



Paolo Creminelli, ICTP (Trieste)

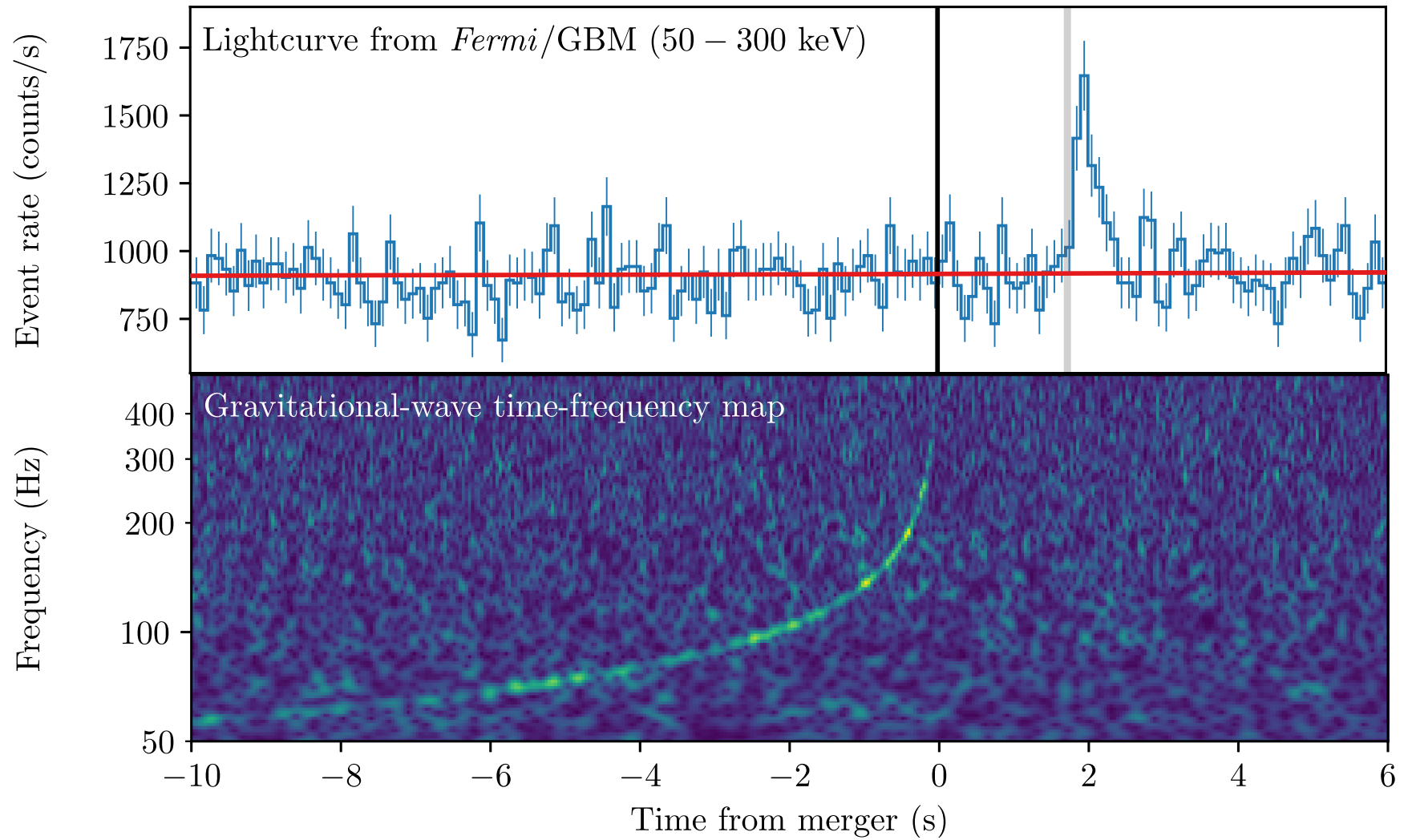
# EFT of Dark Energy and GW Observations

with F. Vernizzi, 1710.05877, with M. Lewandowski, G. Tambalo and F. Vernizzi, 1809.03484

+ work in progress with G. Tambalo, F. Vernizzi and V. Yingcharoenrat

Saclay, 5 June 2019

# GW170817 = GRB170817A



# GW170817 = GRB170817A

$$-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$$

- Previous tight limit for GWs slower than light, Cherenkov radiation  
One-sided and only valid for high energy

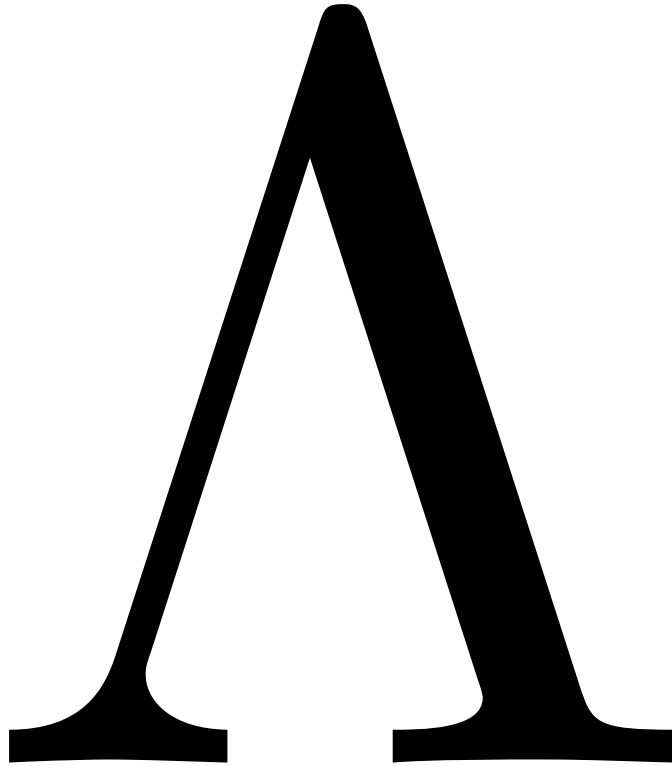
- Over  $\sim$  cosmological distances: 40 Mpc

Screening can (probably) be neglected

- Low energy:  $\lambda \sim 10\,000$  km

Reasonable one can use the same EFT as for cosmo scales

The Universe accelerates



# Dark Energy = Lorentz-violating Medium



In general the speed of GWs is different from photons,  
GWs can be absorbed and have dispersion

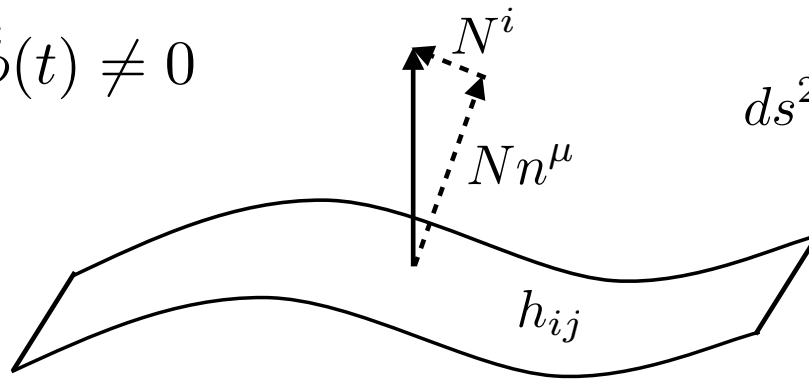
Use GWs to probe Dark Energy as light probes a material

# EFT of Dark Energy

PC, Luty, Nicolis, Senatore 03  
+ Vernizzi, Piazza + many others

Parametrization of possible deviations from CC

$$\dot{\phi}(t) \neq 0$$



$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

$$S = \int d^4x \sqrt{-g} L[t; N, K_j^i, {}^{(3)}R_j^i, \dots] \quad g^{00} = -N^{-2}$$

Action contains all possible scalars under spatial diffs, ordered by number of perturbations and derivatives

Assume **universal metric coupled to SM and DM**

# EFT of Dark Energy

Focus on theories with second order EOM

$$\begin{aligned}
 S = \int d^4x \sqrt{-g} & \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\
 & - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \\
 & \left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].
 \end{aligned}$$

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu, \quad \delta \mathcal{G}_2 \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R / 2,$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho.$$

$$m_2^4 = \alpha_K H^2 M_*^2$$

For LSS we are interested in the regime  $\alpha \sim 0.1$

# EFT of Dark Energy

Quintessence and Brans-Dicke

Speed of sound  
of DE

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\ \left. - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \right. \\ \left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].$$

DGP and braiding

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu, \quad \delta \mathcal{G}_2 \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R / 2,$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho.$$



# EFT of Dark Energy

Galileon, Horndeski and Beyond Horndeski

$$\begin{aligned}
 S = \int d^4x \sqrt{-g} & \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\
 & - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \\
 & \left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].
 \end{aligned}$$

Non-linear terms, screening

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu, \quad \delta \mathcal{G}_2 \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R / 2,$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho.$$

**Horndeski:** most generic theory with 2<sup>nd</sup> order EOM  $\tilde{m}_4^2 = m_4^2$   $\tilde{m}_6 = m_6$

**Beyond Horndeski:** more than 2 derivatives, but degenerate with no ghost

# Some examples

$$-\frac{m_3^3}{2} \delta K \delta g^{00}$$

**Braiding**, the scalar mixes with gravity

Deffayet, Pujolas, Sawicki, Vikman 10

$$\delta g^{00} \rightarrow -2(\dot{\pi} - \Phi), \quad \delta K \rightarrow -(3\dot{\Psi} + a^{-2}\nabla^2\pi)$$

Different from usual Brans-Dicke, e.g.  $\Phi = \Psi$

$$+\frac{\tilde{m}_4^2}{2} \delta g^{00} R$$

Modifications **inside** matter:  
violation of Vainshtein screening

Kobayashi, Watanabe, Yamauchi 14

$$\frac{d\Phi}{dr} = G_N \left( \frac{\mathcal{M}}{r^2} - \epsilon \frac{d^2\mathcal{M}}{dr^2} \right)$$

D'amico, Huang, Mancarella, Vernizzi 14

$$\mathcal{L} = \frac{1}{2} \left\{ \left( 1 + \frac{c_s^2}{c_m^2} \lambda^2 \right) \dot{\pi}_c^2 - c_s^2 (\nabla \pi_c)^2 + \dot{v}_c^2 - c_m^2 (\nabla v_c)^2 + 2 \frac{c_s}{c_m} \lambda \dot{v}_c \dot{\pi}_c \right\}$$

**Kinetic matter  
mixing**

# EFT of Dark Energy

This term changes the speed of GWs

$$\begin{aligned}
 S = \int d^4x \sqrt{-g} & \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\
 & - \frac{m_3^3}{2} \delta K \delta g^{00} - \underbrace{m_4^2 \delta \mathcal{K}_2}_{\text{circled}} - \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \\
 & \left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].
 \end{aligned}$$

$$\begin{aligned}
 \dot{\gamma}_{ij}^2 \subset \delta \mathcal{K}_2 & \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu, & \delta \mathcal{G}_2 & \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R / 2, \\
 \delta \mathcal{K}_3 & \equiv \delta K^3 - 3 \delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho.
 \end{aligned}$$

One can modify a bit the solution, say

changing DM abundance. Background for:  $\delta g_{\text{bkgd}}^{00}$   $\delta K_{\text{bkgd}} \sim \delta H$

# EFT of Dark Energy

$$\begin{aligned}
 S = \int d^4x \sqrt{-g} & \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\
 & - \frac{m_3^3}{2} \delta K \delta g^{00} - \cancel{m_4^2 \delta \mathcal{K}_2} + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \\
 & \left. - \cancel{\frac{m_6}{3} \delta \mathcal{K}_3} - \cancel{\tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2} - \cancel{\frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3} \right].
 \end{aligned}$$

= 0 Horndeski

$$\begin{aligned}
 \dot{\gamma}_{ij}^2 \subset \delta \mathcal{K}_2 & \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu, & \delta \mathcal{G}_2 & \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R / 2, \\
 \delta \mathcal{K}_3 & \equiv \delta K^3 - 3 \delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho.
 \end{aligned}$$

see also Sakstein, Jain 17, Ezquiaga, Zumalacarregui 17,  
Baker etal 17, Copeland etal 18

# Caveat

De Rham, Melville 18

We do not really know whether the EFT of cosmological scales applies to LIGO/Virgo

The theory may break down (new states appear) at energy scales **parametrically** lower than the cut-off ( $\sim 1000$  km)

The speed of GWs may go back to  $c$  at “short” scale

- Naively requires new physics at scales of order  $10^8 \times 1000$  km to satisfy constraints
- How to reconcile with local tests of gravity?
- Can we say something general about the UV completion? Analogous to frequency dependent index of refraction: Kramers-Kronig?

# Covariant theory

Horndeski 74

Gleyzes, Langlois, Piazza, Vernizzi 14

$$S = \int d^4x \sqrt{-g} \sum_I L_I$$

$$L_2 \equiv G_2(\phi, X), \quad L_3 \equiv G_3(\phi, X) \square \phi,$$

$$L_4 \equiv G_4(\phi, X) {}^{(4)}R - 2G_{4,X}(\phi, X) (\square \phi^2 - \phi^{\mu\nu} \phi_{\mu\nu})$$

$$+ \underline{F_4(\phi, X)} \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'},$$

$$L_5 \equiv G_5(\phi, X) {}^{(4)}G_{\mu\nu} \phi^{\mu\nu}$$

$$+ \frac{1}{3} G_{5,X}(\phi, X) (\square \phi^3 - 3 \square \phi \phi_{\mu\nu} \phi^{\mu\nu} + 2 \phi_{\mu\nu} \phi^{\mu\sigma} \phi^\nu{}_\sigma)$$

$$+ \underline{F_5(\phi, X)} \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} \phi_{\sigma\sigma'},$$

Horndeski

**Beyond** Horndeski

Degeneracy Constraint:  $XG_{5,X}F_4 = 3F_5 [G_4 - 2XG_{4,X} - (X/2)G_{5,\phi}]$

$$c_T = 1 \quad m_4^2 = X^2 F_4 - 3H\dot{\phi}X^2 F_5 - [2XG_{4,X} + XG_{5,\phi} + (H\dot{\phi} - \ddot{\phi})XG_{5,X}] = 0$$

$$\Rightarrow \quad G_{5,X} = 0, \quad F_5 = 0, \quad 2G_{4,X} - XF_4 + G_{5,\phi} = 0$$

$$L_{c_T=1} = G_2(\phi, X) + G_3(\phi, X) \square \phi + B_4(\phi, X) {}^{(4)}R$$

$$- \frac{4}{X} B_{4,X}(\phi, X) (\phi^\mu \phi^\nu \phi_{\mu\nu} \square \phi - \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu}),$$

# Radiative stability

Some operators must be set to zero: is this choice stable?

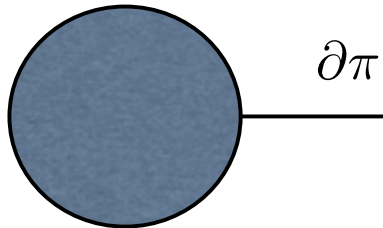
- Approximate Galilean invariance  $\phi \rightarrow \phi + b_\mu x^\mu$

$$\mathcal{L}_3 = (\partial\phi)^2 [\Phi] ,$$

$$\mathcal{L}_4 = (\partial\phi)^2 ([\Phi]^2 - [\Phi^2]) ,$$

$$\mathcal{L}_5 = (\partial\phi)^2 ([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3])$$

- Non renormalization of Galileons [Luty, Porrati, Rattazzi 03](#)

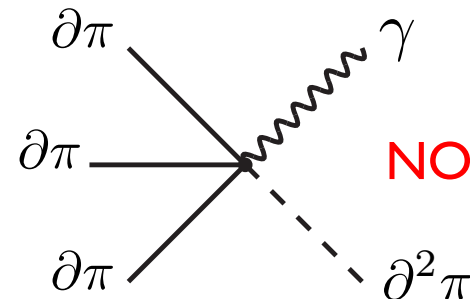


$$\partial^\mu \pi_{\text{ext}} \partial_\mu \pi_{\text{int}} \square_4 \pi_{\text{int}} = \partial^\mu \pi_{\text{ext}} \partial_\nu \left[ \partial_\mu \pi_{\text{int}} \partial_\nu \pi_{\text{int}} - \frac{1}{2} \eta_{\mu\nu} \partial^\rho \pi_{\text{int}} \partial_\rho \pi_{\text{int}} \right]$$

- Broken by gravity

The particular coupling giving 2<sup>nd</sup> order EOM keeps approximate Galilean invariance

[Pirstkhalava, Santoni, Trincherini, Vernizzi 15](#)



# Radiative stability

$$\Lambda_3 \sim (M_P H_0^2)^{1/3} \sim 1000 \text{ km}$$

$$\Lambda_2 \sim (M_P H_0)^{1/2} \sim 0.1 \text{ mm}$$

$$\mathcal{L}_2^{\text{WBG}} = \Lambda_2^4 G_2(X) ,$$

$$\mathcal{L}_3^{\text{WBG}} = \frac{\Lambda_2^4}{\Lambda_3^3} G_3(X) [\Phi] ,$$

$$\mathcal{L}_4^{\text{WBG}} = \frac{\Lambda_2^8}{\Lambda_3^6} G_4(X) R + 2 \frac{\Lambda_2^4}{\Lambda_3^6} G_{4X}(X) ([\Phi]^2 - [\Phi^2]) ,$$

$$\mathcal{L}_5^{\text{WBG}} = \frac{\Lambda_2^8}{\Lambda_3^9} G_5(X) G_{\mu\nu} \Phi^{\mu\nu} - \frac{\Lambda_2^4}{3\Lambda_3^9} G_{5X}(X) ([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3])$$

$$\delta c_n \sim (\Lambda_3/\Lambda_2)^4 \sim 10^{-40} \ll 10^{-15}$$

**The tuning is stable**

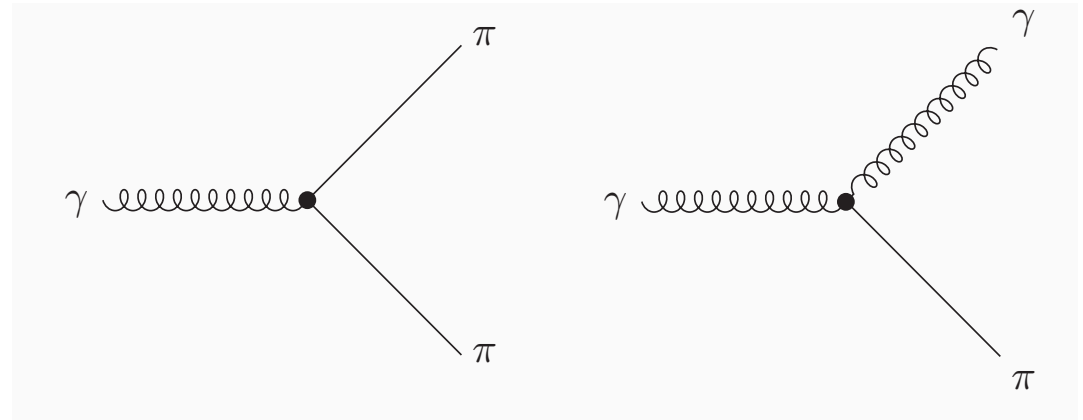
Same holds for Beyond Horndeski theories



# Graviton decay into dark energy

Light is also absorbed as it travels in a material

The (spontaneous) **breaking of Lorentz invariance allows graviton decay**



$$\frac{\tilde{m}_4^2}{2} \delta g^{00} \left( {}^{(3)}R - \delta \mathcal{K}_2 \right)$$

Solving constraints: 
$$\frac{M_\star^2 \tilde{m}_4^2}{M_\star^2 + 2\tilde{m}_4^2} \int d^4x a \ddot{\gamma}_{ij} \partial_i \pi \partial_j \pi$$

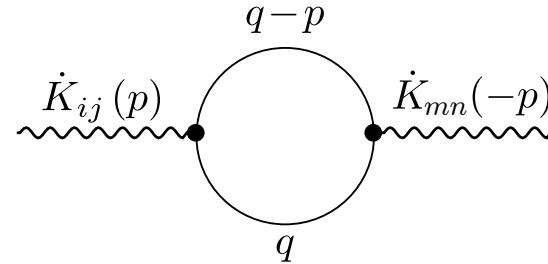
Non-relativistic kinematic: 
$$\Gamma_\gamma = \frac{p^7 (1 - c_s^2)^2}{480 \pi c_s^7 \Lambda^6} \quad \Lambda \sim (H_0^2 M_P)^{1/3}$$

$$\alpha_H \equiv \frac{2\tilde{m}_4^2}{M_{\text{Pl}}^2} \lesssim 10^{-10}$$

irrelevant for LSS observations  
(unless  $c_s=1$  with great precision)

# Higher-derivative terms and dispersion

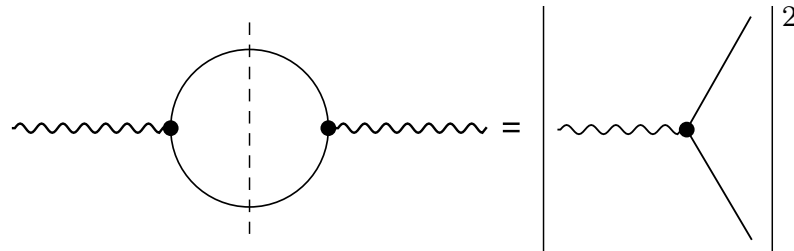
Loops generate higher-derivative corrections



$$S_{\text{eff}} = \frac{M_{\text{Pl}}^2}{480\pi^2\Lambda_*^6 c_s^7} \int \frac{d^4 p}{(2\pi)^4} \bar{p}^4 \dot{K}_{ij}(p) \dot{K}_{ij}(-p) \left[ \frac{1}{\epsilon} + \frac{23}{15} - \frac{\gamma_E}{2} - \frac{1}{2} \log \left( \frac{\bar{p}^2}{4\pi c_s^2 \mu^2} - i\epsilon \right) \right]$$

$$\omega^2 = \mathbf{k}^2 - \frac{\mathbf{k}^8 (1 - c_s^2)^2}{480\pi^2 \Lambda_*^6 c_s^7} \log \left( -(1 - c_s^2) \frac{\mathbf{k}^2}{\mu_0^2} - i\epsilon \right)$$

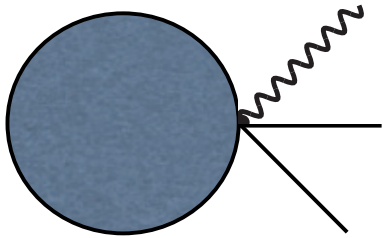
It applies to  $c_s > 1$  as well. For  $c_s = 1$  one still has power divergences



It is inconsistent to look at Horndeski only

# Generic loops

External graviton lines



$$\Lambda_3^4 F \left( \frac{(\partial\phi)^2}{\Lambda_2^4}, \frac{\partial^2\phi}{\Lambda_3^3}, \frac{\gamma^{(c)}}{\Lambda_3}, \frac{\partial}{\Lambda_3} \right)$$

Quartic/quintic Horndeski give large  
higher derivative terms: not exp viable

**Cubic case** scales in a **different** way

$$HM_{\text{Pl}}^2 \delta g^{00} \delta K \sim HM_{\text{Pl}}^2 \dot{\pi} \partial_i \partial_j \pi \gamma_{ij} \quad \Lambda_2 \equiv (H_0 M_{\text{Pl}})^{1/2}$$

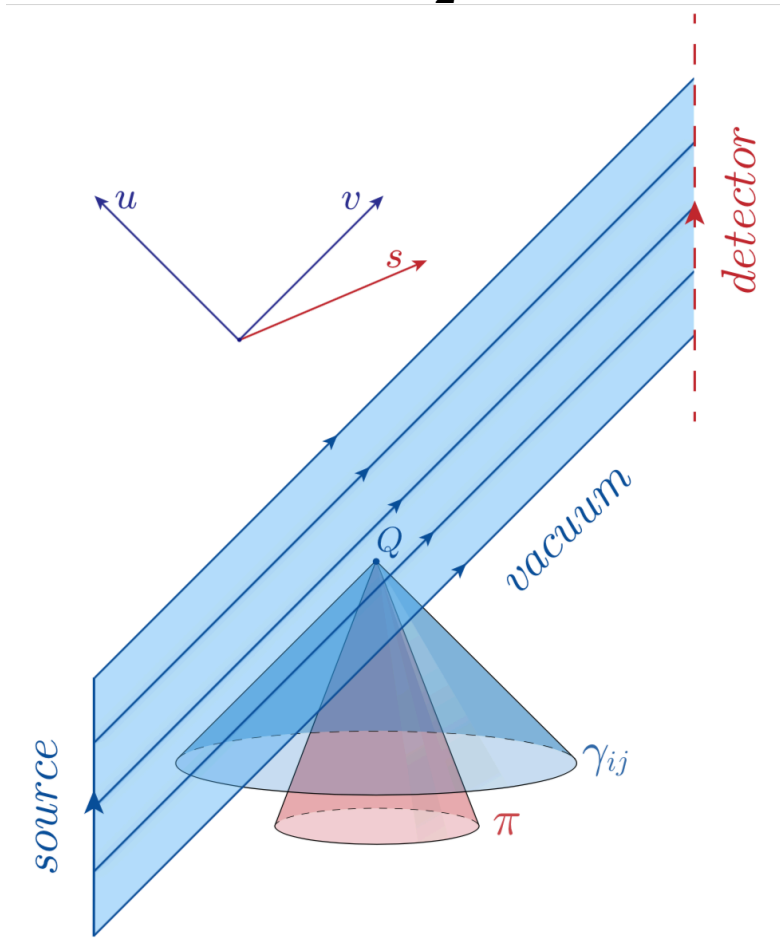
A leg with one derivative must go inside

# Coherent decay

The decay of  $\gamma$  enhanced by the **large occupation number** of GW  
 $\sim$  preheating

$$\mathcal{L}_{m_3} = -\frac{1}{2}\sqrt{-g} m_3^3 \delta K \delta g^{00}$$

$$\ddot{\pi} - c_s^2 \nabla^2 \pi + \frac{2}{\Lambda^2} \dot{\gamma}^{ij} \partial_i \partial_j \pi = 0$$



$$\ddot{\pi} - c_s^2 \nabla^2 \pi + c_s^2 \beta \cos[\omega(t - z)] (\partial_x^2 - \partial_y^2) \pi = 0$$

$$\beta \equiv \frac{2\omega M_{\text{Pl}} h_0^+}{c_s^2 |\Lambda^2|}$$

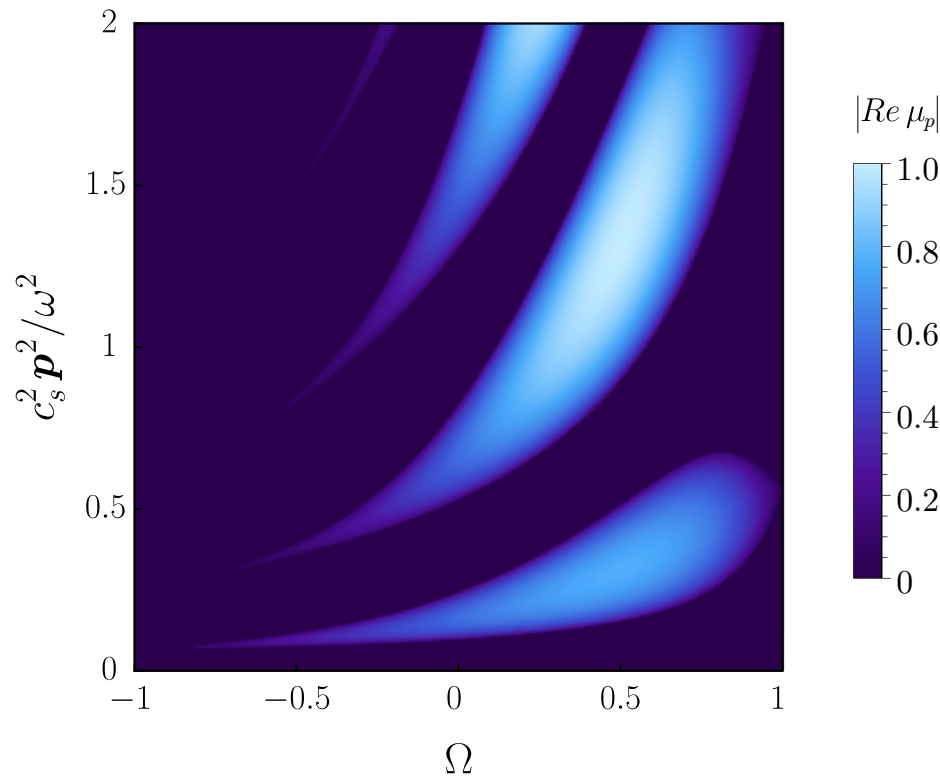
$$u = t - z, \quad s = -t + c_s^{-2} z, \quad v = t + z$$

Similar to a Mathieu equation:  
**parametric resonance**

# Narrow resonance ( $\beta \ll 1$ )

$$\frac{d^2}{d\tau^2}\varphi + [A - 2q \cos(2\tau)]\varphi = 0$$

$$\tau \equiv \frac{\omega u}{2}$$



$$A = 4 \frac{c_s^2 \mathbf{p}^2}{\omega^2} \frac{(1 - c_s \Omega)^2}{(1 - c_s^2)^2} = \frac{4p_u^2}{\omega^2},$$

$$q = 2\beta \frac{c_s^2 \mathbf{p}^2}{\omega^2} \frac{(1 - \Omega^2) \cos(2\phi)}{1 - c_s^2}.$$

**Backreaction on  $\gamma$**

$$\ddot{\gamma}_{ij} - \nabla^2 \gamma_{ij} + \frac{2}{\Lambda^2} \Lambda_{ij,kl} \partial_t (\partial_k \pi \partial_l \pi) = 0$$

# Saddle point

$$J(\tau) = \int g(X) e^{\tau f(X)} d^3 X \quad J(\tau) \approx \frac{1}{\sqrt{\det(-\partial_i \partial_j f(X_0))}} \left( \frac{2\pi}{\tau} \right)^{3/2} g(X_0) e^{\tau f(X_0)}$$

$$\Delta\gamma_{ij}(u, v) \simeq -\frac{v}{4\Lambda^2} \frac{(1 - c_s^2)^2}{c_s^5 \sqrt{\beta}} \frac{\omega^{5/2}}{(8u\pi)^{3/2}} \sin\left(\omega u + \frac{\beta}{2}\right) \exp\left(\frac{\beta}{4}\omega u\right) \epsilon_{ij}^+$$

- Same direction and polarisation
- At higher order in  $\beta$  one gets harmonics.  
They enter in the obs window before: **precursors**

# $\pi$ interactions

**BUT** one expects the exp growth is quenched when  $\pi$  self-interactions are relevant

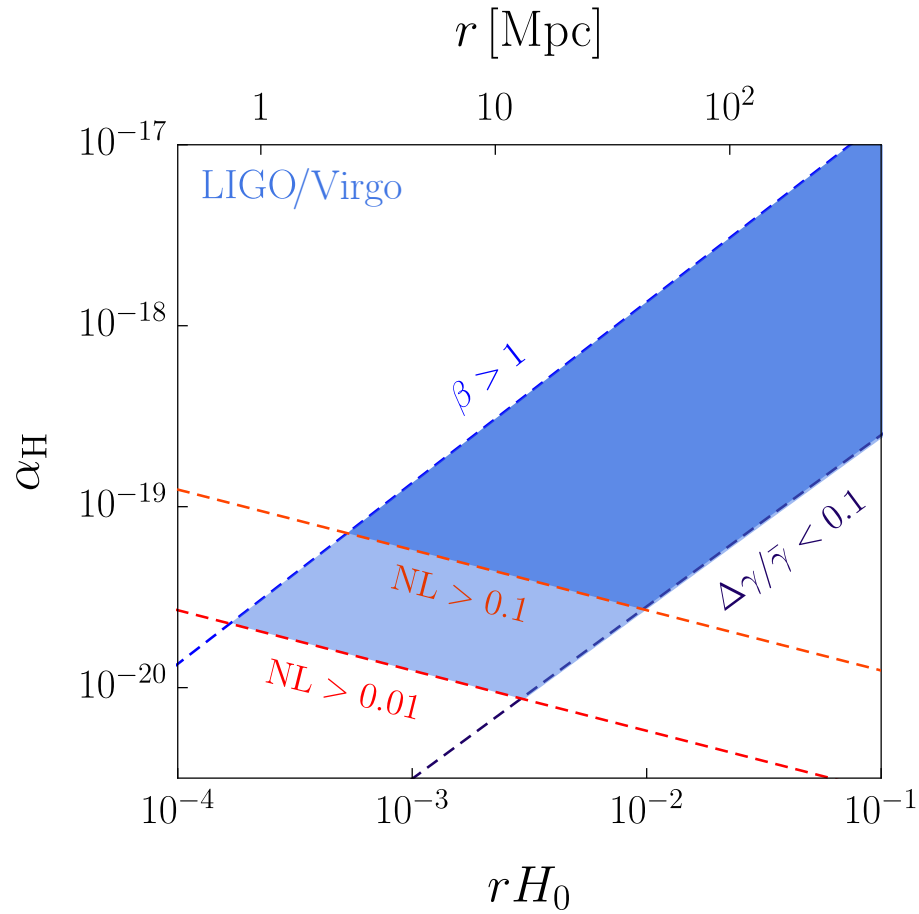
$$\frac{1}{\Lambda_3^3} \nabla^2 \pi (\partial \pi)^2 \quad \text{VS} \quad \frac{1}{\Lambda_2^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi$$

- Kills the effect!? Simulations  $\sim$  preheating
- For  $\tilde{m}_4^2$  self-interactions are small: same scale  $\Lambda_* \gg \Lambda_3$

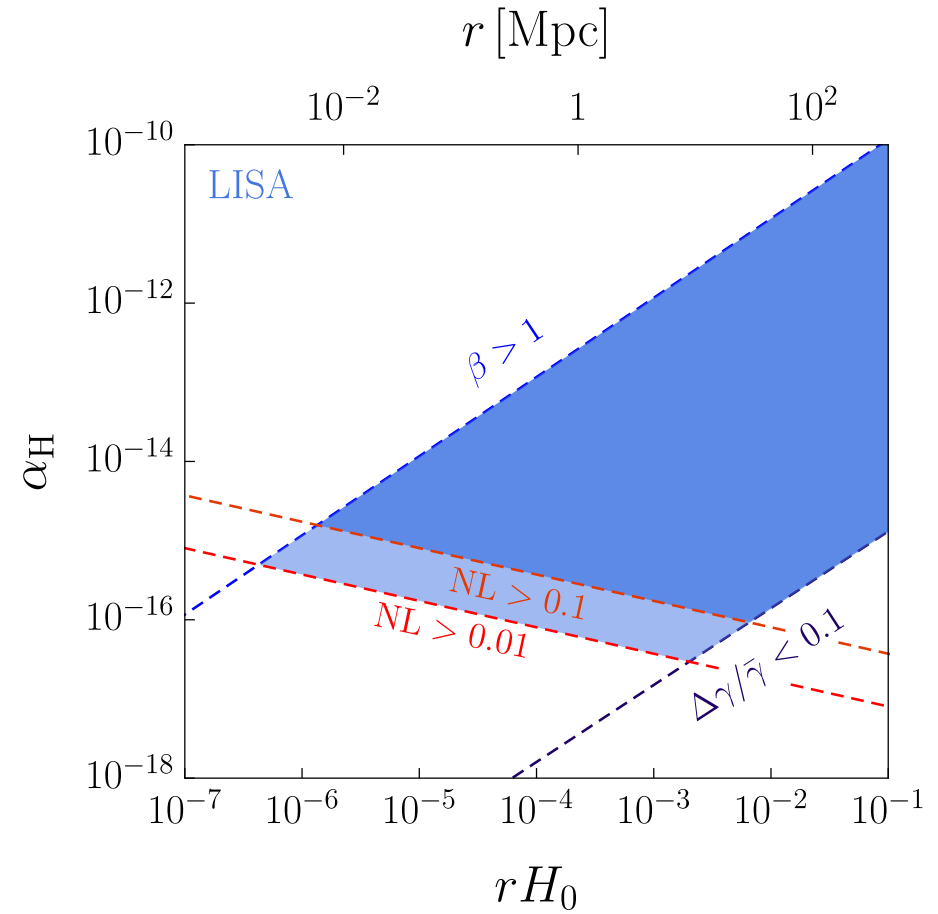
On saddle point, accidental cancellation due to Galileon structure

$$\frac{\Delta \gamma}{\bar{\gamma}} \sim \frac{v \omega (\partial_i \pi)^2}{\Lambda_*^3 M_{\text{Pl}} h_0^+} \lesssim \beta c_s^3 \alpha(v H_0) \frac{H_0}{\omega h_0^+} \sqrt{\beta \tau}$$

# GW modification



$f = 30 \text{ Hz}, M_c = 1.2 M_{\text{sun}}$



$f = 10^{-2} \text{ Hz}, M_c = 30 M_{\text{sun}}$

Perturbative bound:  $\alpha_H < 10^{-10}$



# $\beta > 1$ : $\pi$ instability

$$\ddot{\pi} - c_s^2 \nabla^2 \pi + c_s^2 \beta \cos[\omega(t - z)] (\partial_x^2 - \partial_y^2) \pi = 0$$

Gradient instability

- Generically present for  $m_3$
- To be contrasted with non-linear stability of cubic Galileon

Nicolis Rattazzi 04

$$\mathcal{L}_{(2)} = Z^{\mu\nu} \partial_\mu \pi \partial_\nu \pi$$

Eom imply eigenvalues of the matrix  $Z^{\mu\nu}$  do not flip sign

- Fate of instability is **UV sensitive**. Does it affect GWs?

For  $\beta > 1$  one has to consider  $\pi$  background generated by  $\gamma\gamma\pi$ : this modifies  $\pi$  eom.

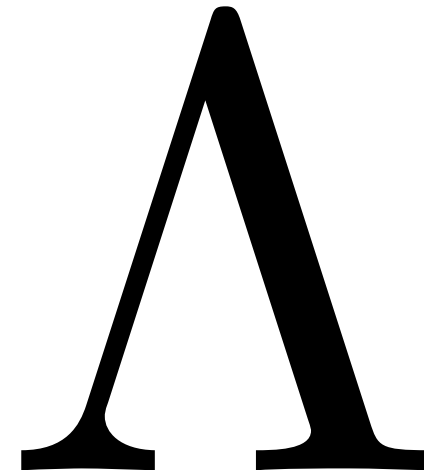
One gets a system similar to Nicolis-Rattazzi 04, but still unstable since  $Z^{\mu\nu}$  is not diagonalizable

...in progress

# Conclusions

GWs probe Dark Energy as light probes a material

- Measurement speed of GWs
- Constraints from perturbative graviton decay and dispersion
- Resonant graviton decay
- Instability due to GW
- Caveats...



**Backup slides**

# Why consider so complicated theories ??

To modify gravity one has to introduce extra dof

Scalars will play with the graviton through  $\partial_\mu \partial_\nu \pi$

- Massive gravity. Longitudinal mode  $g_{\mu\nu} \supset \partial_\mu \partial_\nu \pi$  E.g. De Rham, Gabadadze, Tolley 10
- DGP model



5D Minkowski bulk

Dvali, Gabadadze, Porrati 00

Brane bending mode

$$g_{5\mu} \sim \partial_\mu \pi$$



Actions for scalars with many derivatives  
(but 2<sup>nd</sup> order equations)

# Another caveat

Copeland, Kopp, Padilla,  
Saffin, Skordis 18

We imposed that  $c_T=1$  is robust to changes of  $H(t)$  and  $\phi(t)$ ,  
but there are possible cancellations that we missed

$$\begin{aligned}
 S = \int d^4x \sqrt{-g} & \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\
 \rightarrow & -\frac{m_3^3}{2} \delta K \delta g^{00} - \cancel{m_4^2 \delta \mathcal{K}_2} + \cancel{\frac{\tilde{m}_4^2}{2} \delta g^{00} R} - \cancel{\frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2} \\
 & \left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \cancel{\frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3} \right].
 \end{aligned}$$

The cancellation does not work in the presence of curvature  
and thus in the perturbed universe

# Beyond Beyond Horndeski: **DHOST**

Even more general theories propagating a single dof

A combination of:  $\int d^4x \sqrt{-g} \frac{M^2}{2} \left( -\frac{2}{3} \alpha_L \delta K^2 + 4\beta_1 \delta KV + \beta_2 V^2 + \beta_3 a_i a^i \right)$

These do not affect GWs on any background

Can be obtained by:  $g_{\mu\nu} \rightarrow C(\phi, X) g_{\mu\nu}$

$$\begin{aligned} L_{c_T=1} = & \tilde{B}_2 + \tilde{B}_3 \square \phi + CB_4 {}^{(4)}R - \frac{4CB_{4,X}}{X} \phi^\mu \phi^\nu \phi_{\mu\nu} \square \phi \\ & + \left( \frac{4CB_{4,X}}{X} + \frac{6B_4 C_{,X}^2}{C} + 8C_{,X} B_{4,X} \right) \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu} \\ & - \frac{8C_{,X} B_{4,X}}{X} (\phi_\mu \phi^{\mu\nu} \phi_\nu)^2 . \end{aligned}$$