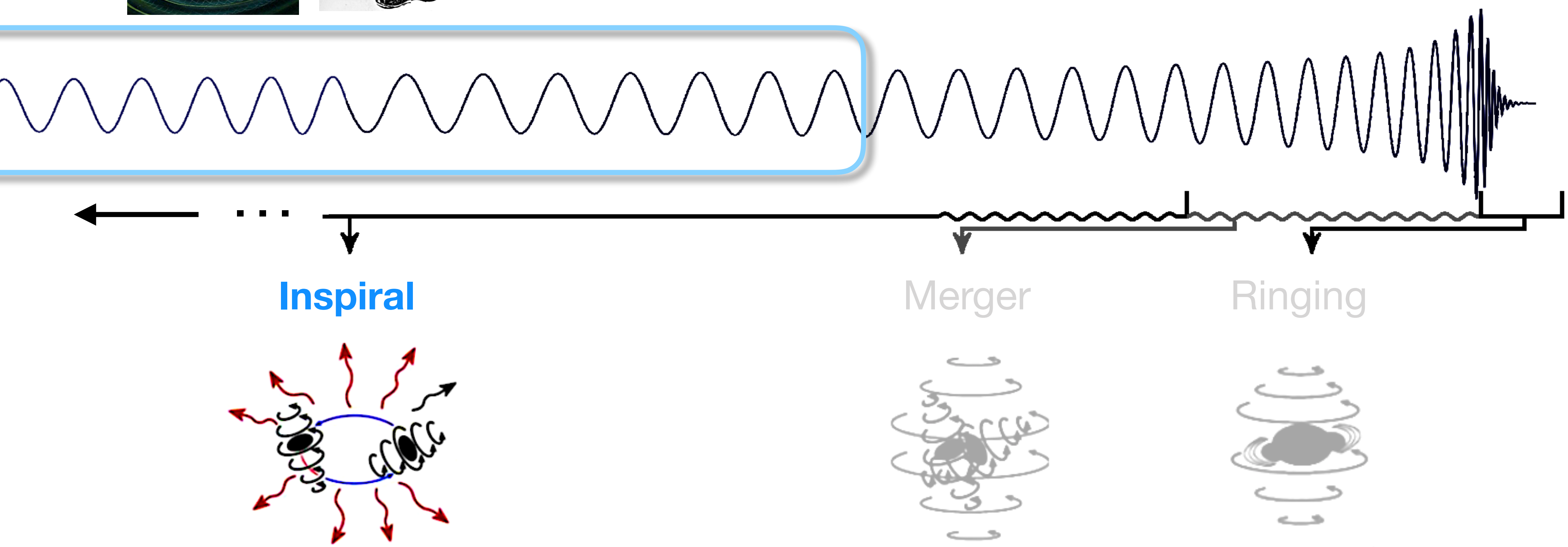


$$R_{im} = \sum_{l} \frac{\partial \Gamma^{l}_{im}}{\partial x_l} + \sum_{\rho l} \Gamma^{\rho}_{il} \Gamma^{l}_{m\rho} = -\kappa \left(T_{im} - \frac{1}{2} g_{im} T \right)$$



$$R_{im} = \sum_{l} \frac{\partial \Gamma^{l}_{im}}{\partial x_{l}} + \sum_{\rho l} \Gamma^{\rho}_{il} \Gamma^{l}_{m\rho} = -\varkappa \left(T_{im} - \frac{1}{2} g_{im} T \right)$$



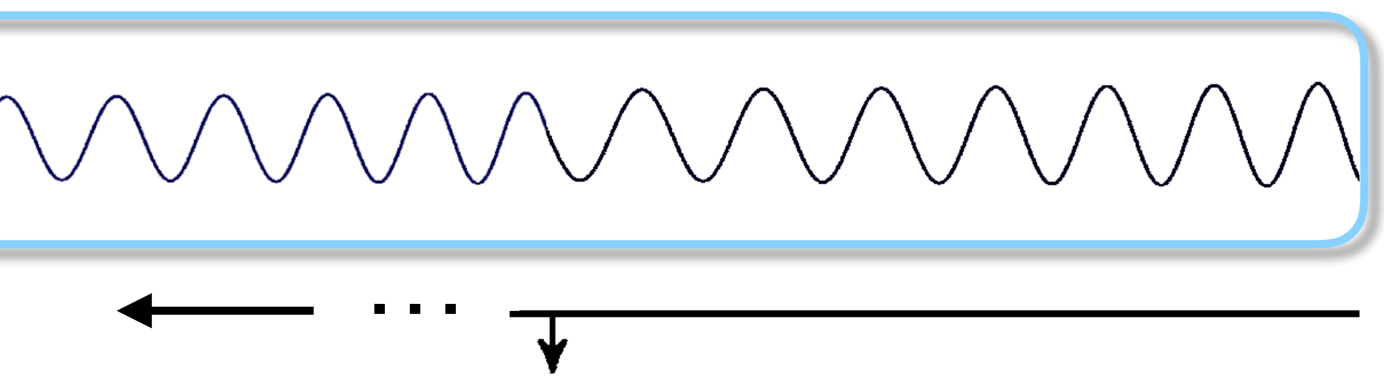


Analytic/PN expansion
(Approx. but fast)

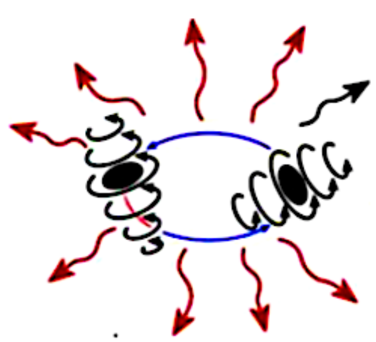
1000+ cycles in band @ Design-Sensitivity

100+ events per year!

State of the Art



Inspiral



3.5PN order

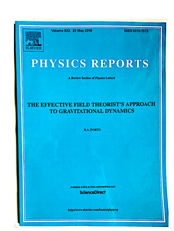
$$\frac{\dot{\omega}}{\omega^2} = \underbrace{\frac{96}{5}\nu x^{5/2}}\left\{1 + \overbrace{\cdots + [\cdots] x^{7/2}}\right\}$$

$$4\pi R^2 \bar{\mathcal{G}} = \frac{\varkappa}{40\pi}\left[\sum_{\mu\nu} \dddot{J}_{\mu\nu}^{2} - \frac{1}{3}\left(\sum_{\mu} \dddot{J}_{\mu\mu}\right)^2\right]$$

$\nu \sim m_2/m_1$

$x \sim (v/c)^2$

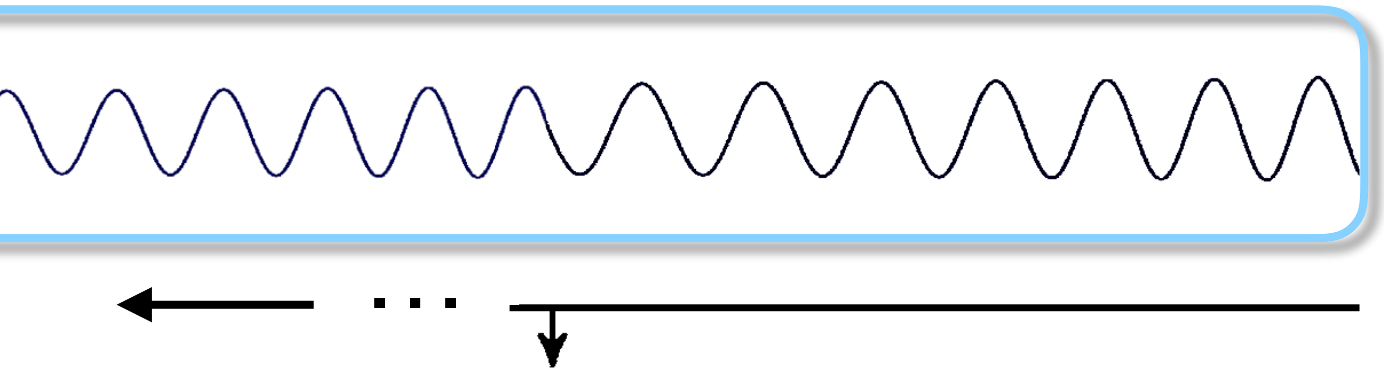
The effective field theorist's approach to gravitational dynamics
Physics Reports
Rafael A. Porto
Volume 633, 20 May 2016, Pages 1-104



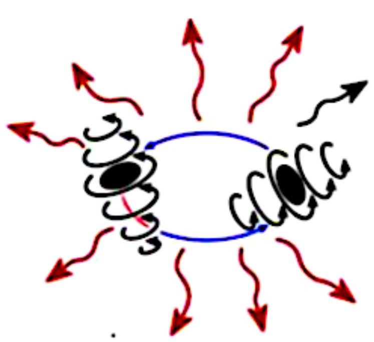
et al.

Blanchet, Damour, Faye et al. (harmonic)
Damour, Jaranowski, Schaefer, et al. (ADM)

Are we ready for the future?



Inspiral



3.5PN order

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \overbrace{\cdots + [\cdots]x^{7/2}}\right\}$$

The effective field theorist's approach to gravitational dynamics

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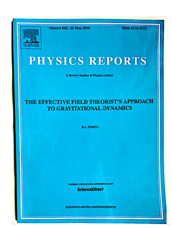
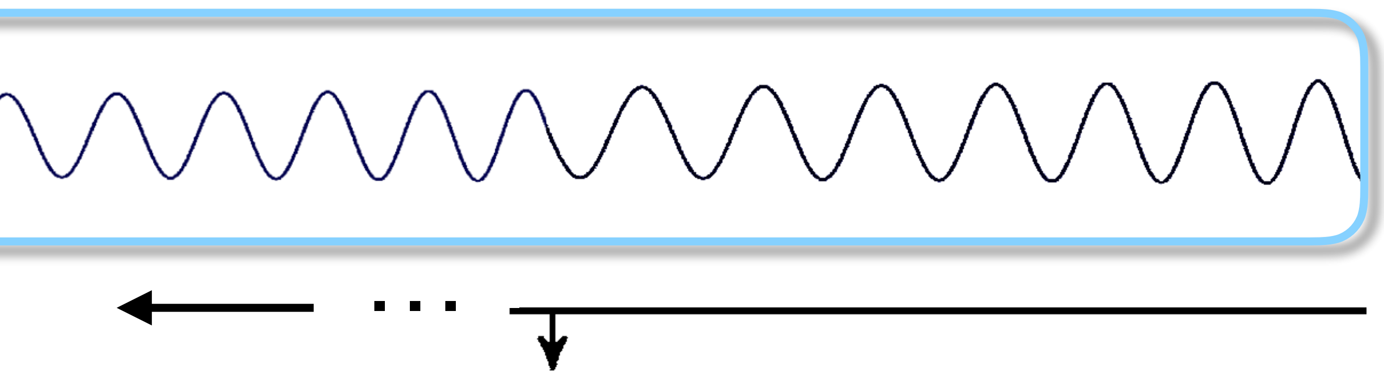


et al.

Blanchet, Damour, Faye et al. (harmonic)

Damour, Jaranowski, Schaefer, et al. (ADM)

Theoretical uncertainties dominate over planned empirical reach



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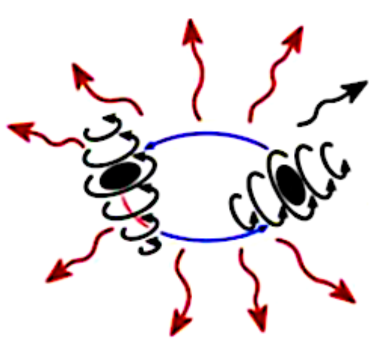


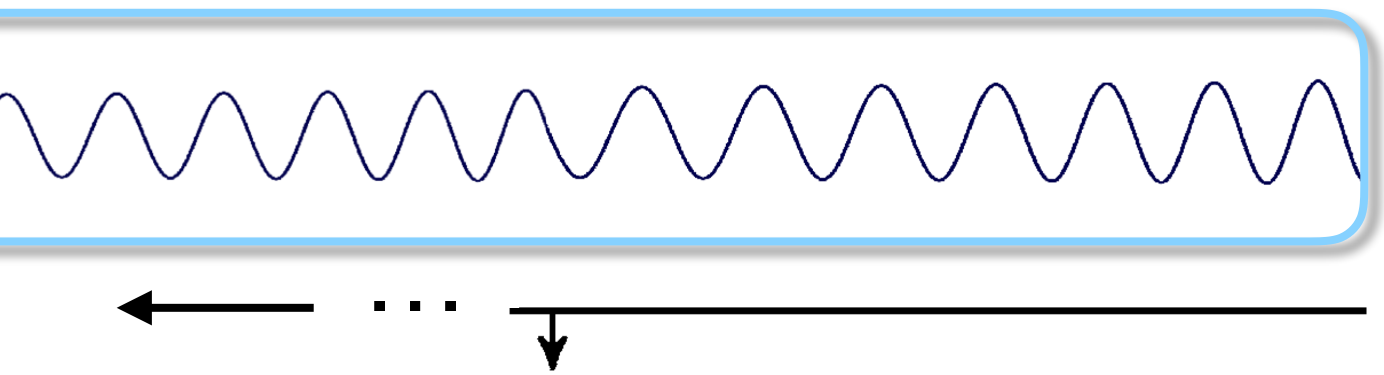
NOT GOOD ENOUGH

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \overbrace{\cdots + [\cdots]x^{7/2}}\right\}$$

- Gravitational-wave experiments on ground and in space require more accurate waveform models: new theoretical challenges and opportunities.

A. Buonanno (QCD meets Gravity 18')

We haven’t reached the analytic precision to distinguish between compact bodies!



‘New Physics’
Threshold

Inspiral

$$Q_{ij} = C_E E_{ij}$$

(‘Susceptibility’)

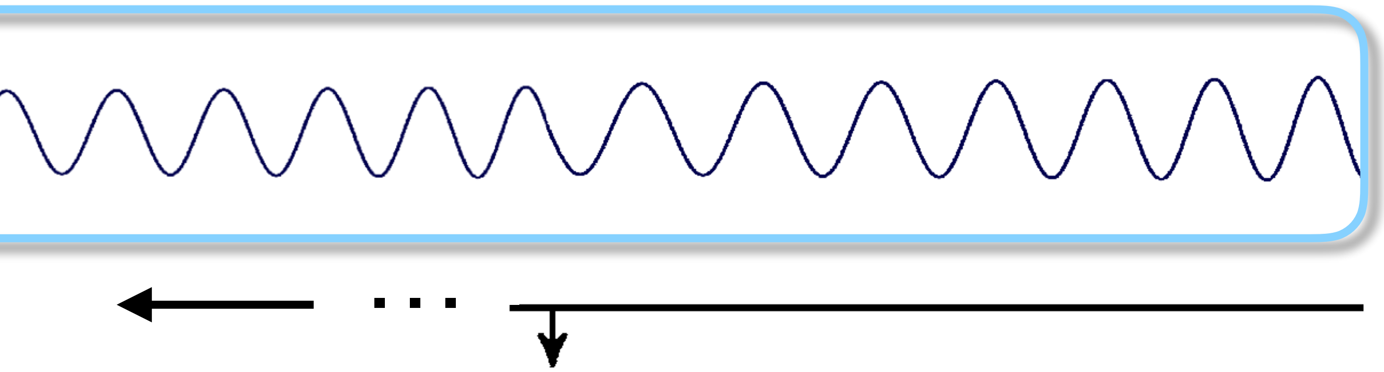
$$C_E \sim R^5 \longrightarrow \left(\frac{R}{r}\right)^5 \sim v^{10}$$

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

N^5LO
5PN

$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v)$$

We haven’t reached the analytic precision to distinguish between compact bodies!



Inspiral

$$C^{\rm bh}_{E(B)}(\mu) = 0$$

‘New Physics’ **Threshold**

$$\frac{\dot\omega}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

N^5LO

5PN

Fortschr. Phys. 64, No. 10, 723–729 (2016) / DOI 10.1002/prop.201600064

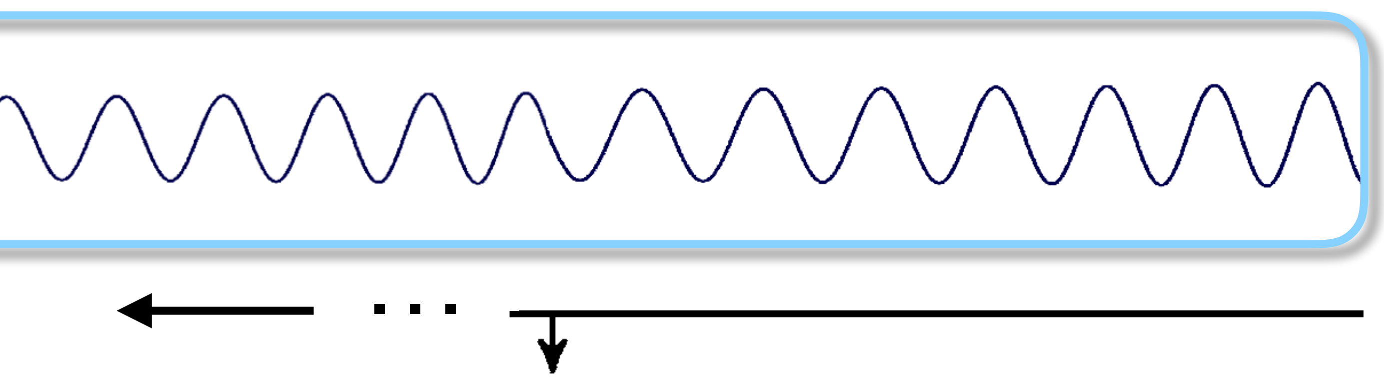
The tune of love and the *nature(ness)* of spacetime

*Rafael A. Porto**

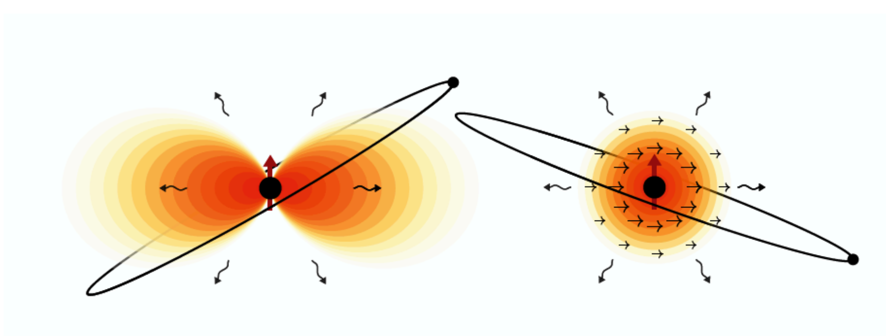
$$\Psi(v) = \Psi_{\rm PP}(v) +$$

QM: See Ira’s talk

We haven't reached the analytic precision to distinguish between compact bodies!



Inspiral



'New Physics' Threshold

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

N^5LO
5PN

$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v)$$

Probing ultralight bosons with binary black holes

Daniel Baumann, Horng Sheng Chia, and Rafael A. Porto

Phys. Rev. D 99, 044001 (2019)

Published February 4, 2019

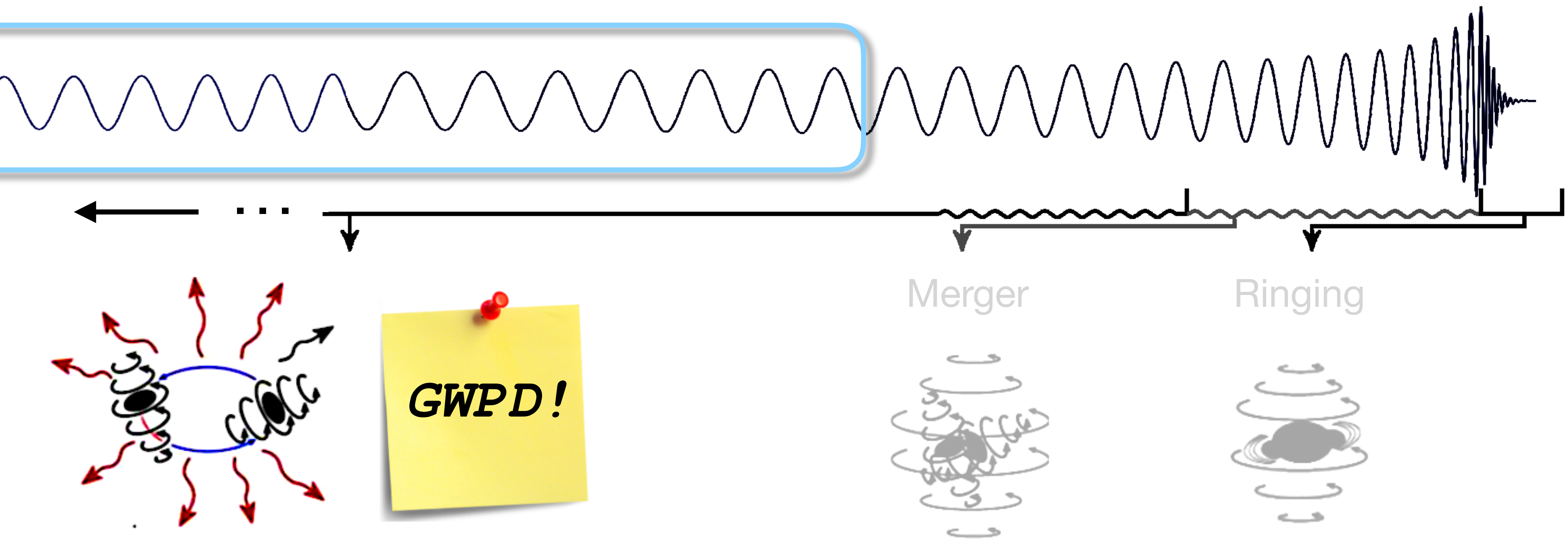
Black Holes Could Reveal New Ultralight Particles

(See other talks for various probes of light particles)

Extremely accurate Post-Newtonian waveforms

1000+ cycles in band @ Design-Sensitivity

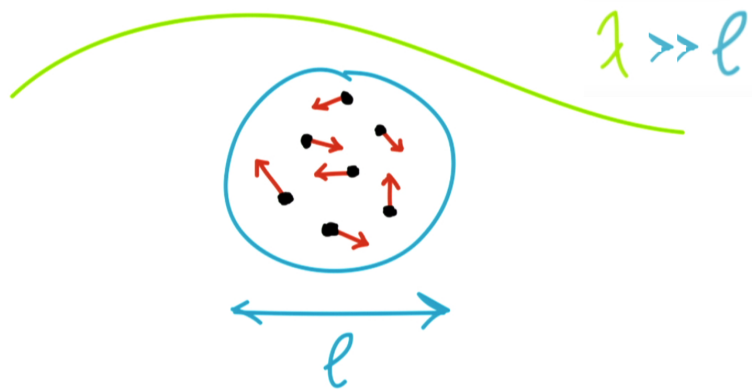
100+ events per year!

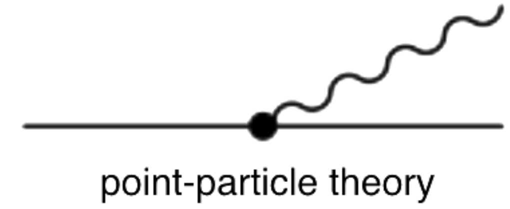


GW Precision Data (**GWPD**) *TM*

Goldberger Rothstein (2004)
RAP (2005)
Galley, Leibovich, Ross
Foffa, Sturani, …

EFT1: Finite size

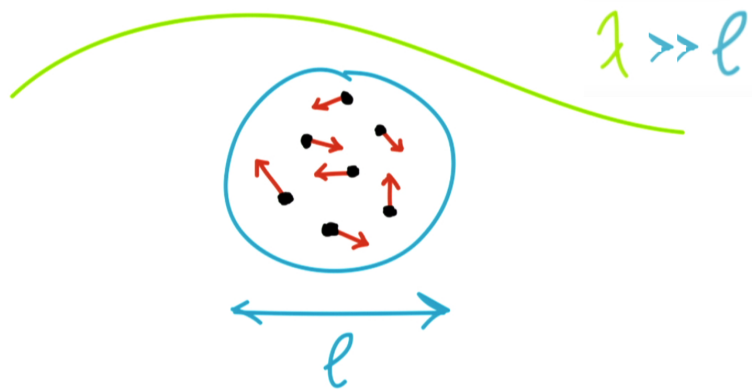


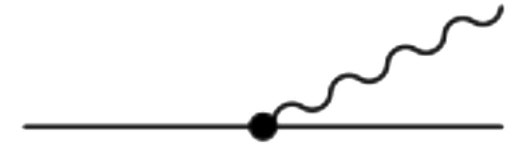


$$\ldots = \frac{1}{2} Q_{ij} E^{ij} + \ldots$$

Goldberger Rothstein (2004)
RAP (2005)
Galley, Leibovich, Ross
Foffa, Sturani, …

EFT1: Finite size





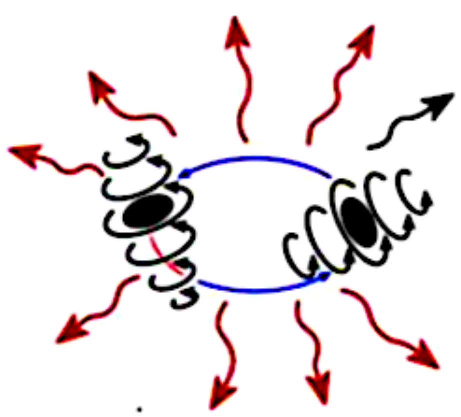
$$\ldots = \frac{1}{2} Q_{ij} E^{ij} + \ldots$$

$$e^{iW} = \int Dh\, e^{i(S_{\rm EH}[h] + S_{\rm pp}[h, x_a])}$$

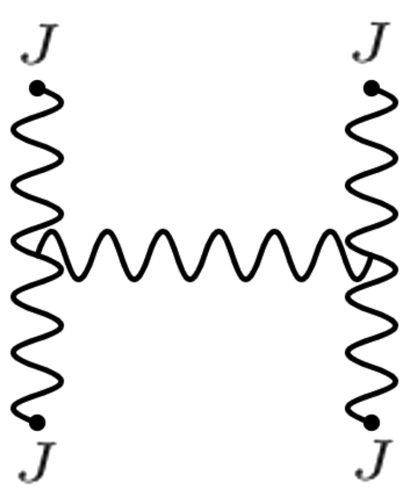
fully relativistic

$$\underbrace{\mathrm{Re}\, W[x_a]}_{\text{binding}} + i\, \underbrace{\mathrm{Im}\, W[x_a]}_{\text{radiation}}$$

Classical **optical theorem with Feynman b.c.**



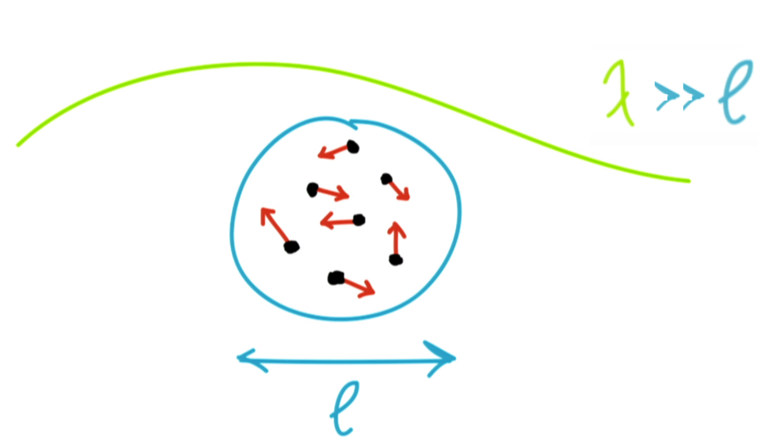
$$\log \langle 0|0\rangle^J =$$



UV Divergences: (localized sources)

Goldberger Rothstein (2004)
RAP (2005)
Galley, Leibovich, Ross
Foffa, Sturani, …

EFT1: Finite size



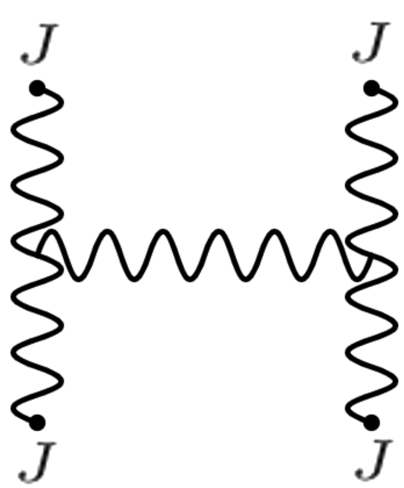
→

$$\ldots = \frac{1}{2} Q_{ij} E^{ij} + \ldots$$

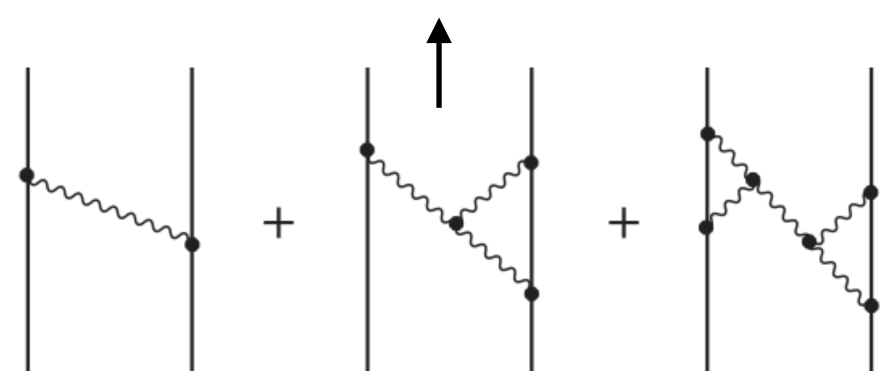
$$e^{iW} = \int Dh\, e^{i(S_{\rm EH}[h] + S_{\rm pp}[h, x_a])}$$

$$\underbrace{\mathbf{Re}\, \boldsymbol{W}[\boldsymbol{x_a}]}_{\textbf{binding}} +$$

$$\log \langle 0|0\rangle^J =$$



UV Divergences: localized sources →

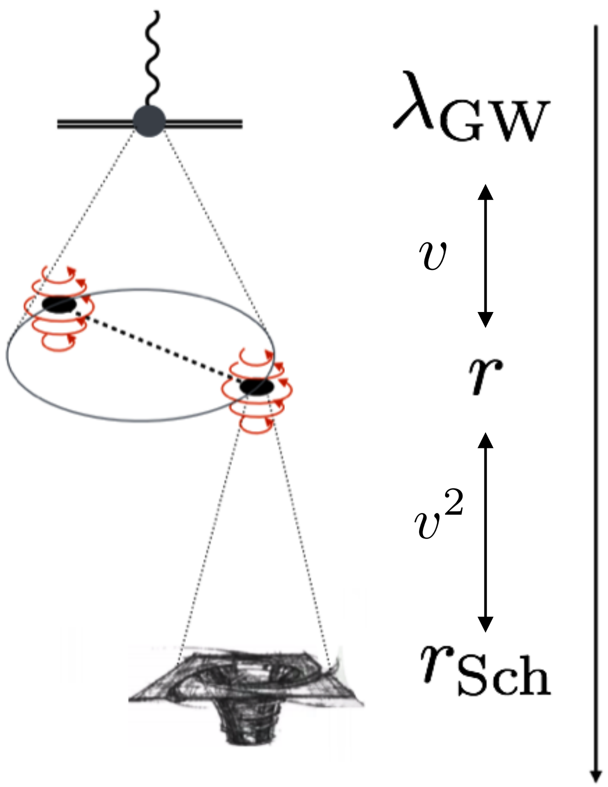


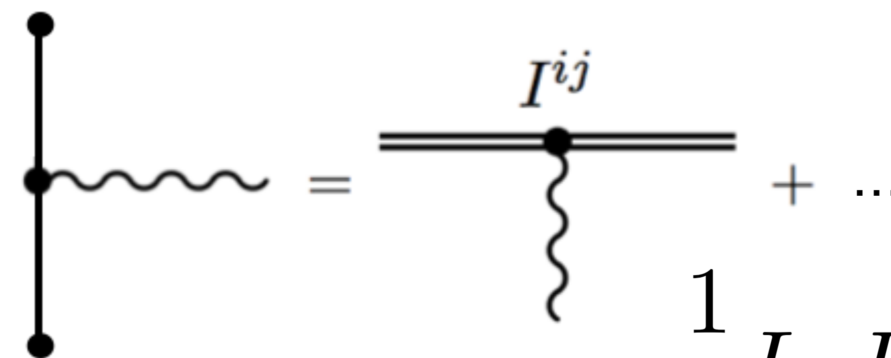
e.g. Duff (70's); Damour et al. (90's)

$$S_{\rm red}(T) = \frac{1}{2} T\, G\, T + V_3(G\,T, G\,T, G\,T) + \ldots$$

EFT2: NRGR (similar to NRQCD)

Goldberger Rothstein (2004)
RAP (2005)
Galley, Leibovich, Ross
Foffa, Sturani, …





$$\frac{1}{2} I_{ij} E^{ij}$$

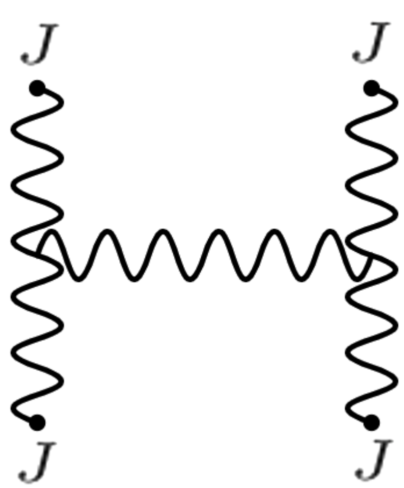
binary

potential & radiation

$$e^{iW} = \int Dh\, e^{i(S_{\rm EH}[h] + S_{\rm pp}[h, x_a])}$$

$$\underbrace{\mathrm{Re}\, W[x_a]}_{\text{binding}} + i\, \underbrace{\mathrm{Im}\, W[x_a]}_{\text{radiation}}$$

$$\log \langle 0|0\rangle^J =$$



$$\frac{1}{p_0^2 - p^2} = \frac{1}{p^2}\left(-1 + p_0^2/p^2 + \cdots\right)$$

(includes RR!)

Neill Rothstein (2013)
Cheung Rothstein Solon (2017)
Bern et al. (2019)
...

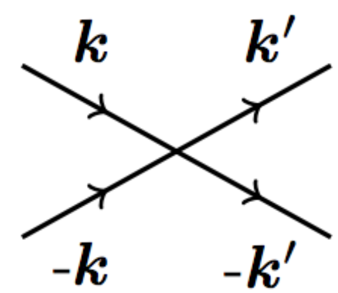
Amplitudes (On-shell)

PRECISION GRAVITY: FROM THE LHC TO LISA

26 August - 20 September 2019 MIAPP Munich Institute for Astro- and Particle Physics

John Joseph Carrasco, Ilya Mandel, Donal O'Connell, Rafael Porto, Fabian Schmidt

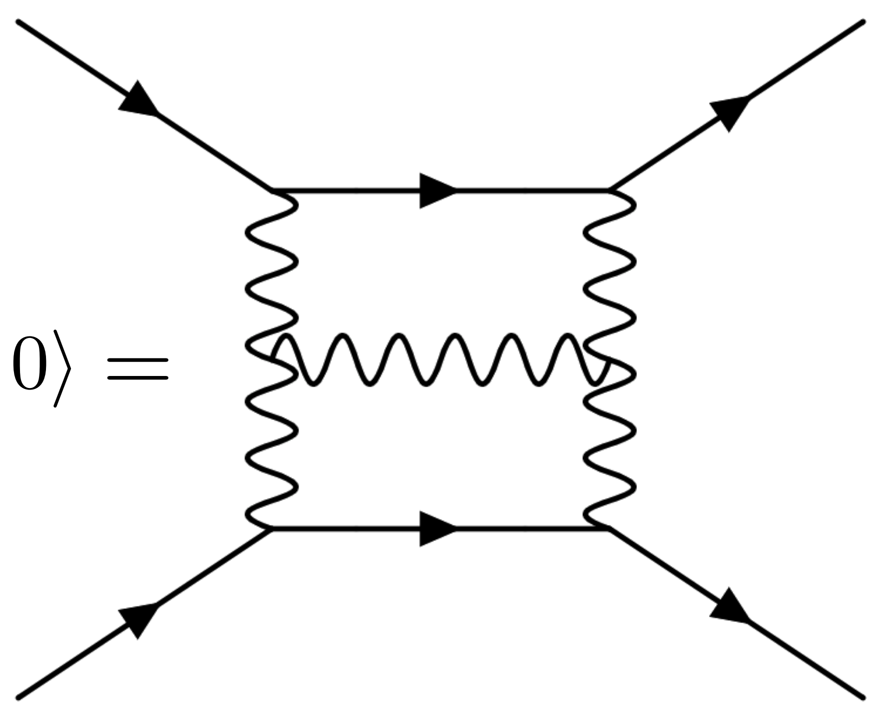
(See Pierre's talk)



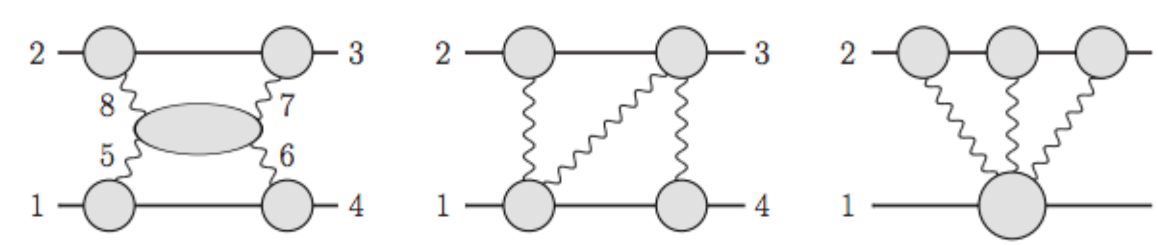
$$= -iV(\boldsymbol{k},\boldsymbol{k}') = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$\int D[\text{matter}]\, Dh\, e^{i(S_{\rm EH}[h]+S_{\rm pp}[h,\text{matter}])}$$

$$\langle p_1, p_2, 0 | p_3, p_4, 0\rangle =$$



Double Copy & Unitarity methods



3PM (G^3) potential region to all orders in velocity (Subset of full PN)

Latest: Binding energy to 4PN

$$
\begin{aligned}
E^{\mathrm{4PN}} = -\frac{\mu c^2 x}{2}\Bigg\{ & 1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2 \\
& + \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3 \\
& + \Bigg(-\frac{3969}{128} + \left[-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_{\mathrm{E}} + \frac{448}{15}\ln(16x)\right]\nu \\
& \quad + \left[-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right]\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\Bigg)x^4\Bigg\}
\end{aligned}
$$

$\nu \sim m_2/m_1$

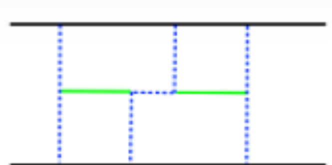
$x \sim (v/c)^2$

Damour Jaranowski Schafer (2014)
Blanchet et al. (2018)

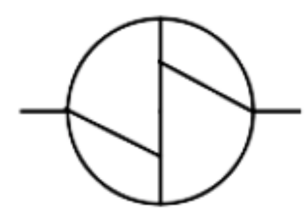
Galley RAP Leibovich Ross (2016)
Foffa Sturani Mastrolia Sturm (2016)
RAP Rothstein (2017)
RAP (2017)
Foffa Sturani (2019)
Foffa RAP Sturani Rothstein (2019)

Challenging computations

$$E^{4\mathrm{PN}} = -\frac{\mu c^2 x}{2}\Big\{1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2 + \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3 + \left(-\frac{3969}{128} + \left[-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16x)\right]\nu + \left[-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right]\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\right)x^4\Big\}$$



$$= -2\,\mathrm{i}\,(8\pi G_N)^5 \left(\frac{(d-2)}{(d-1)}\, m_1 m_2\right)^3$$



$$[N_{49}]$$

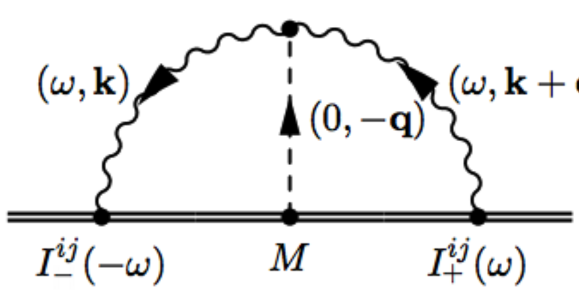
$$[N_{49}] \equiv \int_{k_1,k_2,k_3,k_4} \frac{N_{49}}{k_1^2\; p_2^2\; k_3^2\; p_4^2\; k_{12}^2\; k_{13}^2\; k_{23}^2\; k_{24}^2\; k_{34}^2}\,,$$

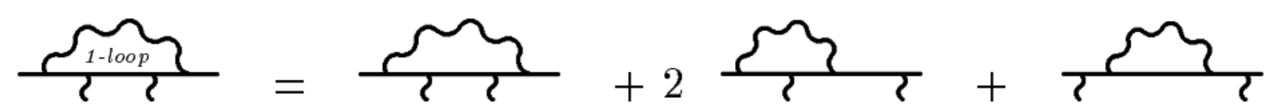
Foffa Sturani Mastrolia Sturm (2016,2019)

$$\mathcal{V}_{\mathrm{static}}^{(5\mathrm{PN})} = \frac{5}{16}\frac{G_N^6 m_1^6 m_2}{r^6} + \frac{91}{6}\frac{G_N^6 m_1^5 m_2^2}{r^6} + \frac{653}{6}\frac{G_N^6 m_1^4 m_2^3}{r^6} + (m_1 \leftrightarrow m_2)$$

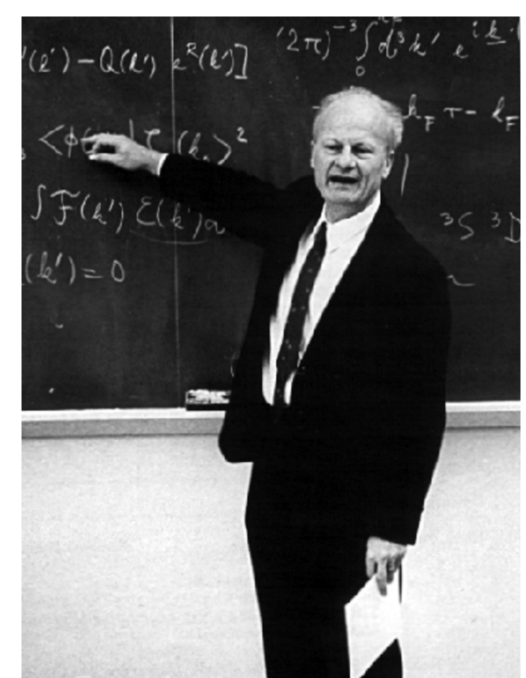
There are ‘IR’ logs (before finite size effects!)

$$E^{4\mathrm{PN}} = -\frac{\mu c^2 x}{2}\left\{1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2 \right.$$
$$+ \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3$$
$$+ \left(-\frac{3969}{128} + \left[-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16x)\right]\nu\right.$$
$$\left.\left. + \left[-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right]\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\right)x^4\right\}$$





Galley RAP Leibovich Ross (2016)



PHYSICAL REVIEW D **96,** 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

Rafael A. Porto

PHYSICAL REVIEW D **89**, 064058 (2014)

Nonlocal-in-time action for the fourth post-Newtonian conservative dynamics of two-body systems

T. Damour, P. Jaranowski, and G. Schäfer,

$$H_{4\mathrm{PN}}^{\text{near-zone (s)}}[\mathbf{x}_a, \mathbf{p}_a] = H_{4\mathrm{PN}}^{\mathrm{loc0}}[\mathbf{x}_a, \mathbf{p}_a] + F[\mathbf{x}_a, \mathbf{p}_a]\left(\ln\frac{r_{12}}{s} + C\right)$$

Ambiguity associated to **IR divergences**
(Similar to Lamb shift…soon)

Fixed by comparison
with self-force

$$C = -\frac{1681}{1536}$$

It wasn't determined from first principles with PN framework!

Fokker action of nonspinning compact binaries at the fourth post-Newtonian approximation

Laura Bernard, Luc Blanchet, Alejandro Bohé, Guillaume Faye, and Sylvain Marsat
Phys. Rev. D **93**, 084037 – Published 20 April 2016

lowever, we find that it differs from the recently published result derived within the ADM Hamiltonian formulation of general relativity [T. Damour, P. Jaranowski, and G. Schäfer, Phys. Rev. D **89**, 064058 (2014)]. More work is needed to understand this discrepancy.

PHYSICAL REVIEW D **93,** 084014 (2016)

Conservative dynamics of two-body systems at the fourth post-Newtonian approximation of general relativity

T. Damour, P. Jaranowski, and G. Schäfer,

(iii) several claims in a recent harmonic-coordinates Fokker-action computation [L. Bernard *et al.*, arXiv:1512.02876v2 [gr-qc]] are incorrect, but can be corrected by the addition of a couple of *ambiguity parameters* linked to subtleties in the regularization of infrared and ultraviolet

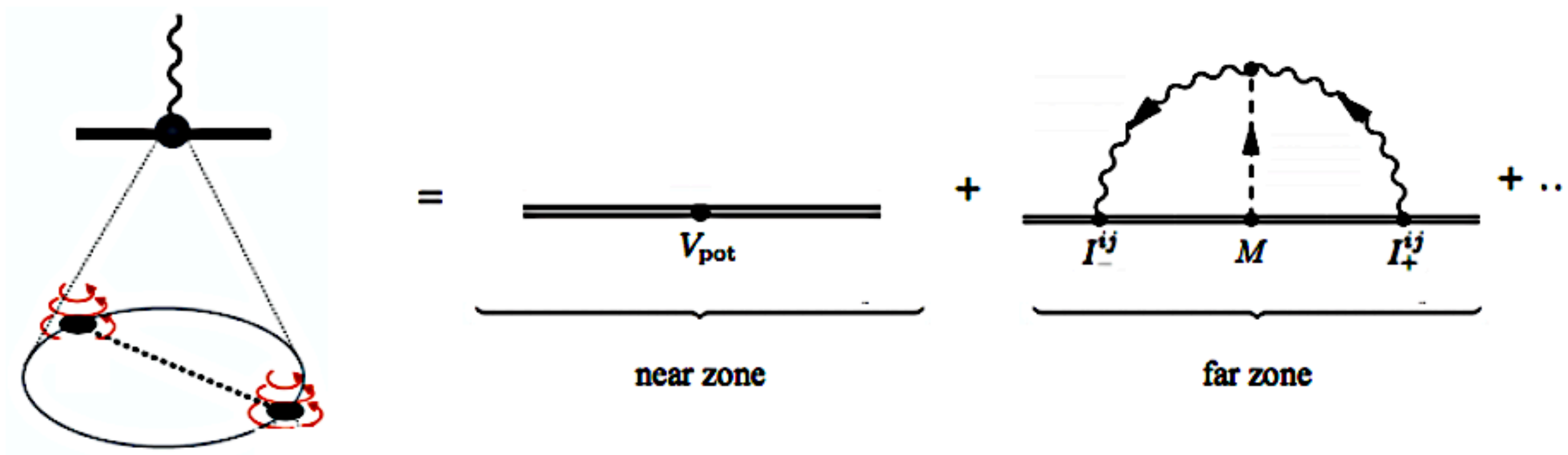
VII. SUGGESTION FOR ADDING MORE IR AMBIGUITY PARAMETERS IN REF. [21]

$$(a,b,c)^{\rm new}_{\rm B^3FM} = (a,b,c)_{\rm B^3FM} + \Delta C\,\frac{16}{15}\,(-11,12,0).$$

PHYSICAL REVIEW D **96**, 024062 (2017)

Apparent ambiguities in the post-Newtonian expansion for binary systems

Rafael A. Porto[1] and Ira Z. Rothstein[2]



$$\frac{2G_N^2 M}{5} I^{(3)ij} I^{(3)ij} \left(-\frac{1}{\epsilon_{\rm IR}} + 2\log(\mu r) + \cdots\right) + \left(\frac{1}{\epsilon_{\rm UV}} + 2\log(\Omega/\mu) + \cdots\right)$$

IR/UV cancelation
There are no ambiguities!

PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

PHYSICAL REVIEW D **96**, 024062 (2017)

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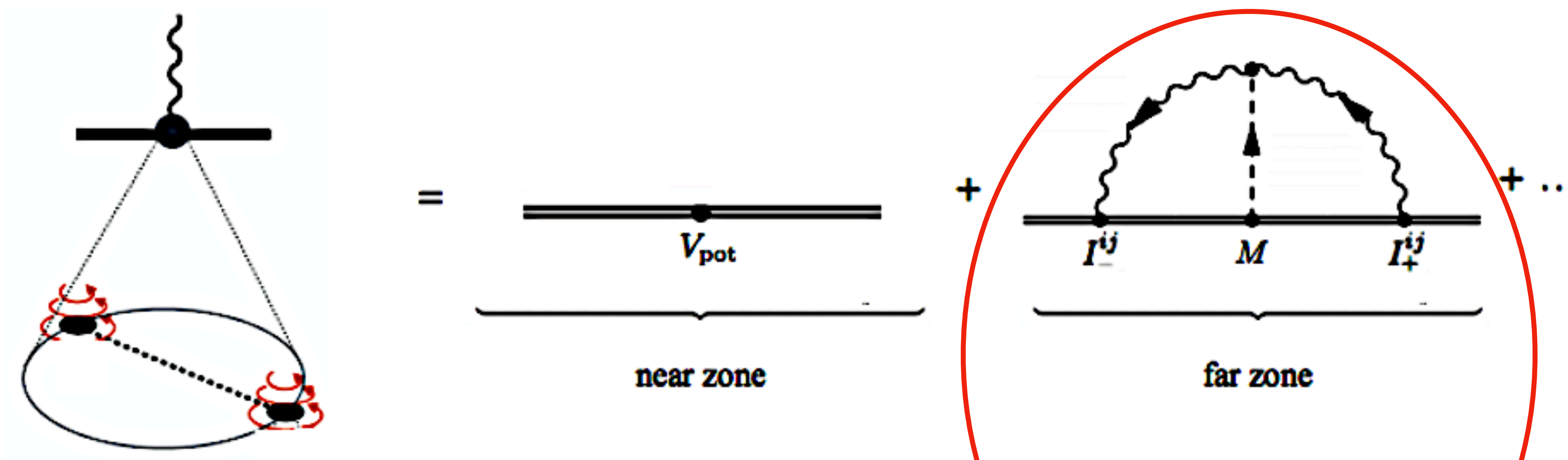


$$\frac{2G_N^2 M}{5} I^{(3)ij} I^{(3)ij} \left(-\frac{1}{\epsilon_{\rm IR}} + 2\log(\mu r) + \cdots\right) + \left(\frac{1}{\epsilon_{\rm UV}} + 2\log(\Omega/\mu) + \cdots\right)$$

IR/UV cancelation
There are no ambiguities!

PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

$$\mu \frac{d}{d\mu} V_{\rm ren}(\mu) = \frac{2G_N^2 M}{5} I^{ij(3)}(t) I^{ij(3)}(t)$$

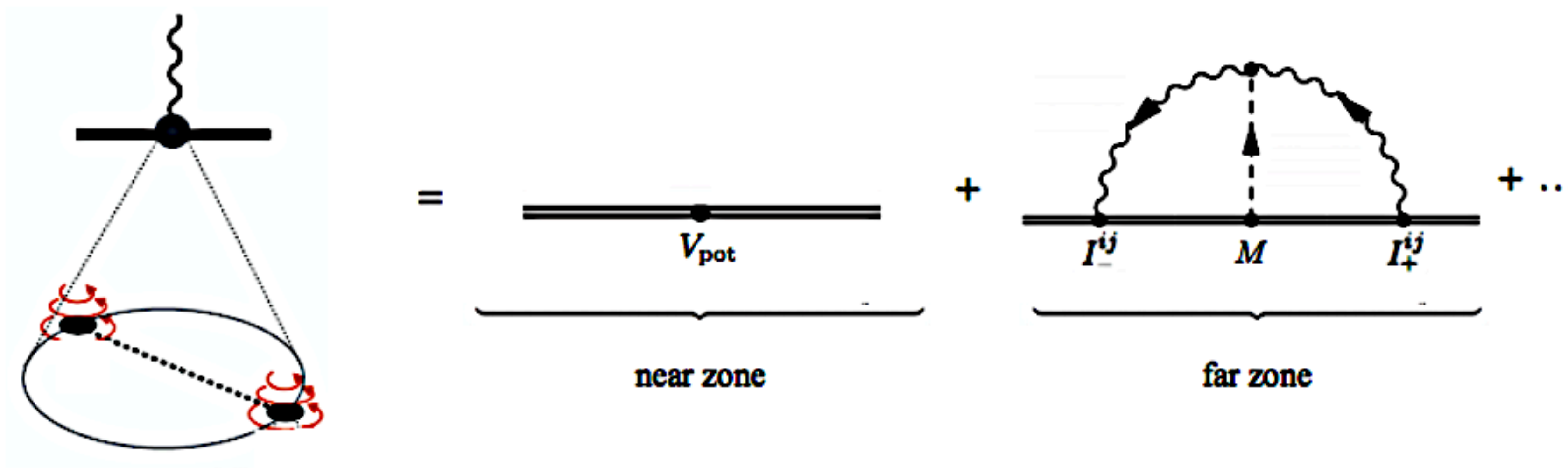
Universal log
in binding energy

$$E_{\log} = -2G_N^2 M \langle I^{ij(3)}(t) I^{ij(3)}(t)\rangle \log v$$

PHYSICAL REVIEW D 96, 024062 (2017)

Apparent ambiguities in the post-Newtonian expansion for binary systems

Rafael A. Porto[1] and Ira Z. Rothstein[2]



PHYSICAL REVIEW D 96, 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

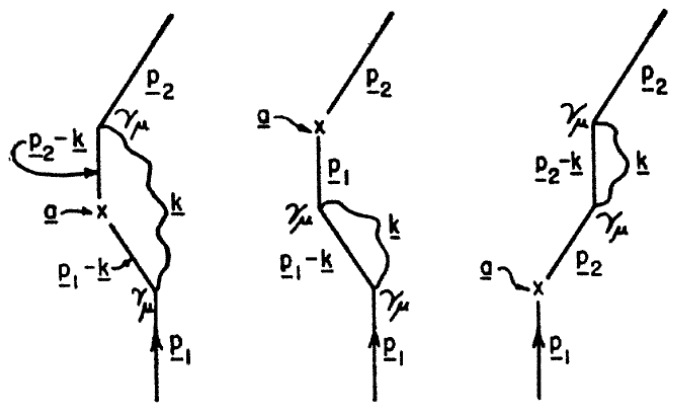
Rafael A. Porto

$$\begin{aligned}\delta E_{n,\ell} &= (\delta E_{n,\ell})_{US} + (\delta E_{n,\ell})_{cV} + \cdots \\ &= \frac{2\alpha_e}{3\pi}\left[\frac{5}{6}e^2\frac{|\psi_{n,\ell}(\boldsymbol{x}=0)|^2}{2m_e^2} - \sum_{m\neq n,\ell}\left\langle n,\ell\left|\frac{\boldsymbol{p}}{m_e}\right|m,\ell\right\rangle^2 (E_m-E_n)\log\frac{2|E_n-E_m|}{m_e}\right] + \\ &\quad + \frac{4\alpha_e^2}{3m_e^2}\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}\right)|\psi_{n,\ell}(\boldsymbol{x}=0)|^2\,.\end{aligned}$$

correct value w/out ambiguities!

IR/UV cancelation in dim. reg. (non-trivial in other schemes)

$$\Delta H=\frac{e^2}{2\pi\hbar c}\left\{-\frac{\hbar e i}{2\mu c}\beta\boldsymbol{\alpha}\cdot\nabla\varphi+\frac{2\hbar^2 e}{3\mu^2c^2}(\nabla^2\varphi)\right.$$
$$\left.\times\left(\ln\frac{\mu c}{2\hbar k_{\min}}+\frac{5}{8}\right)\right\}, \quad (19)$$



a. Eq.12 b. Eq.13 c. Eq.14

Space-Time Approach to Quantum Electrodynamics

R. P. Feynman
Department of Physics, Cornell University, Ithaca, New York

(Received May 9, 1949)

Lamb shift as interpreted in more detail in B.[13]

[13] That the result given in B in Eq. (19) was in error was repeatedly pointed out to the author, in private communication, by V. F. Weisskopf and J. B. French, as their calculation, completed simultaneously with the author's early in 1948, gave a different result. French has finally shown that although the expression for the radiationless scattering B, Eq. (18) or (24) above is correct, it was incorrectly joined onto Bethe's non-relativistic result. He shows that the relation $\ln 2k_{\max}-1=\ln\lambda_{\min}$ used by the author should have been $\ln 2k_{\max}-5/6=\ln\lambda_{\min}$. This results in adding a term $-(1/6)$ to the logarithm in B, Eq. (19) so that the result now agrees with that of J. B. French and V. F. Weisskopf,

PHYSICAL REVIEW D **96,** 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

Rafael A. Porto

$$\begin{aligned}\delta E_{n,\ell} &= (\delta E_{n,\ell})_{US}+(\delta E_{n,\ell})_{cV}+\cdots\\ &= \frac{2\alpha_e}{3\pi}\left[\frac{5}{6}e^2\frac{|\psi_{n,\ell}(\boldsymbol{x}=0)|^2}{2m_e^2}-\sum_{m\neq n,\ell}\left\langle n,\ell\left|\frac{\boldsymbol{p}}{m_e}\right|m,\ell\right\rangle^2(E_m-E_n)\log\frac{2|E_n-E_m|}{m_e}\right]+\\ &\quad+\frac{4\alpha_e^2}{3m_e^2}\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}\right)|\psi_{n,\ell}(\boldsymbol{x}=0)|^2\,.\end{aligned}$$

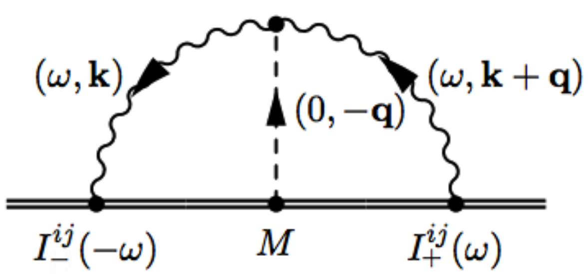
correct value w/out ambiguities!

PHYSICAL REVIEW D **93,** 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

$$W_{\rm tail}[\boldsymbol{x}_a^{\pm}] = \frac{2G_N^2 M}{5}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\omega^6\, I_-^{ij}(-\omega) I_+^{ij}(\omega)\left[-\frac{1}{(d-4)_{\rm UV}} - \gamma_E + \log\pi - \log\frac{\omega^2}{\mu^2} + \frac{41}{30} + i\pi\,{\rm sign}(\omega)\right].$$



"log+5/6"

dissipative term

PHYSICAL REVIEW D **97,** 044023 (2018)

Ambiguity-free completion of the equations of motion of compact binary systems at the fourth post-Newtonian order

Tanguy Marchand,[1,2,*] Laura Bernard,[3,†] Luc Blanchet,[1,‡] and Guillaume Faye[1,§]

V. DETERMINATION OF THE AMBIGUITY PARAMETERS

Remarkably, the value $\kappa = \frac{41}{60}$ we have obtained in our result for the tail [see Eq. (4.13)], agrees with the result found by Galley *et al* [10] in their computation of the tail term in d

Conservative dynamics of binary systems to fourth Post-Newtonian order in the EFT approach II: Renormalized Lagrangian

Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

$$\underbrace{\left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(near+self)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{c.t.\,(near)}}\right)}_{\text{near zone renormalization (self-energies important!)}} + \underbrace{\left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(IR\,near+self-ZB)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(far)}}\right)}_{\text{cancelation of near/far IR/UV spurious poles*}} \to \text{finite}\,,$$

near zone renormalization (self-energies important!)

cancelation of near/far IR/UV spurious poles*

***Zero-bin subtraction (scale-less integrals)**

$$I_{\mathrm{ZB}}\,[n_1,n_2] = \int_{\boldsymbol{k}} \frac{1}{[\boldsymbol{k}^2]^{n_1}[\boldsymbol{p}^2]^{n_2}} \xrightarrow{(n_1=3/2,n_2=1/2)} |\boldsymbol{p}|^{-1}\int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^3} = \frac{i}{16\pi|\boldsymbol{p}|}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right)$$

Conservative dynamics of binary systems to fourth Post-Newtonian order in the EFT approach II: Renormalized Lagrangian

Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

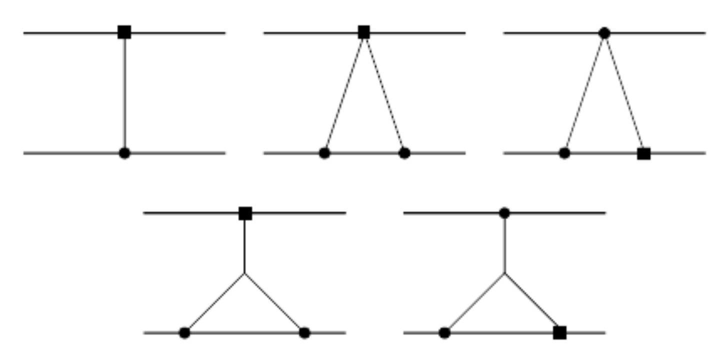
$$\left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(near+self)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{c.t.\,(near)}}\right) + \left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(IR\,near+self-ZB)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(far)}}\right) \to \mathrm{finite}\,,$$

$$S_{\mathrm{pp}}[x_a^\alpha(\tau_a)] = \sum_a \int d\tau_a \left(-m_a + \sum_i c_i \mathcal{O}_i\left[x_a^\alpha(\tau_a), \dot{x}_a^\alpha(\tau_a), \cdots; g_{\mu\nu}, \partial_\beta g_{\mu\nu}, \cdots\right]\right)$$

diff invariance + RPI (in dim. reg.)

Effective action to 4PN order:

$$\begin{aligned} S_{\mathrm{pp}}[x_a^\alpha(\tau_a)] \;&=\; \sum_a \int d\tau_a \Bigg[-m_a + \left(c^{(a)}_{a\dot{v},\,\mathrm{ren}}(\mu) - \frac{11}{3}\frac{G^2 m_a^2}{\epsilon_{\mathrm{UV}}}\right) g_{\mu\nu} a_a^\mu \dot{v}_a^\nu \\ &+\; \left(c^{(a)}_{V,\,\mathrm{ren}}(\mu) + \frac{G^2 m_a^2}{\epsilon_{\mathrm{UV}}}\right) R_{\mu\nu} v_a^\mu v_a^\nu\Bigg]\,. \end{aligned}$$



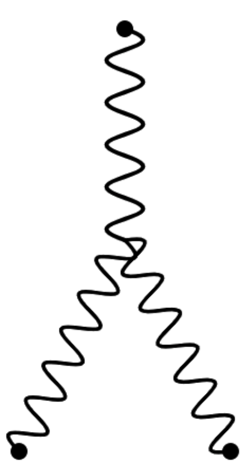
The operators beyond minimal coupling can be **removed by field-redefinitions** until 5PN (no spin)

No renormalization scheme-dependence (no UV ambiguities)

Conservative dynamics of binary systems to fourth Post-Newtonian order in the EFT approach II: Renormalized Lagrangian

Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

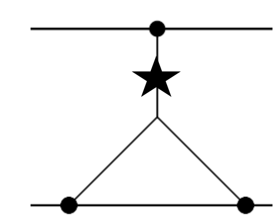
$$\left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(near+self)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{c.t.\,(near)}}\right) + \left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(IR\,near+self-ZB)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(far)}}\right) \to \text{finite}\,,$$



Potential $\quad \boldsymbol{a}_1^i \boldsymbol{a}_2^j \int dt \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{\boldsymbol{p}^i \boldsymbol{p}^j}{|\boldsymbol{p}|^5} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}_1(t)-\boldsymbol{x}_2(t))}$

$$\frac{1}{p_0^2 - p^2} = \frac{1}{p^2}\left(-1 + p_0^2/p^2 + \cdots\right)$$

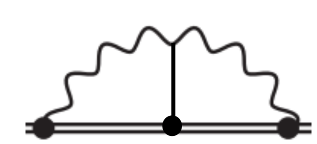
★



IR divergent

cancel out!*

Radiation $\quad \int \frac{d\omega}{\pi} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{d^3\boldsymbol{k}}{(2\pi)^3} \boldsymbol{x}_1^i(\omega) \boldsymbol{x}_1^j(\omega) \frac{\boldsymbol{k}^i \boldsymbol{k}^j}{\boldsymbol{p}^2 k^2 (p+k)^2}$



UV divergent

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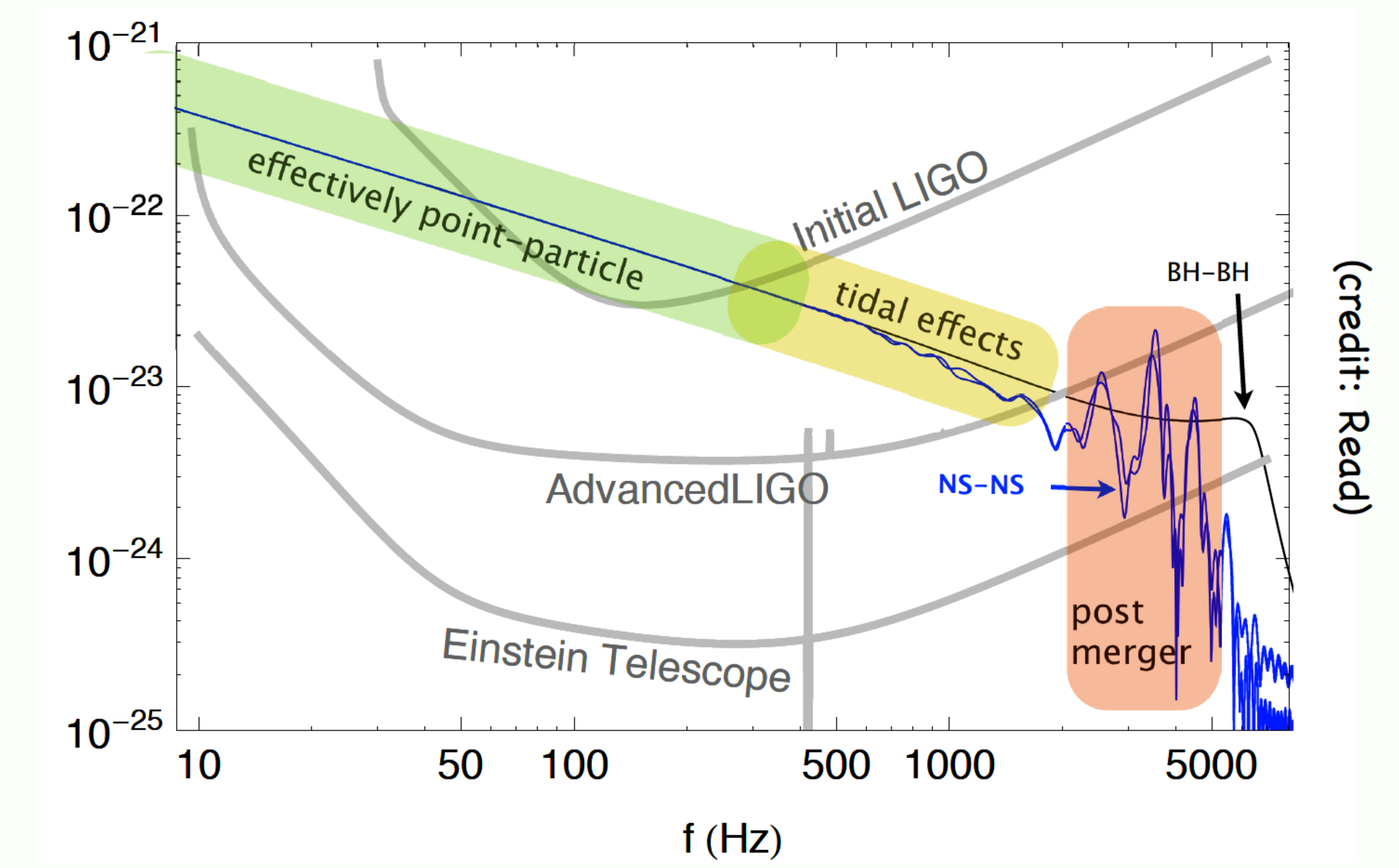
$$I_{\mathrm{ZB}}[n_1, n_2] = \int_{\boldsymbol{k}} \frac{1}{[\boldsymbol{k}^2]^{n_1} [\boldsymbol{p}^2]^{n_2}} \xrightarrow{(n_1=3/2,\, n_2=1/2)} |\boldsymbol{p}|^{-1} \int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^3} = \frac{i}{16\pi|\boldsymbol{p}|}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right)$$



"Waveforms will be far more complex and carry more information than expected. Improved modeling will be needed for extracting the GW's information" ***<u>1993</u>***

Kip Thorne 'The last 3 minutes' paper

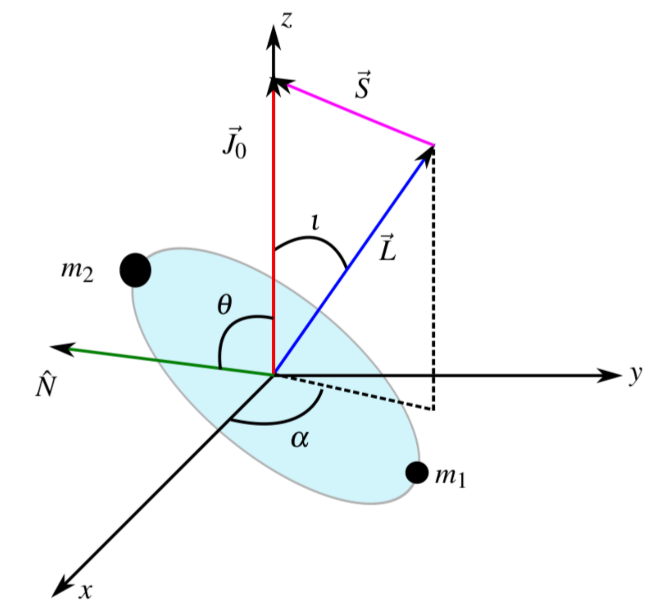
<u>20+ years prior to first detection!</u>

The last three minutes: Issues in gravitational-wave measurements of coalescing compact binaries

Curt Cutler, Theocharis A. Apostolatos, Lars Bildsten, Lee Smauel Finn, Eanna E. Flanagan, Daniel Kennefick, Dragoljub M. Markovic, Amos Ori, Eric Poisson, Gerald Jay Sussman, and Kip S. Thorne
Phys. Rev. Lett. **70**, 2984 – Published 17 May 1993

Knowledge at the time!

$$\frac{d\mathcal{N}_{\rm cyc}}{d\ln f} = \frac{5}{96\pi}\frac{1}{\mu M^{2/3}(\pi f)^{5/3}}\left\{1+\left(\frac{743}{336}+\frac{11}{4}\frac{\mu}{M}\right)x - [4\pi + \text{S.O.}]x^{1.5} + [\text{S.S.}]x^{2} + O(x^{2.5})\right\}.$$



PRL 97, 021101 (2006) PHYSICAL REVIEW LETTERS week ending 14 JULY 2006

Calculation of the First Nonlinear Contribution to the General-Relativistic Spin-Spin Interaction for Binary Systems

Rafael A. Porto and Ira Z. Rothstein

$$\frac{dN_{\rm cyc}}{d\ln f} = \frac{5}{96\pi}\frac{1}{\mu M^{2/3}(\pi f)^{5/3}}\left\{1 + \left(\frac{743}{336} + \frac{11}{4}\frac{\mu}{M}\right)x \right.$$
$$\left. -[4\pi + \mathrm{S.O.}]x^{1.5} + [\mathrm{S.S.}]x^2 + [\mathrm{S.O.}]x^{2.5} + [\mathrm{S.S.}]x^3\right.$$

Spin induced multipole moments for the gravitational wave flux from binary inspirals to third Post-Newtonian order

Rafael A. Porto[a,b,c], Andreas Ross[d,e] and Ira Z. Rothstein[e]

Published 2 March 2011 • Journal of Cosmology and Astroparticle Physics, Volume 2011,

Conservative dynamics of binary systems to fourth Post-Newtonian order in the EFT approach II: Renormalized Lagrangian

Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

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$$\left. -[4\pi+\text{S.O.}]x^{1.5}+[\text{S.S.}]x^2+[\text{S.O.}]x^{2.5}+[\text{S.S.}]x^3+O(x^4)\right.$$

"... + Log + 41/30"

PHYSICAL REVIEW D **93,** 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

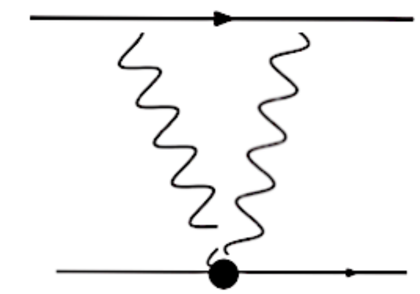
Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

Damour Jaranowski Schafer (2014,2016)
Blanchet Faye et al. (2015,2018)

Are we ready for the future?

NO!

First **non-trivial** operator in EFT!

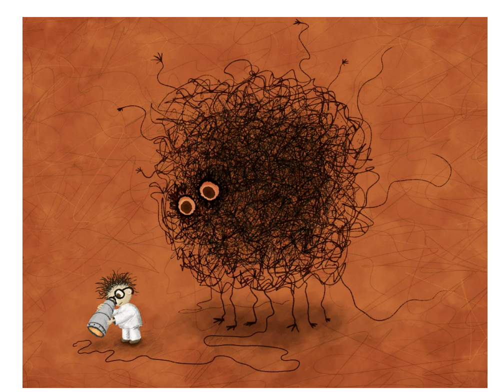


?

$$\frac{dN_{\rm cyc}}{d\ln f} = \frac{5}{96\pi}\frac{1}{\mu M^{2/3}(\pi f)^{5/3}}\left\{1 + \left(\frac{743}{336} + \frac{11}{4}\frac{\mu}{M}\right)x - [4\pi + \text{S.O.}]x^{1.5} + [\text{S.S.}]x^2 + [\text{S.O.}]x^{2.5} + [\text{S.S.}]x^3 + O(x^4) + O(x^5)\right\}.$$

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

Gravitational-wave observations alone are able to measure the masses of the two objects and set a lower limit on their compactness, but the results presented here do not exclude objects more compact than neutron stars such as quark stars, black holes, or more exotic objects [57–61].



Le Monde

no.203.078 01.01.2025

EinsTein Reloaded!

LHC to LISA

New era of foundational investigations established through GWPD.

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\Big\{1 + \cdots + [\cdots]x^{7/2} + [\cdots]x^4 + [\cdots]x^5\Big\}$$

New particles discovered!

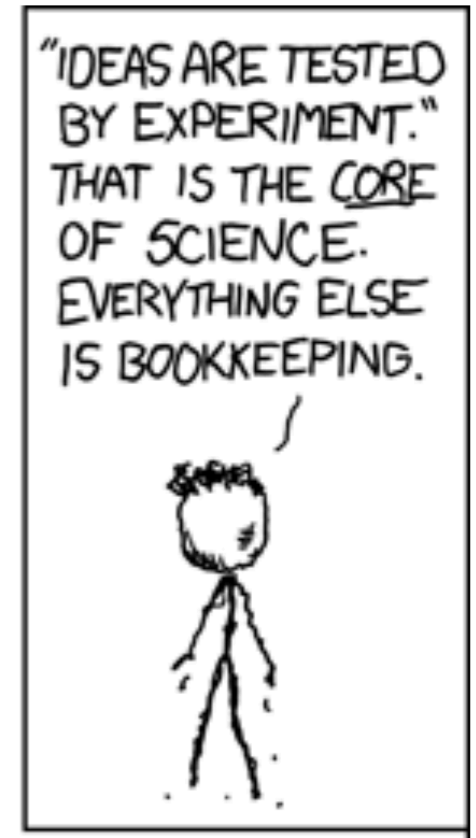
New objects found!

Neutron stars unveiled!

Experts Clash Over Project To Detect Gravity Wave

Physicists say device could help them fathom black holes, but others fault its price.

Extra Slides



"New directions in science are launched by new tools much more often than by new concepts. The effect of a concept-driven revolution is to explain old things in a new way. The effect of a tool-driven revolution is to discover new things that have to be explained"

Freeman Dyson, "Imagined Worlds"

‘That’s nice, but what can you do with it?’

Key contributions to State-of-the-art

* **General Relativity and Gravitation: A Centennial Perspective**

Chapter 6: **Sources of Gravitational Waves: Theory and Observations**

Alessandra Buonanno and B.S. Sathyaprakash

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + \underbrace{\cdots + [\cdots]x^{7/2}}\right\}$$

* the EFT approach has extended the knowledge of the conservative dynamics and multipole moments to high PN orders [134–145].

[134] **Porto, R. A.** 2006. *Phys. Rev. D*, **73**, 104031.
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[139] **Porto, R. A.**, Ross, A., Rothstein, I. Z. 2011. *JCAP*, **1103**, 009.
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[142] Levi, M. 2012. *Phys. Rev. D*, **85**, 064043.
[143] Hergt, S., Steinhoff, J., Schaefer, G. 2012. *Annals Phys.*, **327**, 1494–1537.
[144] Hergt, S., Steinhoff, J., Schaefer, G. 2014. *J.Phys.Conf.Ser.*, **484**, 012018.
[145] **Porto, R. A.**, Ross, A., Rothstein, I. Z. 2012. *JCAP*, **1209**, 028.

ON THE MOTION OF PARTICLES IN GENERAL RELATIVITY THEORY

A. EINSTEIN and L. INFELD

1. Introduction. The gravitational field manifests itself in the motion of bodies. Therefore the problem of determining the motion of such bodies from the field equations alone is of fundamental importance. This problem was solved for the first time some ten years ago and the equations of motion for two particles were then deduced [1]. A more general and simplified version of this problem was given shortly thereafter [2].

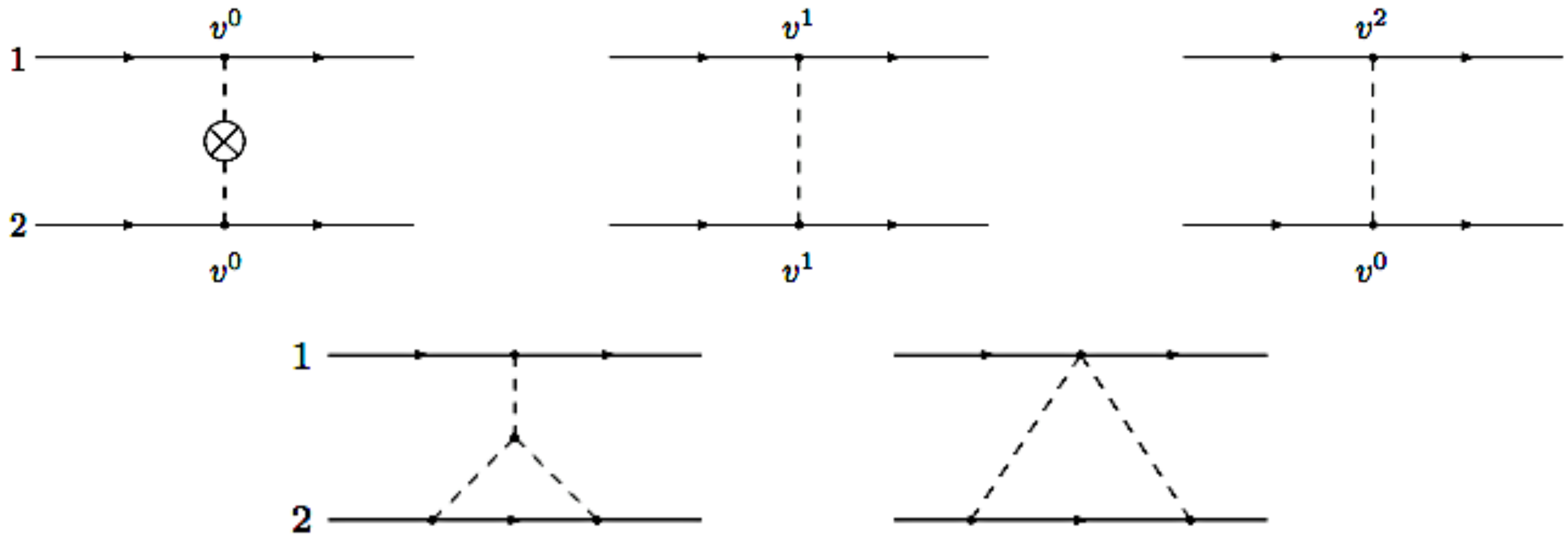
Mr. Lewison pointed out to us, that from our approximation procedure, it does not follow that the field equations can be solved up to an arbitrarily high approximation. This is indeed true. We believe that the present work not only removes this difficulty, but that it gives a new and deeper insight into the problem of motion. From the logical point of view the present theory is considerably simpler and clearer than the old one. But as always, we must pay for these logical simplifications by prolonging the chain of technical argument.

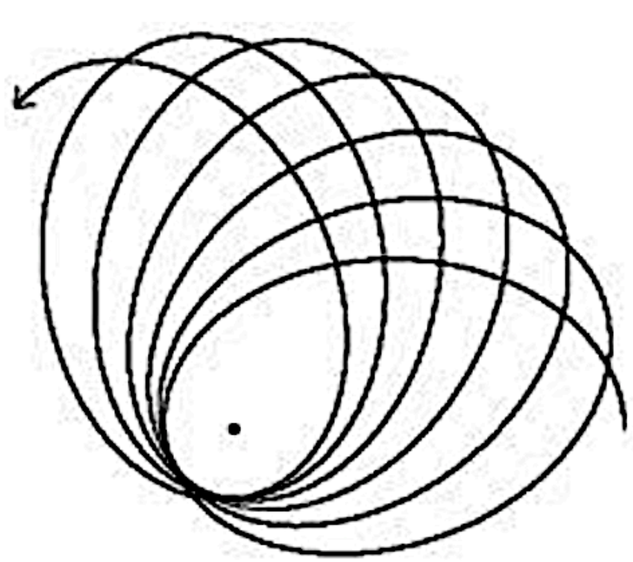
TABLE OF SURFACE INTEGRALS FOR $\int \overset{1}{\underset{6}{\Lambda}}{}_{ms} n_s \, dS$

No.	Expression	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	Result	Remarks
1	$\overset{1}{m} g_{,s} \dot{\eta}^s \dot{\eta}^m$	$-\frac{16}{3}$				$-\frac{4}{3}$		$-\frac{8}{3}$	$-\frac{4}{15}$		$\frac{8}{15}$	$\frac{4}{15}$				$\frac{4}{5}$			-8	$\tilde{g}_{,s} = -2\overset{2}{m} \frac{\partial \overset{1}{r}}{\partial \eta^s}$
2	$\overset{1}{m} \tilde{g} \ddot{\eta}^m$	-2						-4	$-\frac{4}{3}$			$-\frac{29}{3}$	3	$\frac{11}{3}$	$-\frac{5}{3}$	$\frac{2}{3}$	$\frac{32}{3}$	$-\frac{22}{3}$	-8	$\tilde{g} = -\frac{2\overset{2}{m}}{r};\ \ddot{\eta}^m = -\frac{1}{2}\tilde{g}_{,m}$
3	$\overset{1}{m} \tilde{g}_{,m} \dot{\eta}^s \dot{\eta}^s$	1					$-\frac{4}{3}$	$-\frac{4}{5}$			$\frac{8}{5}$	$\frac{4}{5}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{4}{15}$			2	$\tilde{g}_{,m} = -2\overset{1}{m}\ddot{\eta}^m$
4	$\overset{1}{m} \tilde{g}_{,m} \dot{\zeta}^s \dot{\zeta}^s$											2	$\frac{1}{3}$	1	$-\frac{1}{3}$				3	
5	$\overset{1}{m} \tilde{g}_{,m} \tilde{f}$	$\frac{4}{3}$				2	$\frac{2}{3}$					$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$				5	$\tilde{g}_{,m}\tilde{f} = -\tilde{g}\tilde{f}_{,m};\ \tilde{f} = -\frac{2\overset{1}{m}}{r}$
6	$\overset{1}{m}\overset{2}{m}\tilde{r}_{,00m}$											-2							-2	$\tilde{r}_{,00m} = (\overset{2}{r}_{,00m})$ for $x^s = \eta^s$ *
7	$\overset{1}{m} \tilde{g}_{,s} \dot{\zeta}^m \dot{\eta}^s$	$\frac{16}{5}$				$\frac{8}{3}$	$\frac{4}{5}$	$\frac{4}{3}$											8	
8	$\overset{1}{m} \tilde{g}_{,s} \dot{\zeta}^s \dot{\eta}^m$	$\frac{16}{5}$					$-\frac{8}{15}$	4	$\frac{4}{3}$	-2									6	
9	$\overset{1}{m} \tilde{g}_{,m} \dot{\eta}^s \dot{\zeta}^s$	$-\frac{32}{15}$		$-\frac{16}{3}$	$-\frac{8}{3}$		$\frac{4}{5}$		$\frac{4}{3}$										-8	
10	$\overset{1}{m} \tilde{g}_{,s} \dot{\zeta}^s \dot{\zeta}^m$	$-\frac{8}{3}$				-4	$-\frac{4}{3}$												-8	

* $\tilde{r}_{,00m} = \frac{\partial^3 r}{\partial \eta^s \partial \eta^r \partial \eta^m} \dot{\zeta}^s \dot{\zeta}^r$, as $\frac{\partial^2 r}{\partial \eta^s \partial \eta^m} \frac{\partial \overset{1}{r}}{\partial \eta^s} = 0$.

Feynman's EFT computation
(Anomalous shift at 1PN)





Perihelion of Mercury:
One of the first confirmations
of Einstein theory

Precision gravity!
(Jupiter is the leading effect)

Feynman's Gravity (ala QFT)

QUANTUM THEORY OF GRAVITATION*

By R. P. FEYNMAN

(Received July 3, 1963)

Møller: May I, as a non-expert, ask you a very simple and perhaps foolish question. Is this theory really Einstein's theory of gravitation in the sense that if you would have here many gravitons the equations would go over into the usual field equations of Einstein?

Feynman: Absolutely.

[...] gravitational radiation when two stars — excuse me, two particles — go by each other, to any order you want (not for stars, then they have to be particles of specified properties; because obviously the rate of radiation of the gravity depends on the give of the starstides are produced). If you do a real problem with real physical things in in then I'm sure we have the right method that belongs to the gravity theory. There's no question about that.

5PN threshold!