Leonardo Senatore (Stanford)

$\lambda\phi^4$ in de Sitter

with V. Gorbenko

in progress

Leonardo Senatore (Stanford)

The Effective Field Theory of Large-Scale Structure applied to Data: the analysis of BOSS

Guido d'Amico, Jerome Gleyzes, Nickolas Kockron, Dida Markovic, Leonardo Senatore, Pierre Zhang, Florian Beutler, Hector Gill-Marin

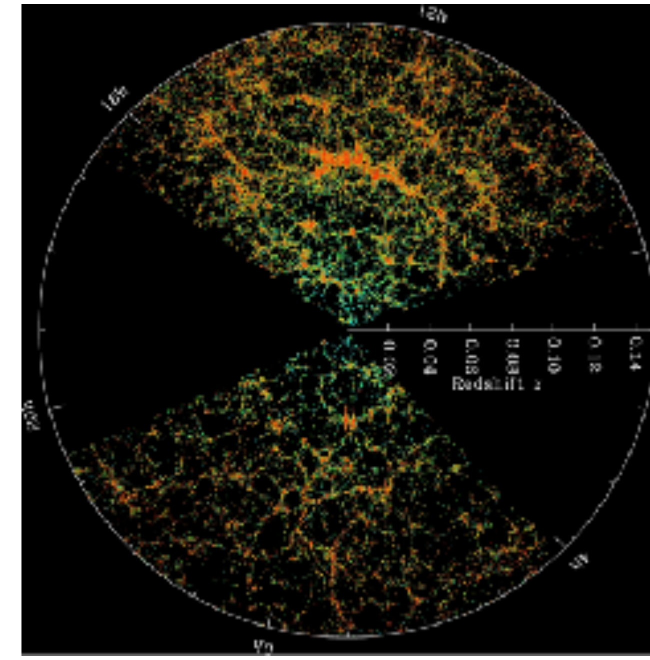
in progress

The future of data-driven cosmology

- To me, the future of data-driven cosmology is in our capability of understanding large-scale structure data.
- In fact, information comes from cosmological modes, and they are there.

- This is why I have devoted so much effort in the last few years to the development of the EFTofSS

A long, long, lonely journey



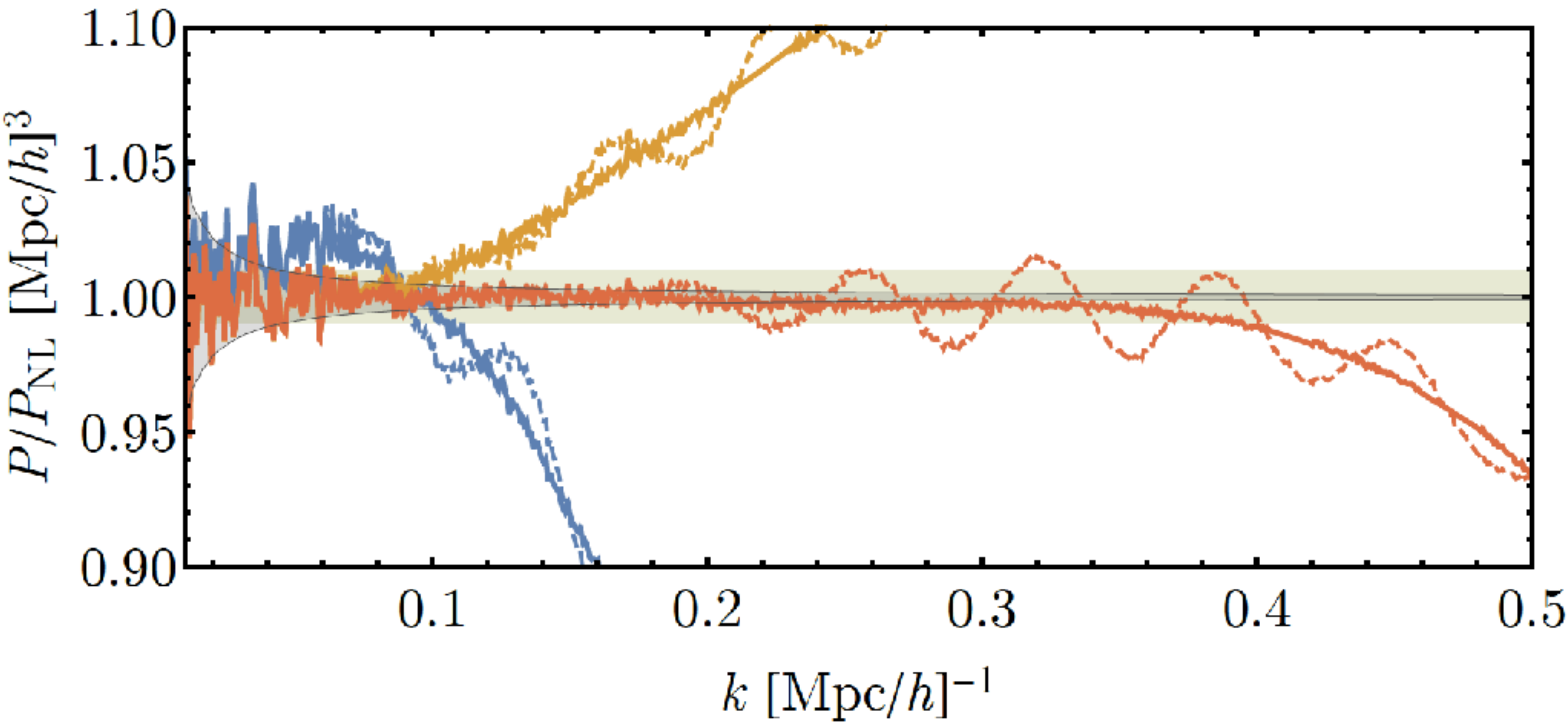
- Correlations of Galaxy density in Redshfit space
- In terms of Correlations of Galaxy density and velocity in real space + EFT parameters
- In terms of Correlation of dark matter and tidal tensors, etc. + EFT parameters
- Dark matter correlations from fluid equations + EFT parameters

- $\rightarrow$ IR-resummation

IR-resummation and the BAO peak

- It works very well

with Zaldarriaga **1404**
with Trevisan **JCAP1805**



- Similarly well in redshift space with Lewandowski et al **1512**
- Subsequent approximation of this formula, such as the wiggle-nowiggle one.

Galaxies in the EFTofLSS

Senatore **1406**

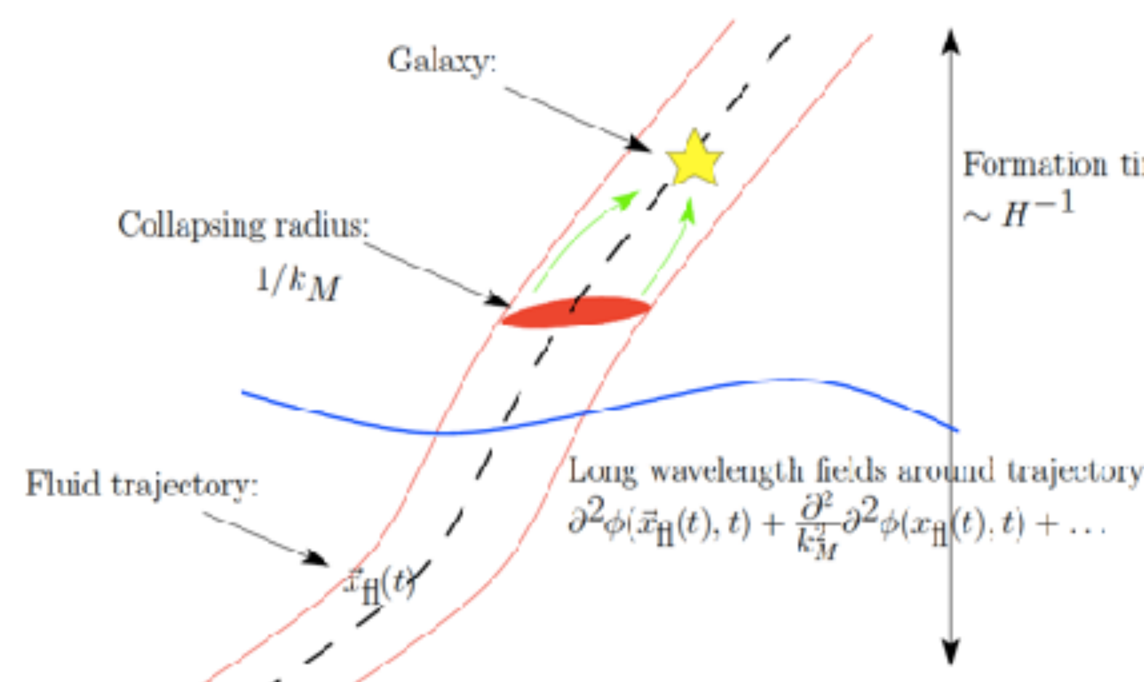
- Fist, original, correct parametrization of biases.

$$n_{\rm gal}(\vec{x},t) = f_{\text{very complicated}}\left[\{H(t'),\Omega_{\rm dm}(t'),\ldots,\rho_{\rm dm}(x',t'),\rho_b(x',t'),\ldots,m_e,m_p,g_{ew},\ldots\}\big|_{\text{on past light cone}}\right]$$

$$\Downarrow \quad \text{Taylor Expansion}$$

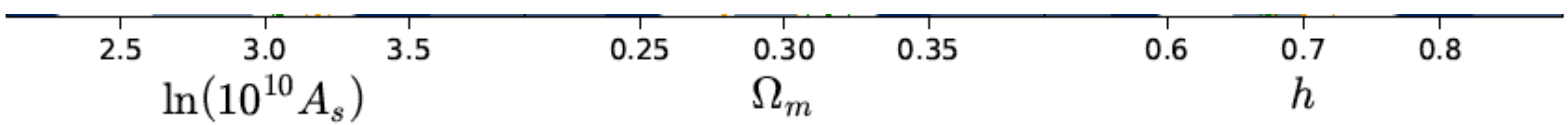
$$\begin{aligned}
\delta_M(\vec{x},t) \simeq \int^t dt'\, H(t') \Bigg[& \bar{c}_{\partial^2\phi}(t,t')\,\frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} \\
& + \bar{c}_{\partial_i v^i}(t,t')\,\frac{\partial_i v^i(\vec{x}_{\rm fl},t')}{H(t')} + \bar{c}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t,t')\,\frac{\partial_i\partial_j\phi(\vec{x}_{\rm fl},t')}{H(t')^2}\frac{\partial^i\partial^j\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \ldots \\
& + c_\epsilon(t,t')\,\epsilon(\vec{x}_{\rm fl},t') + c_{\epsilon\partial^2\phi}(t,t')\,\epsilon(\vec{x}_{\rm fl},t')\frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \ldots \\
& + c_{\partial^4\phi}(t,t')\,\frac{\partial^2_{x_{\rm fl}}}{k_{\rm M}{}^2}\frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \ldots \Bigg].
\end{aligned}$$

- all terms allowed by symmetries are present
- all physical effects are included
- former ones are not complete
- subsequent ones are a change of basis of this



Analysis of the SDSS/BOSS data

–*Preliminary* results of the power spectrum analysis of the CMASS NGC sample



- We assume flat $\Lambda\mathrm{CDM}$ and Planck's n_s & Ω_b/Ω_m
- and measure $A_s,\ \Omega_m,\ H_0,\ b_1 \quad \leftrightarrow \quad f,\ \sigma_8,\ H_0,\ b_1$
- These *preliminary* results, if confirmed, tell us that there is the potentiality of much improving the whole *legacy* of SDSS.
- These results are so important that Matias set up an independent group to check them

An EFT for testing extensions of GR with Gravitational Waves

with Endlich, Huang, Gorbenko

2017

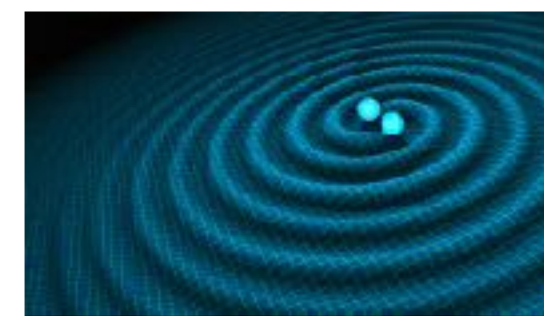
An EFT for probing extensions of GR at LIGO

- The most general such a Lagrangian is

$$S_{\text{eff}} = 2M_{\text{pl}}^2 \int d^4x\sqrt{-g} \left(-R + \frac{\mathcal{C}^2}{\Lambda^6} + \frac{\tilde{\mathcal{C}}^2}{\tilde{\Lambda}^6} + \frac{\tilde{\mathcal{C}}\mathcal{C}}{\Lambda_-^6} + \dots\right)$$

$$\mathcal{C} \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}, \quad \tilde{\mathcal{C}} \equiv R_{\alpha\beta\gamma\delta}\,\epsilon^{\alpha\beta}{}_{\mu\nu}\,R^{\mu\nu\gamma\delta}\ ,$$

- No superluminality $\sim\Rightarrow \quad \text{No } R^3_{\mu\nu\rho\sigma}$ Camanho, Edelstain, Maldacena, Zhiboeadov **2016**



- Testable at LIGO $\Rightarrow \quad \Lambda \sim 10^{-1}\ \text{Km}^{-1}$
- Not ruled out by GR tests

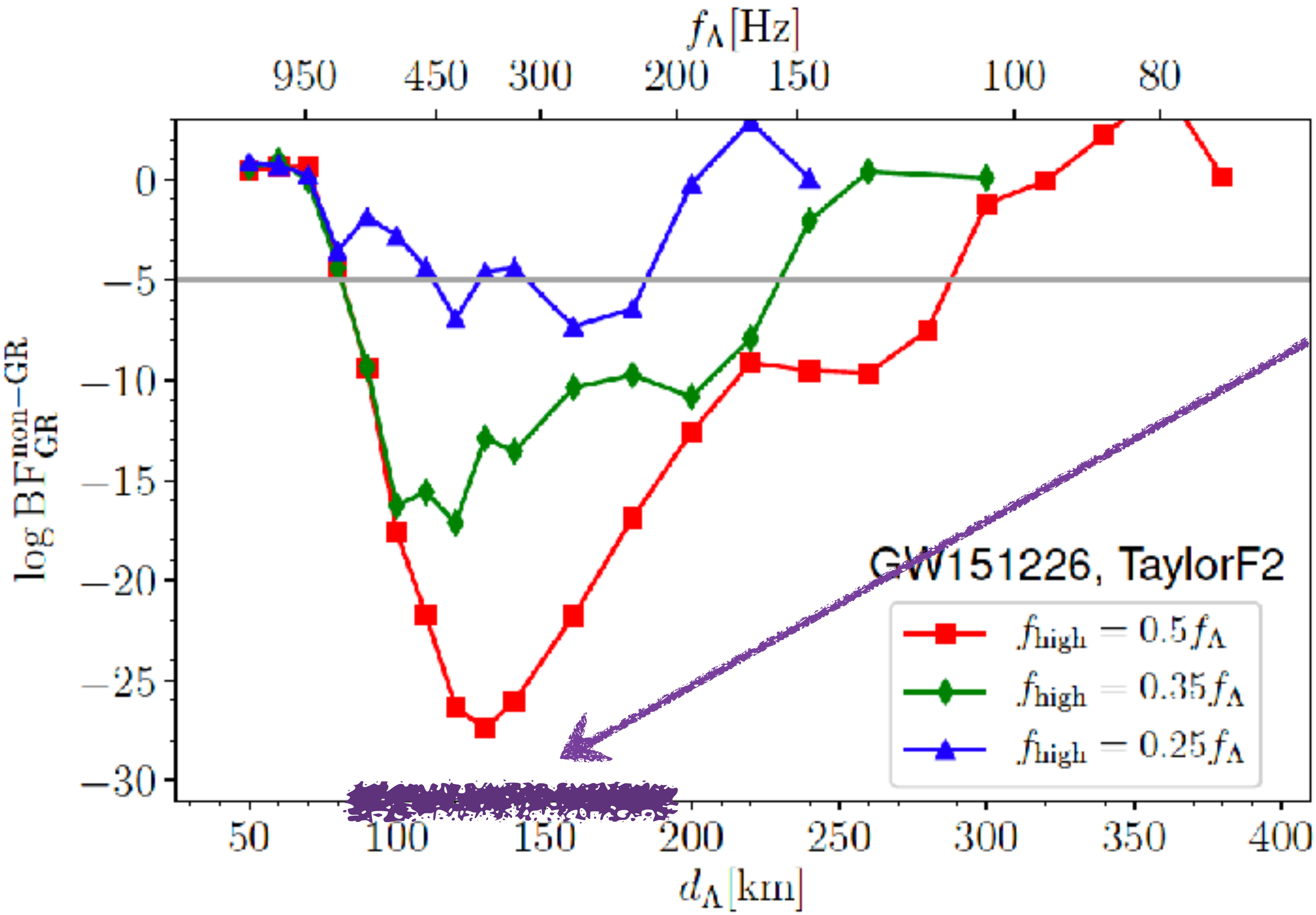
$$\Rightarrow \quad \delta g^{\mu\nu}T_{\mu\nu} = \text{as in GR} \quad \& \quad \text{amplitudes saturate when UV enters}$$

- In this way, information from LIGO can be mapped into parameters of a fundamental physics Lagrangian
  - instead of into some arbitrary and potentially-unphysical rescaling of the post-Newtonian parameters of the templates $\omega(r) = \sum_n c_n\, v^n$

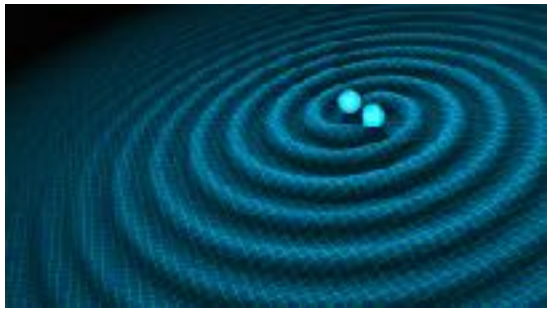
First bound from LIGO data

- LIGO members, preliminary

with Sennett, Brito, Buonanno, Gorbenko, Senatore
in progress



Excluded



$$S_{\mathrm{eff}} = 2M_{\mathrm{pl}}^2 \int d^4x\sqrt{-g}\,\left(-R + \frac{\mathcal{C}^2}{\Lambda^6} + \frac{\tilde{\mathcal{C}}^2}{\tilde{\Lambda}^6} + \frac{\tilde{\mathcal{C}}\mathcal{C}}{\Lambda_-^6} + \dots\right)$$

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Introduction

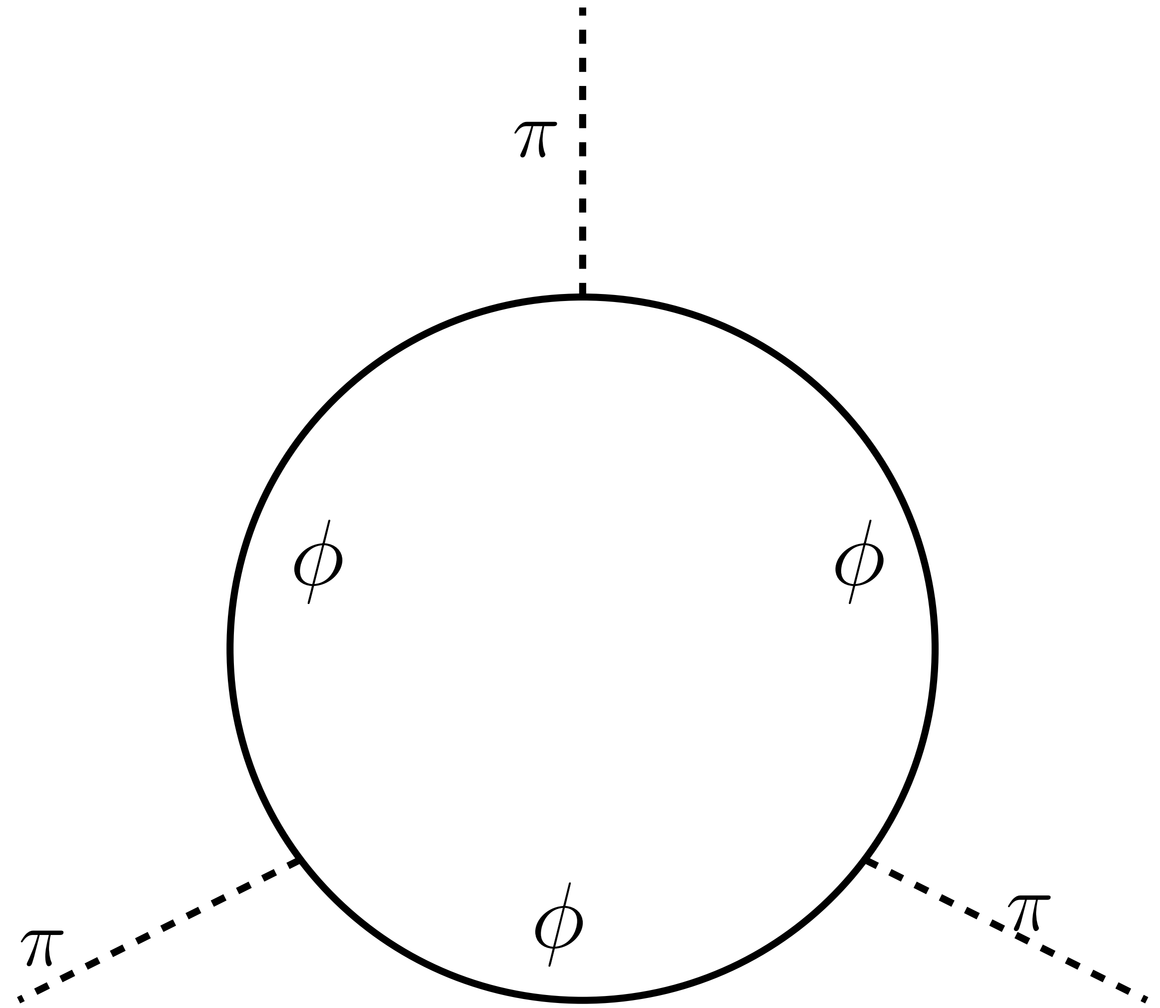
- Massless $\lambda\,\phi^4$ is IR-divergent in dS: we do not know how to make predictions
- Why do we care?
  - 1) This is somewhat embarrassing

Introduction

– Massless $\lambda\,\phi^4$ is IR-divergent in dS: we do not know how to make predictions

– Why do we care?

  – 2) It could have phenomenological consequences, for example to Black Holes from inflation, or to non-Gaussianities



Introduction

– Massless $\lambda\,\phi^4$ is IR-divergent in dS: we do not know how to make predictions

– Why do we care?

– 3) Since inflation is most probably the theory of the early universe, we should be able to understand its radiative corrections

- For single-field inflation, we have a satisfactory and complete understanding

with Zaldarriaga **JHEP 2010, JHEP 2012, JCAP 2012, JHEP 2013**
with Pimentel and Zaldarriaga **JHEP 2012**

$$\langle \zeta_k^2 \rangle \quad \supset \quad \log\left(\frac{H}{\mu}\right)\ , \quad Ht\ , \quad \log(kL)$$

Logarithmic Running
After S. Weinberg **2008** had begun the exploration,
finding $\log\,(k/\mu)$

- but not for non-derivatively coupled multifield

see KITP video of 2015 String Program

Introduction

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$$\langle\zeta_k^2\rangle \quad \supset \quad \log\left(\frac{H}{\mu}\right)\ , \quad Ht\ , \quad \log(kL)$$

Out-of-Horizon time-dependence:
absent, to all loops

- but not for non-derivatively coupled multifield

see KITP video of 2015 String Program

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$$\langle \zeta_k^2 \rangle \quad \supset \quad \log\left(\frac{H}{\mu}\right) \ , \quad H t \ , \quad \log(kL)$$

Logarithmic dependence on IR-cutoff of the universe: absent once define observable quantities.

- but not for non-derivatively coupled multifield

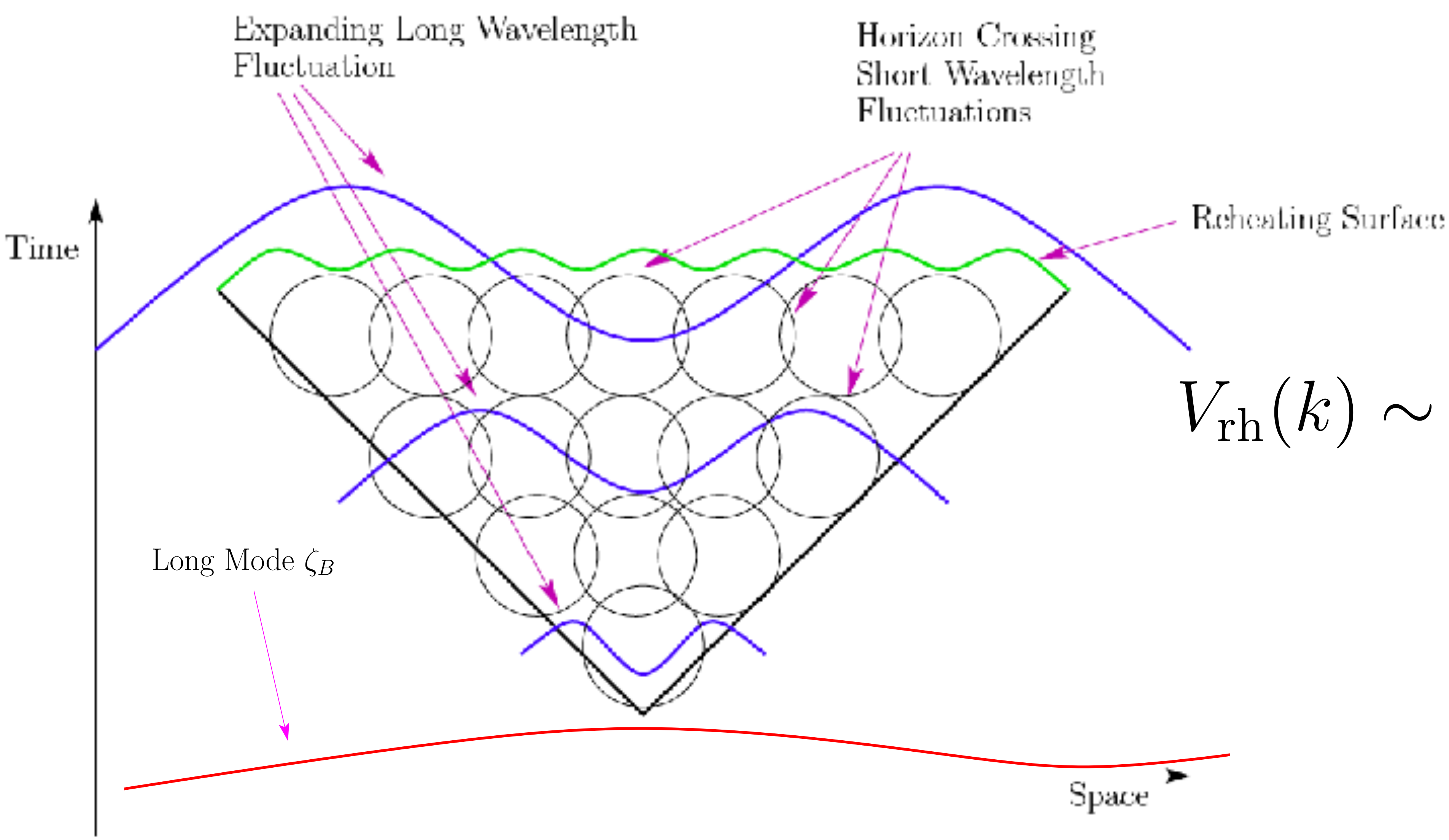
see KITP video of 2015 String Program

Introduction

– Massless $\lambda\,\phi^4$ is IR-divergent in dS: we do not know how to make predictions

– Why do we care?

  – 4) Quantum-enhanced expansion and eternal inflation



$$V_{\rm rh}(k) \sim \left(1+\langle\zeta^2\rangle\right)\frac{e^{3N_e(k)}}{H^3}$$

Introduction

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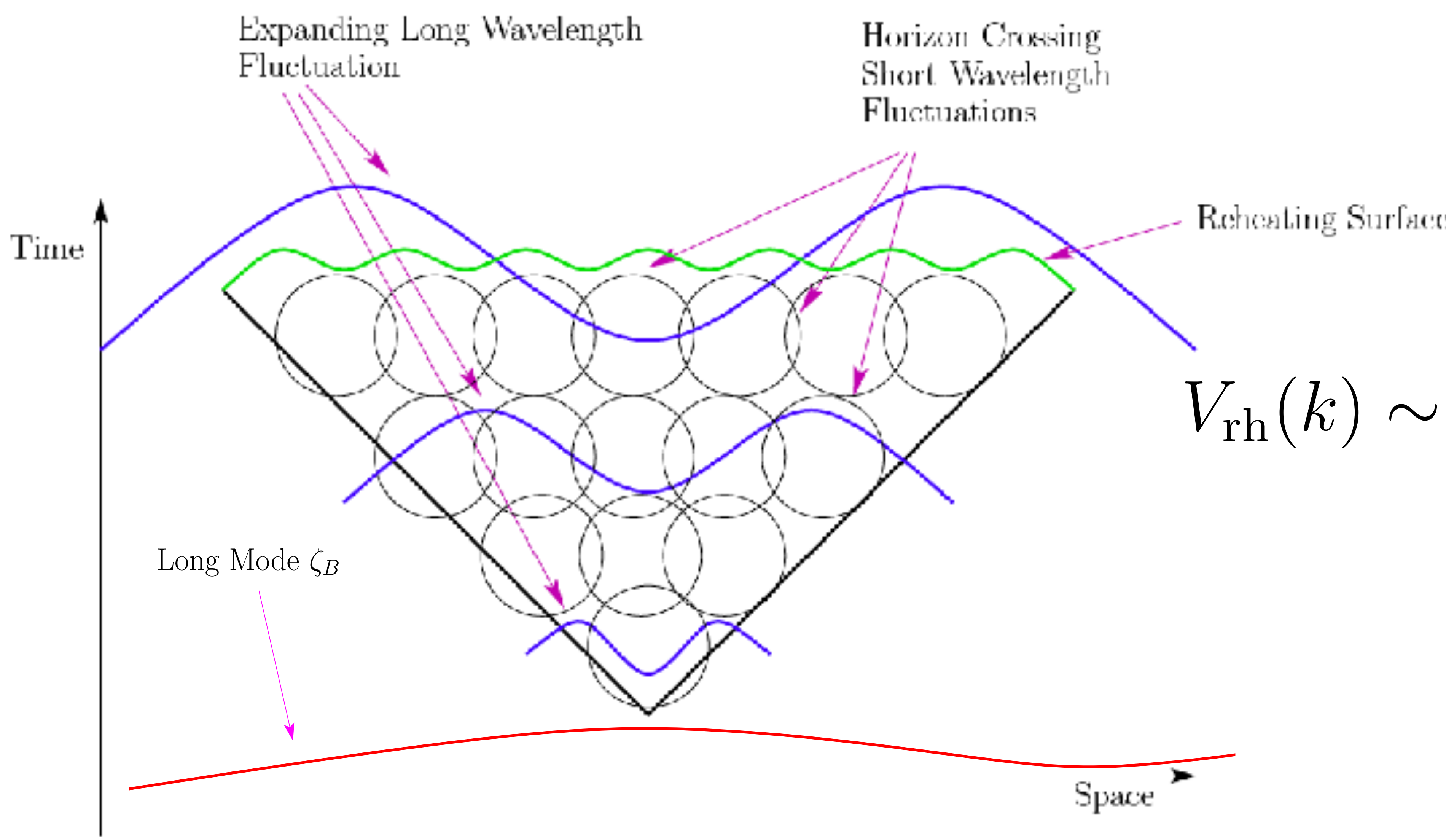

$$V_{\rm rh}(k) \sim \left(1+\langle\zeta^2\rangle\right)\frac{e^{3N_e(k)}}{H^3}$$

reheating volume
at classical level

Introduction

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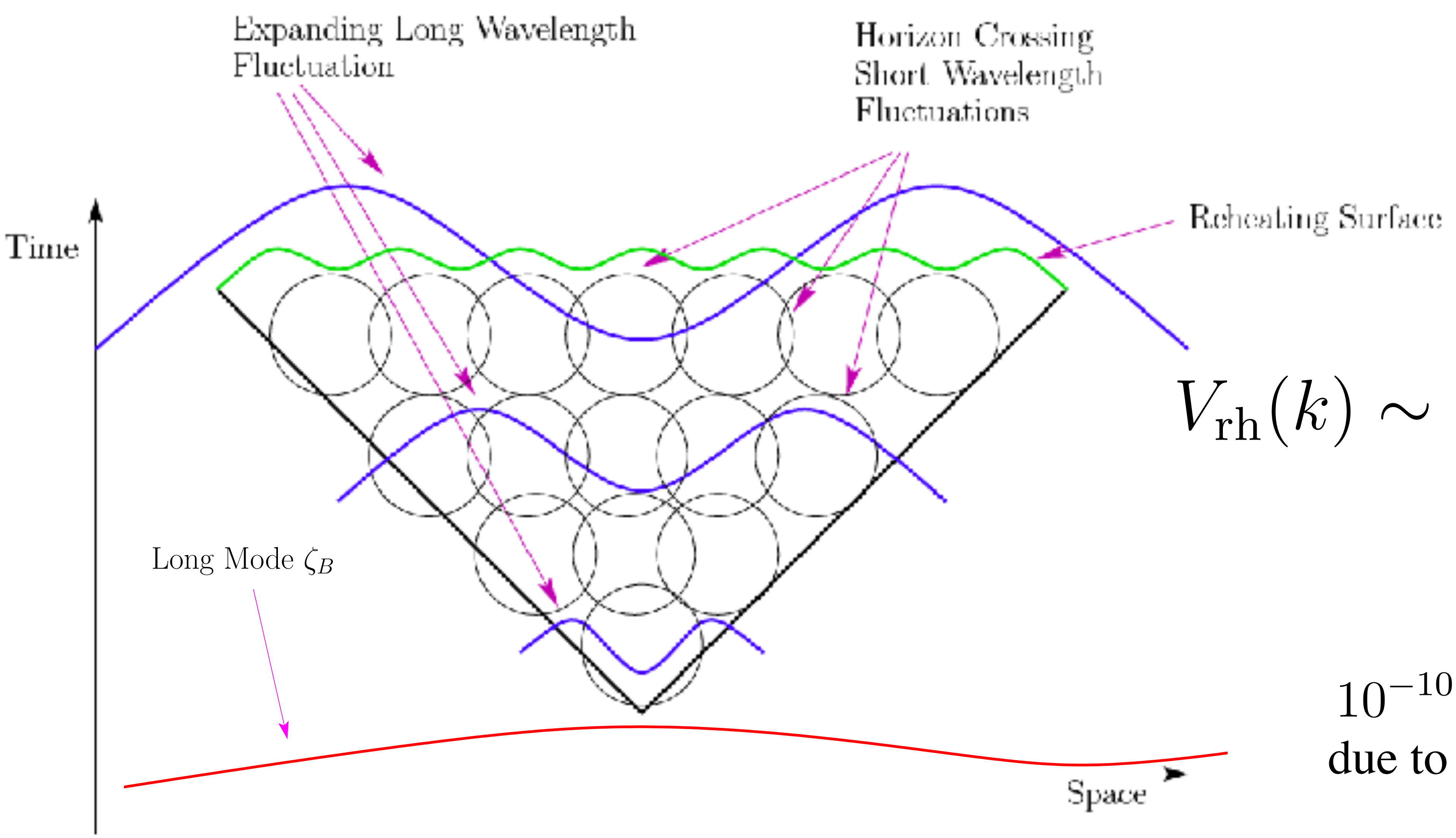


$$V_{\rm rh}(k) \sim \left(1 + \langle \zeta^2 \rangle\right) \frac{e^{3N_e(k)}}{H^3}$$

10^{-10} quantum enhancement
due to shorter modes

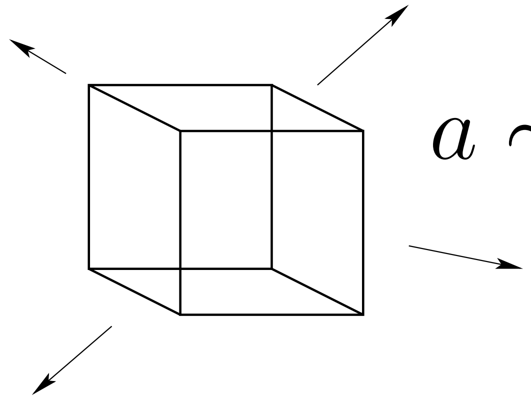
Introduction

– Massless $\lambda\,\phi^4$ is IR-divergent in dS: we do not know how to make predictions

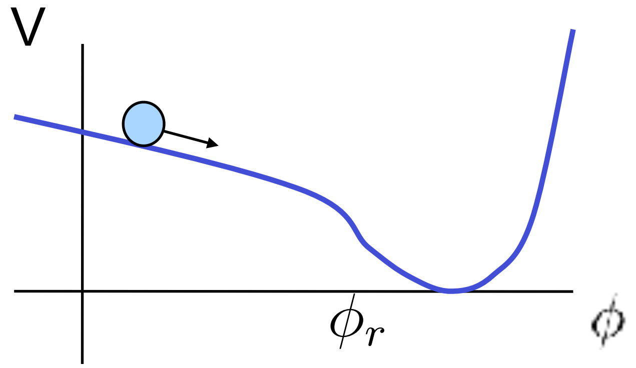
– Why do we care?

  – 4) Slow-Roll Eternal Inflation

    • This is a different application: let us elaborate



$$a \sim e^{Ht}$$



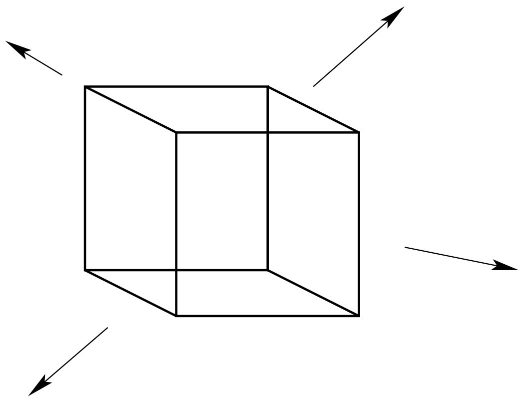
$$\zeta \sim H\frac{\delta\phi}{\dot\phi}$$

$$V_{\rm rh}(k) \sim \left(1+\langle\zeta^2\rangle\right)\frac{e^{3N_e(k)}}{H^3}$$

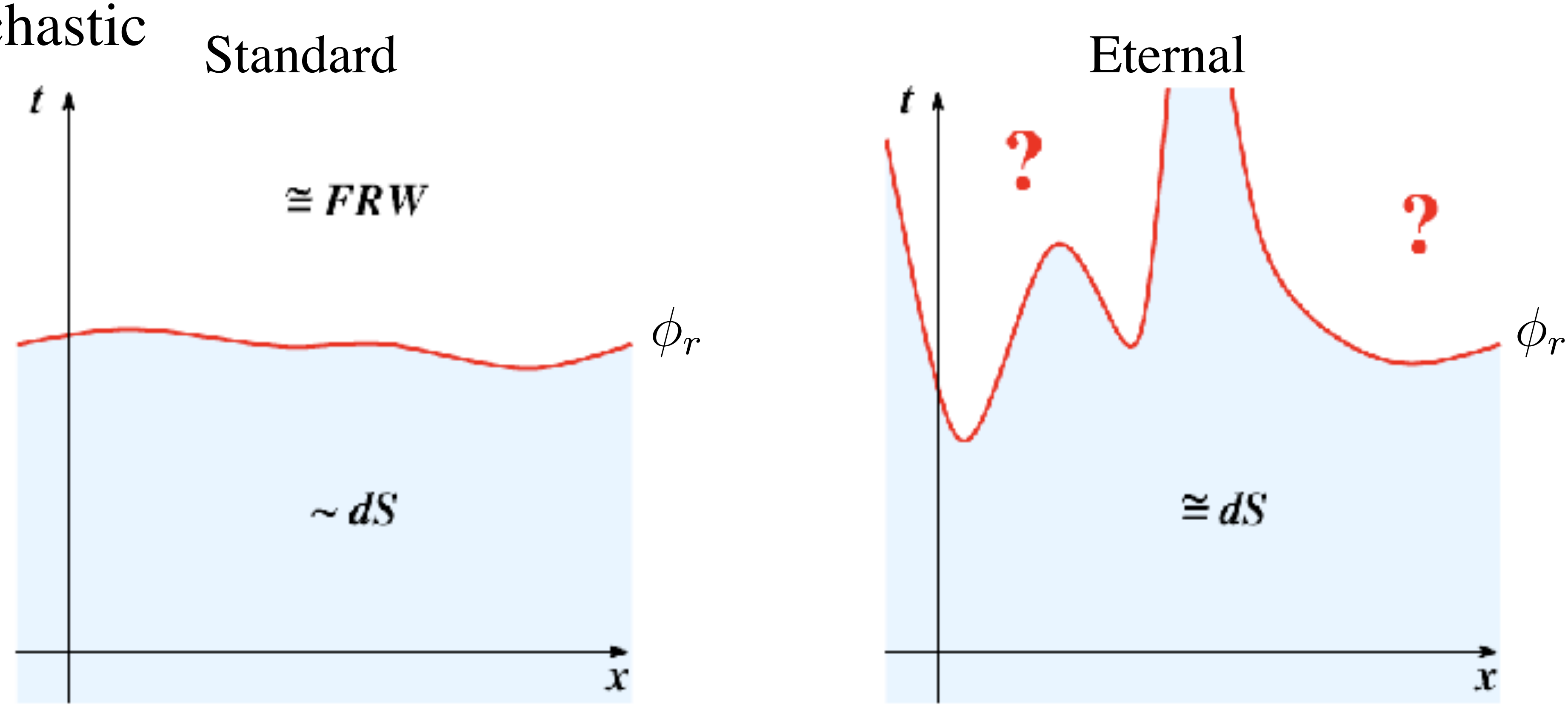
~ 1

Slow-Roll Eternal Inflation

- The spacetime becomes stochastic

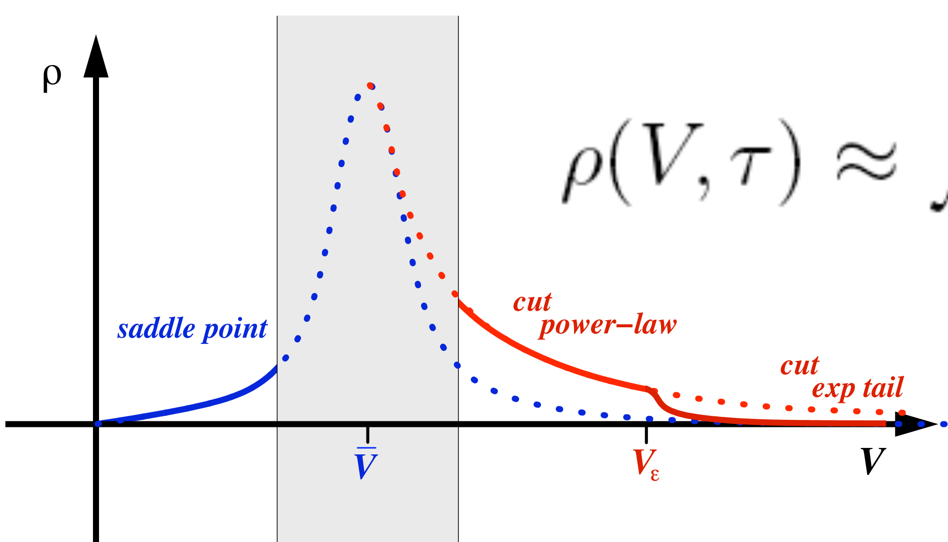


$$\Omega = \frac{\dot{\phi}^2}{H^4}$$



- Much more radical quantum effect on spacetime than the Black-hole evaporation

- Discovered in the 80's, we provided a first rigorous quantitative understanding



$$\rho(V,\tau) \approx \mathcal{N} e^{-\Omega\left[\frac{3N}{2}\left(1+\sqrt{1-\frac{1}{\Omega}}\right)-3N_c\right]^2}$$

With Creminelli, Dubovsky, Nicolis and Zaldarriaga **JHEP 2008**
With Dubovsky and Villadoro **JHEP2009; JHEP2012**

$$e^{3N_e} \to e^{\mathbf{6}N_e}$$

In particular: $P\left[V_{\text{finite}} > e^{S_{\text{dS}}/2}\right] \lesssim e^{-S_{\text{dS}}}$

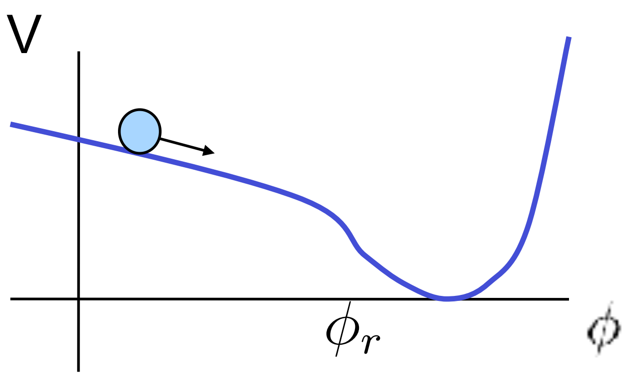
Slow-Roll Eternal Inflation

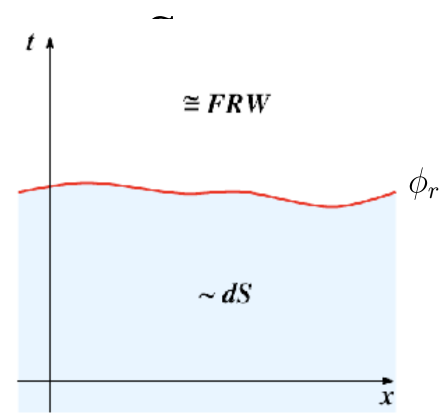
- The initial discovery and our quantitative understanding were based on a so-called Stochastic equation for inflationary fluctuations initially proposed and developed by Starobinsky in the 80's.

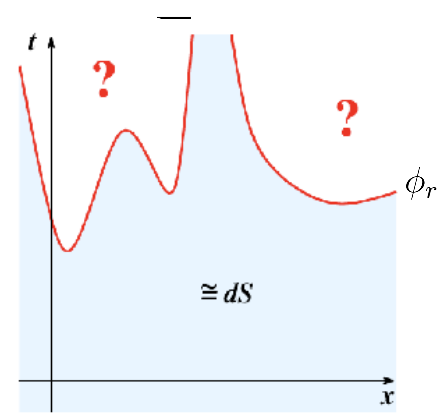
- According to which the probability distribution of the quantum fluctuations satisfies a Fokker-Planck equation.

Starobinsky, **1980's**

$$\frac{\partial}{\partial t}P(\phi(\vec{x}))=H^2\frac{\partial^2}{\partial\phi(\vec{x})^2}P(\phi(\vec{x}))+\frac{\partial}{\partial\phi(\vec{x})}\left(V'(\phi(\vec{x}))P(\phi(\vec{x}))\right)$$







- As we will see, this same equation is supposed to solve non-perturbatively $\lambda\phi^4$ in dS

- Lacking: a satisfactory derivation, an understanding of if this is a toy model or the leading expansion in \`*something*', and, if it is the second, if it can be made a precise approach.

Introduction

- Massless $\lambda\,\phi^4$ is IR-divergent in dS: we do not know how to make predictions
- Why do we care?
  - 5) Polyakov worries that dS is unstable due to *these* radiative corrections
    - he worries dS invariance is spontaneously broken by these IR divergencies.
      - apparently, he does not believe Starobinsky equation
      - and many prestigious people are uncomfortable with it

Introduction

- Massless $\lambda\,\phi^4$ is IR-divergent in dS: we do not know how to make predictions
- Why do we care?
  - 6) In agreement with Polyakov, so far gravity has kept its most stunning surprises in the IR
    - Black Evaporation and information loss
    - dS entropy

Summary of Introduction

- Both
  - –Massless $\lambda\,\phi^4$ in dS
  - –Slow-Roll eternal inflation
- are non-perturbative phenomena
- The Stochastic equation $\frac{\partial}{\partial t}P(\phi(\vec{x})) = H^2\frac{\partial^2}{\partial\phi(\vec{x})^2}P(\phi(\vec{x})) + \frac{\partial}{\partial\phi(\vec{x})}\left(V'(\phi(\vec{x}))P(\phi(\vec{x}))\right)$
  - –might provide a way to solve for them
  - –but lacks a systematic derivation and proof of accuracy
- Modulo some further checks/subtleties, here we will prove that that equation is the leading-order truncation of a generalized equation, from which we can derive arbitrary accurate results.
  - – Proving the existence slow-roll eternal inflation
  - – Solving $\lambda\,\phi^4$ in de Sitter
- Of course, the value of these results depend on how much you already believed in the stochastic approach: a toy model? order one correct? parametrically and systematically correct?

Let us start

IR-divergencies

– Consider a massive field in dS

$$\langle \phi_k^2 \rangle \sim e^{-3Ht} \left[H^{(1)}_{\nu=\sqrt{9/4-m^2/H^2}} \left(\frac{k}{aH} \right) \right]^2 \qquad a \sim e^{Ht}$$

– Look at late times, assuming the mass is small:

$$\langle \phi_k^2 \rangle \to \frac{H^2}{k^3} \left(1 + \frac{m^2}{H^2} \log \left(\frac{k}{a(t)H} \right) \right)$$

– This perturbative expansion, obviously, breaks at late times $\frac{k}{a(t)H} \sim e^{-\frac{H^2}{m^2}}$

- Notice that this is the answer we would have gotten if we had treated the mass as a perturbation: at late times:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{k^2}{a^2}\phi = m^2\phi \quad \Rightarrow \quad \phi^{(1)} \sim \int dt' \, \frac{1}{3H} m^2 \phi^{(0)}(t') \sim \phi^{(0)} \, \frac{m^2}{H^2} H \, t$$

IR-divergencies

– Consider a massive field in dS

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IR-divergencies

– Consider a massive field in dS

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IR-divergencies

– Now, consider massless $\lambda\,\phi^4$

– Solving perturbatively

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \frac{k^2}{a^2}\phi_k = \lambda[\phi^3]_k \quad \Rightarrow \quad \phi_k^{(1)} \sim \int dt'\,\frac{1}{3H}\lambda\left[\phi^{(0)3}\right]_k$$

– Surely, at loop level, among the many contributions, there will be this one

$$\Rightarrow \quad \phi_k^{(1)} \sim \int dt'\,\frac{1}{3H}\lambda\langle\phi^{(0)2}\rangle\phi_k^{(0)} \propto \lambda t$$

– IR divergent

- It is not just some simple mean field term:

$$\langle\phi_k^2\rangle \supset \langle\phi_k^{(1)2}\rangle \supset \int dt' \int dt''\,\lambda^2 \int d^3q_{\rm long}\,\langle\phi^{(0)}(q_{\rm long})^2\phi^{(0)}(q_{\rm long})^2\rangle\,\langle\phi_k^{(0)2}\rangle \propto \lambda^2 t^2$$

IR-divergencies

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– Solving perturbatively

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IR-divergencies

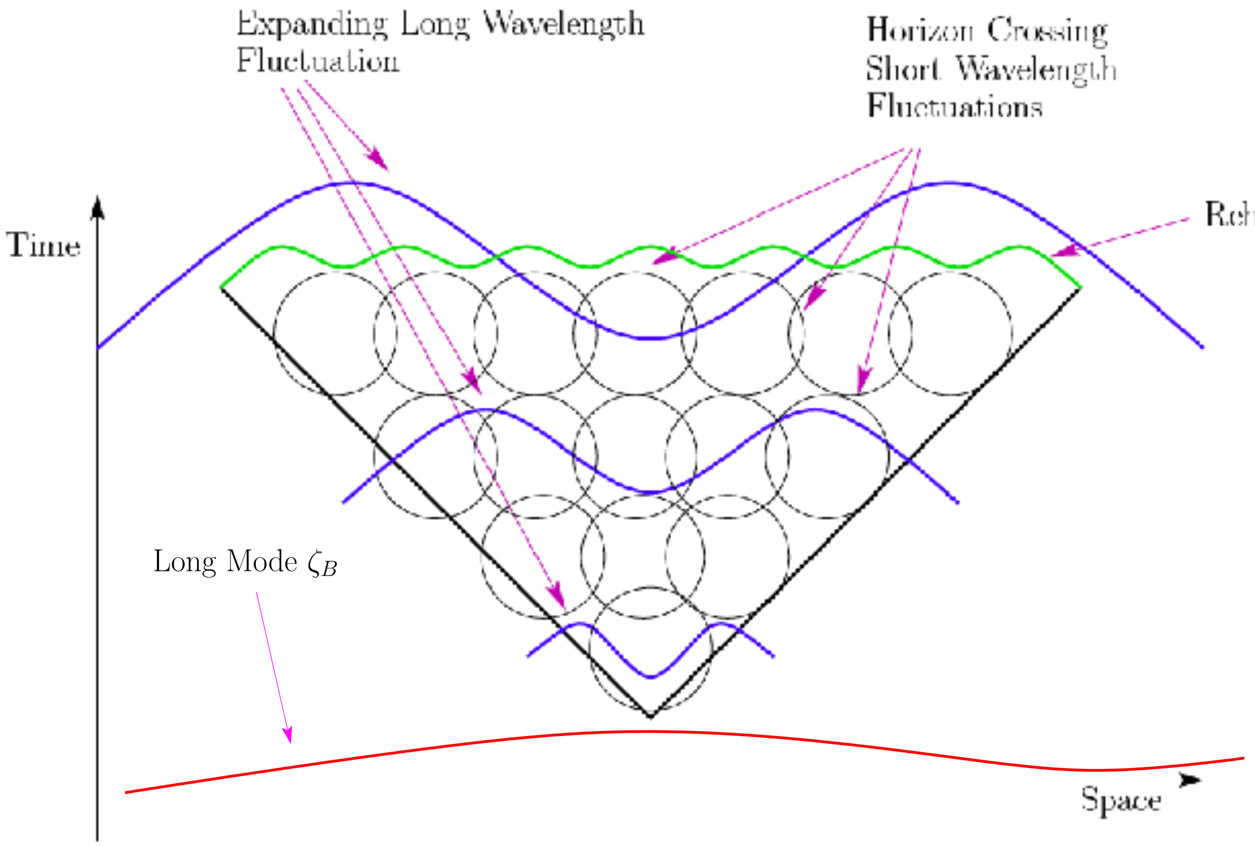
– This is what happens when one does the full calculation (see for example Burgess et al. **0912**)

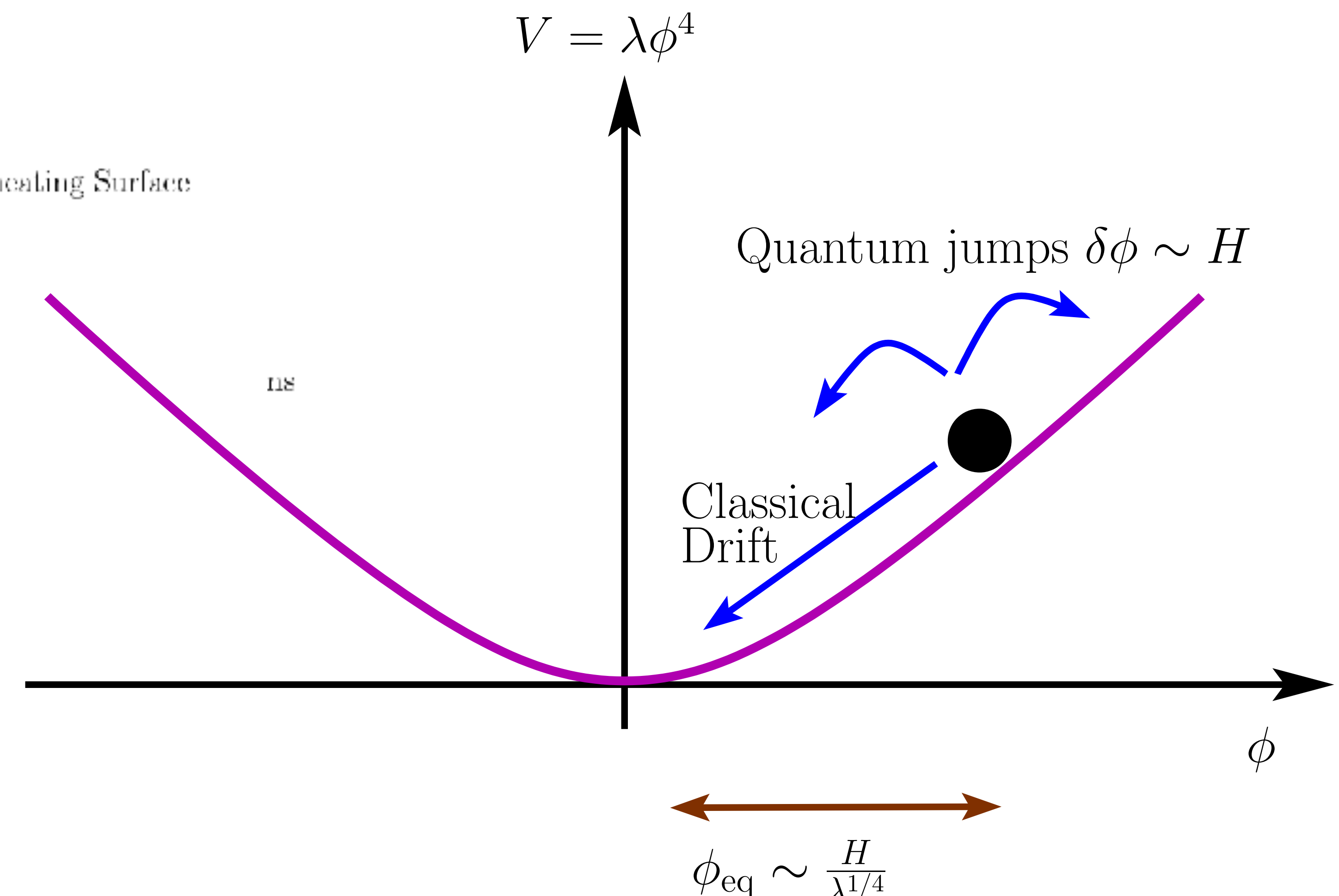
$$\langle \mathcal{O}(t)\rangle = \left\langle \mathrm{in} \left| \left[\overline{T} \exp\left(i \int_{t_{\mathrm{in}}}^{t} \mathrm{d}t'\, \mathcal{H}(t') \right) \right] \mathcal{O}(t) \left[T \exp\left(-i \int_{t_{\mathrm{in}}}^{t} \mathrm{d}t'\, \mathcal{H}(t') \right) \right] \right| \mathrm{in} \right\rangle ,$$

$$\Rightarrow \quad \langle \phi_k^2 \rangle \sim \lambda \log\left(\frac{k}{a(t) H} \right) \qquad\qquad a \sim e^{Ht}$$

Intuition

– What is going on?





– there is an $\mathcal{O}(1)$ change to perturbative evolution: $\Longrightarrow$ solution is non perturbative

– We expect a diffusion-upwards, stopped by a drift-downwards, reaching a sort of equilibrium distribution with

$$\text{Energy} \sim H \quad \Rightarrow \quad V(\phi) = \lambda\phi^4 \sim H^4 \quad \Rightarrow \phi \sim \frac{H}{\lambda^{1/4}}$$

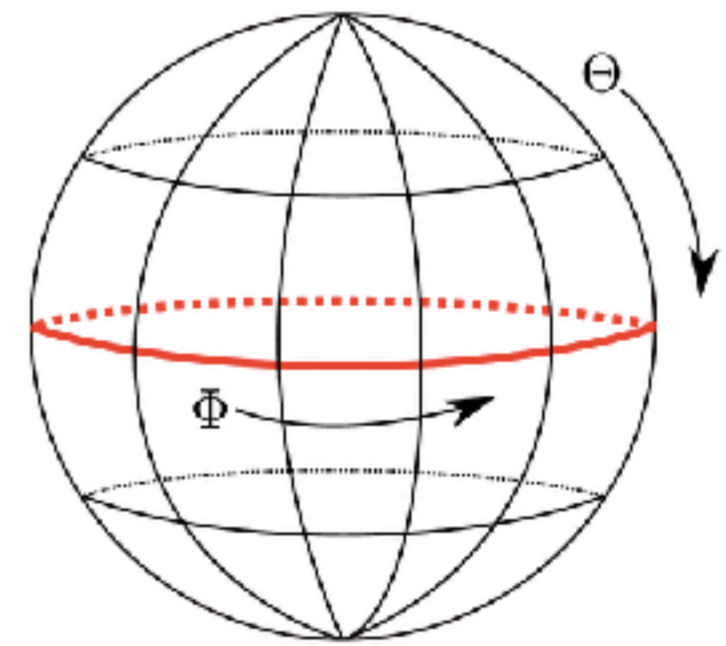
– How to obtain a rigorous calculation? with arbitrary precision?

Euclidian Space

– Common lore: upon Euclidean rotation, dS is a sphere, so there are no IR-divergencies

– It is true that there are no IR divergencies in Euclidean space

  – and that just the zero mode is non-perturbative.



– But when the resulting correlation functions are rotated back, they become IR divergent again.

– See for ex:

Marolf, Morrison **2010, 2011**
Rajaraman, **2011**
Beneke, Moch **2012**
Lopez Nacir, Mazzitelli, Trobetta **2016, 2016, 2018**

– No known solution from Euclidean space. In fact, not even a partial improvement. But maybe improvement can be done also on this side.

A Generalized Stochastic Approach

– We are going to define a rigorous formalism to solve the problem, that, at zeroth order in all the expansion parameters we will identify and introduce, reduces to the remarkable Stochastic approach of Starobinsky.

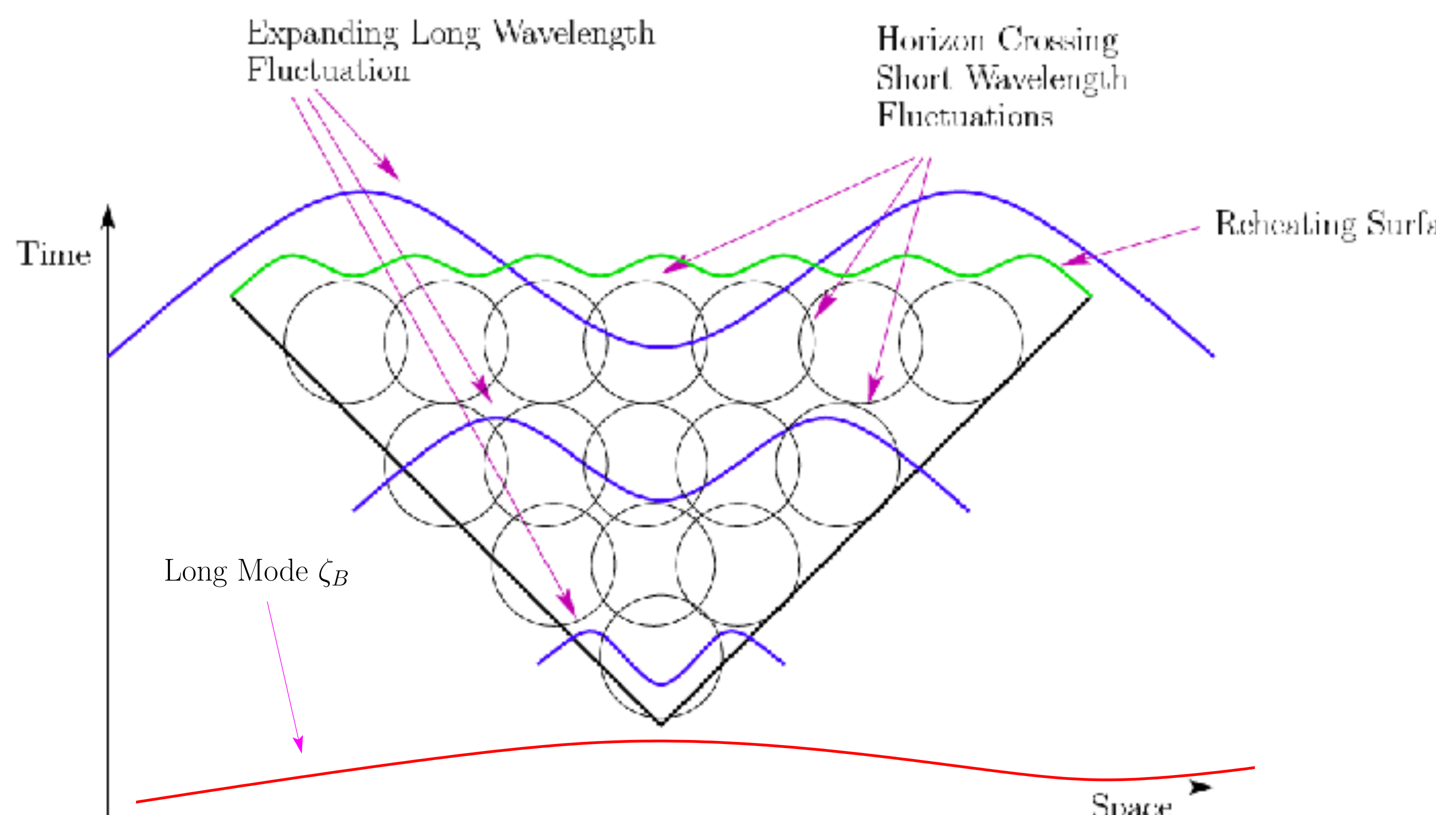
- We will use two crucial simplifications, around which we will expand, that allow us to solve a non-perturbative quantum problem in curved spacetime.

A Generalized Stochastic Approach

– Two crucial expansion parameters:

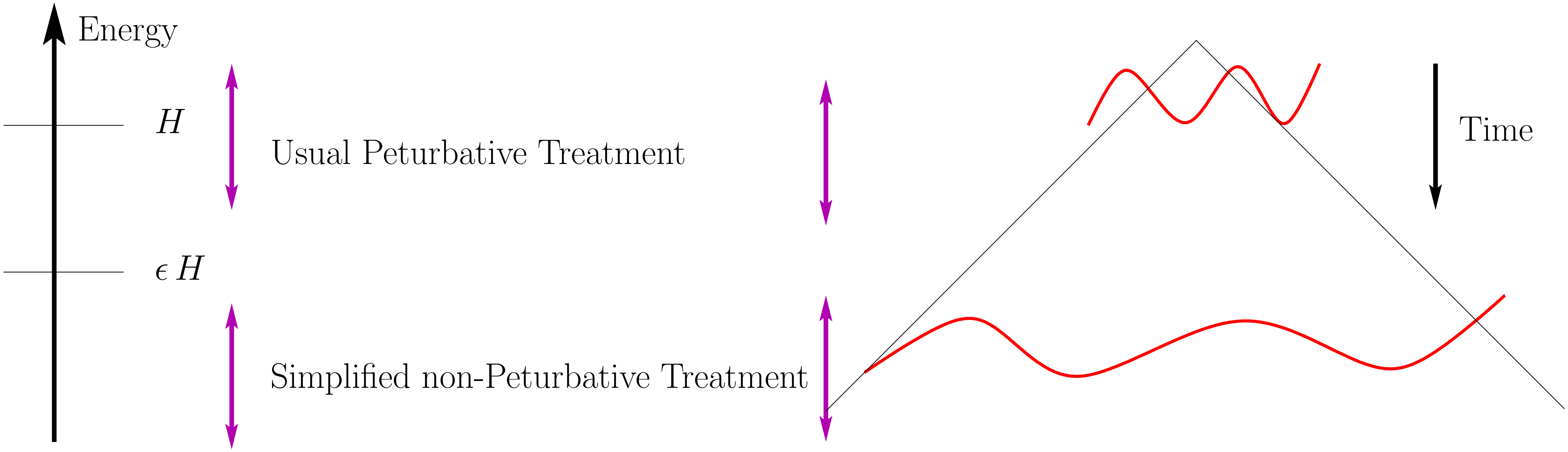
– (1): outside of horizon, gradients are negligible. We can expand around an exactly local-in-space evolution.

– (2): perturbativity of coupling constant: $\lambda \ll 1 \quad \Rightarrow \quad \sqrt{\lambda} \ll 1$



A Generalized Stochastic Approach

– Main idea:



– separate long and short modes at an *artificial* fixed physical scale:

- in terms of wavenumbers it is a time-dependent scale $k = \Lambda(t) = \epsilon a(t) H \ , \quad \epsilon \ll 1$
- modes move from `short' to `long'

– for short modes: use usual quantum perturbation theory: $\lambda t \sim \lambda \log \epsilon \ll 1$

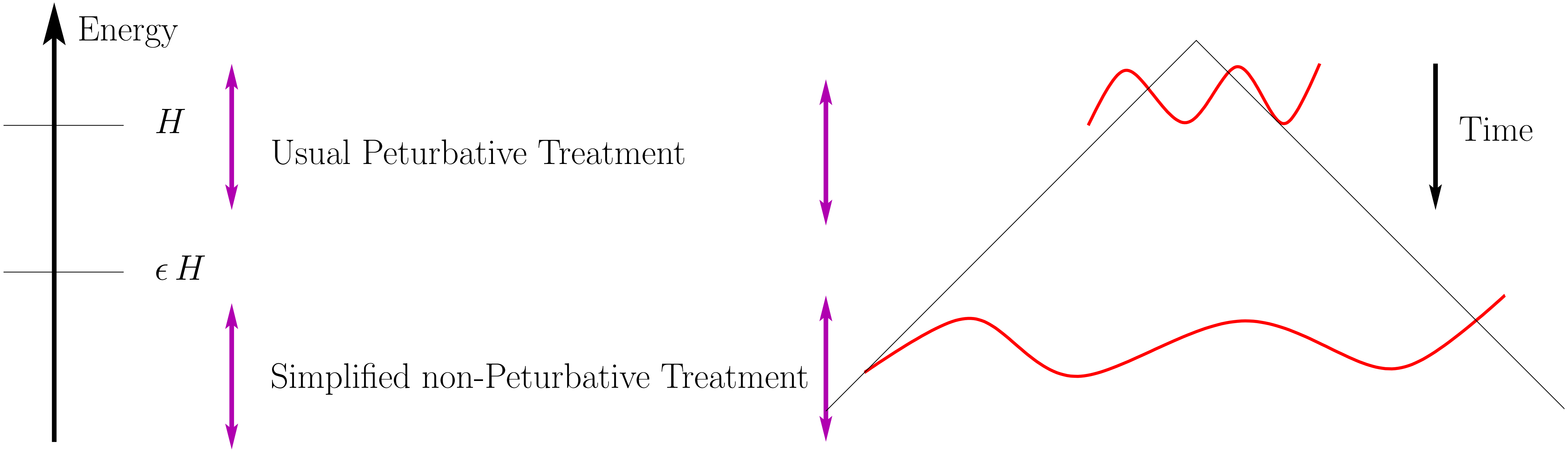
$$\langle \mathcal{O}(t) \rangle = \left\langle \mathrm{in} \left| \left[\overline{T} \exp \left(i \int_{t_{\mathrm{in}}}^{t} \mathrm{d}t' \, \mathcal{H}(t') \right) \right] \mathcal{O}(t) \left[T \exp \left(-i \int_{t_{\mathrm{in}}}^{t} \mathrm{d}t' \, \mathcal{H}(t') \right) \right] \right| \mathrm{in} \right\rangle ,$$

– for long modes, use the helps of the two expansion (2,3) above: expansion in

$$\hbar \sim \epsilon \ll 1 \ , \quad \lambda^{1/2} \ll 1$$

A Generalized Stochastic Approach

–Main idea:



–Modes, with time, pass from `short' to `long' regime.

– Optimal ϵ : equalize two expansion parameters:

$$\lambda \log \epsilon \sim \lambda^{1/2} \quad \Rightarrow \quad \epsilon \sim e^{-\frac{1}{\lambda^{1/2}}} \lll\ll 1$$

– $\Longrightarrow$ quantum and gradient corrections can be made much smaller than $\sqrt{\lambda}$ corrections

What we wish to compute

– We wish to compute $\langle \phi(x_1) \dots \phi(x_n) \rangle$

– This is given by:

$$\langle \phi(x_1) \dots \phi(x_n) \rangle = \int \mathcal{D}\phi \; \Psi^\star[\phi] \Psi[\phi] \; \phi(x_1) \dots \phi(x_n) = \int \mathcal{D}\phi \; \bar{\rho}[\phi] \; \phi(x_1) \dots \phi(x_n)$$

– with $\bar{\rho}[\phi] = \Psi^\star[\phi] \Psi[\phi]$

– Formally, I could do the integral over the intermediate points, and write

$$\langle \phi(x_1) \dots \phi(x_n) \rangle = \int d\phi_1 \dots d\phi_n \; \bar{\rho}(\phi_1, \dots, \phi_n) \; \phi_1 \dots \phi_n$$

– where $\bar{\rho}(\phi_1, \dots, \phi_n) = \int \mathcal{D}\phi \; \delta^{(1)}(\phi_1 - \phi(x_1)) \dots \delta^{(1)}(\phi_n - \phi(x_n)) \; \bar{\rho}[\phi]$

– the problem is that it is hard to make such a path integral.

– Instead, our strategy is to find an equation that is satisfied by $\bar{\rho}(\phi_1, \dots, \phi_n)$

Solving for the field density

– Let us start with $\bar{\rho}[\phi] = \Psi^\star[\phi]\Psi[\phi]$, which satisfies the following equation:

$$\frac{\partial \bar{\rho}[\phi,t]}{\partial t} = -\frac{i}{2a^3}\int d^3x \frac{\delta}{\delta\phi(\vec{x})}\left(\Psi[\phi]^*\frac{\delta}{\delta\phi(\vec{x})}\Psi[\phi] - \Psi[\phi]\frac{\delta}{\delta\phi}\Psi^*[\phi]\right)$$

– which is functional and not even closed.

– But we actually can compute the wavefunction in dS.

– This will allow us to manipulate this equation.

Solving for the wavefunction

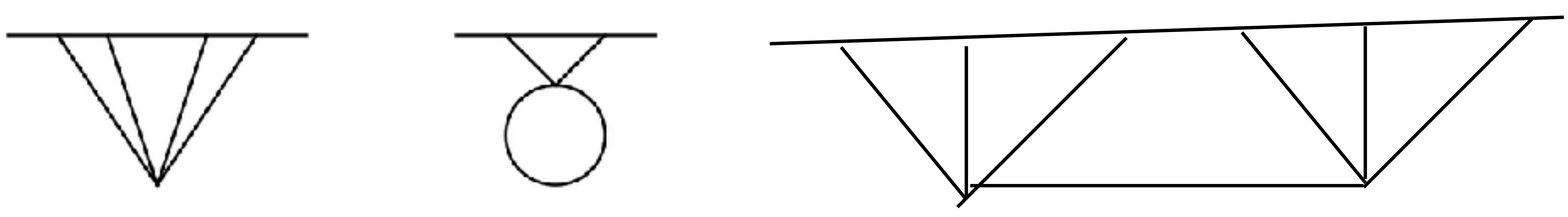
- Some relevant literature has already emphasized how to compute the wavefunction in dS. For example Nima et al. **…, 2017, 2018, …** For $\lambda\phi^4$, it has been computed by Anninos, Anous, Freedman, Kostantinidis, **2015**

- The perturbative structure is extremely different than for correlation functions, because of the different boundary conditions the propagators have. Indeed, there are two propagators:

- Bulk-to-Bulk: $G(k, x_1, x_2; x_c)$

- Bulk-to-Boundary (the `transfer function'): $K(k, x; x_c)$

- Both propagators are regular for $k \to 0$, so there are no IR-divergencies (of course, they come back once one tries to compute correlation functions)



Our Strategy

– we separate for long and short modes. In fact short modes are perturbative.

– So, we split all the modes in short and long:

$$\phi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, \Omega_{\Lambda(t)}(k)\;\; e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k}) + \int \frac{d^3k}{(2\pi)^3}\;\left(1-\Omega_{\Lambda(t)}(k)\right)\;\; e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k}) \equiv \phi_\ell(\vec{x}) + \phi_s(\vec{x})\;,$$

– with

$$\Omega_{\Lambda(t)}(k) = \begin{cases} 1 & \text{for } k \le \Lambda(t), \\ 0 & \text{for } \; k \ge (1+\delta)\Lambda(t)\,. \end{cases}$$

– smooth and wide enough

$$\Lambda(t) = \varepsilon a(t) H$$

$$\frac{e^{-\frac{1}{\sqrt{\lambda}}}}{\epsilon} \ll \delta \ll \sqrt{\lambda},$$

– Effective long-Density Function

$$\bar{P}_\ell[\phi_\ell, t] = \int \mathcal{D}\phi\;\delta\left[\phi_\ell(\vec{x}) - \int d^3k\;\Omega_{\Lambda(t)} e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right]\;\bar{P}[\phi, t]$$

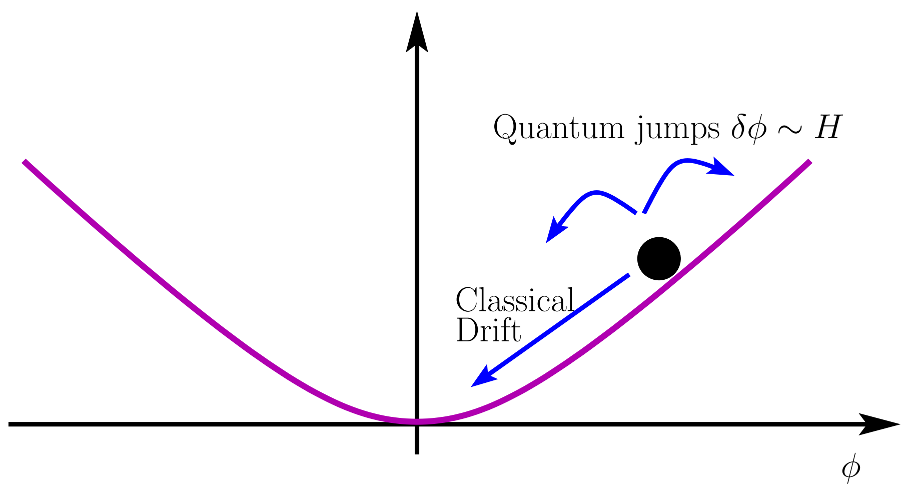
Effective Quasi-Probability for long modes

– Given the effective probability for the long modes

$$\bar{P}_\ell[\phi_\ell, t] = \int \mathcal{D}\phi \; \delta\left[\phi_\ell(\vec{x}) - \int d^3k \; \Omega_{\Lambda(t)} e^{i\vec{k}\cdot\vec{x}} \phi(\vec{k})\right] \; \bar{P}[\phi, t]$$

– Let us find the effective time-evolution for the long modes ,

$$\frac{\partial \bar{P}_\ell[\phi_\ell]}{\partial t} = \text{Drift} + \text{Diff} \; .$$



– Let us start with the Drift:

$$\text{Drift} = \int \mathcal{D}\phi \; \delta\left[\phi_\ell(\vec{x}) - \int d^3k \; \Omega_{\Lambda(t)} e^{i\vec{k}\cdot\vec{x}} \phi(\vec{k})\right] \; \frac{\partial \bar{P}[\phi, t]}{\partial t}$$

$$= -\int \mathcal{D}\phi \; \delta\left[\phi_\ell(\vec{x}) - \int d^3k \; \Omega_{\Lambda(t)} e^{i\vec{k}\cdot\vec{x}} \phi(\vec{k})\right] \; \frac{i}{2a^3} \int d^3x \frac{\delta}{\delta\phi(\vec{x})} \left(\Psi[\phi]^* \frac{\delta}{\delta\phi(\vec{x})} \Psi[\phi] - \Psi[\phi] \frac{\delta}{\delta\phi} \Psi^*[\phi]\right)$$

Effective Quasi-Probability for long modes

– Given the effective probability for the long modes

$$\bar{P}_\ell[\phi_\ell, t] = \int \mathcal{D}\phi\ \delta\left[\phi_\ell(\vec{x}) - \int d^3k\ \Omega_{\Lambda(t)} e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right]\ \bar{P}[\phi, t]$$

Keep long modes fixed

– Let us find the effective time-evolution for the long modes ,

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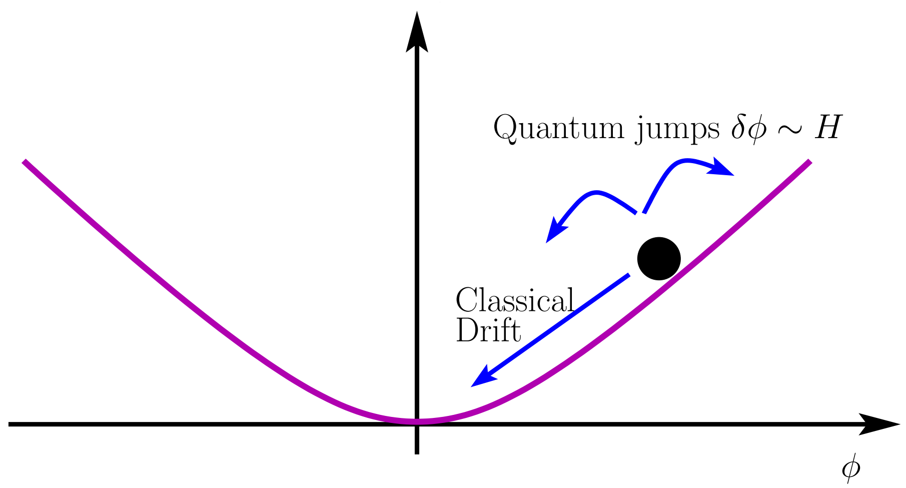

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Effective Quasi-Probability for long modes

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$$\bar{P}_\ell[\phi_\ell, t] = \int \mathcal{D}\phi\ \delta\left[\phi_\ell(\vec{x}) - \int d^3k\ \Omega_{\Lambda(t)} e^{i\vec{k}\cdot\vec{x}} \phi(\vec{k})\right]\ \bar{P}[\phi, t]$$

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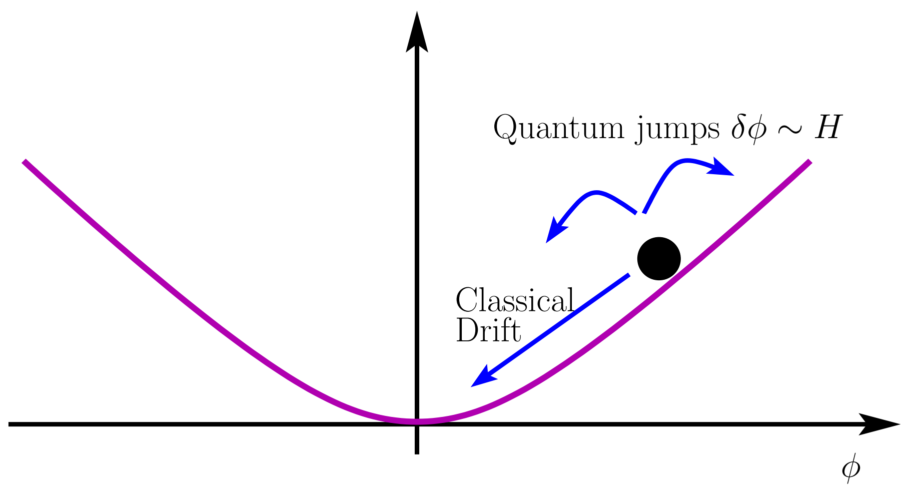

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Effective Quasi-Probability for long modes

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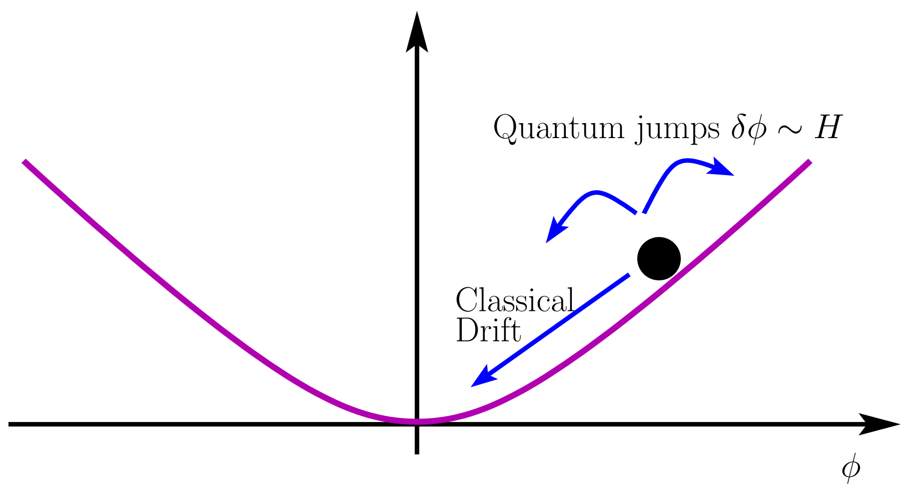

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Effective Quasi-Probability for long modes

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$$\bar{P}_\ell[\phi_\ell, t] = \int \mathcal{D}\phi\ \delta\left[\phi_\ell(\vec{x}) - \int d^3k\ \Omega_{\Lambda(t)} e^{i\vec{k}\cdot\vec{x}} \phi(\vec{k})\right]\ \bar{P}[\phi, t]$$

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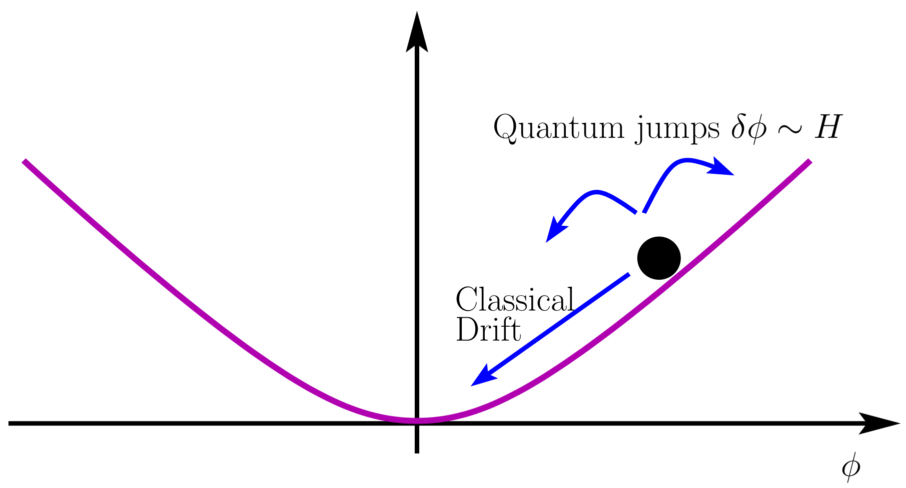

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Effective Probability for long modes

– Upon integration by parts and switching the derivative of the $\delta-$function:

$$\text{Drift} = -\int \mathcal{D}\phi\;\delta\left[\phi_\ell(\vec{x}) - \int d^3k\;\Omega_{\Lambda(t)}(k)e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right]\;\frac{i}{2a^3}\int d^3x\frac{\delta}{\delta\phi(\vec{x})}\left(\Psi[\phi]^*\frac{\delta}{\delta\phi(\vec{x})}\Psi[\phi] - \Psi[\phi]\frac{\delta}{\delta\phi}\Psi^*[\phi]\right)$$

$$= \frac{i}{2a^3}\int d^3x d^3x'\int d^3k\;\Omega_{\Lambda(t)}(k)(k)e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\frac{\delta}{\delta\phi_\ell(\vec{x})}$$

$$\times\int \mathcal{D}\phi\;\delta\left[\phi_\ell(\vec{x}) - \int d^3k\;\Omega_{\Lambda(t)}(k)e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right]\;\left(\Psi[\phi]^*\frac{\delta}{\delta\phi(\vec{x}')}\Psi[\phi] - \Psi[\phi]\frac{\delta}{\delta\phi(\vec{x}')}\Psi^*[\phi]\right) =$$

$$= \int d^3x\frac{\delta}{\delta\phi_\ell(\vec{x})}\int \mathcal{D}\phi\;\delta\left[\phi_\ell(\vec{x}) - \int d^3k\;\Omega_{\Lambda(t)}(k)e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right]\;\left[\mathrm{Re}\left(\frac{\Pi[\phi(\vec{x})]}{a^3}\right)\right]_\Lambda\;\bar{\rho}[\phi]$$

– where we wrote $\quad \Pi[\phi(\vec{x})]\Psi[\phi] = -i\dfrac{\delta}{\delta\phi(\vec{x})}\Psi[\phi]$

– The last path integral is nothing but the expectation value of the long-component of $\Pi(\phi)$ with a fixed long-background.

$$\text{Drift} = \int d^3x\frac{\delta}{\delta\phi_\ell(\vec{x})}\left(\left\langle\left[\mathrm{Re}\left(\frac{\Pi(\phi(\vec{x}))}{a^3}\right)\right]_\Lambda\right\rangle_{\phi_\ell}\;\bar{\rho}_\ell[\phi_\ell]\right)$$

Effective Probability for long modes

– Upon integration by parts and switching the derivative of the $\delta-$function:

$$\text{Drift} = -\int \mathcal{D}\phi\, \delta\left[\phi_\ell(\vec{x}) - \int d^3k\, \Omega_{\Lambda(t)}(k) e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right] \frac{i}{2a^3}\int d^3x \frac{\delta}{\delta\phi(\vec{x})}\left(\Psi[\phi]^*\frac{\delta}{\delta\phi(\vec{x})}\Psi[\phi] - \Psi[\phi]\frac{\delta}{\delta\phi}\Psi^*[\phi]\right)$$

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$$\times \int \mathcal{D}\phi\, \delta\left[\phi_\ell(\vec{x}) - \int d^3k\, \Omega_{\Lambda(t)}(k) e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right]\left(\Psi[\phi]^*\frac{\delta}{\delta\phi(\vec{x}')}\Psi[\phi] - \Psi[\phi]\frac{\delta}{\delta\phi(\vec{x}')}\Psi^*[\phi]\right) =$$

$$= \int d^3x \frac{\delta}{\delta\phi_\ell(\vec{x})}\int \mathcal{D}\phi\, \delta\left[\phi_\ell(\vec{x}) - \int d^3k\, \Omega_{\Lambda(t)}(k) e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right]\left[\mathrm{Re}\left(\frac{\Pi[\phi(\vec{x})]}{a^3}\right)\right]_\Lambda \bar{\rho}[\phi]$$

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$$\Pi[\phi(\vec{x})]\Psi[\phi] = -i\frac{\delta}{\delta\phi(\vec{x})}\Psi[\phi]$$

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$$\text{Drift} = \int d^3x \frac{\delta}{\delta\phi_\ell(\vec{x})}\left(\left\langle\left[\mathrm{Re}\left(\frac{\Pi(\phi(\vec{x}))}{a^3}\right)\right]_\Lambda\right\rangle_{\phi_\ell}\bar{\rho}_\ell[\phi_\ell]\right)$$

Effective Probability for long modes

– Upon integration by parts and switching the derivative of the $\delta-$function:

$$\text{Drift} = -\int \mathcal{D}\phi\ \delta\left[\phi_\ell(\vec{x}) - \int d^3k\ \Omega_{\Lambda(t)}(k) e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right]\ \frac{i}{2a^3}\int d^3x \frac{\delta}{\delta\phi(\vec{x})}\left(\Psi[\phi]^*\frac{\delta}{\delta\phi(\vec{x})}\Psi[\phi] - \Psi[\phi]\frac{\delta}{\delta\phi}\Psi^*[\phi]\right)$$

$$= \frac{i}{2a^3}\int d^3x d^3x' \int d^3k\ \Omega_{\Lambda(t)}(k)(k) e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\frac{\delta}{\delta\phi_\ell(\vec{x})}$$

$$\times \int \mathcal{D}\phi\ \delta\left[\phi_\ell(\vec{x}) - \int d^3k\ \Omega_{\Lambda(t)}(k) e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right]\ \left(\Psi[\phi]^*\frac{\delta}{\delta\phi(\vec{x}')}\Psi[\phi] - \Psi[\phi]\frac{\delta}{\delta\phi(\vec{x}')}\Psi^*[\phi]\right) =$$

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– The last path integral is nothing but the expectation value of the long-component of $\Pi(\phi)$ with a fixed long-background.

$$\text{Drift} = \int d^3x \frac{\delta}{\delta\phi_\ell(\vec{x})}\left(\left\langle\left[\text{Re}\left(\frac{\Pi(\phi(\vec{x}))}{a^3}\right)\right]_\Lambda\right\rangle_{\phi_\ell} \bar{\rho}_\ell[\phi_\ell]\right)$$

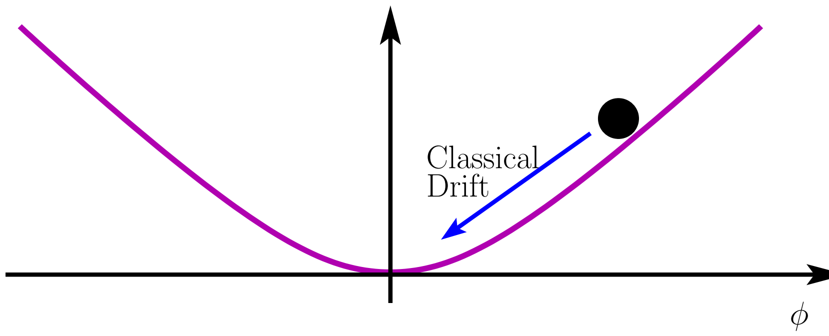
Effective Quasi-Probability for long modes

– Therefore:

$$\text{Drift} = \int d^3x \frac{\delta}{\delta\phi_\ell(\vec{x})} \left(\left\langle \left[\text{Re}\left(\frac{\Pi(\phi(\vec{x}))}{a^3} \right) \right]_\Lambda \right\rangle_{\phi_\ell} \bar{\rho}_\ell[\phi_\ell] \right)$$

– The expectation value over the short modes can be computed using perturbative methods.

$$\langle \mathcal{O}(t) \rangle = \left\langle \text{in} \left| \left[\overline{T} \exp\left(i \int_{t_{\text{in}}}^{t} \mathrm{d}t' \, \mathcal{H}(t') \right) \right] \mathcal{O}(t) \left[T \exp\left(-i \int_{t_{\text{in}}}^{t} \mathrm{d}t' \, \mathcal{H}(t') \right) \right] \right| \text{in} \right\rangle ,$$



– This is all well assuming we know the functional form of

$$\Pi[\phi(\vec{x})] = \frac{\delta}{\delta\phi(\vec{x})} \Psi[\phi]$$

– Before that, let us do the diffusion

$$\frac{\partial}{\partial t} P_l^{(w)}(\gamma_l, t) = \text{Diffusion} + \text{Drift}$$

Effective Quasi-Probability for long modes

– Diffusion term: it arises because our cutoff is time-dependent (modes become long)

$$\text{Diff} = \int \mathcal{D}\phi \; \frac{\partial}{\partial t}\delta\left[\phi_\ell(\vec{x}) - \int d^3k \; \Omega_{\Lambda(t)} e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})\right] \; \bar{P}[\phi, t]$$

New modes enter the long theory

Keep long modes fixed

Effective Quasi-Probability for long modes

– Diffusion term: it arises because our cutoff is time-dependent (modes become long)

$$\text{Diffus.} = \left(\int \mathcal{D}\phi \left(\int d^3x \, \frac{\partial}{\partial \phi_\ell(\vec{x})}\delta^{(1)}\left[\phi_\ell(\vec{y}) - \int_0^{\Lambda(t)} \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{y}}\phi(\vec{k})\right] \times \left(-\frac{\partial}{\partial t}\Delta\phi(\vec{x})\right)\, P^{(w)}[\gamma, t] + \right.\right.$$

$$\left. + \int d^3x \int d^3x' \, \frac{1}{2}\frac{\partial^2}{\partial\phi_\ell(\vec{x})\partial\phi_{\ell,b}(\vec{x}')}\delta^{(1)}\left[\phi_\ell(\vec{y}) - \int_0^{\Lambda(t)} \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{y}}\phi(\vec{k})\right] \times \frac{\partial}{\partial t}\left(\Delta\phi(\vec{x})\Delta\phi(\vec{x}')\right)\, \bar{P}[\phi, t]\right)$$

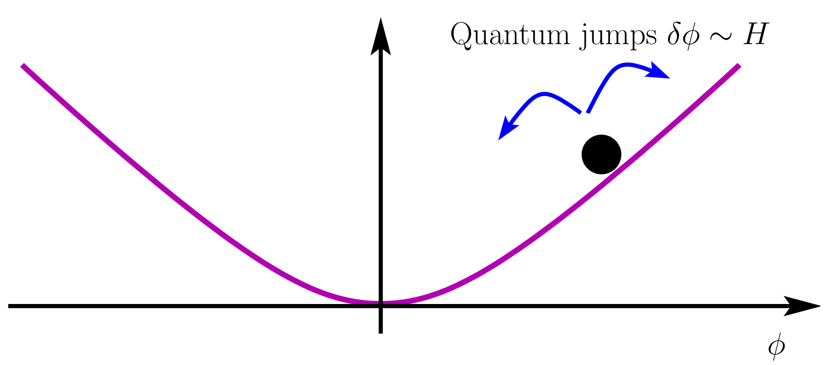
$$\times (1 + O(\delta)) =$$

$$= \left(\int d^3x \, \frac{\partial}{\partial\phi_\ell(\vec{x})}\left\langle\left(-\frac{\partial}{\partial t}\Delta\phi(\vec{x})\right)\right\rangle_{\phi_\ell} \bar{P}_\ell[\phi_\ell, t] + \right.$$

$$\left. + \int d^3x \int d^3x' \, \frac{1}{2}\frac{\partial^2}{\partial\phi_\ell(\vec{x})\partial\phi_{\ell,b}(\vec{x}')}\left\langle\frac{\partial}{\partial t}\left(\Delta\phi(\vec{x})\Delta\phi(\vec{x}')\right)\right\rangle_{\phi_\ell} \bar{P}[\phi_\ell, t]\right) \times (1 + O(\delta))$$

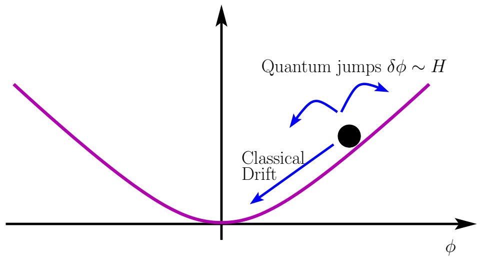
– where $\quad \Delta\phi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, \Omega_{\Lambda(t)}(k)\, e^{i\vec{k}\cdot\vec{x}}\phi(\vec{k})$,: short modes entering the long theory.

–. $\quad \langle\Delta\phi^3\rangle \sim \mathcal{O}(\delta)$



Effective Quasi-Probability for long modes

– In summary: to all orders in λ & ϵ and leading in δ, we have obtained the following effective equation. It is Fokker-Plank-like, but it has differences



$$\frac{\partial \bar{P}_\ell[\phi_\ell]}{\partial t} = \text{Drift} + \text{Diff.}\ .$$

$$\text{Drift} = \int d^3x \frac{\delta}{\delta\phi_\ell(\vec{x})} \left(\left\langle \left[\text{Re}\left(\frac{\Pi(\phi(\vec{x}))}{a^3} \right) \right]_\Lambda \right\rangle_{\phi_\ell} \bar{\rho}_\ell[\phi_\ell] \right)$$

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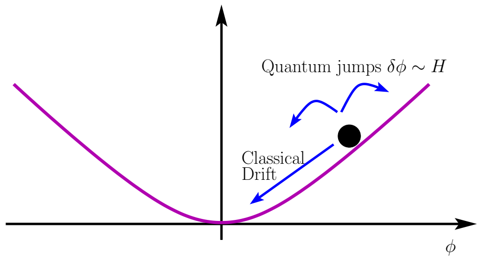
$$\left. + \int d^3x \ \int d^3x' \ \frac{1}{2} \frac{\partial^2}{\partial\phi_\ell(\vec{x})\partial\phi_{\ell,b}(\vec{x}')} \left\langle \frac{\partial}{\partial t} \left(\Delta\phi(\vec{x})\Delta\phi(\vec{x}') \right) \right\rangle_{\phi_\ell} \bar{P}[\phi_\ell, t] \right)$$

– there is a tadpole-diffusion term, and in principle higher order terms.

– Strategy: compute these expectation values for the short modes in perturbation theory with a given background for the long modes in expansion in $\lambda t \sim \lambda \log \epsilon \ll 1, \sqrt{\lambda} \ll 1$, and solve this functional Fokker-Planck-like equation containing only long modes.

Effective Quasi-Probability for long modes

– In summary: to all orders in λ & ϵ and leading in δ, we have obtained the following effective equation. It is Fokker-Plank-like, but it has differences



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The Momentum

– So far, we glossed on how to obtain $\Pi[\phi(\vec{x})] = \dfrac{\delta}{\delta\phi(\vec{x})}\Psi[\phi]$

– Naively, given that the long theory is strongly coupled, how can we obtain that.

– It turns out that one can reliably compute $\Pi[\phi(\vec{x})]$

– In fact, it was already computed, almost entirely, by Anninos, Anous, Freedman, Kostantinidis, **2015**

– We focus on the long momentum, as this is the one for which perturbation theory is not obvious. We assume the scaling $\phi_\ell \sim \dfrac{1}{\lambda^{1/4}}$, for the counting.

– The wavefunction reads

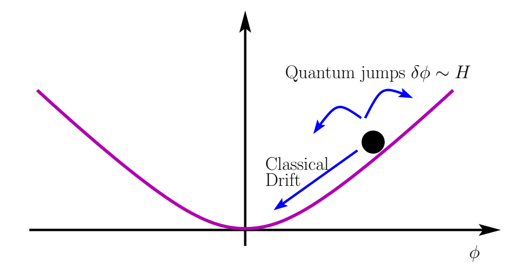
$$\Psi[\phi] \sim \mathrm{Exp}\left(-i\, a(t)^3\left(\frac{\lambda}{12H}\phi(\vec{x})^4 - i\frac{\lambda^2}{54H^4}\phi(\vec{x})^6\right) + \mathcal{O}(\epsilon, \epsilon^3(\lambda\log(k\eta))^n)\right)$$

$$\Rightarrow \quad \frac{\Pi[\phi]}{a^3} = -\frac{\lambda}{3H}\phi(\vec{x})^3 - \frac{\lambda^2}{9H^3}\phi(\vec{x})^5 + \mathcal{O}(\epsilon) = \mathrm{slow-roll\ solution} + \mathcal{O}(\epsilon)$$

– Notice that the correction in $\epsilon^3(\lambda\log(k\eta))^n$ are under control because, as we will see, the long modes decay at long wavenumber, but in general the large phase makes expansion ok

Effective Quasi-Probability for long modes

– In summary: to all orders in λ & ϵ and leading in δ, we have obtained the following effective equation. It is Fokker-Plank-like, but it has differences



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One-location

Effective Quasi-Probability

– For simplicity, we begin to expand in the number of locations, as evolution is quasi local, thanks to dS (this is the opposite of what we do for perturbative theories in Minkowski).

– We define the one-location probability distribution:

$$\bar{P}_1(\phi_\ell(\vec{x}_1) = \phi_1) = \int \mathcal{D}\phi_\ell \; \delta^{(1)}\left[\phi_1 - \phi_\ell(\vec{x}_1)\right] \bar{P}_\ell[\phi_\ell]$$

Keep long fields at one point fixed

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– The resulting equation can be easily derived:

$$\frac{\partial \bar{P}_{\ell,1}(\phi_1,t)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial \phi_1^2}\left(\left\langle \frac{\partial}{\partial t}\Delta\phi(\vec{x}_1)^2 \right\rangle_{\phi_1} \bar{P}_{\ell,1}(\phi_1,t)\right) - \frac{\partial}{\partial \phi_1}\left(\left\langle \left[\frac{\Pi(\phi(\vec{x}_1))}{a^2}\right]_\Lambda \right\rangle_{\phi_1} \bar{P}_{\ell,1}(\phi_1,t)\right)$$

– We dropped the term $\left\langle \left(-\frac{\partial}{\partial t}\Delta\phi(\vec{x})\right) \right\rangle_{\phi_\ell}$ because it will not contribute at the order at which we will compute (by translation invariance, it requires many long modes for it not to vanish).

– The expectation value of the short modes on the long depends only on the field at the same location (this is true only for smooth window function)

– The expectation values can be computed using ordinary perturbation theory.

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Diffusion Drift

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– The expectation values can be computed using ordinary perturbation theory.

Solving at one-location: leading order

– We compute the various ingredients, assuming counting $\phi_\ell \sim \frac{1}{\lambda^{1/4}}$

$$\langle\hat{\phi}(\vec{k},t)\hat{\phi}(-\vec{k},t)\rangle' = \frac{H^2}{2k^3}\left(1+O\left(\sqrt{\lambda}\right)\right)$$

$$\left\langle\left[\frac{\Pi(\phi(\vec{x}_1))}{a^2}\right]_\Lambda\right\rangle_{\phi_1} = -\frac{\lambda}{3H}\phi_1^3$$

- We obtain

$$\frac{\partial\bar{P}_{\ell,1}(\phi_1)}{\partial t} = \Gamma_{\phi_1}\bar{P}_{\ell,1}(\phi_1,t)\left(1+\mathcal{O}(\lambda^{1/2},\delta,\epsilon^2)\right)$$

$$\Gamma_\phi = -\frac{\partial}{\partial\phi}\left(\frac{\lambda}{3H}\phi^3\right) + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\phi^2}$$

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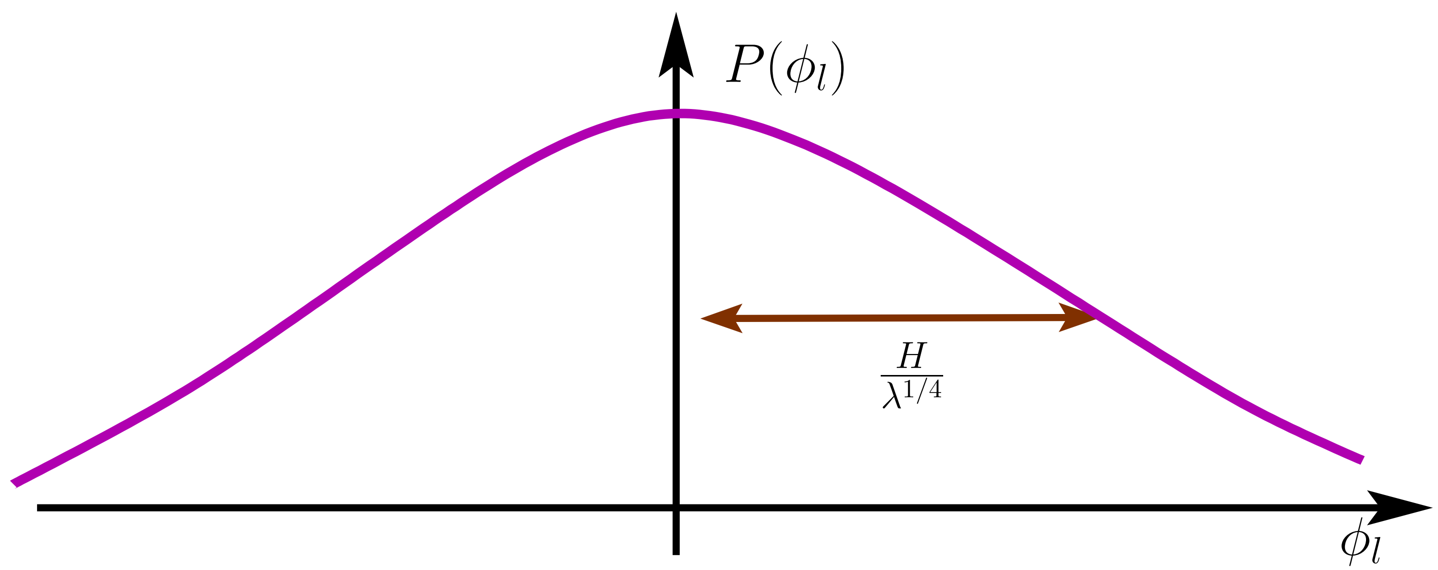
$$\Gamma_\phi = -\frac{\partial}{\partial \phi}\left(\frac{\lambda}{3H}\phi^3\right) + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial \phi^2}$$

– This is the famous Starobinsky equations, but now it is rigorously derived with control of approximation and, as we will see, we can include them.

– There is an equilibrium, i.e. static, solution:

$$\frac{\partial^2}{\partial \phi_l^2}\left[D(\phi_l)\bar{P}_{1,\,\mathrm{eq}}(\phi_l)\right] - \frac{\partial}{\partial \phi_l}\left[F(\phi_l)\bar{P}_{1,\,\mathrm{eq}}(\phi_l)\right] = 0 \quad \Rightarrow \quad \bar{P}_{1,\,\mathrm{eq}}(\phi_l) = e^{-\frac{\lambda \phi_l^4}{H^4}}$$

- This is a static solution, with $\phi_l \sim \dfrac{H}{\lambda^{1/4}}$: so our ϕ counting is correct
- Studying time-dep. solution, we see it is an attractor



Solving at one-location: sub-leading order

– We start again from

$$\frac{\partial \bar{P}_{\ell,1}(\phi_1,t)}{\partial t}=\frac{1}{2}\frac{\partial^2}{\partial\phi_1^2}\left(\left\langle\frac{\partial}{\partial t}\Delta\phi(\vec{x}_1)^2\right\rangle_{\phi_1}\bar{P}_{\ell,1}(\phi_1,t)\right)-\frac{\partial}{\partial\phi_1}\left(\left\langle\left[\frac{\Pi(\phi(\vec{x}_1))}{a^2}\right]_\Lambda\right\rangle_{\phi_1}\bar{P}_{\ell,1}(\phi_1,t)\right)$$

– with counting $\quad \phi_1\ \sim\ H\lambda^{-1/4},\quad \phi_s\ \sim\ H,$

– and compute to next order the various expectation values

– drift term:

$$\left\langle\left[\frac{\Pi(\phi(\vec{x}_1))}{a^2}\right]_\Lambda\right\rangle_{\phi_1}=-\frac{\lambda}{3H}\phi_1^3-\frac{\lambda^2}{9H^3}\phi_1^5-\frac{\lambda}{H}\langle\phi_s(\vec{x})^2\rangle\phi_1$$

– there is a mass on long modes from short modes and there is the quintic potential

$$\left\langle\hat{\phi}_s(x_1^\mu)^2\right\rangle=-\frac{H^2}{4\pi^2}\log\epsilon$$

– diffusion term: there is a mass induced by the long modes on the shorts:

$$\delta m_s^2=3\lambda\phi_1^2$$

–

$$\left\langle\hat{\phi}(\vec{k}_s,t)\hat{\phi}(-\vec{k}_s,t)\right\rangle'_{\phi_1}=\frac{H^2}{2k_s^3}\left(1+\log\epsilon\frac{2\delta m_s^2}{3H^2}\right)+O(\lambda)$$

Solving at one-location: sub-leading order

– Summary so far:

$$\frac{\partial \bar{P}_{\ell,1}(\phi_1,t)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial \phi_1^2}\left(\left\langle \frac{\partial}{\partial t}\Delta\phi(\vec{x}_1)^2 \right\rangle_{\phi_1} \bar{P}_{\ell,1}(\phi_1,t)\right) - \frac{\partial}{\partial \phi_1}\left(\left\langle \left[\frac{\Pi(\phi(\vec{x}_1))}{a^2}\right]_\Lambda \right\rangle_{\phi_1} \bar{P}_{\ell,1}(\phi_1,t)\right)$$

– where

$$\left\langle \frac{\partial}{\partial t}\Delta\phi(\vec{x}_1)^2 \right\rangle_{\phi_1} = H^3\left(1+\lambda\frac{\phi_1^2}{H^2}\log(\epsilon)\right) ,$$

$$\left\langle \left[\frac{\Pi(\phi(\vec{x}_1))}{a^2}\right]_\Lambda \right\rangle_{\phi_1} = -\frac{\lambda}{H}\phi_1^3 - \frac{\lambda^2}{H^3}\phi_1^5 - \lambda H \log\epsilon\, \phi_1$$

Effective mass

Solving at one-location: sub-leading order

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dependence on $\log\epsilon$ is unphysical

Subleading order

– Now we can solve the same Fokker-Planck-like equation as before, including subleading terms. We find:

$$\bar{P}_{1,\,\mathrm{eq}}(\phi_l) = e^{-\frac{\lambda\phi_l^4}{H^4}} \left(1 + (1-\log(\epsilon))\,\frac{\lambda\phi_l^2}{H} + \frac{\lambda^2\phi_l^6}{H^6}\,(-1+\log(\epsilon))\right)$$

$$\Rightarrow \quad \langle \phi_l(\vec{x})^n \rangle = \int d\phi_l \; \bar{P}_{1,\,\mathrm{eq}}(\phi_l)\,\phi_l^n = \text{depends on } \log(\epsilon)$$ unphysical!

– This is ok, because $\langle \phi_l(\vec{x})^n \rangle$ is UV sensitive

– What is physical is $\langle \phi(\vec{x})^n \rangle = \langle (\phi_s(\vec{x}) + \phi_l(\vec{x}))^n \rangle$

– Counting: $\phi_s \sim H \sim \phi_l \cdot \lambda^{1/4} \quad \Rightarrow \quad \dfrac{\phi_s^2}{\phi_l^2} \sim \sqrt{\lambda}$

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Result for 1-location

– Using

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$$\langle\phi(\vec{x},t)^{2n}\rangle = \left(\frac{H}{\lambda^{1/4}}\right)^{2n}\frac{2^{8-\frac{n}{2}}3^{\frac{n}{2}-2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{n}{2}+\frac{1}{4}\right)}{49\pi^n\Gamma\left(-\frac{7}{4}\right)^2}$$
$$\times\left(1-\sqrt{\lambda}\frac{\left(16\sqrt{2}\pi\Gamma\left(\frac{n}{2}+\frac{1}{4}\right)+3(2n-3)\Gamma\left(-\frac{3}{4}\right)^2\Gamma\left(\frac{n}{2}+\frac{3}{4}\right)\right)}{32\sqrt{6}\pi\Gamma\left(\frac{1}{4}\right)^2\Gamma\left(\frac{n}{2}+\frac{1}{4}\right)}+\mathcal{O}(\sqrt{\lambda}^2,\epsilon)\right)$$

– The $\log(\epsilon)$ cancelled!

– Strictly speaking, this expression is still Minkowski UV sensitive, but the UV sensitivity is subleading in $\sqrt{\lambda}$ already from $n \geq 2$

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2-locations

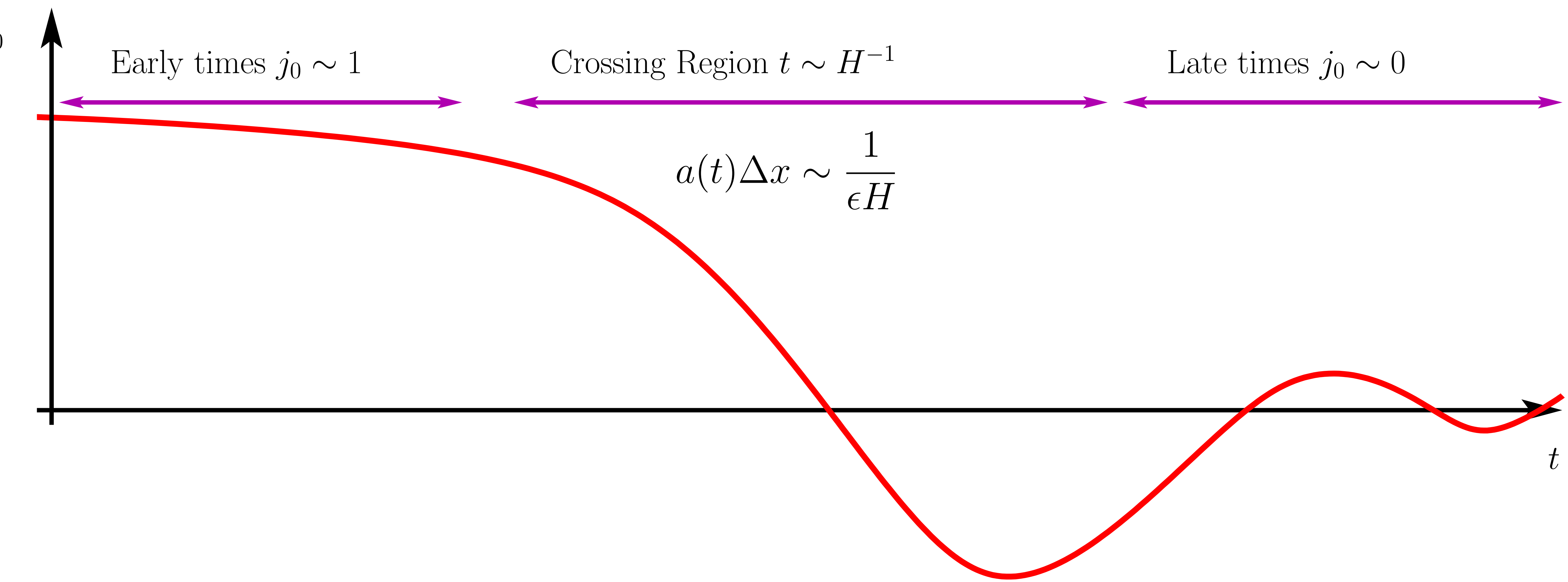
2-locations n-point function

– There is an analogous Fokker-Planck-like equation for the distribution at 2-points:

$$\frac{\partial}{\partial t} P_2(\phi_1, \phi_2, \Delta x, t) = \left(\Gamma_{\phi_1} + \Gamma_{\phi_2}\right) P_2 + j_0\left(\epsilon a(t) H \Delta x\right) \frac{\partial^2}{\partial \phi_1 \partial \phi_2} P_2$$

where $$\frac{\partial}{\partial t} P_1(\phi_1, t) = \Gamma_{\phi_1} P_1 = \frac{\partial^2}{\partial \phi_1^2} P_1 + \frac{\partial}{\partial \phi_1}\left(V'(\phi_1) P_1\right)$$

– The last term strongly depends on distance



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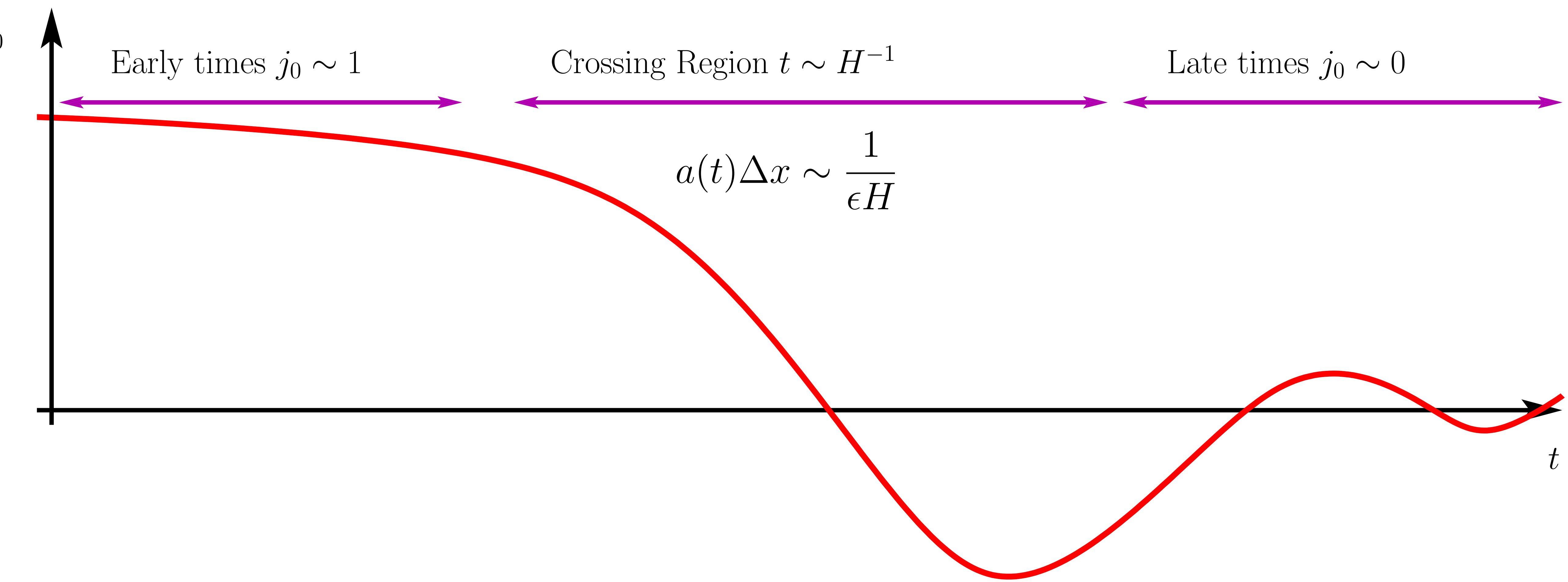
where $$\frac{\partial}{\partial t}P_1(\phi_1,t) = \Gamma_{\phi_1}P_1 = \frac{\partial^2}{\partial\phi_1^2}P_1 + \frac{\partial}{\partial\phi_1}\left(V'(\phi_1)P_1\right)$$

– The last term strongly depends on distance



2-locations n-point function

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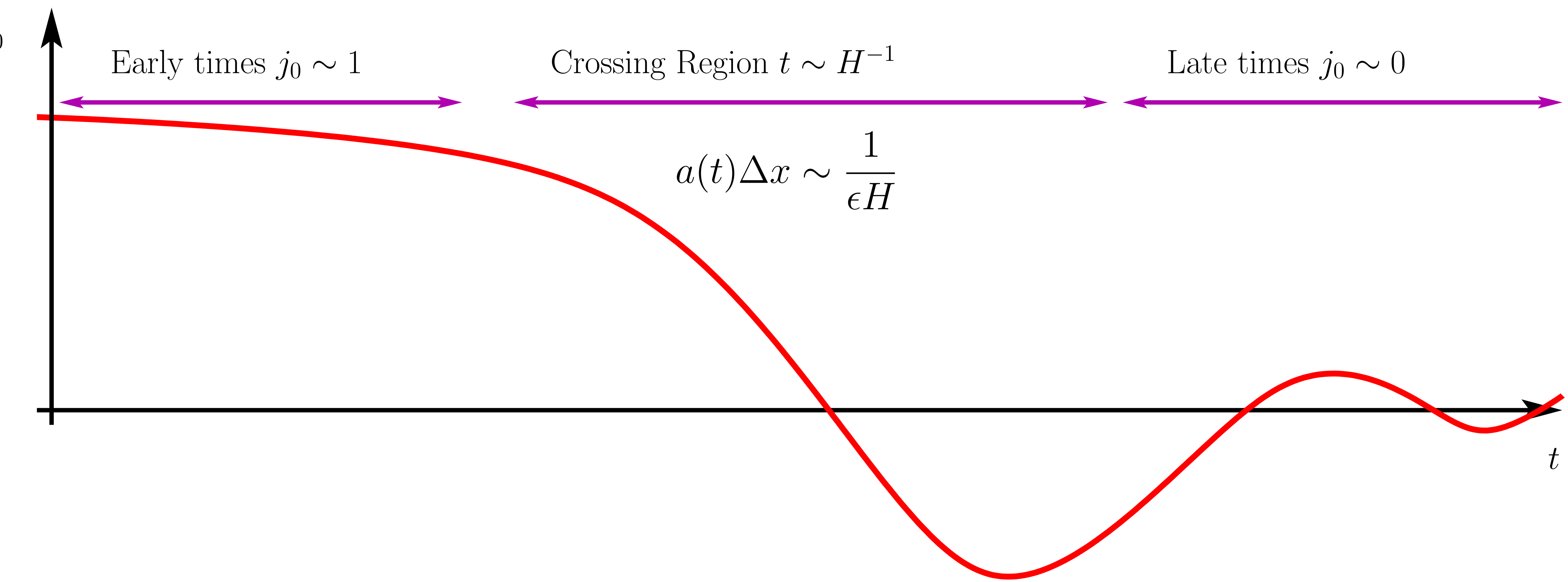

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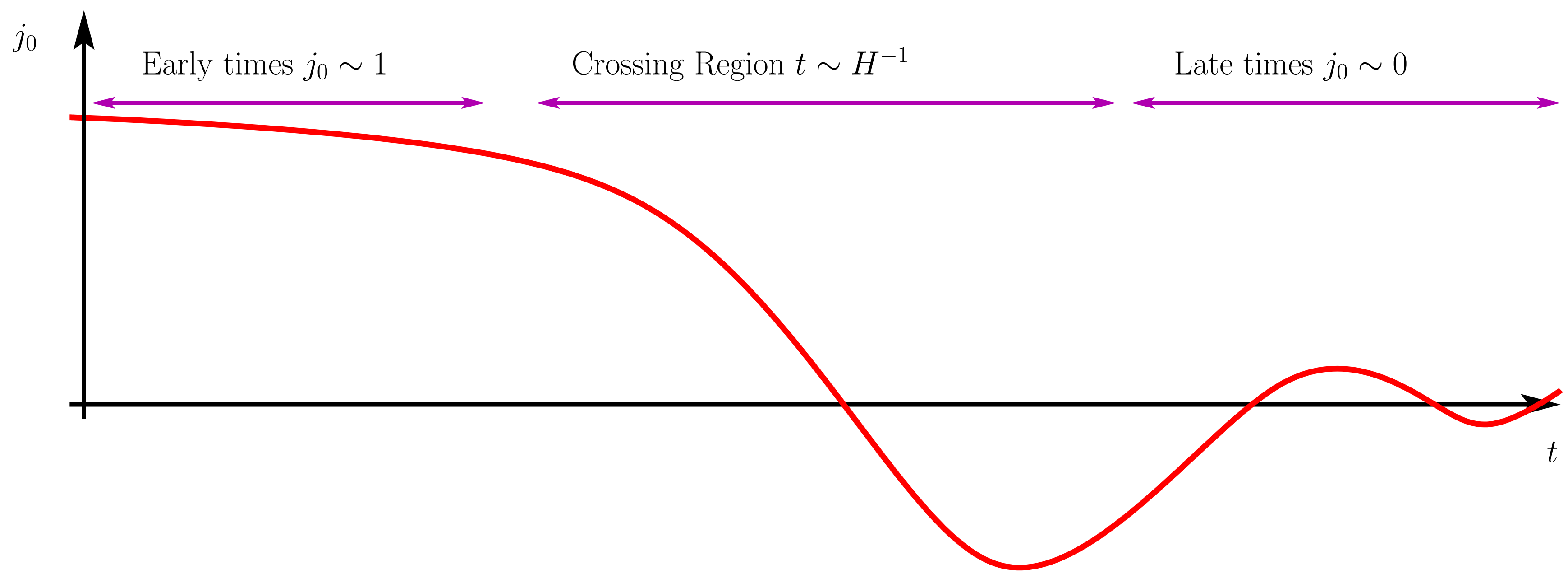

– At early times, solutions is $P_2(\phi_1, \phi_2, t_{\text{early}}) \sim \delta^{(1)}(\phi_1 - \phi_2)\ P_{\text{eq},1}(\phi_1)$

– At late times is $P_2(\phi_1, \phi_2,\ t_{\text{late}}) \sim P_{1,a}(\phi_1,\ t_{\text{late}})\ P_{1,b}(\phi_2\, t_{\text{late}})$

– The time scale of diff equation is $H^{-1}/\sqrt{\lambda}$, but crossing time $H^{-1} \ll H^{-1}/\sqrt{\lambda}$

– $\Rightarrow$ we can glue using `sudden perturbation theory', which corresponds to expansion in $\sqrt{\lambda}$ (by coincidence)

de Sitter invariance

– There is an analogous Fokker-Planck-like equation for the distribution at 2-points:

$$\frac{\partial}{\partial t}P_2(\phi_1,\phi_2,\Delta x,t) = \left(\Gamma_{\phi_1}+\Gamma_{\phi_2}\right)P_2 + j_0\left(\epsilon a(t) H \Delta x\right)\frac{\partial^2}{\partial\phi_1\partial\phi_2}P_2$$

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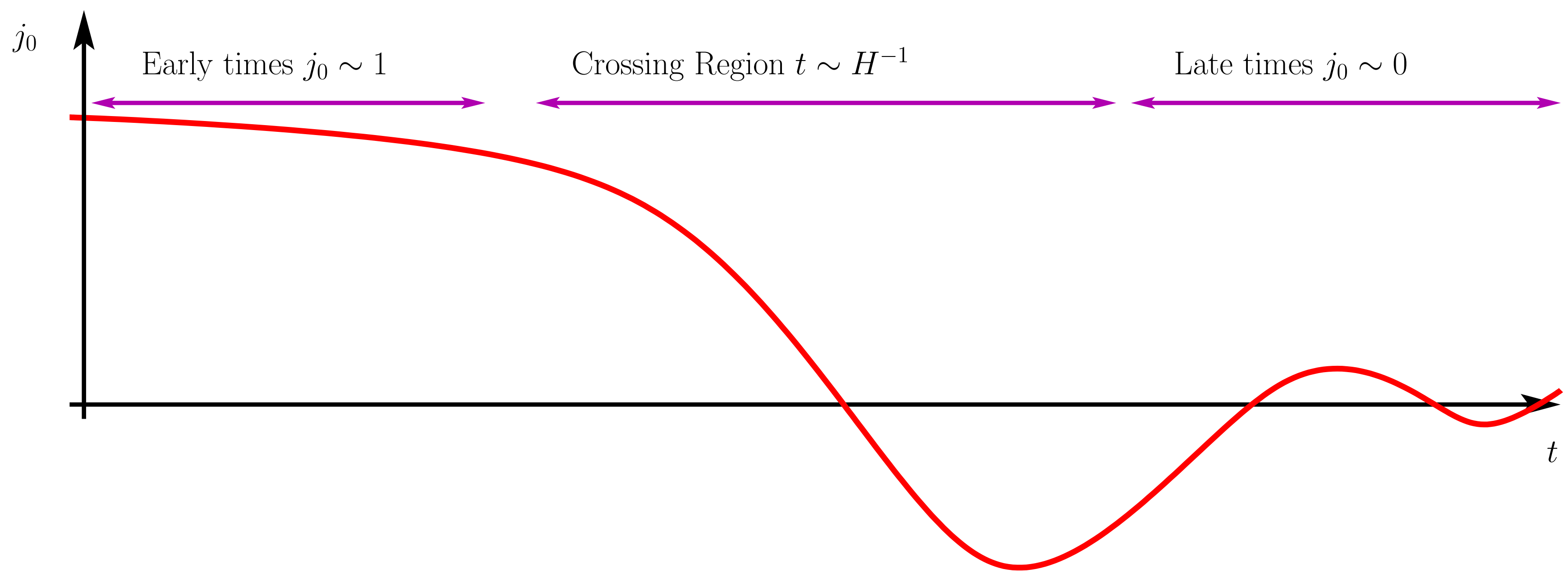


– We find that the solution is de Sitter invariant.

$$\langle\phi(\vec{x}_1,t_1)^n\phi(\vec{x}_2,t_2)^m\rangle = f_{nm}(z)$$

where $\quad z^2 = \cosh(t_1+t_2) - H^2 e^{H(t_1+t_2)}|\vec{x}_1-\vec{x}_2|$

de Sitter Invariant

– Correlation functions at different spacetime-points

– under perturbative control: decay at long distances, signaling stability

– they are de Sitter invariant

$$\langle \phi(\vec{x}_1, t_1)^n \phi(\vec{x}_2, t_2)^m \rangle = f_{nm}(z) \sim z^{-\sqrt{\lambda}}, \quad z \to \infty$$

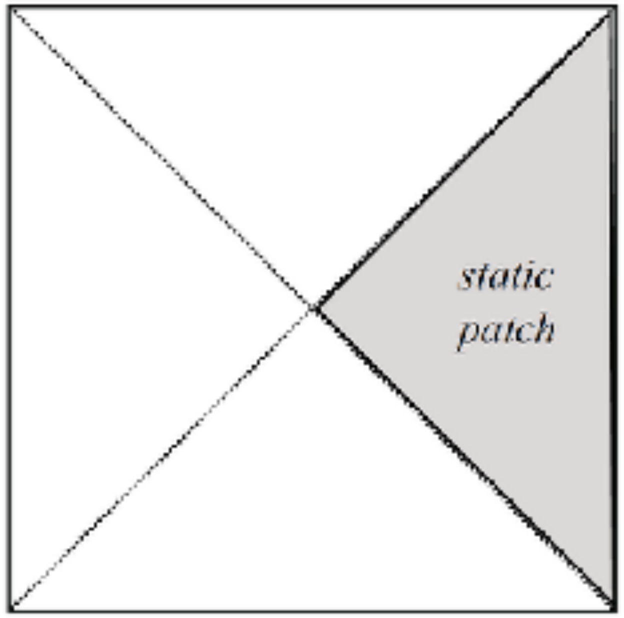
$$\text{where} \quad z^2 = \cosh(t_1 + t_2) - H^2 e^{H(t_1+t_2)} |\vec{x}_1 - \vec{x}_2|^2$$

Thermality

– Correlation functions at different spacetime-points

$$\langle \phi(\vec{x}_1, t_1)^n \phi(\vec{x}_2, t_2)^m \rangle = f_{nm}(z) \sim z^{-\sqrt{\lambda}}, \quad z \to \infty$$

– Restricted to static patch, they satisfy thermality with $T_{\rm dS} = \frac{H}{2\pi}$



- i.e. the KMS condition - certain periodicity in imaginary time

$$\langle \phi(x_1, t_1) \phi(x_2, t_2 + iT_{dS}^{-1}) \rangle = \langle \phi(x_2, t_2) \phi(x_1, t_1) \rangle$$

- This is not obviously true, since the leading term by itself does not satisfy it. KMS condition requires particular coefficient of the subheading term:
- $$f_{nm}(z) \sim z^{-\sqrt{\lambda}}(1 - i\pi\sqrt{\lambda})$$, which we also computed.

Sharp Window Function

– If we used a sharp window function in Fourier space, the splitting of the modes would be non-local in Real space.

– This implies that the dependence of the short modes on the long ones is non-local.

$$\frac{\partial \bar{P}_{\ell,1}(\phi_1,t)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial \phi_1^2}\left(\left\langle \frac{\partial}{\partial t}\Delta\phi(\vec{x}_1)^2\right\rangle_{\phi_1} \bar{P}_{\ell,1}(\phi_1,t)\right) - \frac{\partial}{\partial \phi_1}\left(\left\langle \left[\frac{\Pi(\phi(\vec{x}_1))}{a^2}\right]_\Lambda\right\rangle_{\phi_1} \bar{P}_{\ell,1}(\phi_1,t)\right)$$

All points

– But the dependence on the long modes points is perturbative, and so one can use the lower order solutions to compute the next-order diffusion and drift coefficients.

– We do this, and we get the same answer.

Conclusion

– Modulo some further checks/subtleties, we have developed a formalism to compute correlation functions of $\lambda\,\phi^4$ in dS

– manifest expansion in

$$\sqrt{\lambda} \quad \& \quad \epsilon \quad \& \quad \delta$$

– the solution is remarkably non-perturbative, and yet we can solve it

– at zeroth order in everything, we obtain the old Starobinsky result.

- but now we know it is correct, that it is the truncation of `something', and we know what `something' is, and we can systematically compute the corrections (which we do).

- all radiative corrections in dS and inflation are understood, and well behaved

– no instability in the rigid limit

with Zaldarriaga **JHEP 2010, JHEP 2012, JCAP 2012, JHEP 2013**
with Pimentel and Zaldarriaga **JHEP 2012**
& this work