From Symmetries to Cosmological Observables

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  - curved universes
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Motivations

Motivations

- More models of inflation and Dark Energy than stars in the sky. How to proceed?

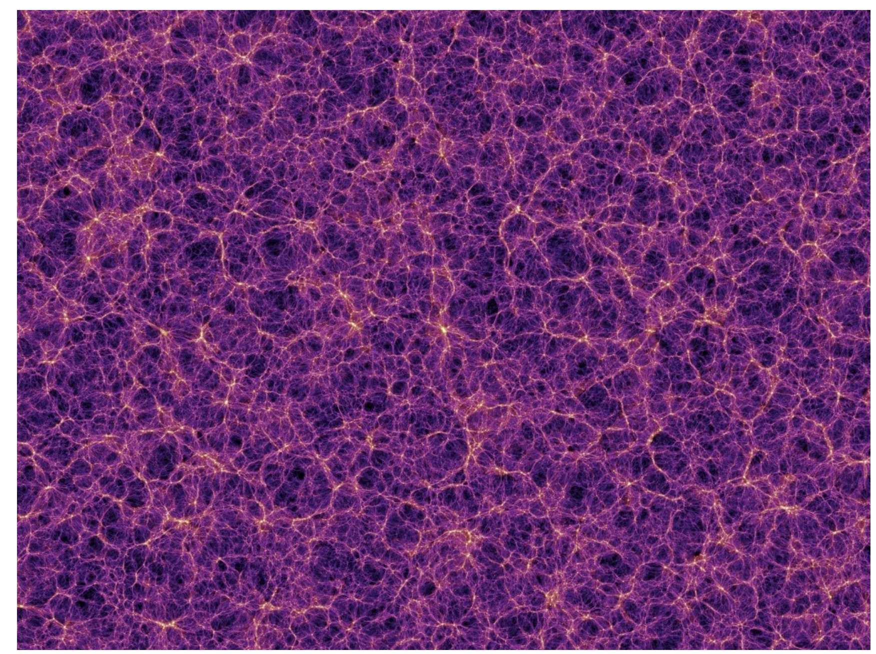
"*Use the symmetry, Luke!*" [Obi-Wan Kenobi '77]

- Idea: constrain and/or rule out classes of models that share the same symmetries
- *How do we go from (spacetime and internal) symmetries to observables in cosmology*?

Cosmo observables

- In cosmology we measure (mostly equal time) *in-in 3D correlators* of *perturbative* quantum gravity

$$\langle \mathcal{O}_1(\vec{p}_1)\ldots\mathcal{O}_n(\vec{p}_n)\rangle$$



- contrast this with particle physics, where we access flat space in-out correlators (scattering amplitudes)

Symmetries in cosmology

- In cosmology:
  - Time translations and boosts are broken (and gauged)

$$ISO(3,1) \quad \rightarrow \quad ISO(3)$$

  - Non-trivial space-time symmetries exist, as e.g. in warped Dirac-Born-Infeld inflation
  - Spacetime symmetries can emerge non-trivially as in (super)fluids, (super)solids, etc

Linearly realised syms

- Linearly realised symmetries (unbroken) have simple consequences

$$\sum_{a=1}^{n} L_a \langle \mathcal{O}(\mathbf{k}_1)\mathcal{O}(\mathbf{k}_2)\ldots\mathcal{O}(\mathbf{k}_n)\rangle = 0\,,$$

- E.g. for translations

$$L_a = \mathbf{k}_a \quad \Rightarrow \quad \sum_a \mathbf{k}_a = 0$$

Non-linearly realised syms

- Non-linearly realised symmetries (e.g. broken syms) lead to soft theorems

$$\lim_{\mathbf{q}\to 0} \frac{\langle \mathcal{O}(\mathbf{q})\mathcal{O}(\mathbf{k}_1)\ldots\mathcal{O}(\mathbf{k}_n)\rangle'}{\langle \mathcal{O}(\mathbf{q})\mathcal{O}(\mathbf{q})\rangle'} = \sum_{a=1}^{n} L_a \langle \mathcal{O}(\mathbf{k}_1)\ldots\mathcal{O}(\mathbf{k}_n)\rangle' ,$$

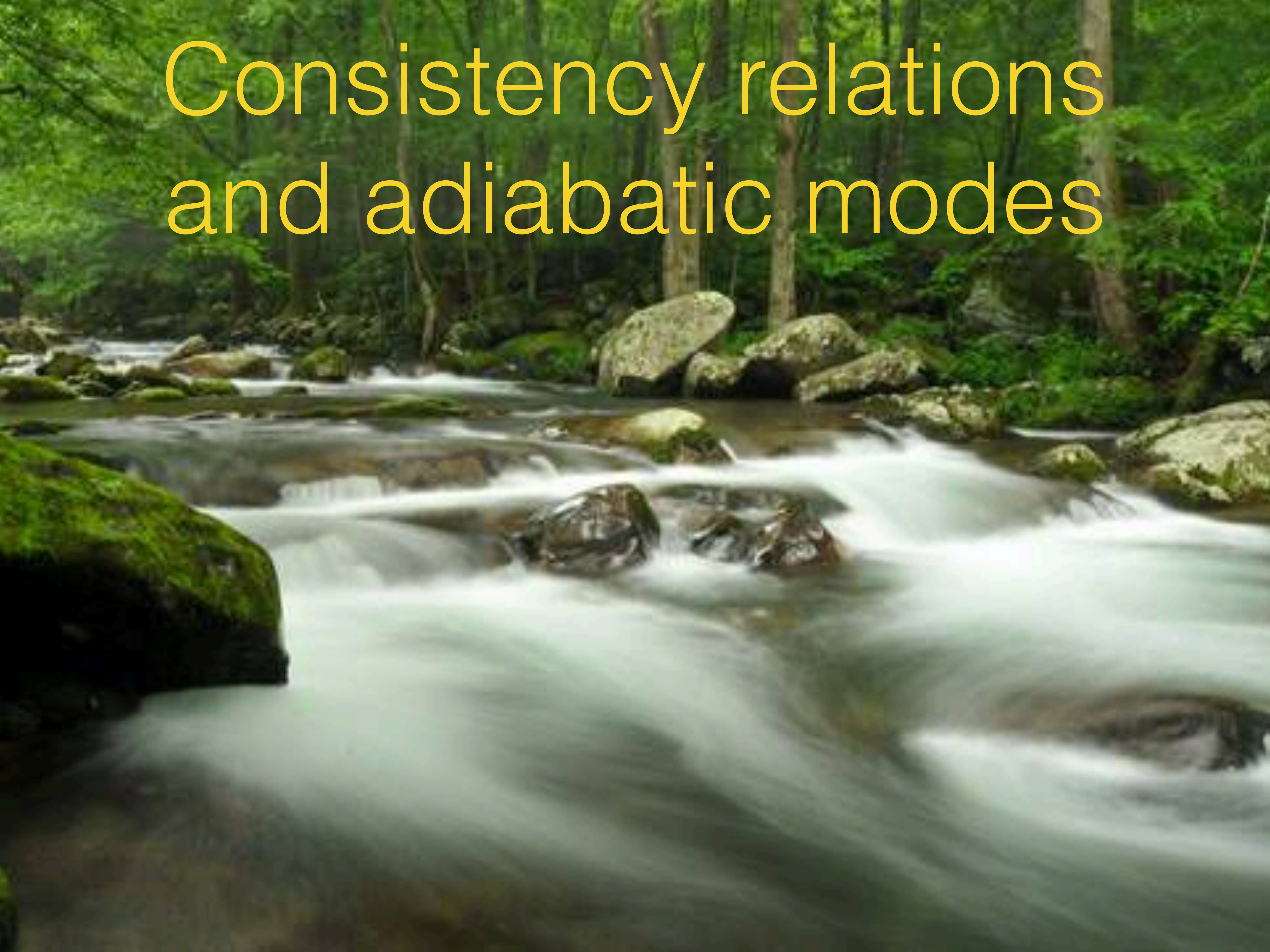
- Relate n+1 to n point functions

Soft theorems

$$\frac{\text{Soft theorems}}{\text{Cosmo correlators}} = \frac{\text{Non-linear symmetries}}{\text{Lagrangians}}$$

- Soft theorems are useful when comparing with data because they constraint observables directly
- But they are also useful theoretically, as they play the same role that symmetries play for Lagrangians
- Soft theorems extend our understanding of syms beyond the crutch of Lagrangians, gauge symmetries and fields

Consistency relations and adiabatic modes



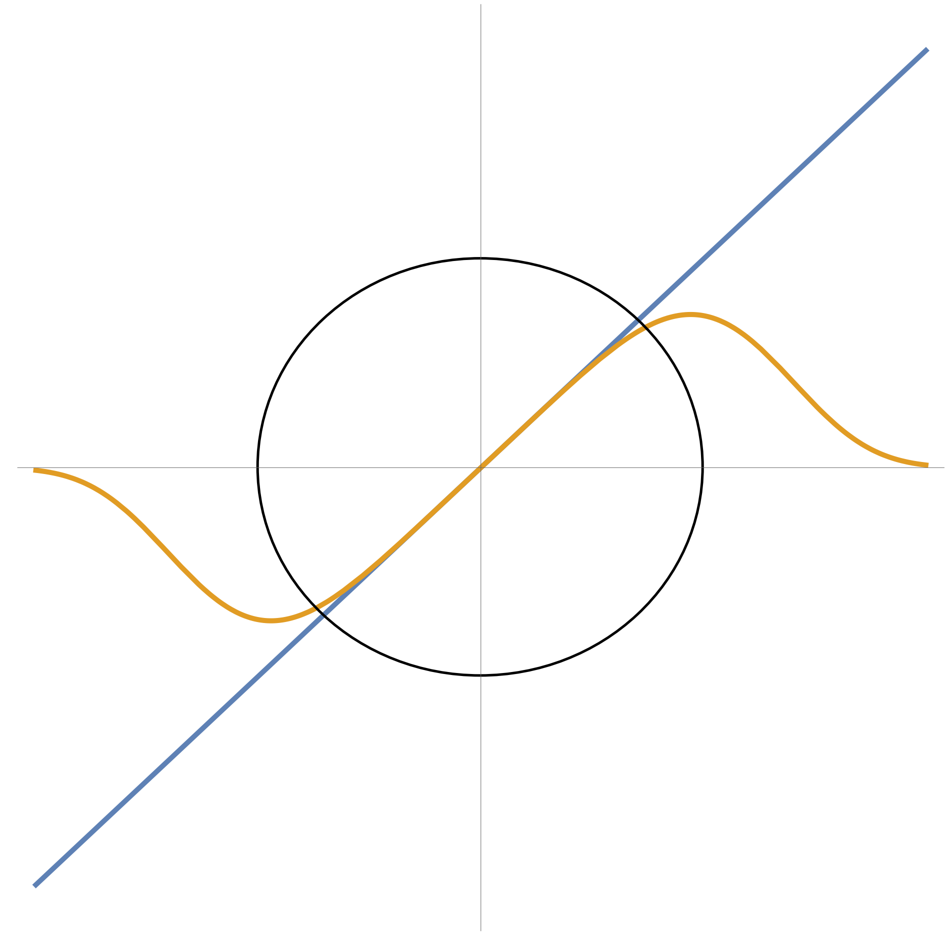
Adiabatic perturbations

- primordial pert's are adiabatic to few % [Planck]

$$\left(\frac{\delta\rho}{\bar\rho+\bar p}\right)_{DM} = \left(\frac{\delta\rho}{\bar\rho+\bar p}\right)_{b} = \left(\frac{\delta\rho}{\bar\rho+\bar p}\right)_{\gamma} = \left(\frac{\delta\rho}{\bar\rho+\bar p}\right)_{\nu} = \dots$$

- Explaining it might be a legacy of our generation

Weinberg's Adiabatic modes



- Consider classical GR perturbations around FLRW
- Fix the gauge for pert's that vanish at infinity, e.g. Newton gauge
- A large diff. generates a new solution that does not vanish at infinity
- Two (non-decaying) solutions survive to finite momentum: the adiabatic mode and gravitational waves
- *If there is only one scalar mode, it is the adiabatic mode*

Weinberg's adiabatic modes

- Definition: "*Adiabatic modes are physical perturbations that are locally indistinguishable from a change of coordinates (gauge transformation)*"

- Adiabatic modes locally perturb all tensors in the same way

$$\left(\frac{\delta\rho}{\bar\rho+\bar p}\right)_{DM}=\left(\frac{\delta\rho}{\bar\rho+\bar p}\right)_{b}=\left(\frac{\delta\rho}{\bar\rho+\bar p}\right)_{\gamma}=\left(\frac{\delta\rho}{\bar\rho+\bar p}\right)_{\nu}=\dots$$

- All cosmo pert's we have observed in our universe are adiabatic modes!

- Primordial gravity waves are adiabatic modes

Remarks

- Classical level: Adiabatic modes provide model-independent solutions to cosmological perturbation theory

- Quantum level: Adiabatic modes are related to non-linearly realised symmetries, which yield soft theorems, valid non-perturbatively

- Connections: Adiabatic modes provide a bridge to Strominger's IR triangle (Soft theorem - memory - asymptotic symmetries)

The vanilla example

- Consider a single-clock accelerated cosmology, e.g. single field slow-roll inflation
- Fix the gauge for small diffs, e.g. comoving (a.k.a. "zeta") gauge
- There are infinite residual large diffs. One is a dilation

$$x^i \to x^i(1+\lambda)$$

- Under this diff, zeta changes as

$$\Delta\mathcal{R}(t,\mathbf{k}) = -\frac{\omega_{ii}}{3}(2\pi)^3\delta^3(\mathbf{k}) + \frac{\omega_{ii}}{3}\left(3+\mathbf{k}\cdot\partial_{\mathbf{k}}\right)\mathcal{R}(t,\mathbf{k})\,.$$

Maldacena's consistency relation

- There are many ways to derive the soft theorem: Ward-Takahashi identities, Operator Product Expansion, Background wave, wave function, …

- E.g. the WT identity $i\langle[Q,\mathcal{O}]\rangle = \langle\Delta\mathcal{O}\rangle$

  leads to

$$\lim_{\mathbf{q}\to 0}\langle\mathcal{R}(\mathbf{q})\mathcal{R}(\mathbf{k})\mathcal{R}(\mathbf{k}')\rangle' = (1-n_s)P_{\mathcal{R}}(k)P_{\mathcal{R}}(q)$$

Applications

- This soft theorem can be tested observationally and it is promoted to a *consistency relation*

$$\lim_{\mathbf{q}\to 0}\langle \mathcal{R}(\mathbf{q})\mathcal{R}(\mathbf{k})\mathcal{R}(\mathbf{k}')\rangle' = (1-n_s)P_{\mathcal{R}}(k)P_{\mathcal{R}}(q)$$

- It is our best hope to test single vs multifield inflation

- Almost all CMB and LSS experiments attempt to test this relation, e.g. via temperature anisotropies, spectral distortions, galaxy bias, etc

The gravitational floor

- Single canonical field inflation has the lowest amount of *primordial* non-Gaussianity, which comes exclusively from the un-avoidable non-linearities of gravity

- The ground floor for non-Gaussianity? $\epsilon \ll \eta \sim (1-n_s)$

$$B_{singlefield} \sim (1-n_s)B^{loc} + \epsilon B^{equi}$$

- The leading term is an artifact of using comoving coordinates, and the first locally measurable effect is

$$\langle \mathcal{R}(q)\mathcal{R}(k)\mathcal{R}(k)\rangle \to (n_s-1)P(q)P(k)\left[\cancel{1} + \frac{q^2}{k^2} + \dots\right]$$

“Violations”

- Sometimes the (not always explicit) assumptions going into the derivation of this consistency relation are violated:
  - Spatially curved universe
  - non-attractor models
  - solids and non-standard symmetry breaking
- Modified “Early-late” relations can be derived [Hui, Joyce & Wong ‘18]

Soft Theorems for curved universes

Curved universes

- The most generic homogeneous and isotropic spacetime famously allows for spatial curvature

$$ds^2 = -dt^2 + a^2 \frac{dx^2}{(1 + Kx^2/4)^2}$$

- This is strongly constrained by observations

$$\frac{|K|}{H_0^2} < 10^{-3}$$

- Yet, we know there is a lower bound from superHubble perturbations!

$$K = \nabla^2 \mathcal{R} \quad \Rightarrow \quad \frac{|K|}{H_0^2} \gtrsim 10^{-4}$$

Soft theorems

- Is Maldacena's consistency relation still valid in a curved universe??
- If not, is there another soft theorem that replaces/ corrects it??
- The answer is no to both questions [Jazayeri, EP & Supel, to appear]

(in)Direct computation

- We could compute directly the squeezed bispectrum in a curved universe. It's a long an tricky calculation…

- But we can also read off the result from the flat universe *trispectrum,* using the fact that a long mode is locally equivalent to spatial curvature

$$B_K(q_L, k_S, k_S) \simeq \lim_{q_U \to 0} \frac{T^{(q_U^2)}(q_U, q_L, k_S, k_S)}{P(q_U)}$$

Result

- Both contact interaction and scalar exchange diagrams in the trispectrum contribute to the correction to the squeezed bispectrum

$$B_K \sim P_L P_S \left[(1-n_s) + \frac{(1-n_s)}{c_s^2} \frac{K}{q_l^2} + \frac{\epsilon + (1-c_s^2)}{c_s^4} \frac{K}{k_s^2} \right]$$

Maldacena's — scalar exchange — contact interaction

Violation

- So the flat space consistency relation *is violated* in curved space

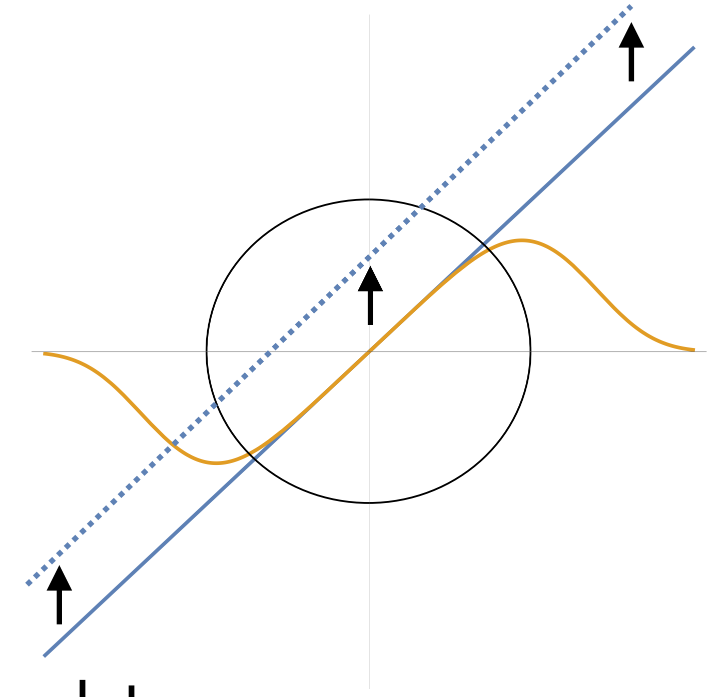
- The violation is model dependent, so there should not be any improved soft theorem!

- The size of the violation is small in canonical models, but potentially measurable if small cs

$$B_K - B \sim P_L P_S \left[\frac{(1-n_s)}{c_s^2} \frac{K}{q_L^2} + \frac{1}{c_s^4} \frac{K}{k_S^2} \right]$$

$$\frac{K}{k_S^2} \ll \frac{K}{q_L^2} \ll 10^{-3}$$

Absence of adiabatic modes

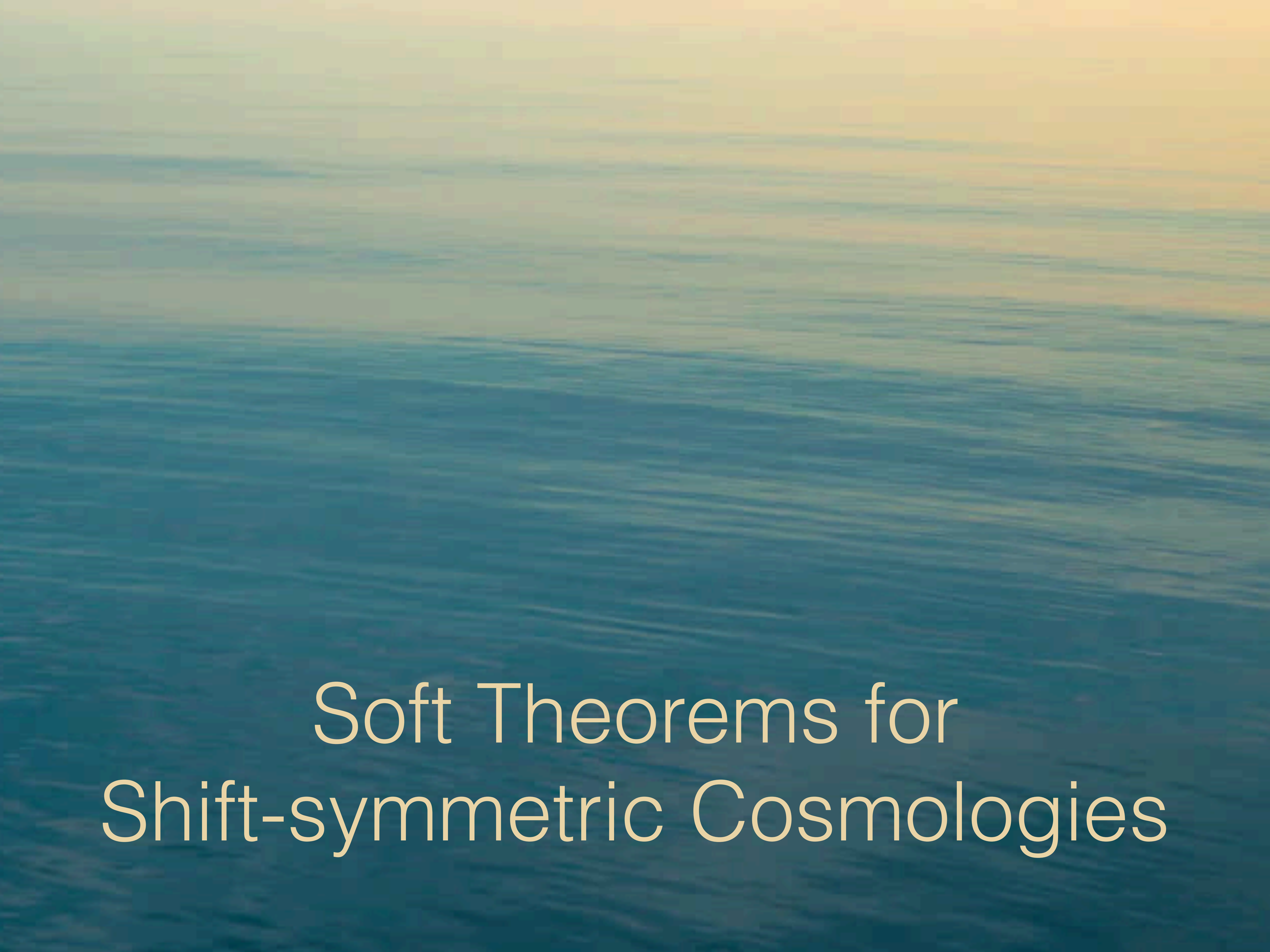


- The lack of a soft theorem can be related to the absence of adiabatic modes in curved universes

- Adiabatic modes have a difference time dependence from physical modes at order K

$$\ddot{\mathcal{R}}_{Ad} + 3H\dot{\mathcal{R}}_{Ad} - \frac{K}{a^2}\mathcal{R}_{Ad} = 0$$

- The same is true for tensor modes


Soft Theorems for
Shift-symmetric Cosmologies

UV sensitivity

- The mechanism of inflation is incredibly UV-sensitive

$$V(\phi) \supset \sum_{n=0}^{\infty} \frac{\phi^n}{\Lambda^{n-4}}$$

- Even dimension 5 and 6 operators suppressed by Mpl change completely the predictions

$$V = V_{sr} + \frac{\phi^2}{\Lambda^2} V_{sr} + \cdots \Rightarrow \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \sim \eta_{sr} + \mathcal{O}(1)$$

Shift symmetry

- The low energy EFT that describe inflation is obtained integrating out UV degrees of freedom at the cutoff scale, Λ<Mpl

- One generically expects (hierarchy problem)

$$V \supset \Lambda^2 \phi^2 + \frac{\phi^6}{\Lambda^2} + \dots$$

- a *shift symmetry* helps by forbidding dangerous operators

$$\phi \to \phi + \mathrm{const.}$$

Quantum gravity

- To be useful for inflation, the shift symmetry must be broken *above* Mpl, so it must be respected by the UV-completion of gravity

- Many conjectures (swampland, Weak Gravity, …) that such any global (shift-)symmetry must be broken at the Planck scale, but still no conclusive evidence

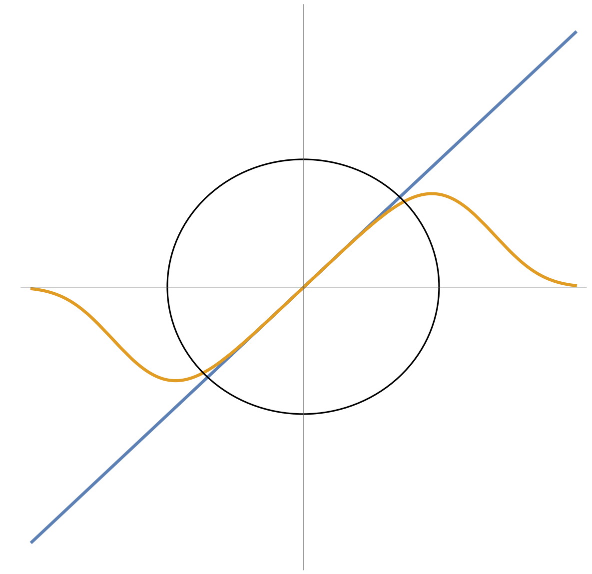
- Idea: *can we establish observationally/ experimentally whether a Shift symmetry is at play in nature?*

Observables?

- Despite being ubiquitous in cosmological constructions (e.g. inflation and dark energy), *we still did not know the generic observable consequences of a shift symmetry until 2017!*

- Contrast this with particle physics where there is a well-understood route from symmetries to observables

Shifty Adiabatic Modes



- In the presence of internal symmetries, we have new *generalised adiabatic modes*
- Def: "*Generalised adiabatic modes are physical perturbations that are locally indistinguishable from a change of coordinates plus an internal symmetry*"
- We can use this to find new (classical) solutions of shift symmetric cosmologies and new Soft theorems

Shift Adiabatic Modes

- Consider general shift symmetric theories w gravity

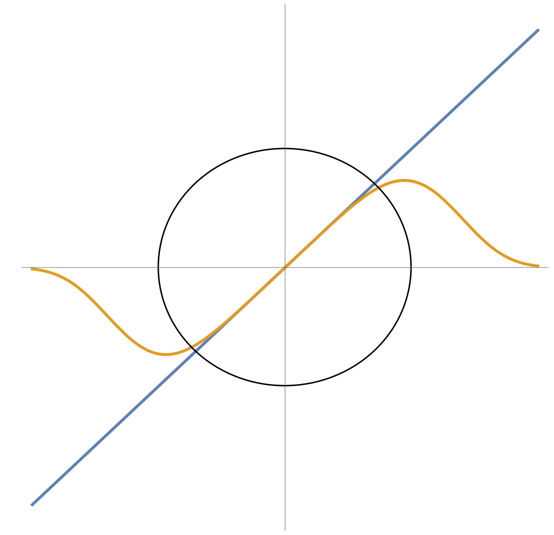
$$\mathcal{L} = \frac{M_{pl}^2}{2} R + P(X) + G(X)\Box\phi + \dots\,,\ X \equiv -\frac{1}{2}(\nabla\Phi)^2$$

- Fix comoving gauge

$$\Phi(x^\mu) = \bar{\Phi}(t)$$

$$ds^2 = -\left(1+\delta N\right)^2 dt^2 + a^2 e^{2\zeta}\delta_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

Extension to finite momentum



- The theory is invariant under one time diff, but this mode does not continue to finite momentum

- Mixing the "shift symmetric diff" with space diffs we can create physical modes that solve the EoM

$$\xi^\mu = \{\frac{c}{\dot{\bar{\bar{\Phi}}}}, c\,\lambda(t) x^i\}$$

$$\lambda(t) = C_1 - \int^t dt' \left(\frac{\dot{H}}{\dot{\bar{\bar{\Phi}}}} + (\Theta - H)\,\frac{\ddot{\bar{\bar{\Phi}}}}{\dot{\bar{\bar{\Phi}}}^2} \right)$$

Operator Product Expansion

- Use the generic OPE in Fourier space

$$\zeta_{\vec{k}-\frac{1}{2}\vec{q}}\zeta_{-\vec{k}-\frac{1}{2}\vec{q}} \xrightarrow{\vec{q}\to 0} P(k)(2\pi)^3\delta^3(\vec{k}) + f(k)\zeta_{-\vec{q}} + g(k)\dot{\zeta}_{-\vec{q}} + \mathcal{O}(q\zeta, \zeta^2)$$

- Two unknown functions, *f* and *g*
- In slow-roll inflation one neglects the time derivative because it decays

Symmetries

- We use the two symmetries we have: Weinberg's adiabatic mode and the Shifty adiabatic mode

$$\zeta_{\mathbf{k}} \to \zeta_{\mathbf{k}} + \lambda (2\pi)^3 \delta^3(\mathbf{k}) - \lambda \left(3 + \mathbf{k} \cdot \partial_{\mathbf{k}}\right) \zeta_{\mathbf{k}}$$

$$\zeta_{\mathbf{k}} \to \zeta_{\mathbf{k}} + c\,\lambda(t) \left((2\pi)^3 \delta^3(\mathbf{k}) - \left(3 + \mathbf{k} \cdot \partial_{\mathbf{k}}\right) \zeta_{\mathbf{k}}\right) + \frac{c}{\dot{\bar{\Phi}}} \left(H (2\pi)^3 \delta^3(\mathbf{k}) + \dot{\zeta}_{\mathbf{k}}\right)$$

- These fix f and g

$$f(k) = (1 - n_s) P(k)$$

$$g(k) = \frac{1}{\Theta} \frac{\dot{\bar{\Phi}}}{\ddot{\bar{\Phi}}} \left[(1 - n_s) P(k) H - \dot{P}(k)\right]$$

Squeezed bispectrum

- For the 3 pt function (bispectrum), we *generalise* Maldacena famous consistency relation

$$\lim_{\mathbf{q}\to 0}\langle\zeta_{\mathbf{q}}\zeta_{\mathbf{k}-\frac{1}{2}\mathbf{q}}\zeta_{-\mathbf{k}-\frac{1}{2}\mathbf{q}}\rangle'$$

$$= -\frac{\dot{\bar{\Phi}}\dot{P}(q)}{2\ddot{\bar{\Phi}}\Theta}\left[(n_s-1)HP(k)+\dot{P}(k)\right]$$

$$+(1-n_s)P(k)P(q)$$

Applications

- It would be nice to apply our result to shift symmetric models of inflation. Unfortunately:

- *No-go theorem*: slow-roll inflation cannot be driven by any number of exactly shift-symmetric scalar fields, to leading order in derivatives.

- No explicit models are known to higher order in derivatives

Ultra-Slow-Roll inflation

- E.g, consider the simples shift symmetric theory

$$\mathcal{L} = -\partial\phi^2 - V_0$$

- This drives Ultra Slow Roll inflation

$$\epsilon \sim \frac{1}{a^6}\,, \quad \eta \sim -6$$

- Surprisingly, this model still respects scale invariance!
- This single-field model famously “violates” Maldacena’s single consistency relation because perturbations do not freeze out

$$P_{\mathcal{R}} \sim a^6$$

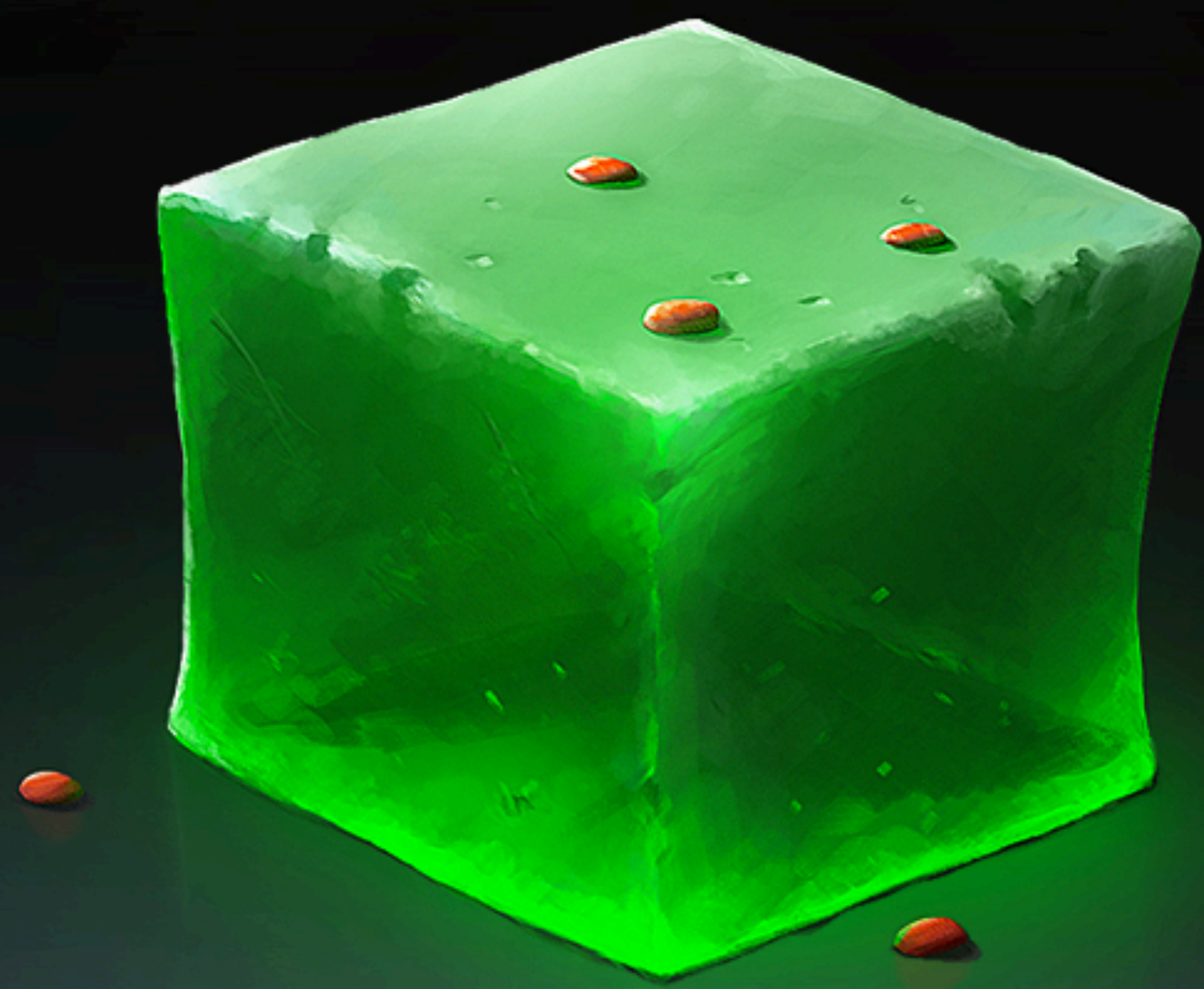
Remarks

- Our correction dominates the squeezed limit
- It's easy to check that our relation is satisfied
- Unfortunately, ours are not "consistency relations" because the right hand side is not observable

$$\lim_{\mathbf{q}\to 0}\langle\zeta_{\mathbf{q}}\zeta_{\mathbf{k}-\frac{1}{2}\mathbf{q}}\zeta_{-\mathbf{k}-\frac{1}{2}\mathbf{q}}\rangle'$$

$$= -\frac{\dot{\bar{\Phi}}\dot{P}(q)}{2\ddot{\bar{\Phi}}\Theta}\left[(n_s-1)HP(k)+\dot{P}(k)\right]$$

$$+(1-n_s)P(k)P(q)$$



Soft theorems for Solid Inflation

Solids

- Solids are characterised by the symmetry breaking pattern

$$ISO(3)_{space} \times ISO(3)_{internal} \rightarrow ISO(3)_{diagonal}$$

- e.g. three scalar field with vevs

$$\langle \phi^i \rangle = x^i$$

- Solids can drive inflation [Gruzinov '04; Endlich, Nicolis & Wang '12]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 R + F(X, Y, Z) \right] ,$$

$$B_{IJ} \equiv g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J , \quad X \equiv [B] , \quad Y \equiv \frac{[B^2]}{[B]^2} , \quad Z \equiv \frac{[B^3]}{[B]^3} ,$$

Solid inflation

- Because the symmetry breaking pattern is very different from standard inflation, the prediction are also very different

- Correlators violate Maldacena's generalised consistency relations. Direct computations:

$$B(q_L \ll k_S) \to -\frac{20 F_Y}{F} \frac{1 - 3\cos^2\theta}{\epsilon c_L^2} P_L P_S + \text{slow roll}$$

Soft theorems

- The squeezed limit can be generically expanded in Legendre polynomials. The leading terms in the squeezed limit are [similar results derived by Creminelli et al '17]

$$\langle \zeta_{\mathbf{q}} \zeta_{\mathbf{k}-\frac{1}{2}\mathbf{q}} \zeta_{-\mathbf{k}-\frac{1}{2}\mathbf{q}} \rangle = P_q P_k \left[a_0^{(0)}(k) + a_2^{(0)}(k) P_2(\cos\theta) + \mathcal{O}(q^2) \right]$$

$$\langle \gamma^s_{\mathbf{q}} \zeta_{\mathbf{k}-\frac{1}{2}\mathbf{q}} \zeta_{-\mathbf{k}-\frac{1}{2}\mathbf{q}} \rangle = P^\gamma_q P_k \epsilon^s_{ij}(\hat{q}) \hat{k}^i \hat{k}^j \left[b_0^{(0)}(k) + \mathcal{O}(q^2) \right]$$

$$a_0^{(0)} = (n_s - 1)$$

$$2 b_0^{(0)}(k) + \frac{1}{2} a_2^{(0)}(k) = 3 + (1 - n_s)$$

Summary

- Soft theorems are the observational consequence of non-linearly realised symmetries
- Maldacena's consistency relation is violated in curved universes
- In Ultra-slow roll an improved soft theorem can be derived using shift symmetry
- Soft theorems probe the symmetry breaking pattern during inflation