An EFT approach to black hole perturbations in scalar tensor theories

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based on
1708.09391 + work in progress
in collaboration with
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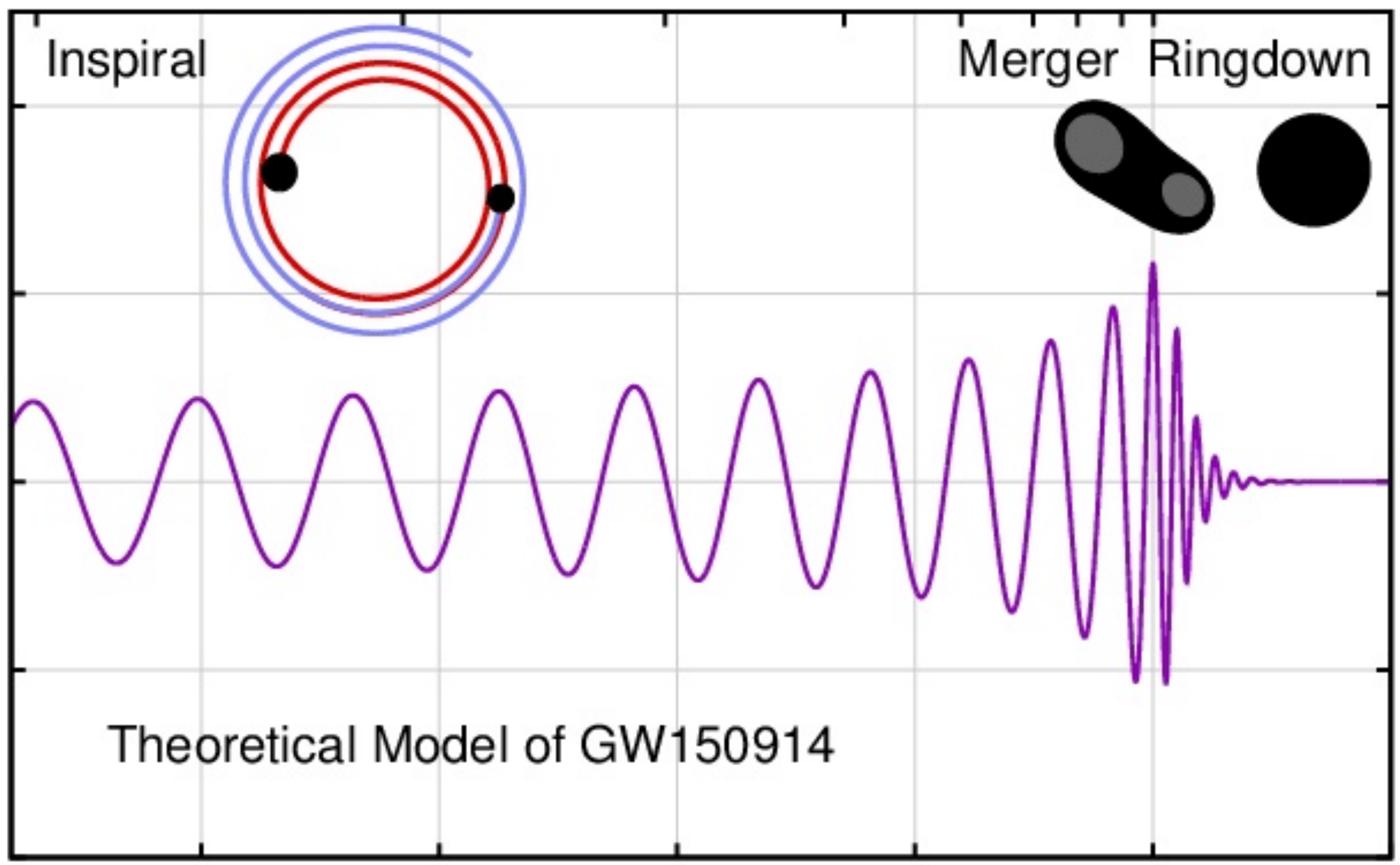
Question: GW observations $\longrightarrow$ model-independent constraints on new d.o.f's in the gravitational sector?



Source: Horst *et al.* [1607.05251]

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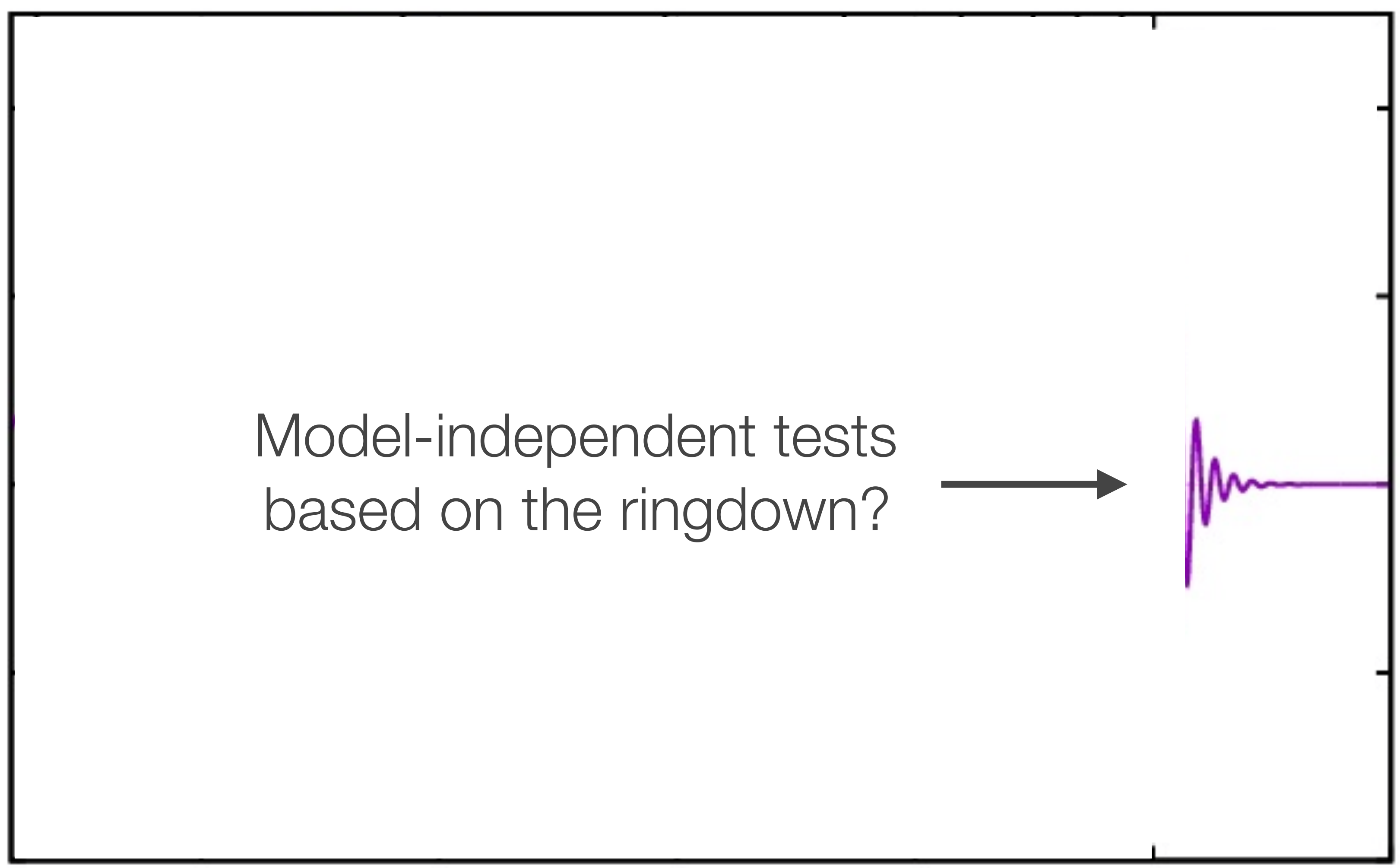

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Why Ringdown?

Theory: usually studied on a model-by-model basis: a more systematic approach is needed.

Experiment: 3rd-gen & space-based detectors will enable "black hole spectroscopy”.

This talk: EFT for ringdown of static, hairy black holes in scalar-tensor theories

Schwarzschild Ringdown

- BH perturbation theory: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \Delta g_{\mu\nu}$
- Break $\Delta g_{\mu\nu}$ into irreducible components:

FRW	$e^{i\mathbf{x}\cdot\mathbf{k}}$	scalars, vectors, tensors
Schwarzschild	$e^{-i\omega t},\ Y_\ell^m(\theta,\phi)$	parity eigenstates

- Parity eigenstates:

$$\Delta g_{\mu\nu} = \begin{cases} \text{3 odd} & \rightarrow \quad \text{1 d.o.f.} \\ \text{7 even} & \rightarrow \quad \text{1 d.o.f.} \end{cases}$$

Schwarzschild Ringdown

- Linearized equations reduce to:

$$\left[-\frac{d^2}{d\tilde{r}^2} + V_i(\tilde{r})\right]\psi_i = \omega^2\psi_i, \qquad i = \text{even, odd}$$

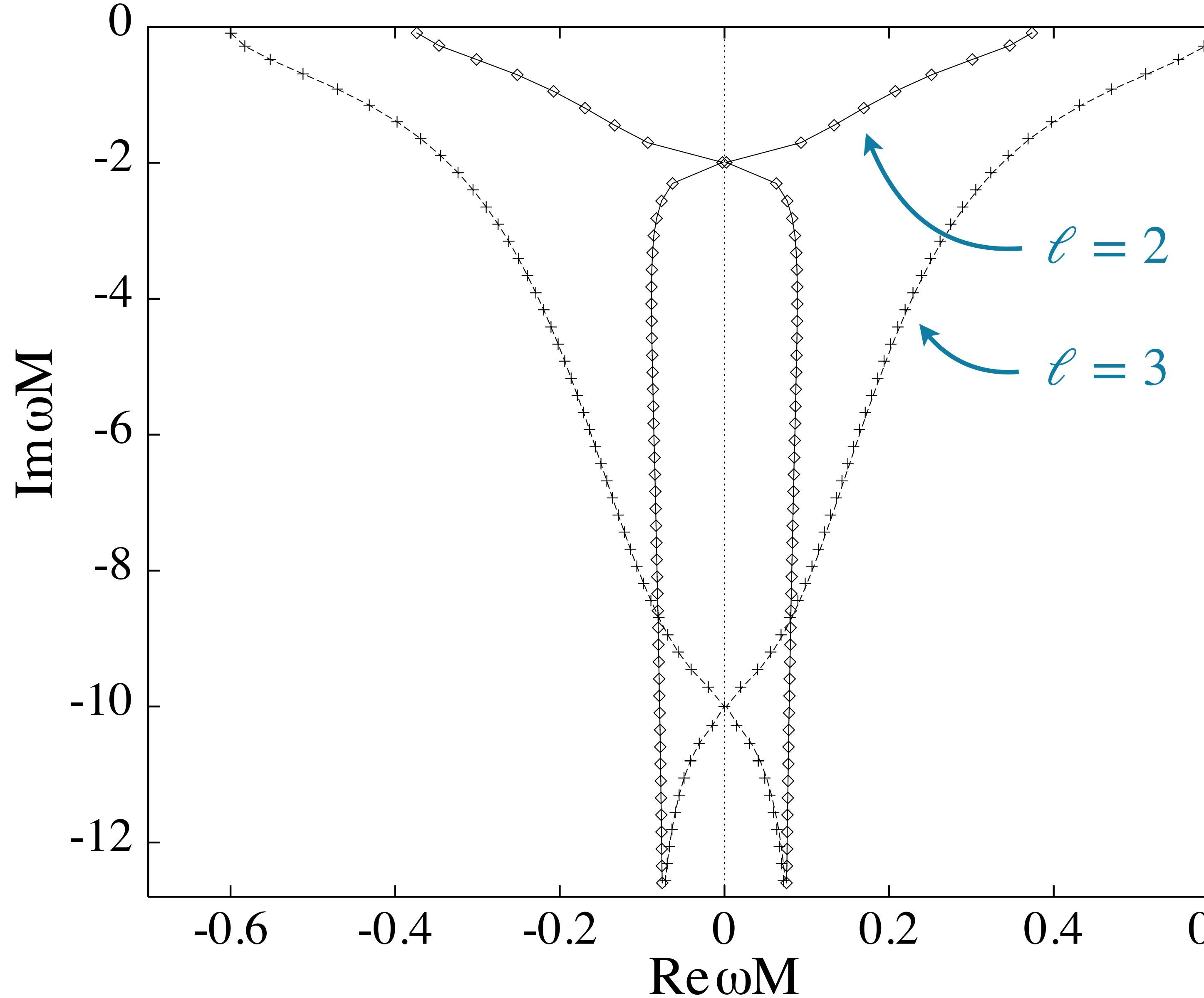
- Out-going boundary conditions:

$$\psi_i \sim e^{-i\omega(t\mp\tilde{r})} \quad \text{for} \quad \tilde{r} \to \pm\infty$$

- Solutions exist for complex quasi-normal frequencies $\omega_{n,\ell}$

- **Note:** “electric-magnetic duality” $\longrightarrow$ isospectrality in d=4

Schwarzschild Ringdown



Source: Kokkotas & Schmidt 99

Modifications of QNM spectrum

in scalar-tensor theories

1. New d.o.f. $\longrightarrow$ new frequencies

2. GR frequencies are shifted

3. Isospectrality is broken

4. even/odd mixing

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Require a scalar hair

[Ferreira, Lagos & Tattersall 17]

More on Scalar Hair

- No-go theorems for static, asymptotically flat BHs are based on <u>assumptions</u>

- Scalar hair allowed theoretically:

e.g. $$L = \frac{M_p^2}{2} R - \frac{1}{2}(\partial\Phi)^2 - V(\Phi) + \Delta L$$

$$\Delta L = \begin{cases} \alpha\,\Phi \tilde{R}^{\mu\nu\lambda\rho} R_{\mu\nu\lambda\rho} & \text{[Alexander \& Yunes 07]} \\ \alpha\,\Phi (R^{\mu\nu\lambda\rho} R_{\mu\nu\lambda\rho} - 4R^{\mu\nu} R_{\mu\nu} + R^2) & \text{[Sotiriou 15]} \end{cases}$$

- Let's test scalar hair!

Effective action

- Blueprint: EFT of inflation / dark energy

[Cheung, Creminelli, Fitzpatrick, Kaplan & Senatore 08]

[Gubitosi, Vernizzi & Piazza 12]

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- Work in unitary gauge: $\Delta\Phi \;\to\; \Delta\Phi - \xi^r \partial_r \bar{\Phi} \equiv 0$

- Effective action invariant under (t, θ, ϕ) – diffs :

$$S = \int d^4x \sqrt{-g}\, L\left(g^{\mu\nu}, \epsilon^{\mu\nu\lambda\rho}, R_{\mu\nu\alpha\beta}, \nabla_\mu, g^{rr}, K_{\mu\nu}, r\right)$$

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- Expand building blocks around background:

$$\text{e.g.} \qquad g^{rr} = \bar{g}^{rr} + \delta g^{rr}, \quad K_{\mu\nu} = \bar{K}_{\mu\nu} + \delta K_{\mu\nu}, \quad \ldots$$

Effective action

$$
\begin{aligned}
S = \int d^4x\,\sqrt{-g}\Big[\frac{M^2(r)}{2}R - f_1(r) - f_2(r)\delta g^{rr} - f_3(r)\bar{K}_{\mu\nu}\delta K^{\mu\nu} \\
+ c_1(r)(\delta g^{rr})^2 + c_2(r)\delta g^{rr}\delta K + c_3(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta K^{\mu\nu} \\
+ \ (11 \text{ terms})\ + \ldots\Big]
\end{aligned}
$$

Effective action

Removable with
conformal transformation

Fixed by the
background metric

$$S = \int d^4x \sqrt{-g}\Big[\frac{M^2(r)}{2}R - f_1(r) - f_2(r)\delta g^{rr} - f_3(r)\bar{K}_{\mu\nu}\delta K^{\mu\nu}$$

$$+ c_1(r)(\delta g^{rr})^2 + c_2(r)\delta g^{rr}\delta K + c_3(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta K^{\mu\nu}$$

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Effective action → QNMs

- In principle:

Finite # of terms in the effective action for perturbations

↓

Schrödinger-like equation for even/odd modes

Effective action → QNMs

- In principle:

  Finite # of terms in the effective action for perturbations

  ↓

  Schrödinger-like equation for even/odd modes

- In practice:

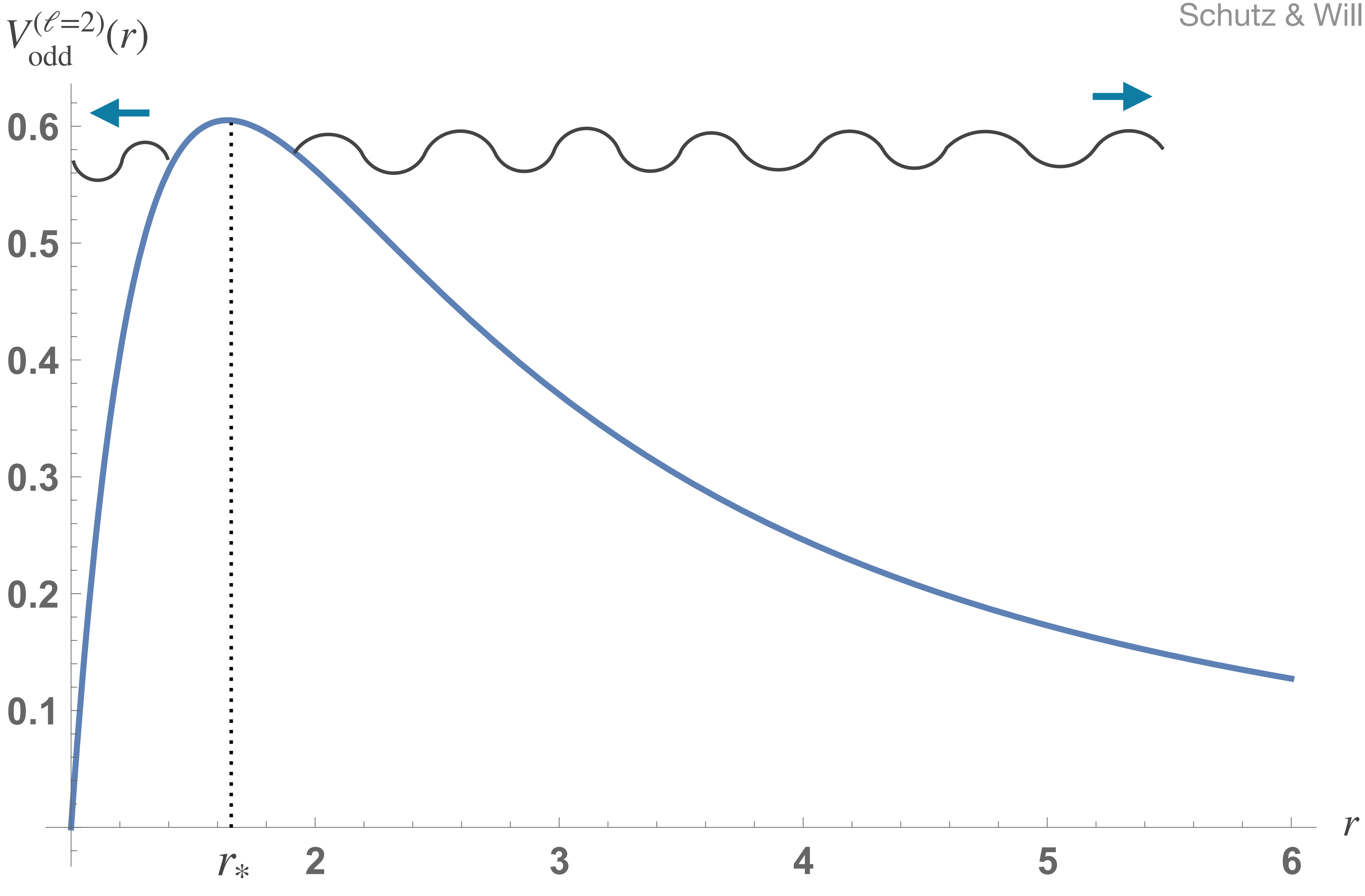
  EFT coefficients are arbitrary functions of r

  ↓

  Arbitrary QNM potentials

WKB approximation

[Mashhoon 83,
Schutz & Will 85]



WKB approximation

- Lowest order WKB approximation:

[Mashhoon 83,
Schutz & Will 85]

$$\left.\frac{\omega^2 - V_\ell}{\sqrt{-2\,\partial_{\tilde{r}}^2 V_\ell}}\right|_{r=r_*(\ell)} \simeq -i(n+1/2) \qquad \longrightarrow \qquad \omega_{n,\ell}$$

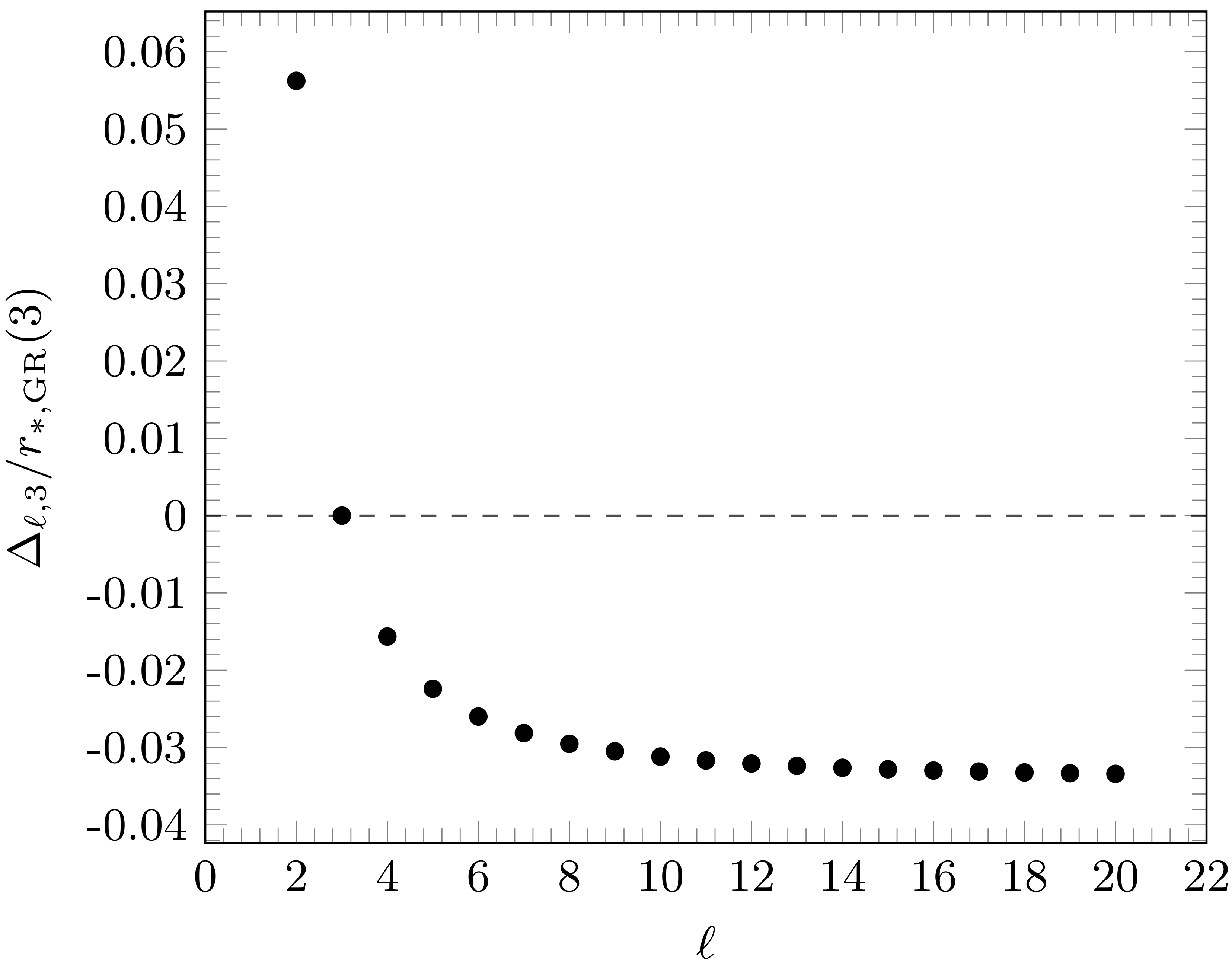
- Schwarzschild: 3% accuracy on Re[$\omega_{0,3}$], 0.5% on Im[$\omega_{0,3}$]

- Accuracy improves (worsens) for higher ℓ (n)

- Can be extended to higher orders

[Iyer & Will 87]

Light-ring Expansion



Light-ring Expansion

- Quasi-Schwarzschild approximation: the position of the light ring is close to its GR value

$$r_*(\ell) = r_{*,GR}(\ell) + \delta r_*(\ell), \qquad \text{with} \qquad \delta r_* \ll r_{*,GR}$$

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- Higher order: express $\delta r_*(\ell)$ in terms of derivatives of V:

$$\left.\frac{dV_\ell}{dr}\right|_{r_*=r_{*,GR}(3)+\Delta_{\ell,3}+\delta r_*} = 0 \quad \rightarrow \quad \Delta_{\ell,3} + \delta r_* \simeq -\left.\frac{\partial_r V_\ell}{\partial_r^2 V_\ell}\right|_{r=r_{*,\mathrm{GR}}(3)}$$

Hairy BHs vs Inflation

Inflation	QNMs of Hairy BHs
scalar=“clock”	scalar=“hair”
quasi-deSitter	quasi-Schwarzschild
Slow-roll expansion	Light-ring expansion
Derivatives of inflation potential	Derivatives of QNM potential
horizon-crossing: $k/a(t_*) = H(t_*)$	maximum of potential: $r_*(\ell)$
EFT of inflation	Our EFT

Odd Sector

- Effective action at lowest order in derivatives:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - f_1(r) - f_2(r) \delta g^{rr} \right]$$

- Background metric:

$$d\bar{s}^2 = -a(r)^2 dt^2 + \frac{dr^2}{b(r)^2} + r^2 d\Omega^2$$

- EFT coefficients:

$$f_1(r) = \frac{M_p^2(1 - b^2 - 2rbb')}{r^2}, \qquad f_2(r) = \frac{M_p^2}{r} \left(\frac{a'}{a} - \frac{b'}{b} \right)$$

Odd Sector

- Linearized equations $\longrightarrow$ Schrödinger-like equation

$$\left[-\frac{d^2}{d\tilde{r}^2} + V\right]\psi = \omega^2\psi$$

w/ $$V = \frac{a^2b^2}{r}\left[\frac{a'}{a} + \frac{b'}{b} - \frac{2}{r} - \frac{(\ell-1)(\ell+2)}{rb^2}\right], \qquad \frac{d\tilde{r}}{dr} = \frac{1}{ab}$$

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- Lowest order WKB formula + light ring expansion:

$$\omega_{n,\ell} = F(n, \ell, a_*, a'_*, b_*, b'_*, b''_*, b^{(3)}_*, b^{(4)}_*)$$

- <u>3rd-Gen + LISA</u>: $\omega_{0,\ell}$ potentially up to $\ell \sim 7$ [Baibhav & Berti 18]

Conclusions

- Introduced EFT for QNMs of BHs with scalar hair
- Model-indep. parametrization of deviations from QNMs of GR
- Close analogy between inflation and QNMs of hairy BH
- Formal applications, *e.g.* wormhole stability [Franciolini, Hui, Penco, Santoni & Trincherini, 18]]
- Next steps:

  Extend to spinning solutions

  Include matter couplings

  Reconsider no-hair theorems

EFT of multi-field inflation [Senatore & Zaldarriaga, 10]
EFT of dark energy [Gubitosi, Piazza & Vernizzi, 12]

Thank you.

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