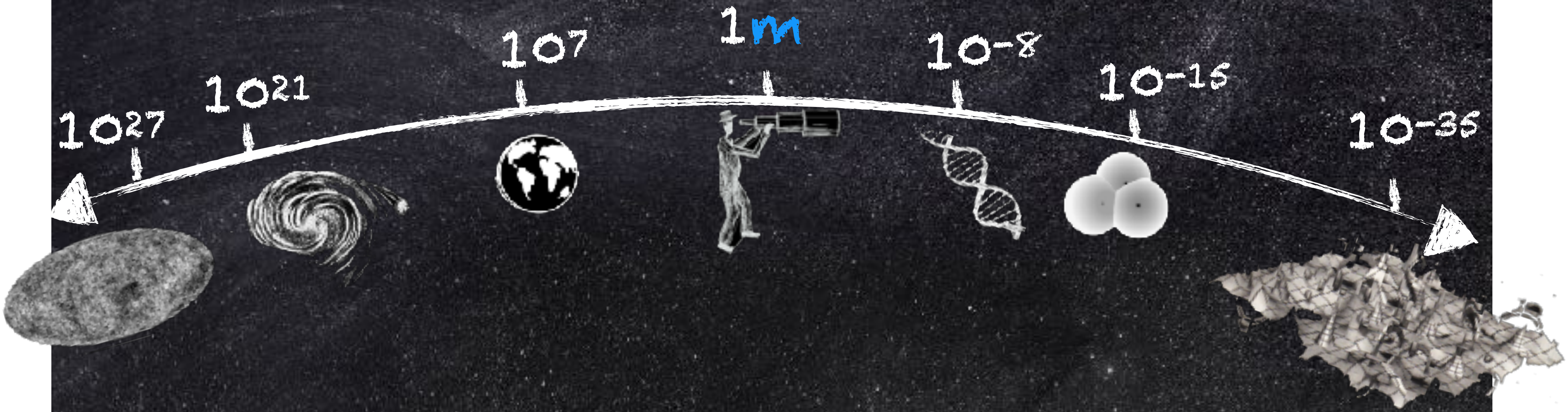
Beyond and Beyond the SM, Effectively

(actually: positivity constraints)

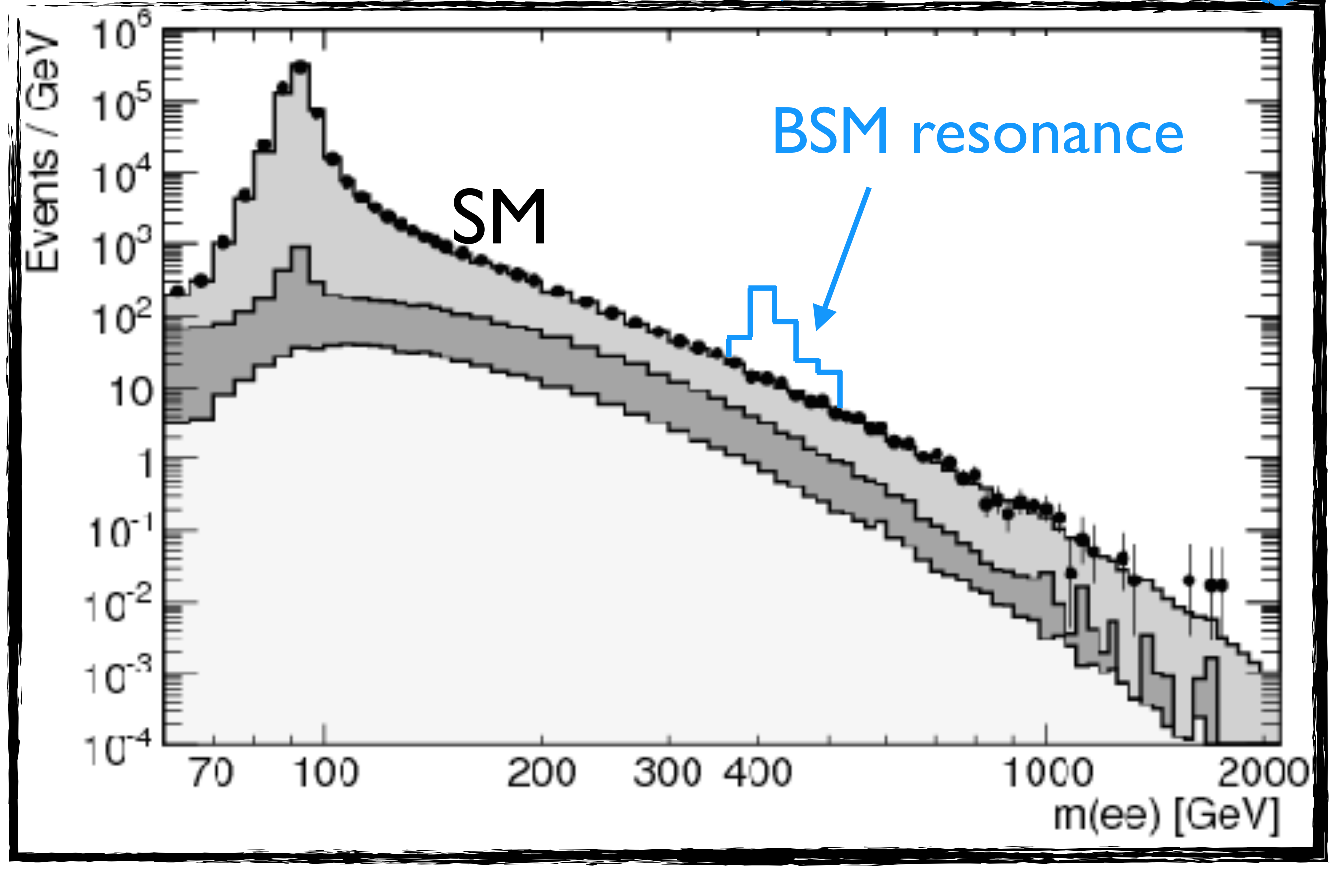


Francesco Riva
(UNIGE)

In collaboration with
Bellazzini, Elias-Miro, Riembau, Rattazzi, in progress
Bellazzini 1806.09640
Bellazzini, Sgarlata, Serra 1706.03070, 1710.02539, 1903.08664

EFT in Particle Phenomenology

LHC Exploration so far: Search for new light particles



Energy frontier (13 TeV)

- Experimentally: First accessible signal/Easy to study
- Theoretically: Weakly coupled, well studied

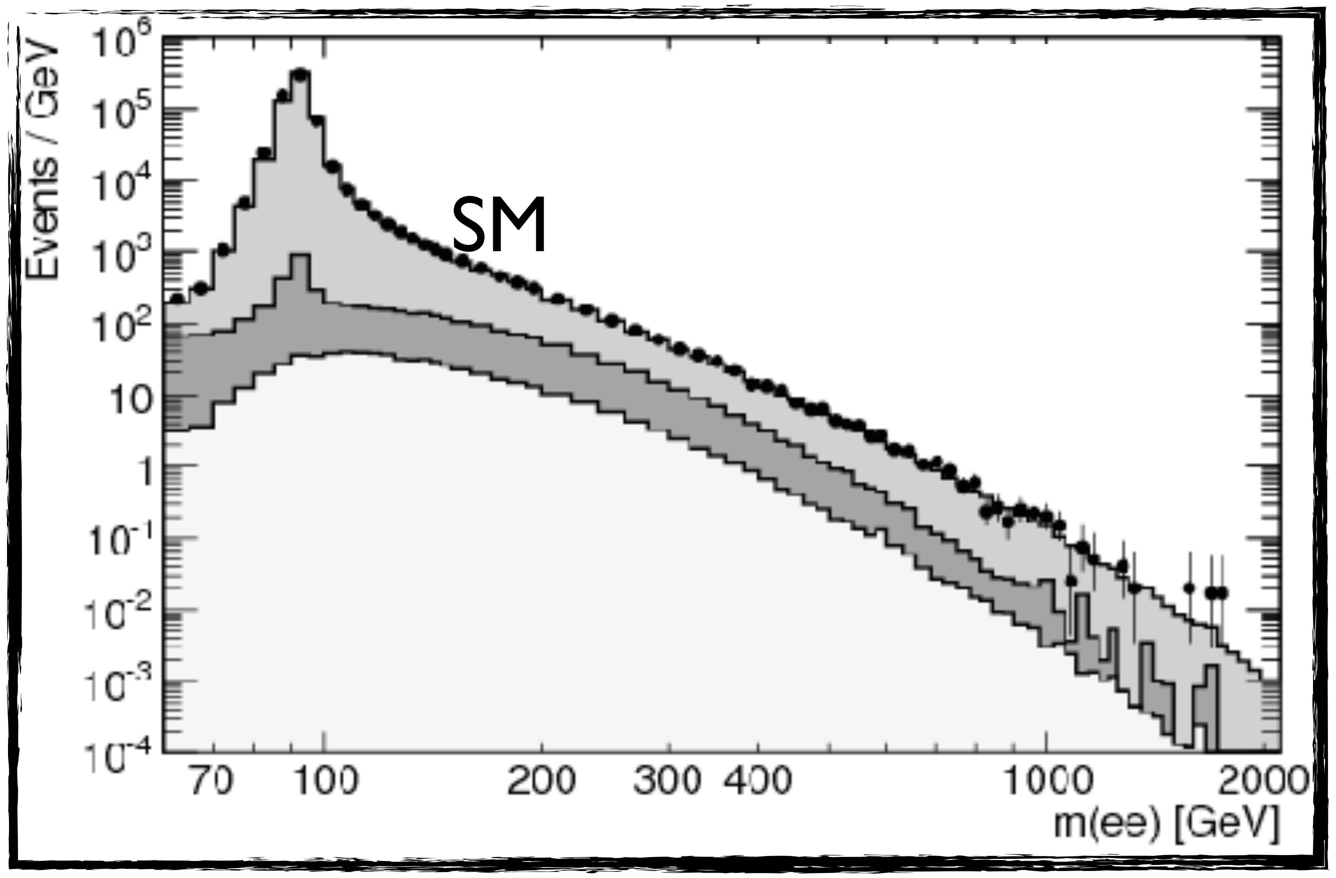
EFT in Particle Phenomenology

Future LHC Exploration: Standard Model Precision Tests

(2035: 3000 fb-1)

intensity

(2016: 40 fb-1)



EFT in Particle Phenomenology

Future LHC Exploration: Standard Model Precision Tests

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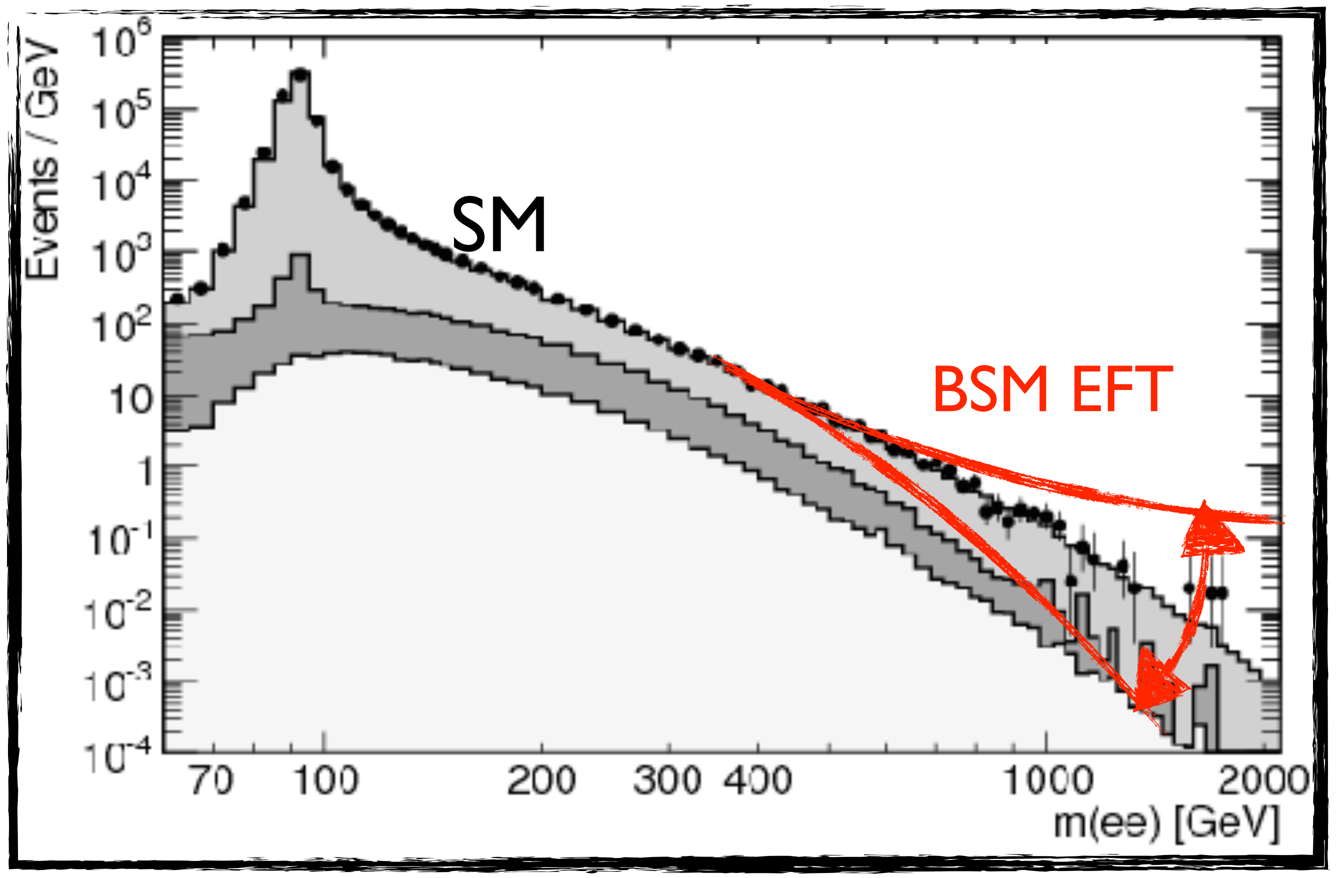

EFT in Particle Phenomenology

Future LHC Exploration: Standard Model Precision Tests

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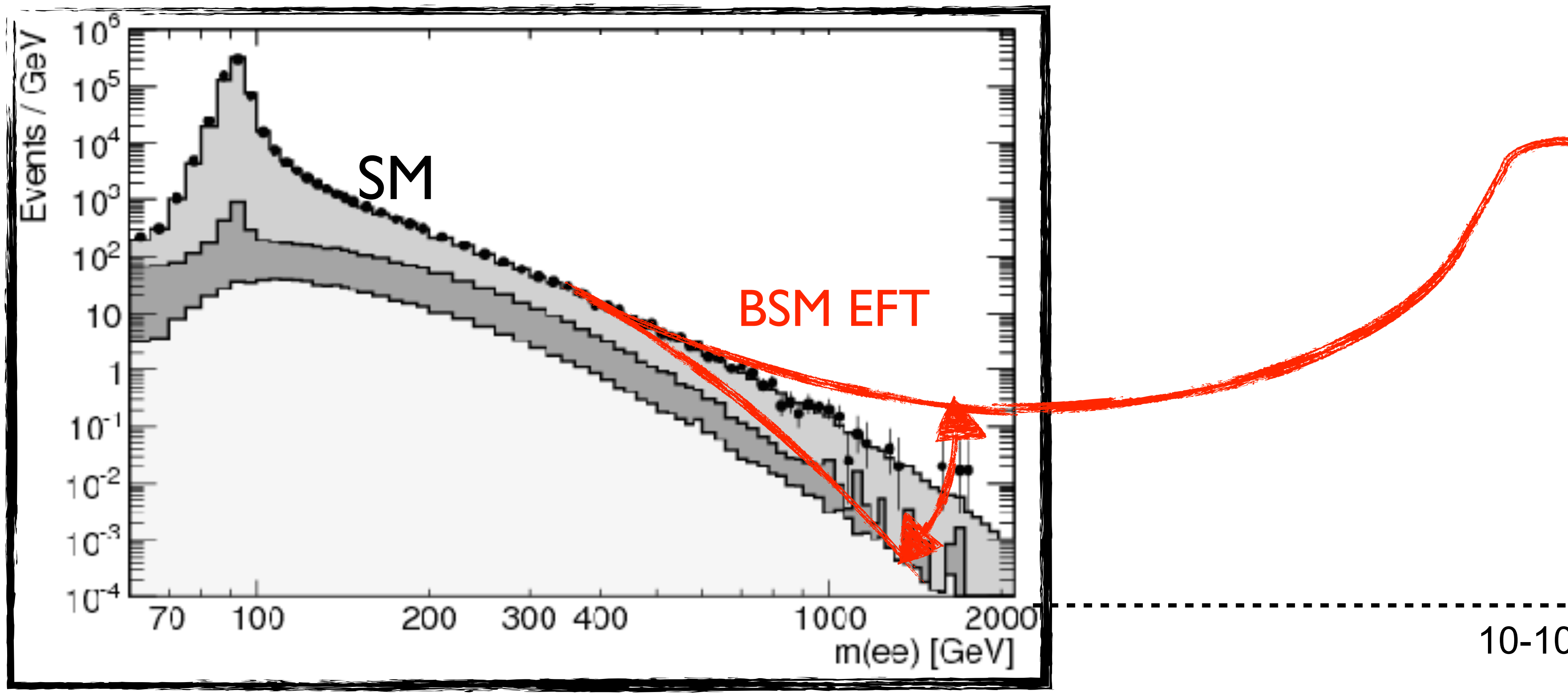

10-10

EFT in Particle Phenomenology

Future LHC Exploration: Standard Model Precision Tests

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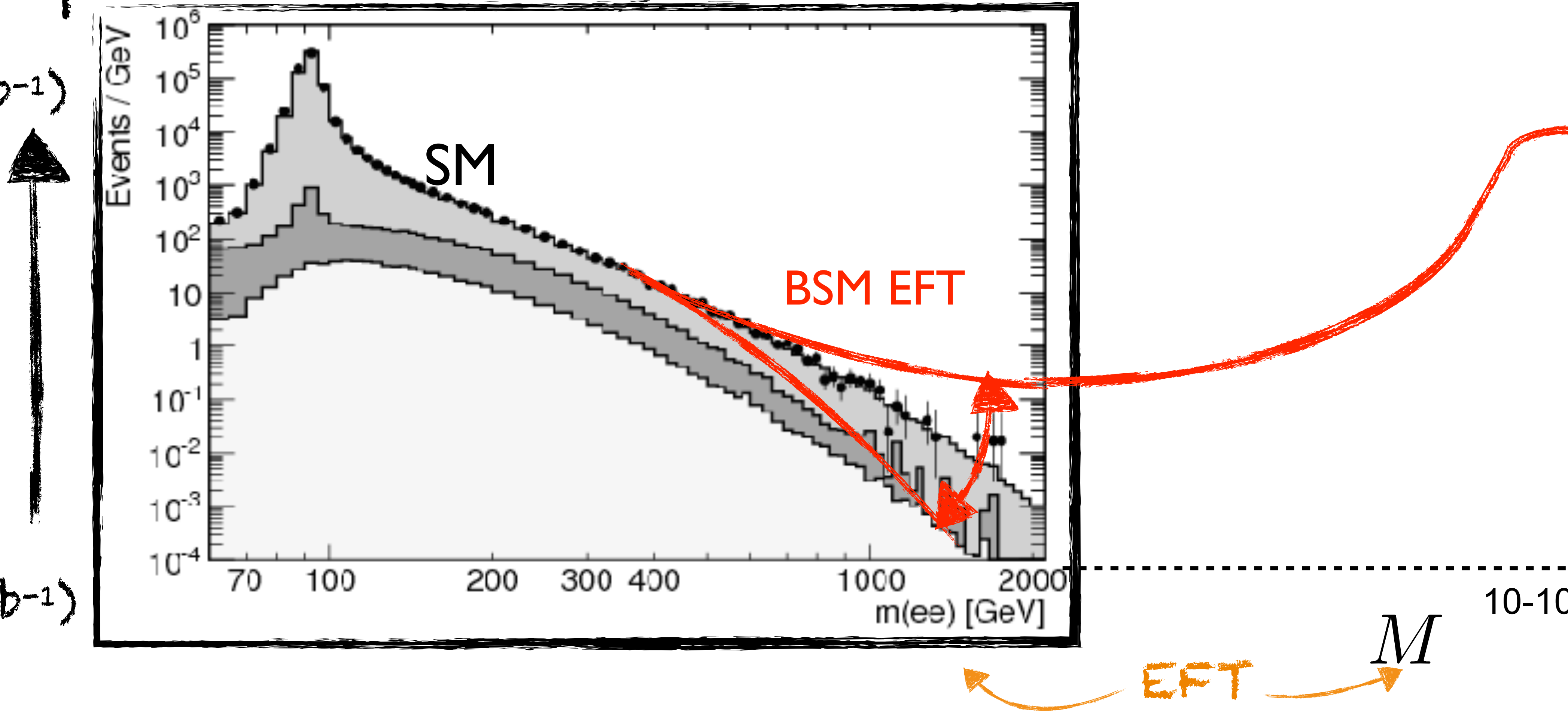
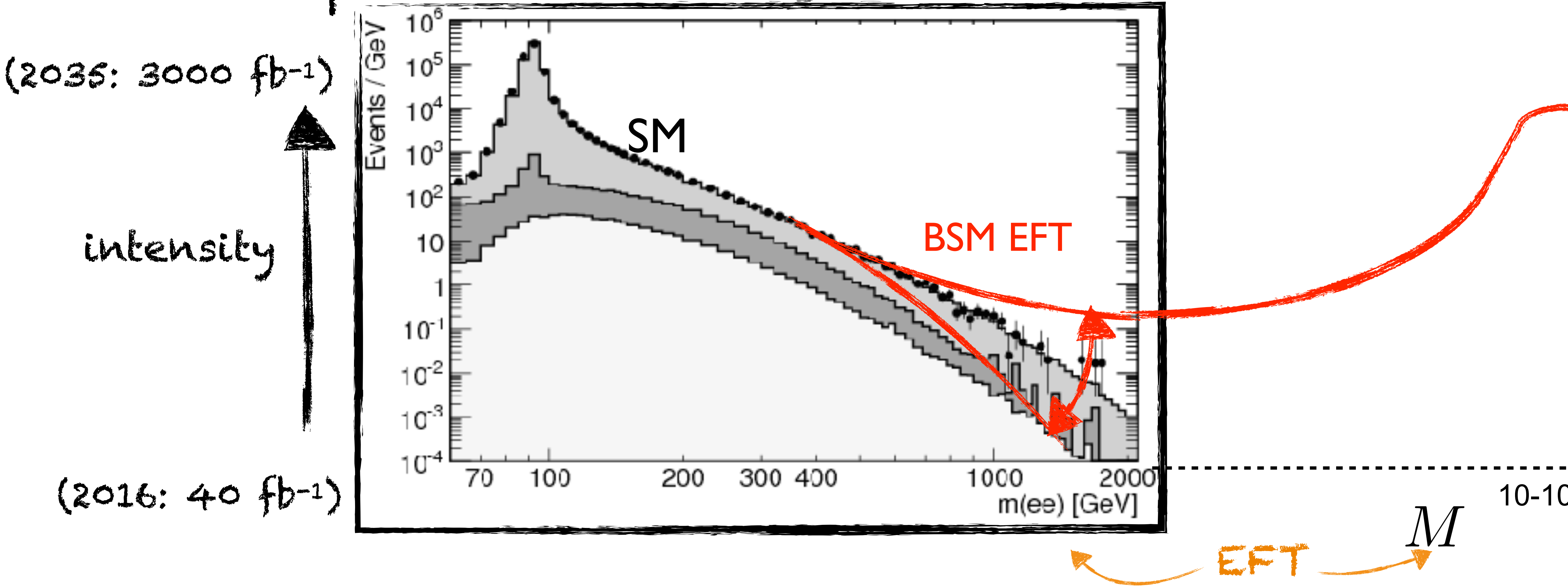


EFT in Particle Phenomenology

Future LHC Exploration: Standard Model Precision Tests



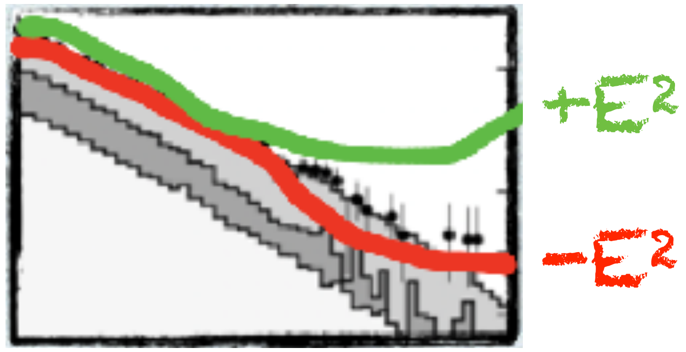
- **Theoretically:** Strongly coupled $\delta A_{2\to 2} \sim g_*^2 \dfrac{E^2}{M^2}$
- **Experimentally:** small statistics, challenging, big improvements

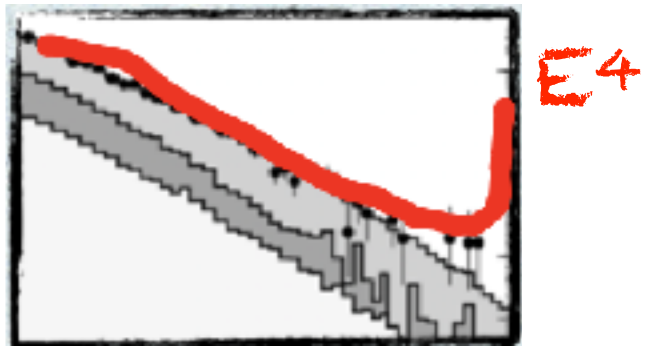
Simple, well-defined, context where EFTs more and more necessary focus on 2>2 processes

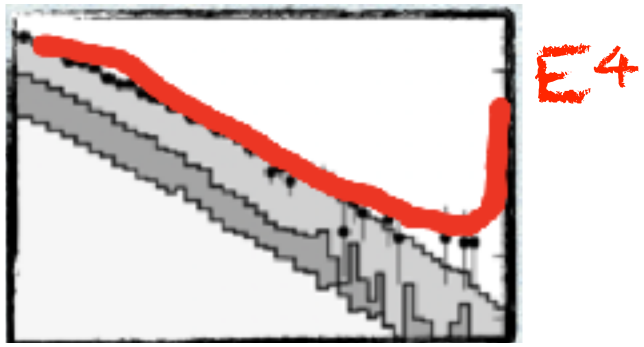
most about scalars, something not

EFT in Particle Phenomenology

Important to understand what EFTs possible.







Negative SM/BSM interference

Softer BSM

No SM/BSM interference

BSM harder to observe / better hidden

$$\mathcal{A}^{tree}_{2\to 2} \sim \underset{\substack{\uparrow \\ \text{dim-4} \\ \text{operators}}}{c_0} + \underset{\substack{\uparrow \\ \text{dim-6} \\ \text{operators}}}{c_2} \frac{E^2}{M^2} + c_4 \frac{E^4}{M^4} + c_6 \frac{E^6}{M^6} + \cdots$$

What signs and relative sizes of operator coefficients possible?

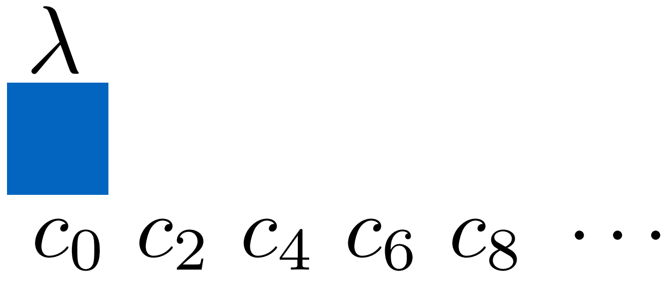
EFT Point of view

$$\mathcal{A}_{2\to 2}^{tree} \sim c_0 + c_2 \frac{E^2}{M^2} + c_4 \frac{E^4}{M^4} + c_6 \frac{E^6}{M^6} + \cdots$$

Symmetries and selection rules shape different c_i patterns

$\lambda\phi^4$-theory

$$\mathcal{L} = \lambda|\phi|^4$$



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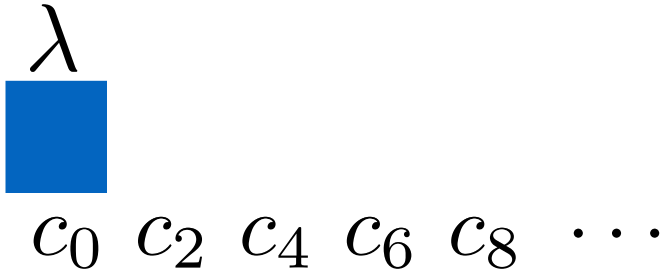
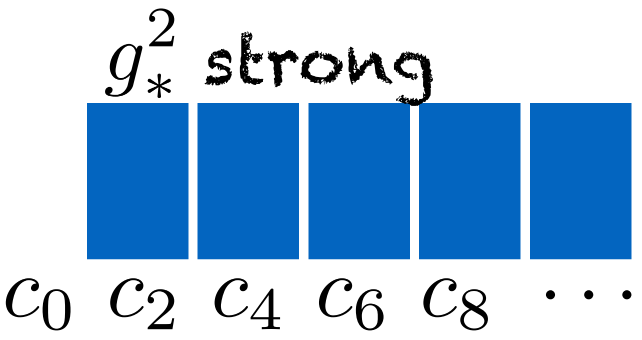
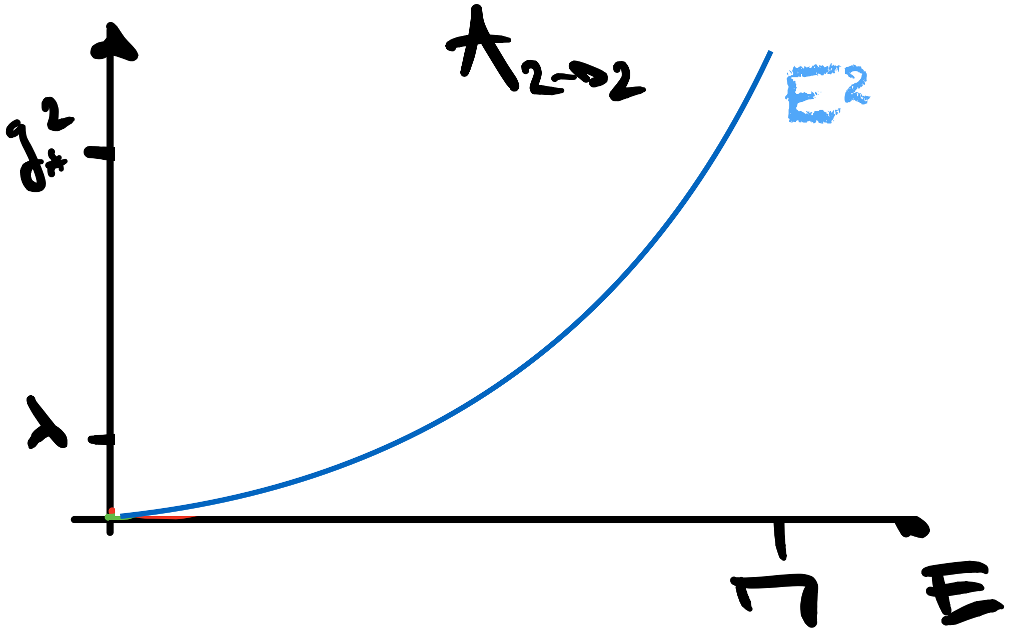


Goldstones

(non-linear global symmetry)

$$\mathcal{L} = \frac{g_*^2}{M^2}(\phi^* \overleftrightarrow{\partial} \phi)^2$$





g_*^2

EFT Point of view

$$\mathcal{A}^{tree}_{2\to 2} \sim c_0 + c_2 \frac{E^2}{M^2} + c_4 \frac{E^4}{M^4} + c_6 \frac{E^6}{M^6} + \cdots$$

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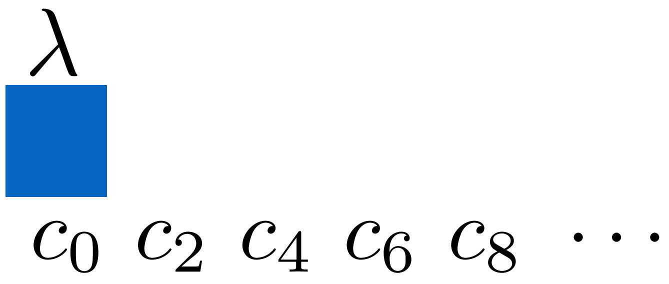

Pseudo

Goldstones

(non-linear global symmetry)

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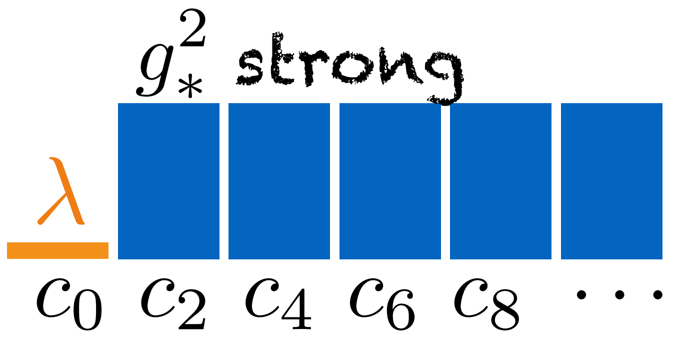

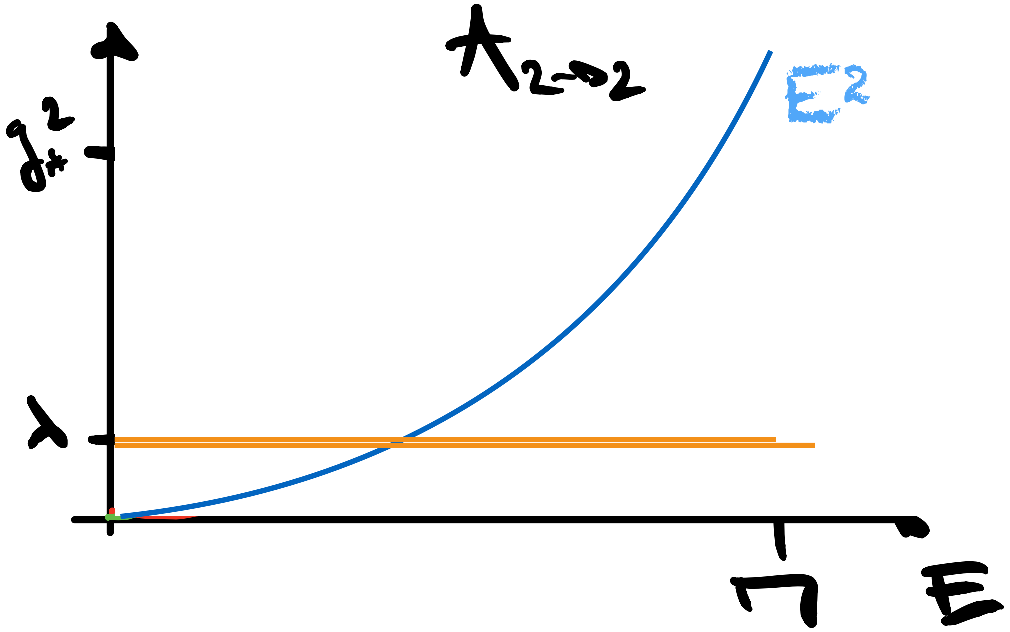

g_*^2

EFT Point of view

$$\mathcal{A}^{tree}_{2\to2} \sim c_0 + c_2\frac{E^2}{M^2} + c_4\frac{E^4}{M^4} + c_6\frac{E^6}{M^6} + \cdots$$

Symmetries and selection rules shape different c_i patterns

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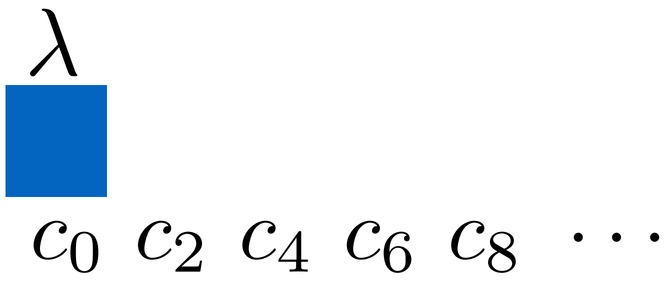

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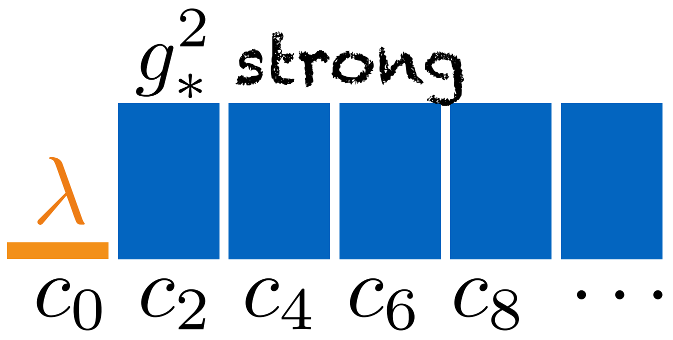

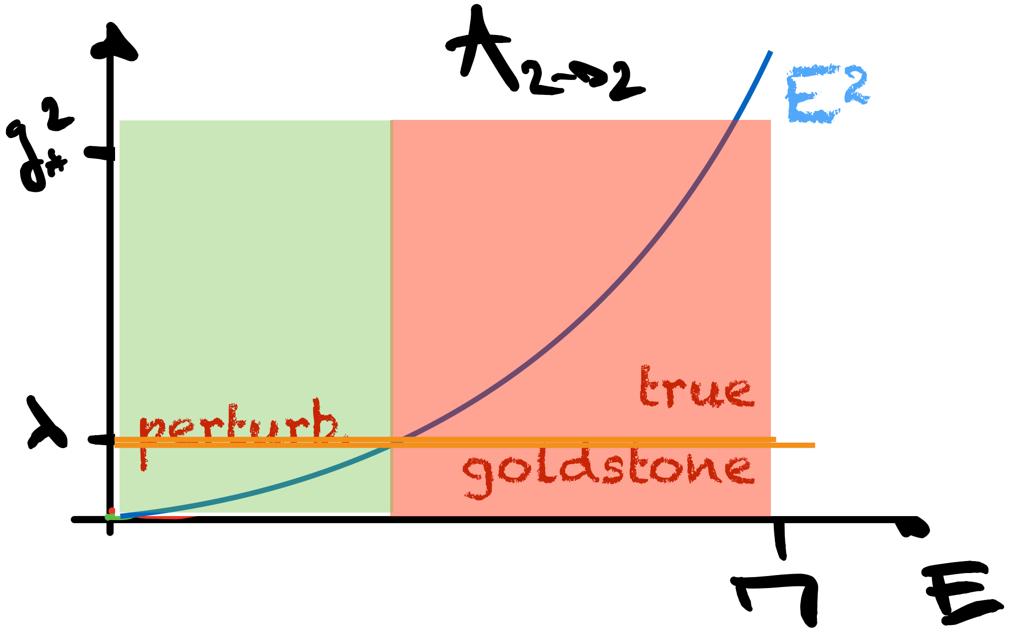

g_*^2

EFT Point of view

$$\mathcal{A}_{2\to 2}^{tree} \sim c_0 + c_2 \frac{E^2}{M^2} + c_4 \frac{E^4}{M^4} + c_6 \frac{E^6}{M^6} + \cdots$$

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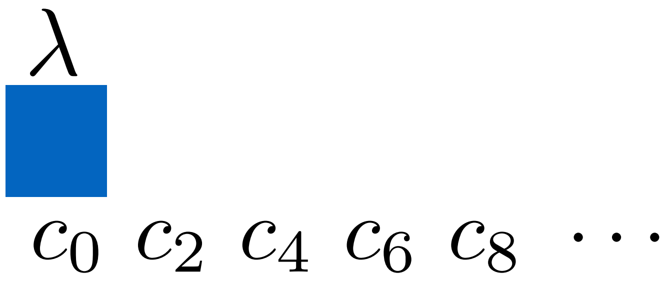

Pseudo

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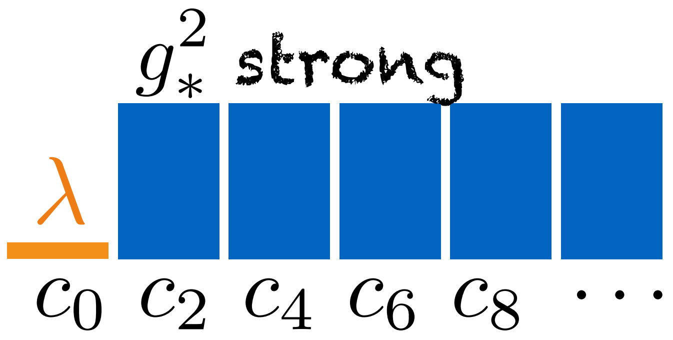
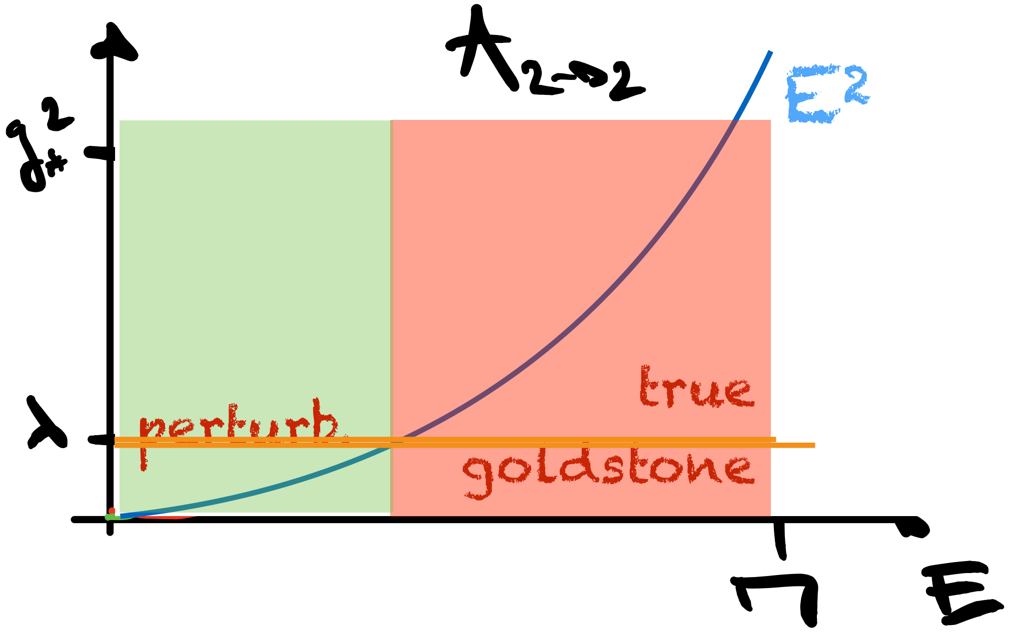
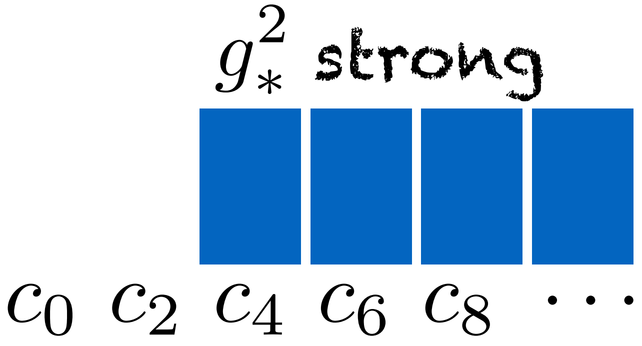




Goldstinos

(non-linear SUSY)

$$\mathcal{L} = \frac{g_*^2}{M^4}(\partial_\mu \bar{\psi}\gamma^\mu \psi)^2$$



EFT Point of view

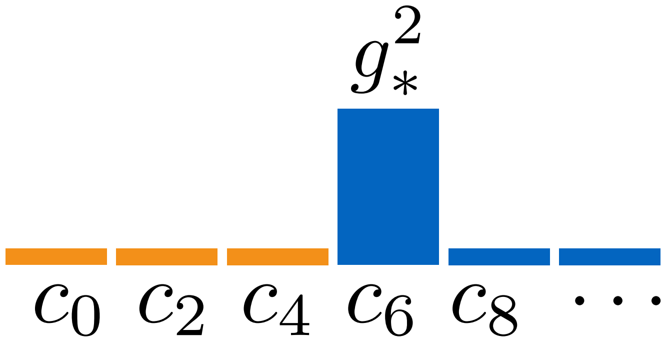
$$\mathcal{A}^{tree}_{2\to 2} \sim c_0 + c_2 \frac{E^2}{M^2} + c_4 \frac{E^4}{M^4} + c_6 \frac{E^6}{M^6} + \cdots$$

Non-linearly realised symmetries ▶ arbitrarily soft EFTs

Galileon

$$\pi \to \pi + c_\mu x^\mu + d$$

$$\mathcal{L} = \frac{g_*^2}{M^6} (\partial_\mu \pi \partial^\mu \pi) \Box (\partial_\nu \pi \partial^\nu \pi)$$



Nicolis,Rattazzi,Trincherini'08

EFT Point of view

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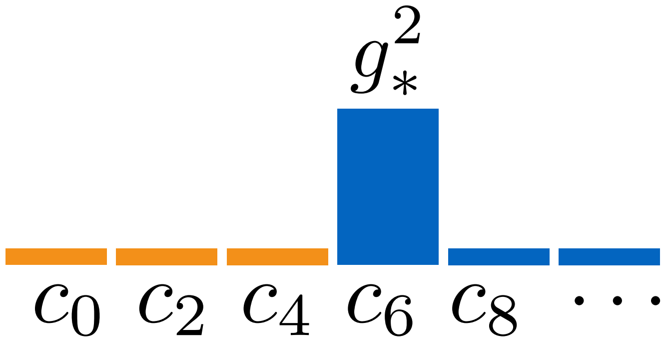
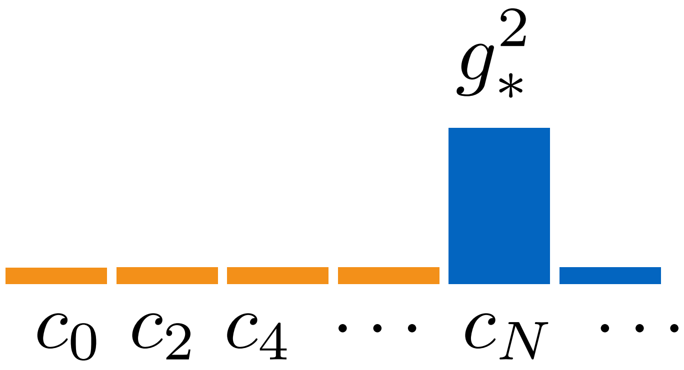


Nicolis,Rattazzi,Trincherini'08

Extended Shift symm.

$$\phi \longmapsto \phi + c^{(0)} + c_\mu^{(1)} x^\mu + c_{\mu\nu}^{(2)} x^\mu x^\nu + \cdots + c_{\mu_1 \cdots \mu_N}^{(N)} x^{\mu_1} \cdots x^{\mu_N}$$



Hinbterbichler,Joyce'14

EFT Point of view

$$\mathcal{A}^{tree}_{2\to 2} \sim c_0 + c_2\frac{E^2}{M^2} + c_4\frac{E^4}{M^4} + c_6\frac{E^6}{M^6} + \cdots$$

Non-linearly realised symmetries ▸ arbitrarily soft EFTs

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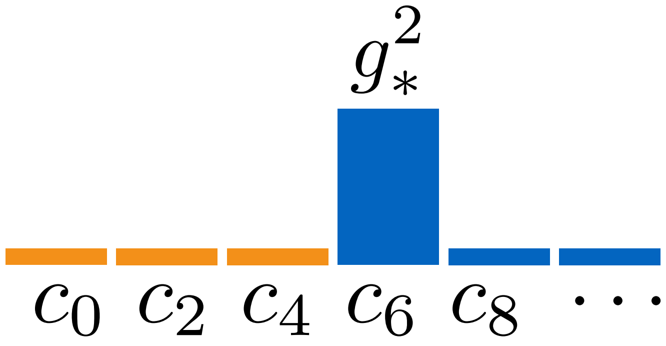
$$\pi \to \pi + c_\mu x^\mu + d$$

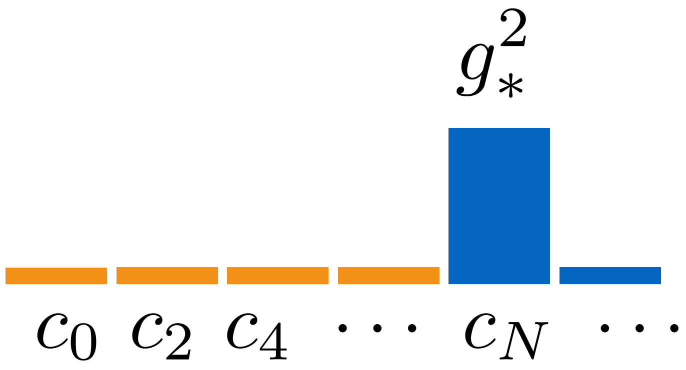
$$\mathcal{L} = \frac{g_*^2}{M^6}(\partial_\mu\pi\partial^\mu\pi)\Box(\partial_\nu\pi\partial^\nu\pi)$$



Nicolis,Rattazzi,Trincherini'08

Extended Shift symm.

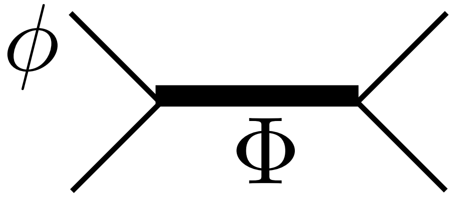
$$\phi \longmapsto \phi + c^{(0)} + c^{(1)}_{\mu}x^\mu + c^{(2)}_{\mu\nu}x^\mu x^\nu + \cdots + c^{(N)}_{\mu_1\cdots\mu_N}x^{\mu_1}\cdots x^{\mu_N}$$

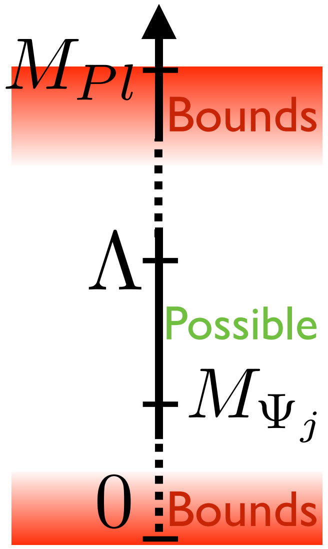


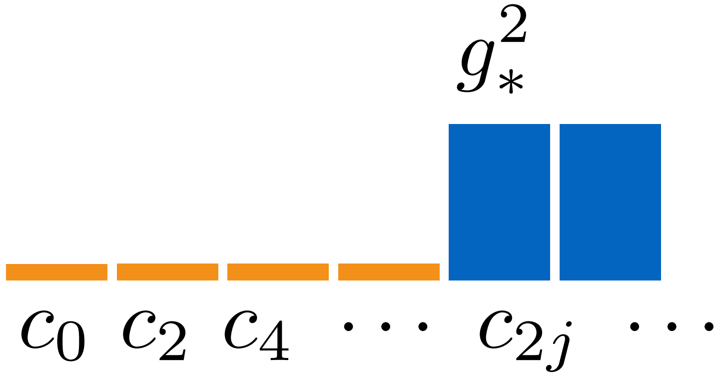
Hinbterbichler,Joyce'14

Higher-Spin below cutoff

$$\mathcal{L} = \Phi^{\mu_1\cdots\mu_j}\,\phi^*\overleftrightarrow{\partial}_{\mu_1}\cdots\overleftrightarrow{\partial}_{\mu_j}\phi$$







Porrati,Rahman'08
Bonifacio,Hinterbichler'18
Afkhami-Jetti,Kundu,Tajdini'18

Is any EFT Energy-running UV plausible?

PART I - Tree Level

UV-IR Connection

Adams,Arkani-Hamed,Dubovsky,Nicolis,Rattazzi'06

Study forward (t=0) amplitude

$$\mathcal{A}_{2\to 2}(s) = c_0 + c_2\cancel{\frac{s+t+u}{M^2}} + c4\frac{s^2}{M^4} + c_6\cancel{\frac{stu}{M^2}} + c_8\frac{s^2}{M^4} + \cdots \qquad \text{for} \qquad s \in \mathbb{C}$$

UV-IR Connection

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Broad UV-Assumptions:

UV-IR Connection

Adams,Arkani-Hamed,Dubovsky,Nicolis,Rattazzi'06

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Broad UV-Assumptions:

Analyticity, Crossing, Unitarity, Locality

UV-IR Connection

Adams,Arkani-Hamed,Dubovsky,Nicolis,Rattazzi'06

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$$\mathcal{A}_{2\to2}(s) = c_0 + c_2\cancel{\frac{s+t+u}{M^2}} + c4\frac{s^2}{M^4} + \cancel{c_6\frac{stu}{M^2}} + c_8\frac{s^2}{M^4} + \cdots \quad \text{for} \quad s \in \mathbb{C}$$

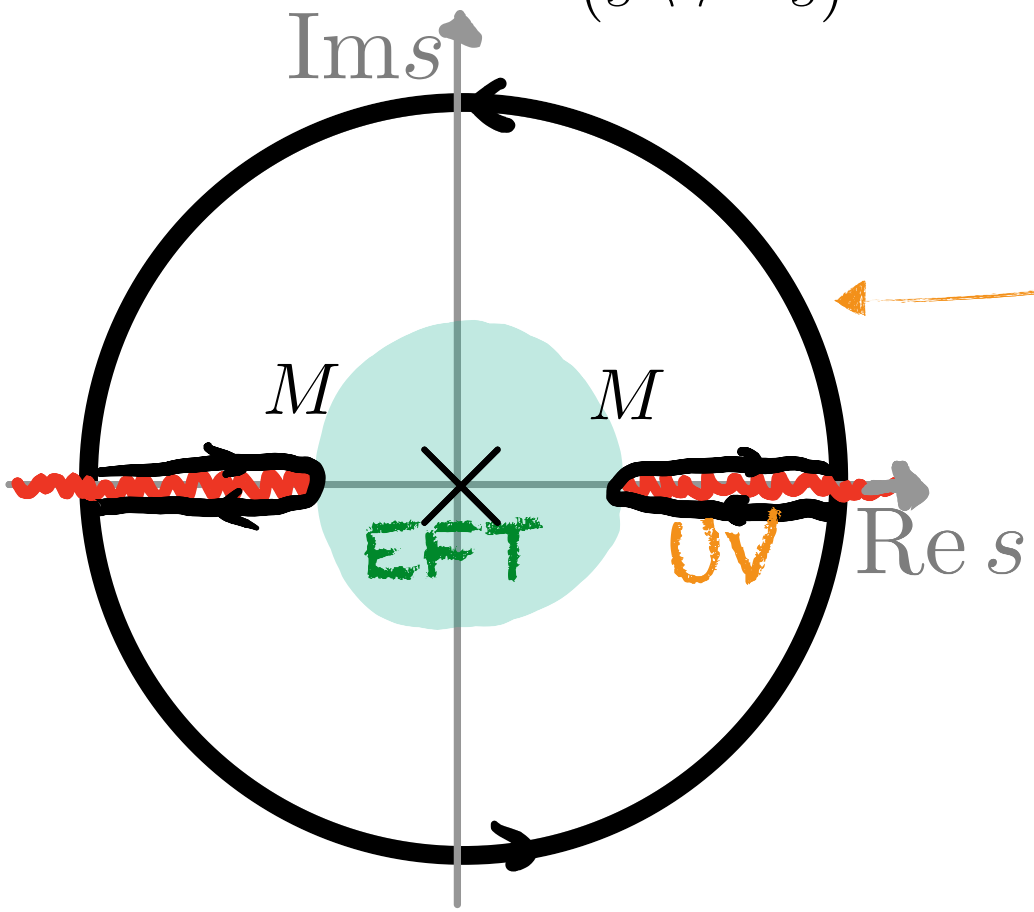
Broad UV-Assumptions:

Analyticity, (Deform Contour)

Crossing, $s \leftrightarrow u$ $\sim (s \leftrightarrow -s)$

Unitarity, $Disc \sim s\sigma_{Tot}(s) > 0$

Locality Froissart-Martin $\lesssim s\log s$



Study $\dfrac{\mathcal{A}_{2\to2}(s)}{P_n(s)}$

Drop big contour for $P_n(s) \sim s^n \gtrsim s^3$ $(s \to \infty)$

UV-IR Connection

Adams,Arkani-Hamed,Dubovsky,Nicolis,Rattazzi'06

Study forward (t=0) amplitude

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Broad UV-Assumptions:

Analyticity, (Deform Contour)

Crossing, $s \leftrightarrow u$ $\sim (s \leftrightarrow -s)$

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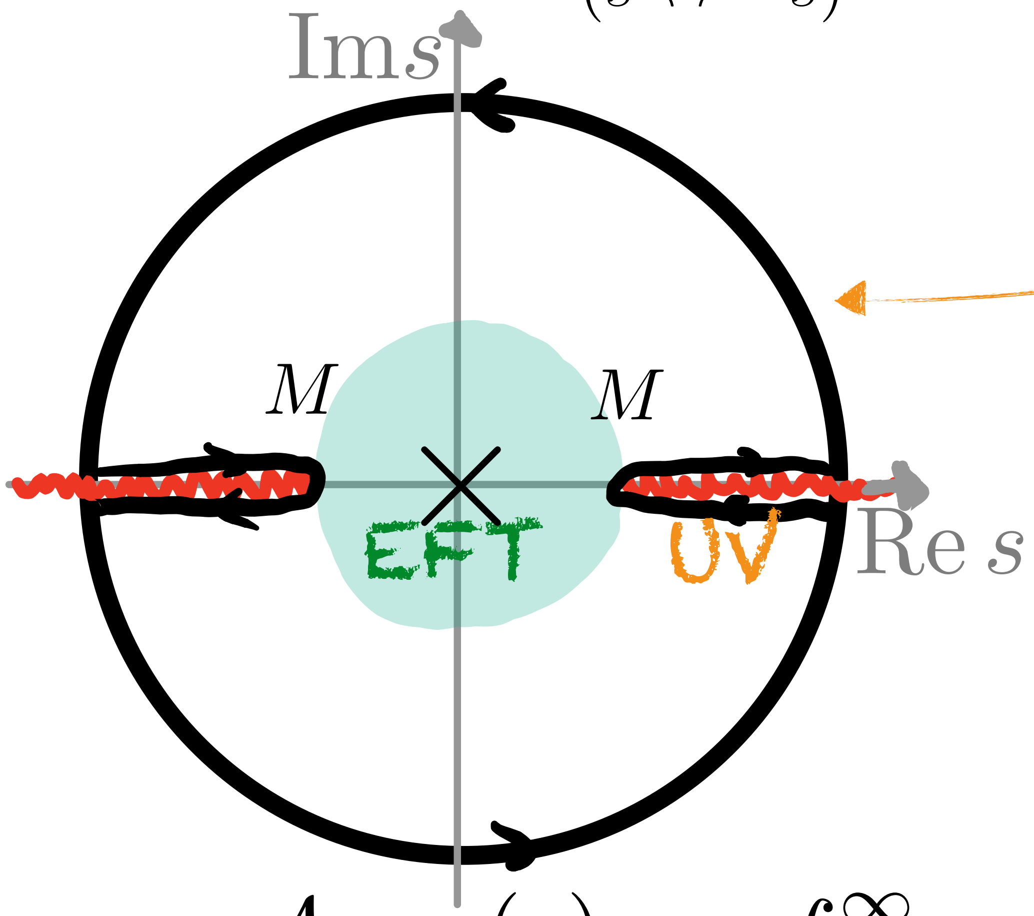


Study $\dfrac{\mathcal{A}_{2\to2}(s)}{P_n(s)}$

Drop big contour for $P_n(s) \sim s^n \gtrsim s^3$ $(s \to \infty)$

$$c_{2n} = \mathrm{Res}_{s=0}\frac{\mathcal{A}_{2\to2}(s)}{s^n} = \int_M^\infty ds\, s\frac{\sigma_{tot}(s)}{s^n} > 0$$

n>2 odd

Positivity Constraints

Adams,Arkani-Hamed,Dubovsky,Nicolis,Rattazzi'06

- No direct info on effects ≈ E^2 or const (operators d≤6)

- Strictly positive E^4 (and $E^{n>4}$) effects (operators d≥8):

  e.g Goldstino: $i\chi^{j\dagger}\overline{\sigma}^\mu\partial_\mu\chi_j + \frac{1}{F^2}\left(\chi_i^\dagger\partial_\mu\chi_j^\dagger\right)\left(\partial^\mu\chi^i\chi^j\right)$

- Any (non-linear) symmetry forbidding E^4 cannot be exact

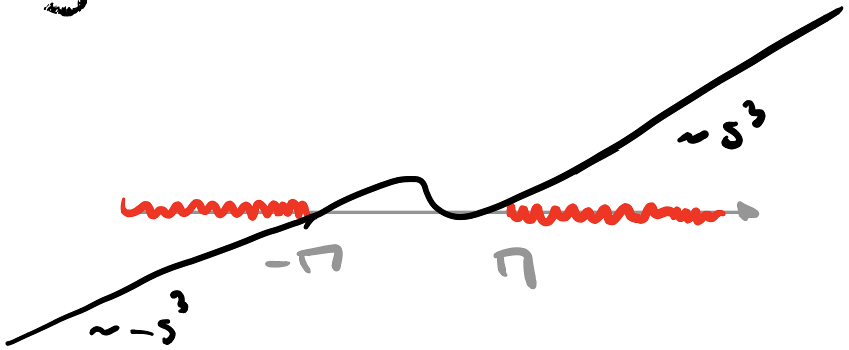
- Small relevant perturbation makes theory OK

  e.g massive Galileon:: $\mathcal{L} = -\frac{1}{2}\left(\partial_\mu\pi\right)^2\left[1 + \frac{c_3}{\Lambda^3}\Box\pi + \frac{c_4}{\Lambda^6}\left(\left(\Box\pi\right)^2 - \left(\partial_\mu\partial_\nu\pi\right)^2\right)\right] - \frac{m^2}{2}\pi^2$

  $\text{Res} = \frac{c_3^2 m^2}{2\Lambda^6} > 0$ (vanishes in exact Galileon limit)

More Positive

More structure accessible by more general $P_n(s)$

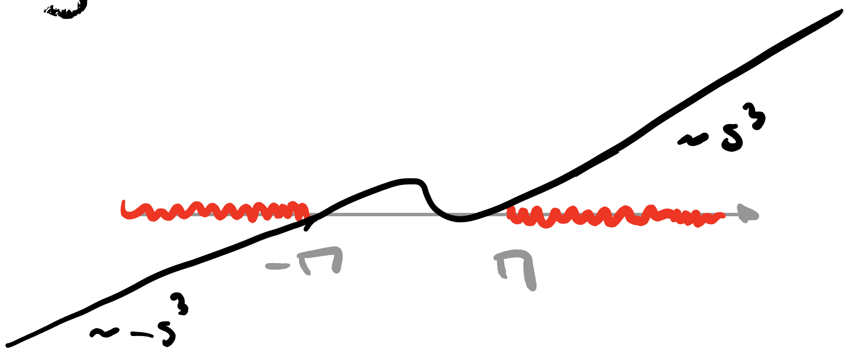


- Must be odd order n≥3
- $\frac{1}{P_n(s)} - \frac{1}{P_n(-s)} > 0 \qquad (s > M^2)$

- e.g: $P_n(s) = s(s+R^2)$

More Positive

More structure accessible by more general $P_n(s)$



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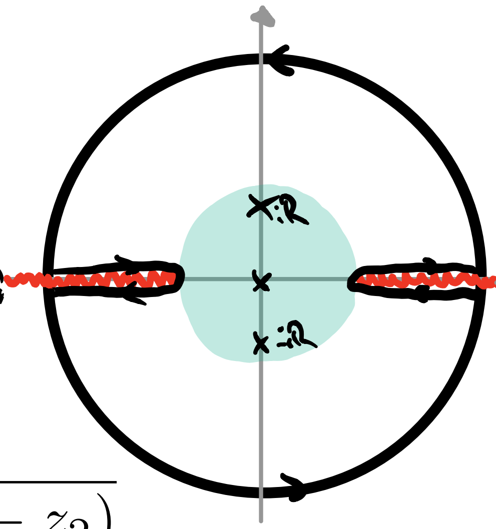
- e.g: $P_n(s) = s(s + R^2)$

Gives positivity constraints on sum of residues



$$\sum \mathrm{Res}_{z_1,z_2,z_3} \frac{A}{P_n} = \frac{A\,(z_1)}{(z_1 - z_2)\,(z_1 - z_3)} + \frac{A\,(z_2)}{(z_2 - z_1)\,(z_2 - z_3)} + \frac{A\,(z_3)}{(z_3 - z_1)\,(z_3 - z_2)}$$

$$\mathcal{A}_{2\to 2}(s) = c_0 + c_4 \frac{s^2}{M^4} + c_8 \frac{s^2}{M^4} + \cdots$$

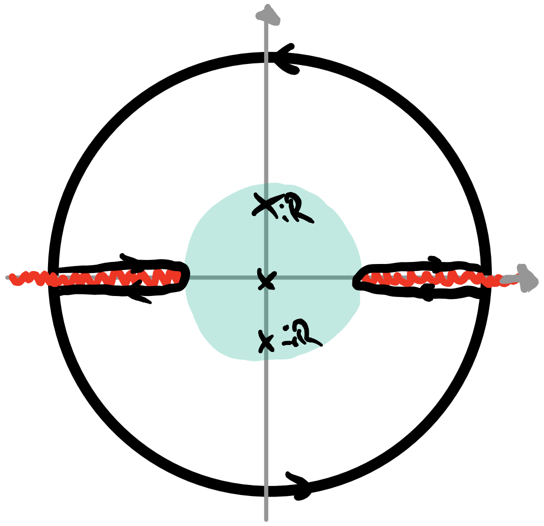
$$\boxed{c_4 > \frac{R^2}{M^4} c_8 + \cdots}$$

and similar for $c_{i>8}$

Killing Softly

$R = E^2 \lesssim M^2$

$$\boxed{c_4 > \frac{R^2}{M^4} c_8 + \cdots}$$

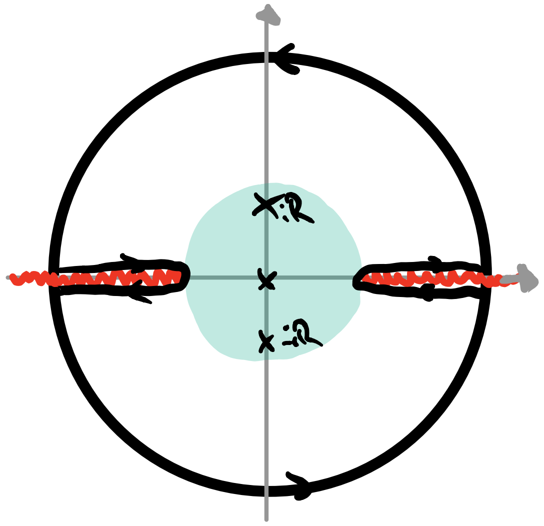


First thought: "this is just EFT validity (higher orders smaller)"

Killing Softly

$R = E^2 \lesssim M^2$

$$\boxed{c_4 > \frac{R^2}{M^4} c_8 + \cdots}$$



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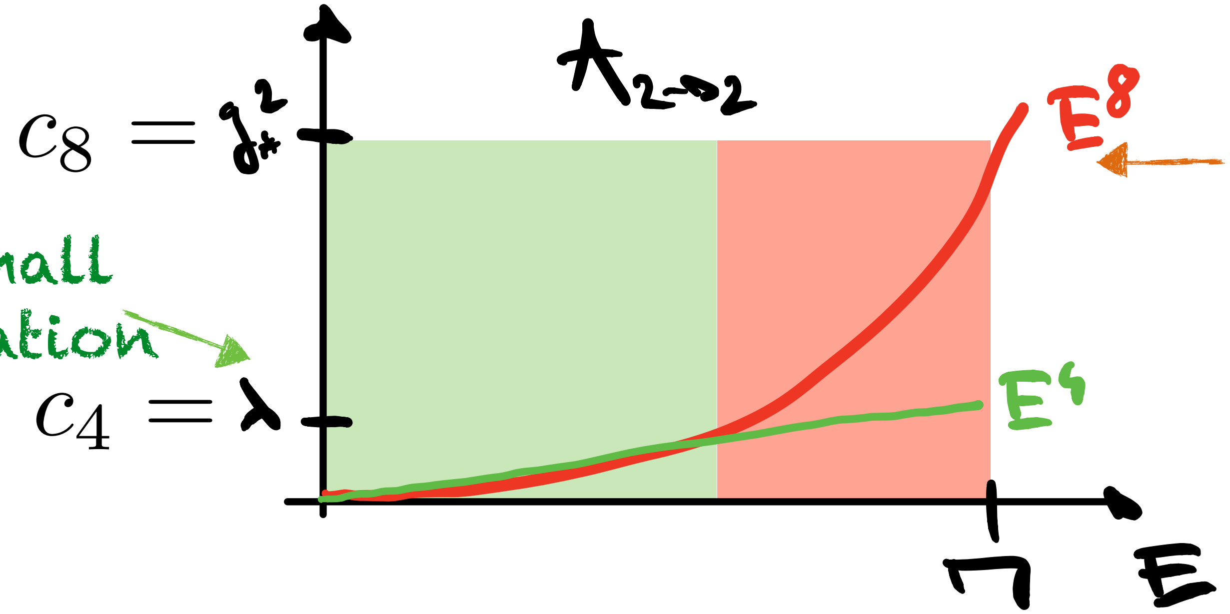

UV: the true goldstone nature emerges

IR dominated by small "relevant" E^4 perturbation

For the red regime to exist: $c_8 \frac{E^8}{M^8} > c_4 \frac{E^4}{M^4}$

Killing Softly

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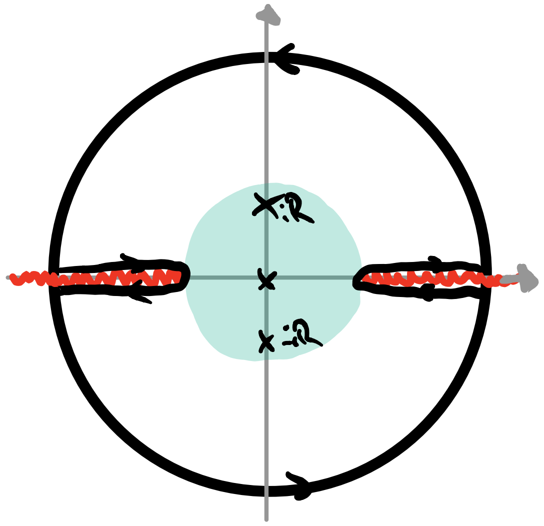

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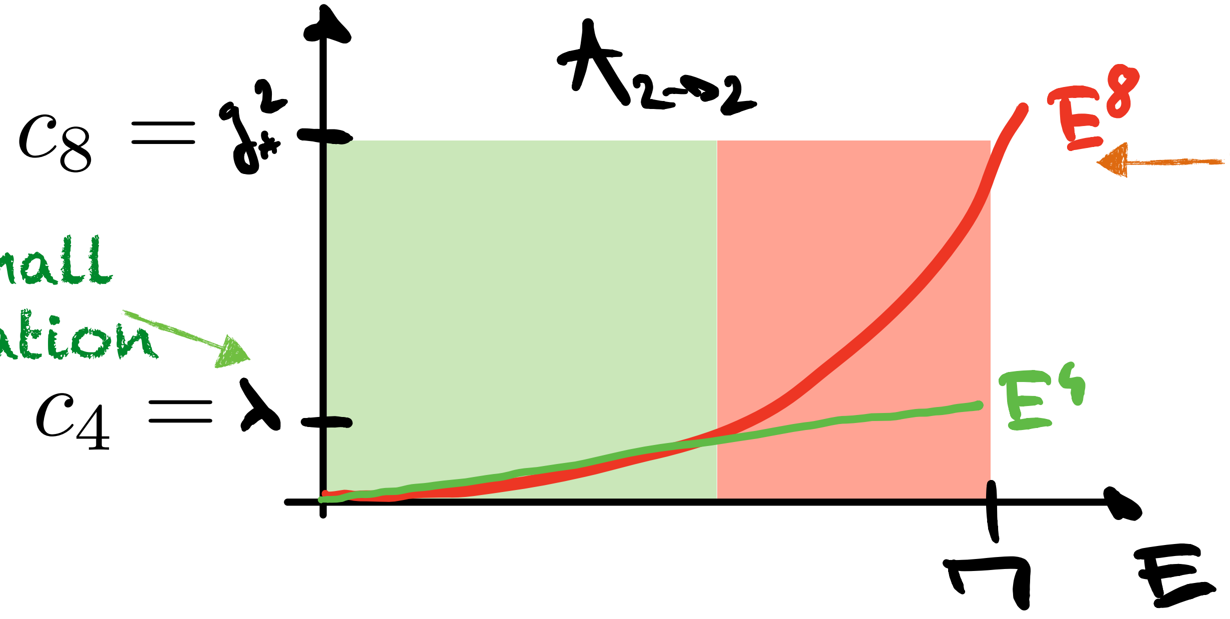

IR dominated by small "relevant" E^4 perturbation

UV: the true goldstone nature emerges

For the red regime to exist: $c_8 \frac{E^8}{M^8} > c_4 \frac{E^4}{M^4}$

This is forbidden: the "small" perturbation is always bigger than the "large" effect!

Killing Softly

Exemple: Extended Shift Symmetries

Hinbterbichler,Joyce'14

$$\phi \longmapsto \phi + c^{(0)} + c^{(1)}_{\mu} x^{\mu} + c^{(2)}_{\mu\nu} x^{\mu} x^{\nu} + \cdots + c^{(N)}_{\mu_1 \cdots \mu_N} x^{\mu_1} \cdots x^{\mu_N}$$

perturbed by small mass $m_\phi \ll M$ (cutoff)

N=3: $$\mathcal{A}^{(3)} = (\lambda_3^{(3)})^2 \frac{9m_\phi^{10}}{8} \left(3m_\phi^8 + 4m_\phi^6 s + \overset{c_4}{\boxed{47m_\phi^4 s^2}} - 24m_\phi^2 s^3 \overset{c_8}{\boxed{+ 3s^4)}}\right.$$

Positive coefficients

(compatible with positivity using $P_n(s)=s^n$)

Killing Softly

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Positive coefficients

(compatible with positivity using $P_n(s)=s^n$)

$$c_4 > \frac{E_{max}^4}{M^4} c_8 \quad ▶ \quad E_{max} < \left(\frac{47}{3}\right)^{1/4} m_\phi \approx 2m_\phi$$

$$N = 5 \quad E_{max} < \left(\frac{687}{379}\right)^{1/4} m_\phi \approx 1.2m_\phi$$

EFT regime shrinks to zero

$$N = 7 \quad E_{max} < \left(\frac{10927}{13051}\right)^{1/4} m_\phi \approx 0.95m_\phi$$

...still, terms $(stu)^n$ vanish at t=0 and elude positivity...

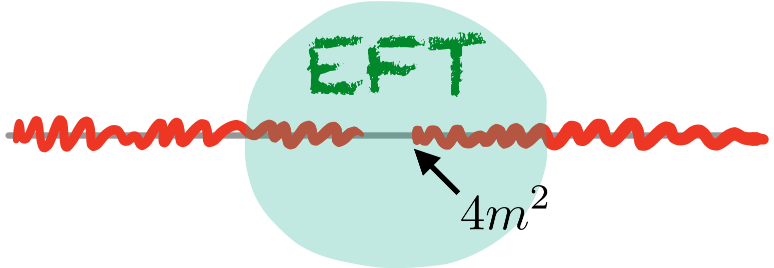
In Progress

PART II - Loop Level

Positive Arches

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR

In Progress

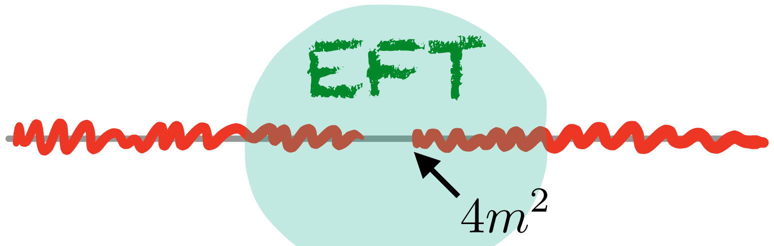


- Non-analyticity within EFT (calculable) regime
- m=0 limit: upper and lower plane disconnected
  - → IR regulator or study a different object...

Positive Arches

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR

In Progress



- Non-analyticity within EFT (calculable) regime
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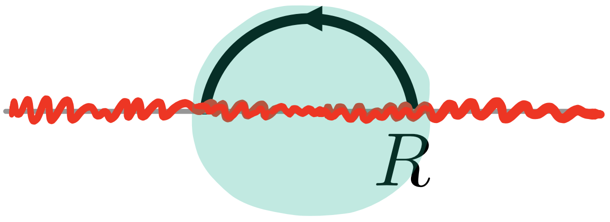
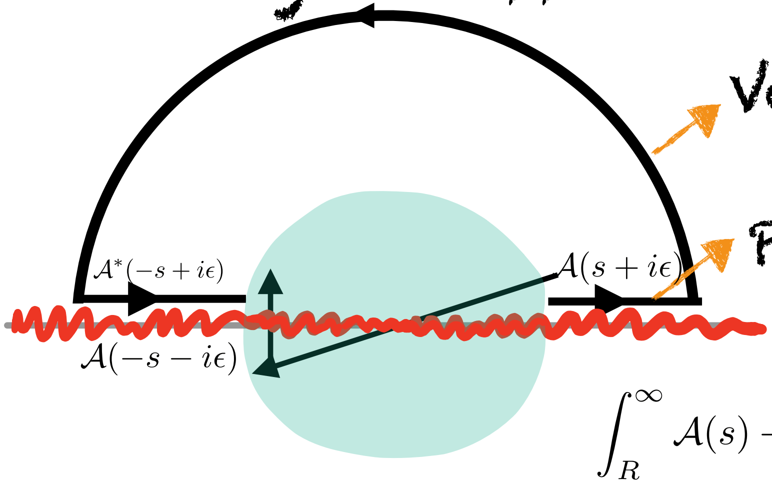
→ IR regulator or study a different object...

$$A(n,R)=\int_{arch}\frac{\mathcal{A}}{P_n}ds=$$



$$=$$



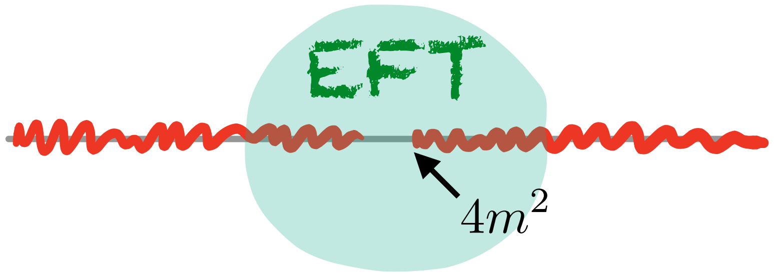
$s\sigma(s)$

$$\int_R^\infty \mathcal{A}(s)+\int_{-\infty}^{-R}\mathcal{A}^*(-s)=\int_R^\infty 2iIm\mathcal{A}(s)$$

Positive Arches

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR

In Progress



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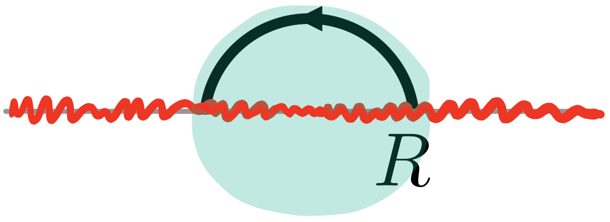
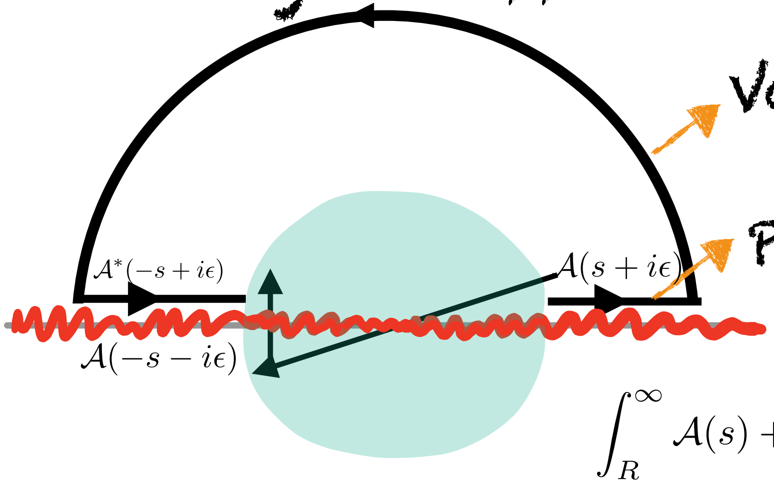


$=$



Vanishes with $P_n(s) \gtrsim s^3$

Positive

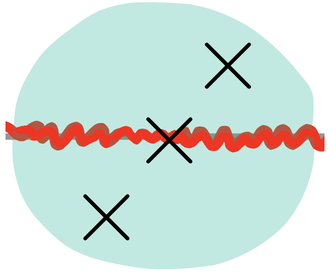
$s\sigma(s)$

$$\int_R^\infty \mathcal{A}(s) + \int_{-\infty}^{-R} \mathcal{A}^*(-s) = \int_R^\infty 2iIm\mathcal{A}(s)$$

Polynomial $P_n(s)$ must satisfy

$$\frac{1}{P_n(s)} + \frac{1}{P_n(-s)} = 0$$

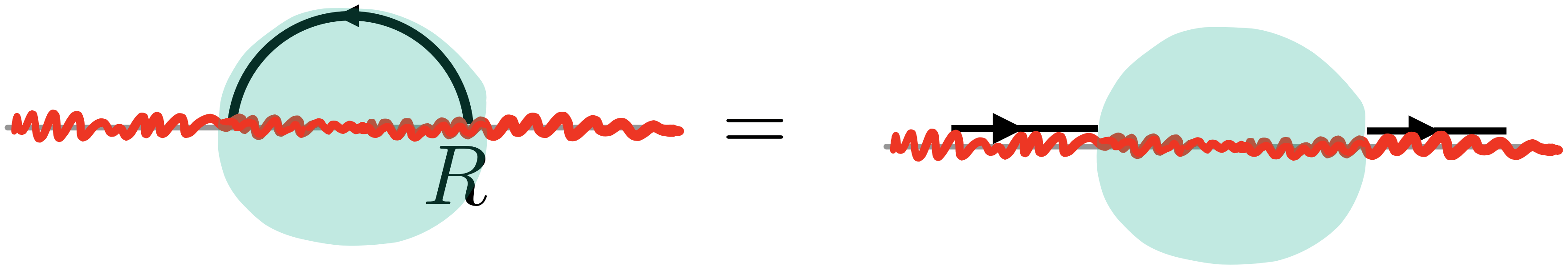
▶ $P_n(s) = s(s^2 - z^2)\cdots$



Bellazzini,Elias-Miro,Rattazzi,Riembau,FR

Properties

In Progress



1) $A(n,R) > 0$ (=integral over a positive quantity)

2) $\frac{d}{dR}A(n,R) \leq 0$ (less integral over a positive quantity)

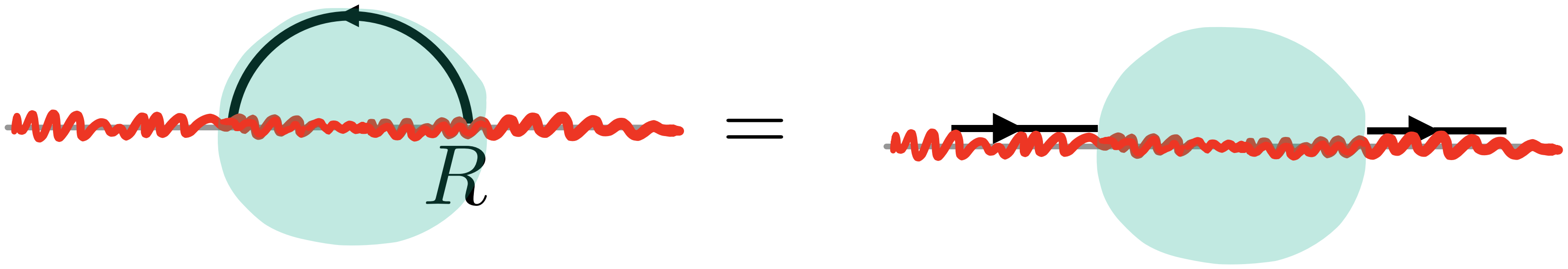
3) $A(n,R) \geq R^2 A(n+2,R)$ (larger n, smaller integrand)

$$A(n,R) = \frac{1}{R^n}\int_R^\infty \mathcal{A}\frac{R^n}{s^n}$$

Properties

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR

In Progress



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Test these on loop amplitude ansatz for U(1) Goldstone

$$\mathcal{A}(s) = c_4 s^2 + s^4\left(c_8 + \beta_8 \overset{\overset{\log(s)+\log(-s)}{\downarrow}}{L_s}\right) + ic_{10}s^5 + s^6\left(c_{12} + \beta_{12}^1 L_s + \beta_{12}^{2,1}L_s^2 + \beta_{12}^{2,2}\overset{\overset{\log(s)\log(-s)}{\downarrow}}{L_s'}\right) + \cdot\cdot$$

Loop Positivity for U(1) Goldstone

In Progress

Ansatz:

$$\mathcal{A}(s) = c_4 s^2 + s^4\left(c_8 + \beta_8 L_s\right) + i c_{10} s^5 + s^6\left(c_{12} + \beta^1_{12} L_s + \beta^{2,1}_{12} L_s^2 + \beta^{2,2}_{12} L_s'\right) + \cdots$$

Arches:

$$A(3,R) = c_4 + \boxed{R^2\beta_8} + \frac{1}{8}R^8(4\beta^1_{12} - 4\beta^{2,1}_{12} - \beta^{2,2}_{12} + 4(4\beta^{2,1}_{12} + \beta^{2,2}_{12})\log R$$

$$A(5,R) = c_8 + \boxed{2\beta_8 \log R} + R^2\left(\beta^1_{12} - 2\beta^{2,1}_{12} - \frac{\beta^{2,2}_{12}}{2} + (4\beta^{2,1}_{12} + \beta^{2,2}_{12})\log R\right)$$

- $A(n,R) > 0$: coefficients c_i no longer positive (c_4 yes, $\boxed{R=E^2 \to 0}$)

2-loop

Loop Positivity for U(1) Goldstone

In Progress

Ansatz:

$$\mathcal{A}(s) = c_4 s^2 + s^4 \left(c_8 + \beta_8 L_s\right) + i c_{10} s^5 + s^6 \left(c_{12} + \beta_{12}^{1} L_s + \beta_{12}^{2,1} L_s^2 + \beta_{12}^{2,2} L_s'\right) + \cdots$$

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- $\frac{d}{dR} A(n,R) \leq 0$

2-loop

$$\beta_8 + E^4 \beta_{12}^{1} + E^4 (4\beta_{12}^{2,1} + \beta_{12}^{2,2}) \log E^2 < 0 \qquad \forall E < M$$

Loop Positivity for U(1) Goldstone

In Progress

Ansatz:

$$\mathcal{A}(s) = c_4 s^2 + s^4\left(c_8 + \beta_8 L_s\right) + i c_{10} s^5 + s^6\left(c_{12} + \beta_{12}^1 L_s + \beta_{12}^{2,1} L_s^2 + \beta_{12}^{2,2} L_s'\right) + \cdots$$

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$$\overset{\text{1-loop}}{\boxed{\beta_8}} + \boxed{E^4\beta_{12}^1} + E^4(4\beta_{12}^{2,1} + \overset{\text{2-loop}}{\beta_{12}^{2,2}})\log E^2 < 0 \qquad \forall E < M$$

$\beta_8 \leq 0$ Leading running coefficient c_8 increases towards IR

...becomes more positive w.r.t. tree-level approx.

Loop Positivity for U(1) Goldstone

In Progress

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$$\overset{\text{1-loop}}{\boxed{\beta_8}} + \boxed{E^4 \beta_{12}^{1}} + \overset{\text{2-loop}}{E^4 (4\beta_{12}^{2,1} + \beta_{12}^{2,2}) \log E^2} < 0 \qquad \forall E < M$$

$\beta_8 \leq 0$ Leading running coefficient c_8 increases towards IR

...becomes more positive w.r.t. tree-level approx.

1-loop: next running coefficient c_{12} can decrease towards IR

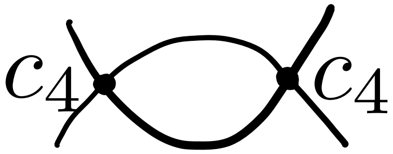
$$\beta_{12}^{1} \leq \frac{|\beta_8|}{E^4}$$

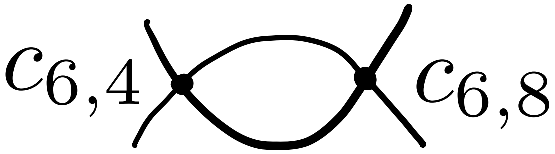
U(1) Goldstone @ 1-loop

$$\boxed{\beta_8} + E^4 \boxed{\beta^1_{12}} < 0$$

Explicit calculation:

$$\beta_8 = -\frac{7c_4^2}{5\pi^2} \checkmark \qquad \beta_{12} = -\frac{23c_6^2}{5\pi^2} - \frac{83c_4c_8}{70\pi^2}$$





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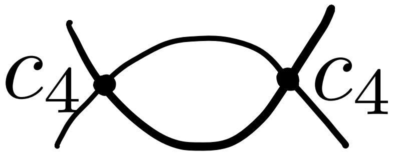
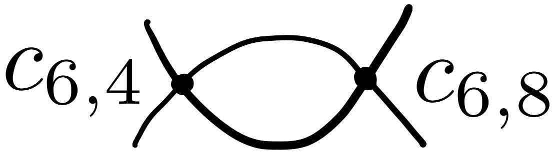
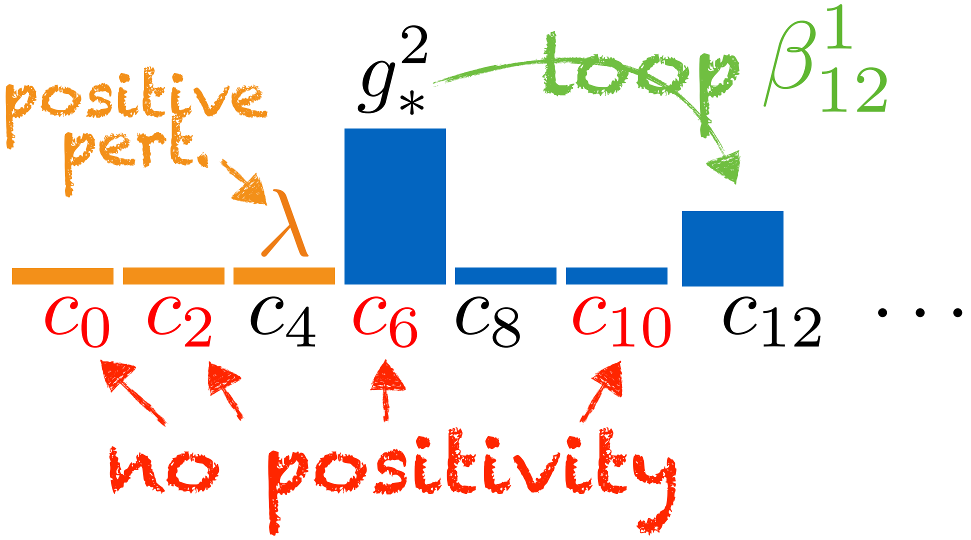




- Contains c6 (unbounded @ tree)... can access Galileon:

$$\mathcal{L} = \frac{g_*^2}{M^6}(\partial_\mu\pi\partial^\mu\pi)\Box(\partial_\nu\pi\partial^\nu\pi) \longrightarrow c_2, \underset{\displaystyle \lambda}{\underset{\downarrow}{\boxed{c_4}}}, c_8 \ll \underset{\displaystyle g_*^2}{\underset{\downarrow}{\boxed{c_6}}}$$



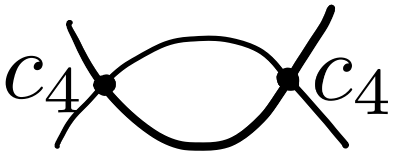
$$\lambda > \frac{3}{640}\frac{g_*^2}{16\pi^2}\left(\frac{E}{M}\right)^8$$

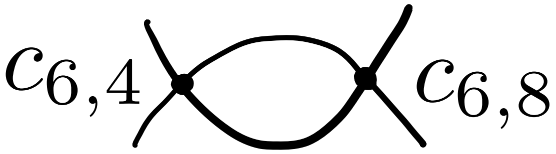
U(1) Goldstone @ 1-loop

$$\boxed{\beta_8} + E^4 \boxed{\beta^1_{12}} < 0$$

Explicit calculation:

$$\beta_8 = -\frac{7c_4^2}{5\pi^2} \checkmark \qquad \beta_{12} = -\frac{23c_6^2}{5\pi^2} - \frac{83c_4c_8}{70\pi^2}$$





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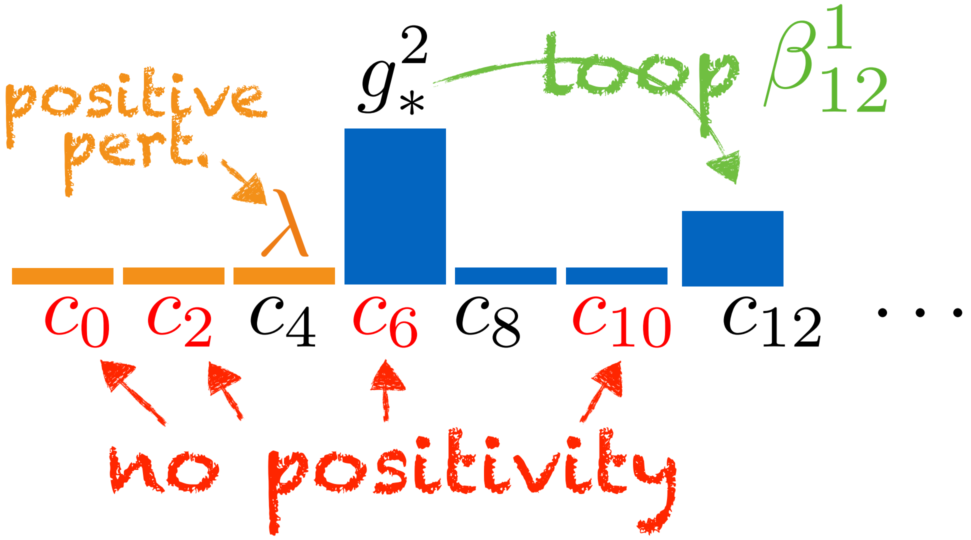


$$\lambda > \frac{3}{640}\frac{g_*^2}{16\pi^2}\left(\frac{E}{M}\right)^8$$

Perturbation at most one-loop factor below stronger effect

Nicolis,Rattazzi,Trincherini'09; Bellazzini,Serra,Sgarlata,FR'17

Similar arguments to access c₂ Distler,Grinstein,Porto,Rothstein'06

See also bounds for $t \neq 0$ deRham,Melville,Tolley,Zhoiu'17

Bellazzini,Serra,Sgarlata,FR'19

Higher Spin

$\Phi^{\mu_1 \cdots \mu_J}$

Study (irrelevant) self interactions in $\mathcal{A}_{\Phi\Phi\to\Phi\Phi}$

longitudinal $\lambda_L \Phi^4$

$(J=3:\ \lambda_1,\lambda_2,\lambda_3)$

$\frac{\mathcal{R}^4}{f_T^{4J}}$ transverse

Bellazzini,Serra,Sgarlata,FR'19

Higher Spin

$$\Phi^{\mu_1\cdots\mu_J}$$

Study (irrelevant) self interactions in $\mathcal{A}_{\Phi\Phi\to\Phi\Phi}$

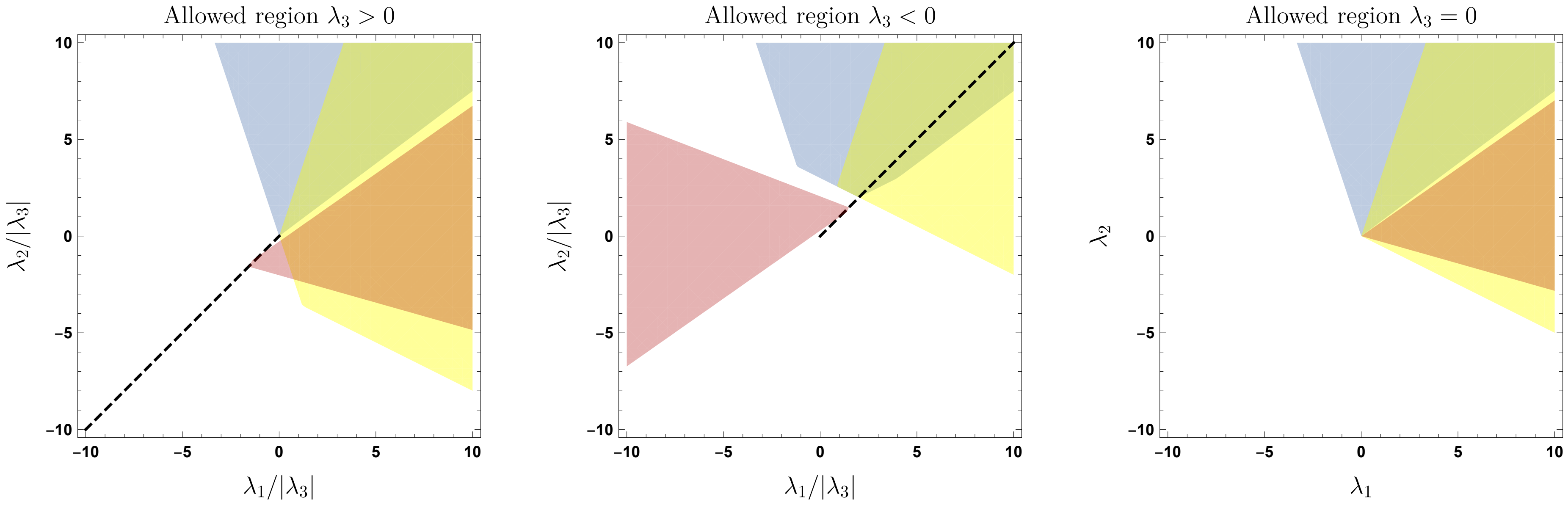
longitudinal $\lambda_L \Phi^4$

$(J=3: \ \lambda_1, \lambda_2, \lambda_3)$

$\dfrac{\mathcal{R}^4}{f_T^{4J}}$ transverse

perturbation: $+m^2\Phi^2$

↓ J=3



Most relevant interactions, perturbed by mass, not positive, excluded

(many helicities, one perturbation = many constraints)
richer perturbation would be ok

Higher Spin

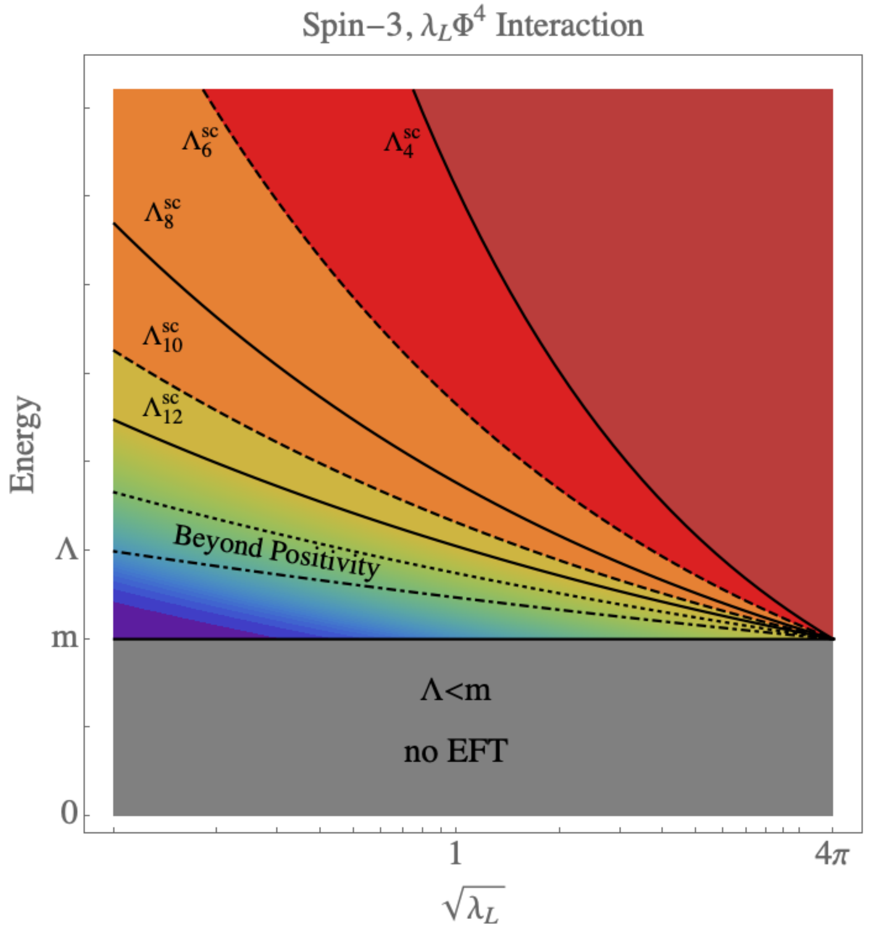
Bellazzini,Serra,Sgarlata,FR'19

Loop arguments stronger:

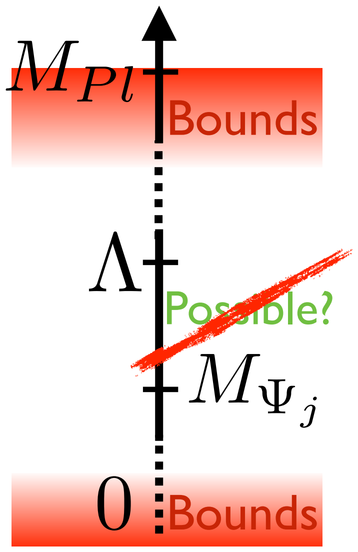
longitudinal $\lambda_L \Phi^4$

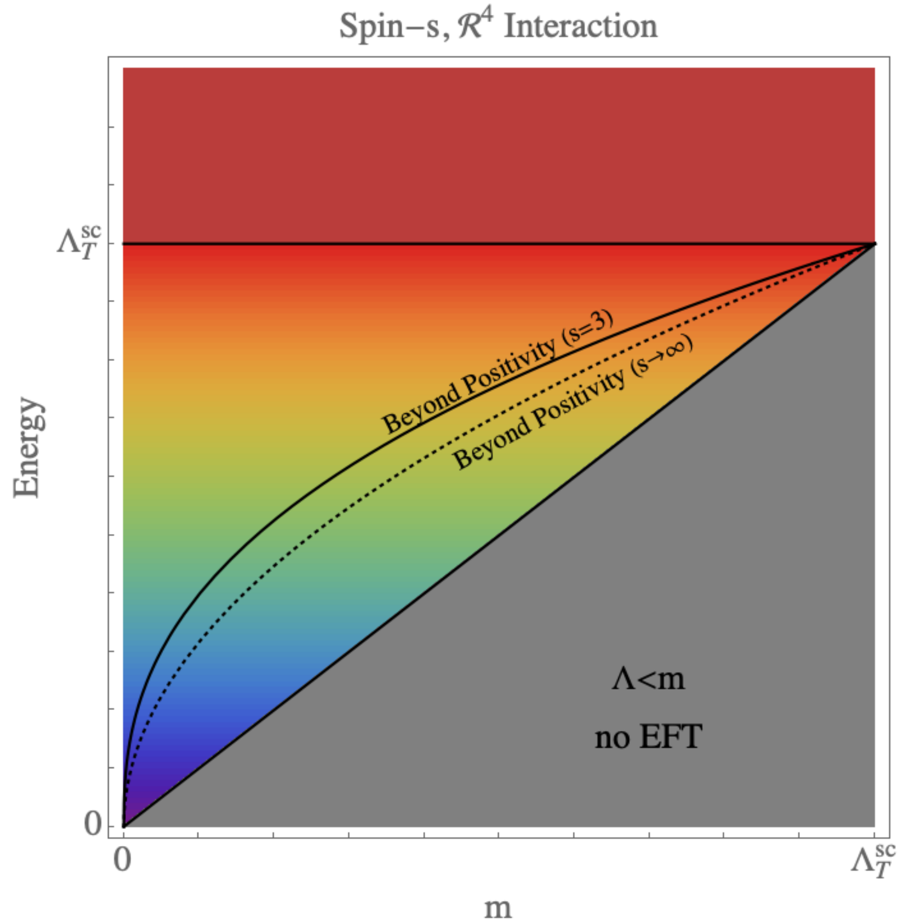
$(J=3:\ \lambda_1,\lambda_2,\lambda_3)$

$\frac{\mathcal{R}^4}{f_T^{4J}}$ transverse



$$\Lambda \lesssim m_\Phi \left(\frac{16\pi^2}{\lambda_L}\right)^{\frac{1}{8J-4}}$$





$$\Lambda \lesssim \Lambda_T^{sc}\left(\frac{m}{\Lambda_T^{sc}}\right)^{8s-4}$$

strong coupling scale

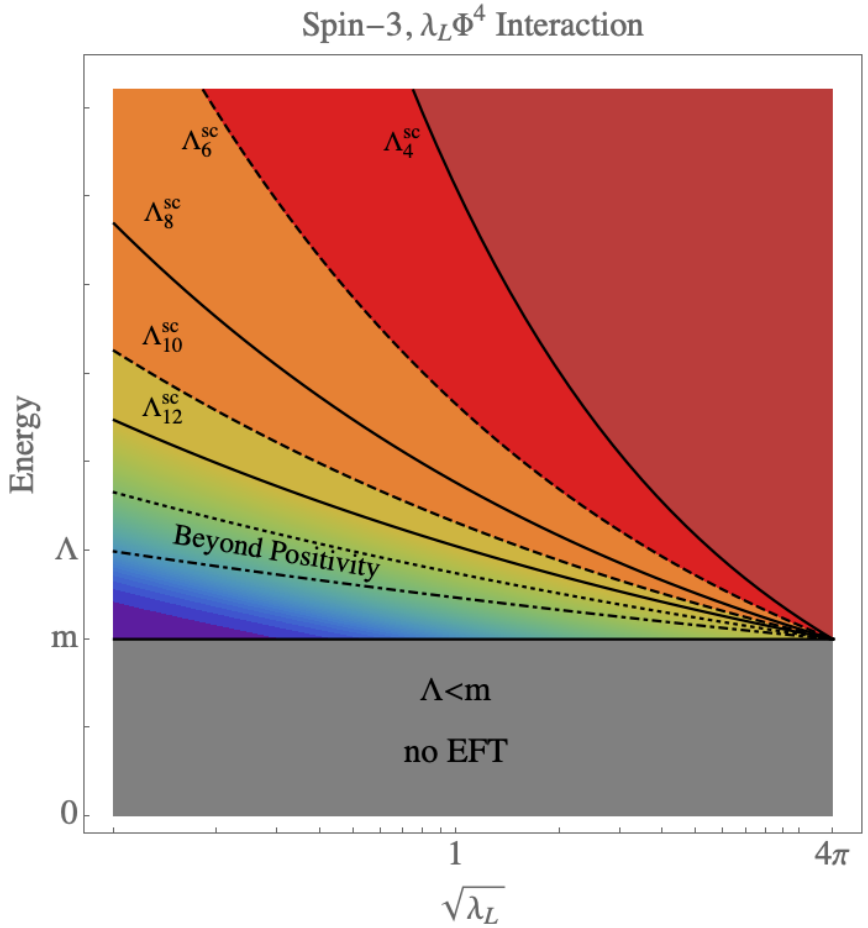
Higher Spin

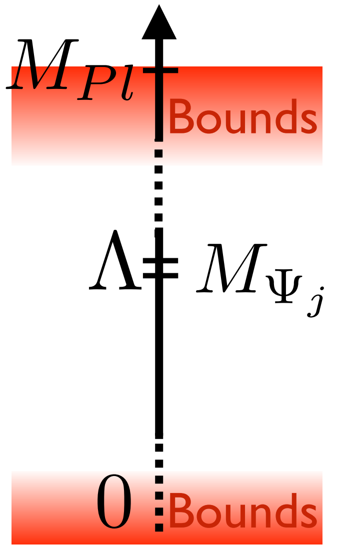
Bellazzini,Serra,Sgarlata,FR'19

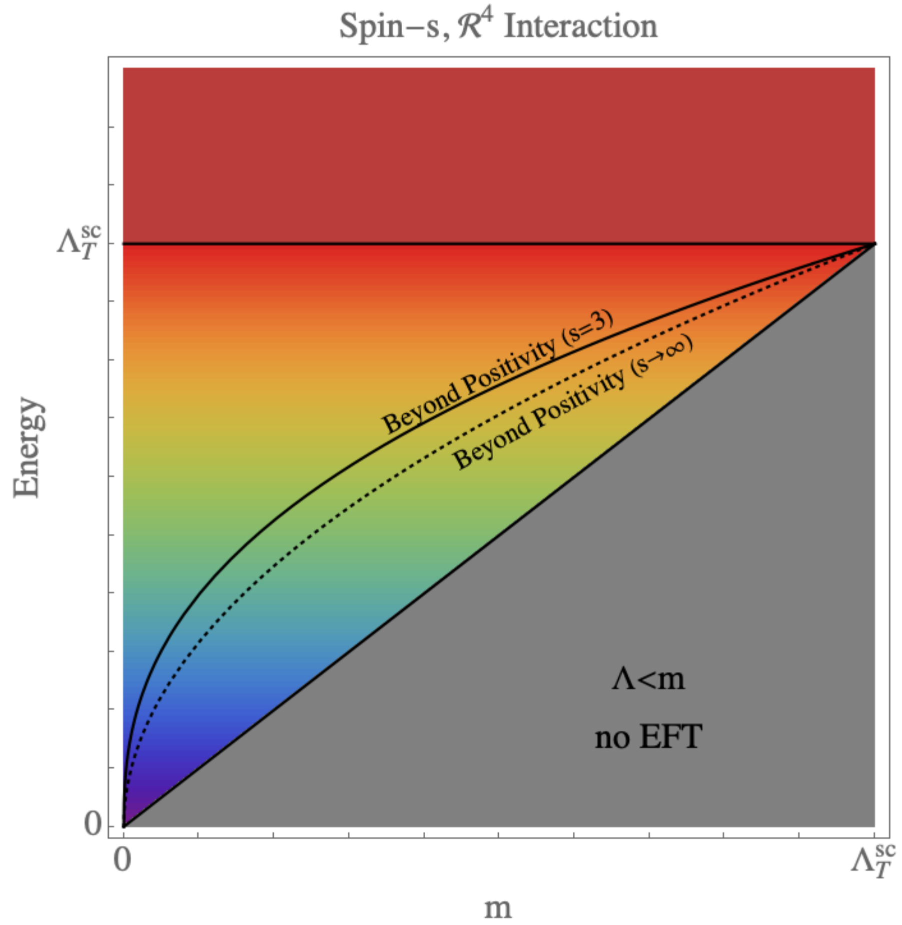
Loop arguments stronger:

longitudinal $\lambda_L \Phi^4$
$(J=3:\ \lambda_1, \lambda_2, \lambda_3)$

$\frac{\mathcal{R}^4}{f_T^{4J}}$ **transverse**


Spin−3, $\lambda_L \Phi^4$ Interaction




Spin−s, $\mathcal{R}^4$ Interaction

$$\Lambda \lesssim m_\Phi \left(\frac{16\pi^2}{\lambda_L}\right)^{\frac{1}{8J-4}}$$

$$\Lambda \lesssim \Lambda_T^{sc} \left(\frac{m}{\Lambda_T^{sc}}\right)^{8s-4}$$

strong coupling scale

HS mass at cutoff (J>>1)...just like in QCD, atoms, strings...

Conclusion/Outlook

Supersoft EFTs $\approx E^{4n}$ have no regime of usage

Supersoft EFTs $\approx E^{4n+2}$ have small regime of usage

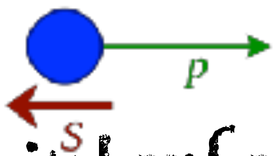
IR running at all orders?

Non-Interference

Azatov,Contino,Machado,FR'16

(2->2,high-E,tree-level)

For $E \gg m_W$ states have well defined helicity



Amplitudes for 2→2 with different total h dont interfere

SM

Any BSM dim-6 operator

helicity

A_4	$\|h(A_4^{\rm SM})\|$	$\|h(A_4^{\rm BSM})\|$
$VVVV$	0	4,2
$VV\phi\phi$	0	2
$VV\psi\psi$	0	2
$V\psi\psi\phi$	0	2
$\psi\psi\psi\psi$	2,0	2,0
$\psi\psi\phi\phi$	0	0
$\phi\phi\phi\phi$	0	0

Non-Interference

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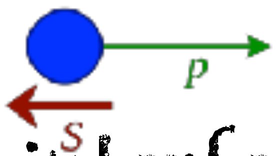

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$\psi\psi\phi\phi$	0	0
$\phi\phi\phi\phi$	0	0

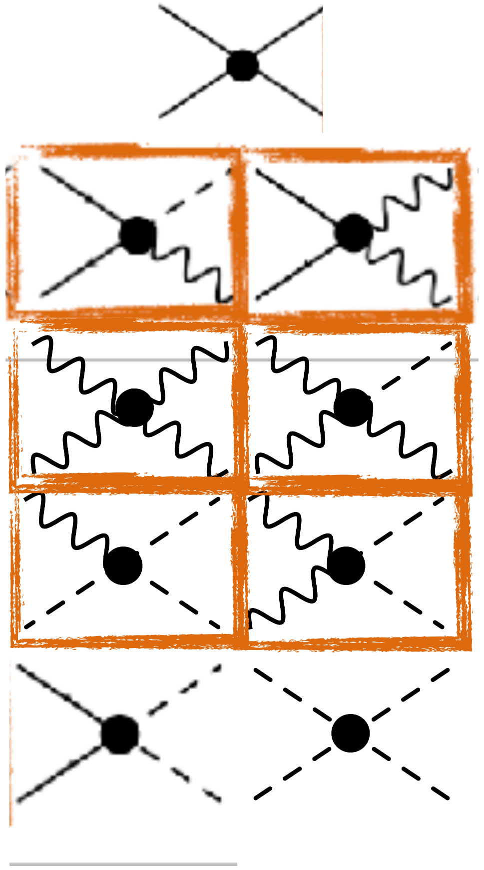
Different helicity

⇩

No-Interference

⇩

Poor Measurement



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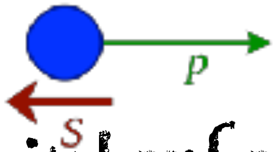

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$\psi\psi\phi\phi$	0	0
$\phi\phi\phi\phi$	0	0

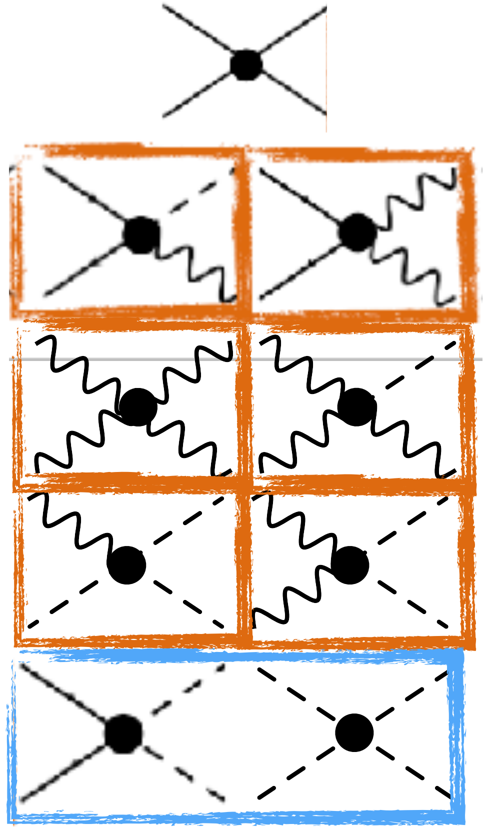
Different helicity

⇩

No-Interference

⇩

Poor Measurement



Small in inclusive xsec

Non-Interference

Azatov,Contino,Machado,FR'16

(2->2,high-E,tree-level)

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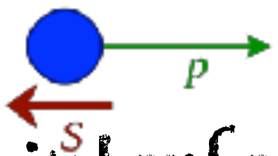

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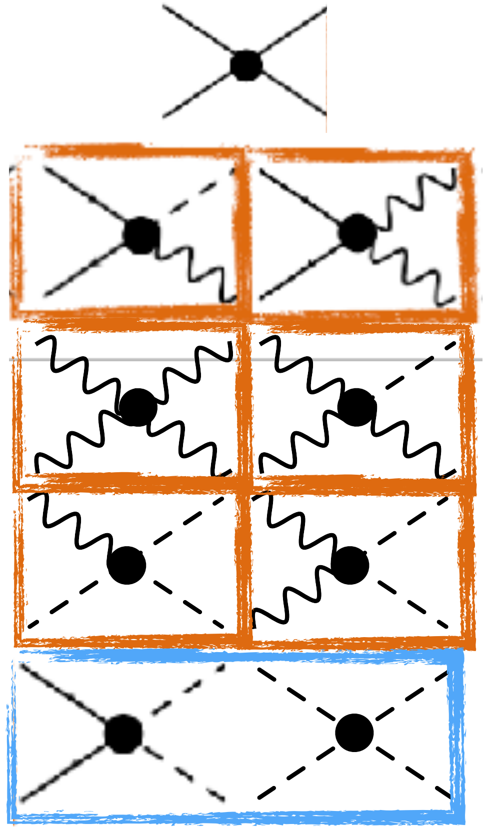
Different helicity

⇩

No-Interference

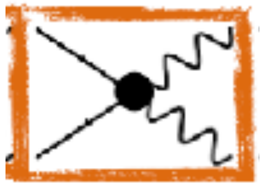
⇩

Poor Measurement

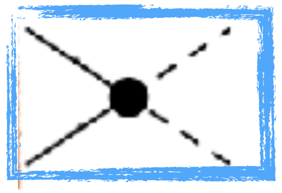


Small in inclusive xsec

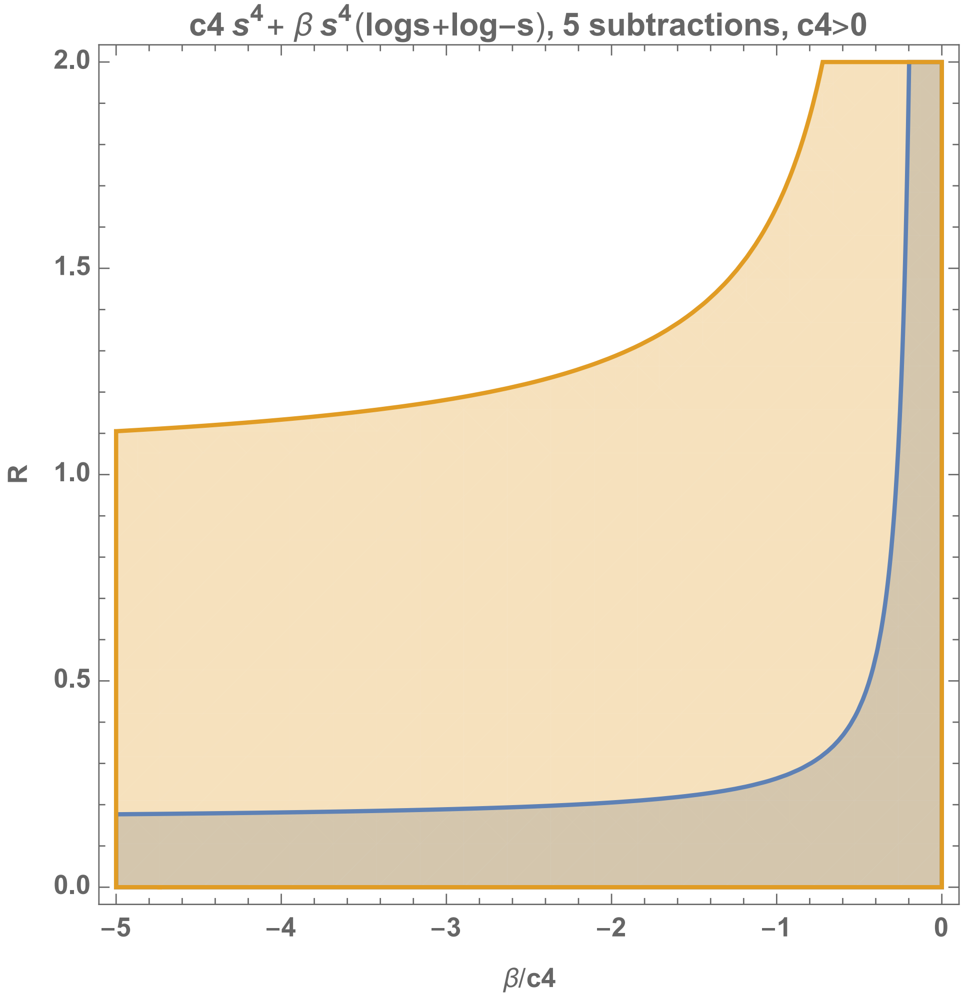
I will discuss:

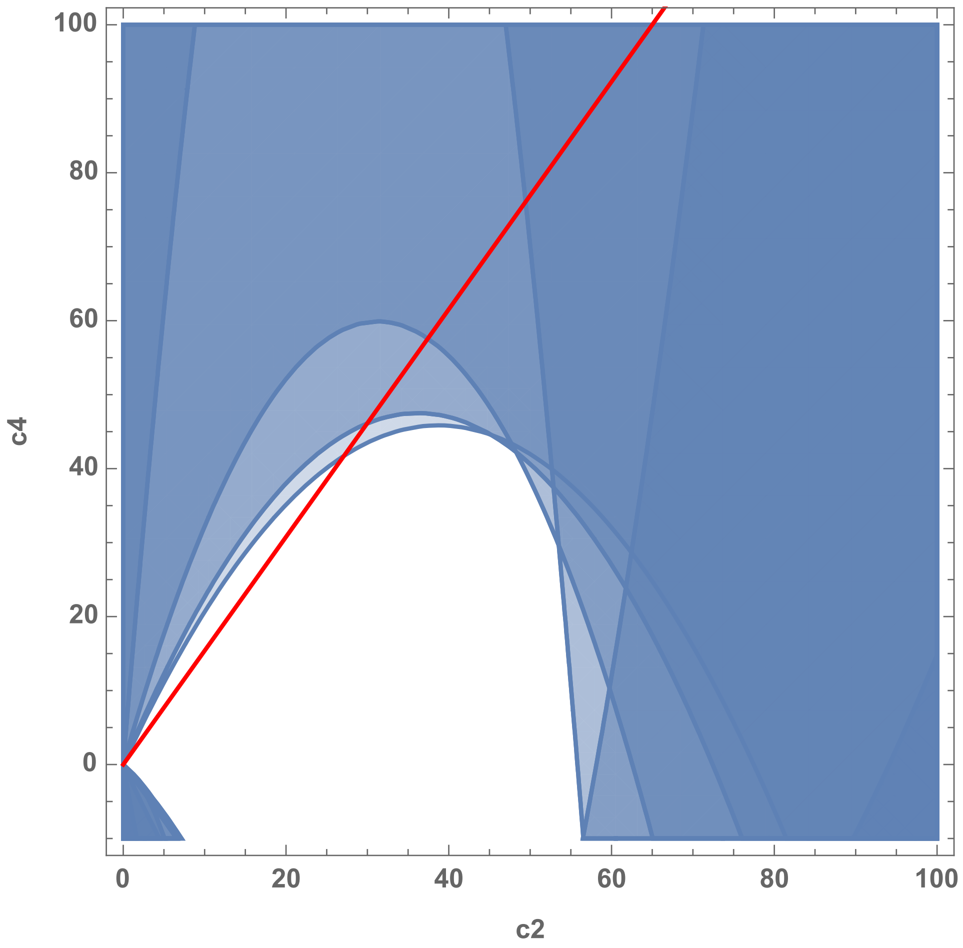


Resurrect Interference



Revitalize Interference


c4 s^4+ β s^4(logs+log−s), 5 subtractions, c4>0
R
2.0
1.5
1.0
0.5
0.0
−5
−4
−3
−2
−1
0
β/c4


c4
100
80
60
40
20
0
0
20
40
60
80
100
c2