Bootstrapping Inflationary Correlators

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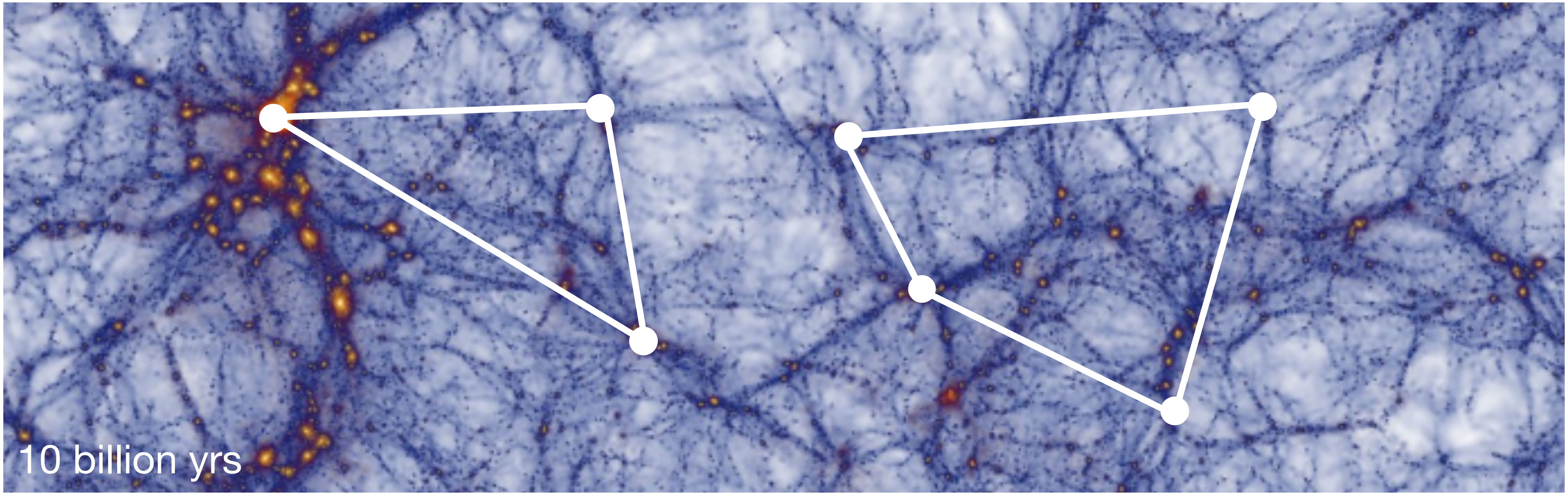
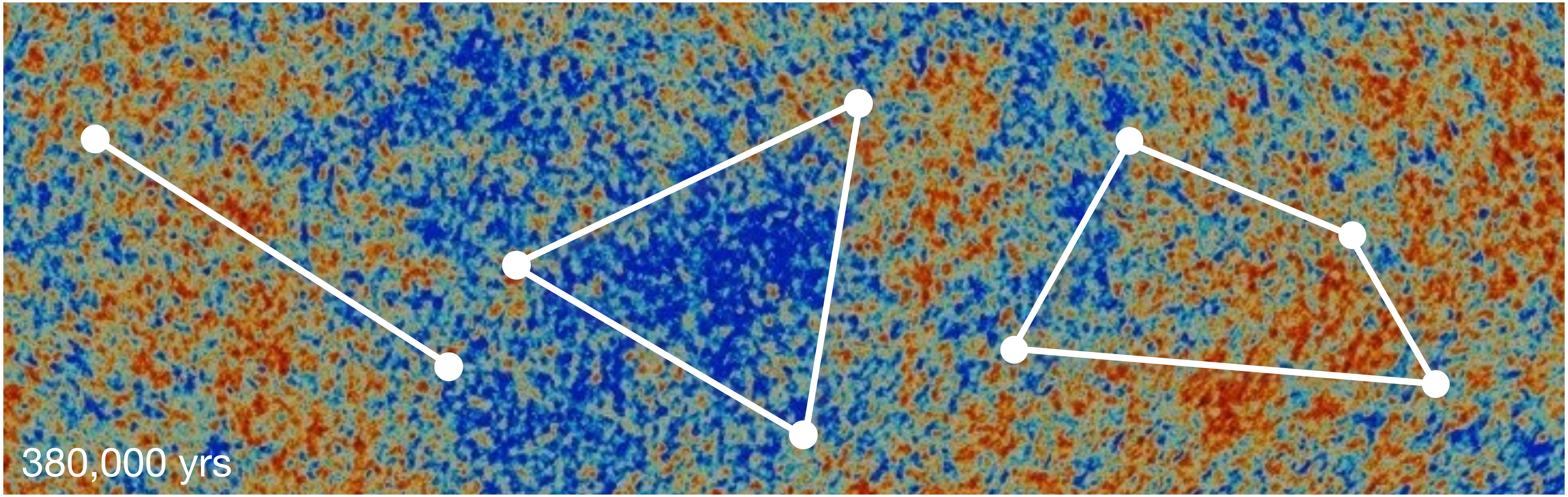
Based on work with

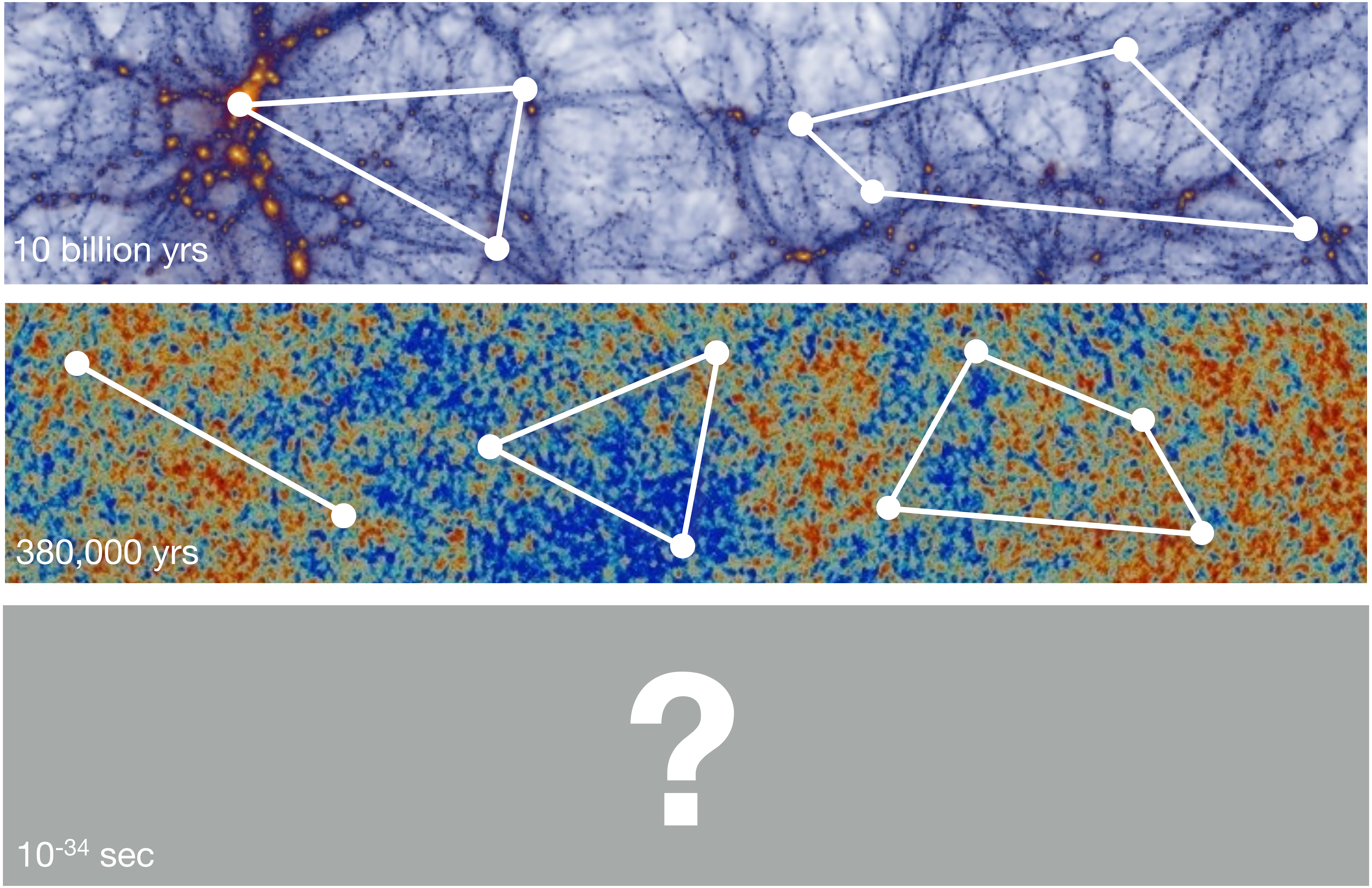
Nima Arkani-Hamed, Hayden Lee and Guilherme Pimentel

The physics of the early universe is encoded in the **spatial correlations** between cosmological structures at late times:



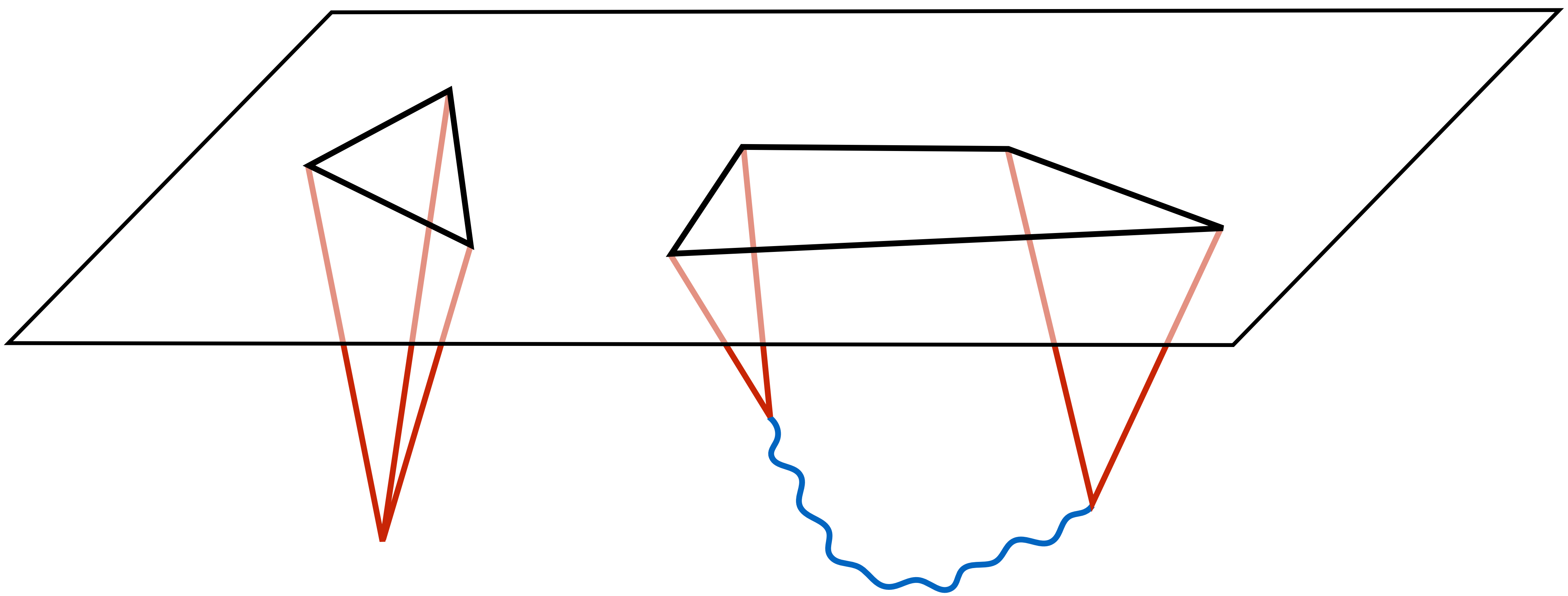


A central challenge of modern cosmology is to construct a **consistent history** of the universe that explains these correlations:



Time Without Time

All cosmological correlations can be traced back to the spacelike boundary of the inflationary quasi-de Sitter spacetime:

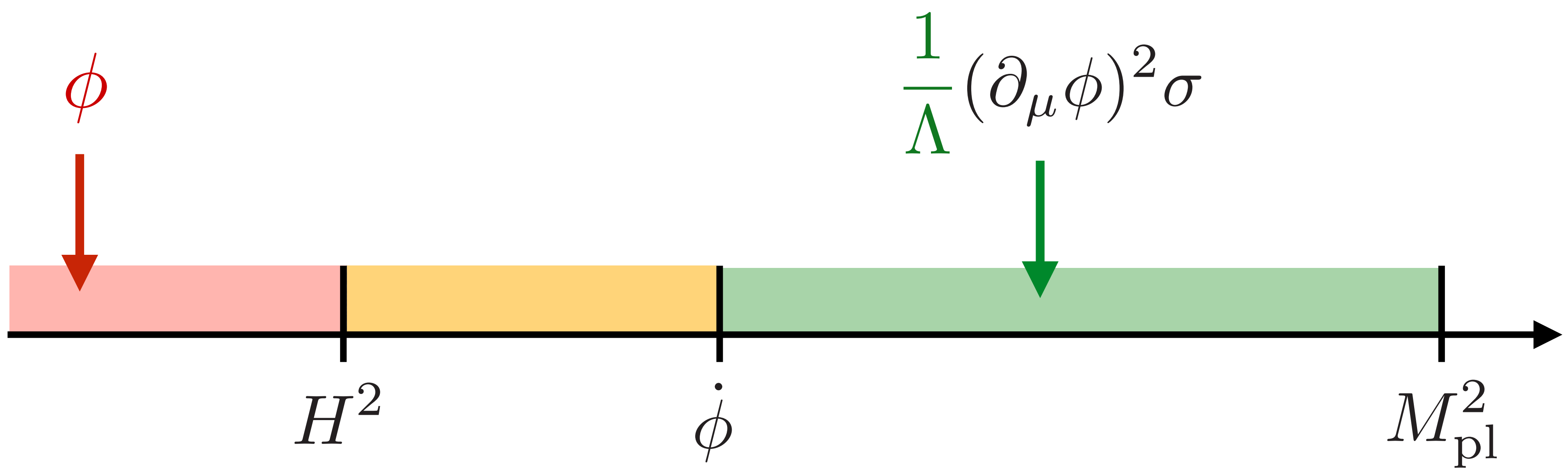


The time dependence of bulk interactions is encoded in the momentum dependence of these boundary correlators.

In this talk, I will describe a new approach to determine these correlations from consistency conditions alone = **bootstrap**.

A Lamppost

We will work under the lamppost of **single-field slow-roll inflation** with **weak couplings** to massive particles:



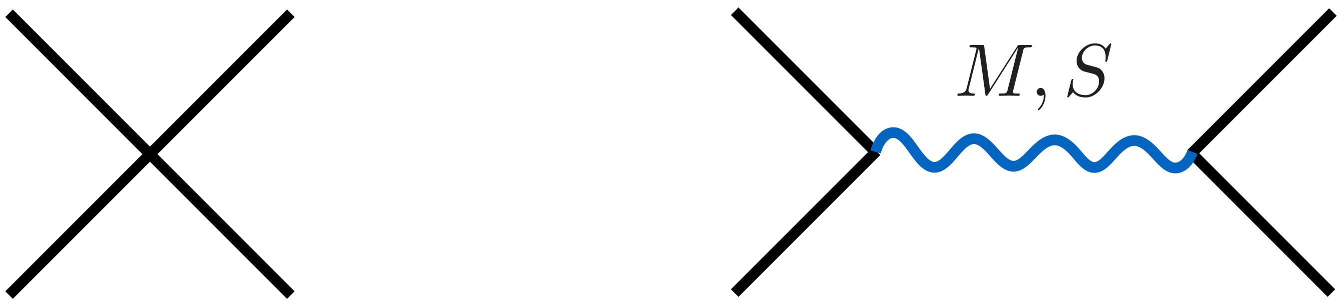
This provide a maximal degree of theoretical control, but limits the strength of the allowed interactions:

$$f_{\rm NL} < 1$$

The S-Matrix Bootstrap

We will take inspiration from the S-matrix bootstrap, where the structure of scattering amplitudes is fixed by **Lorentz invariance**, **locality** and **unitarity**:

$$A(s,t) \quad = \quad \sum a_{nm} s^n t^m \quad + \quad \frac{g^2}{s-M^2} P_S \left(1 + \frac{2t}{M^2}\right)$$

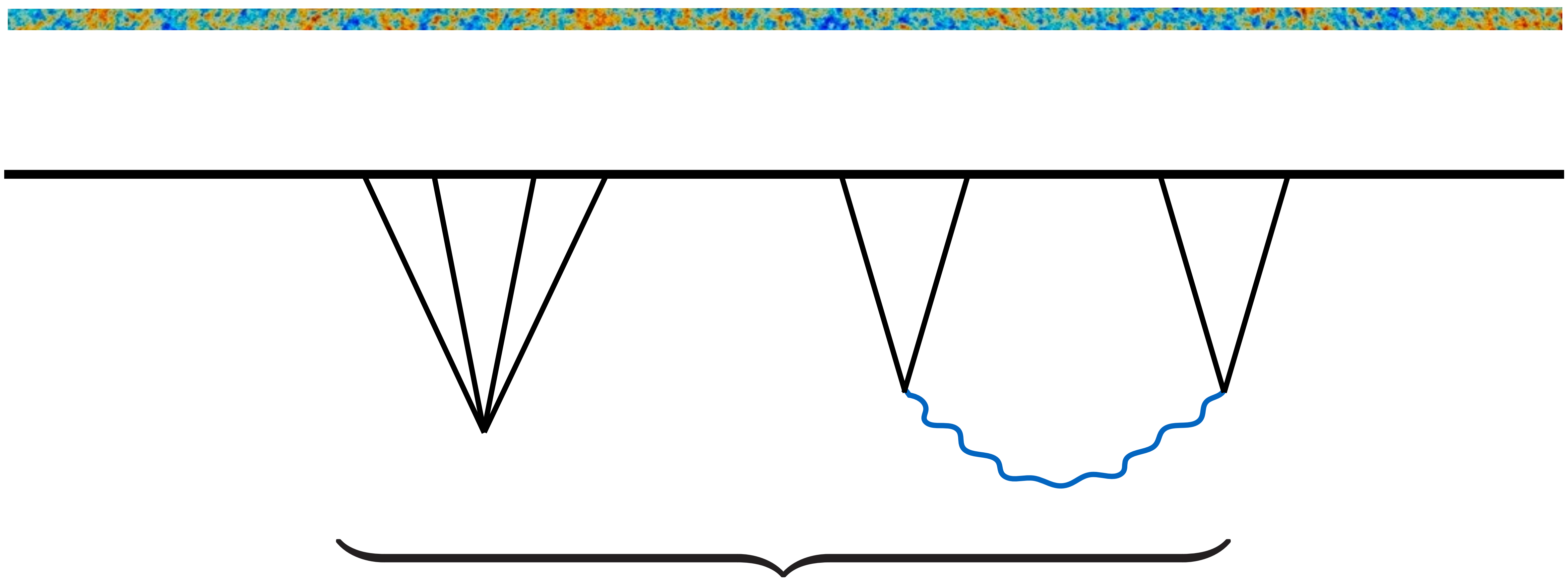


contact interactions

exchange interactions

- No Lagrangian or Feynman diagrams are needed to derive this.
- Basic principles allow only a small menu of possibilities.

The Cosmological Bootstrap

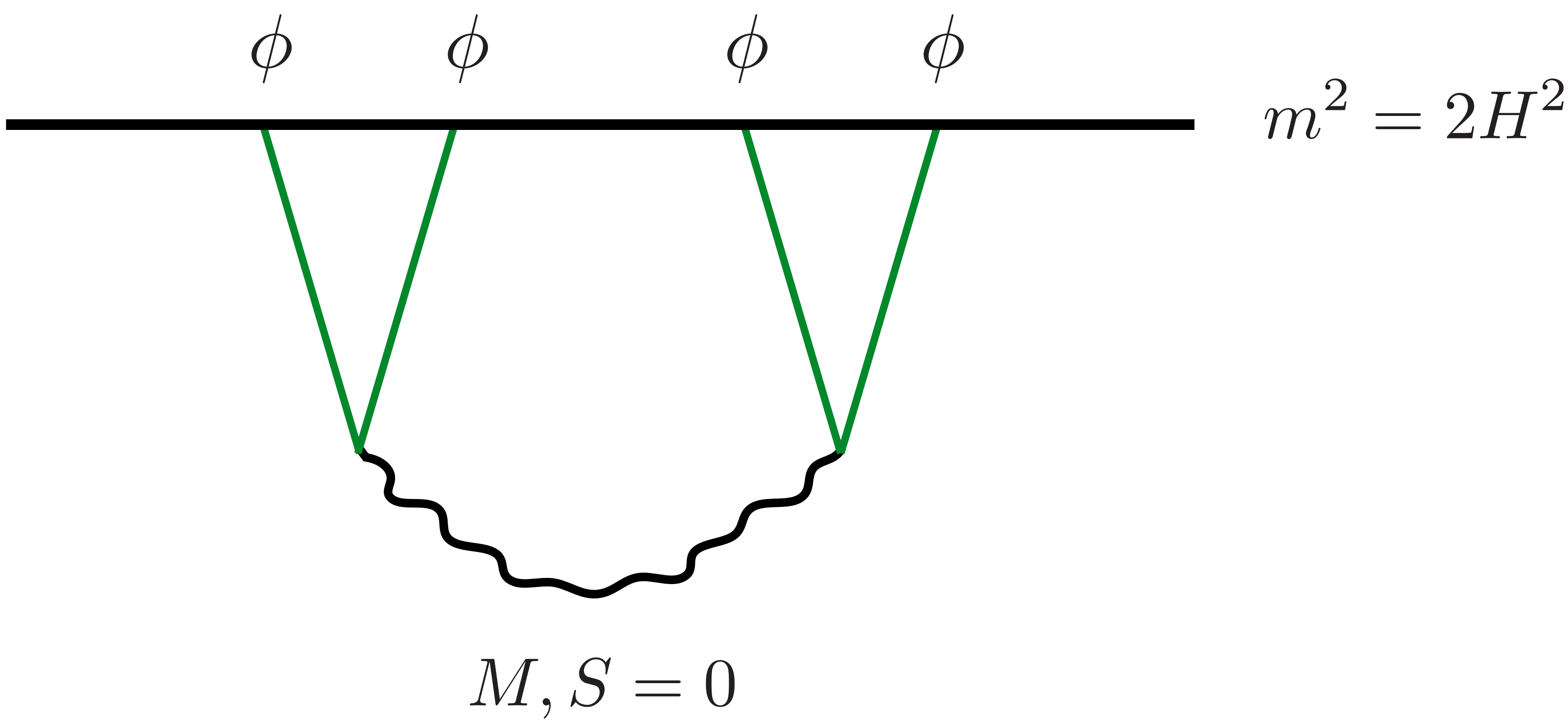


fixed by
symmetries and singularities

- No Lagrangian or Feynman diagrams are needed to derive this.
- Basic principles allow only a small menu of possibilities.

Outline

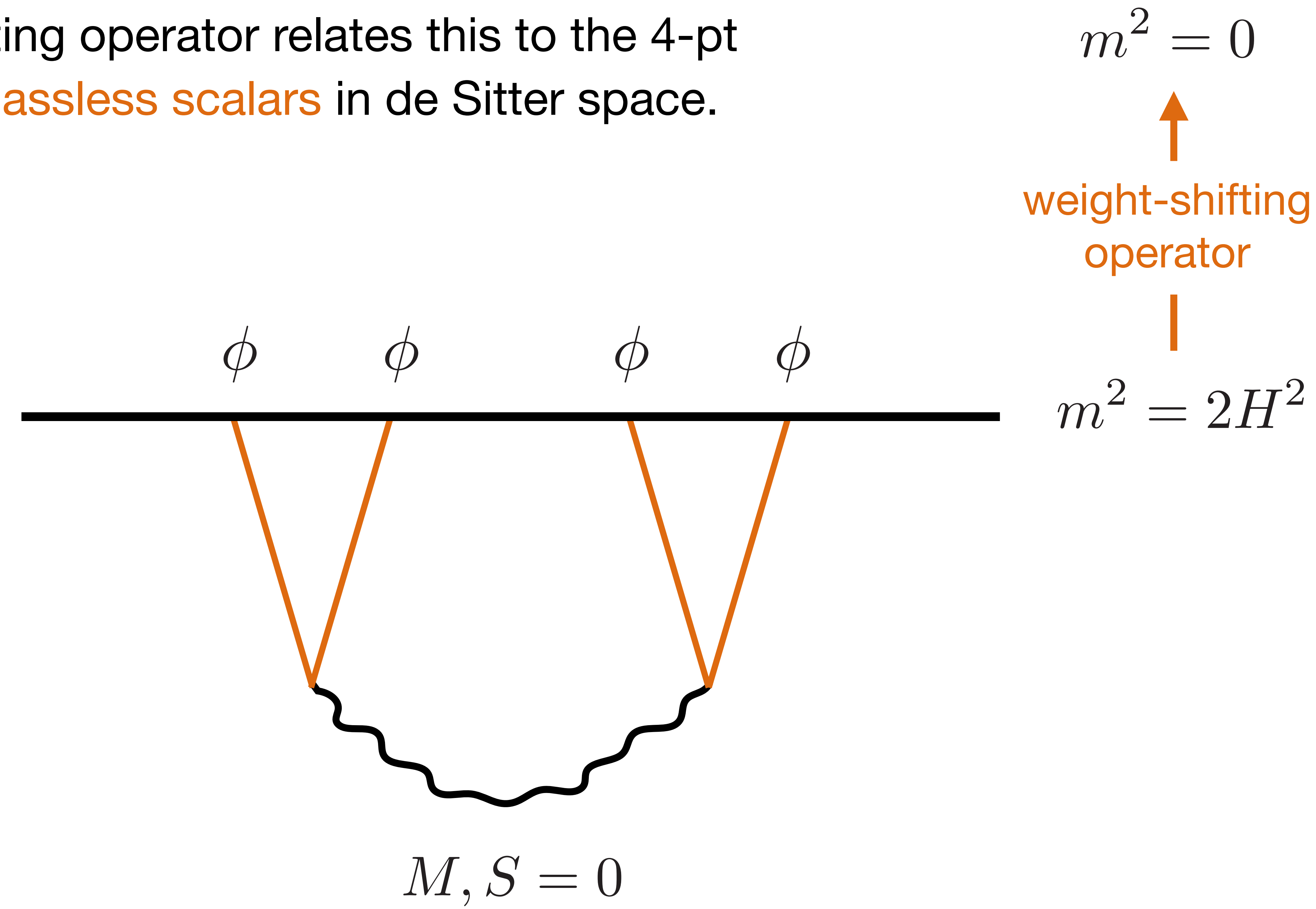
The fundamental object will be the 4-pt function
of conformally-coupled scalars in de Sitter space.



Arkani-Hamed, DB, Lee and Pimentel [2018]

Outline

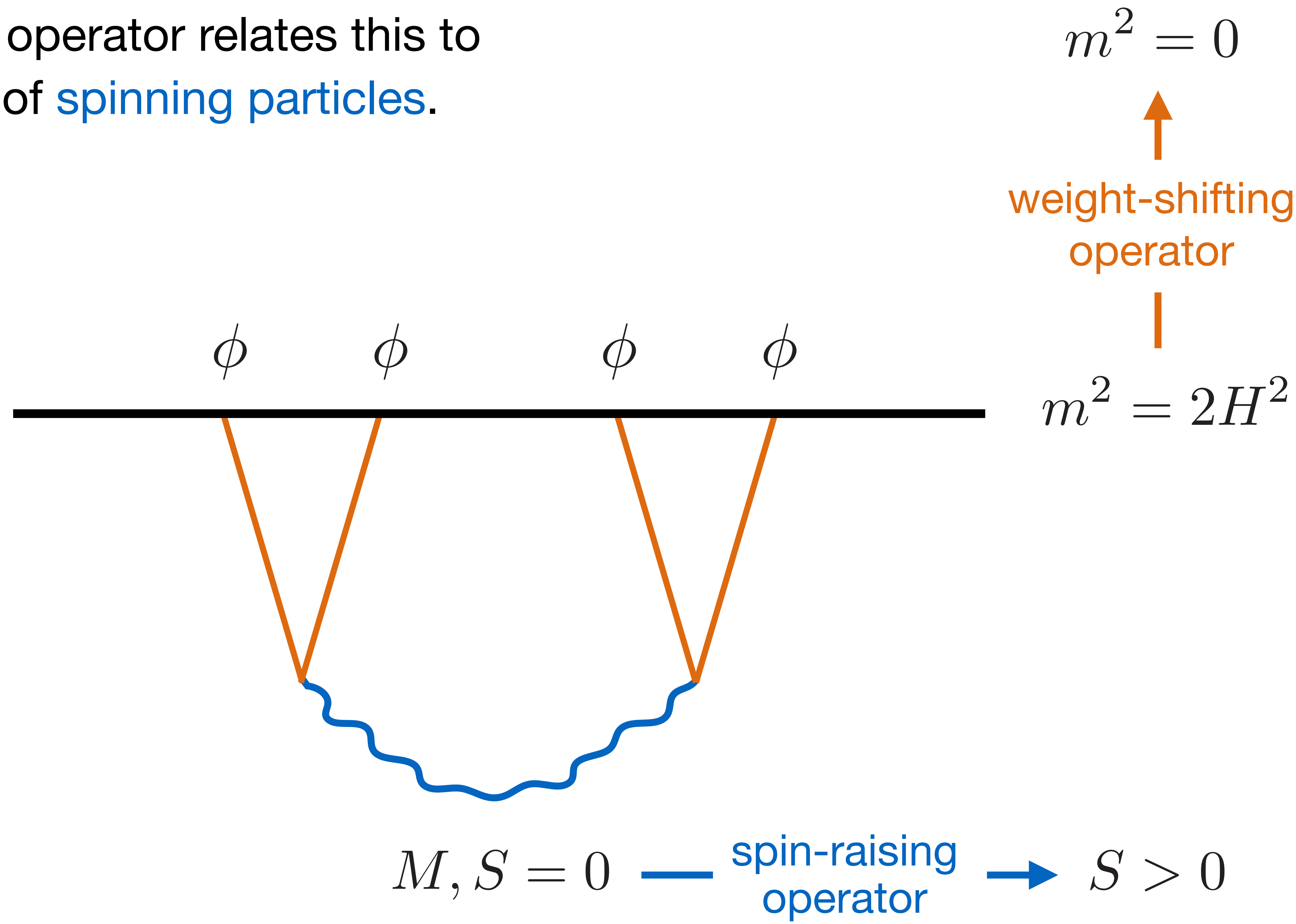
A weight-shifting operator relates this to the 4-pt function of massless scalars in de Sitter space.



Arkani-Hamed, DB, Lee and Pimentel [2018]

Outline

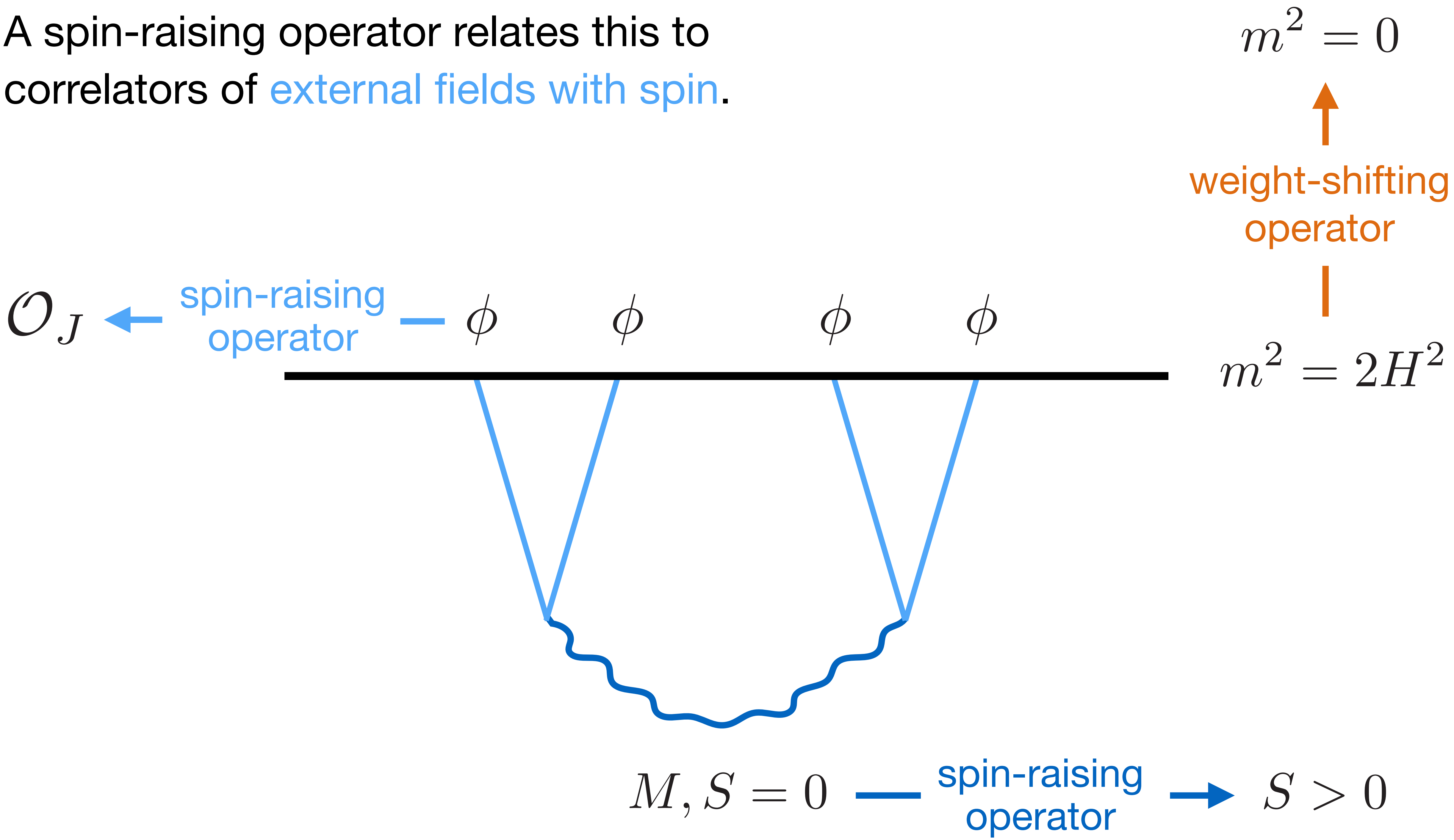
A spin-raising operator relates this to the exchange of spinning particles.



Arkani-Hamed, DB, Lee and Pimentel [2018]

Outline

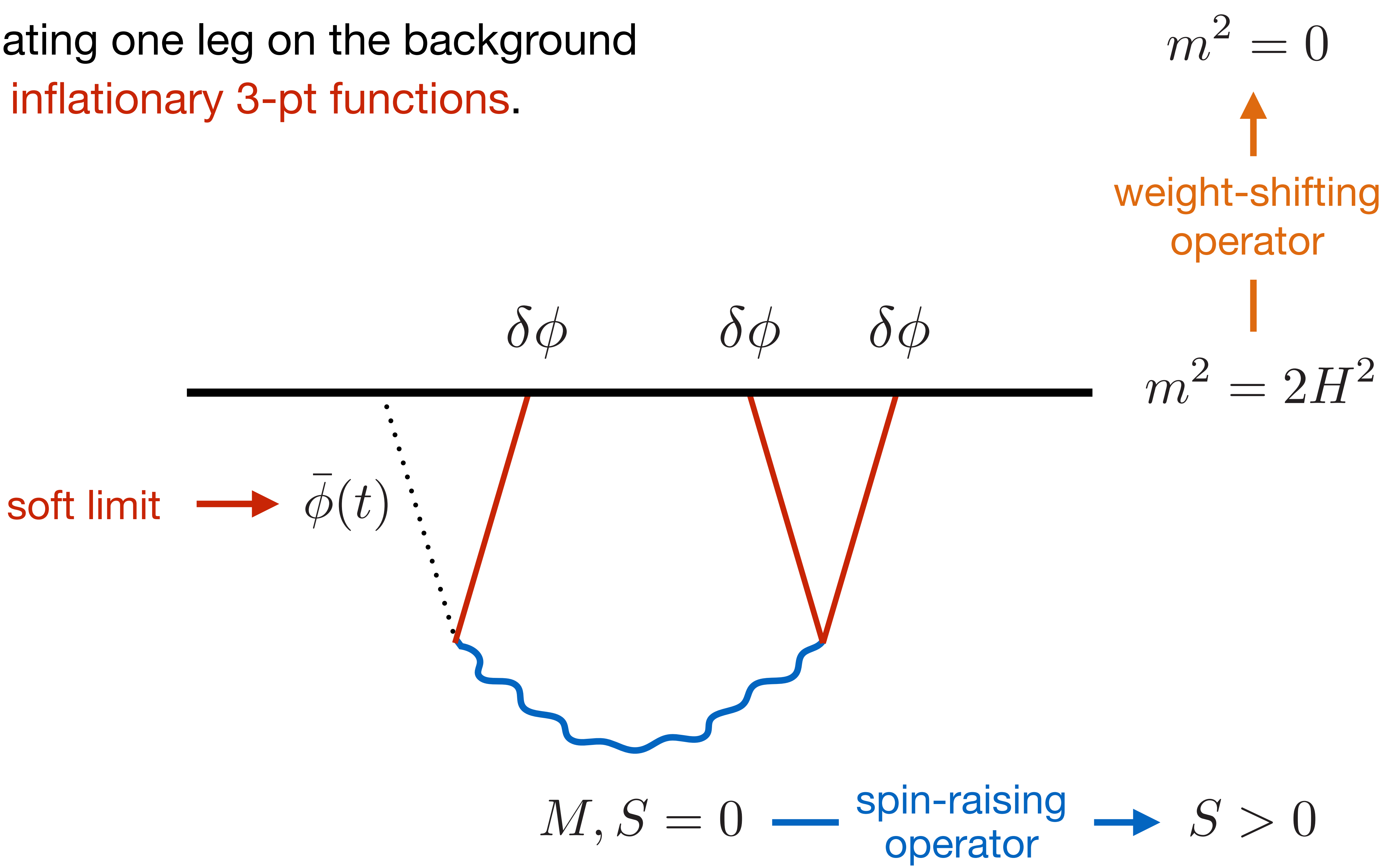
A spin-raising operator relates this to correlators of external fields with spin.



Arkani-Hamed, DB, Lee and Pimentel [2018]

Outline

Evaluating one leg on the background gives inflationary 3-pt functions.



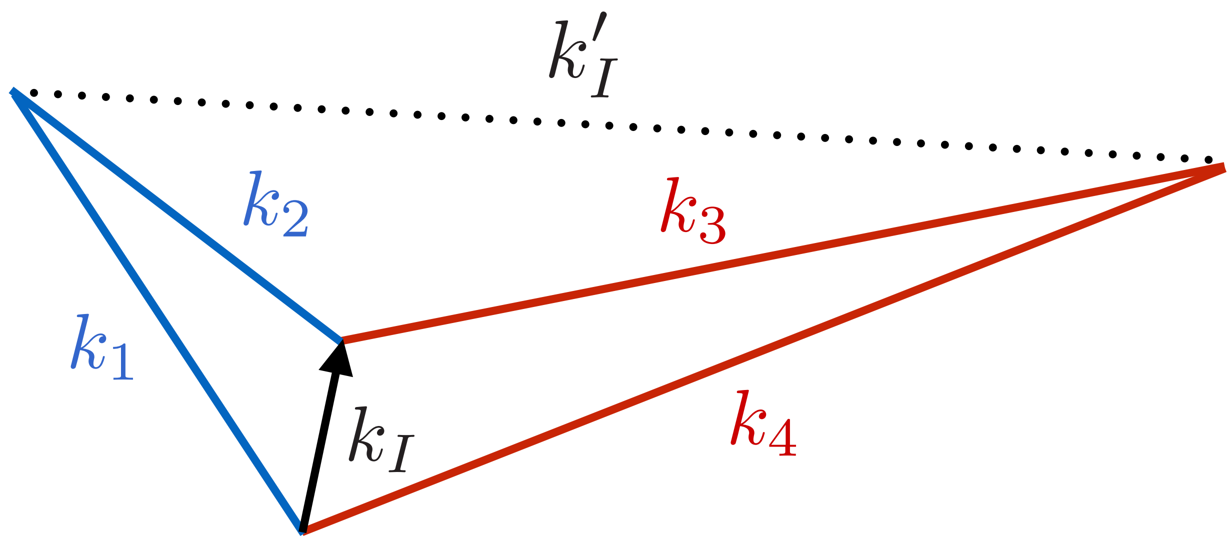
Arkani-Hamed, DB, Lee and Pimentel [2018]

De Sitter Four-Point Functions

Symmetries

Boundary correlators in de Sitter are constrained by conformal symmetry:

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle =$$



rotations + translations

$$= \frac{1}{k_I} \hat{F}(u, v)$$

dilatations

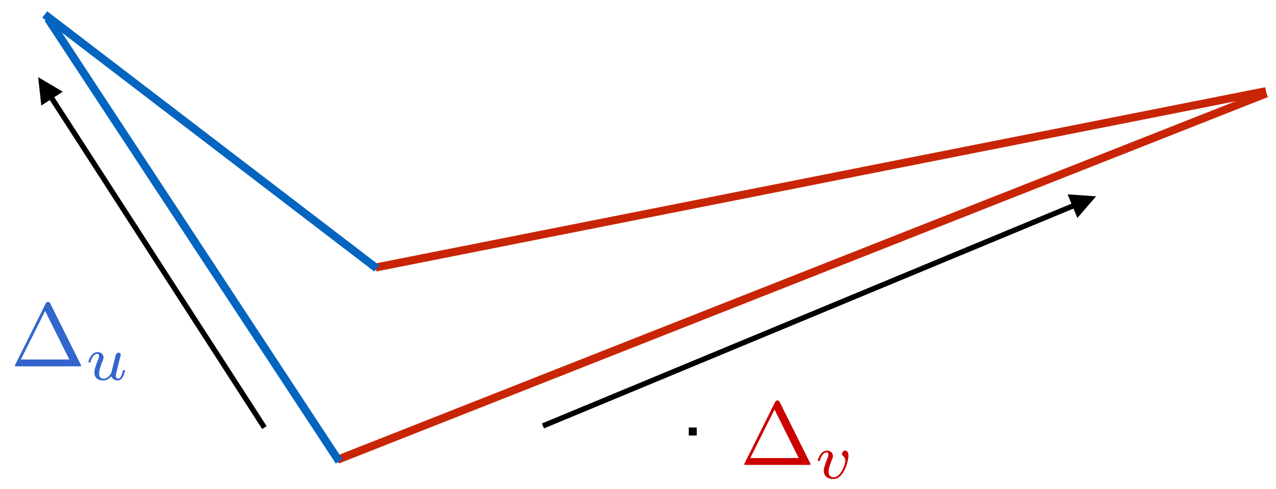
where $u^{-1} \equiv \frac{k_1 + k_2}{k_I}$ and $v^{-1} \equiv \frac{k_3 + k_4}{k_I}$.

Symmetries

The Ward identity of special conformal transformations implies

$$\boxed{(\Delta_u - \Delta_v)\hat{F} = 0}$$

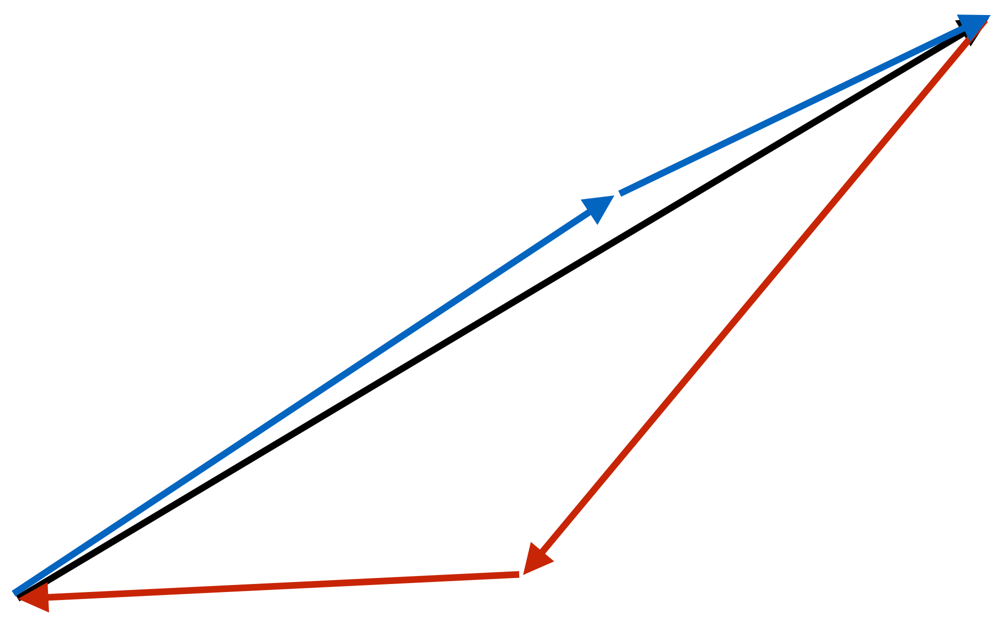
where $\Delta_u \equiv u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$.



Arkani-Hamed, DB, Lee and Pimentel [2018]

Singularities

For the Bunch-Davies vacuum, the solutions should have no singularities in the folded limit:

 = regular

Together with the correct normalization of a factorization channel, this provides the boundary conditions of the problem.

Arkani-Hamed, DB, Lee and Pimentel [2018]

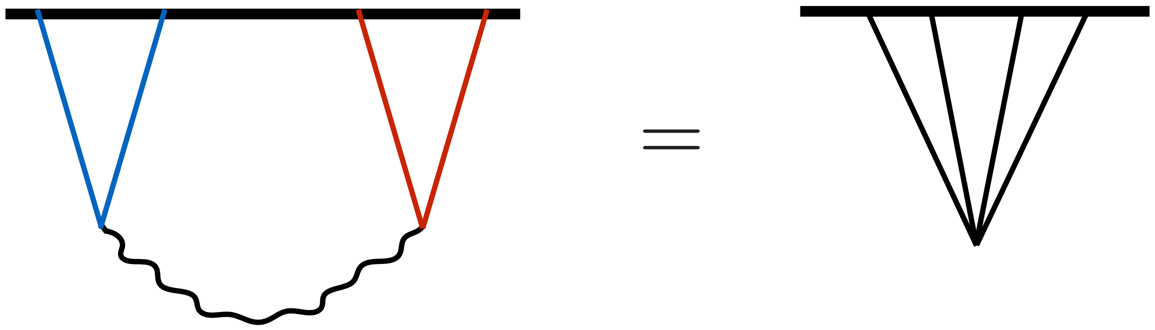
Exchange Interactions

For tree exchange, the conformal Ward identity reduces to:

$$(\Delta_u + M^2)\hat{F} = \hat{F}_c$$
$$(\Delta_v + M^2)\hat{F} = \hat{F}_c$$

where the sources are contact solutions:

$(\Delta_u + M^2)$
$(\Delta_v + M^2)$



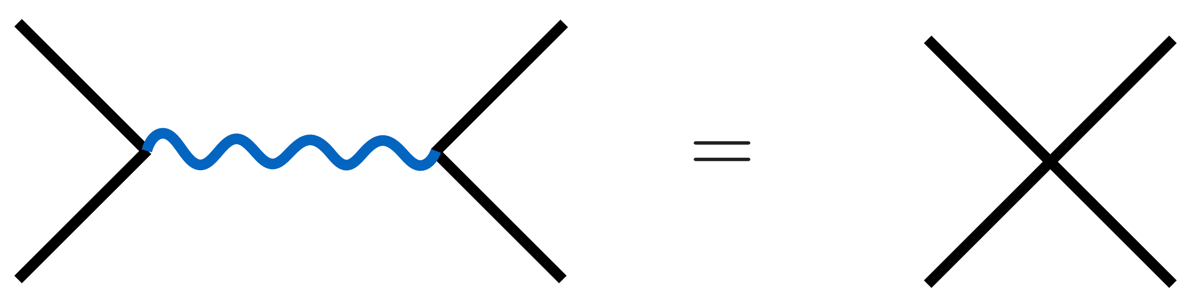
Exchange Interactions

For tree exchange, the conformal Ward identity reduces to:

$$(\Delta_u + M^2)\hat{F} = \hat{F}_c$$
$$(\Delta_v + M^2)\hat{F} = \hat{F}_c$$

This is analogous to the structure for scattering amplitudes.

$(s - M^2)$



EFT Expansion

A formal solution of the conformal Ward identity is

$$\hat{F} = \frac{\hat{F}_c^{(0)}}{\Delta_u + M^2} = \sum_n \frac{1}{n!}\left(-\frac{\Delta_u}{M^2}\right)^n \frac{\hat{F}_c^{(0)}}{M^2} \quad \leftarrow \quad \phi^4 + \frac{(\partial_\mu \phi)^4}{M^4} + \cdots$$

EFT expansion

This misses the effect of particle production!

Particle Production

The boundary conditions of the problem require adding homogeneous solutions that capture the effect of particle production:

For small u, the solution is

analytic

non-analytic

$$\hat{F} = \sum_n \frac{(-1)^n}{(n+\frac{1}{2})^2 + M^2} \left(\frac{u}{v}\right)^{n+1} + \frac{\pi}{\cosh(\pi M)} \frac{\sin(M \log(u/v))}{M}$$

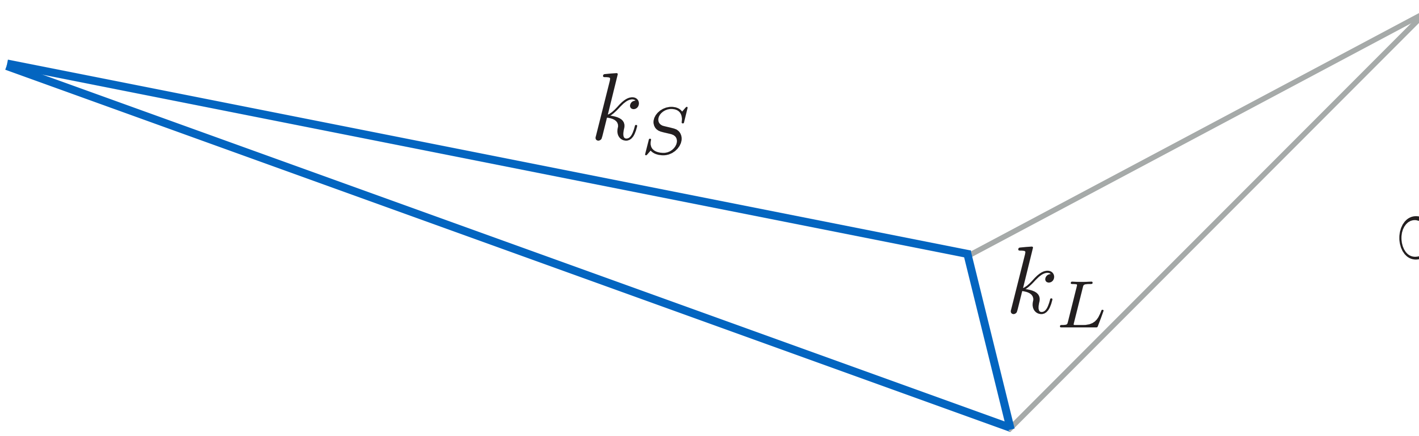
EFT expansion

particle production

The general solution has a similar form.

Soft Limit

The particle production piece dominates in the collapsed limit of the four-point function (or the squeezed limit of the three-point function):



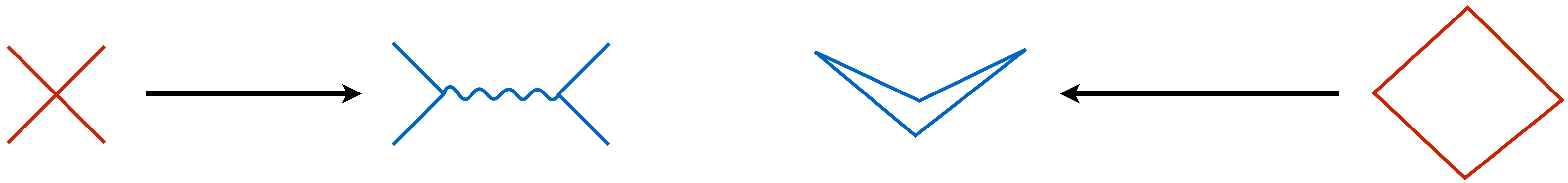
$$\propto \sin\left(\frac{M}{H}\log(k_L/k_S)\right)$$

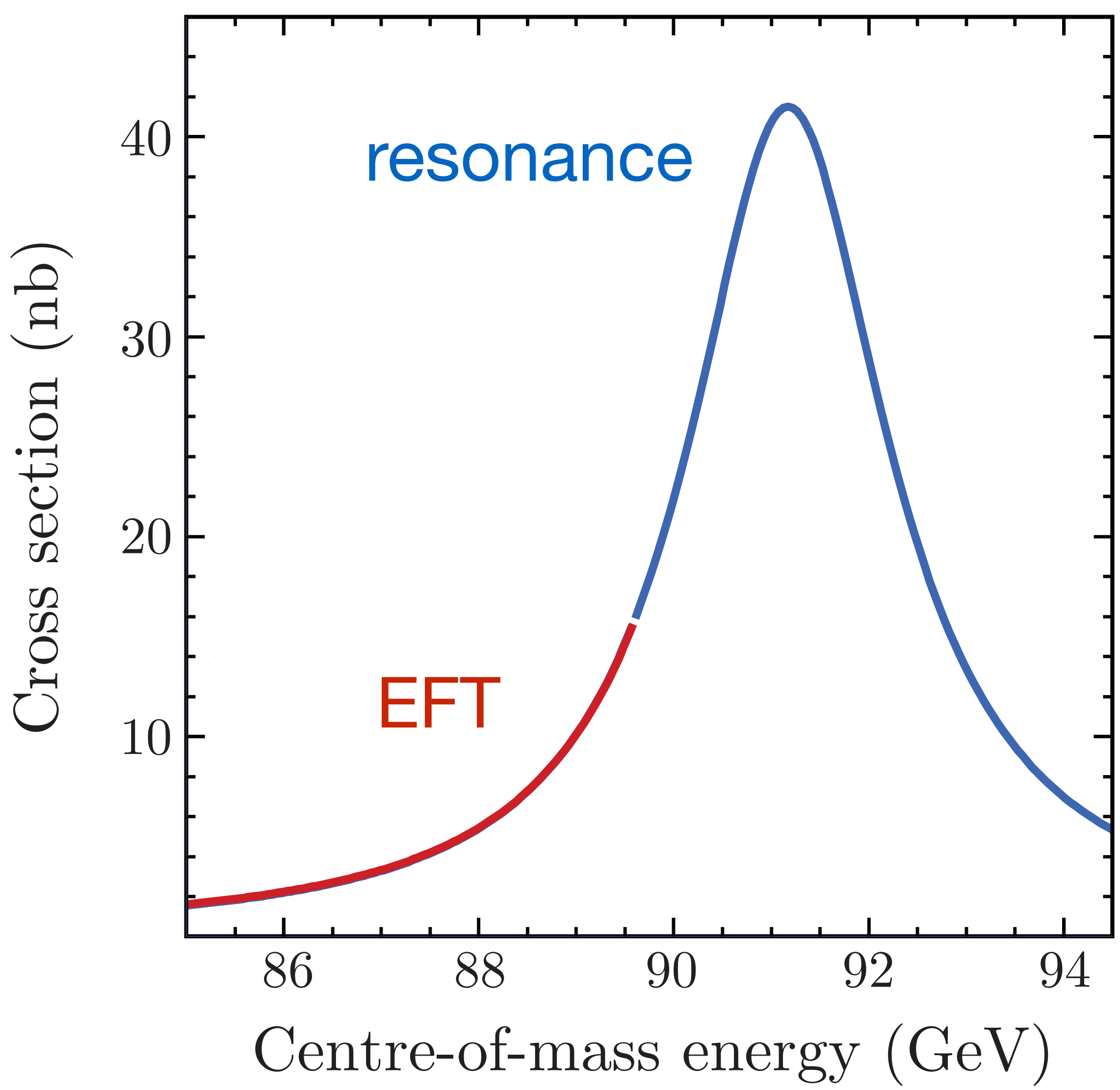
In this limit, the signal oscillates with a frequency given by the mass of the new particles.

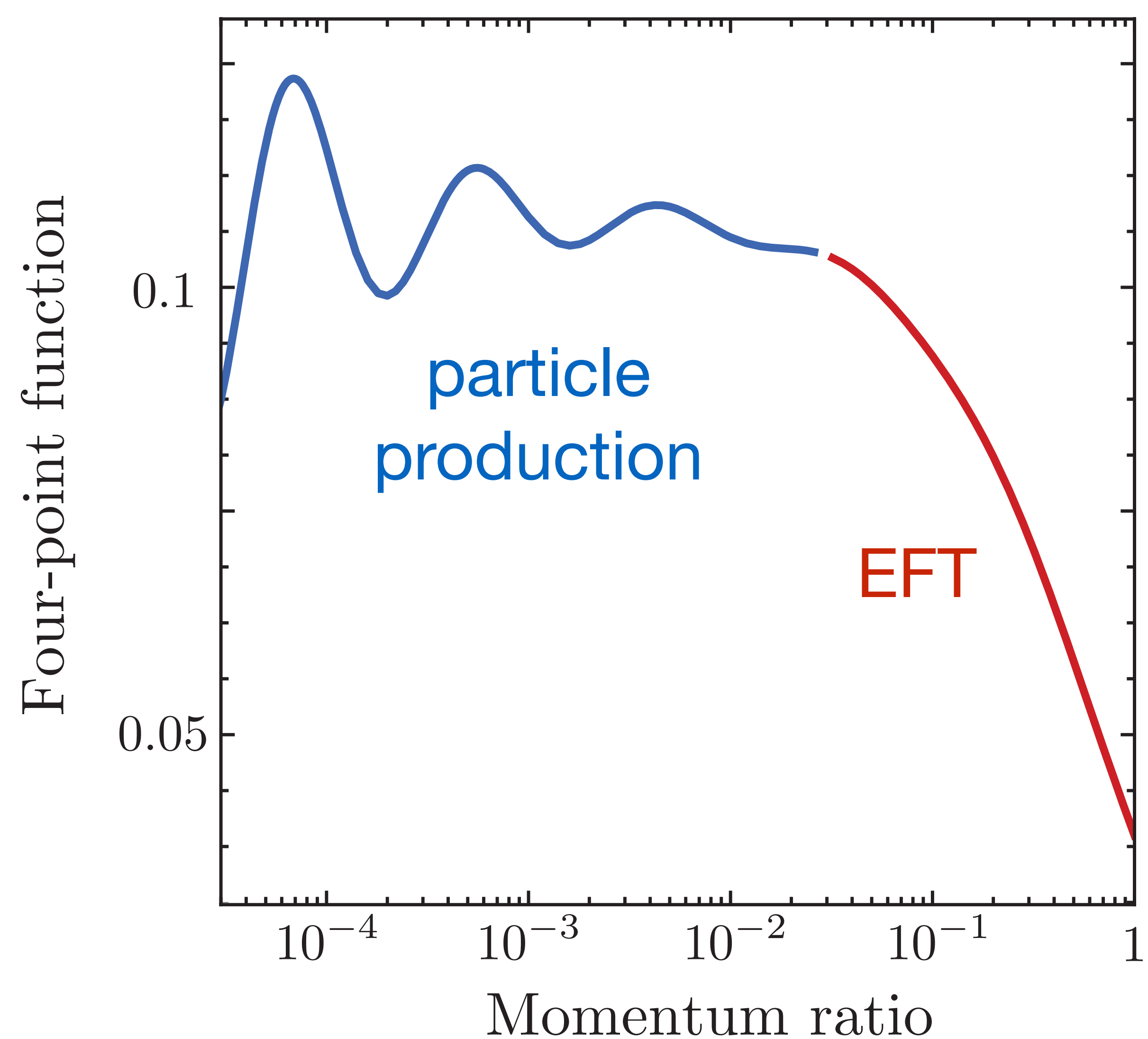
This is the analog of resonances in collider physics.

Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

Particle Spectroscopy







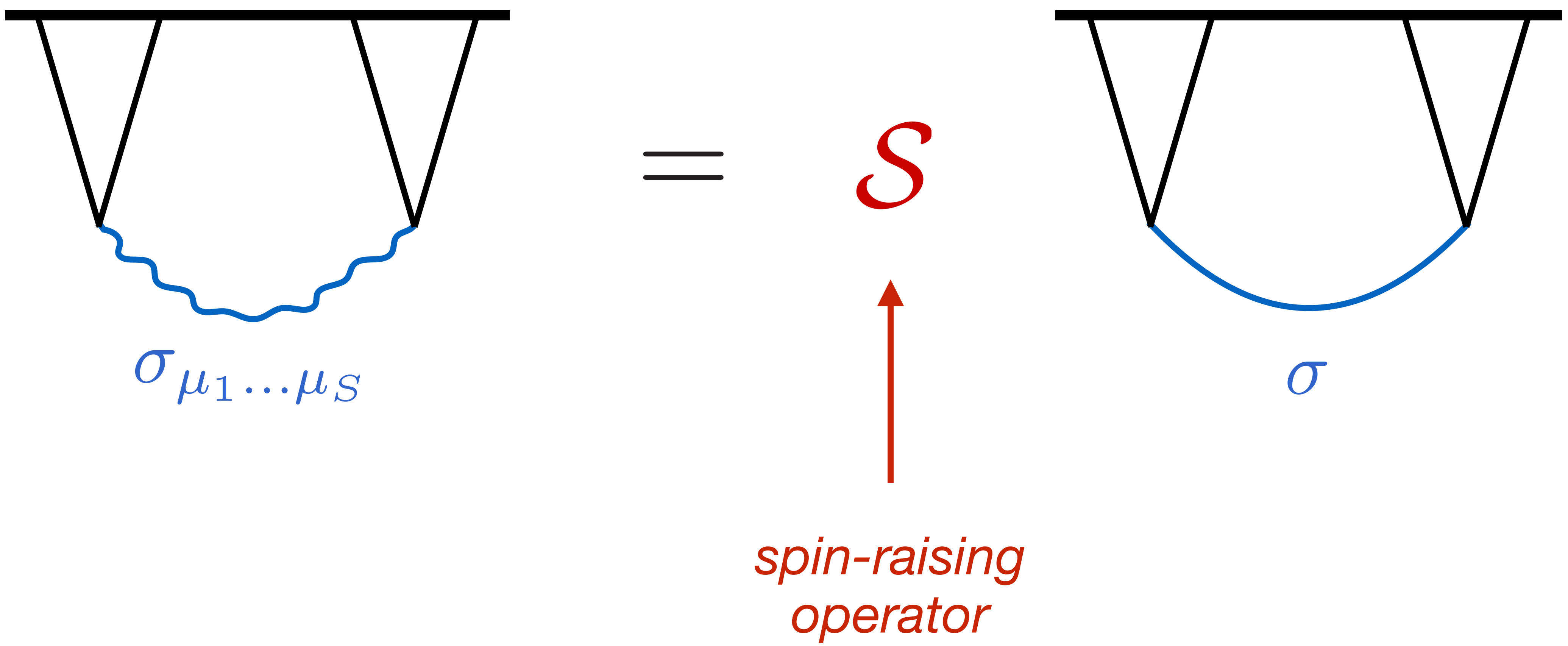
Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

Exchange of Spinning Particles

with Carlos Duaso and Austin Joyce

Strategy

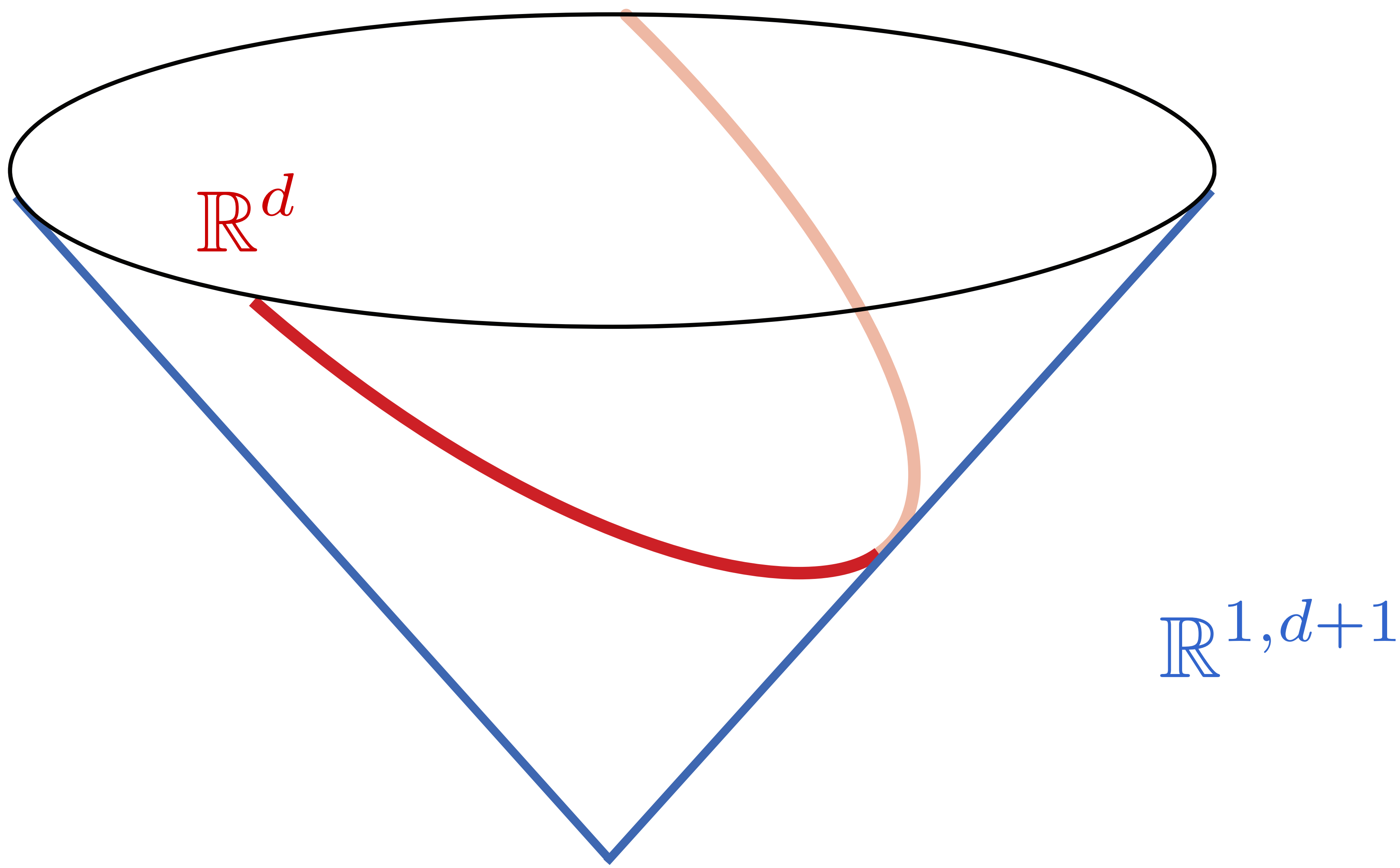
We wish to find differential operators that relate scalar exchange to spin exchange:



It turns out that the spin raising is best implemented in embedding space and then Fourier transformed.

CFTs in Embedding Space

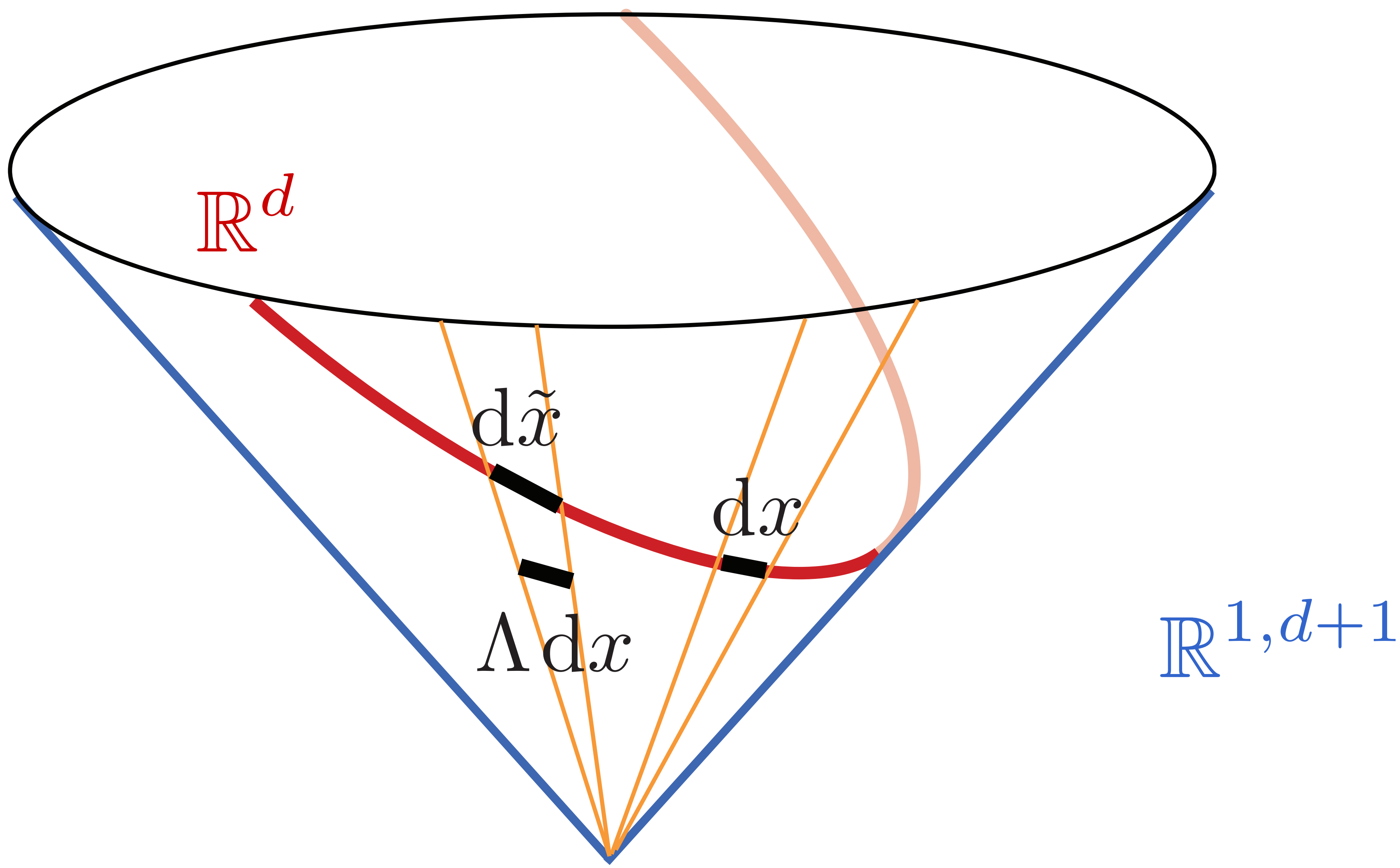
Consider the following embedding of d-dimensional Euclidean space into (d+2)-dimensional Minkowski space:



Dirac [1936]
Costa, Penedones, Poland and Rychkov [2011]

CFTs in Embedding Space

Lorentz transformations in embedding space become conformal transformations on the Euclidean section:



Dirac [1936]
Costa, Penedones, Poland and Rychkov [2011]

CFTs in Embedding Space

Conformal correlators in embedding space are simply the most general Lorentz-invariant expressions with the correct scaling behavior:

$$\langle\phi_1\phi_2\rangle = \frac{1}{X_{12}^{\Delta_1}}\,,$$

$$\langle\phi_1\phi_2\phi_3\rangle = \frac{1}{X_{12}^{(\Delta_1+\Delta_2-\Delta_3)/2}X_{23}^{(\Delta_2+\Delta_3-\Delta_1)/2}X_{31}^{(\Delta_3+\Delta_1-\Delta_2)/2}}\,,$$

$$\langle\phi_1\phi_2\phi_3\phi_4\rangle = f(u,v)\prod_{n<m}^{4}\frac{1}{X_{nm}^{\Delta_n+\Delta_m-\Delta_t/3}}\,,$$

where $X_{mn} \equiv X_n \cdot X_m \;\to\; (x_n - x_m)^2$.

Dirac [1936]
Costa, Penedones, Poland and Rychkov [2011]

Spin-Raising Operator

Correlators of spinning fields can be written in terms of scalar seeds.
For example:

$$\langle \phi\tilde{\phi}\Sigma^M \rangle = \frac{X_1^M X_{23} - X_2^M X_{13}}{(X_{12}X_{23}X_{31})^{1/2}} \langle \phi\tilde{\phi}\Sigma \rangle \;=\; \mathcal{S}^M \langle \phi\tilde{\phi}\Sigma \rangle \,,$$

where

$$\boxed{\mathcal{S}^M \equiv (X_3 \cdot X_2) \frac{\partial}{\partial X_3^M} - X_2^M X_3 \cdot \frac{\partial}{\partial X_3}} \,.$$

In Fourier space, this becomes

$$\boxed{\mathcal{S}^i \equiv (\partial_{k_3^i} - \partial_{k_2^i}) + \frac{k_3^i}{2} (\partial_{k_3^j} - \partial_{k_2^j})(\partial_{k_3^j} - \partial_{k_2^j})} \,.$$

Karateev, Kravchuk and Simmons-Duffin [2018]
Costa, Penedones, Poland and Rychkov [2011]

Bootstrapping Spin Exchange

Using this spin-raising operator, we have

$$\hat{F}_S = \sum_{\lambda=0}^{S} P^{(\lambda)}_{i_1\ldots i_S j_1\ldots j_S} (\mathcal{S}_L^{i_1} \cdots \mathcal{S}_L^{i_S})(\mathcal{S}_R^{j_1} \cdots \mathcal{S}_R^{j_S})\, \hat{F}_0 \,,$$

spin-exchange solution — polarization tensor — spin-raising operator — scalar-exchange solution

which can be written as

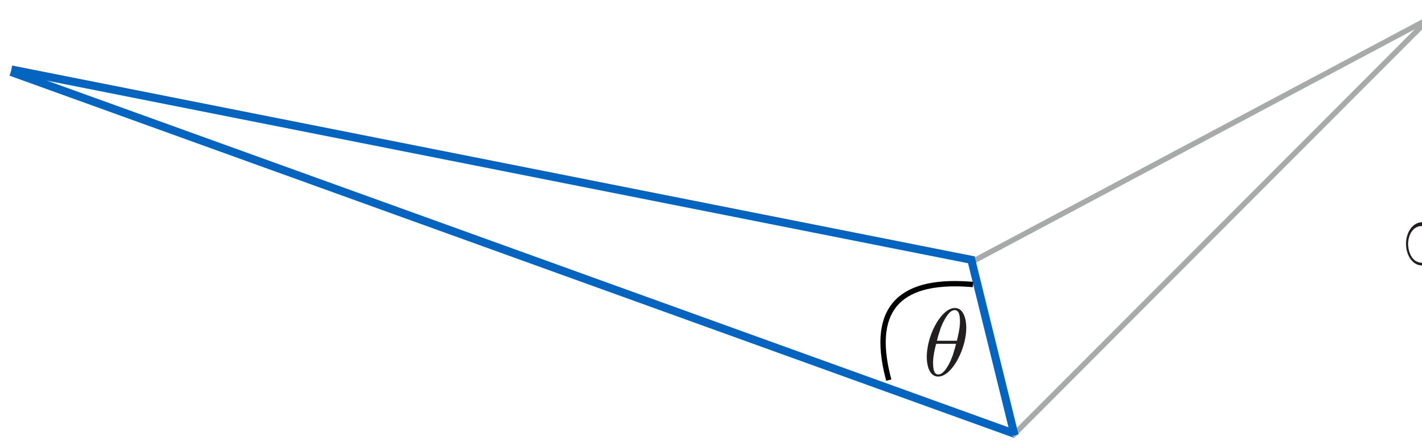
$$\boxed{\hat{F}_S = \sum_{\lambda=0}^{S} \Pi_{S,\lambda}(\text{angles})\, \mathcal{D}^{(S,\lambda)}_{uv} \hat{F}_0}$$

e.g. $\mathcal{D}^{(S,S)}_{uv} \equiv [(uv)^2 \partial_u \partial_v]^S$

Arkani-Hamed, DB, Lee and Pimentel [2018]

Soft Limit

The spin of the new particles is encoded in the angular dependence of the collapsed limit:

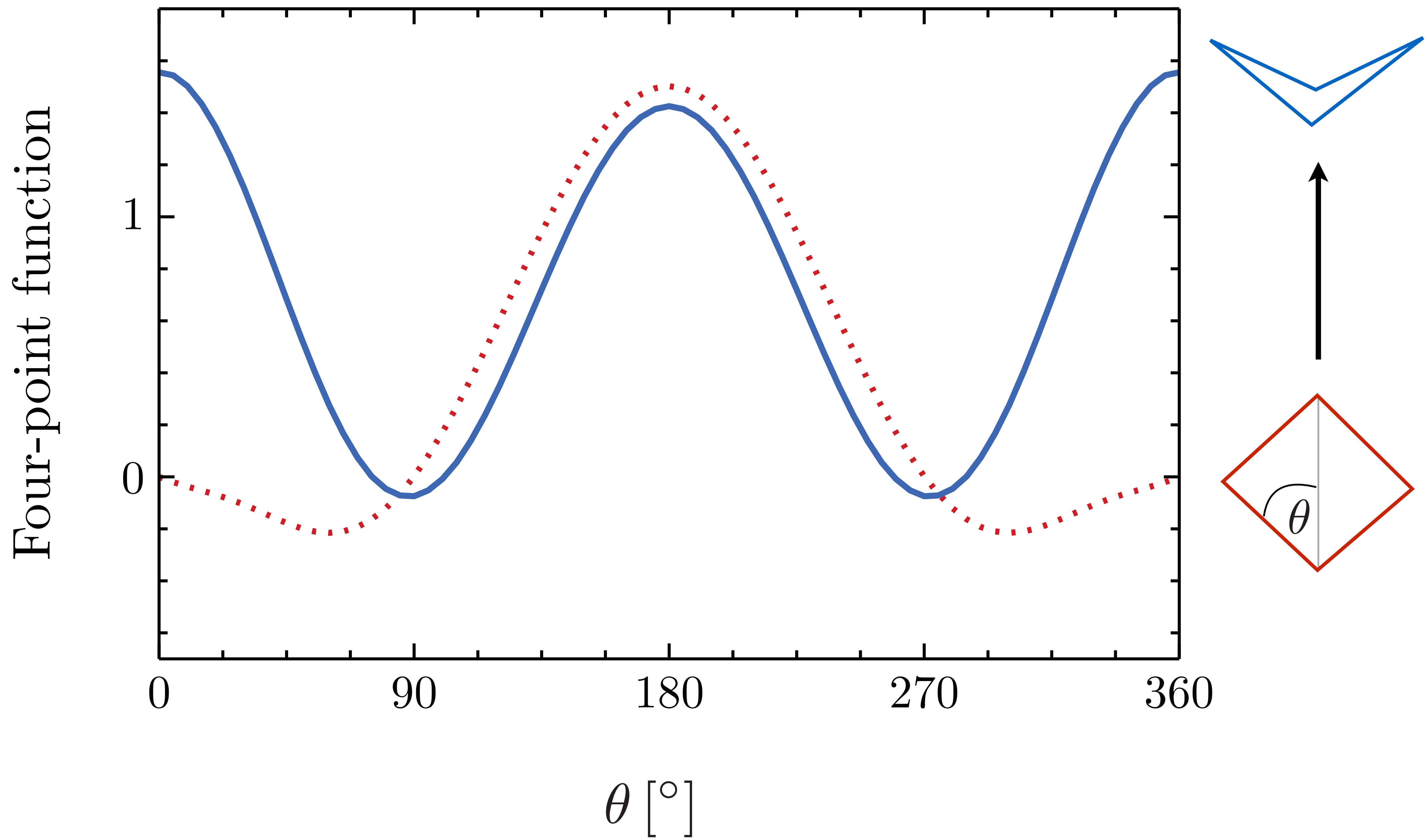


$$\propto P_S(\cos\theta)$$

This is the analog of the angular dependence of the final state particles in collider physics.

Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

Particle Spectroscopy



Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

Inflationary Correlators

Weight-Shifting Operators

Four-point functions with massless external fields (= inflatons) can be obtained by acting with suitable weight-shifting operators:

$$\hat{F}_{m=0} = \mathcal{W}_L \mathcal{W}_R \, \hat{F}_{m=\sqrt{2}H}$$

massless scalars — *weight-shifting operators* — *conformally coupled scalars*

e.g. acting on the scalar exchange solution

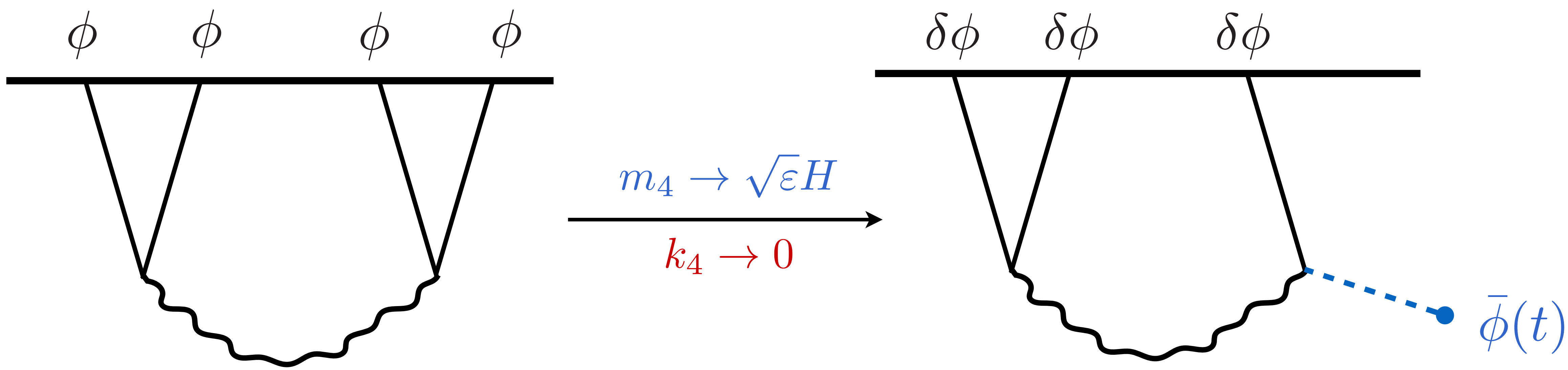
$$\mathcal{W}_L(\cdot) \equiv \frac{1}{2}\left(1 - \frac{k_1 k_2}{k_{12}}\partial_{k_{12}}\right)\left[\frac{1-u^2}{u^2}\partial_u(u\,\cdot)\right]$$

For massless spin-2 exchange, this reproduces the four-point function of slow-roll inflation.

Seery, Sloth and Vernizzi [2008]
Kundu, Shukla and Trivedi [2014]
Arkani-Hamed, DB, Lee and Pimentel [2018]

Inflationary Correlators

To obtain inflationary three-point functions, we evaluate one of the external legs on the time-dependent background:



$$= \mathcal{W}_L \mathcal{W}_R \hat{F}(u,v) \qquad\qquad = \varepsilon\, \mathcal{W}_L \lim_{v\to 1} \hat{F}(u,v)$$

For spin exchange, only the longitudinal mode contributes.

Slow-Roll Inflation

- For the simplest contact interaction, $(\partial_\mu \phi)^4$, this gives

$$B(k_1, k_2, k_3) = \frac{\varepsilon}{E^2}\left(\sum_n k_n^5 + \sum_{n \neq m} (2k_n^4 k_m - 3k_n^3 k_m^2) + \sum_{n \neq m \neq l} (k_n^3 k_m k_l - 4k_n^2 k_m^2 k_l) \right)$$

Creminelli [2003]

- For graviton exchange, this reproduces the standard 3-point function of slow-roll inflation:

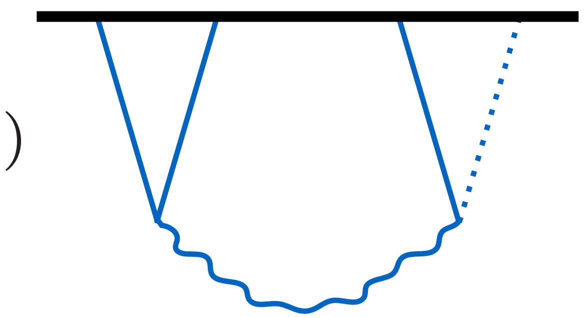
$$B(k_1, k_2, k_3) = \varepsilon \left[\sum_{n \neq m} k_n k_m^2 + \frac{8}{E} \sum_{n > m} k_n^2 k_m^2 \right] + (n_s - 1) \sum_n k_n^3$$

Maldacena [2002]

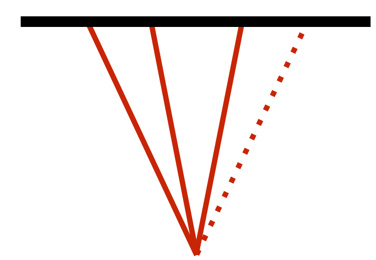
Massive Particles

- The effects of massive particles during inflation are characterized in terms of just two basis functions:

$$B(k_1, k_2, k_3) \;=\; \mathcal{W}_L \left[\sum_S a_S \, \mathcal{S}^{(S)} \right.$$



$$+ \sum_n b_n \, \Delta_u^n$$



$$\left. \right] + \text{perms}$$

This result is valid for all momenta, not just soft limits.

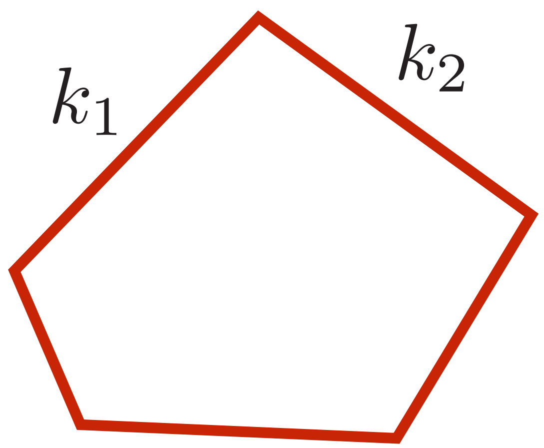
Arkani-Hamed, DB, Lee and Pimentel [2018]

Future Directions

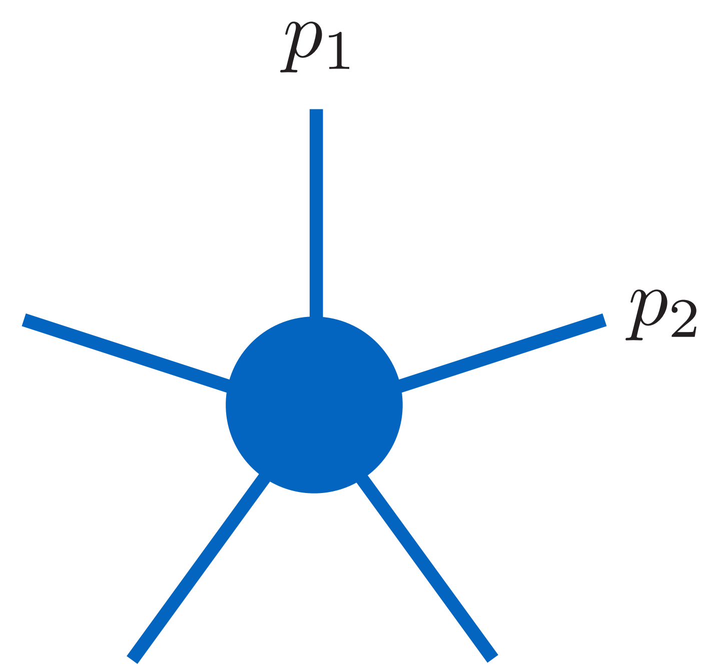
Amplitudes Meet Cosmology

Remarkably, **correlation functions** **contain** **scattering amplitudes**:

$$\lim_{E\to 0}$$



$$\frac{1}{E^p} \times$$



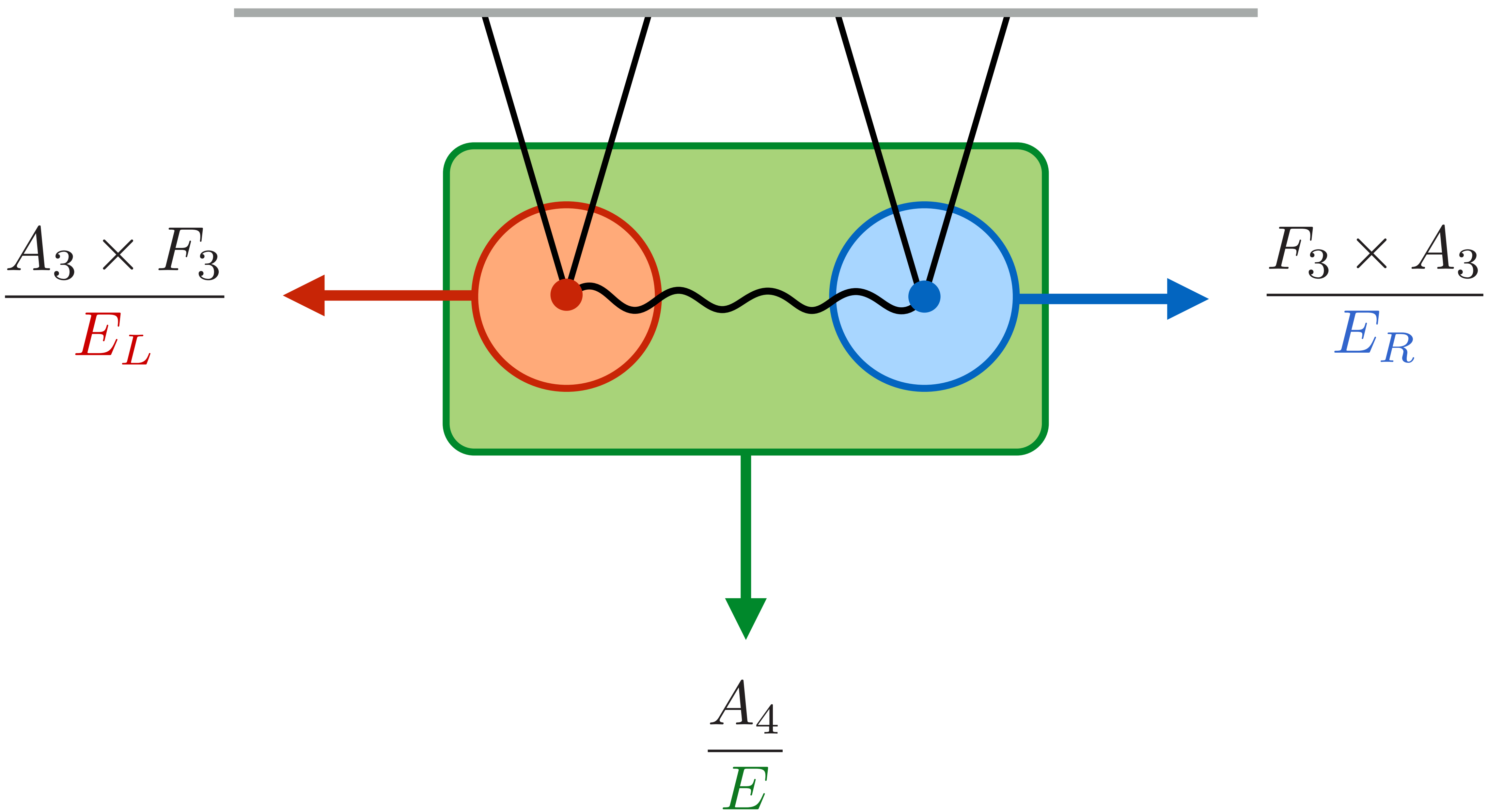
where $E \equiv \sum_n |\vec{k}_n|$.

Raju [2012]
Maldacena and Pimentel [2011]

Insights from modern scattering amplitudes methods should therefore translate to cosmology.

Amplitudes Meet Cosmology

In special limits, correlation functions factorize into products of lower-point amplitudes (and lower-point correlators):



Amplitudes Meet Cosmology

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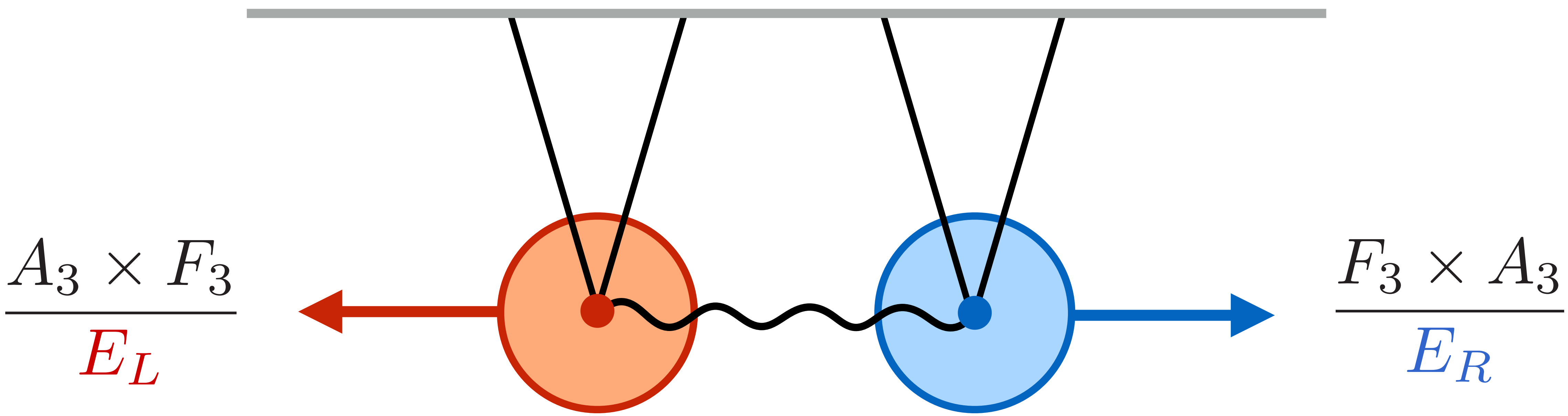


- Correlators of massless spinning particles can be constructed by gluing together these factorization channels.

BCFW [2005]

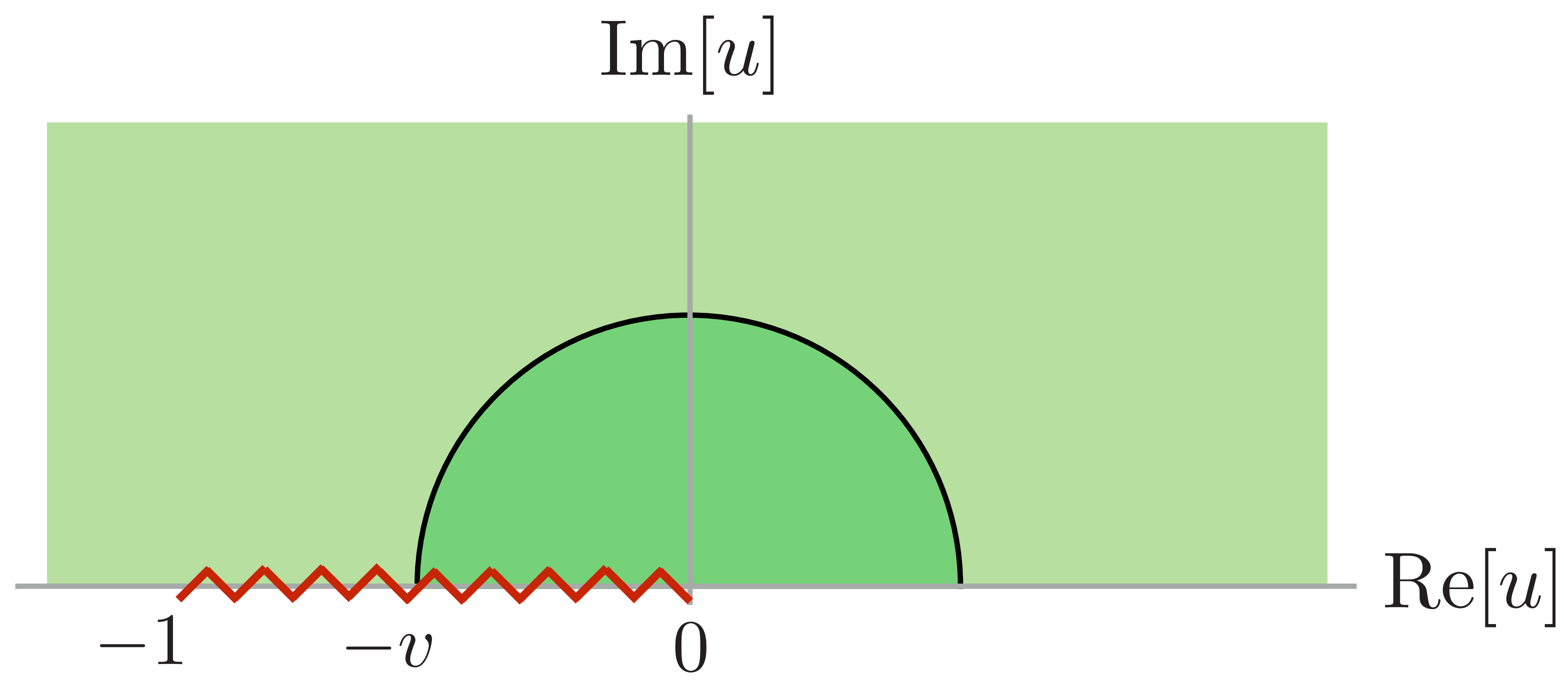
- Not all theories will be consistent with locality.

Benincasa and Cachazo [2007]
McGady and Rodina [2014]


Thank you for your attention!

Analytic Structure

The analytic structure of the solution is



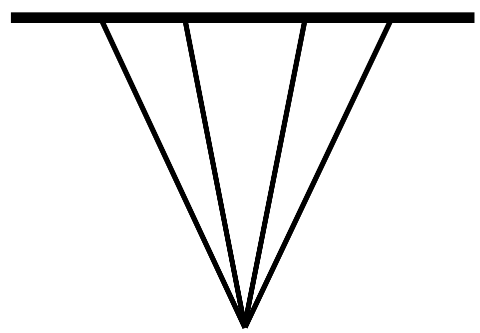
- The branch cut arises from **particle production**.
- The discontinuity across the cut gives the **scattering amplitude**:

$$\lim_{u \to -v} \frac{\mathrm{Disc}[\hat{F}']}{2\pi i} = \frac{1}{(k_1 + k_2)^2 - (\vec{k}_1 + \vec{k}_2)^2} = A_4$$

Contact Interactions

The simplest solutions correspond to **contact interactions**:

$$\hat{F}_c =$$



$$= \sum_n \frac{c_n(u,v)}{E^{2n+1}} \longleftarrow \quad \phi^4, (\partial\phi)^4, \cdots$$

which have poles at vanishing total energy:

$$\frac{E}{k_I} \equiv \frac{k_1 + k_2 + k_3 + k_4}{k_I} = \frac{u+v}{uv} \; .$$

Arkani-Hamed, DB, Lee and Pimentel [2018]